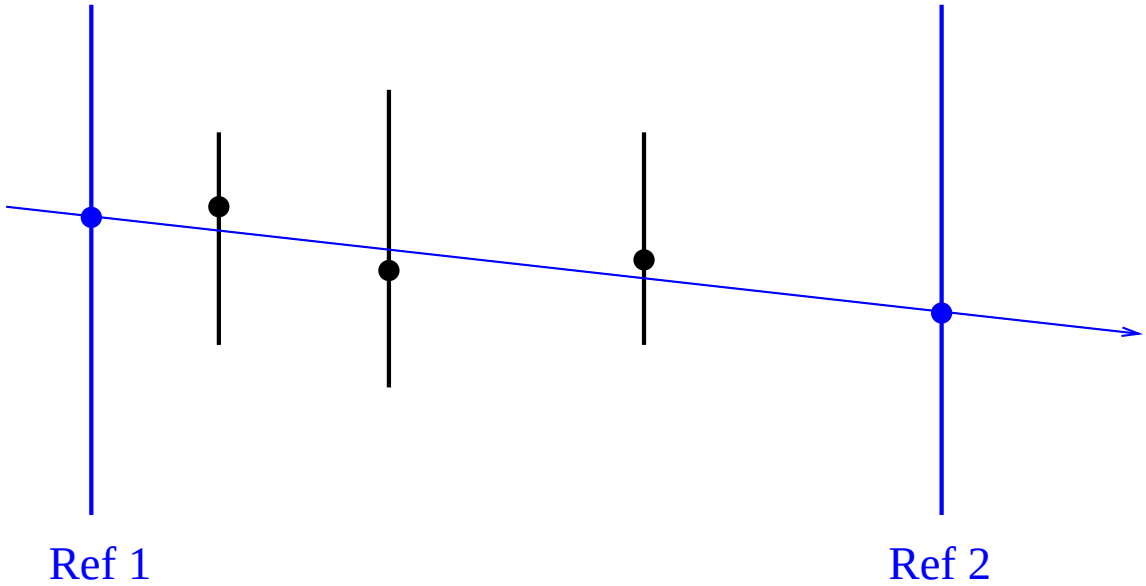
Compass spectrometer alignment method.

- Introduction
- Principle
- Some results

Introduction



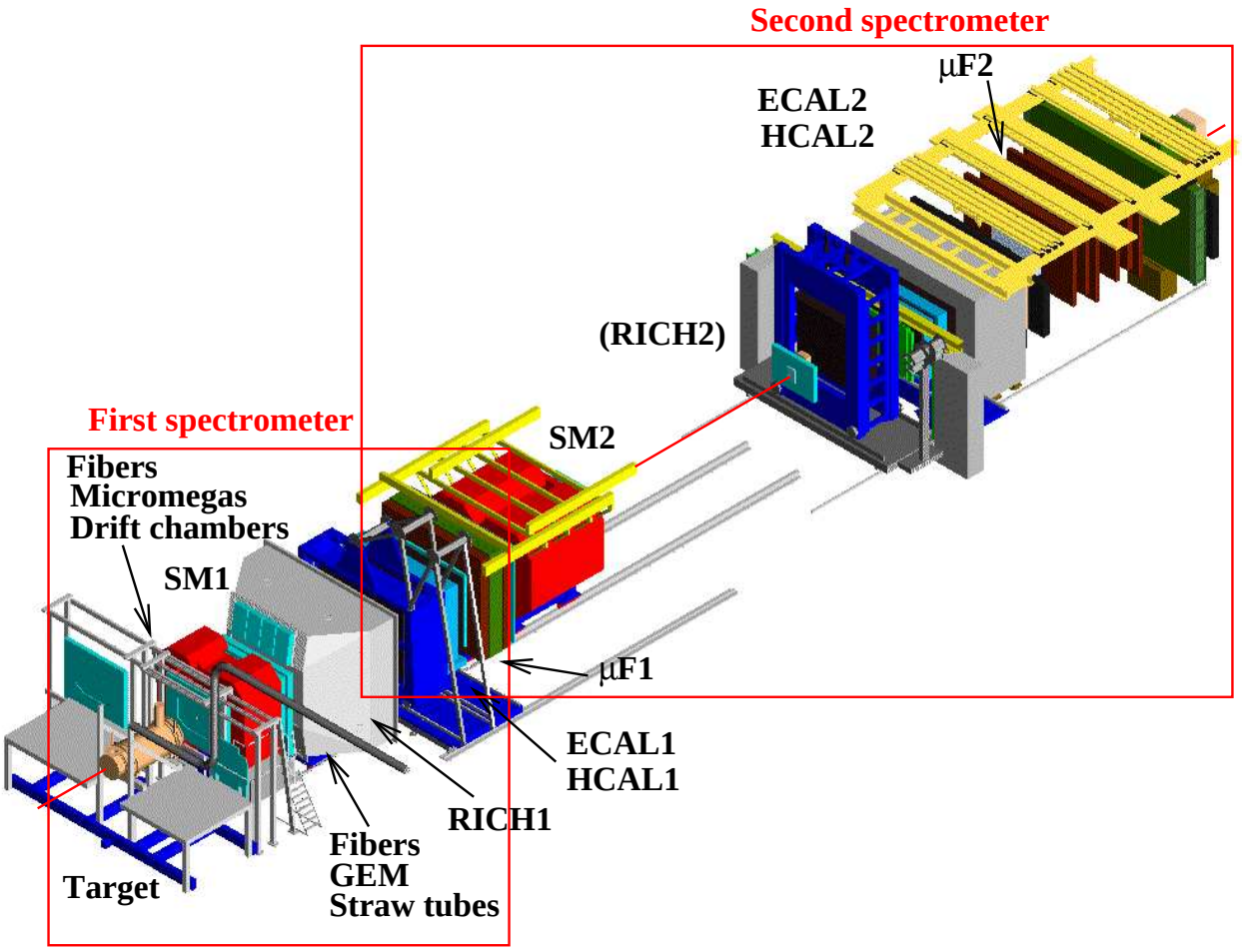
Standard method:

- take **reference** detectors to build tracks
- remove others from tracking
- align removed detectors by hand using residuals

Global method:

- use all detectors in tracking
- fix some detectors for *gauge*
- minimize sum of track χ^2 wrt tracks and alignment parameters.

Compass spectrometer

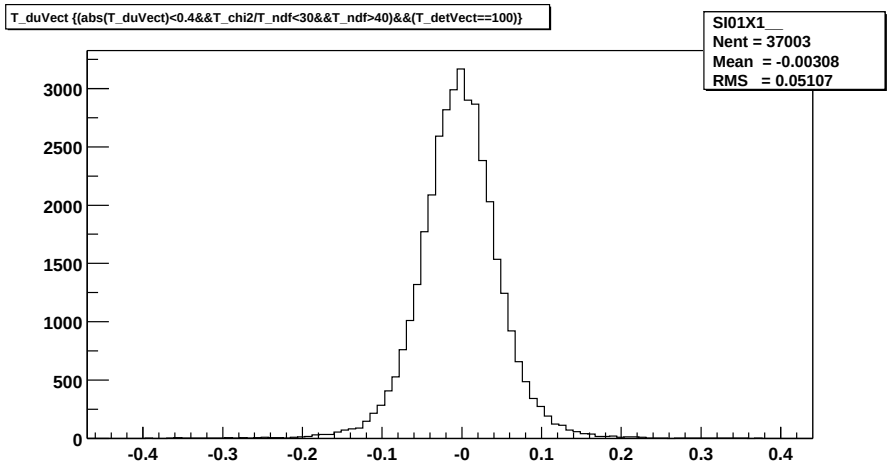


#detectors:	~ 250	
#wire orientations:	14	
detector size:	min $4 \times 4\text{cm}^2$ (FI)	max $5 \times 4\text{m}^2$ (MA)
resolutions:	min $\sim 30\mu\text{m}$ (SI)	max $\sim 3\text{mm}$ (MA)

Criterions

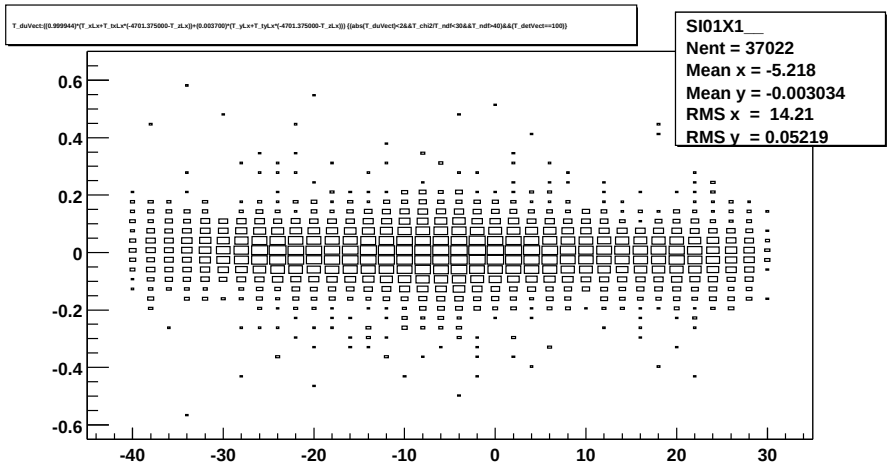
$\langle \Delta U \rangle = 0$

(residuals)



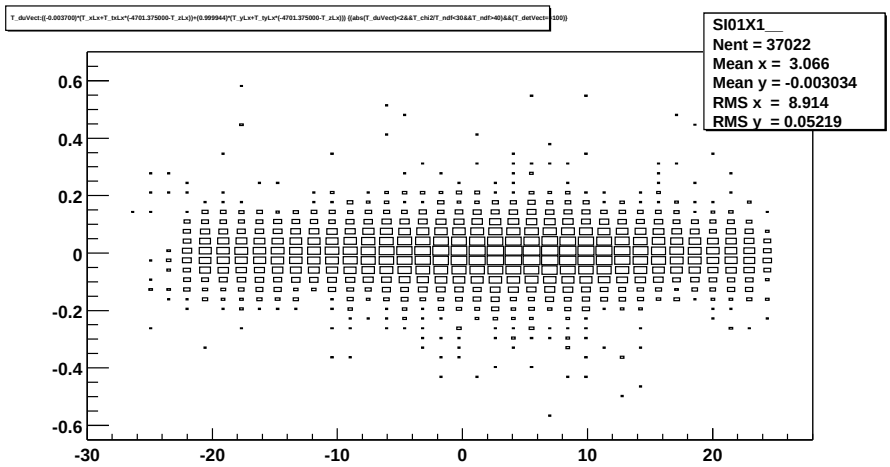
$\partial \langle \Delta U \rangle / \partial U = 0$

(perp. to the wires)



$\partial \langle \Delta U \rangle / \partial V = 0$

(along the wires)



Alignment parameters

Alignment parameters

offset perp. to the wires δU	$\rightarrow \langle \Delta U \rangle$
angle perp. to the beam $\delta\theta$	$\rightarrow \frac{\partial \langle \Delta U \rangle}{\partial U}, \frac{\partial \langle \Delta U \rangle}{\partial V}$
position along the beam δz	$\rightarrow \frac{\partial \langle \Delta U \rangle}{\partial U}$
pitch $P\delta P$	$\rightarrow \frac{\partial \langle \Delta U \rangle}{\partial U}$
angles wrt the beam	not taken into account by present tracking second order corrections

Principle

- *single track* minimization
- *many tracks* minimization
- constraints
- alignment with magnets on

Single track minimization

notation

- U coordinate perp to the wire for all detectors
- α_t track parameters (x, y, t_x, t_y)
- α_a alignment parameters (detector offsets, rotational offsets ...)

track χ^2: $\boxed{\chi^2 = \sum_{\text{det}} \frac{[U_{\text{det}} - U_{\text{calc}}(\alpha_t, \alpha_a)]^2}{\sigma^2}}$

hyp: linear dependency of U upon α $\boxed{U_{\text{calc}} = \sum_i \frac{\partial U}{\partial \alpha_i} \alpha_i}$

χ^2 minimization: $\boxed{0 = -\frac{1}{2}\frac{\partial \chi^2}{\partial \alpha} = \sum_{\text{det}} \frac{1}{\sigma^2} \left(\frac{\partial U}{\partial \alpha} U_{\text{det}} - \sum_i \frac{\partial U}{\partial \alpha} \frac{\partial U}{\partial \alpha_i} \alpha_i \right)}$

and

$$\begin{pmatrix} \sum_{\text{det}} \frac{1}{\sigma^2} \frac{\partial U}{\partial \alpha_1} \frac{\partial U}{\partial \alpha_1} & \cdots & \sum_{\text{det}} \frac{\partial U}{\partial \alpha_1} \frac{\partial U}{\partial \alpha_i} & \cdots \\ \vdots & \ddots & \vdots & \\ \sum_{\text{det}} \frac{\partial U}{\partial \alpha_i} \frac{\partial U}{\partial \alpha_1} & \cdots & \sum_{\text{det}} \frac{\partial U}{\partial \alpha_i} \frac{\partial U}{\partial \alpha_i} & \\ \vdots & & & \ddots \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_i \\ \vdots \end{pmatrix} = \begin{pmatrix} \sum_{\text{det}} \frac{1}{\sigma^2} \frac{\partial U}{\partial \alpha_1} U_{\text{det}} \\ \vdots \\ \sum_{\text{det}} \frac{1}{\sigma^2} \frac{\partial U}{\partial \alpha_i} U_{\text{det}} \\ \vdots \end{pmatrix}$$

Many tracks minimization

To be minimized: $\boxed{\sum_{i=1}^{N} \chi_i^2 \text{ for } N \text{ (many) tracks.}}$

The track parameters α_t are track dependent

The alignment parameters α_a are detector dependent

The derivatives $\partial U/\partial\alpha$ depends upon the detector, evt. the track parameters (non linearities).

$$\left(\begin{array}{c||c|c|c} \Sigma\, C_i & \dots & G_i & \dots \\ \hline\hline \vdots & \ddots & 0 & 0 \\ \hline G_i^T & 0 & \Gamma_i & 0 \\ \hline \vdots & 0 & 0 & \ddots \end{array}\right) \cdot \left(\begin{array}{c} \alpha_a \\ \hline\hline \vdots \\ \hline \alpha_{t,i} \\ \hline \vdots \end{array}\right) = \left(\begin{array}{c} \Sigma\, b_i \\ \hline\hline \vdots \\ \hline \beta_i \\ \hline \vdots \end{array}\right)$$

- C_i, b_i depend upon $\partial U/\partial\alpha_a$ only (for i^{th} track)
- Γ_i, β_i depend upon $\partial U/\partial\alpha_t$ only
- G_i are mixed terms, $(\partial U/\partial\alpha_a)(\partial U/\partial\alpha_t)$.

Many 0 because tracks are independent one to the other.

matrix dimension:

4 parameters/tracks, 20000 tracks

$\sim$ 4 parameters/detectors, 250 detectors

$\boxed{\Rightarrow 4 \times 20000 + 4 \times 250 = 81000}$

Really inverted are 20000 $\underline{4 \times 4}$ (track) matrices + one $\underline{1000 \times 1000}$ (alignment) matrix $\Rightarrow$ all α_a (+ all α_t)

ref: http://www.desy.de/~blobel/wwwmille.html

Matrix inversion

full system:

$$\left(\begin{array}{c||c|c|c} \Sigma\, C_i & \dots & G_i & \dots \\ \hline\hline \vdots & \ddots & 0 & 0 \\ \hline G_i^T & 0 & \Gamma_i & 0 \\ \hline \vdots & 0 & 0 & \ddots \end{array}\right) \cdot \left(\begin{array}{c} \alpha_a \\ \hline\hline \vdots \\ \hline \alpha_{t,i} \\ \hline \vdots \end{array}\right) = \left(\begin{array}{c} \Sigma\, b_i \\ \hline\hline \vdots \\ \hline \beta_i \\ \hline \vdots \end{array}\right)$$

reduced system (alignment parameters):

$$(\alpha_a) = (C')^{-1}\,(b')$$

with

$$C' = \Sigma\, C_i - \Sigma\, G_i \Gamma_i^{-1} G_i^T$$

$$b' = \Sigma\, b_i - \Sigma\, G_i \Gamma_i^{-1} \beta_i$$

tracks parameter (optional):

$$(\alpha_{t,i}) = -\left(\Gamma_i^{-1} G_i^T C'\right)(b) + \left(\Gamma_i^{-1} + \Gamma_i^{-1} G_i^T C' G_i \Gamma_i^{-1}\right)(\beta_i)$$

ref: http://www.desy.de/~blobel/wwwmille.html

Derivatives

$$U_{\text{calc}} = (1+\delta P)\left\{ \begin{array}{ll} & \cos(\theta_{\text{det}}+\delta\theta)\,[x+t_x(z_{\text{det}}+\delta z-z)] \\ + & \sin(\theta_{\text{det}}+\delta\theta)\,[y+t_y(z_{\text{det}}+\delta z-z)] \end{array} \right\} + \delta U$$

Track parameters:

- tracks coordinates at z: (x, y)

  $\partial U/\partial x = \cos\theta_{\text{det}}$

  $\partial U/\partial y = sin\theta_{\text{det}}$

- tracks angle: (t_x, t_y)

  $\partial U/\partial t_x = \cos\theta_{\text{det}}(z_{\text{det}} - z)$

  $\partial U/\partial t_y = \sin\theta_{\text{det}}(z_{\text{det}} - z)$

alignment parameters:

- offset perp. to the wires δU

  $\partial U/\partial\delta U = 1$

- offset along the beam δz

  $\partial U/\partial\delta z = \cos\theta_{\text{det}} t_x + \sin\theta_{\text{det}} t_y$

- rotational offset $\delta\theta$

  $\partial U/\partial\delta\theta = -\sin\theta_{\text{det}}\,[x + t_x(z_{\text{det}} - z)] + \cos\theta_{\text{det}}\,[y + t_y(z_{\text{det}} - z)]$

- pitch offset δP

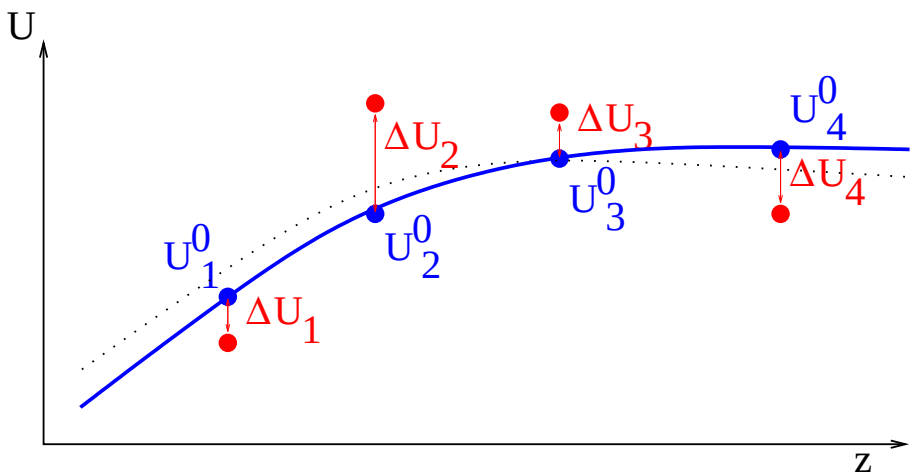
  $\partial U/\partial\delta P = \cos\theta_{\text{det}}\,[x + t_x(z_{\text{det}} - z)] + \sin\theta_{\text{det}}\,[y + t_y(z_{\text{det}} - z)]$

Summary

we need:

- $\sim$ 20000 tracks;
- $\partial U_{\mathrm{trk,det}}/\partial \alpha_{a,\mathrm{det}}$, the derivatives wrt alignment parameters
- $\partial U_{\mathrm{trk,det}}/\partial \alpha_{trk,\mathrm{det}}$, the derivatives wrt track parameters
- some algebrae to perform the matrix inversion.

Alignment with magnets on



No track parametrisation $\Leftarrow$ use output from *raw* tracking (U_i^0)

fit a correction to the track:

$$\chi^2 = \sum_{\text{det}} \frac{\left\{\left[U_{\text{det}} - U^0\right] - \left[U_{\text{calc}} - U^0\right]\right\}^2}{\sigma^2_{\text{det}}}$$

$$\sum_{\text{det}} \frac{\left[\Delta U_{\text{det}} - \Delta U_{\text{calc}}\right]^2}{\sigma^2_{\text{det}}}$$

Assumption: ΔU_{calc} **linear upon** z.

$\Rightarrow$ the expression of the derivatives are the same but using the *local* track parameters:

- x_{det} and y_{det} the position **in the detector**
- $t_{x,\text{det}}$ and $t_{y,\text{det}}$ the **local** track angles

$\Rightarrow$ the *measurement* is now ΔU_{det} instead of U_{det}

Constraints

Fixing detectors

- δU: 4 detectors to be fixed to avoid x, y global translation, rotation.
- $\delta\theta$: 1 detector to be fixed
- δz: 2 detectors to be fixed to avoid translation and scaling along the beam
- δP (pitch): 2 detectors to be fixed to avoid x, y global scaling

Constraints between parameters

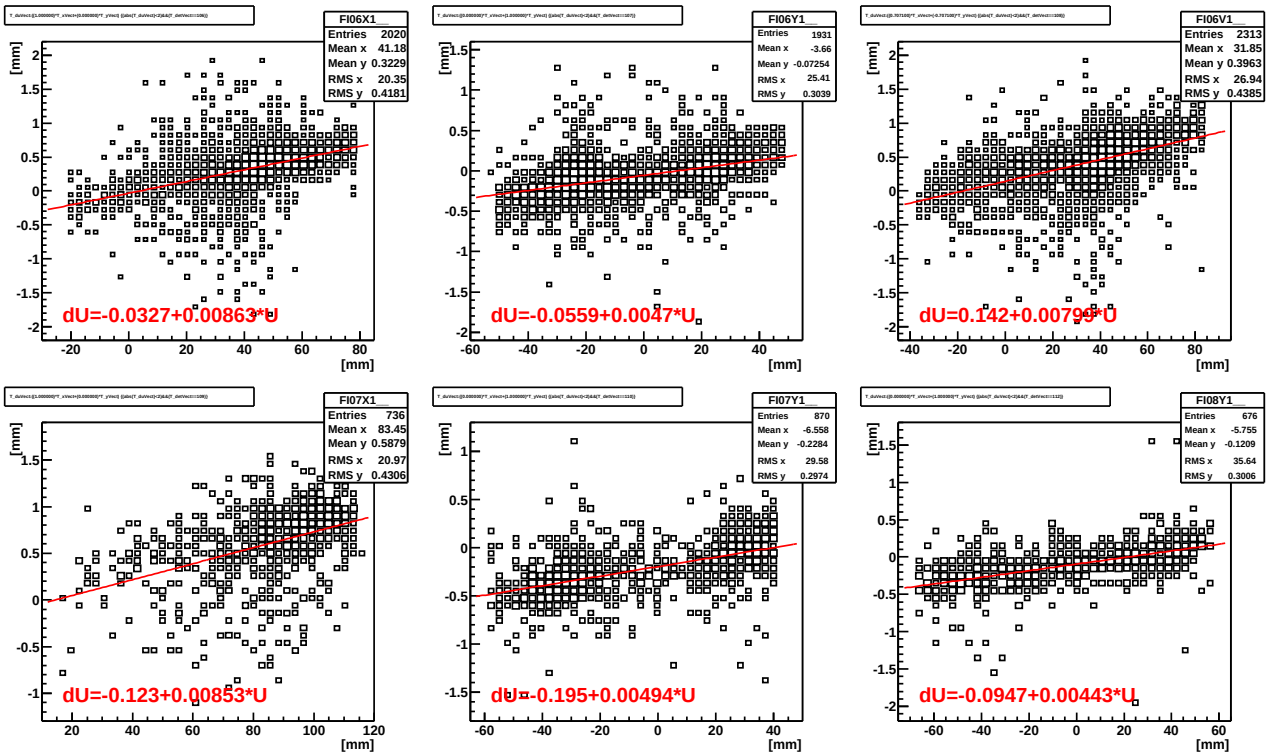
- $\sum_{\text{det}} \delta U_i = 0$ to avoid global translation
- $\sum_{\text{det}} \frac{\delta U_i}{z_i} = 0$ to avoid global rotation
- ...

Results

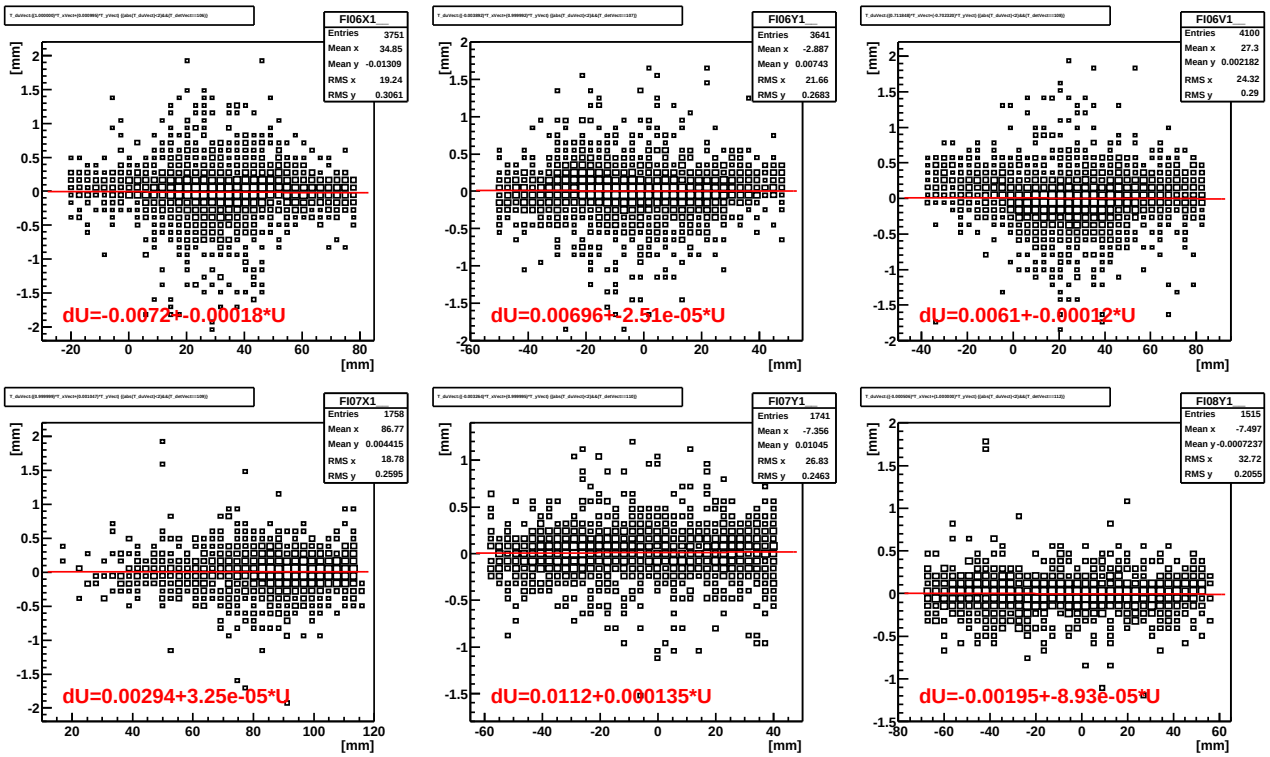
- angular alignment (+pitch adjustment) on fibers
- effect of SM1 magnetic field on gems
- effect of target field rotation on fibers
- *global* performances

angular alignment on fibers

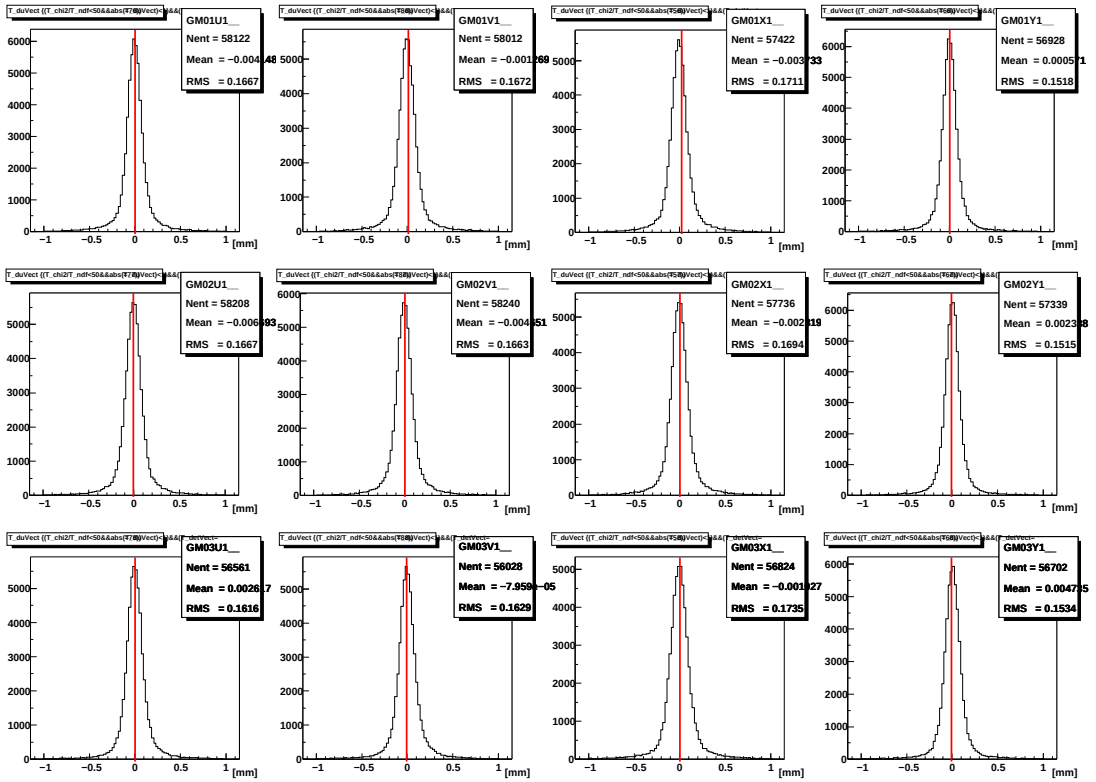
before:

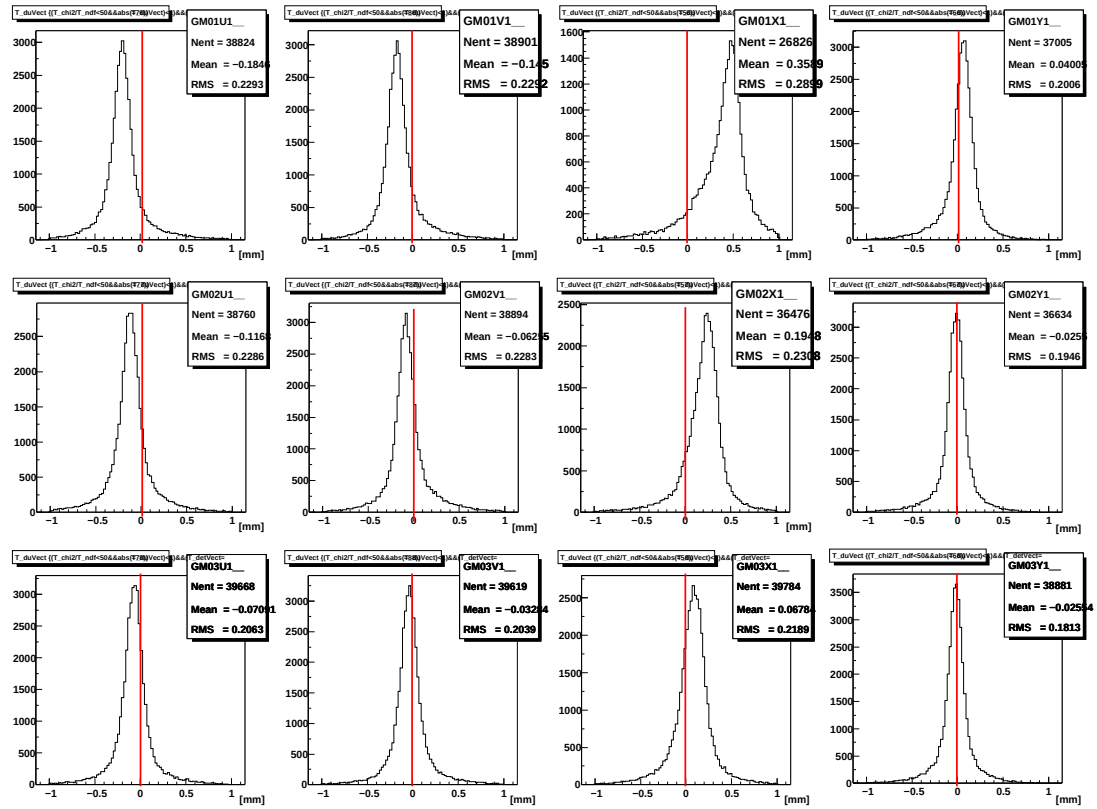


after:

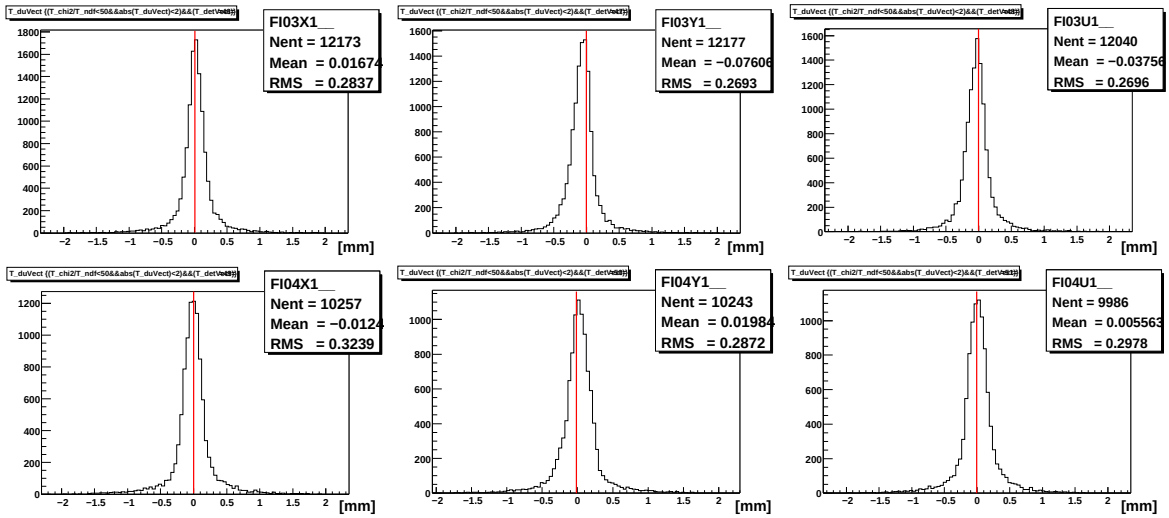


effect of SM1 magnetic field on gems

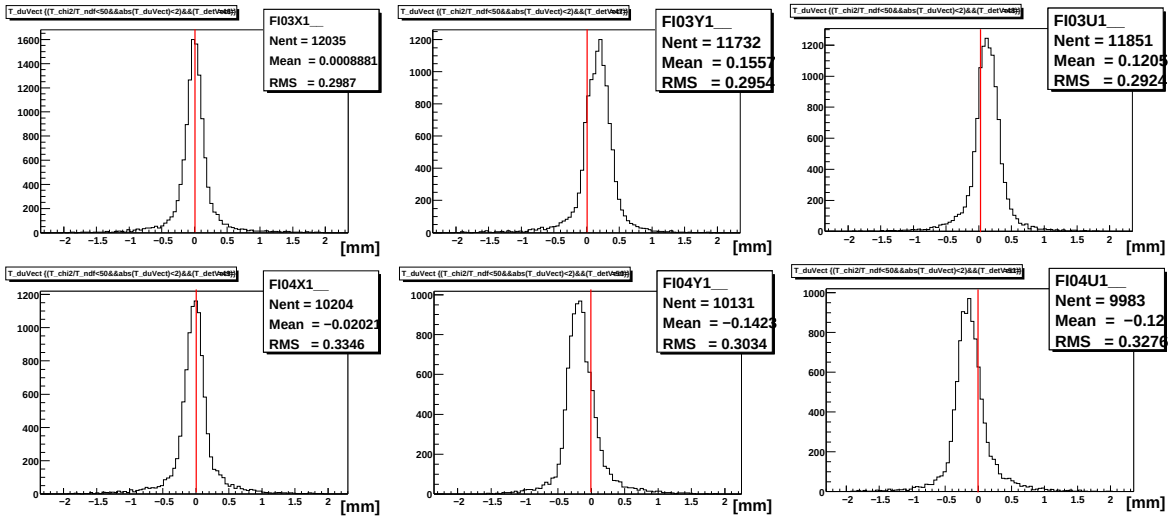




effect of target field rotations on fibers 3 and 4



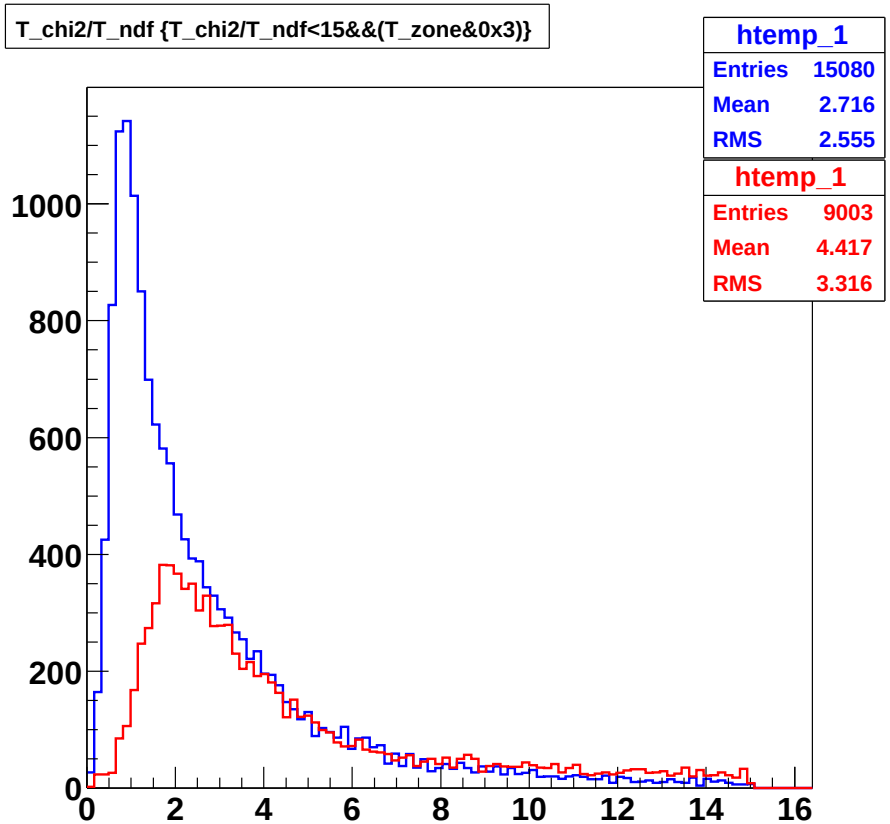
run 22771 - Field scale −



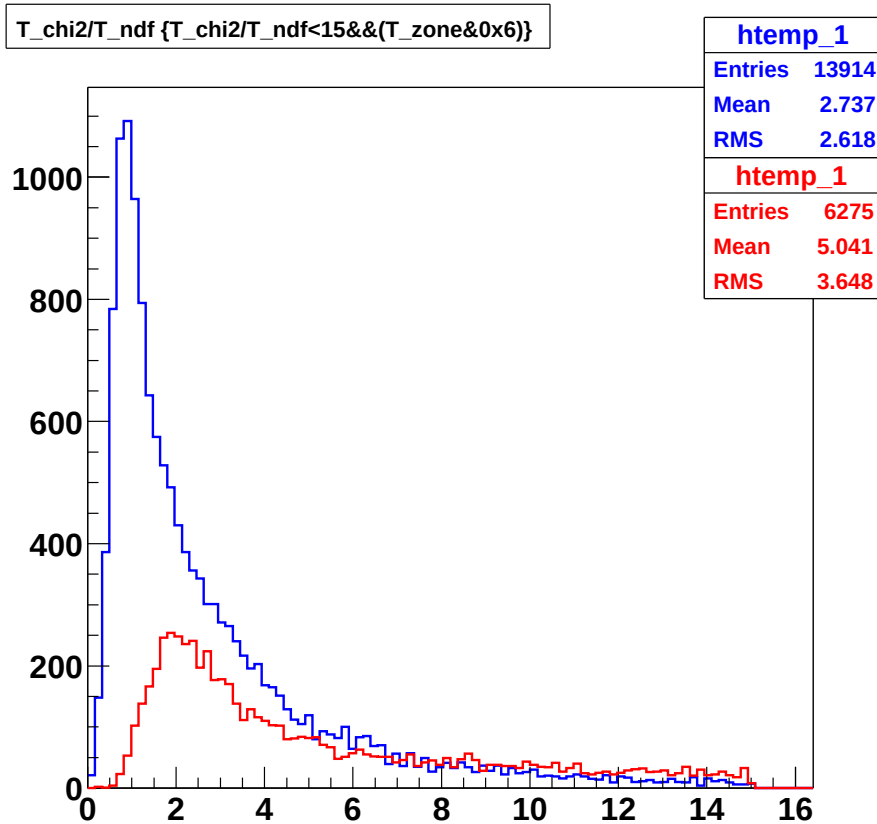
run 22776 - Field scale +

χ^2 performances

Tracks bridged through SM1



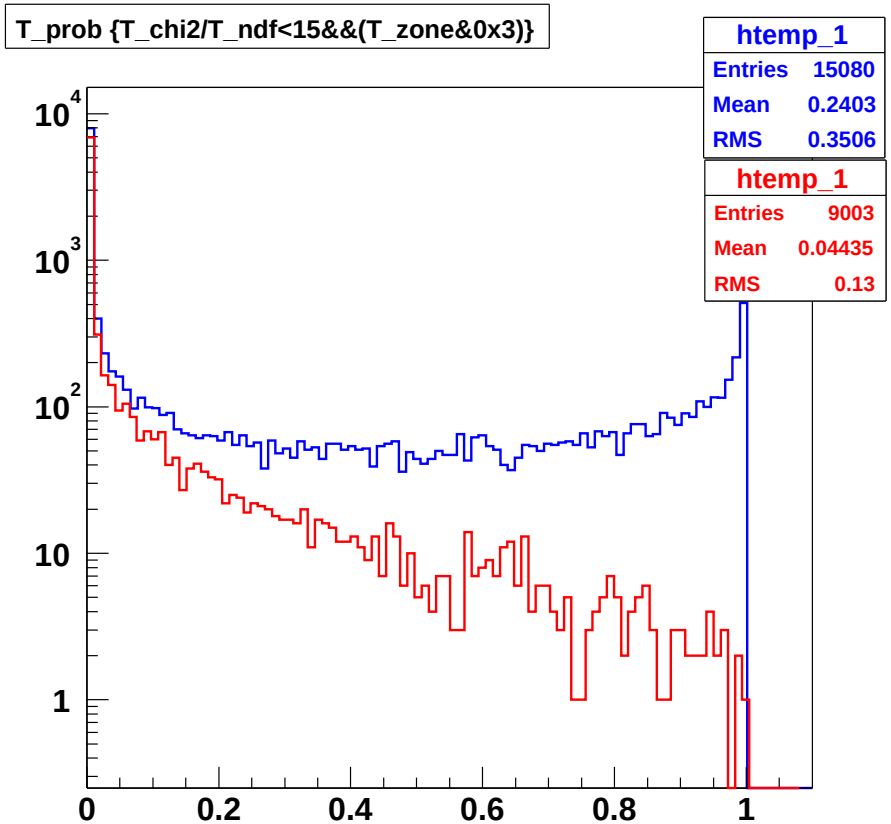
Tracks bridged through SM2



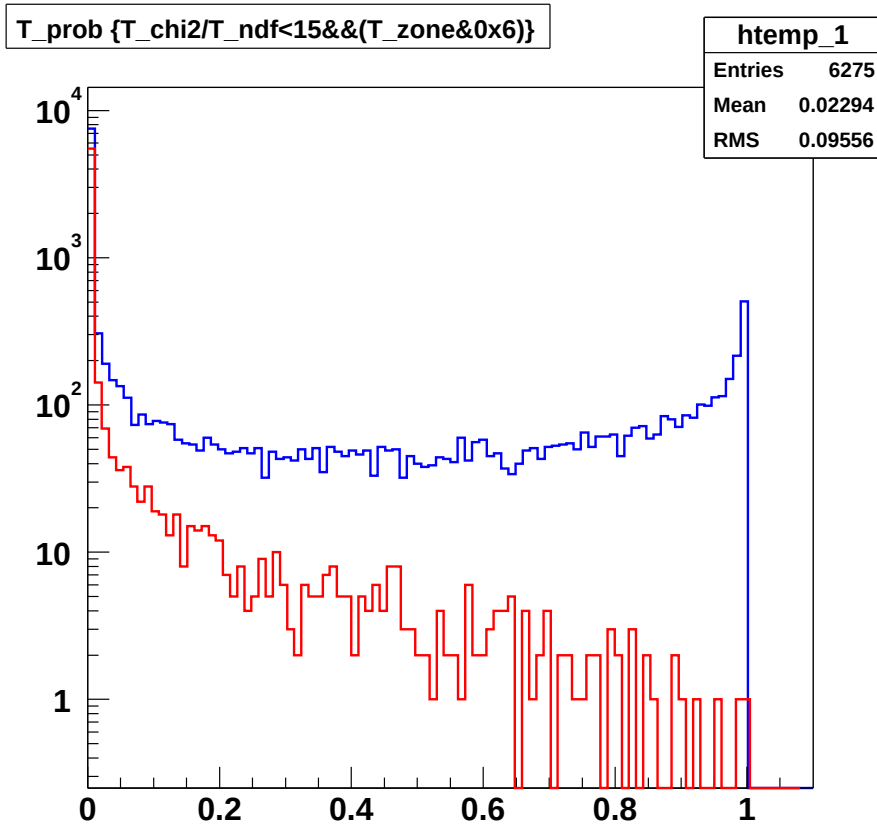
- red: alignment *by hand* on alignment run only (magnets off)
- blue: *this* method on alignment (magnets off) + physics (magnets on) run.

χ^2 performances

Tracks bridged through SM1



Tracks bridged through SM2



- red: alignment *by hand* on alignment run only (magnets off)
- blue: *this* method on alignment (magnets off) + physics (magnets on) run.

global performances

Run	method	$n_{\mathrm{trk}}/n_{\mathrm{evt}}$	$\overline{\chi^2}$	α_{vtx}	$n_{\mathrm{trk}}/n_{\mathrm{vtx}}$	$\alpha_{\mathrm{vtx},\mu'}$
22019 (P2D) -	(1)	1.71	7.98	36.1	2.96	14.2
	(2)	3.09	3.07	45.7	3.38	33.1

- Method (1): alignment *by hand* on alignment run only (magnets off)
- Method (2): *this* method on alignment (magnets off) + physics (magnets on) run.