In-medium QCD cascade: democratic branching and wave turbulence

Jet Quenching at RHIC vs LHC
BNL
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European
Research
Council

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Outline

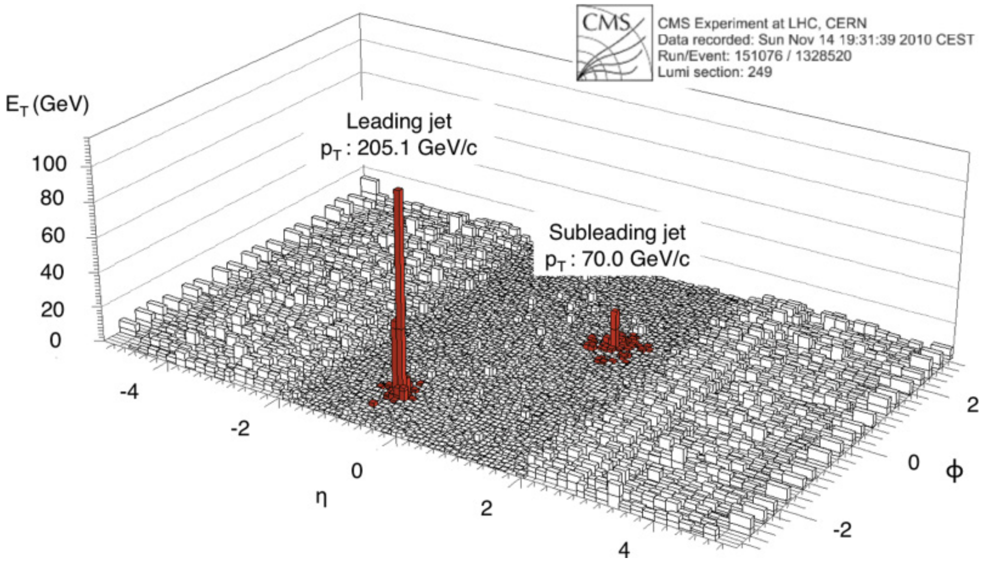
- Phenomenological motivations
- In-medium gluon branching (BDMPSZ mechanism)
- Multiple branching, (de)coherence, in-medium cascade
- In-medium cascade, turbulent flow
- Relevance to di-jet asymmetry
- Conclusion

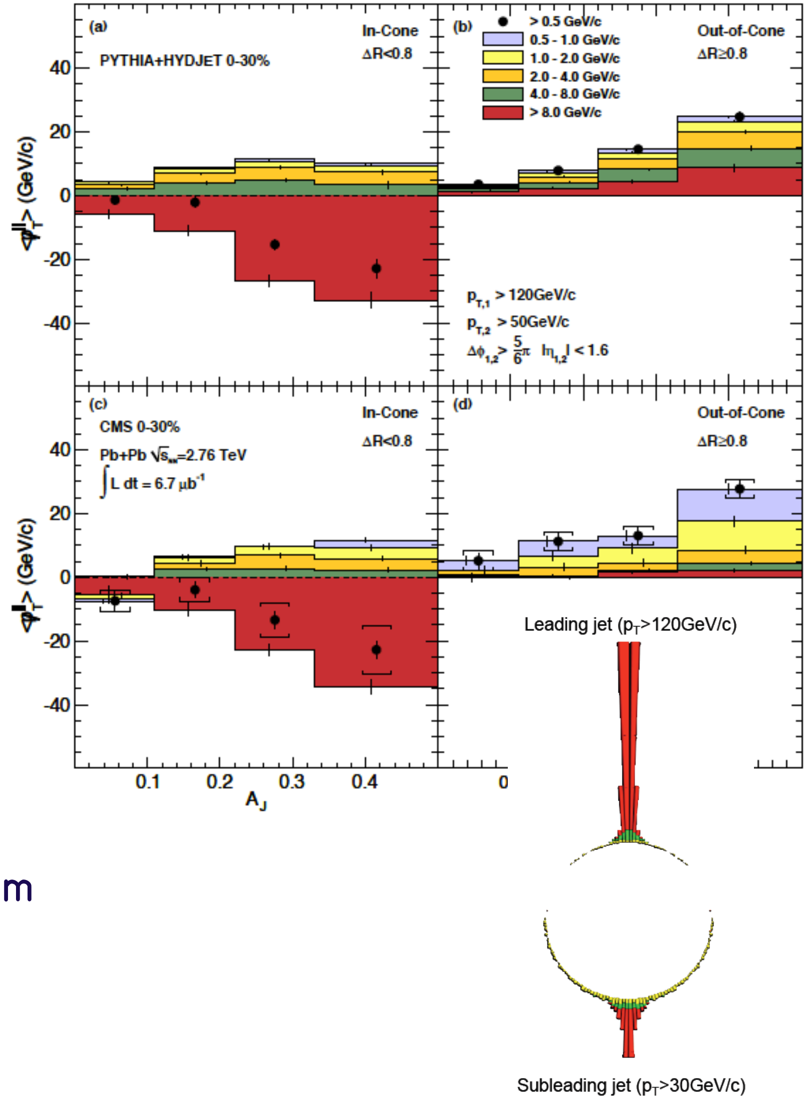
Work done in collaboration with F. Dominguez, E. Iancu and Y. Mehtar-Tani (arXiv:1209.4585, 1301.6102)

Phenomenological motivations

Di-jet asymmetry

there is more to it than just 'jet quenching'...





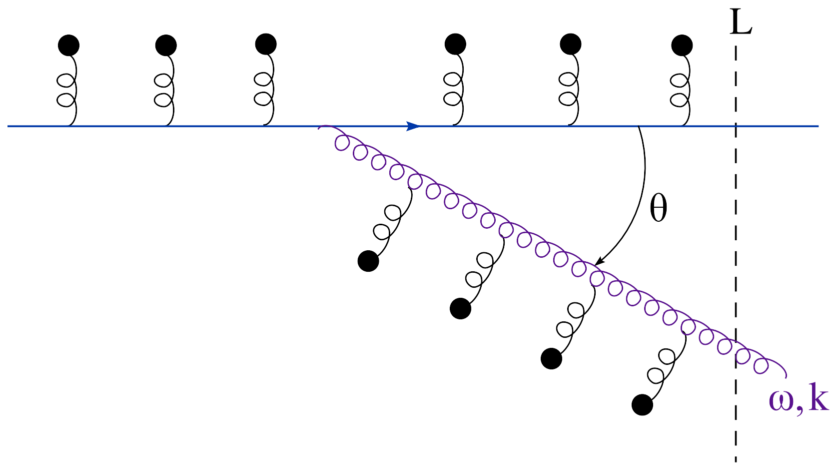
Missing energy is associated with additional radiation of many soft quanta at large angles

Perhaps reflecting a genuine feature of the in-medium QCD cascade (JPB, E. Iancu and Y. Mehtar-Tani, arXiv: 1301.6102)

In-medium parton branching
BDMPSZ mechanism

(Baier, Dokshitzer, Mueller, Peigné, Schiff; Zakharov ~ 1996)

The BDMPSZ mechanism for in-medium branching



Gluon emission is linked to momentum broadening

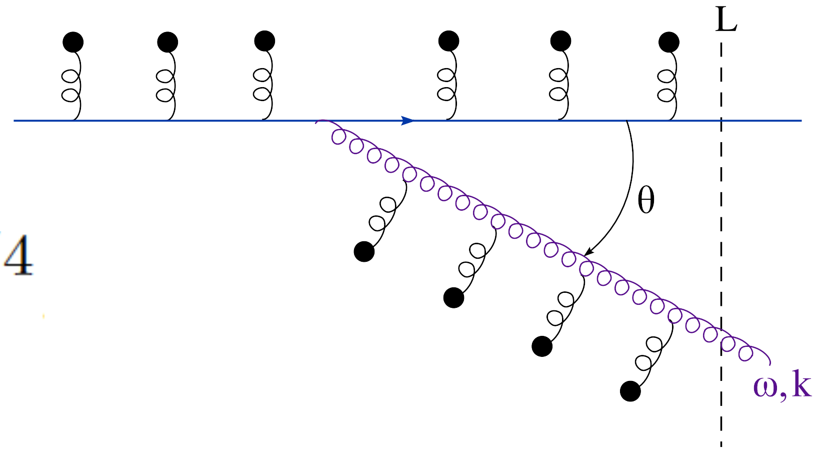
$$\frac{1}{\tau_{\mathrm{f}}} \sim \frac{k_\perp^2}{2\omega} \qquad \Delta k_\perp^2 = \hat{q}\Delta t$$

Time scale for the branching process $\tau_{\mathrm{br}}(\omega) \sim \sqrt{\frac{2\omega}{\hat{q}}}$

Medium of finite extent $\tau_{\mathrm{br}} \lesssim L \Rightarrow \omega \lesssim \omega_c \qquad \omega_c \sim \hat{q}L^2$

Formation time and emission angle

Typical branching kT and angle



$$k_{\rm br}^2 = \hat{q}\tau_{\rm br}$$

$$\theta_{\rm br} \sim k_{\rm br}/\omega \sim (\hat{q}/\omega^3)^{1/4}$$

Hard gluon: small angle, long time

$$\tau_{\rm br} \lesssim L \qquad \omega \lesssim \omega_c \qquad \theta_{\rm br} \gtrsim \theta_c$$

Soft gluon: large angle, short time

$$\tau_{\rm br} \ll L \qquad \omega \ll \omega_c \qquad \theta_{\rm br} \gg \theta_c$$

BDMPSZ spectrum

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \frac{\alpha_s N_c}{\pi} \sqrt{\frac{\omega_c}{\omega}} \equiv \bar{\alpha} \sqrt{\frac{\omega_c}{\omega}} = \bar{\alpha} \frac{L}{\tau_{\mathrm{br}}(\omega)}$$

Hard emissions

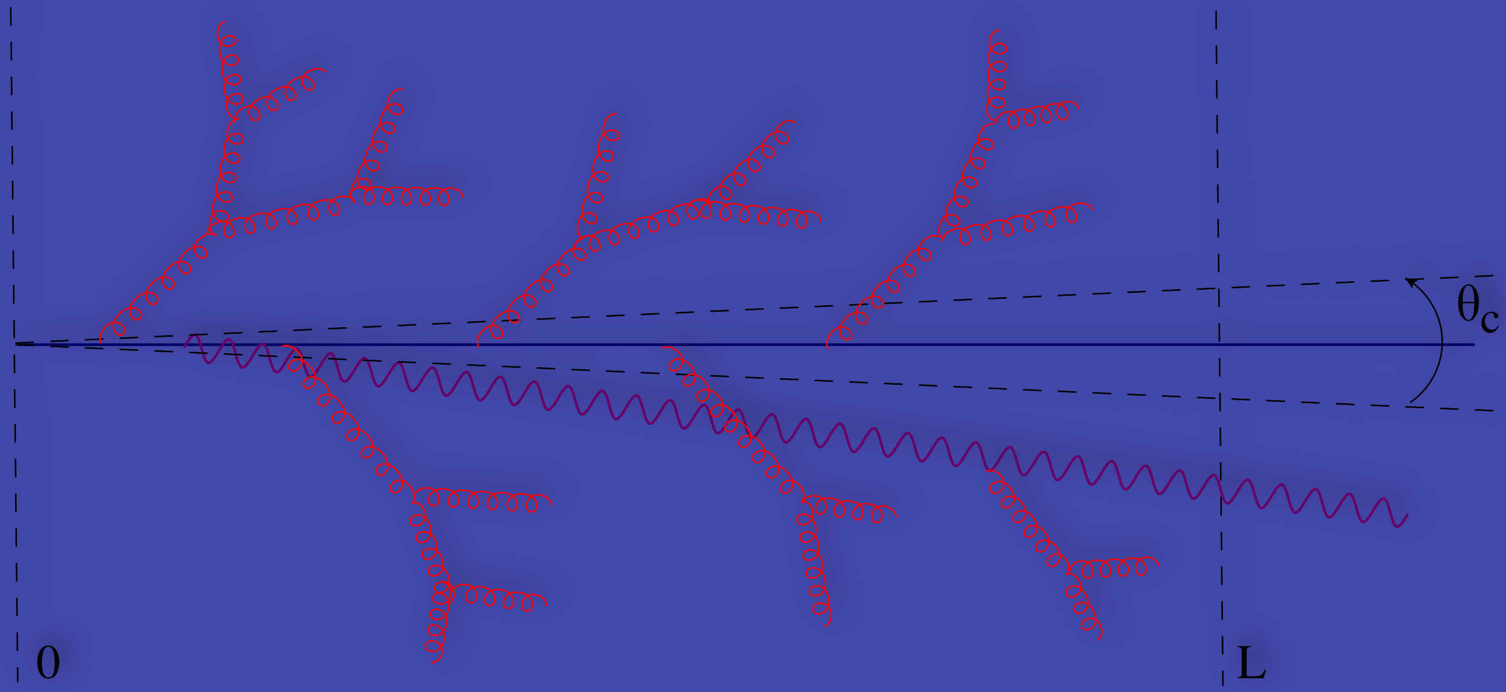
- rare events, with probability $\sim \mathcal{O}(\alpha_s)$
- dominate energy loss: $E_{\mathrm{hard}} \sim \alpha_s \omega_c$
- small angle, not important for di-jet asymmetry

Soft emissions

- frequent, with probability $\sim \mathcal{O}(1)$
- weaker energy loss: $E_{\mathrm{soft}} \sim \alpha_s^2 \omega_c$
- but arbitrary large angles: control di-jet asymmetry

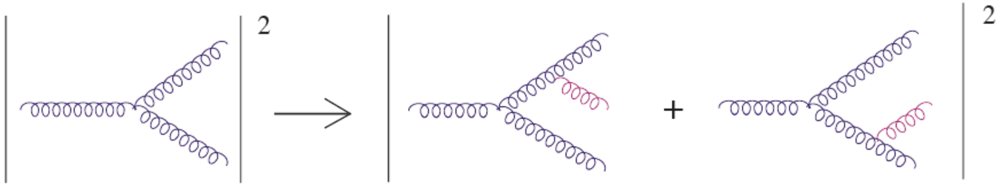
large angles emissions are dominated by soft multiple branchings

Multiple branchings
(de)-coherence
in-medium cascade



Multiple emissions

A priori complicated by interferences



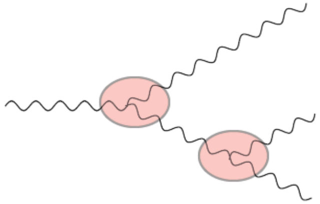
In vacuum, these interferences lead to angular ordering

In medium color coherence is rapidly lost via rescattering

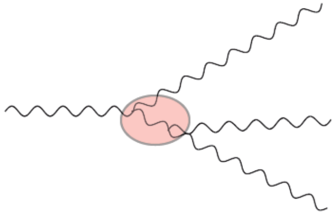
Mehtar-Tani, Salgado, Tywoniuk (1009.2965; 1102.4317)
Iancu, Casalderey-Solana (1106.3864)

In medium, interference effects are subleading

Independent emissions are enhanced by a factor L/τ_f



$\sim \left(\alpha_s \frac{L}{t_f}\right)^2$



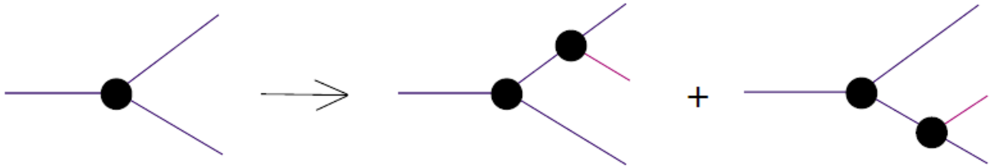
$\sim \alpha_s^2 \frac{L}{t_f}$

JPB, F. Dominguez, E. Iancu, Y. Mehtar-Tani, arXiv: 1209.4585

Resumming the leading terms

When $\bar{\alpha}L/\tau_{br} \sim 1$ all powers of $\bar{\alpha}L/\tau_{br} \sim 1$ need to be resummed.

Since independent emissions dominate, the leading order resummation is equivalent to a probabilistic cascade, with nearly local branchings



Blob: BDMPSZ spectrum

Line: momentum broadening

JPB, Dominguez, Iancu and Mehtar-Tani (arXiv:1209.4585)

Note: already implemented in Monte Carlo codes

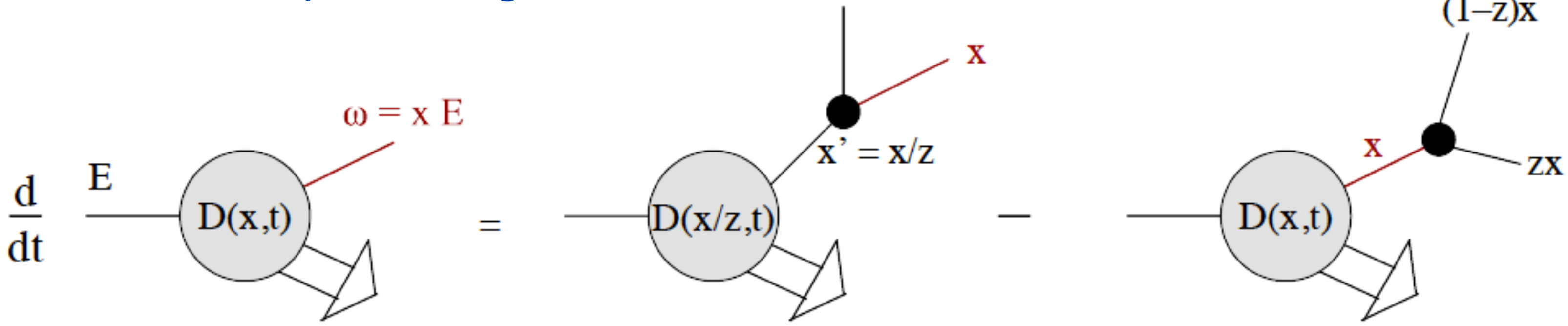
MARTINI (Jeon, Gale, Schenke)
Q_Pythia (Armesto, Salgado et al)
Stachel, Wiedemann, Zapp

Evolution equation for the gluon spectrum

(after integration over kT)

$$D(x,\tau) = x\frac{dN}{dx} \qquad x = \frac{\omega}{E}$$

Probabilistic equation ('gain-loss')



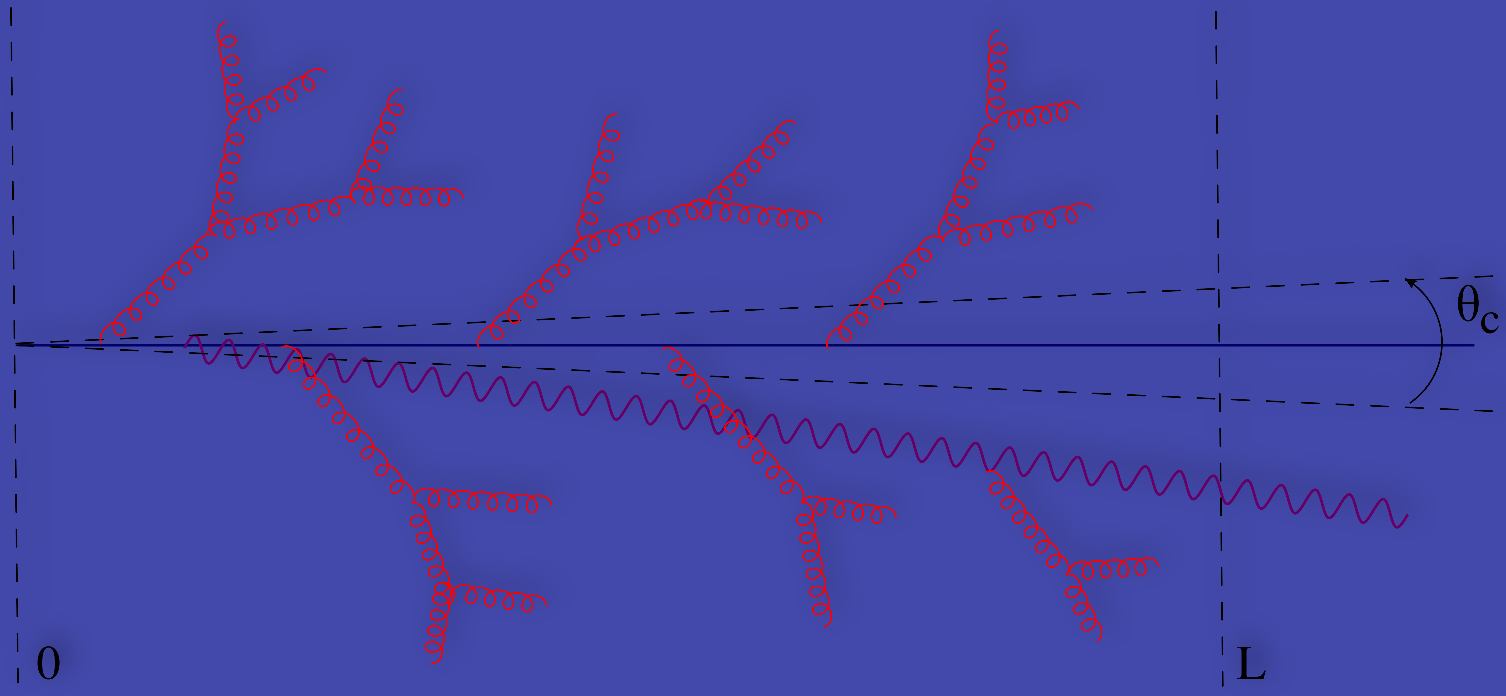
$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\Big(\frac{x}{z},\tau\Big) - \frac{z}{\sqrt{x}}\, D(x,\tau)\right]$$

$$\mathcal{K}(z) = \frac{\bar{\alpha}}{2} \frac{f(z)}{[z(1-z)]^{3/2}}\,, \qquad f(z) = \left[1 - z(1-z)\right]^{5/2}$$

Formally analogous to DGLAP. But very different kernel... and physics.

⟶ **A QCD cascade of a new type**

In-medium QCD cascade
Turbulent flow at small x

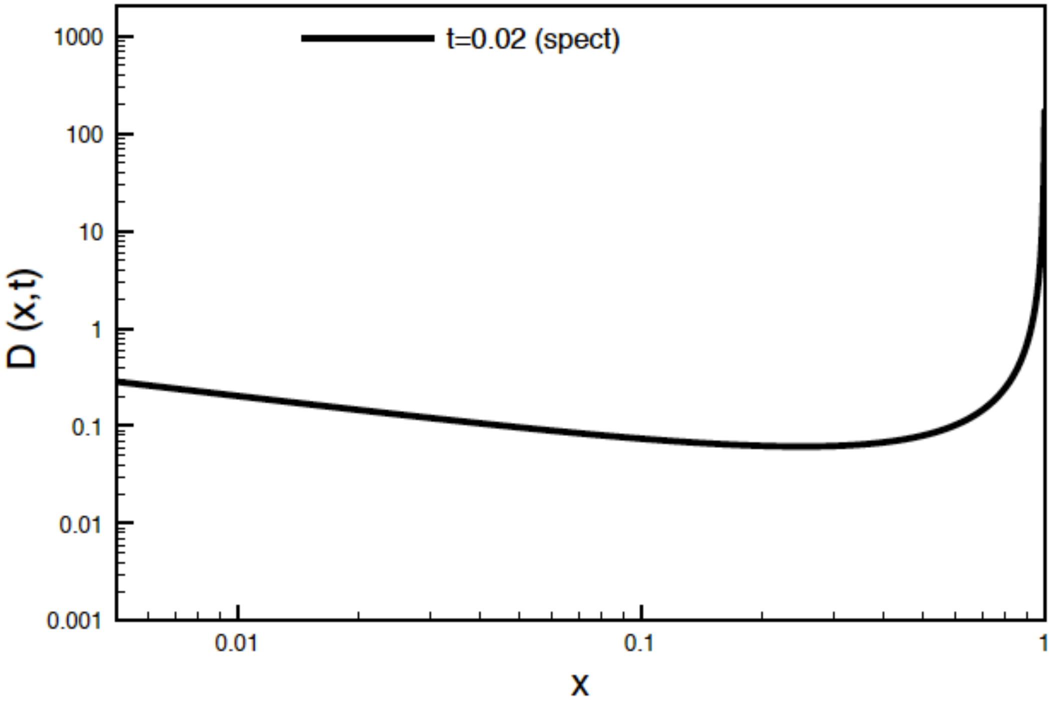


Short times

$$\frac{\partial D(x,\tau)}{\partial \tau} = \int \mathrm{d}z\, \mathcal{K}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right]$$

At short time, single emission by the leading particle $(D_0(\tau = 0, x) = \delta(x-1))$

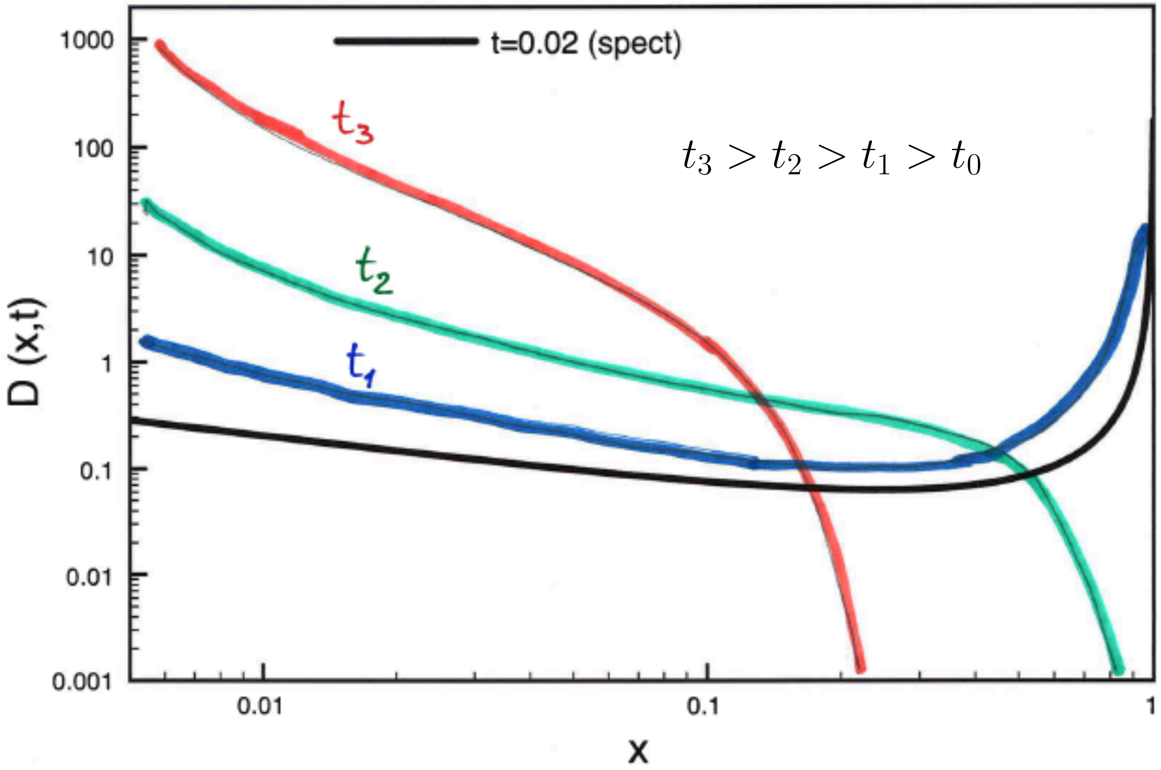
D is the BDMSZ spectrum



How do multiple branchings affect this spectrum ?

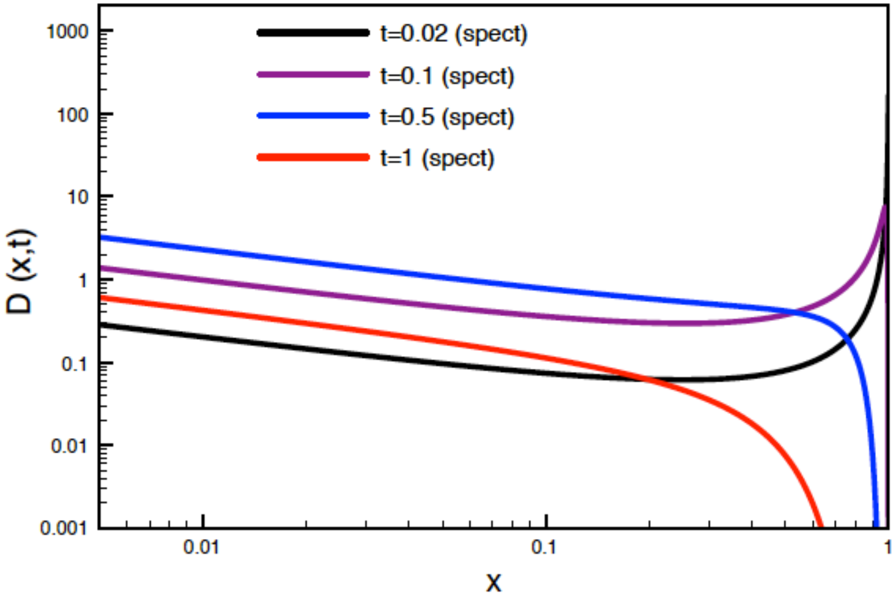
Naively, we could expect the spectrum to be depleted at large x and to increase rapidly at small x, so as to keep the total energy constant

$$\int_0^1 \mathrm{d}x\, D(\tau, x) = 1$$



But this is not what happens !

One finds (exact result) $$D(x,t) \simeq \frac{t}{\sqrt{x}}\, \mathrm{e}^{-\pi t^2} \qquad \text{for } x \ll 1$$

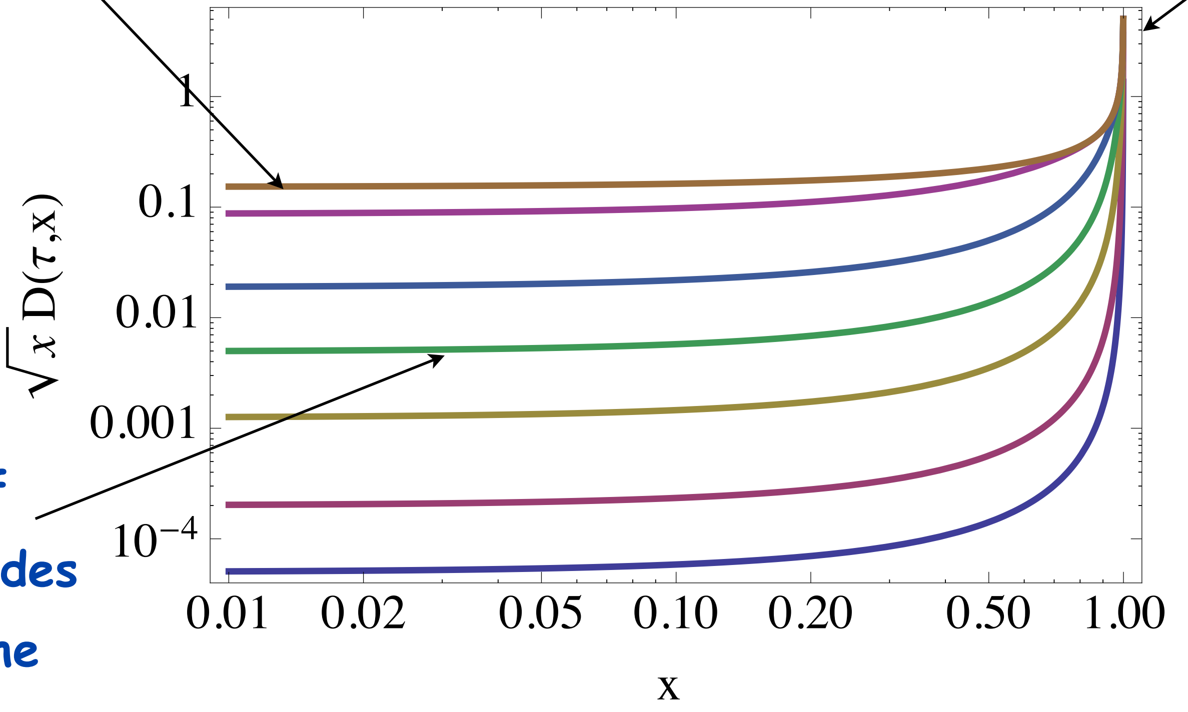


Fine (local) cancellations between gain and loss terms

BDMPS spectrum emerges as a fixed point, scaling, spectrum

Characteristic features of wave turbulence (Kolmogoroz, Zakharov)

The source problem

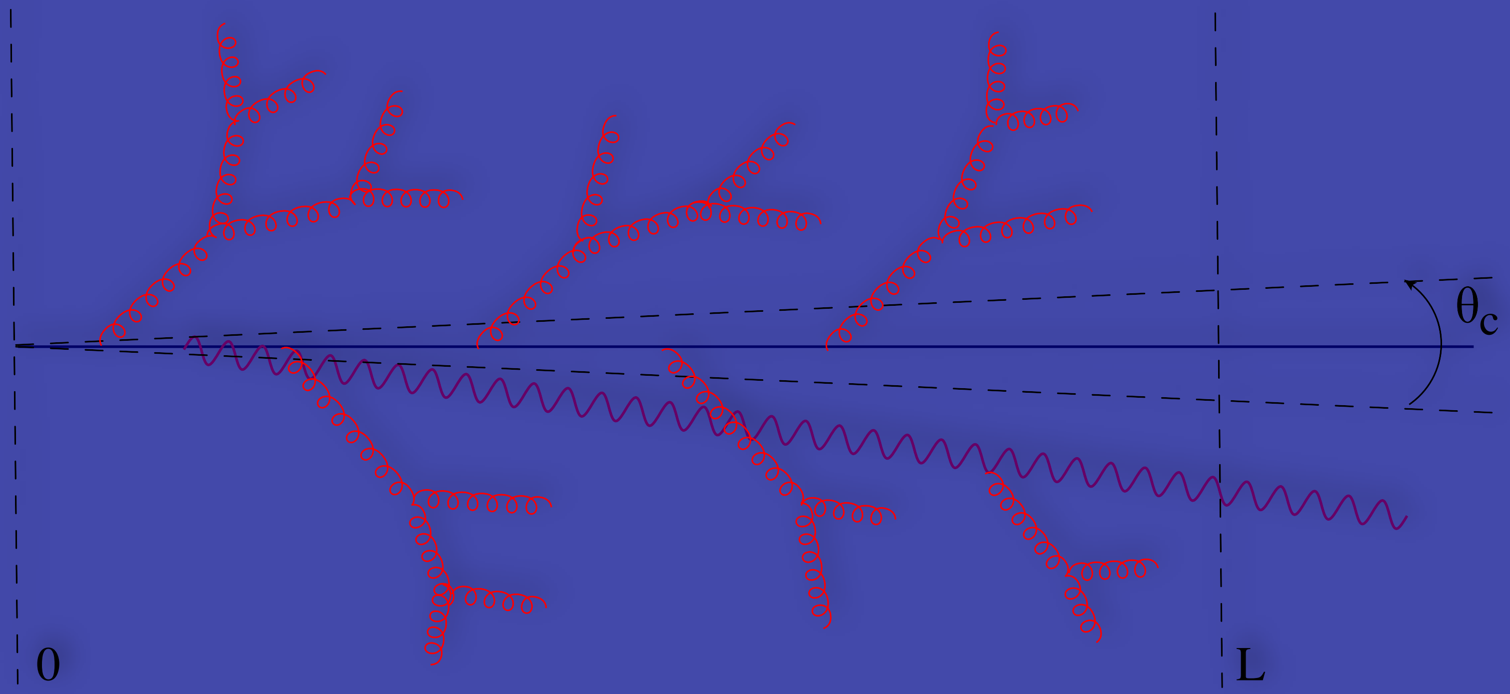


At this (fixed) point ALL the energy flows through the whole system

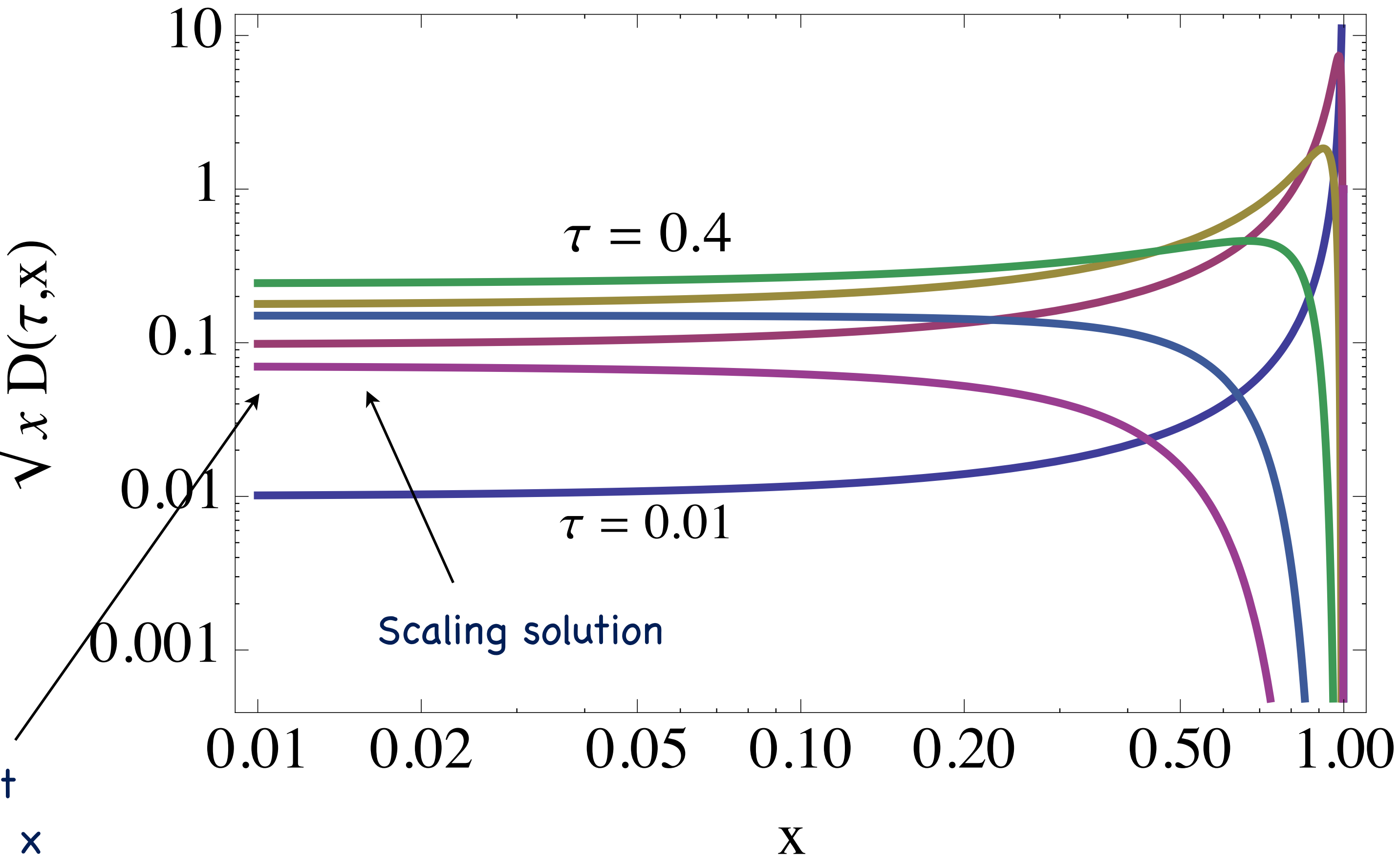
Energy is injected at x=1, at a constant rate

The population of the various x-modes grows, keeping the shape of the spectrum at small x

Relevance to di-jet asymmetry



Evolution of the inclusive spectrum



Scaling solution

Flow of
energy at
very small x

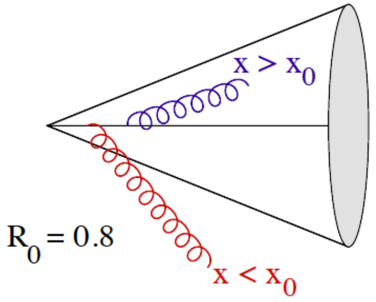
$$\mathcal{E}_{\rm flow} = E\,\frac{\upsilon\tau^2}{2} = \frac{\upsilon}{2}\,\bar{\alpha}^2\omega_c \qquad \omega_c \equiv \frac{\hat{q}L^2}{2} \qquad \upsilon \simeq 5$$

Estimate $\hat{q} = 1\,\mathrm{GeV}^2/\mathrm{fm}$ $\quad \omega_c \simeq 40$ Gev $\quad \bar{\alpha}^2 \simeq 0.1 \quad \mathcal{E}_{\rm flow} \simeq 15\ \mathrm{GeV}$

$L = 4$ fm

J.-P. B., E. Iancu, Y. Mehtar-Tani, arXiv: 1301.6102

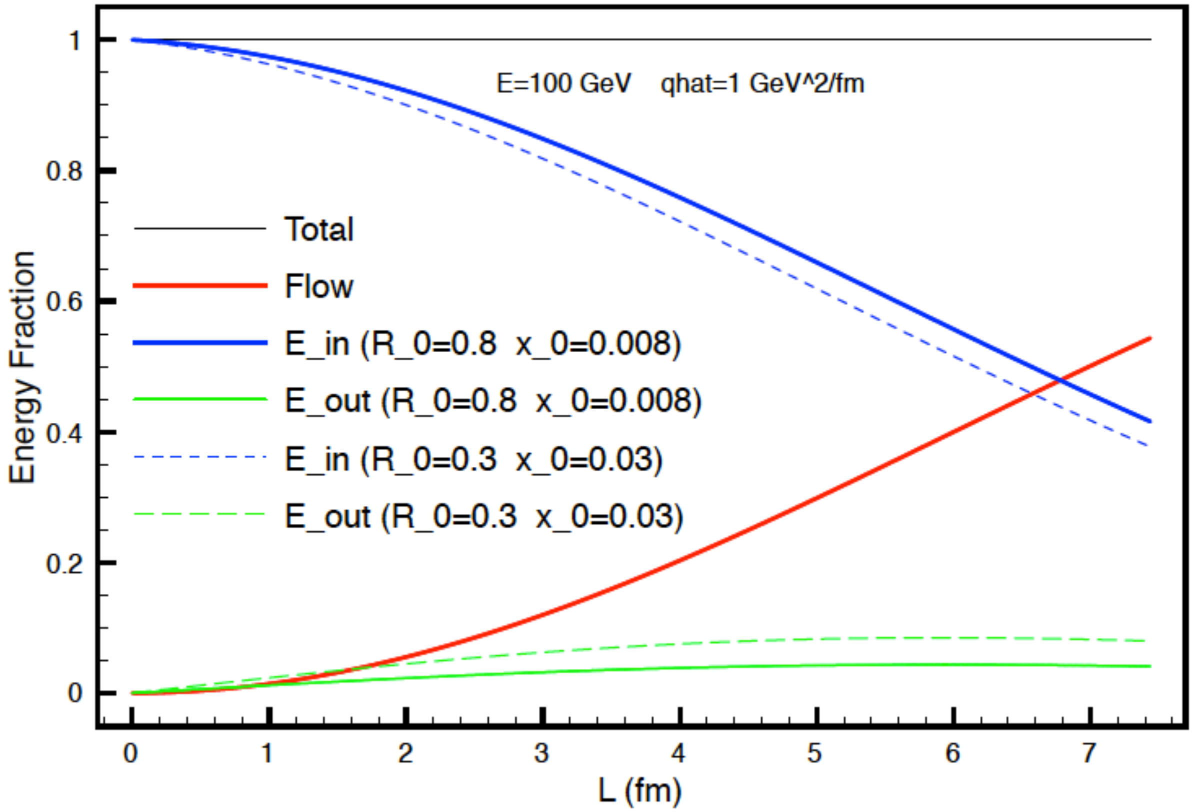
Energy flow at large angle



$E_{\rm in}$ **energy in the jet with x>xo**

$E_{\rm out}$ **energy in the spectrum with x<xo**

$E_{\rm out} + E_{\rm flow}$ **energy out of the jet cone**



Conclusions

In-medium cascade is very different from the in-vacum cascade (no angular ordering, turbulent flow)

Provides a simple and natural mechanism for transfer of jet energy towards very small angles