Jet quenching and fragmentation from an exactly soluble model

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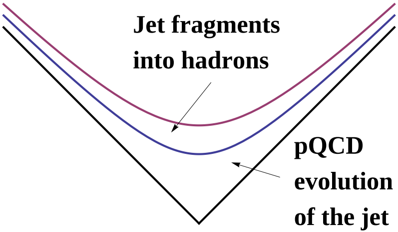
based on *F.L. and D. E. Kharzeev, Int.J.Mod.Phys. E21 (2012) 1250088, arXiv:1111.0493 [hep-ph]*; *D. E. Kharzeev and F.L., to appear in Phys. Rev. D, arXiv:1212.5857 [hep-ph]*

Outline

Jet quenching - General introduction

- It is observed at RHIC and LHC that large p_T hadron yields are suppressed in heavy ion collisions.
- This suppression is attributed to jet quenching.
- Traditional approaches to jet quenching are:
  - Perturbative QCD ($pQCD$)



- At some scale $Q_0^2 \sim 1 - 3$ GeV^2, $pQCD$ is not valid anymore.
- Fragmentation functions are used to describe hadronization phenomenologically.

  - Gauge/Gravity duality

General properties of the model

We want to study nonperturbative effects on jet fragmentation. Properties we would like to have:

- Confinement in the presence of light quarks - screening of color charge.
- Chiral symmetry breaking, topology, anomalies.

Other assumptions

- We study very energetic jets, therefore we assume effective dimensional reduction - namely a $1+1$ dimensional theory, where the spatial direction is along jet axis.
- The picture of confinement we adopt is one based on the Abelian projection - we project the non Abelian theory to N_c (number of colors) Abelian sectors (e.g. condensation of magnetic monopoles).

The Model

In the large N_c approximation we can neglect interference between individual Abelian sectors. We then have N_c independent Abelian sectors in the theory. We assume that the dynamics in these sectors is described by the Schwinger model or QED in $1+1$ dimensions.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - g\gamma^{\mu}A_{\mu} - m_q)\psi$$

It is very well known that this theory is exactly soluble when $m_q = 0$.
Dimensional analysis:

$$[A] = 0, \quad [\psi] = 1/2 \Rightarrow \boxed{[g] = 1}$$

Vector current

$$J_V^{\mu}(x) = \bar{\psi}(x)\gamma^{\mu}\psi(x)$$

Is conserved. For the massless case, $m_q = 0$, the axial current

$$J_A^{\mu}(x) = \bar{\psi}(x)\gamma^{\mu}\gamma^5\psi(x)$$

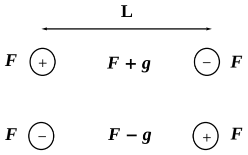
is conserved classically. Quantum corrections give rise to axial anomaly. We will illustrate some of the properties in the following slides.

The theta angle [*Coleman, 1975*]

- In $1+1$ dimensions, in the $A_1 = 0$ gauge, we can write

$$F_{01} = g\partial_1^{-1} j_0 + F$$

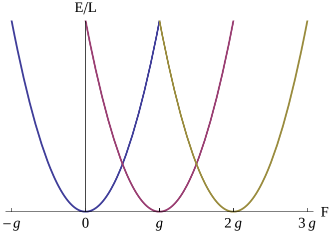
- the constant electric field is allowed in $1+1$ dimensions



- Energy difference

$$\Delta E = \frac{1}{2}\int dx[F_{01}^2 - F^2] = \frac{1}{2}L[(F \pm g)^2 - F^2]$$

- Pair creation favorable for $|F| > \frac{1}{2}g$, until $|F| \leq \frac{1}{2}g$



Ground state energy per unit length.

Physics is periodic with period g.

$$\theta = \frac{2\pi F}{g}$$

Confinement and chiral symmetry breaking

Let's consider two infinitely heavy quarks with charges $\pm g$, separated by $2L$

$$J^0_{ext}(x) = \delta(z+L) - \delta(z-L)$$

The potential can be computed to be

$$\begin{aligned} V(L) &= 2\pi^2 M^2 2L + \frac{g\sqrt{\pi}}{2}\left(1 - e^{-\frac{g}{\sqrt{\pi}}2L}\right) \\ M^2 &\propto m_q g \end{aligned}$$

$m_q = 0$ - Charge screening.
$m_q \neq 0$ - Linear confinement.

Axial anomaly in $1+1$ dimensions has the form

$$\partial_\mu J^\mu_A = \frac{g}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu} = \frac{g}{\pi}F_{01}$$

Using Gauss' theorem

$$\int dzdt\partial_\mu\, J^\mu_A = Q_A(t=\infty) - Q_A(t=-\infty) = N_R - N_L = \frac{g}{2\pi}\int dzdt\; \epsilon^{\mu\nu}F_{\mu\nu}$$

If we have a nonzero background electric field F

$$\boxed{N_R - N_L = \frac{g}{\pi}\int dzdt\; F}$$

Chiral symmetry is not spontaneously broken, but explicitly through anomaly.

Abelian bosonization

In $1+1$ dimensions we can use bosonization

$$J_V^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x) = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi(x)$$

where ϕ is a real scalar field. Using the identity $\gamma^\mu\gamma^5 = -\epsilon^{\mu\nu}\gamma_\nu$, we have

$$J_A^\mu(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x) = \frac{1}{\sqrt{\pi}}\partial^\mu\phi(x)$$

Bosonized Lagrangian can be written as ($m_q = 0$)

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{g}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\phi A_\mu \\
&= \frac{1}{2}F_{01}^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{g}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu A_\mu \\
&= \frac{1}{2}F_{01}^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{g}{\sqrt{\pi}}\phi\, F_{01}
\end{aligned}$$

We can integrate F_{01} (e.g. choose the gauge $A_0 = 0$, Jacobian of $\int \mathcal{D}A_1 \to \int \mathcal{D}F_{01}$ doesn't depend on F_{01}) to get

$$\boxed{\mathcal{L}_{eff} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\frac{g^2}{\pi}\phi^2}$$

This is just a free massive scalar field, with mass $m = \frac{g}{\sqrt{\pi}}$.

Anomaly equation

From Maxwell's equations

$$\partial_1 F^{10} = -\frac{g}{\sqrt{\pi}}\partial_1 \phi \Rightarrow F_{01} = \frac{-g}{\sqrt{\pi}}\phi$$

We have assumed that fields vanish at infinity.
Equation of motion for ϕ is just the Klein-Gordon equation

$$(\Box + \frac{g^2}{\pi})\phi = 0$$

Using bosonization relations

$$\partial_\mu J^\mu_A = \partial_\mu \left(\frac{1}{\sqrt{\pi}}\partial^\mu \phi\right) = \frac{1}{\sqrt{\pi}}\Box\phi$$

Using EOM for ϕ and relation between F_{01} and ϕ, we get

$$\partial_\mu J^\mu_A = \frac{g}{\pi}F_{01} = \frac{g}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}$$

This is the same expression we saw before for the axial anomaly for QED in $1+1$ dimensions.

Adding an external source

Consider a general external source $J^{\mu}_{ext}(x) = j^{\mu}_{ext}(z,t)$. We use the parametrization

$$J^{\mu}_{ext}(x) = -\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_{\nu}\phi_{ext}(x)$$

In the same way as before, we get the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_{\mu}\phi)^2 - \frac{1}{2}\frac{g^2}{\pi}(\phi + \phi_{ext})^2$$

Which gives

$$\boxed{(\Box + m^2)\phi(x) = -m^2\phi_{ext}(x)}$$

- Corresponds to a massive scalar field, coupled to a classical source.
- Coherent particle creation.

Jet fragmentation by quark-antiquark pair production

We consider the source (*Casher, Kogut and Susskind,1974*)

$$J^0_{ext}(x) = \delta(z-t)\theta(z) - \delta(z+t)\theta(-z)$$

Quark anti quark pair, moving back to back.

Using bosonization relations we have

$$\phi_{ext}(x) = -\theta(t-z)\theta(t+z)$$

We therefore have to solve

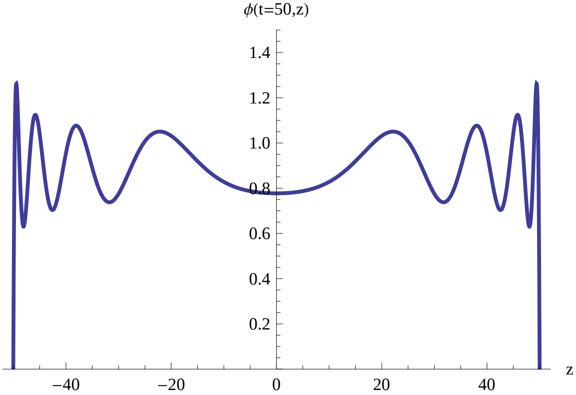
$$(\Box + m^2)\phi = m^2\theta(t-z)\theta(t+z)$$

The solution to the equation of motion is (*F.L. and D. E. Kharzeev, arXiv:1111.0493 [hep-ph]*)

$$\phi(x) = \theta(t+z)\theta(t-z)(1 - J_0(m\sqrt{x^2}))$$

where $x^2 = t^2 - z^2$.

Jet fragmentation (cont'd)



Quark anti quark pair, moving back to back.

Quark anti quark pair, moving back to back.

[*D. E. Kharzeev and F.L., arXiv:1212.5857 [hep-ph]*]

(Anti-)Kinks correpond to (anti-)fermions. Quark fragmentation in the θ-vacuum.

Particle creation by a general source

Let's consider

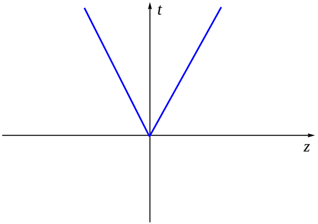
$$(\Box + m^2)\phi(x) = f(x)$$

It is known that

$$\frac{dN}{dp} = \frac{|\tilde{f}(p)|^2}{2E_p},$$
$$\tilde{f}(p) = \int d^2x\, e^{ip\cdot x} f(x), \quad E_p = \sqrt{p^2 + m^2}$$

For jets with finite energy - quarks move with velocity v

$$j^0_{ext}(x) = \delta(z - vt)\theta(z) - \delta(z + vt)\theta(-z)$$



Velocity is calculated from

$$v = \frac{p_q}{E_q} = \frac{p_{jet}}{\sqrt{p_{jet}^2 + Q_0^2}}$$

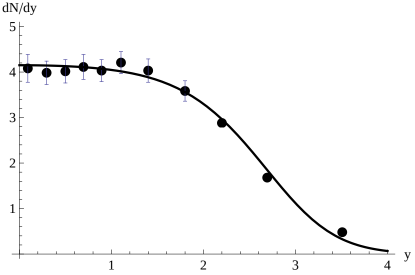
$$\frac{dN}{dp} = 2\pi \frac{v^2 m^4}{E_p(E_p^2 - v^2p^2)^2}$$

We fix Q_0 by comparing our result to experimental data.

Rapidity distribution

We now change variables to $y = \frac{1}{2} \ln \frac{E_p + p}{E_p - p}$

$$\frac{dN}{dy} = 2\pi \frac{v^2}{(\cosh^2 y - v^2 \sinh^2 y)^2}$$

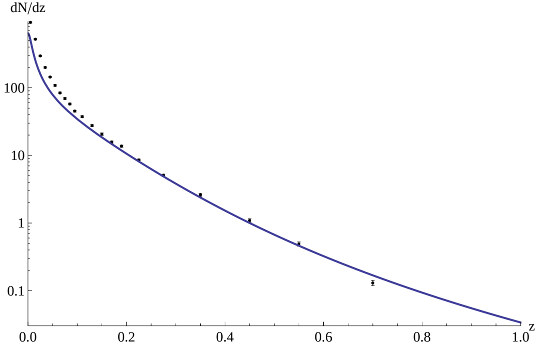


Comparison to experimental data [*Aiphara (TPC/Two Gamma Collaboration), 1988*], for $\sqrt{s} = 29$ GeV

- Q_0 is fixed by above fit. We get $Q_0 \approx 1.8$ GeV.
- Since $\sqrt{s}$ is small, the effect of jet evolution is small and our model fits the data well.

Fragmentation functions

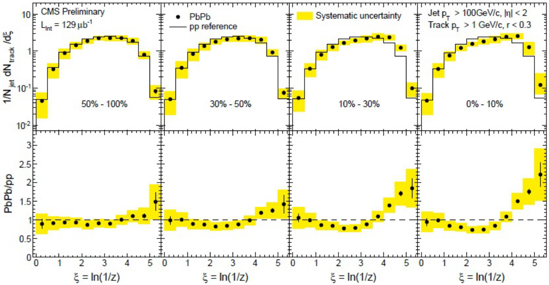
Fragmentation function for e^+e^- annihilation

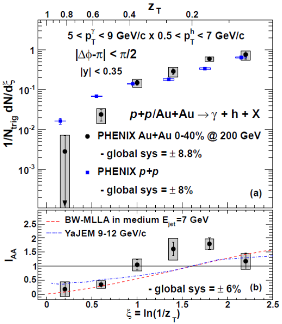


Charged particle distribution for $\sqrt{s} = 201.7$ GeV, [*Abbiendi (OPAL Collaboration), 2003*].

- Reasonable agreement with the data for $\frac{p}{p_{jet}} = z > 0.1$.
- By fitting to data at different center of mass energies $m \simeq 0.6$ GeV.
- Enhancement at small z and suppression at large z are attributed to $pQCD$ evolution of the jet, which is not included in our model.

Medium modified fragmentation functions [*CMS Collaboration, 2012*], [*PHENIX Collaboration, 2012*]



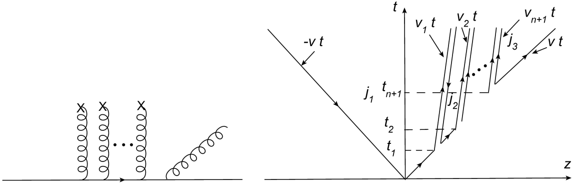


$$z = \frac{p^h}{p^{jet}}, \quad \xi = \ln\frac{1}{z}$$

- There is enhancement of soft particles for central collisions.
- For the most central collisions, we also see a depletion of particles for intermediate ξ.

In-medium scattering

We consider a very simple model: static scatterers, no expansion of the medium. [*D. E. Kharzeev and F.L., arXiv:1212.5857 [hep-ph]*]



We have three types of currents and their Fourier transform is

$$
\begin{aligned}
\tilde{j}_1^0(p) &= \frac{ip}{E_p - vp}\left[\frac{2v}{E_p + vp} - \frac{v - v_1}{E_p - v_1 p} e^{i(E_p - vp)t_1}\right] \\
\tilde{j}_2^0(p) &= \frac{-ip}{E_p - vp}\left[\frac{v - v_2}{E_p - v_2 p} e^{i(E_p - vp)t_2} - \frac{v - v_1}{E_p - v_1 p} e^{i(E_p - vp)t_1}\right] \\
\tilde{j}_3^0(p) &= \frac{ip}{E_p - vp}\frac{v - v_{n+1}}{E_p - v_{n+1} p} e^{i(E_p - vp)t_{n+1}}
\end{aligned}
$$

Nonperturbative LPM effect

Total momentum distribution is given by

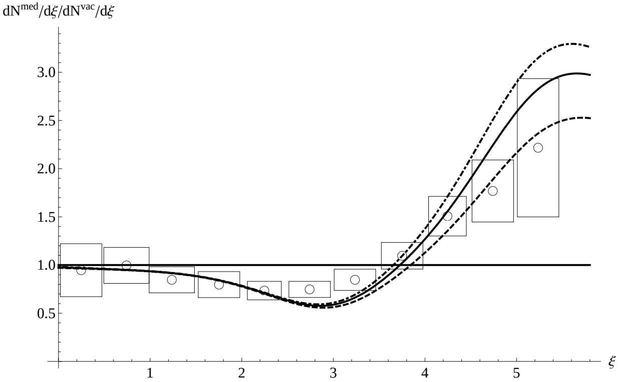
$$\frac{dN^{med}}{dp} = \frac{1}{2E_p}|\tilde{f}(p)|^2 = \frac{1}{2E_p}\left(|\tilde{f}_1(p)|^2 + \sum_{i=1}^{N-1}|\tilde{f}_2(p)|^2 + |\tilde{f}_3(p)|^2\right)$$

where as before we construct f_i's from j_i^0's.
$\tilde{j}_2^0(p)$ responsible for soft radiation. We can define the formation time

$$t_f = \frac{1}{E_p - vp} \simeq \frac{2p}{m^2}$$

Similar to the perturbative result $t_f = \frac{2\omega}{k_\perp^2}$. Radiation is suppressed when mean free path $\lambda = t_2 - t_1 << t_f$ - LPM effect.

Medium fragmentation functions - results



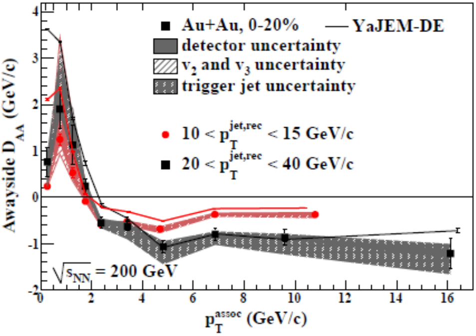
The ratio of in-medium and vacuum fragmentation functions for $p_{jet} = 120$ GeV. The first scattering occurs at $t_1 \simeq 1$ fm, which is the assumed thermalization time. The length of the medium is $L = 5$ fm. The curves correspond to mean free paths of $\lambda = 0.57$, 0.4 and 0.2 fm from top to bottom respectively.

- Scaling for small ξ - Nonperturbative LPM effect.
- Suppression at intermediate ξ is a result of the partial screening of the color charge of the jet by a comoving medium-induced gluon.

Transverse-momentum difference - D_{AA} (Talk by J. Putschke)

We study jet modification using another observable proposed in [*STAR Collaboration, arXiv:1302.6184 [nucl-ex]*]

$$D_{AA}(p) = Y_{Au-Au}(p)\,\langle p\rangle_{Au-Au} - Y_{p-p}(p)\,\langle p\rangle_{p-p}$$

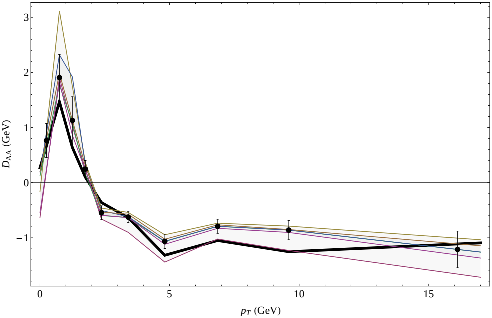


Y are yields in $Au - Au$ and $p - p$ and $\langle p\rangle$ is the average momentum inside a bin.

In our case

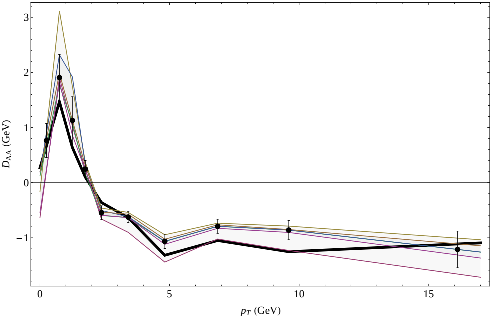
$$D_{AA}(p) = \langle p\rangle \int_{\text{bin with average}\langle p\rangle} dp' \frac{dN^{\text{med}}}{dp'} - \langle p\rangle \int_{\text{bin with average }\langle p\rangle} dp' \frac{dN^{\text{vac}}}{dp'}$$

Transverse-momentum difference - D_{AA} - Results



Circles show the data for the awayside momentum difference for 20<$p_T^{jet,rec} < 40$ GeV and shaded areas show jet energy scale, v2/v3 and detector uncertainties (taken from [*STAR Collaboration, arXiv:1302.6184 [nucl-ex]*]). Calculations are done using $\lambda = 0.4$ fm (solid black line).

Transverse-momentum difference - D_{AA} - Results



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D_{AA} was also addressed in [*T. Renk, Phys.Rev. C87 (2013) 024905*]
In this approach jet broadening contributes to jet modification and energy loss.
In our $1+1$ model we only have energy redistribution in the longitudinal (along jet axis) direction.
Two models could be distinguished experimentally by looking for the modification of the jet cone.

Summary and Outlook

- We considered an effective theory of jet fragmentation based on an exactly soluble model
- This model incorporated confinement in the presence of light quarks, topology and chiral symmetry breaking.
- Topology was shown to be responsible for particle production.

- Would be interesting to investigate further the effects of topology in high energy processes.
- A systematic derivation of the dimensionally reduced theory for high energy processes is needed.