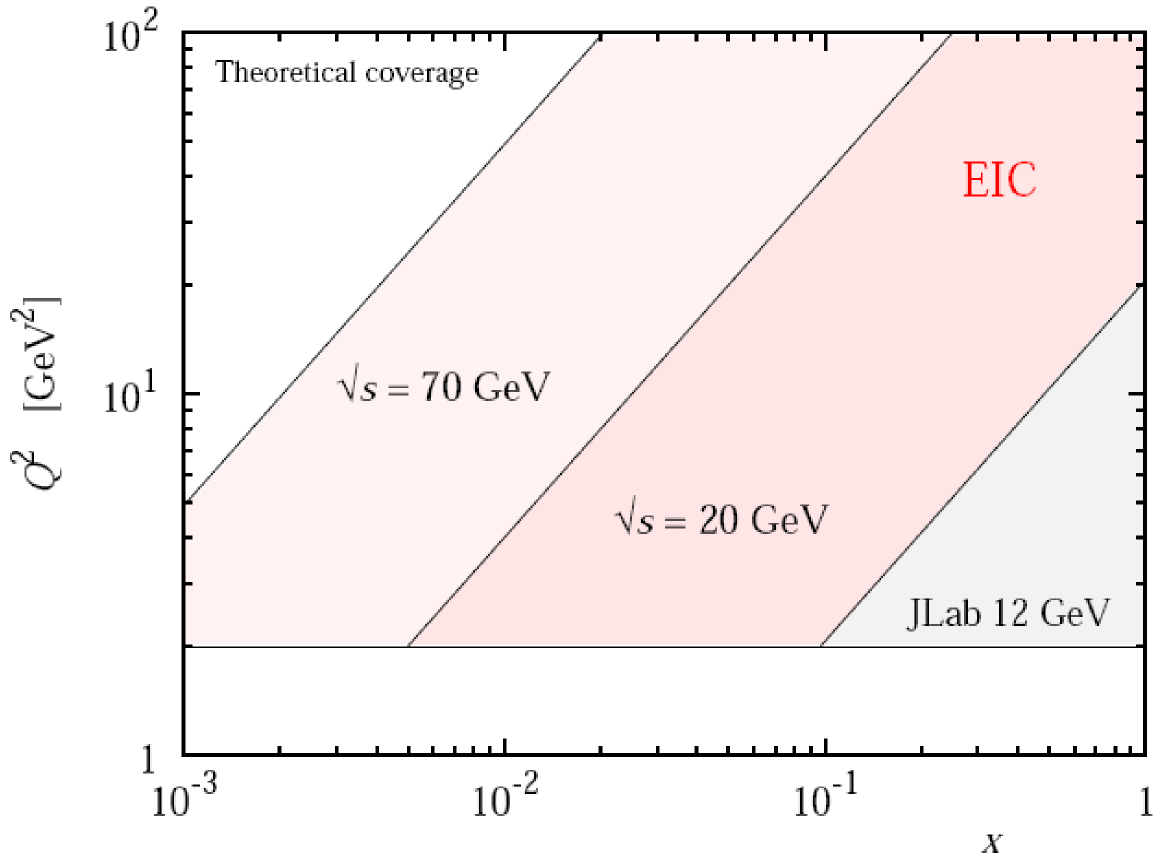
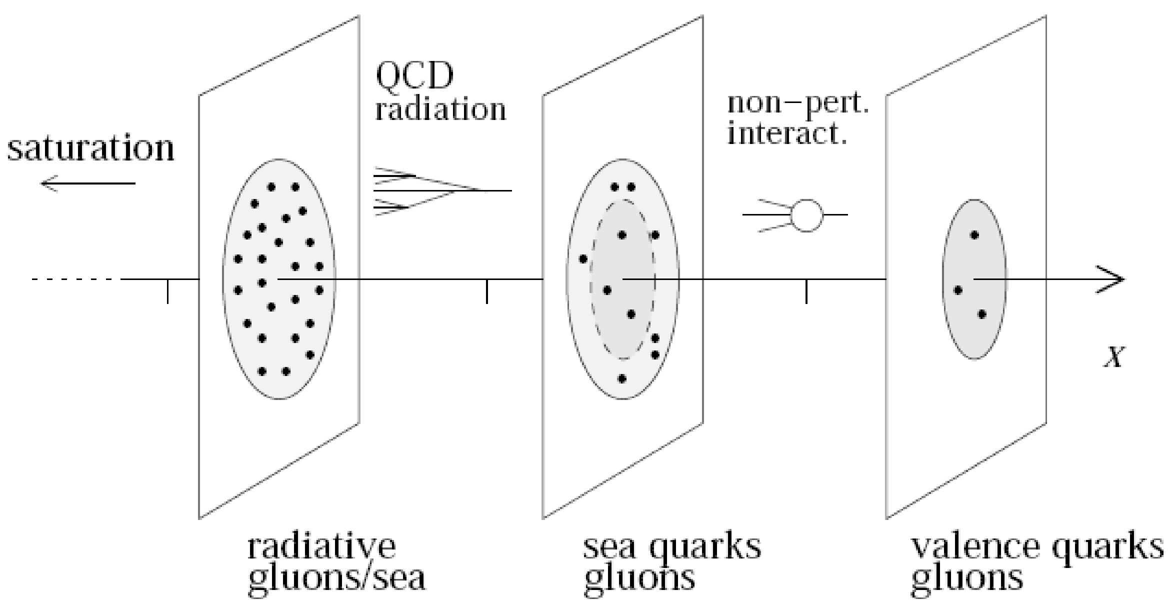
Transversity Distribution and Collins Fragmentation Functions with QCD Evolution

Alexei Prokudin

PENNSTATE Berks 1855

Nucleon landscape





Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How **partons move** and how they are distributed in **space** is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

These distributions are also referred to as **3D (three-dimensional) distributions**

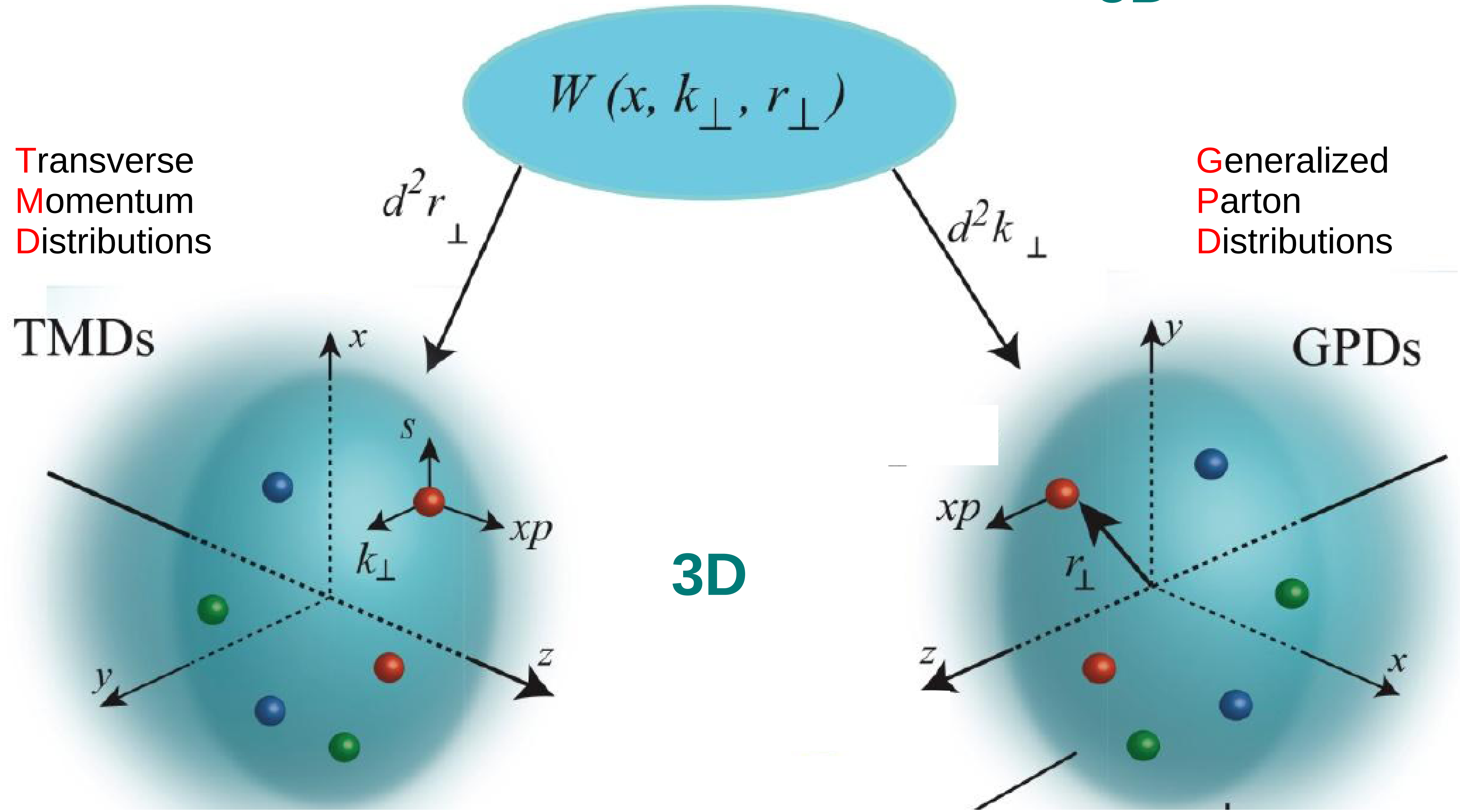
Unified View of Nucleon Structure

Wigner distribution

5D

Transverse
Momentum
Distributions

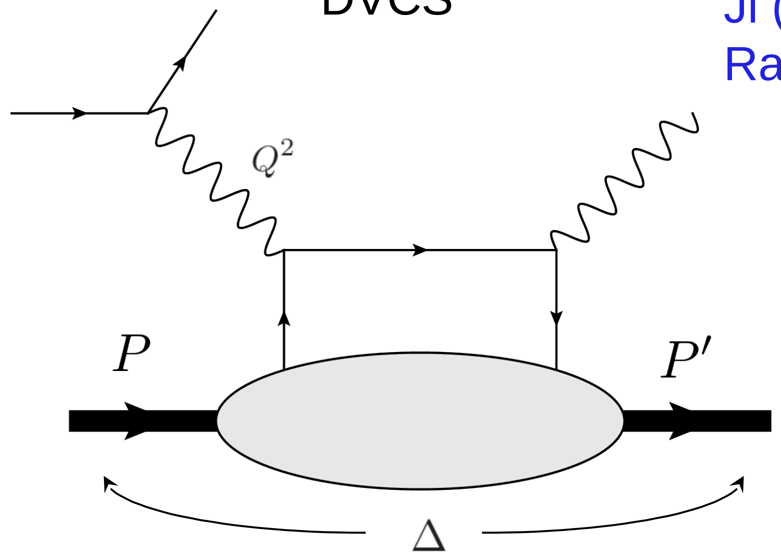
Generalized
Parton
Distributions



GPDs   TMDs

DVCS

Ji (1997)
Radyushkin (1997)



Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

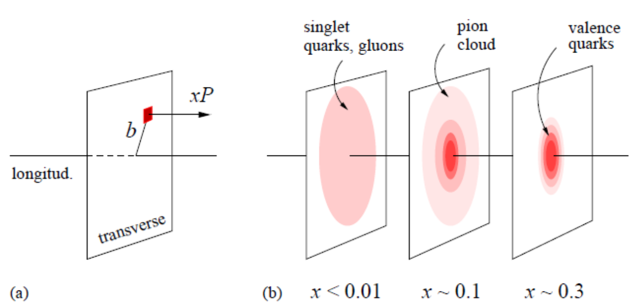
Connection to 3D structure

Burkardt (2000)
Burkardt (2003)

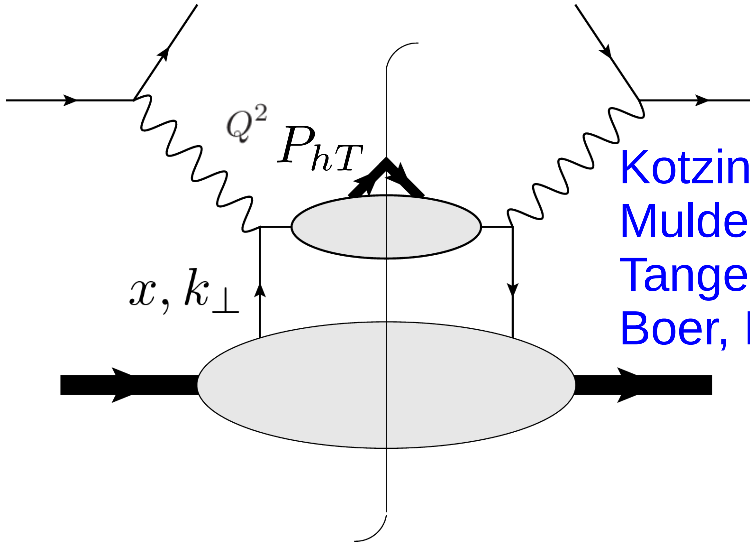
$$\rho(x, \vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

Drell-Yan frame $\Delta^+ = 0$

Weiss (2009)



SIDIS



Kotzinian (1995),
Mulders,
Tangerman (1995),
Boer, Mulders (1998)

Q^2 ensures hard scale, pointlike interaction

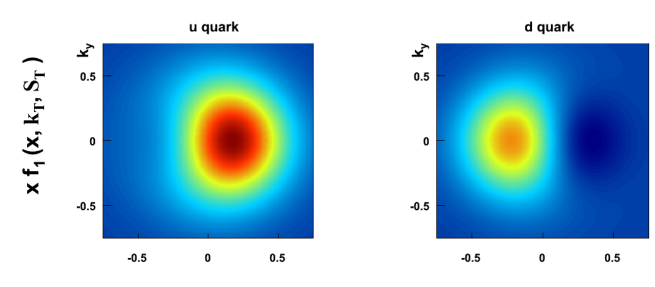
P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure

Ji, Ma, Yuan (2004)
Collins (2011)

$$\tilde{f}(x, \vec{b}_T) = \int d^2k_\perp e^{i\vec{b}_T \cdot \vec{k}_\perp} f(x, \vec{k}_\perp)$$

$\vec{b}_T$ is the transverse separation of parton fields in configuration space



AP (2012)

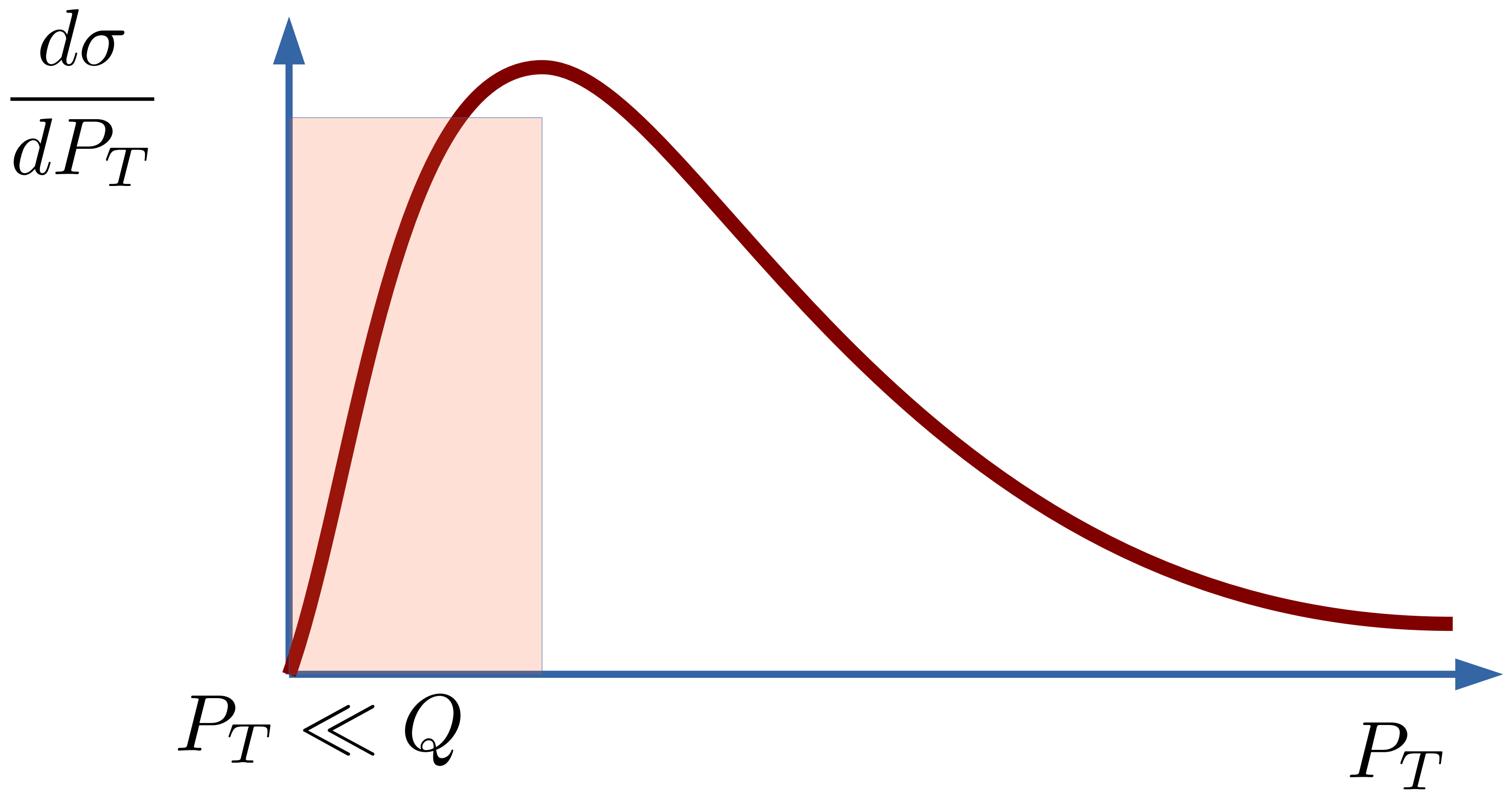
TMD

TMD factorization has a validity region
(two scale problem)

$P_T \ll Q$

$$f(x, k_\perp; Q^2)$$

Access to intrinsic motion



Mulders, Tangerman 1995
Boer, Mulders 1998

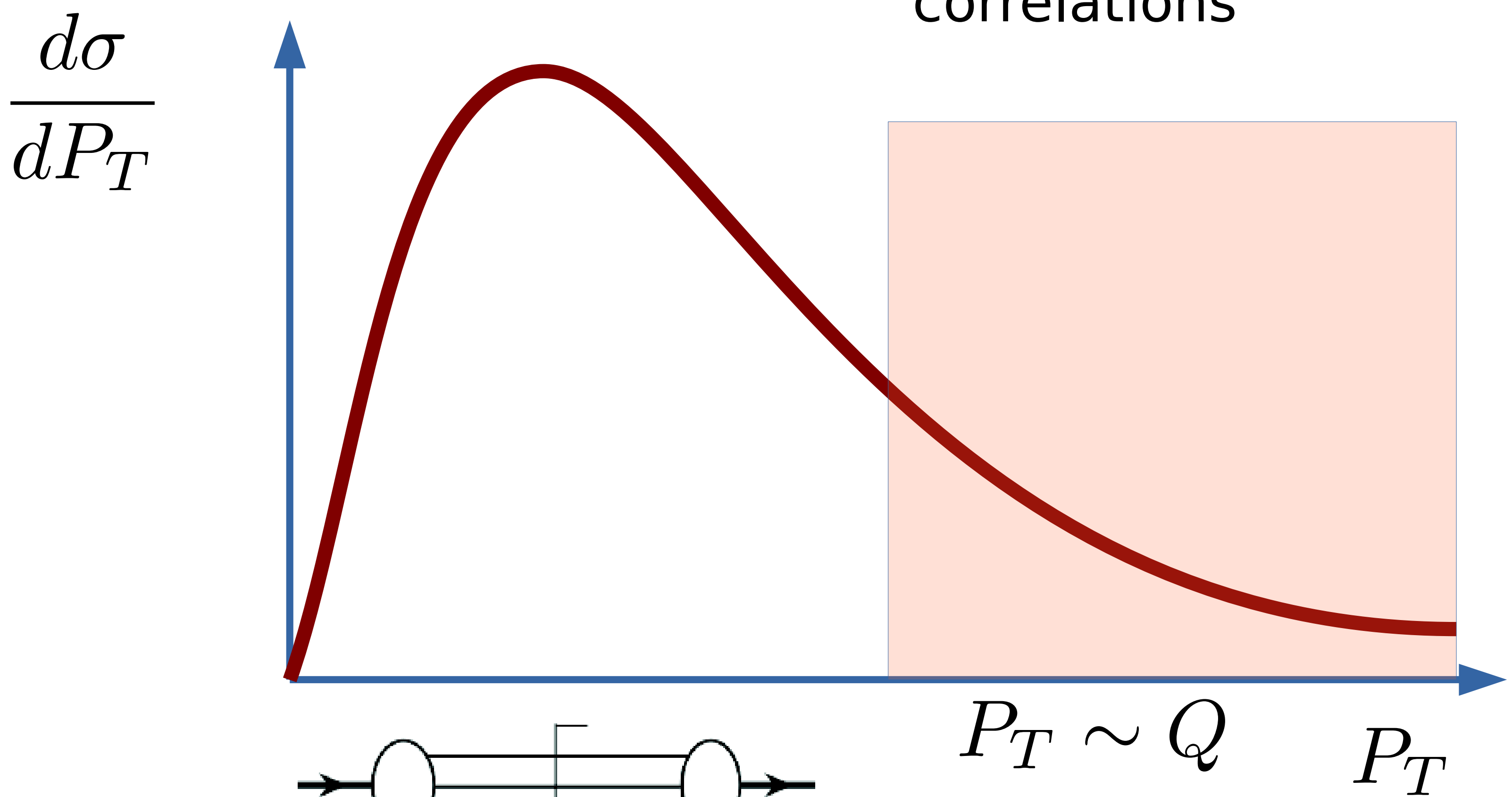
Collinear

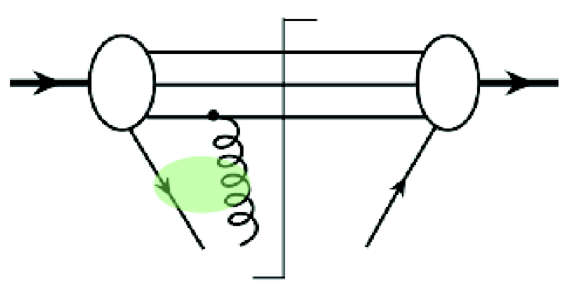
Collinear description will be appropriate in the regime $P_T \sim Q$
(one scale problem)

$$f(x;Q^2)$$

Beyond leading twist:
Twist-3 matrix elements
Efremov Teryaev (1982), Qiu, Sterman (1991)

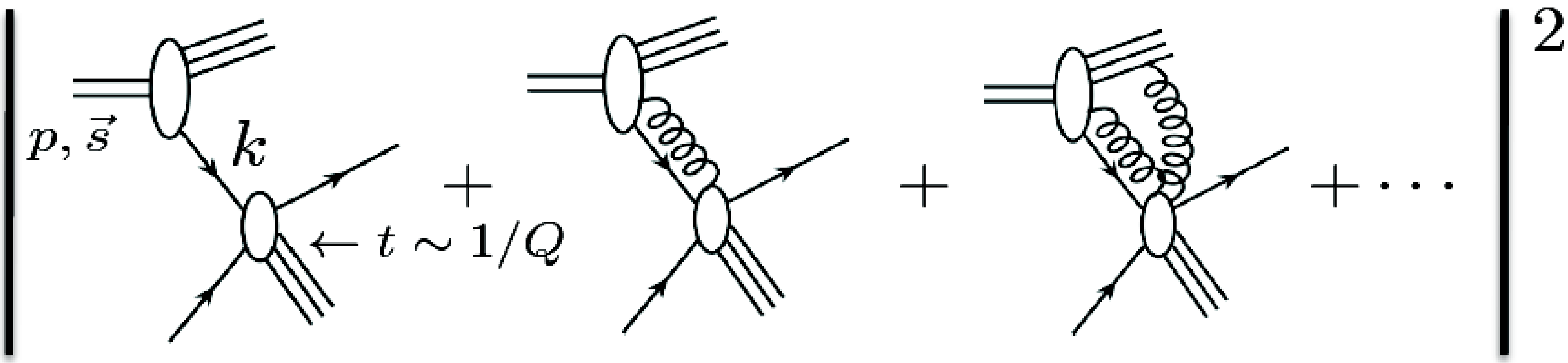
Quantum mechanical multi parton correlations



$T^{(3)}(x, x, S_\perp) \propto$ 

Better understanding of QCD

$\sigma(Q, \vec{s}) \propto$



Multy parton correlations contribute to the cross section.

These correlations are called
Efremov-Teryaev-Qiu-Sterman matrix elements,
They appear at twist-3 level in cross section.

$$\sigma = \sigma^{LT} + \frac{Q_s}{Q}\sigma^{HT} + \ldots$$

$$= H^{LT} \otimes f_2 \otimes f_2 + \frac{Q_s}{Q} H^{HT} \otimes f_3 \otimes f_2 + \ldots$$

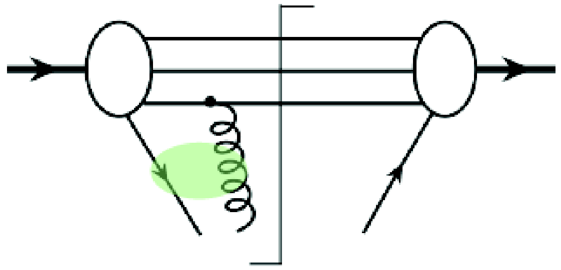
If only one large scale is present in the process, then

$$\begin{aligned} A_N &\propto \sigma(p_T, S_\perp) - \sigma(p_T, -S_\perp) \\ &\propto T^{(3)}(x, x, S_\perp) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x, S_\perp) \otimes \hat{\sigma}_D \otimes D^{(3)}(z, z) + \ldots \end{aligned}$$

Leading power cancels

Twist-3 parton correlation functions

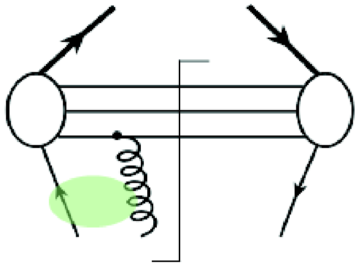
$$T^{(3)}(x, x, S_\perp) \propto$$



Qiu-Sterman 1991

Twist-3 parton fragmentation functions

$$D^{(3)}(z, z) \propto$$



Kang, Yuan, Zhou 2010

Kanazawa, Koike, Metz, Pitonyak (2015) - Towards explanation of SSA asymmetries

No probability interpretation!

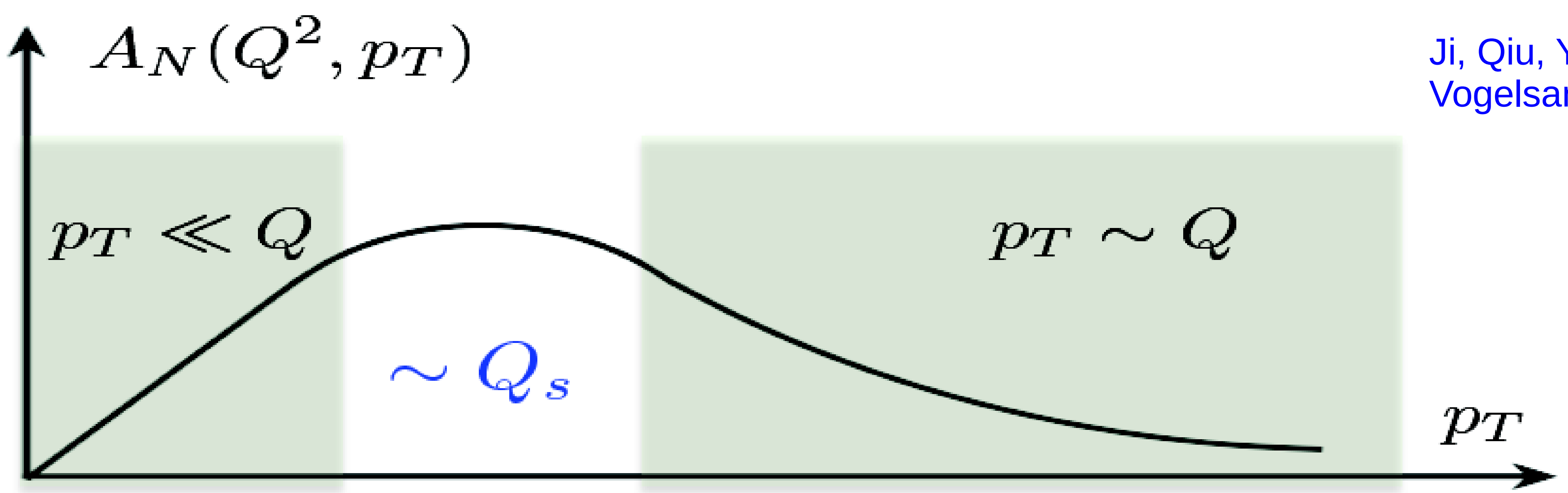
Collinear vs TMD factorization

We can consider two different kinematical regions

$$Q_1, Q_2, \ldots \gg \Lambda_{QCD}$$ Collinear

$$Q_1 \gg Q_2 > \Lambda_{QCD}$$ TMD

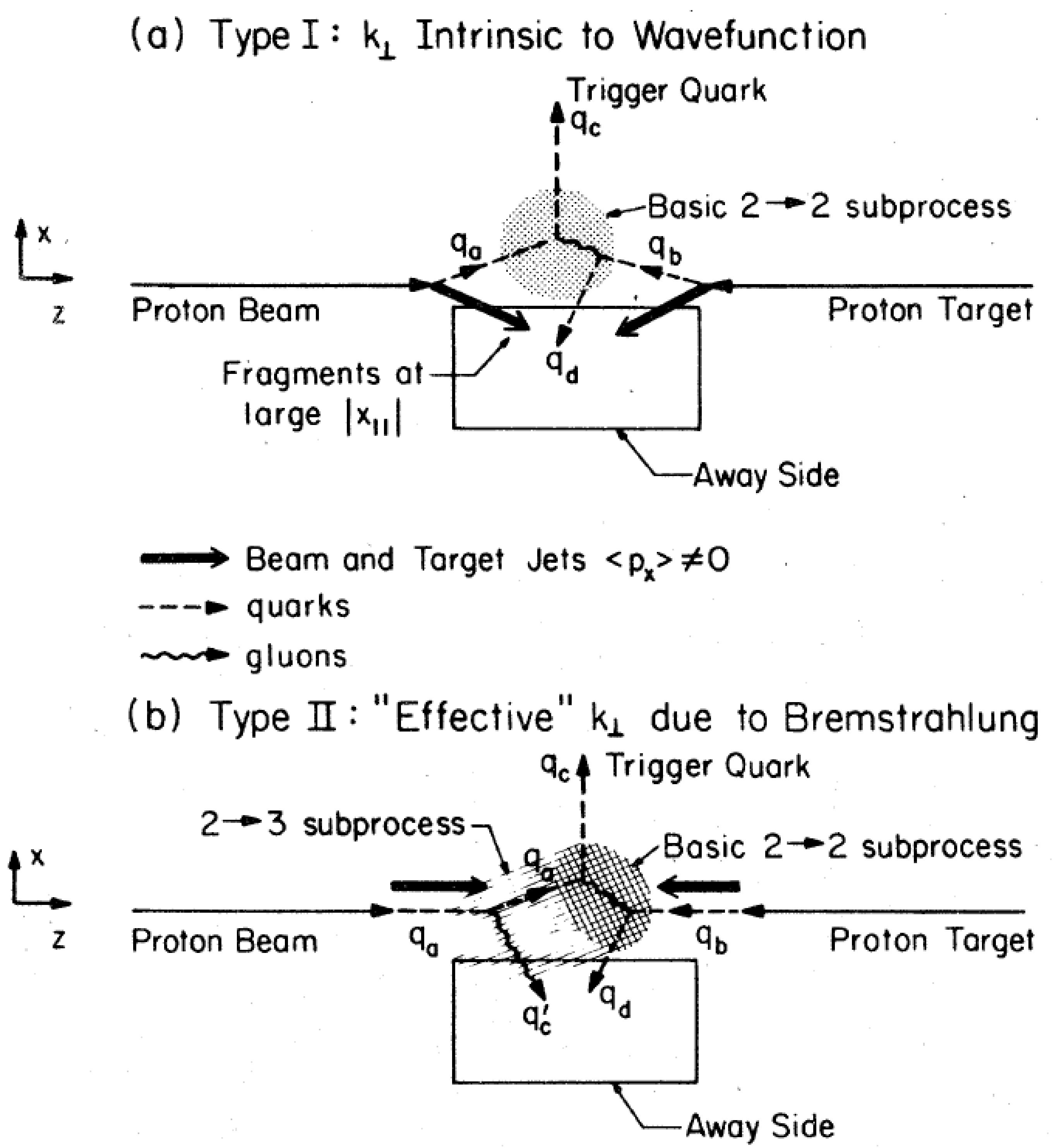
- Twist-3 – integration over parton momenta
- TMD – direct information on partonic transverse motion



Ji, Qiu, Yuan, Vogelsang, Koike

Consistent in the overlap region!

Precursor of TMDs

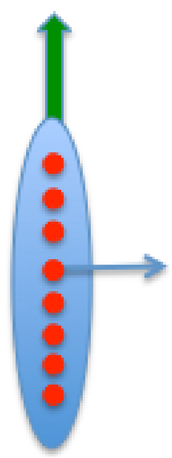


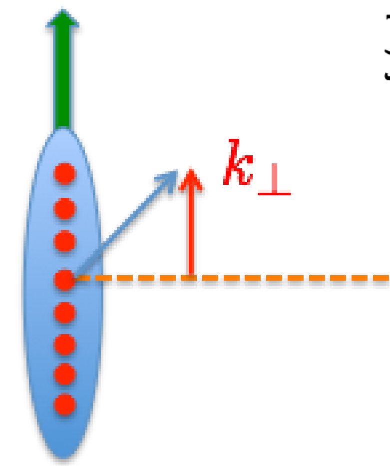
Feynman, Field, Fox (1978)

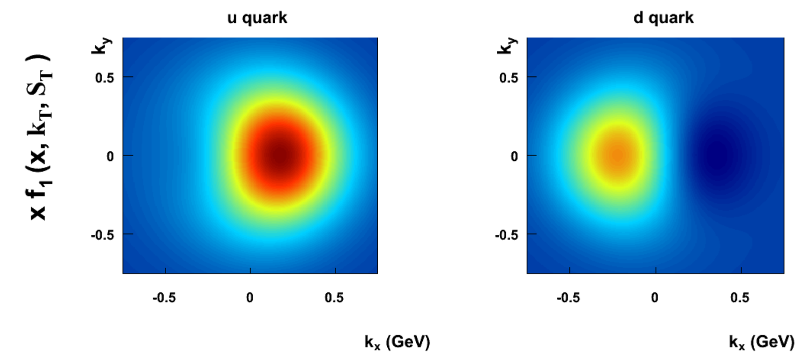
Final transverse momentum is generated either by intrinsic transverse motion or by gluon radiation

From 1-D to 3-D

Hadron structure: one-dimensional picture to three-dimensional tomography







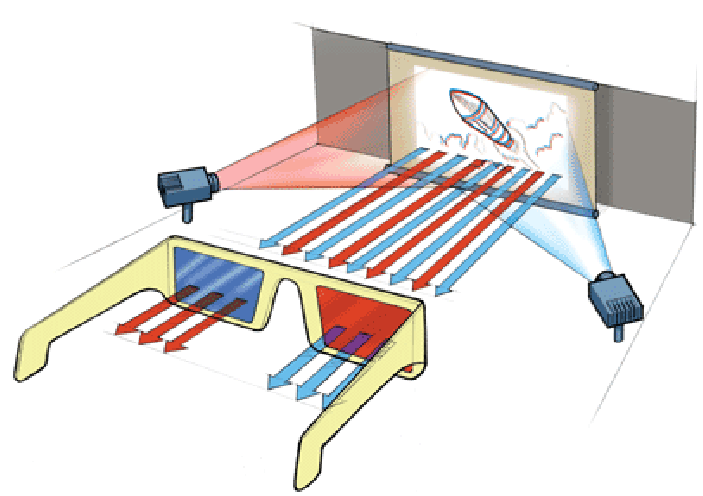
$f(x)$

Collinear PDFs

$f(x, k_\perp)$

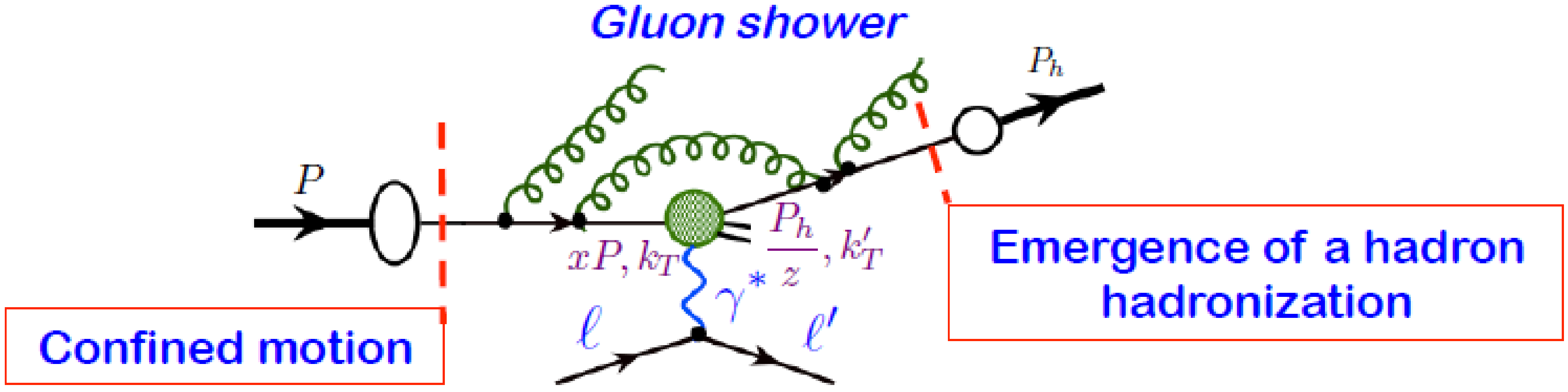
Transverse momentum distributions (TMDs)

Very interesting and non-trivial consequences:
rich QCD dynamics and new insight on hadron structure



Why QCD evolution is interesting?

Study of evolution gives us insight on different aspects and origin
of confined motion of partons, gluon radiation, parton fragmentation



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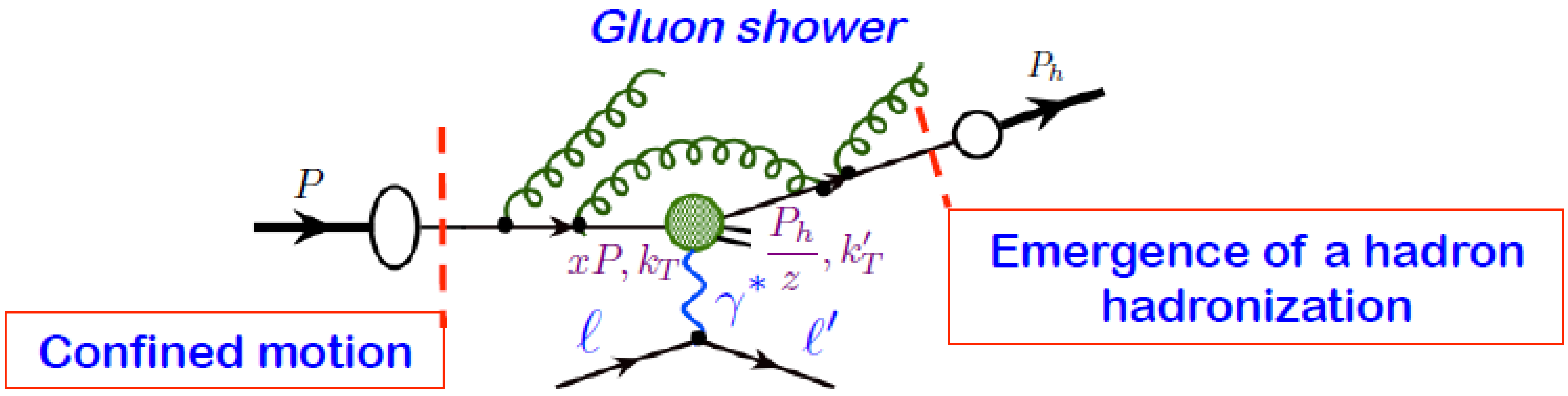


Evolution allows to connect measurements at very different scales

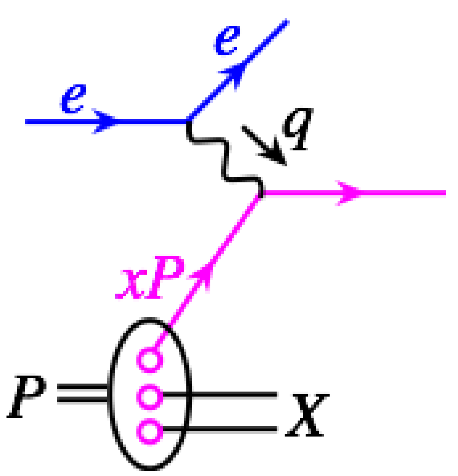
→ Note that DGLAP evolution gives definite predictions that depend only on initial conditions
TMD evolution is more complex and include non perturbative functions, thus results of application of TMD evolution will strongly depend on the choice of non perturbative input

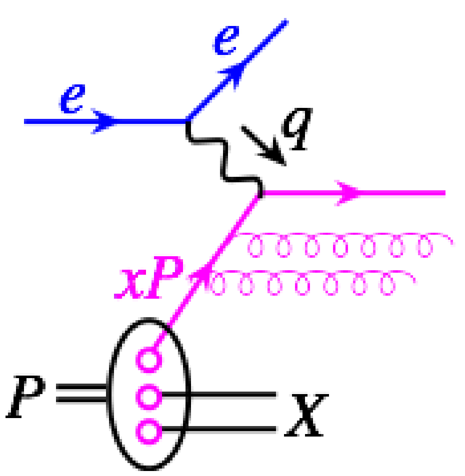
What do we mean by QCD evolution?

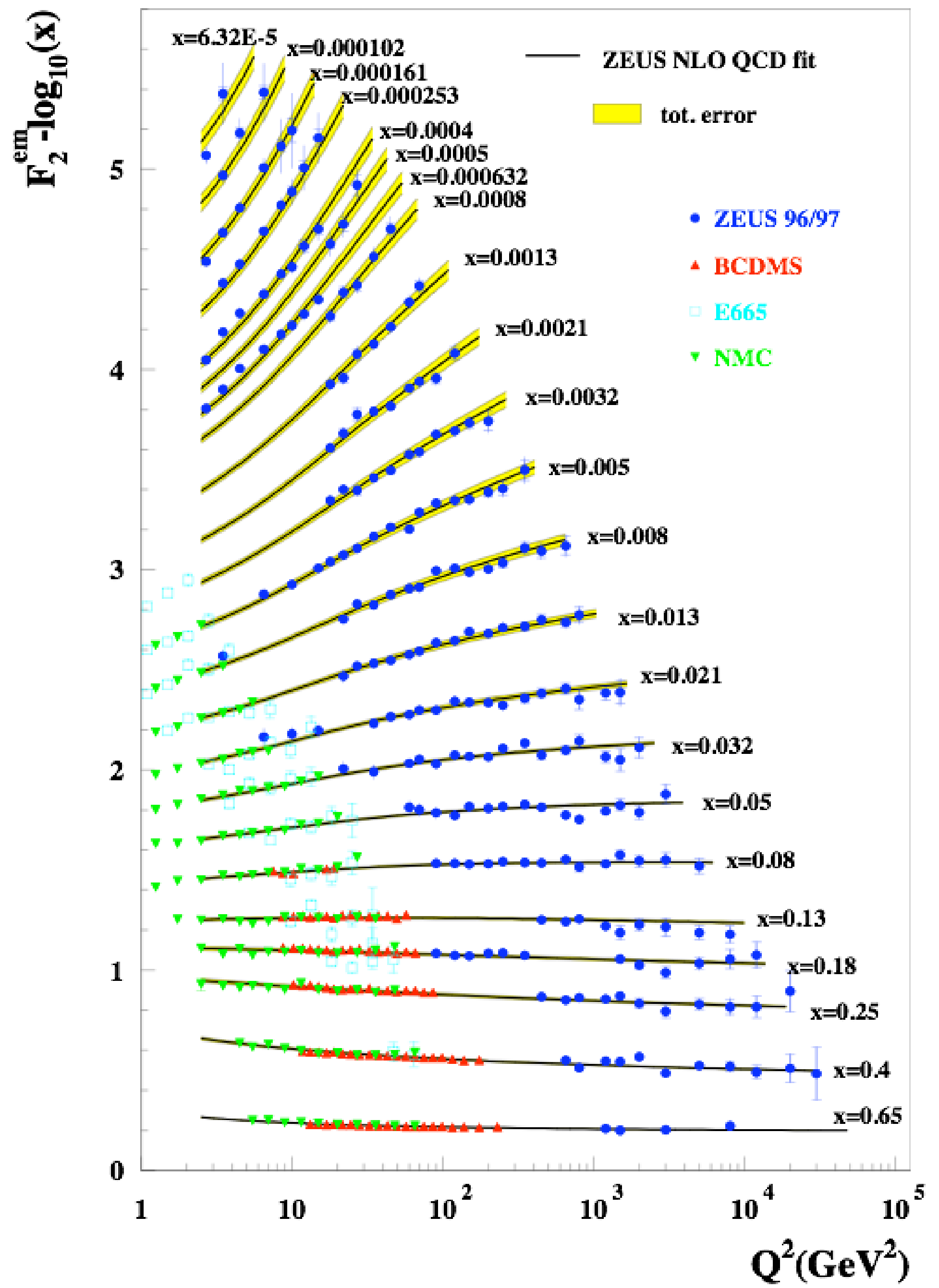
Very well known example:
DGLAP evolution of collinear parton distributions

Take into account perturbative corrections

Single logarithms are resummed order by order in perturbative calculations







$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$

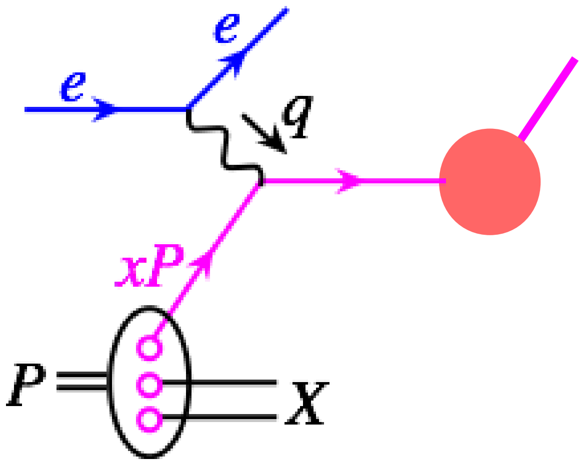
What do we mean by QCD evolution?

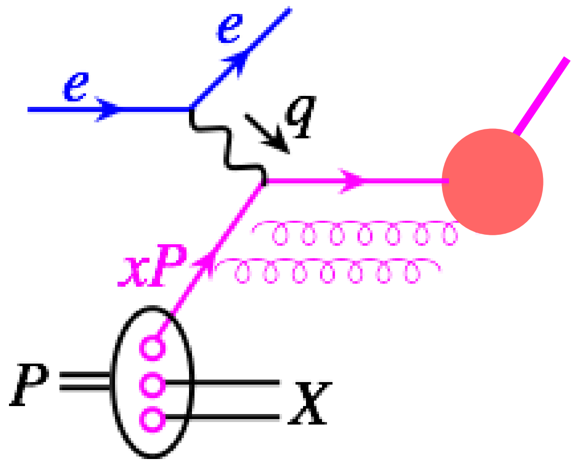
TMD factorization is applicable in case two different scales are observed
in processes such as SIDIS, Drell-Yan, W/Z production in hadron-hadron collisions.
Kinematical regime: $Q_T \ll Q$

For SIDIS Q_T is transverse momentum of final hadron

Again we need to take into account perturbative corrections

Double logarithms are resummed order by order in perturbative calculations

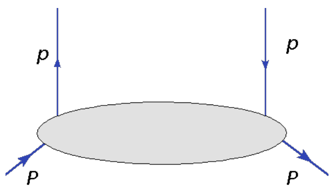




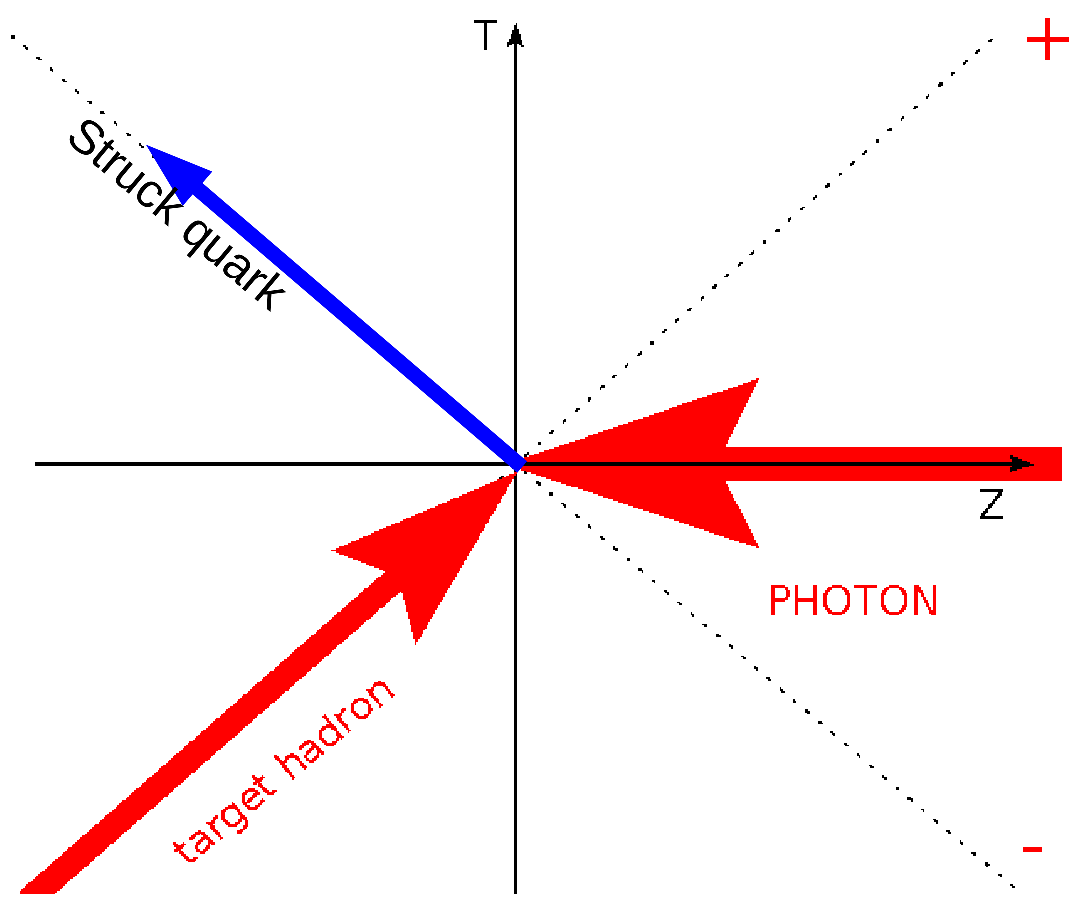
$$\left(\alpha_s \ln^2 \frac{Q^2}{Q_T^2}\right)^n$$

Transverse Momentum Dependent distributions

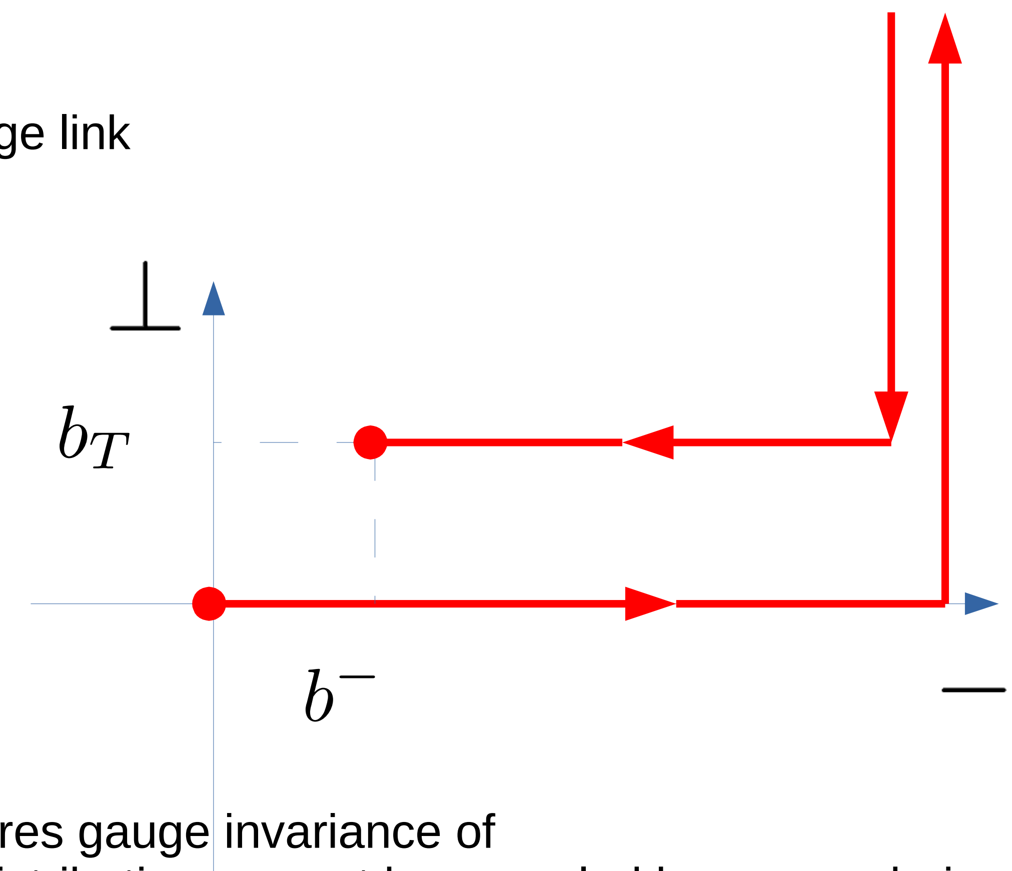
$$\Phi_{ij}(x,\mathbf{k}_\perp) = \int \frac{db^-}{(2\pi)} \frac{d^2 b_T}{(2\pi)^2}\, e^{ixP^+b^- - i\mathbf{k}_\perp \mathbf{b}_T} \langle P,S_P|\bar{\psi}_j(0)\mathcal{U}(\mathbf{0},\mathbf{b})\psi_i(b)|P,S_P\rangle|_{b^+=0}$$



SIDIS in IMF:



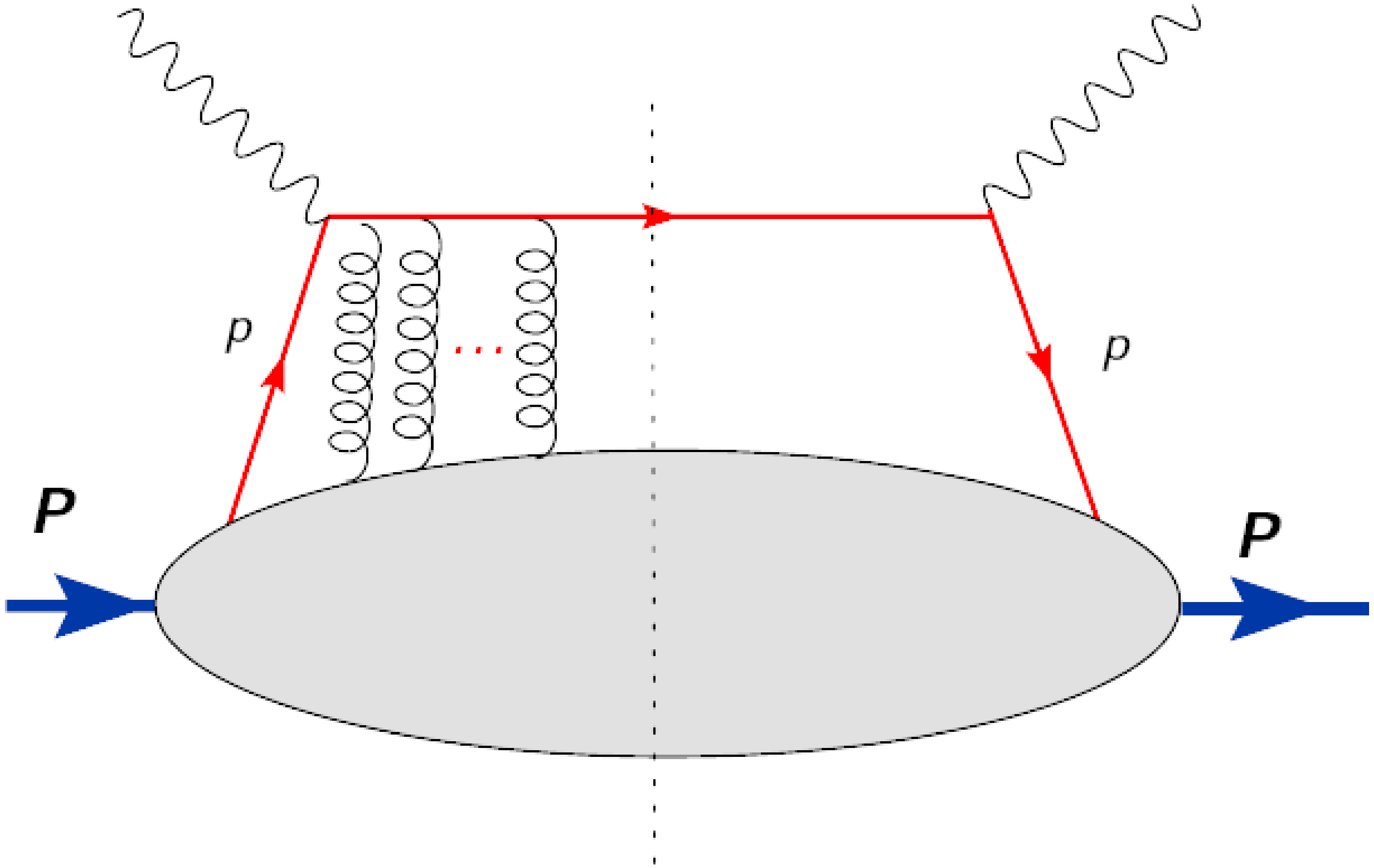
Gauge link



Ensures gauge invariance of
the distribution, cannot be canceled by gauge choice

$$\mathcal{U}(a,b;n) = e^{-ig\int_a^b d\lambda n\cdot A_\alpha(\lambda n)t_\alpha}$$

Origin of the gauge link



N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD Fragmentation Functions

N \ q	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$H_{1T}^{\perp}$	G_{1T}	H_1 $H_{1T}^{\perp}$

8 functions describing fragmentation of a quark into spin ½ hadron

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

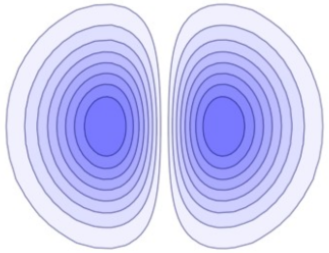
TMD distributions

Three types of modulations

$$f(x, \mathbf{k}_\perp^2)$$

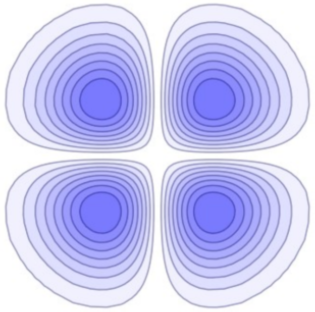
Monopole

$$\frac{\mathbf{k}_{\perp i} S_{Ti}}{M} f(x, \mathbf{k}_\perp^2)$$



Dipole

$$\frac{\mathbf{k}_\perp^i \mathbf{k}_\perp^j - \frac{1}{2}\mathbf{k}_\perp^2 g_T^{ij}}{M^2} f(x, \mathbf{k}_\perp^2)$$



Quadrupole

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

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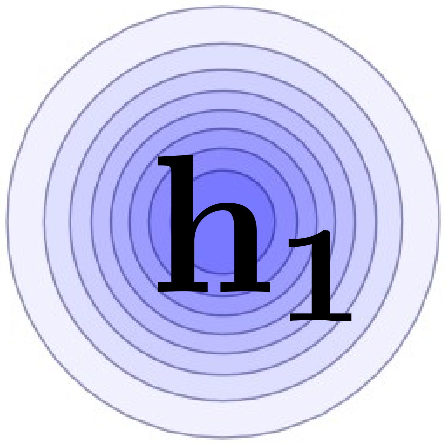
Transversity



TMD distributions

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

Transversity

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of
3 Parton Distributions Functions (PDF) in collinear (kt integrated) case

$$\Phi(x;P,S) = \frac{1}{2}\left\{ f_1(x)\ \not{P} + S_L g_1(x)\gamma_5\ \not{P} + \frac{1}{2} h_1(x)\gamma_5[\not{S}_T,\not{P}]\right\}$$

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Unpolarised PDF

Twist-2 collinear PDFs

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Unpolarised PDF

Helicity distribution

Twist-2 collinear PDFs

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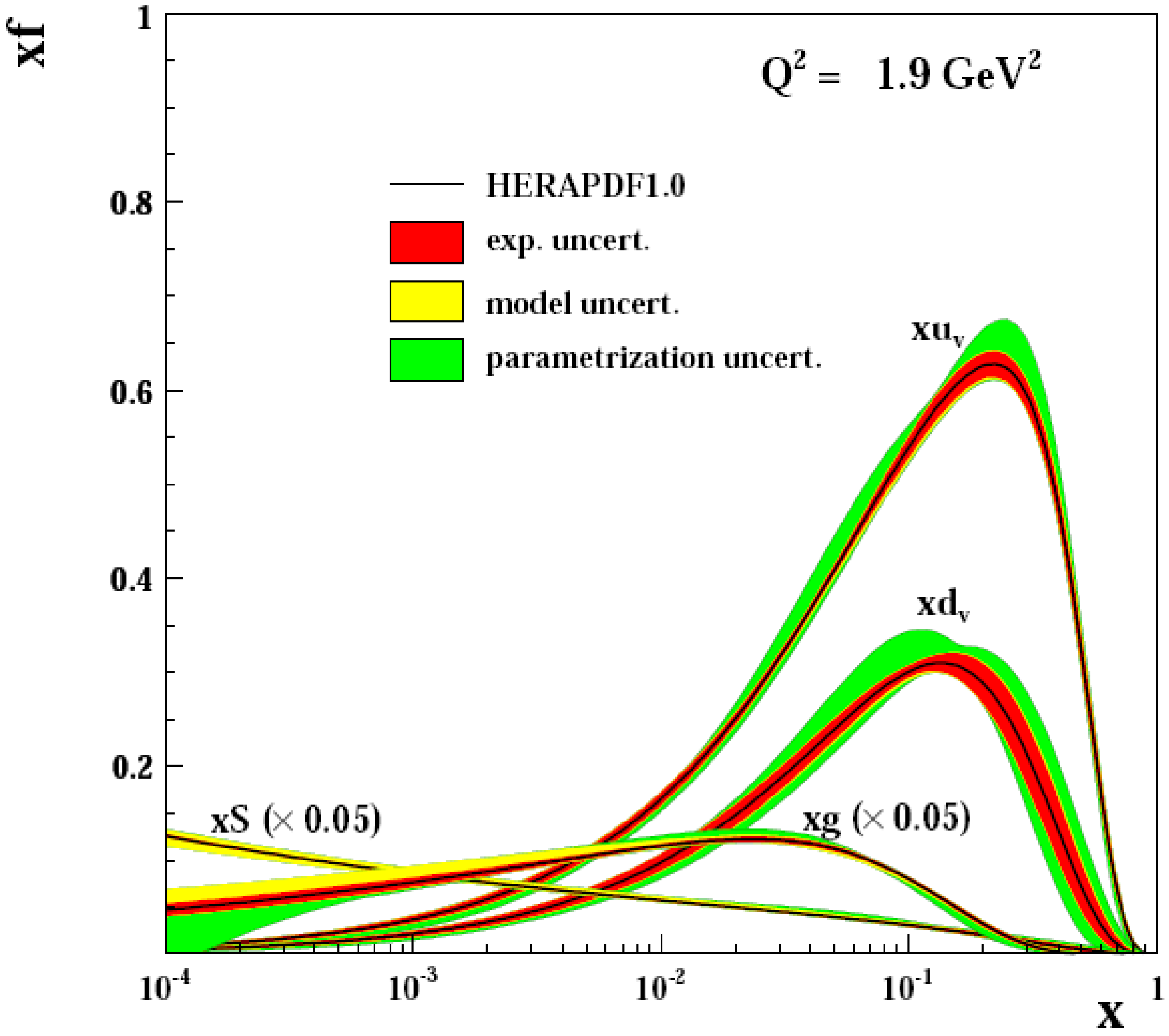
Unpolarised PDF

Helicity distribution

Transversity distribution

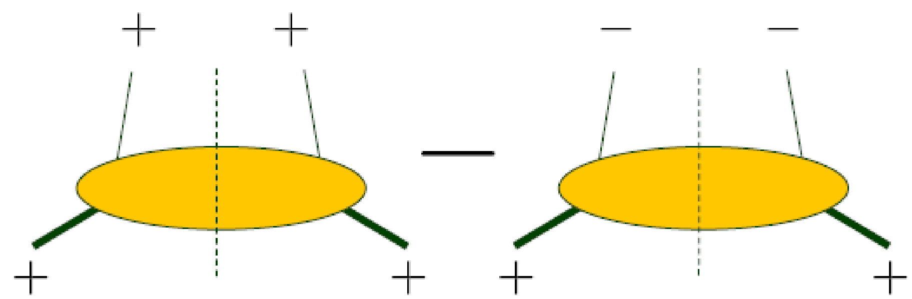
Unpolarised PDFs

Good knowledge of unpolarised
Parton
Distribution
Functions
is acquired

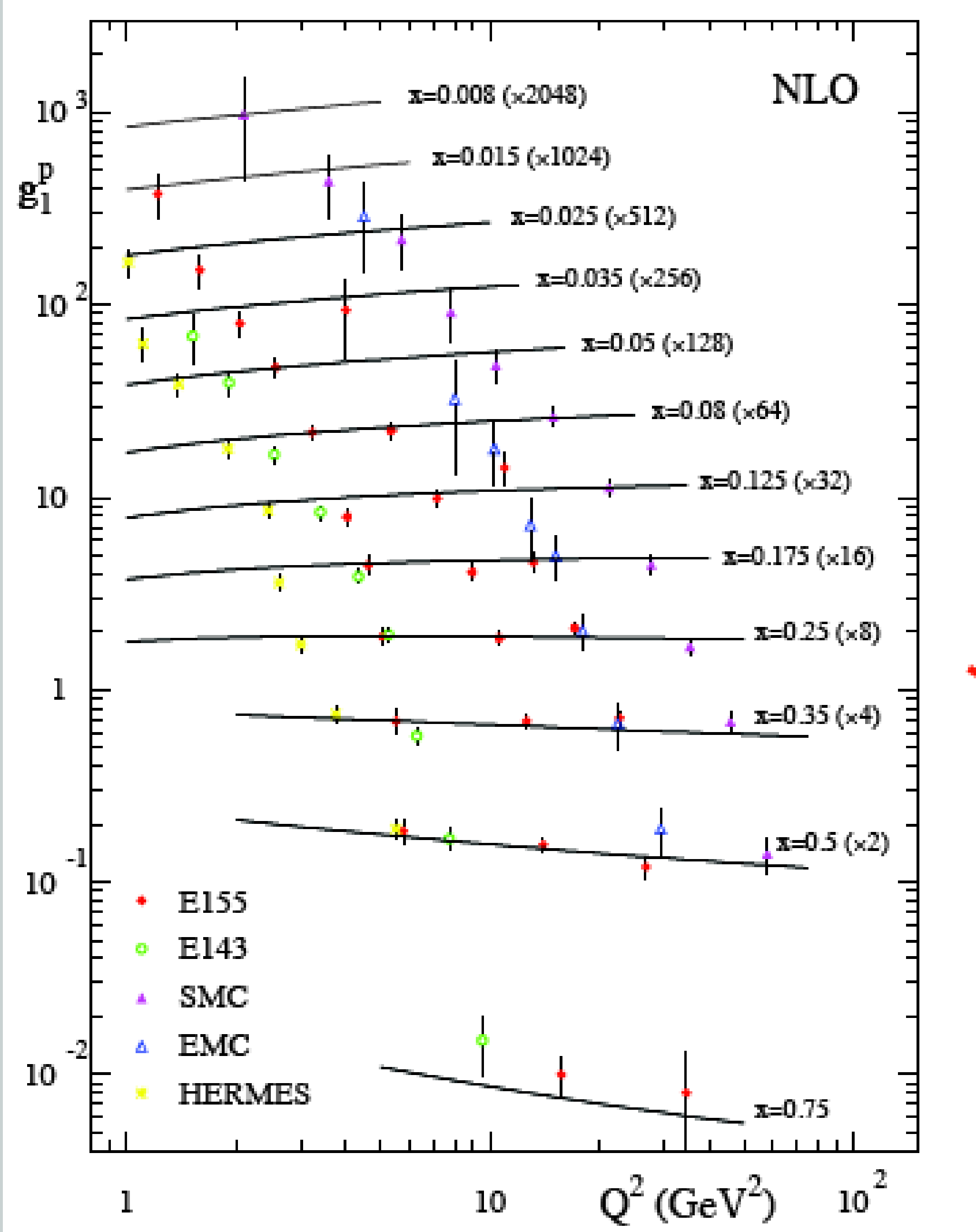


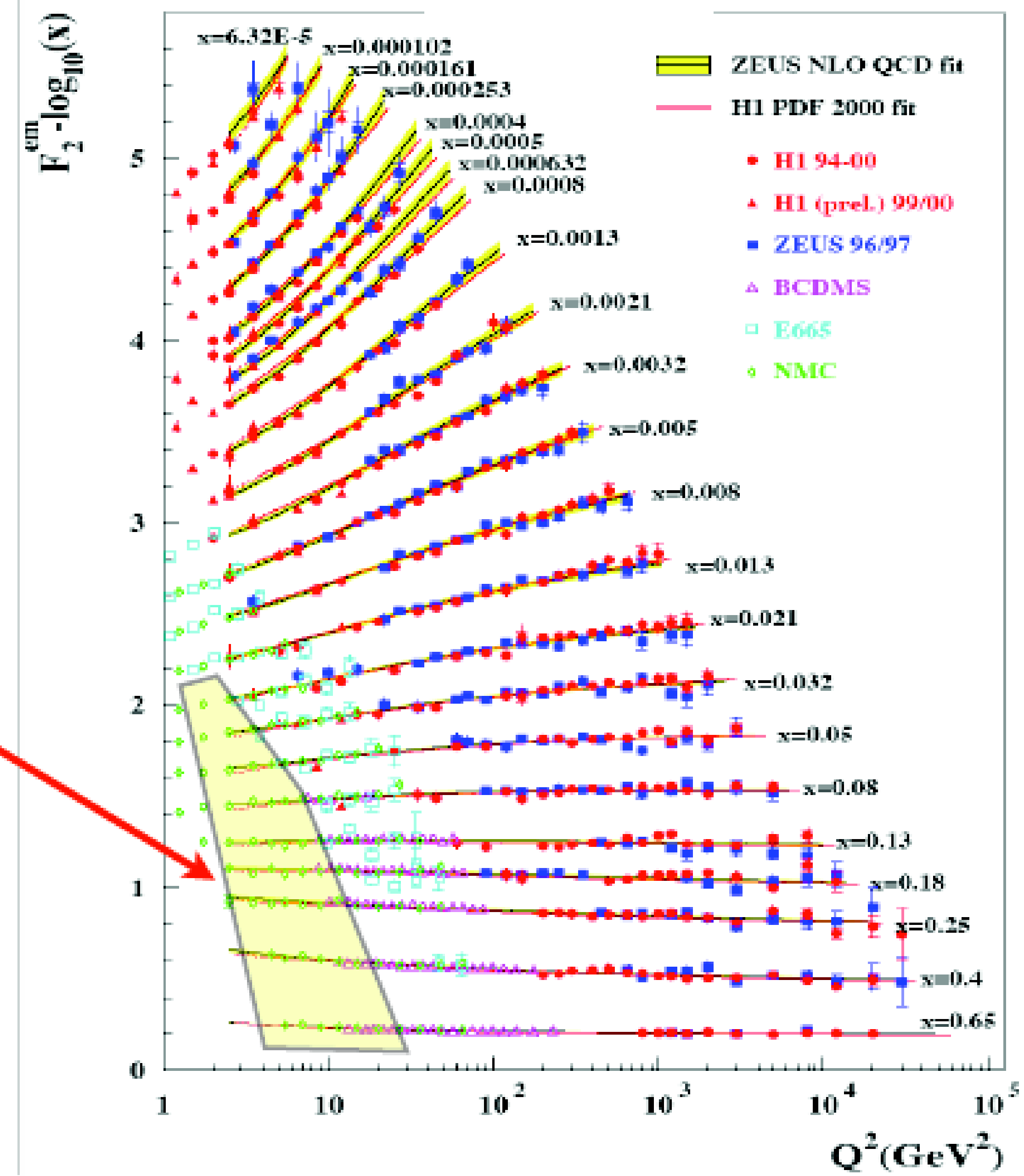
Helicity distributions

Helicity distributions
are relatively well known



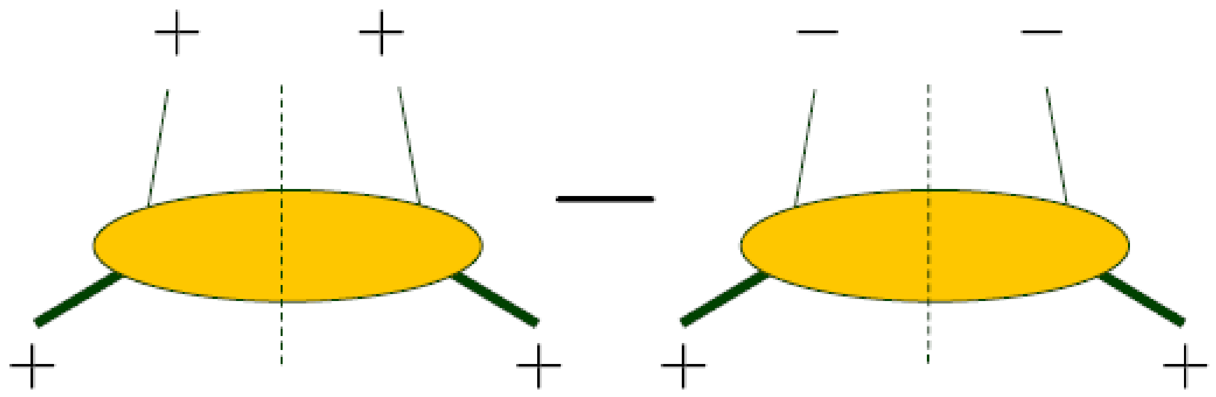
Data from RHIC
is fundamental for
helicity distributions



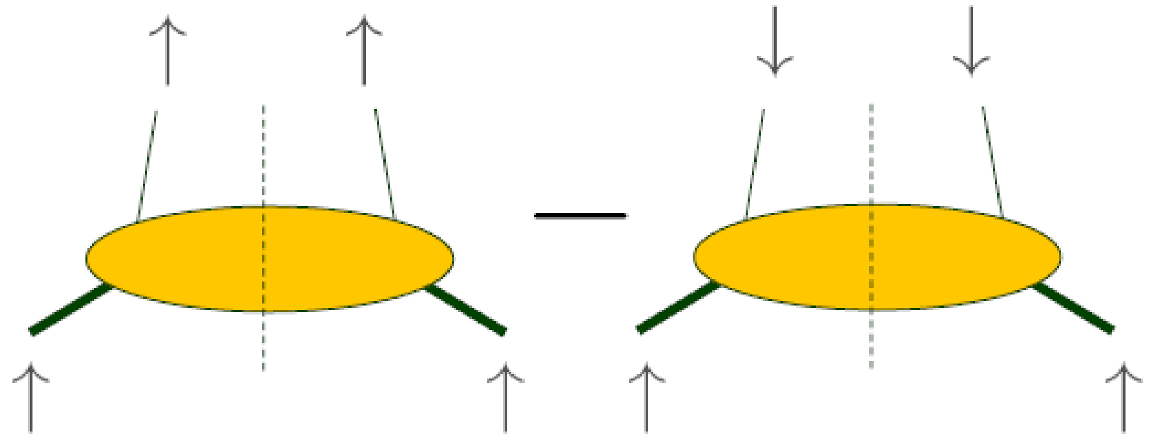


h_1

Helicity distribution



Transversity distribution

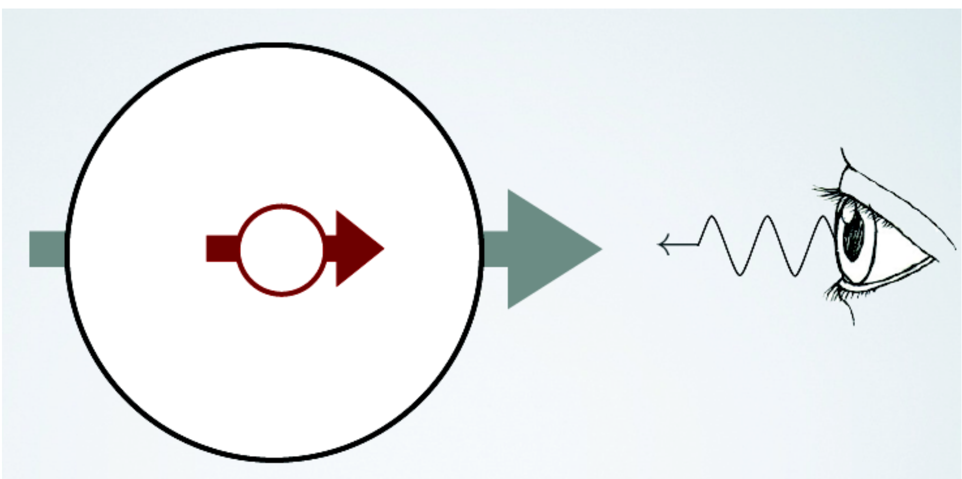


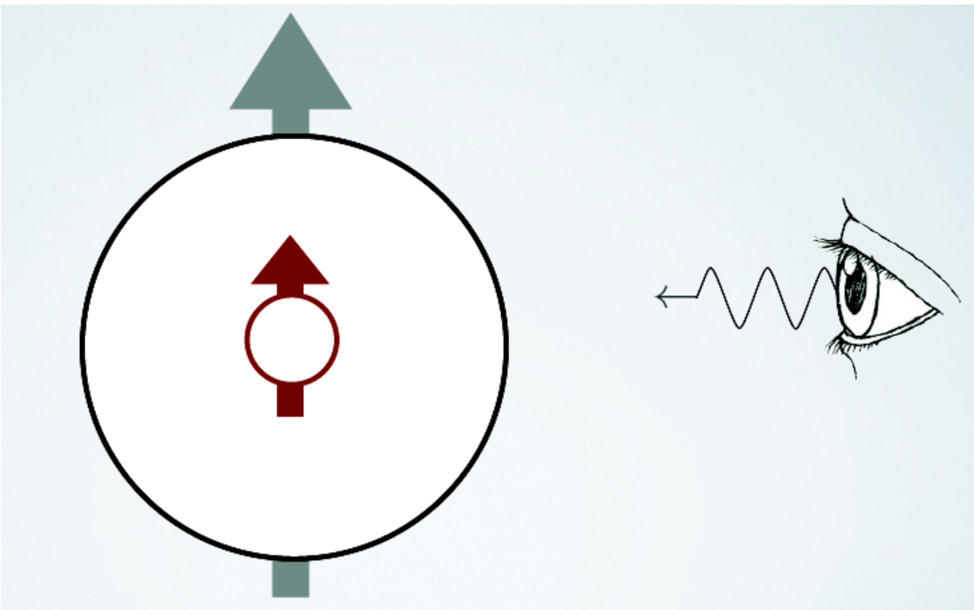
Distribution of transversely polarised quarks inside transversely polarised nucleon

h_1

Helicity distribution

Transversity distribution



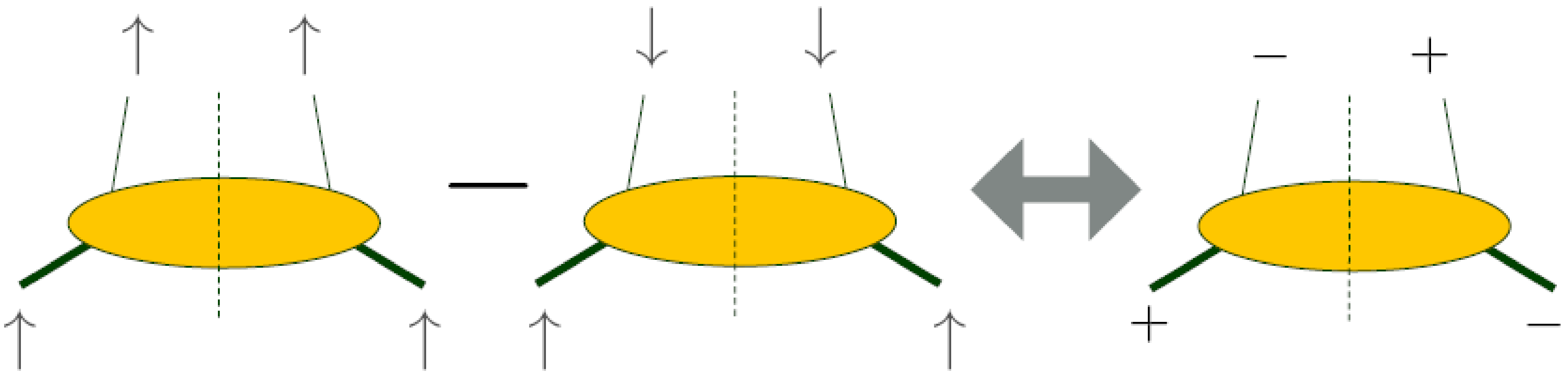


Boost and rotation do not commute → helicity and transversity are different!

Why difficult to measure transversity distribution?

Transversity in helicity basis

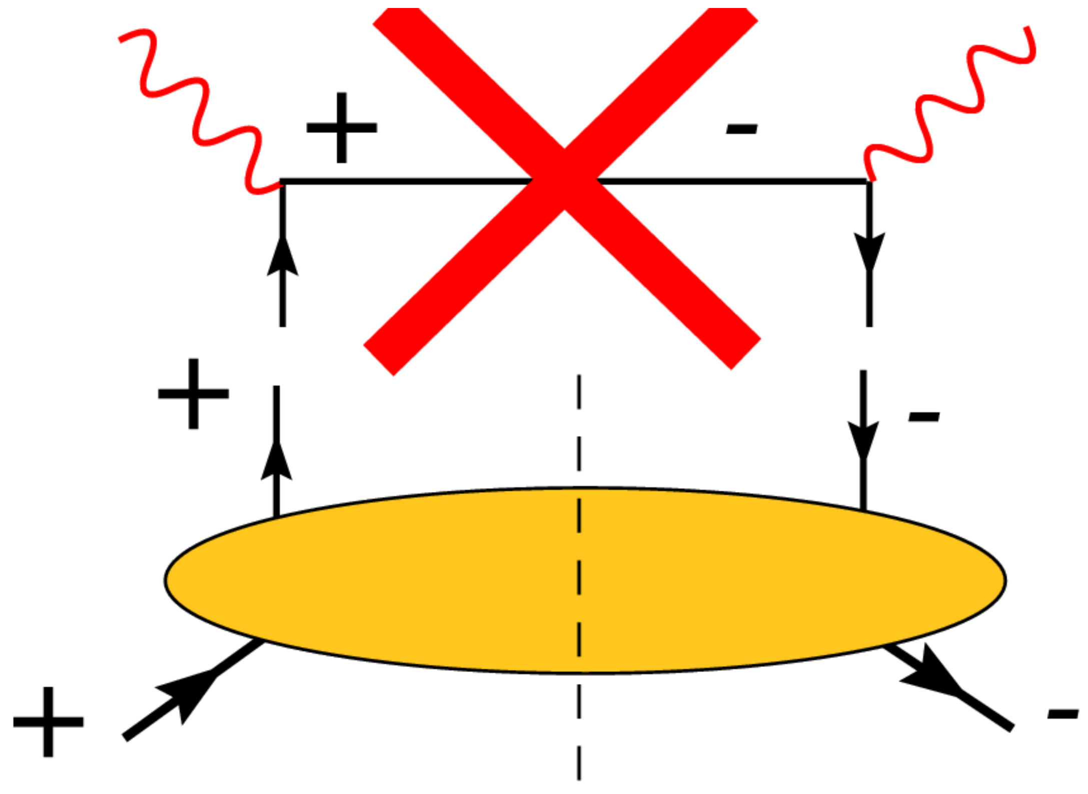
$$|\uparrow,\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$$



Chiral Odd!

Transversity distribution

Chiral Odd: it cannot be measured in Deep Inelastic Scattering process

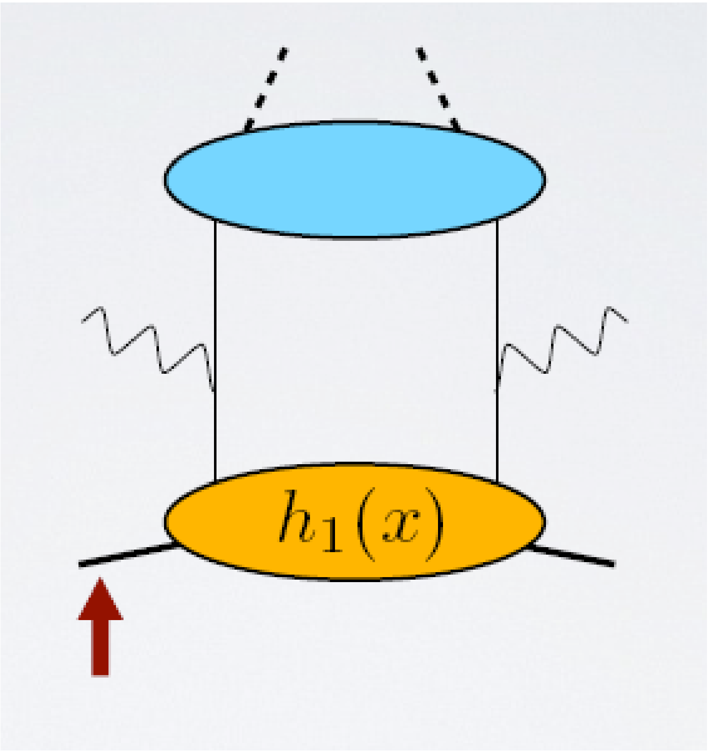


Needs another chiral odd function to be measured

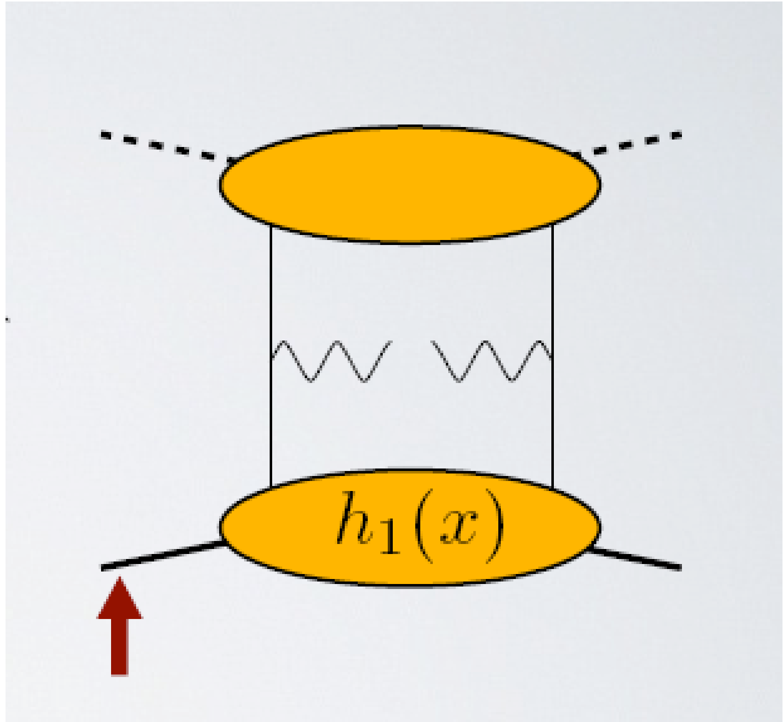
Transversity how to measure?

Transversity needs another chiral odd function to be measured

Semi Inclusive DIS (SIDIS)



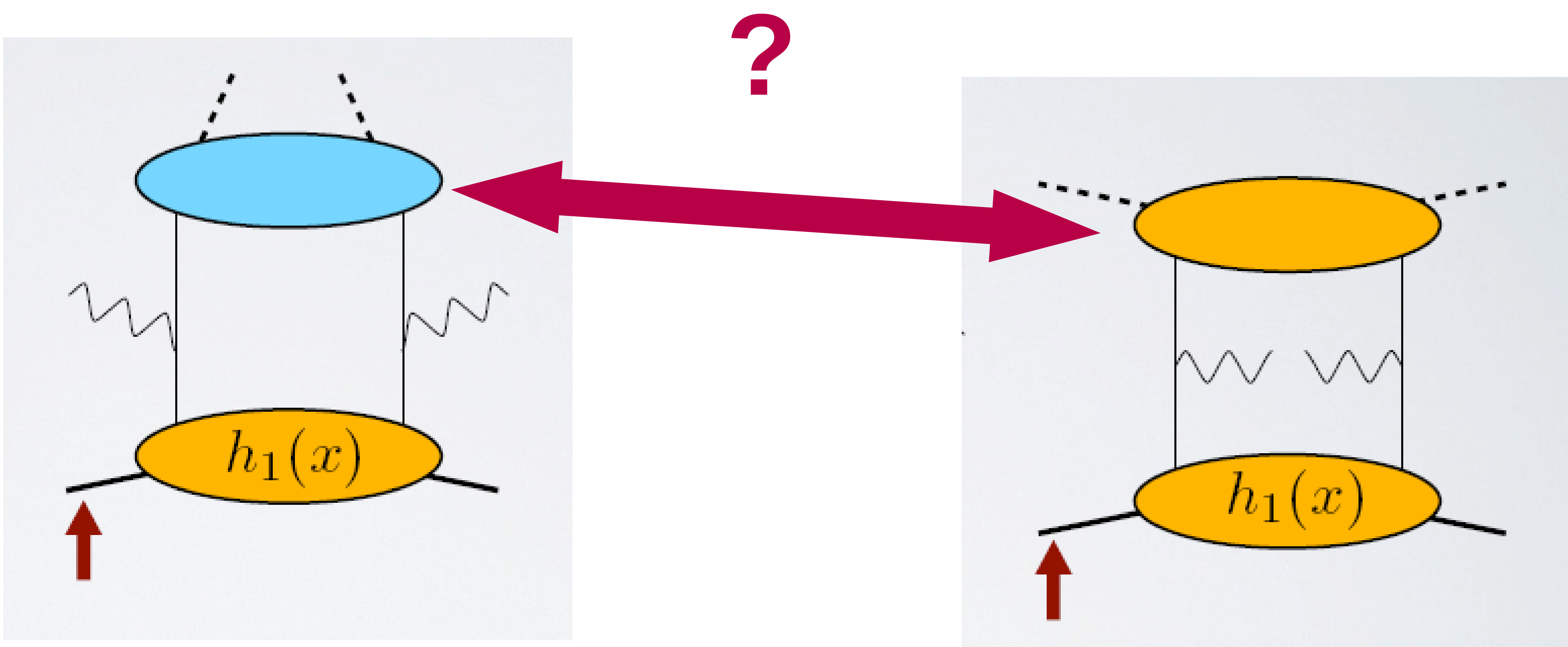
Drell-Yan



Transversity how to measure?

Transversity needs another chiral odd function to be measured

Semi Inclusive DIS (SIDIS) **Drell-Yan**



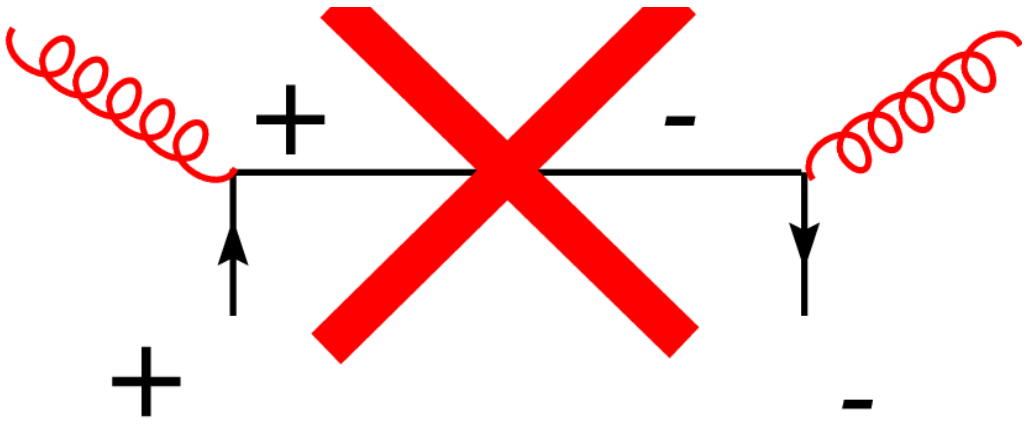
Collins fragmentation function

$$H_1^\perp(z, p_\perp)$$

Transversity or Boer-Mulders function

$$h_1(x) \qquad h_1^\perp(x, k_\perp)$$

QCD Evolution: no gluon contribution in the evolution



$h_1(x, Q^2)$ is suppressed at low x

JLab 12 is an ideal place to measure transversity $\rightarrow$ as JLab explores high x region

Tensor charge

Transversity is the only source of information on tensor charge

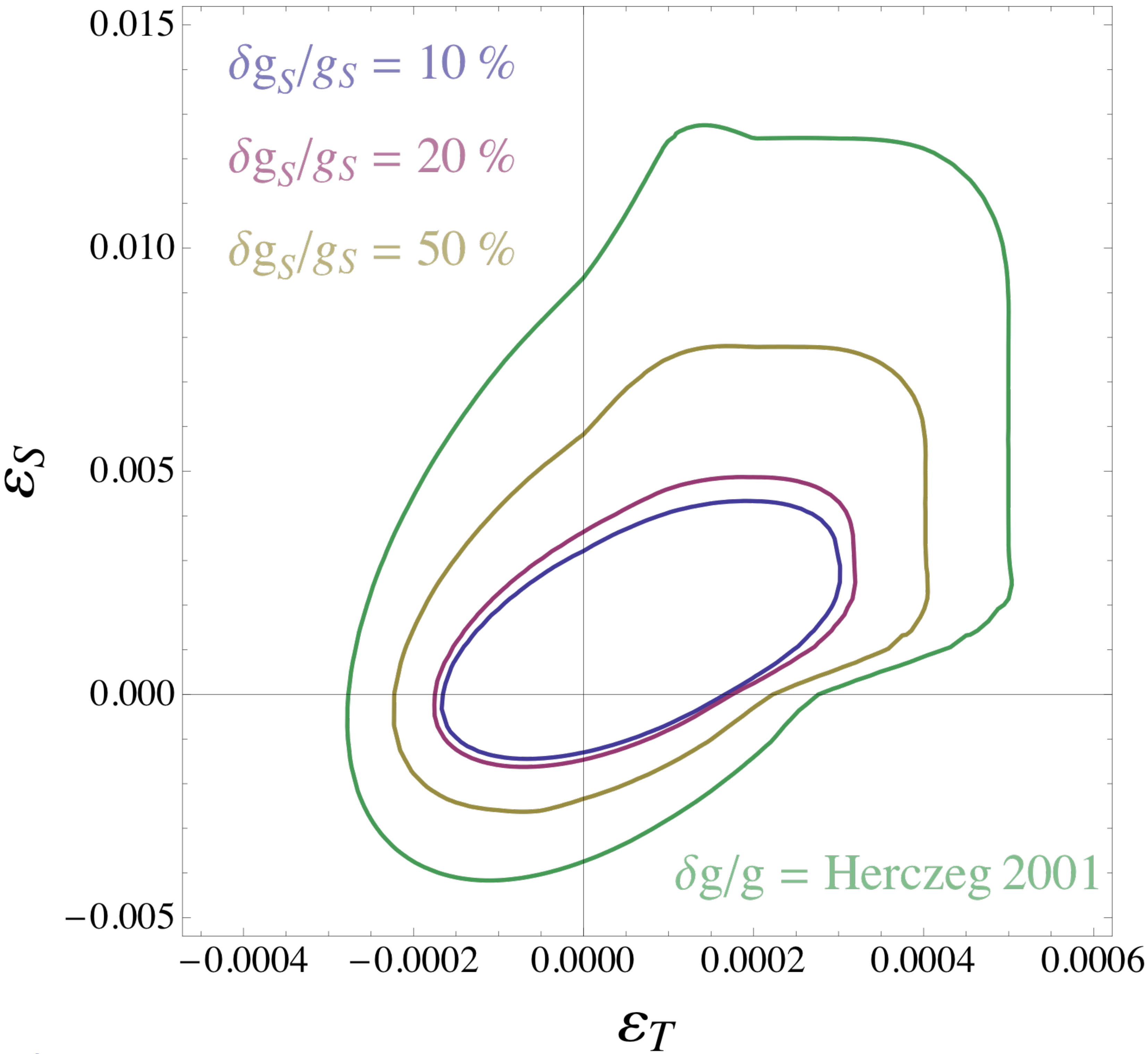
$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

Fundamental quantity

Caveat: no sum rules

Why do we need tensor charge?

Beyond Standard Model searches

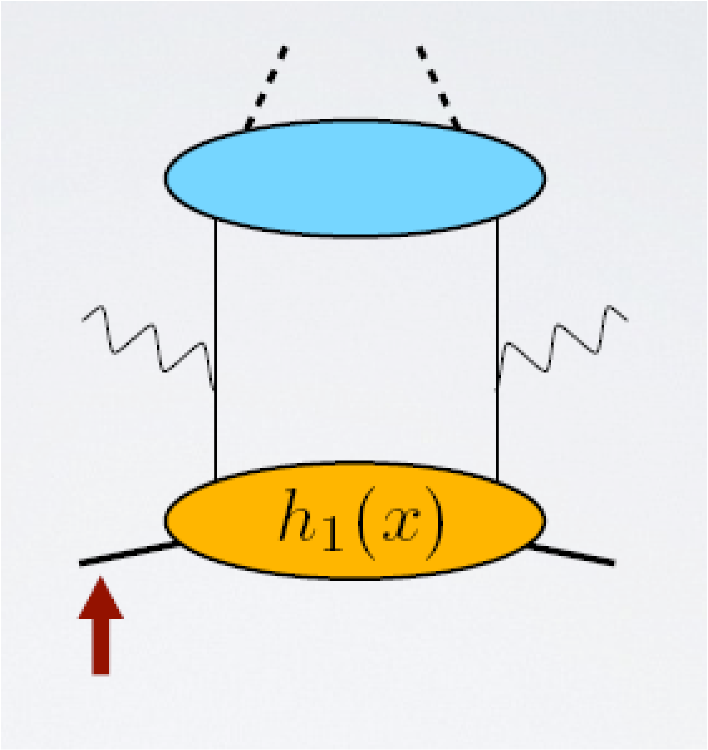


Parameters depend on precision of tensor charge determination

Gupta et al PRD 85 (2012) 054512

$$\epsilon_T \propto m_W^2/\Lambda_T^2$$

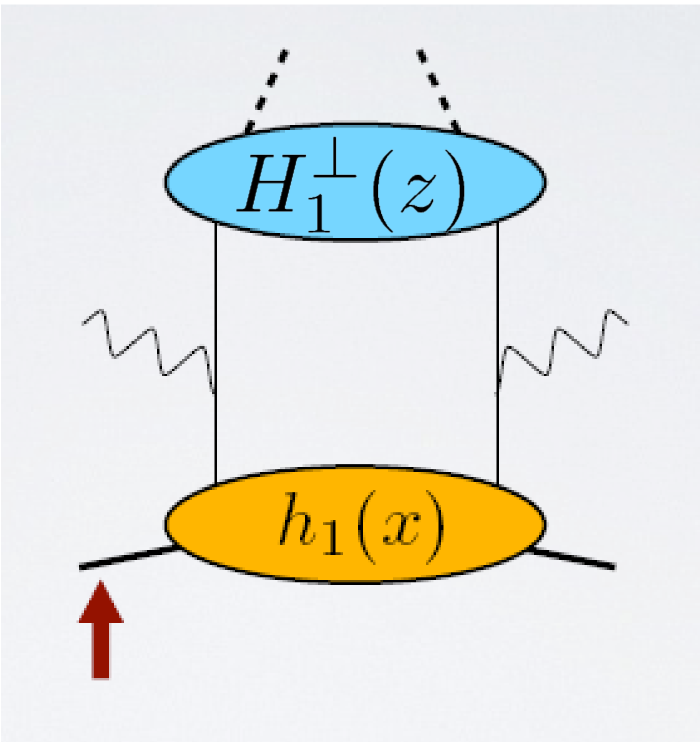
Λ_T is BSM scale



First extraction in 2007, Anselmino et al 07

$$A_{UT}^{\sin(\Phi_h+\Phi_S)} \propto \frac{\sum e_q^2 h_1^q \otimes H_1^{\perp q}}{\sum e_q^2 f_1^q \otimes D_1^q}$$

Transversity from SIDIS



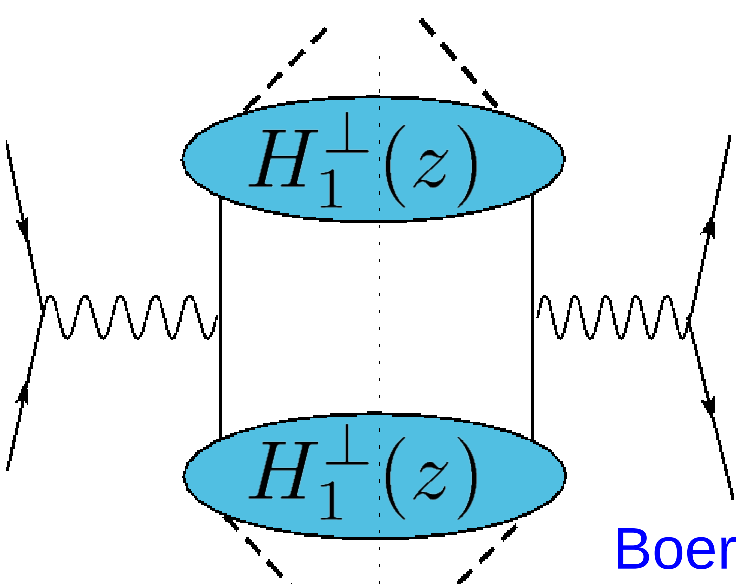
First extraction in 2007, Anselmino et al 07

$$A_{UT}^{\sin(\Phi_h+\Phi_S)} \propto \frac{\sum e_q^2 h_1^q \otimes H_1^{\perp q}}{\sum e_q^2 f_1^q \otimes D_1^q}$$

Two unknowns, transversity $h_1(x)$

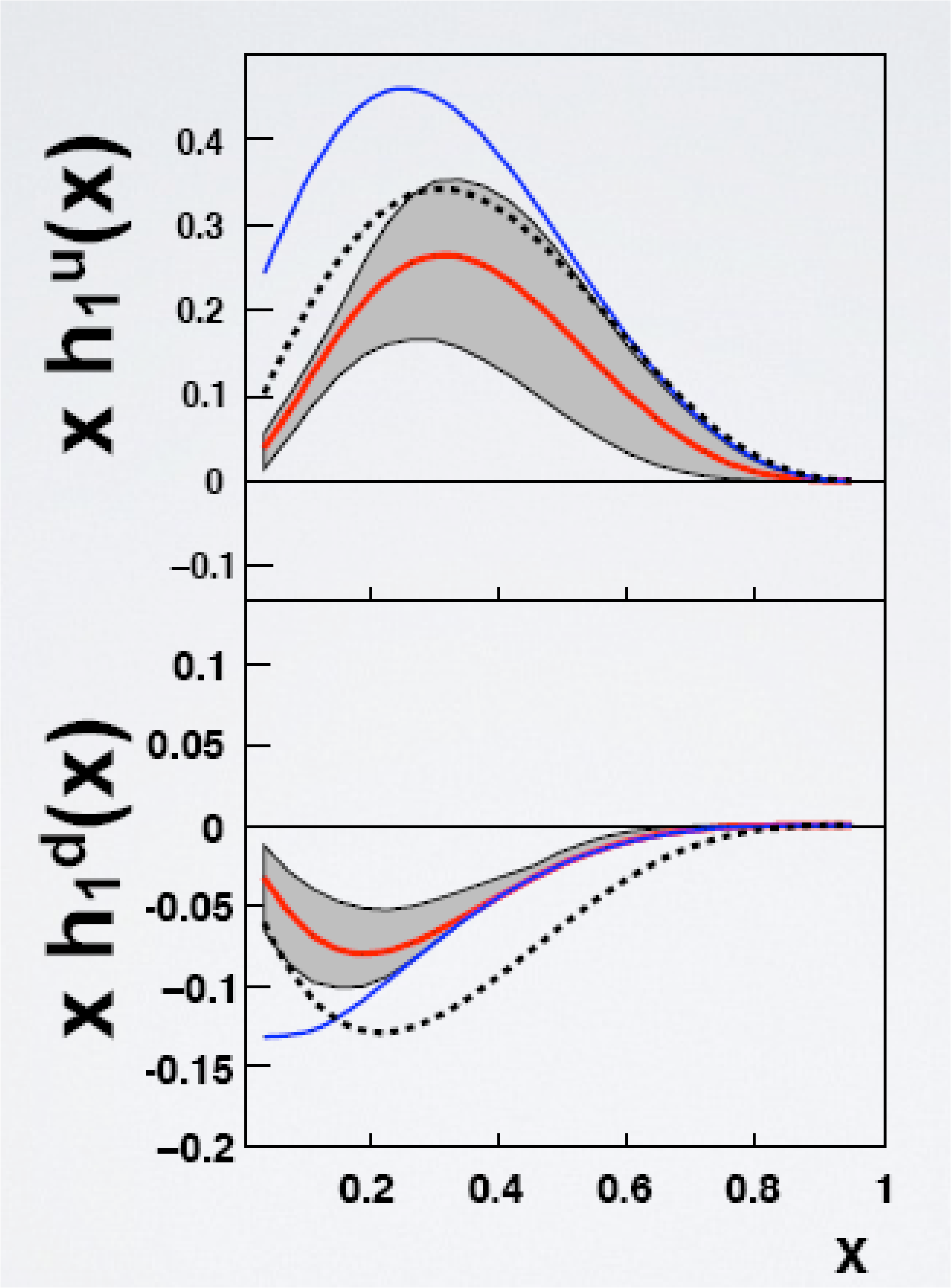
Collins Fragmentation Function $H_1^\perp(z)$

Information on $H_1^\perp(z)$ is available from e^+e^-

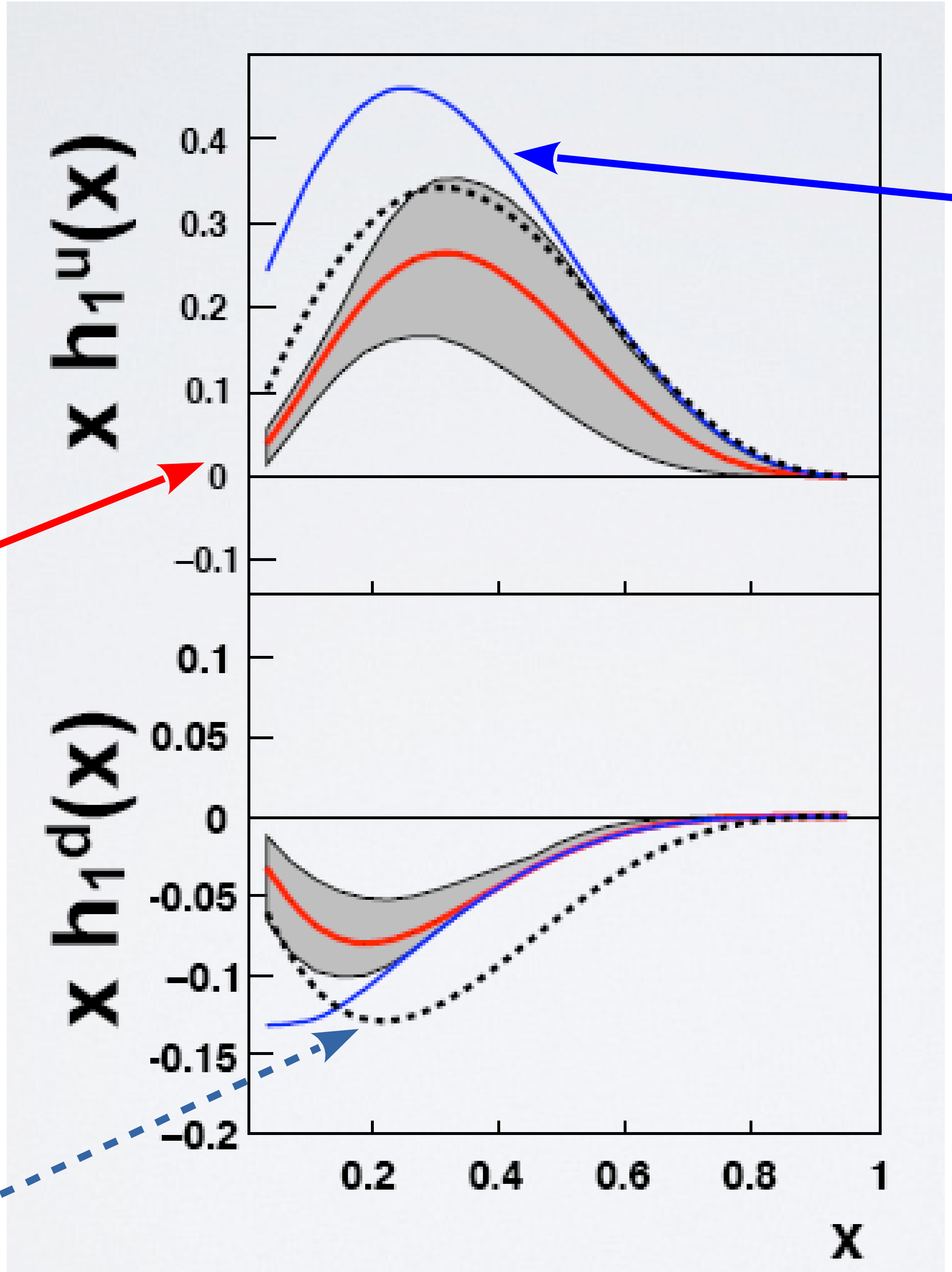


$$A_{e^+e^-} \propto \frac{\sum e_q^2 H_1^{\perp q} \otimes H_1^{\perp \bar{q}}}{\sum e_q^2 D_1^q \otimes D_1^{\bar{q}}}$$

Boer (1999)



Anselmino et al 09



Soffer Bound

$$|h_1(x)| \leq \frac{1}{2}\,(f_1(x) + g_1(x))$$

Valid at LO QCD, Barone 97, Bourelly et al 98

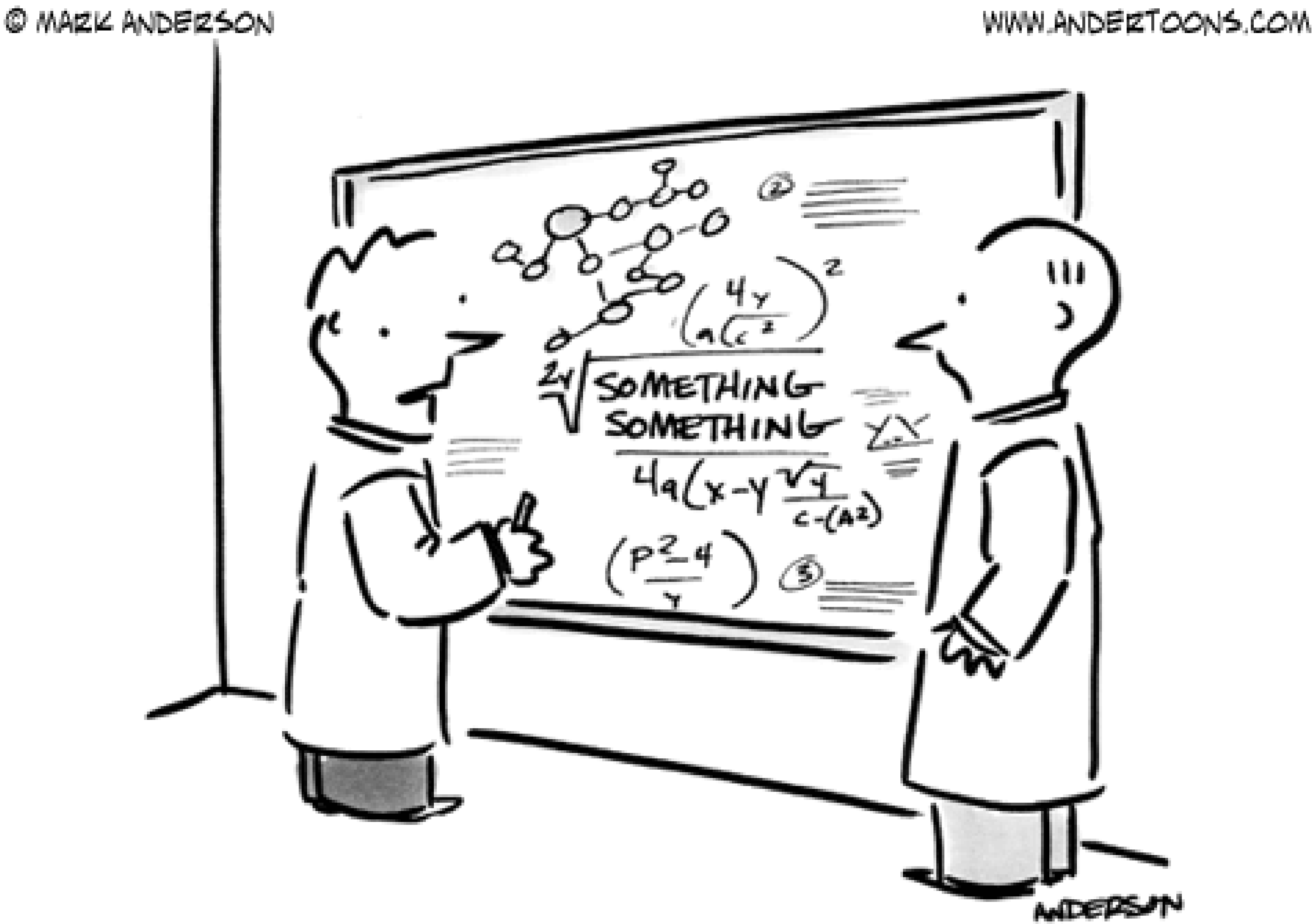
Valid at NLO QCD, Vogelsang 98

Transversity

Helicity

Anselmino et al 09

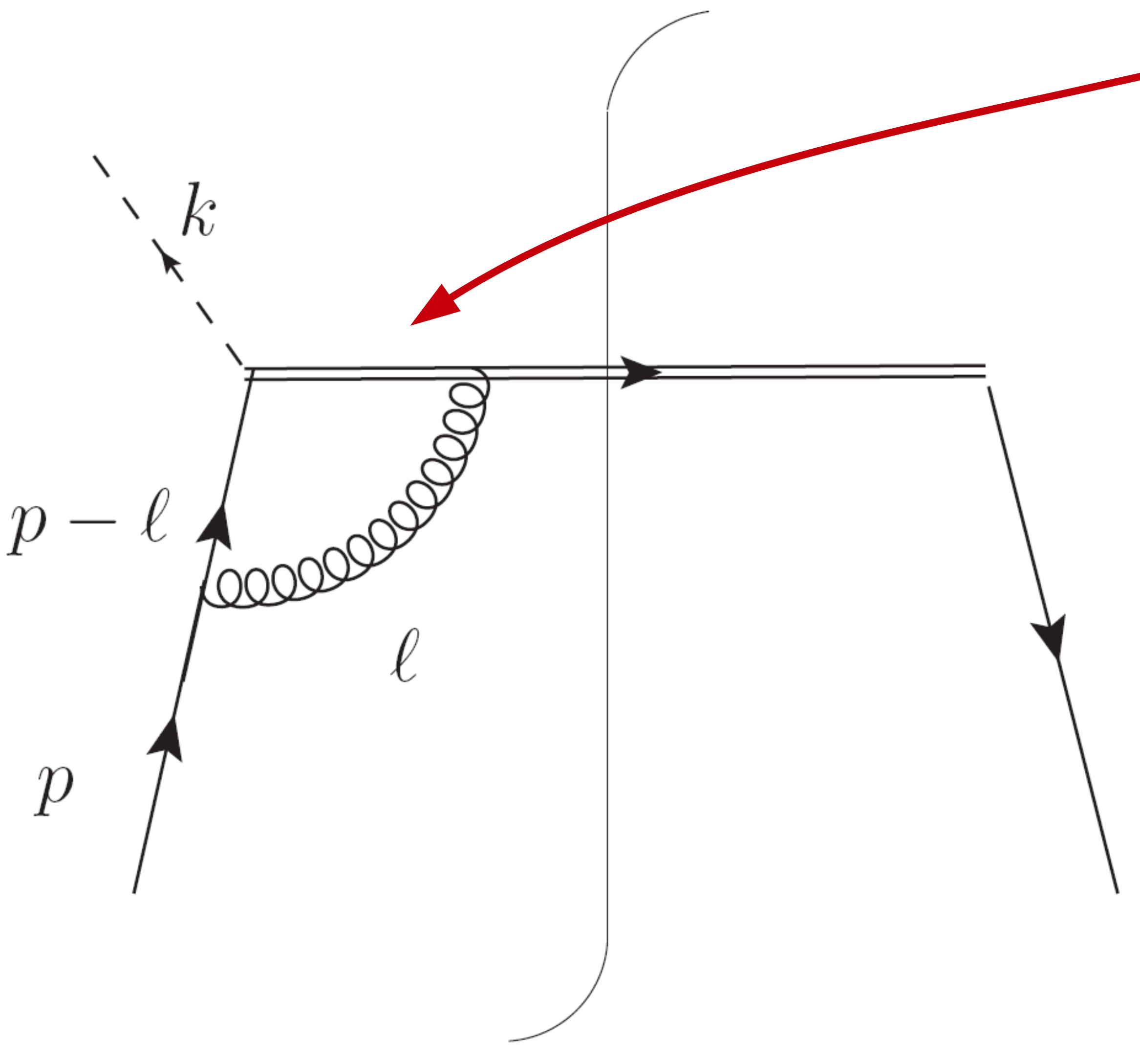
It seems too easy...



PENNSTATE Berks

In fact it is not easy...

If we consider NLO corrections
this diagram diverges

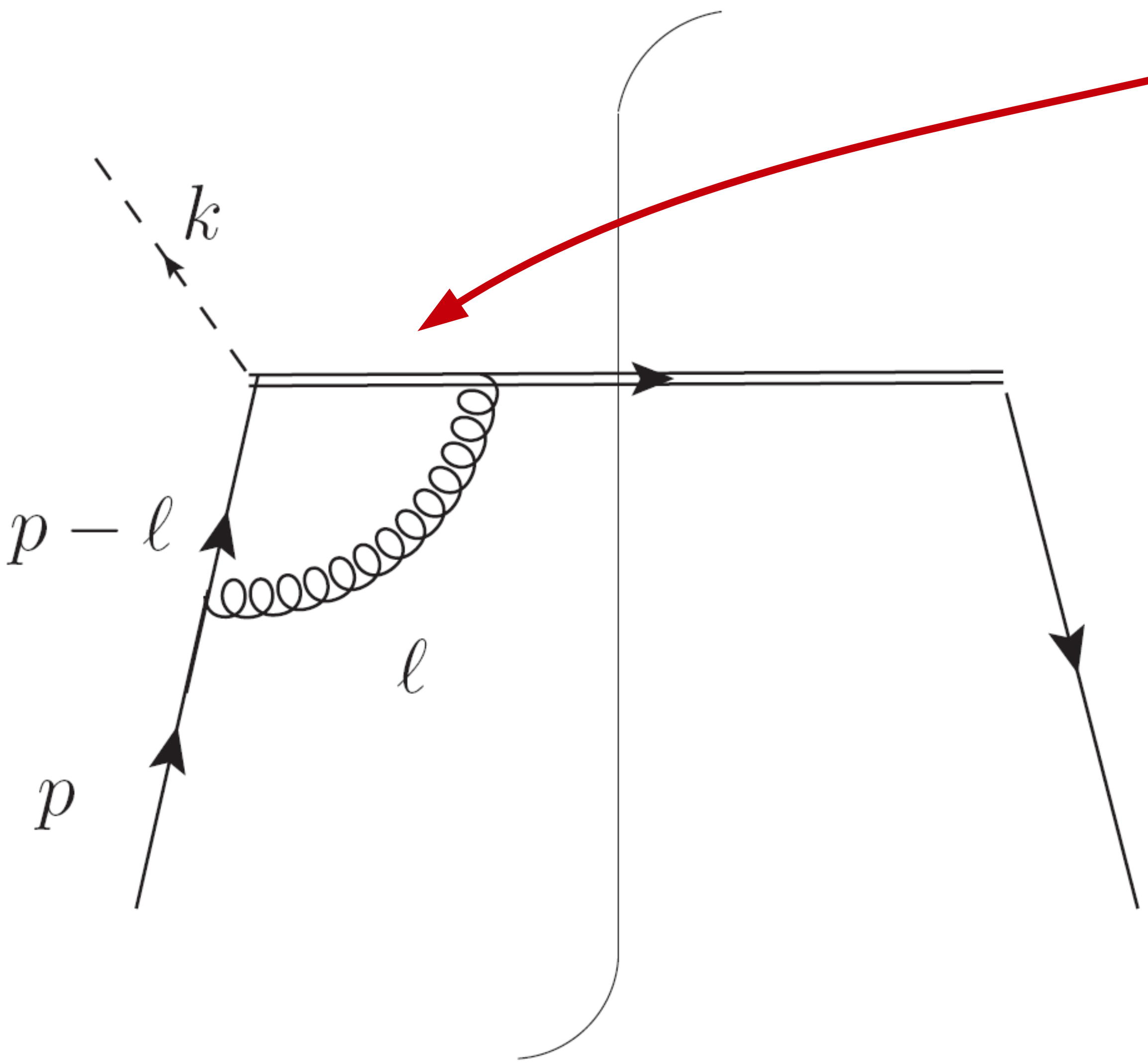


$$\ell = (1-\alpha)p$$

$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

PENNSTATE Berks

In fact it is not easy...



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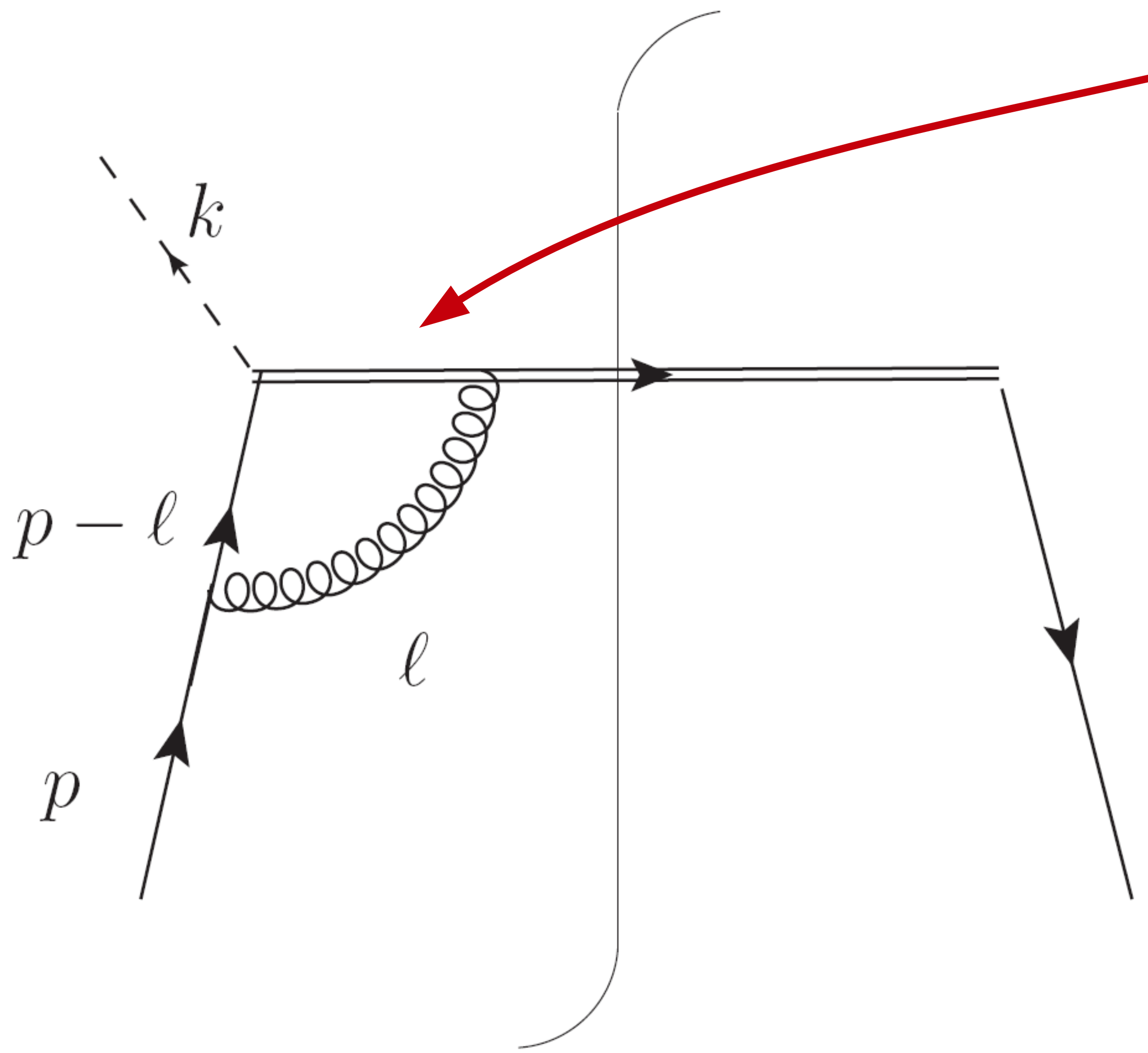


If we consider NLO corrections
this diagram diverges

$$\ell = (1-\alpha)p$$

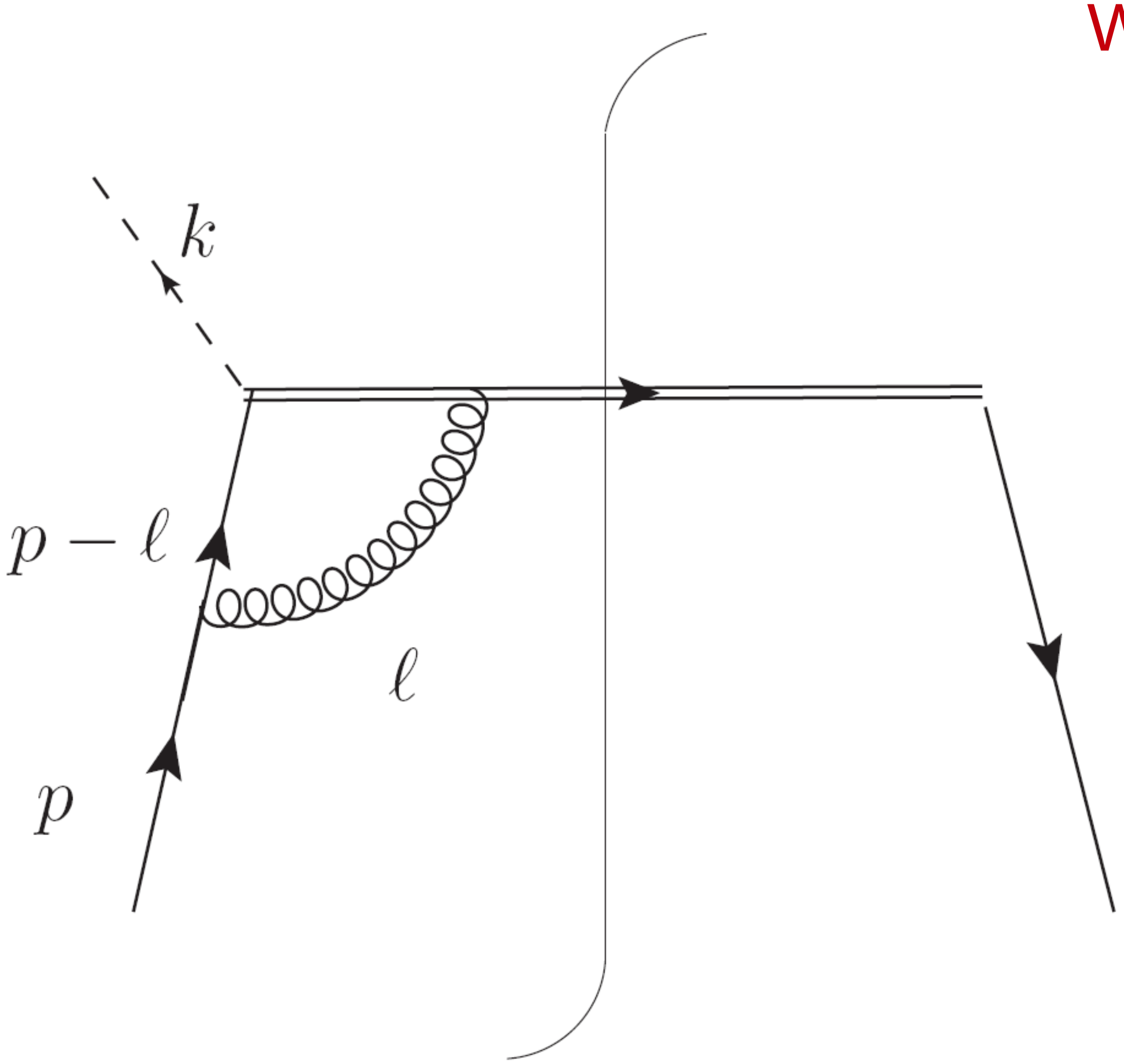
$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

Physics:

The gluon becomes collinear to the Wilson line (struck quark) and its rapidity goes to $-\infty$

"Rapidity divergence"

In fact it is not easy...



We know how to deal with it:

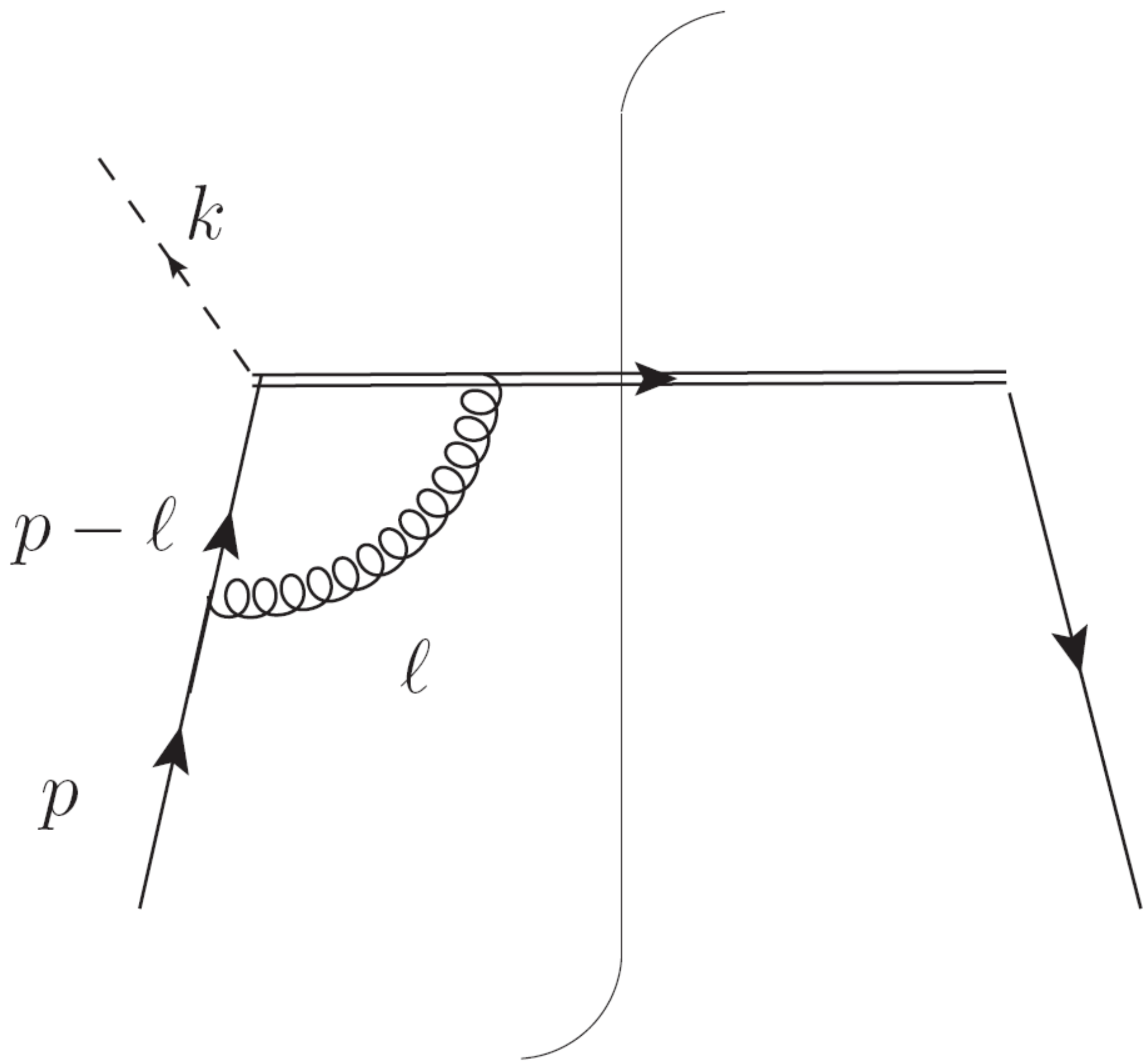
$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha) - T(1)}{(1-\alpha)_+}$$

"+ prescription"

$$T(\alpha = 1) - T(1) = 0$$

In fact it is not easy...



Not working for TMDs:

$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha, k_\perp) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha, k_\perp) - T(1, 0_\perp)}{(1-\alpha)_+}$$

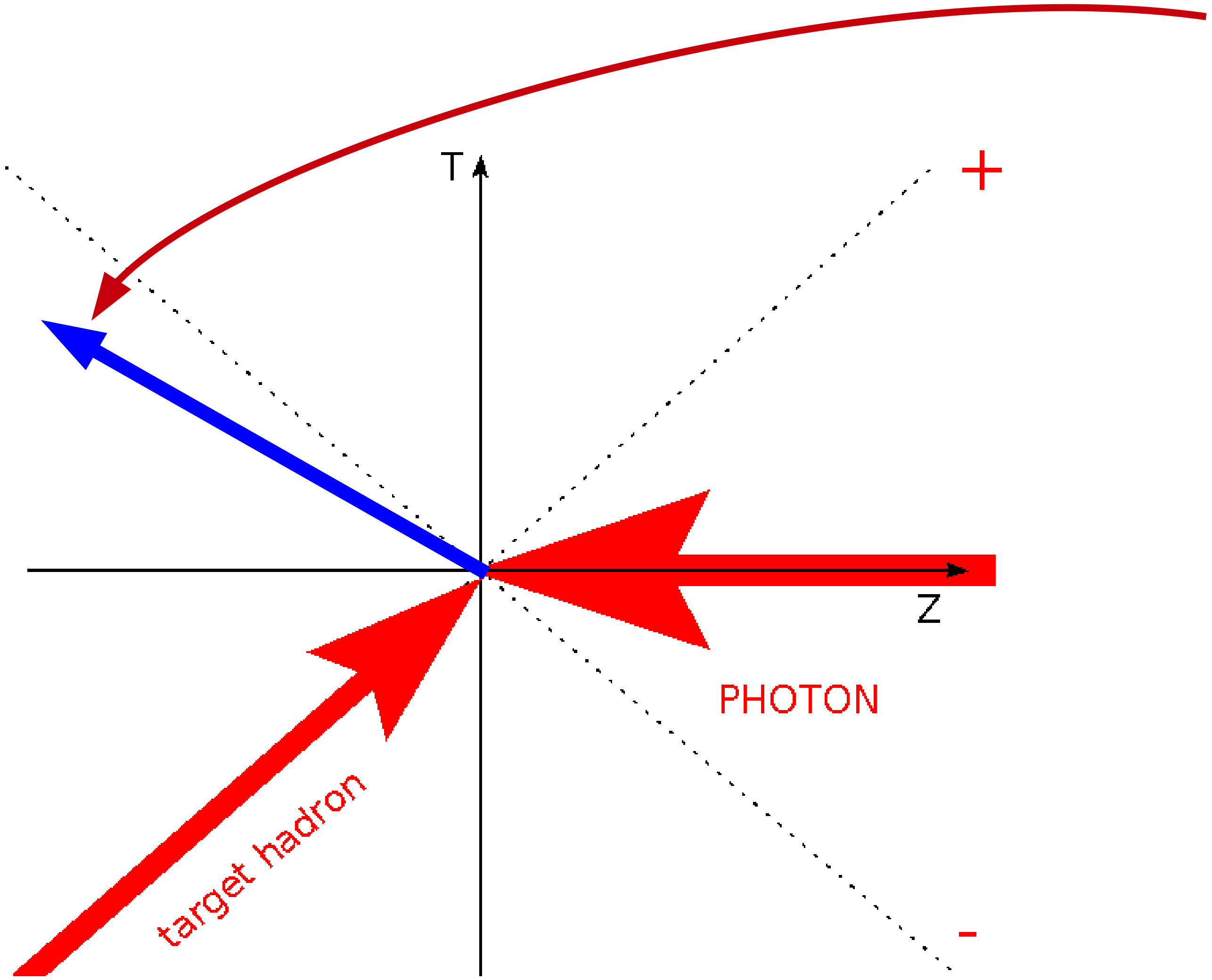
"+ prescription"

$$T(\alpha = 1, k_\perp) - T(1, 0_\perp) \neq 0$$

John Collins, Acta Phys.Polon. B34 (2003) 3103

PENNSTATE Berks 1855

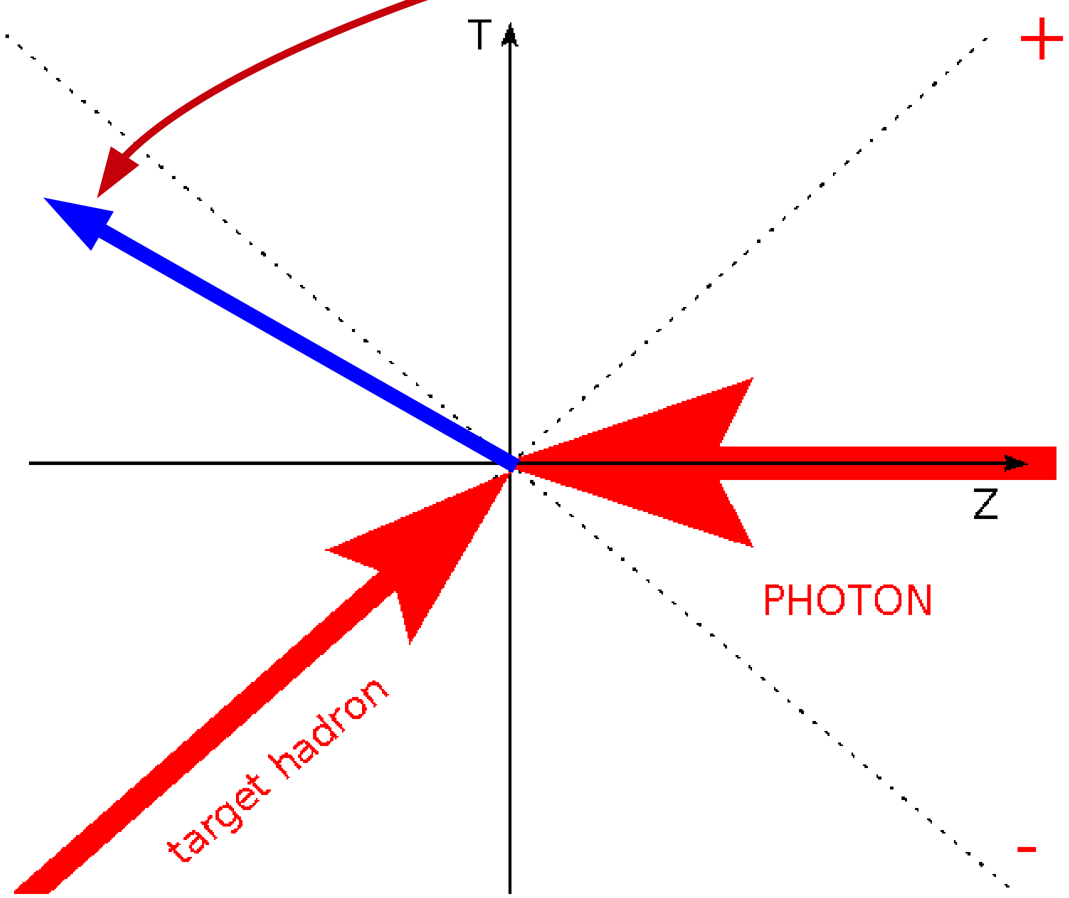
Definition of TMDs



Tilt slightly to regularize rapidity divergence (tilt into the space-like region as otherwise gluons can be still collinear)

Definition of TMDs

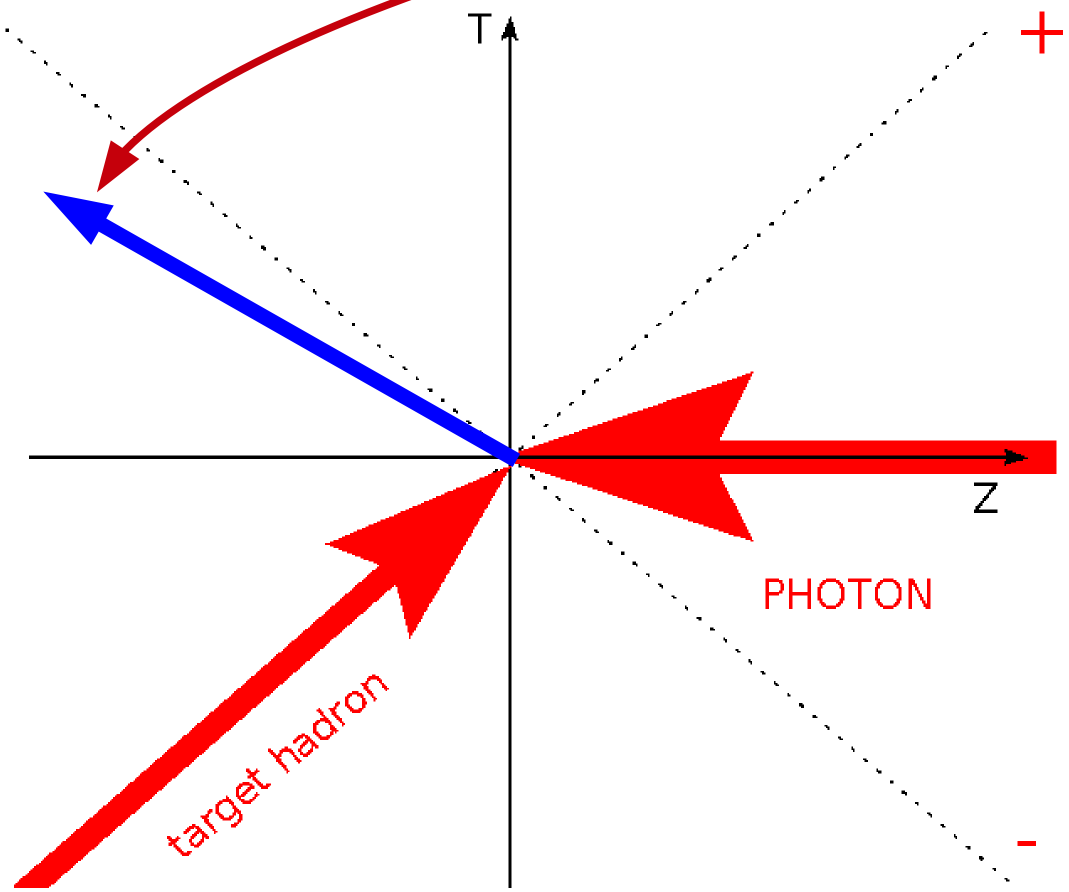
Tilt slightly to regularize rapidity divergence



$$f(x, k_\perp; \mu, \zeta)$$

Definition of TMDs

Tilt slightly to regularize rapidity divergence



$$f(x, k_\perp; \mu, \zeta)$$

Renormalization Group (RG) parameter

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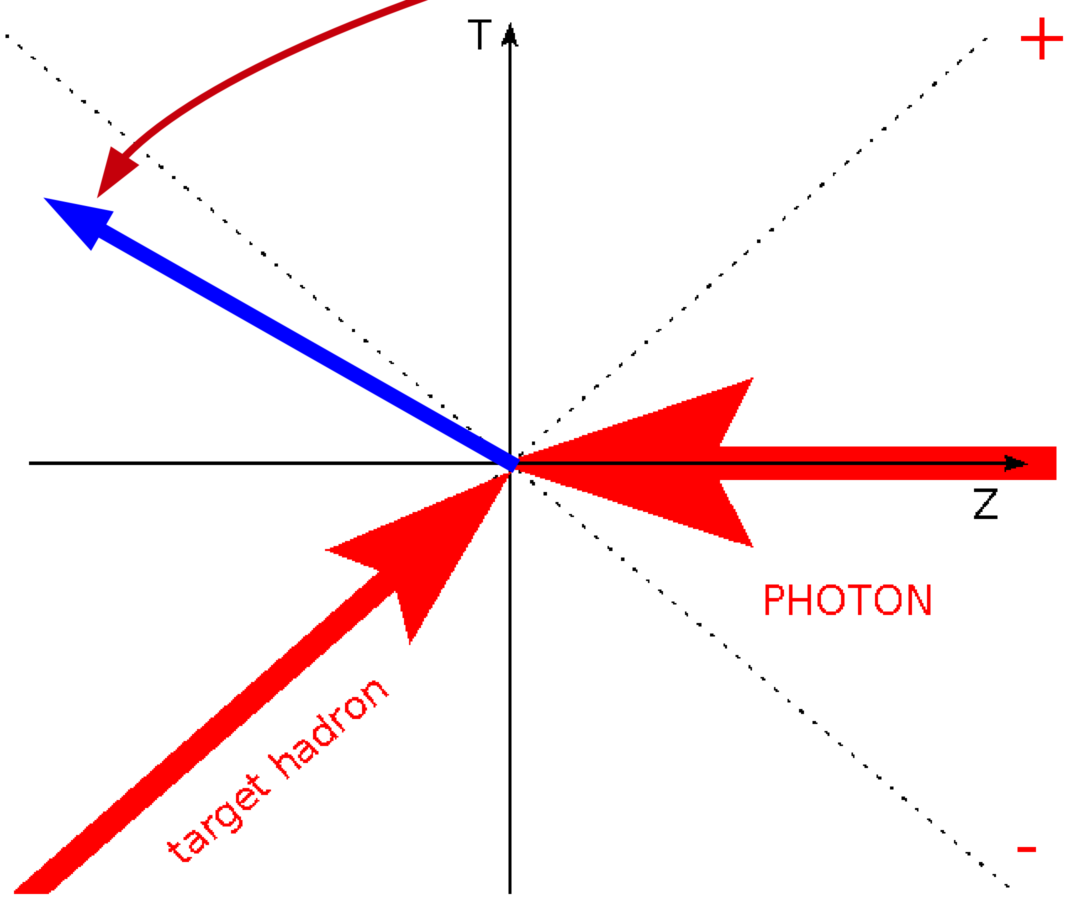
Definition of TMDs



Tilt slightly to regularize rapidity divergence

Additional parameter to cancel rapidity divergence

$$f(x, k_\perp; \mu, \zeta)$$

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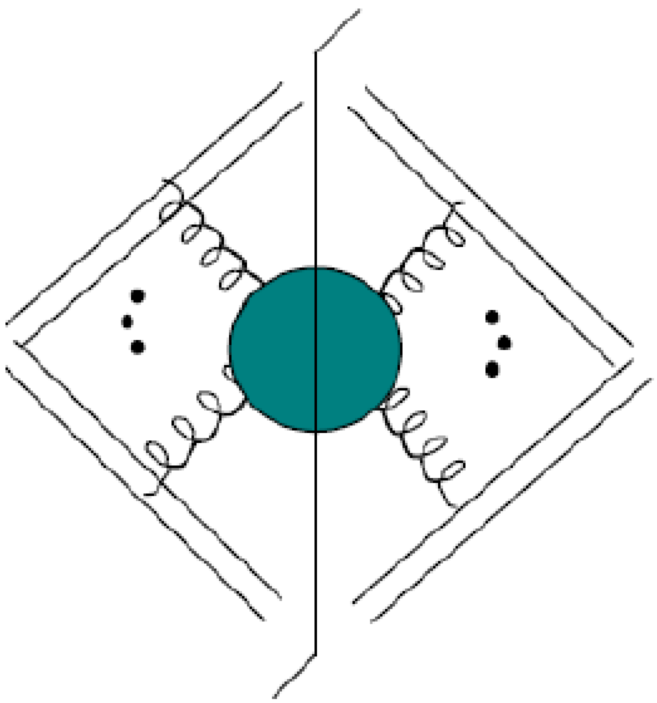
Definition of TMDs



$$= S(y_A, y_B)$$

We also need to add something:
Soft Factor to implement subtractions

Approaches to TMD evolution

Collins-Soper-Sterman (CSS) resummation framework

Collins-Soper-Sterman 1985
ResBos: C.P. Yuan, P. Nadolsky
Qiu-Zhang 1999, Vogelsang etc...
Kang-Xiao-Yuan 2011, Sun-Yuan 2013
Prokudin-Kang-Sun-Yuan 2014

“New” Collins approach

Collins 2011
Aybat-Rogers 2011,
Aybat-Collins-Rogers-Qiu, 2012
Aybat-Prokudin-Rogers 2012
Anselmino-Boglione-Melis 2012
Prokudin-Bacchetta 2013
Echevarria-Idilbi-Kang-Vitev 2014

Soft Collinear Effective Theory (SCET)

Echevarria-Idilbi-Schafer-Scimemi 2012
D'Alesio-Echevarria-Melis-Scimemi 2014

Approaches to TMD evolution

Different approaches are essentially identical

Phenomenological results vary however due to different
treatment of initial conditions

Advantages of TMD formulation of CSS:

→ Universal non perturbative kernel of evolution

→ Description of cross sections in a large region of transverse momenta

TMD evolution

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

John Collins, 2011

$$\frac{d\tilde{K}(b_T, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_T, \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu), \zeta)$$

TMD evolution in a nut shell

TMD functions are measured at scale Q $\qquad f(x, k_\perp; Q)$

Evolution is performed in Fourier space

$$\tilde{f}(x, b_T; Q) = \int d^2 k_\perp e^{-ik_\perp b_T} f(x, k_\perp; Q)$$

Standard CSS formalism, evolution starts from

$$\mu_b = c/b_T, \; c = 2e^{-\gamma_E}$$

in order to allow for OPE of TMDs and relation to collinear functions

$$\tilde{f}(x, b_T; Q) = \tilde{f}(x, b; \mu_b) e^{-S_{pert}(b_T)}$$

$$S_{pert}(b) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right)$$

Perturbative Sudakov factor, serves to evolve TMD from scale μ_b to Q

$$A = \sum_{n=1} \left(\frac{\alpha_s}{\pi} \right)^n A^{(n)} \qquad B = \sum_{n=1} \left(\frac{\alpha_s}{\pi} \right)^n B^{(n)}$$

TMD evolution in a nut shell

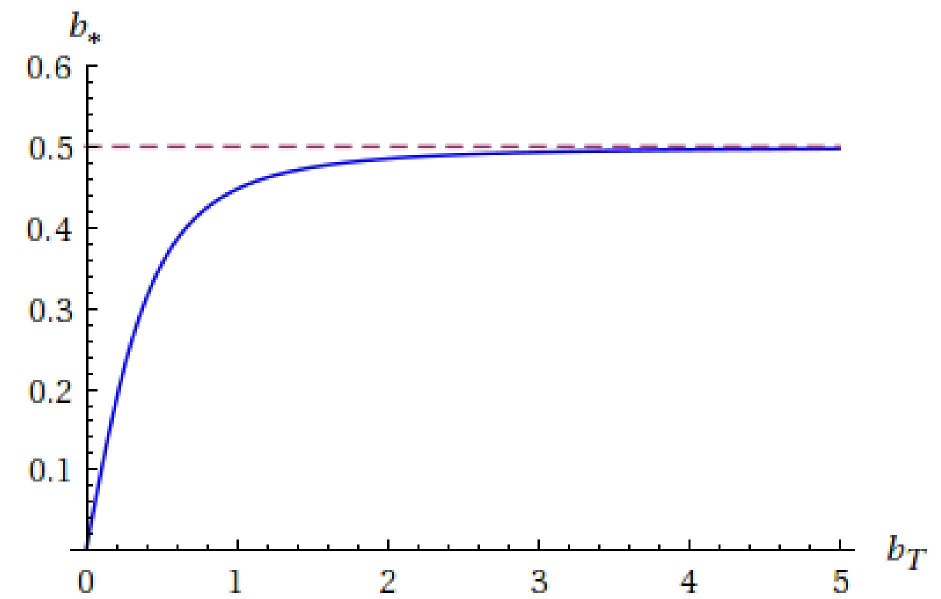
Calculation is perturbative, valid only in region $b \ll 1/\Lambda_{QCD}$

Fourier transform in momentum space involves non-perturbative region

$$f(x, k_\perp; Q) = \int_0^{\infty} \frac{bdb}{2\pi} J_0(k_\perp b) \tilde{f}(x, b; Q)$$

Non perturbative region needs to be treated.
Common method b_* prescription

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$



$$\tilde{f}(x, b; Q) = \tilde{f}(x, b_*; c/b_*) e^{-S_{pert}(b_*)} e^{-S_{NP}(b)}$$

Non perturbative Sudakov factor

TMD evolution in a nut shell

Relation to collinear functions at small values of b:

$$\tilde{f}^j(x, b_*; c/b_*) = \sum_{j'=q,g} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{j/j'}\left(\frac{x}{\hat{x}}, b_*; c/b_*\right) f^{j'}(x; c/b_*)$$

$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi}\right)^n C^{(n)}$$ Wilson coefficient

Collinear PDF

For transversity and helicity TMDs: Bacchetta-Prokudin 2013

For Collins function (relation to collinear **twist-3** function): Yuan-Zhou 2009, Kang 2011

In future also gluon functions will be important

For gluon twist-3 function: Dai-Kang-Prokudin-Vitev 2014

Taking into account Wilson coefficients is very important!
Large K factors of collinear computations between LO and NLO!

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything at NLO:

$$\tilde{C}_{j'/j}(x,\boldsymbol{b}_T;\mu;\zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\left(\frac{1+x^2}{1-x}\right)_+ + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)\ ,$$

$$\Delta\tilde{C}_{j'/j}(x,\boldsymbol{b}_T;\mu;\zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\left(\frac{1+x^2}{1-x}\right)_+ + \frac{1}{2}(1-x) + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)\ ,$$

$$\delta\tilde{C}_{j'/j}(x,\boldsymbol{b}_T;\mu;\zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{ \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\left(\frac{2x}{1-x}\right)_+ + \right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)\ .$$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Simplify: $\mu_b = c/b_T, \; c = 2e^{-\gamma_E}$

$$\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \mathcal{O}(\alpha_s^2).$$

PENNSTATE Berks 1855

TMD evolution in a nut shell

Precision of extraction depends on precision of calculations

Leading Log (LL):	$A^{(1)}$		
Next-to Leading Log (NLL):	$A^{(1,2)}$	$B^{(1)}$	$C^{(1)}$
Next-to-Next-to Leading Log (NNLL):	$A^{(1,2,3)}$	$B^{(1,2)}$	$C^{(1)}$

Precision is important!

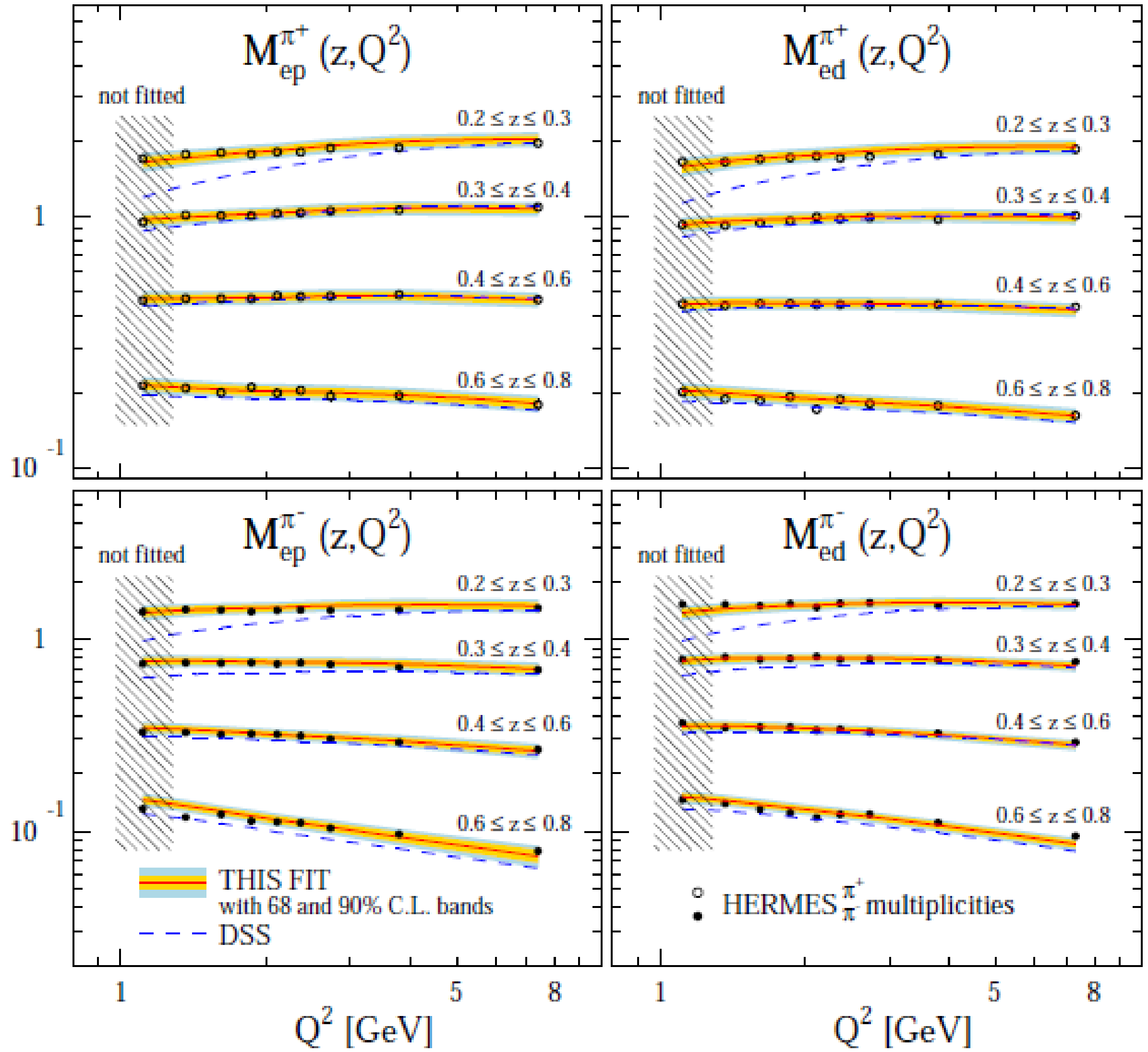
$C^{(1)}$ means that one should use NLO collinear distributions

Why do we need NLO?

Let's compare to HERMES data:

LO – dashed, NLO – solid

Phys.Rev. D91 (2015) 1, 014035



*R. Sassot: "We think that the LO set makes no much sense"**

* unless one wants to give only a *qualitative* description

Transversity at NLL

Kang-Prokudin-Sun-Yuan 2014

First NLL' extraction from the data

$$A^{(1,2)} \quad B^{(1)} \qquad C^{(1)}$$

Collins function is related to twist-3 function

$$\tilde{H}_1^{\perp,\alpha}(z_h, b; \mu_b) \sim \left(\frac{-ib^\alpha}{2z_h}\right) H^{(3)}(z_h; \mu_b)$$

We solve also DGLAP equations for transversity and (diagonal) Collins FF

Yuan-Zhou 2009, Kang 2011

Diagonal part for twist-3 Collins function is:

$$\frac{d}{d\ln\mu^2} H_q^{(3)}(z_h, \mu) = \frac{\alpha_s}{2\pi} P_{i\to q}^{H} \otimes H_i^{(3)}$$

$$P_{i\to q}^{H_1}(\hat{x}) = \delta_{iq} C_F \left(\frac{2\hat{z}}{(1-\hat{z})_+} + \frac{3}{2}\delta(1-\hat{z})\right)$$

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

e+e- data used: BELLE, BABAR including PT dependence – 122 points

$$\chi^2/\text{d.o.f.} \simeq 0.88$$

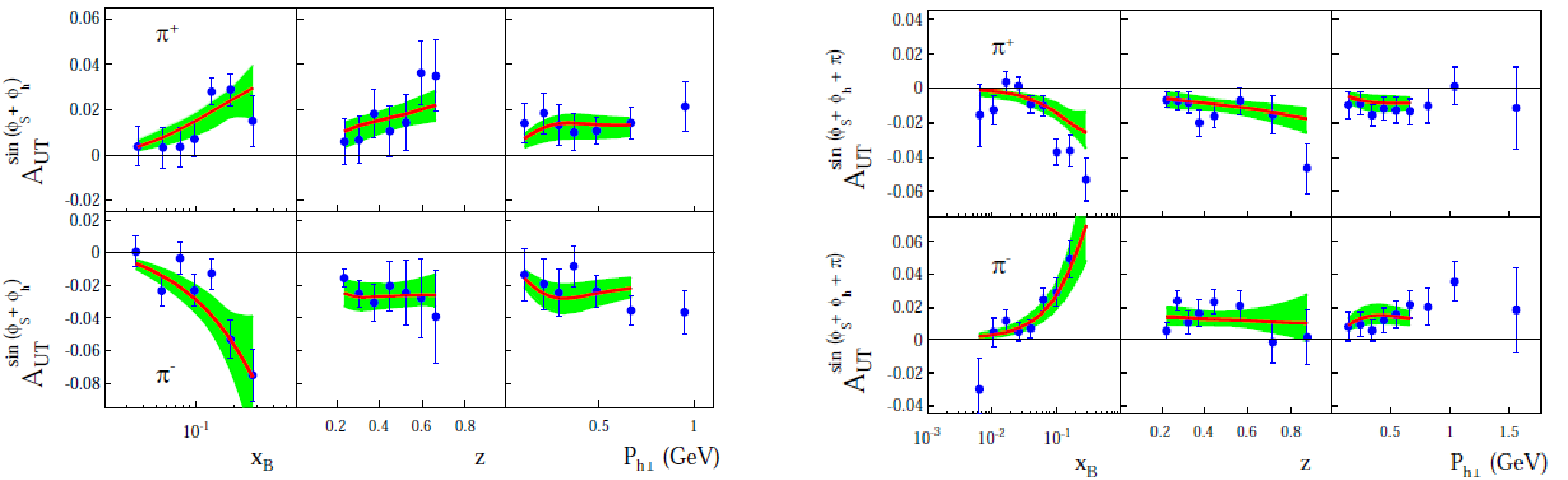
Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

$$\ell P \to \pi^{\pm} X$$

HERMES

COMPASS

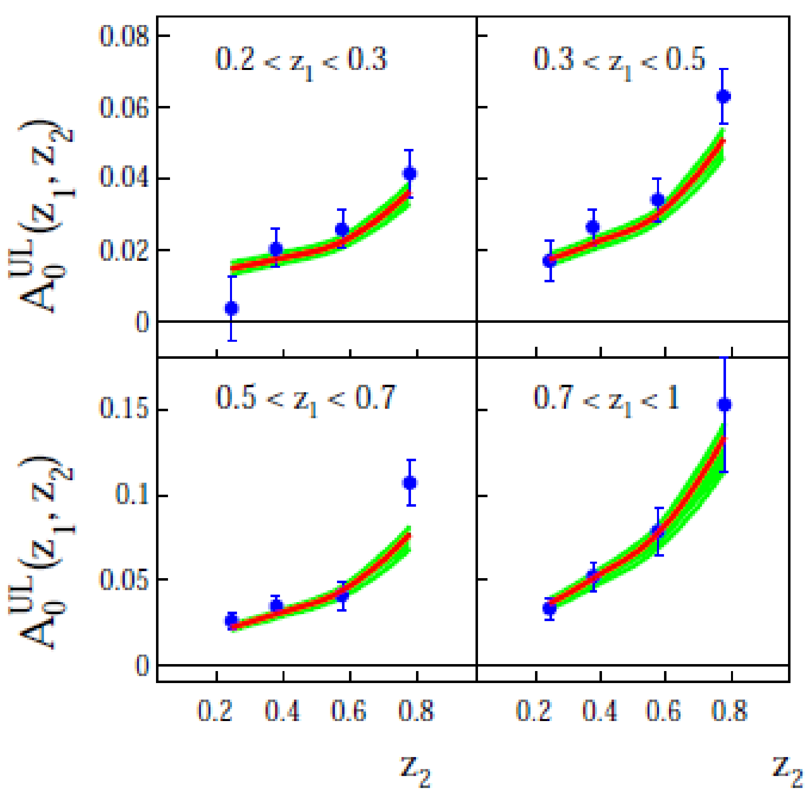


$$1 \lesssim \langle Q^2 \rangle \lesssim \; 6 \text{ GeV}^2$$

$$1 \lesssim \langle Q^2 \rangle \lesssim \; 21 \text{ GeV}^2$$

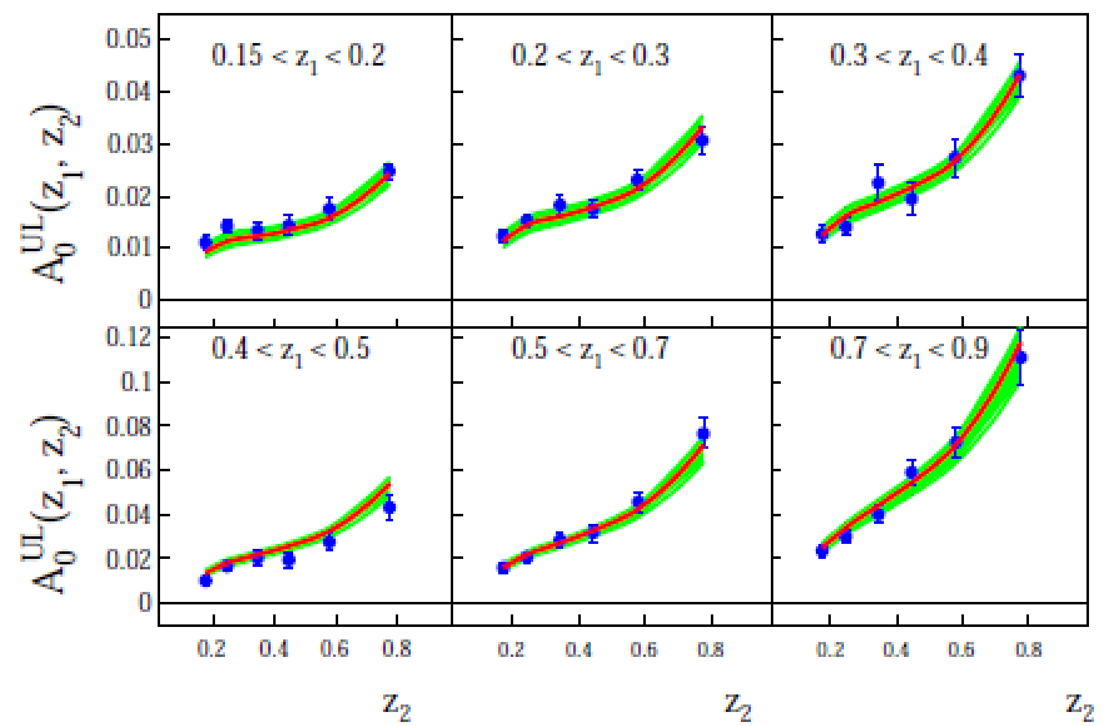
$$e^+e^- \rightarrow \pi\pi X$$

BELLE



$$Q^2 = 110 \text{ GeV}^2$$

BABAR

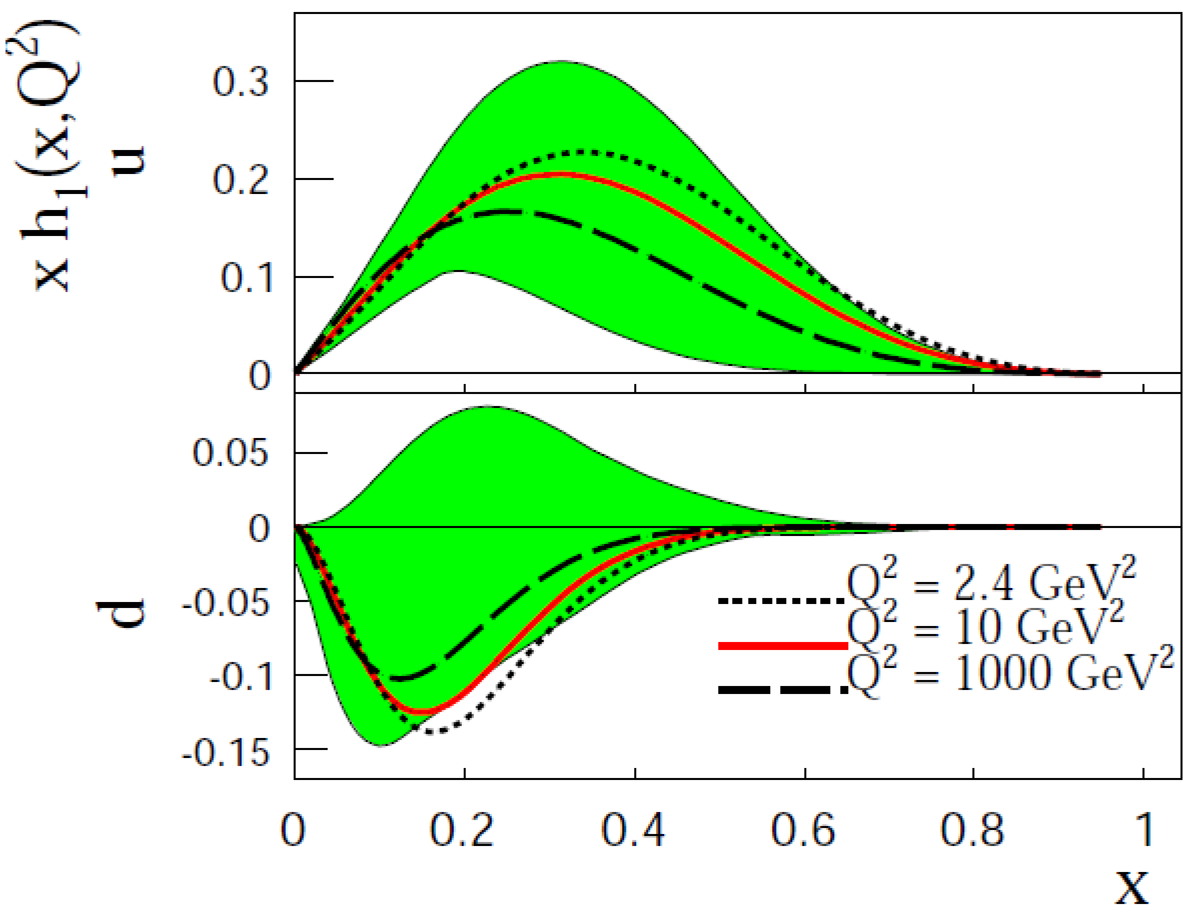


$$Q^2 = 110 \text{ GeV}^2$$

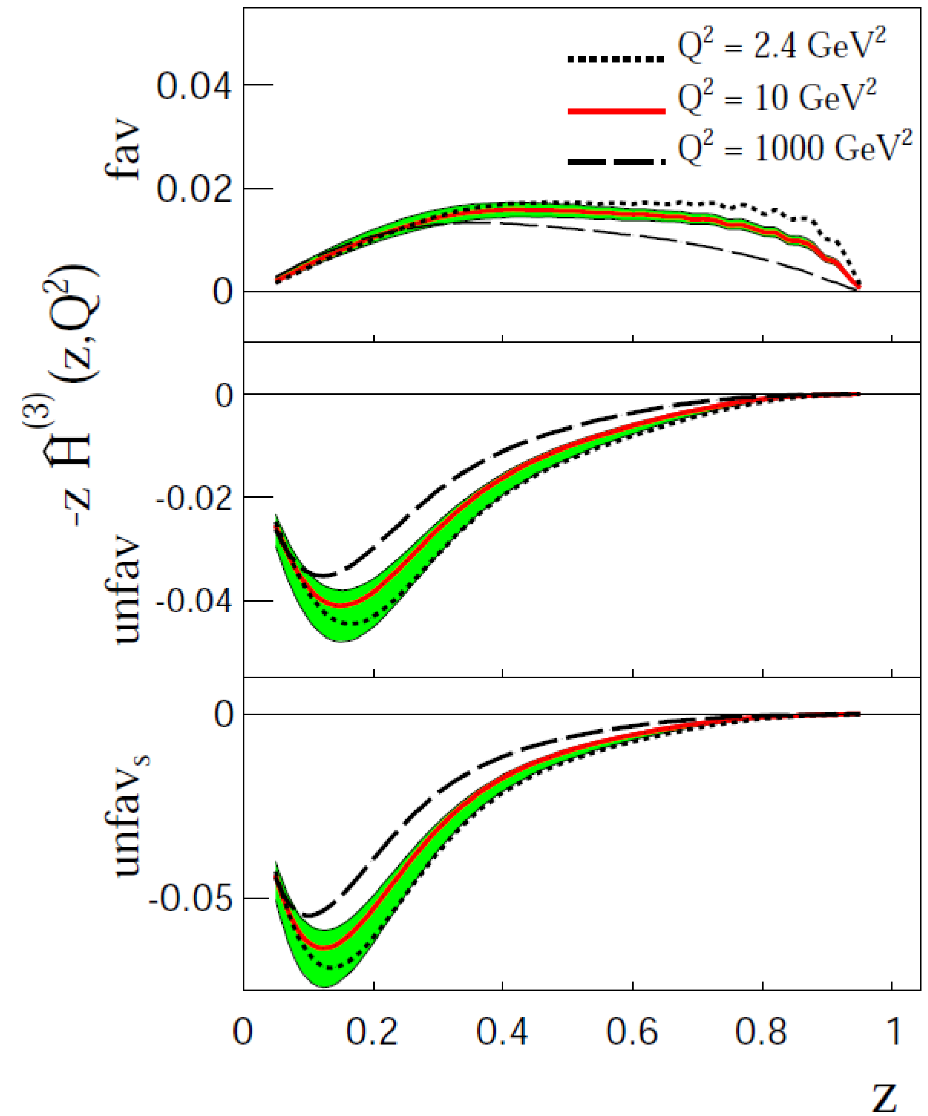
Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

Transversity



Collins



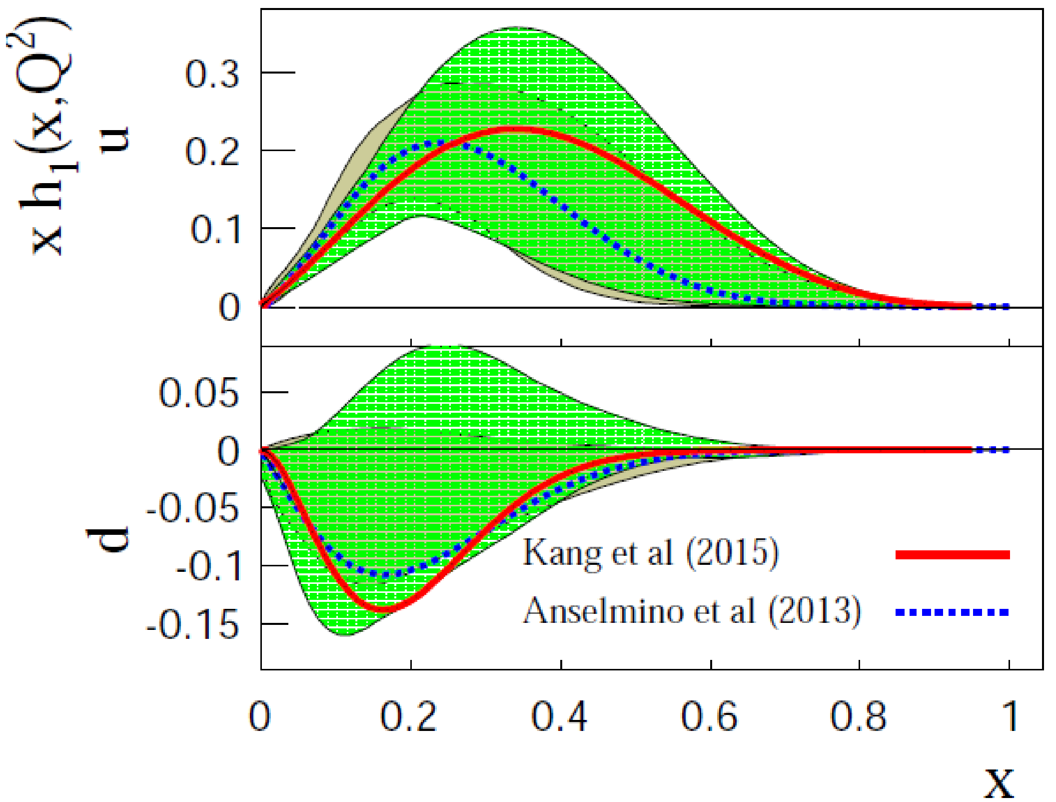
Positive u and negative d transversity

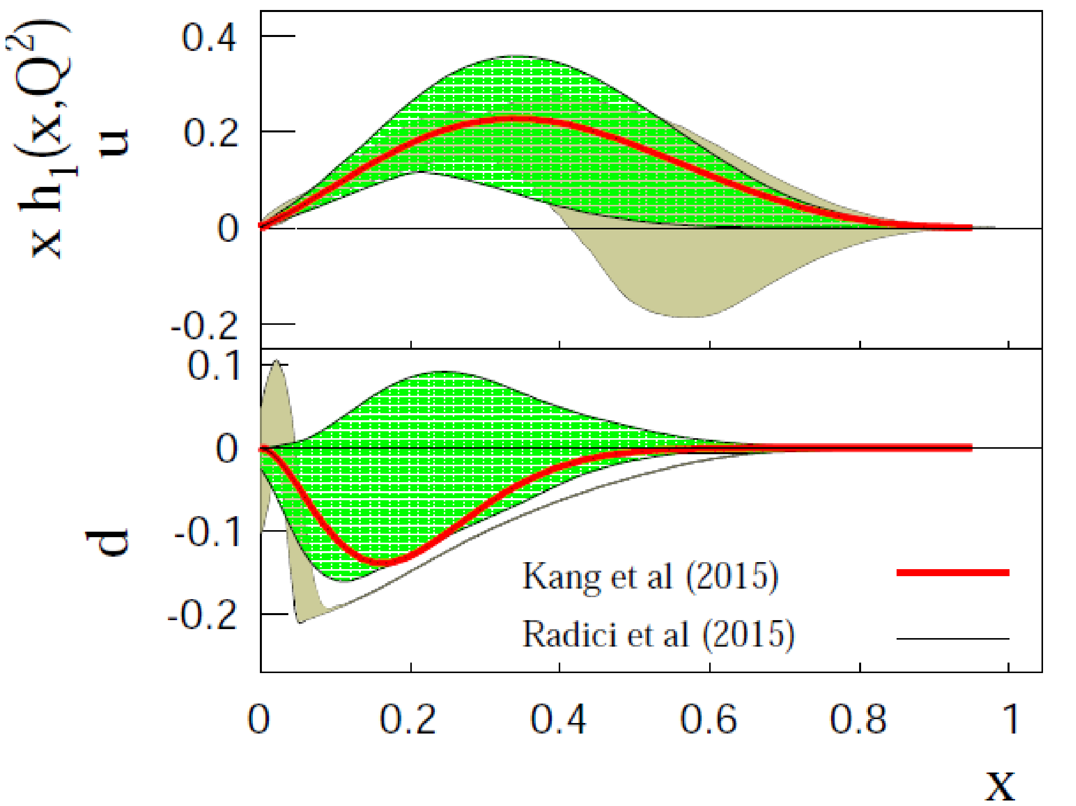
Positive favoured and negative unfavoured Collins FF

Compatible with LO extraction Anselmino et al 2009

Transversity

Kang-Prokudin-Sun-Yuan 2014





Compatible with LO extraction Anselmino et al 2013

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

What are evolution effects?

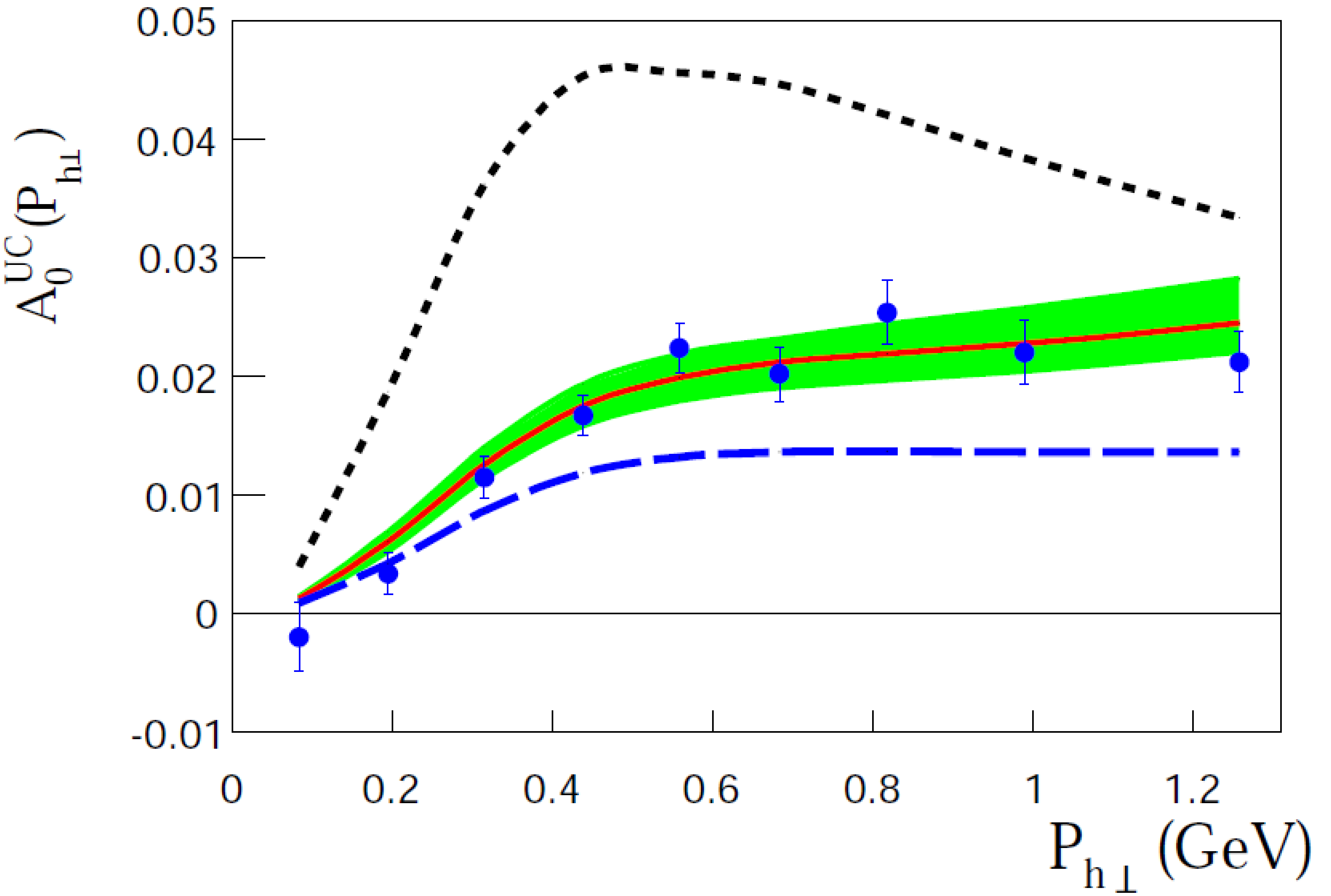
$$e^+e^- \to \pi\pi X$$

No evolution:

LL evolution:

NLL evolution:

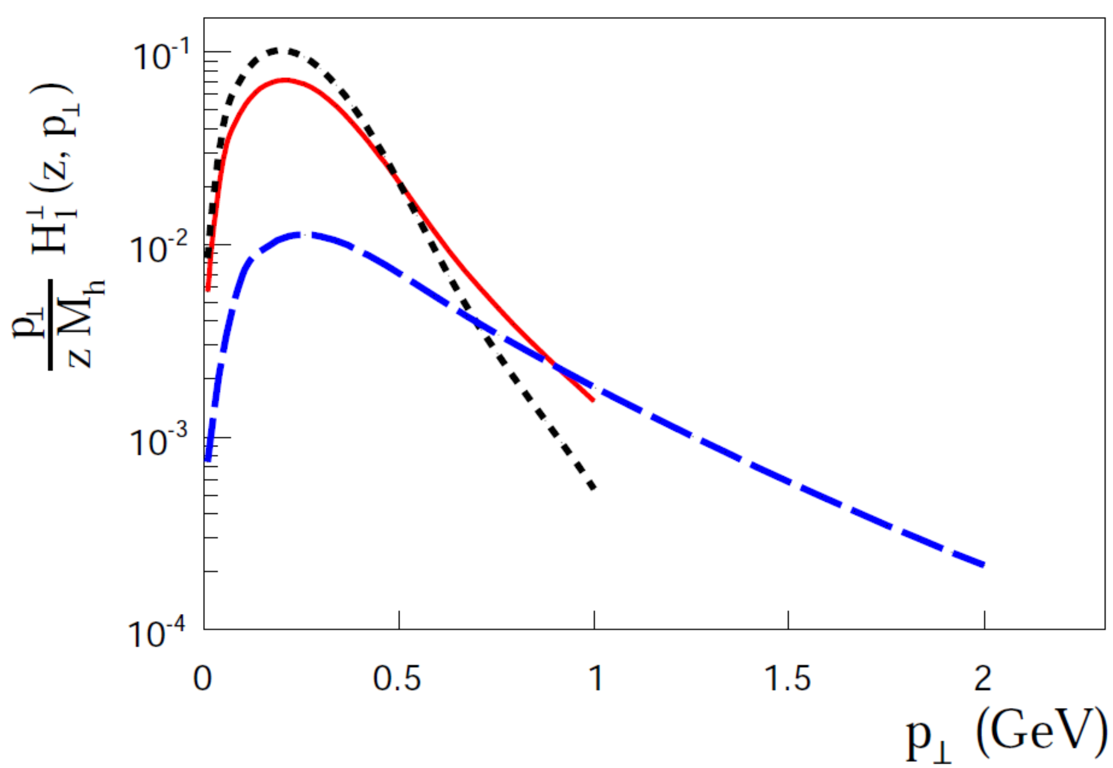
$$Q^2 = 2.4 \text{ GeV}^2$$



Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

It does not mean LO fit is impossible

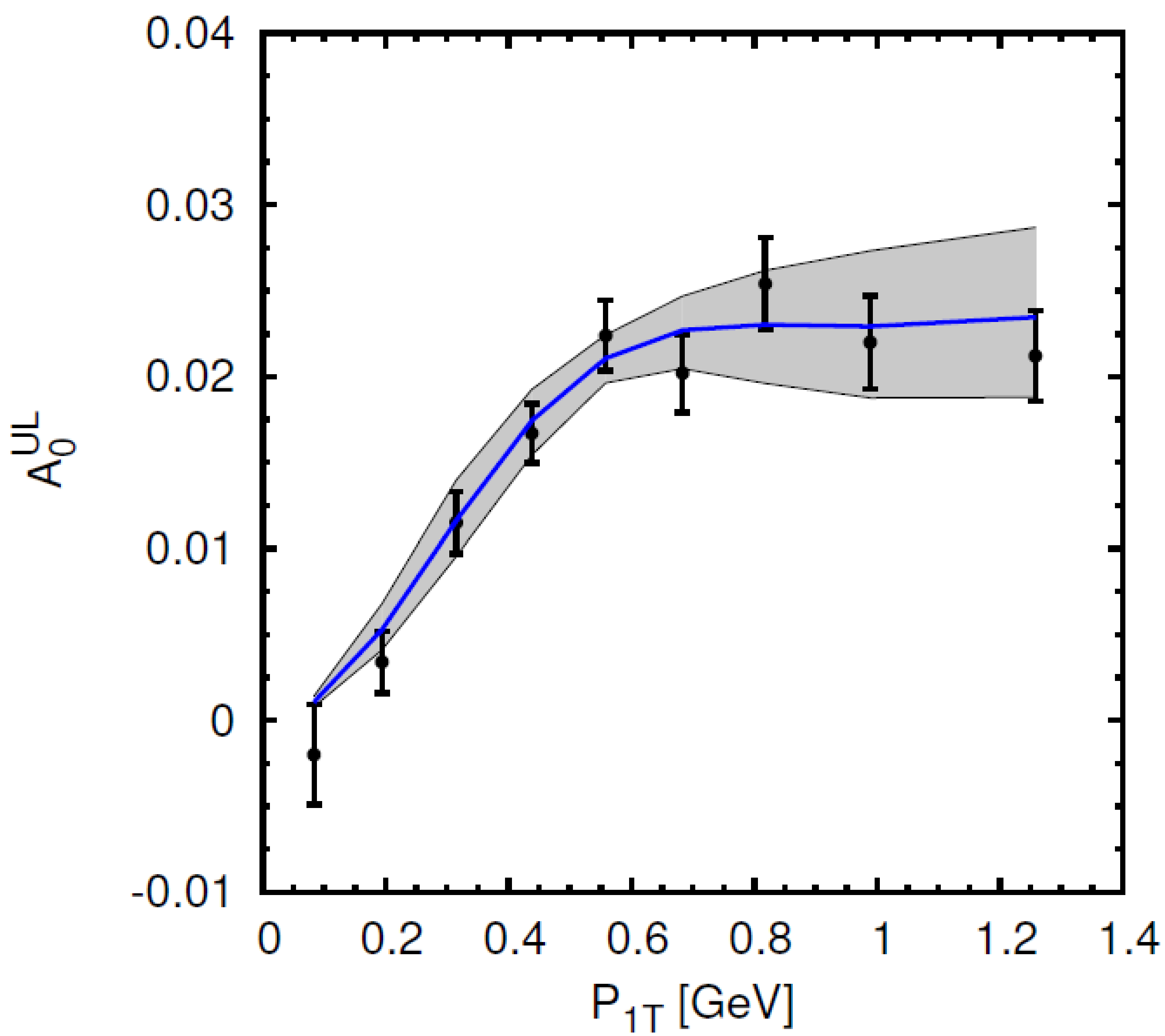


TMD evolution:

$Q^2 = 2.4\ \mathrm{GeV}^2$

$Q^2 = 10\ \mathrm{GeV}^2$

$Q^2 = 1000\ \mathrm{GeV}^2$

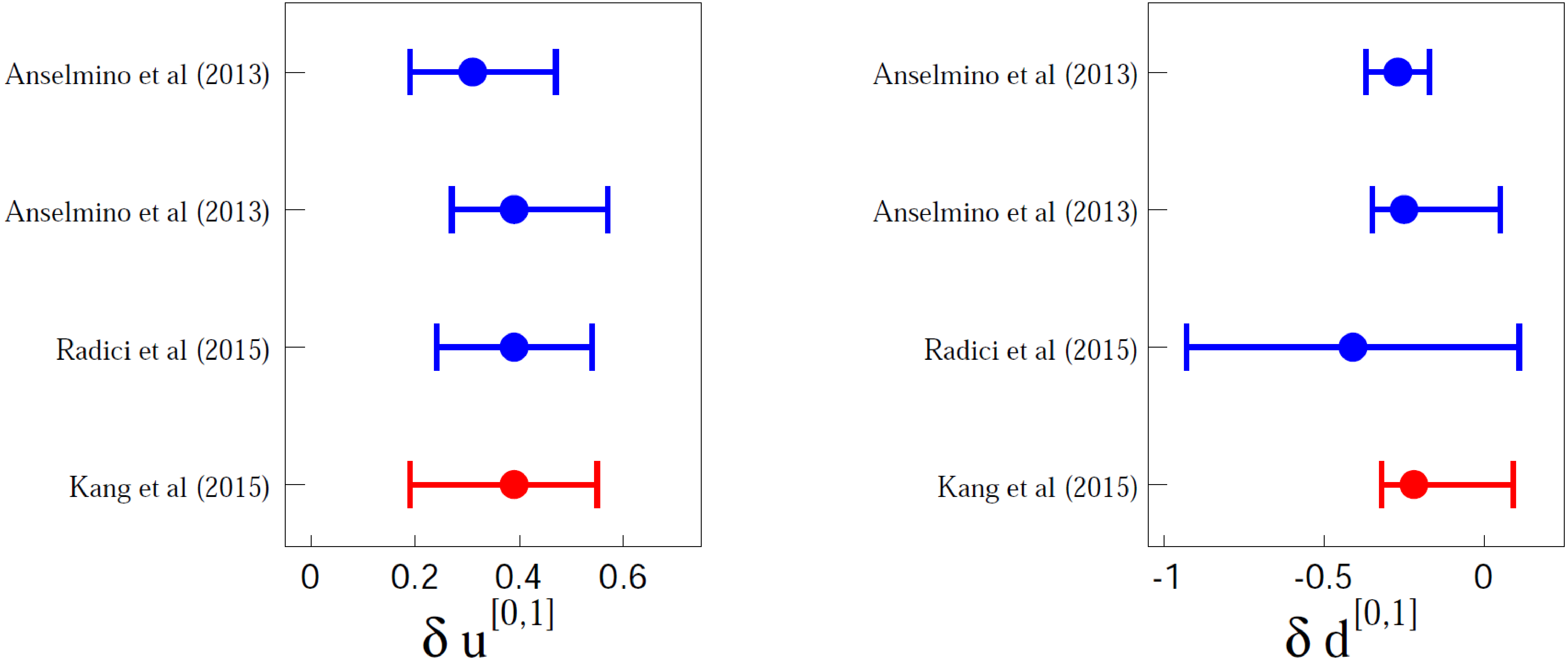


Anselmino et al (2015)

No TMD evolution, LO fit

Tensor charge

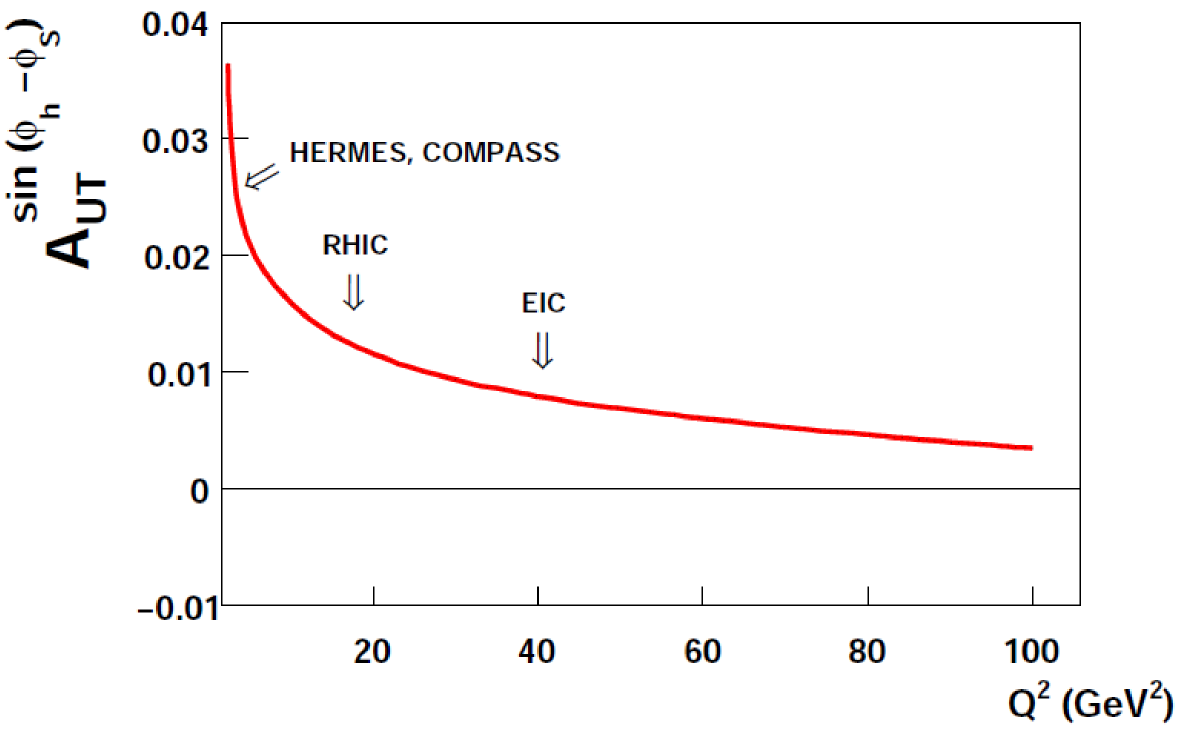
Kang-Prokudin-Sun-Yuan 2014



Is evolution “fast” or “slow”?

Evolution depends of non perturbative Sudakov form factor

Aybat, Prokudin, Rogers (2011)



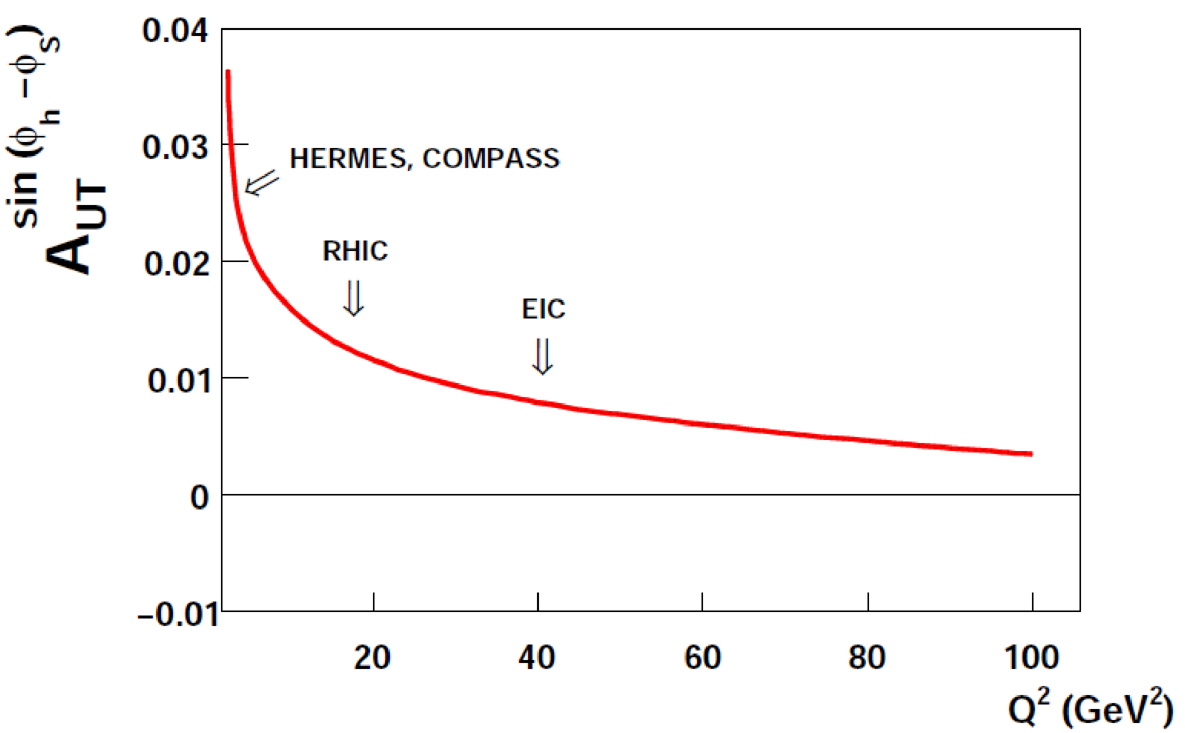
$$S_{NP} \sim g_2 b^2 \ln(Q^2/Q_0^2)$$

g_2 evaluated from high-energy data

Is evolution "fast" or "slow"?

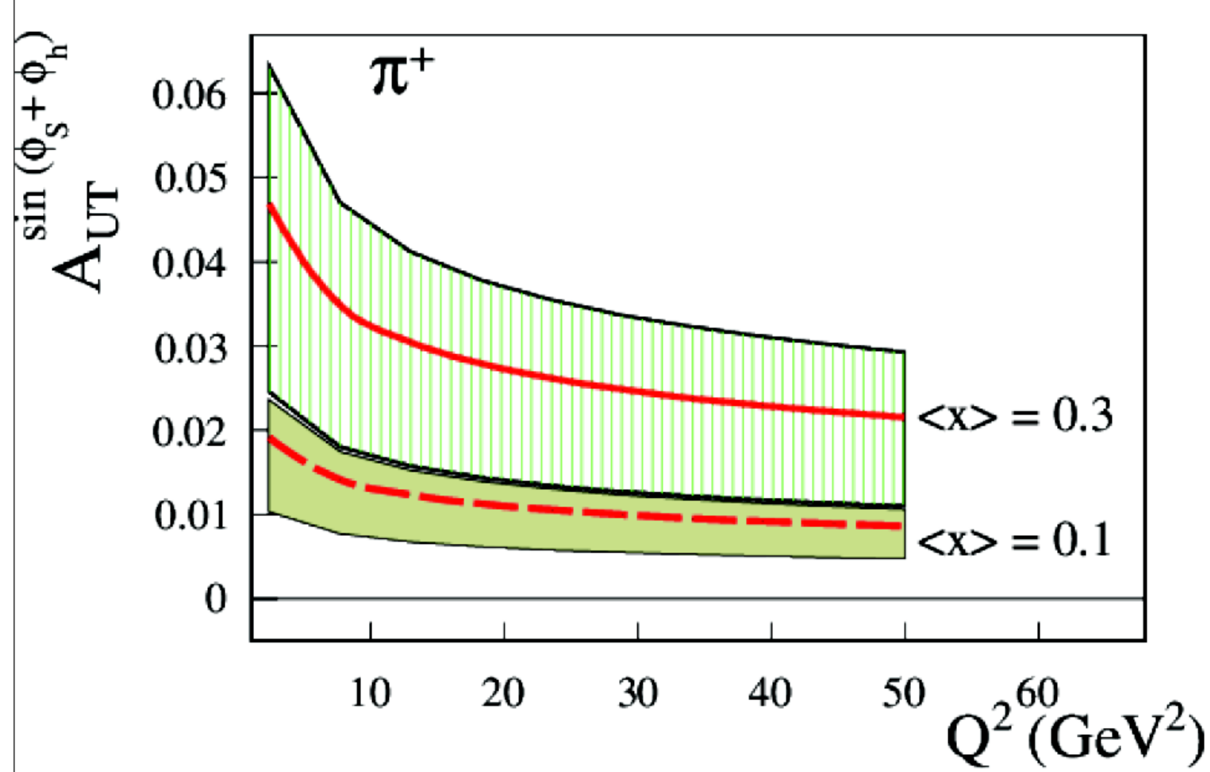
Evolution depends of non perturbative Sudakov form factor

Aybat, Prokudin, Rogers (2011)



$$S_{NP} \sim g_2 b^2 \ln(Q^2/Q_0^2)$$

Kang-Prokudin-Sun-Yuan 2014

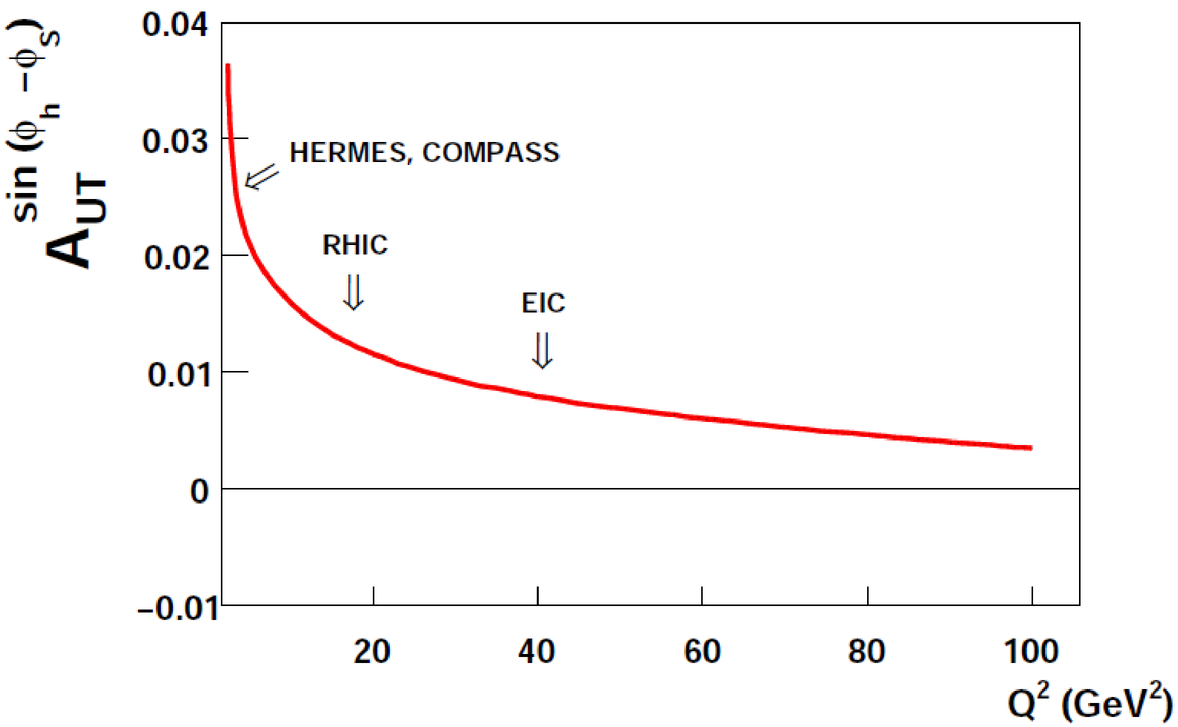


$$S_{NP} \sim g_2 \ln(1 + b^2/b_{max}^2) \ln(Q^2/Q_0^2)$$

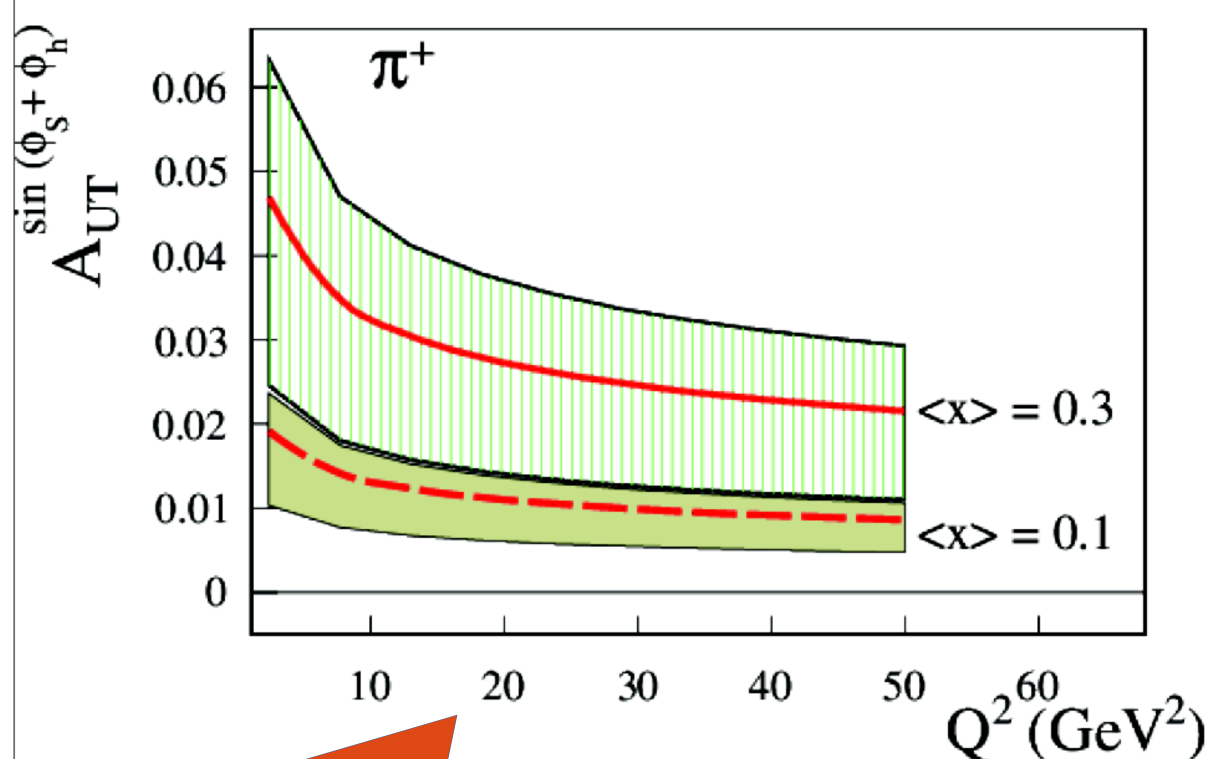
g_2 evaluated from low and high-energy data

Is evolution “fast” or “slow”?

Aybat, Prokudin, Rogers (2011)



Kang-Prokudin-Sun-Yuan 2014



$S_{NP} \sim g_2 b^2$ … $) \ln(Q^2/Q_0^2)$

… high-energy data

The same evolution equations may result in different numerical results.
Global data analysis is needed.

PENNSTATE Berks

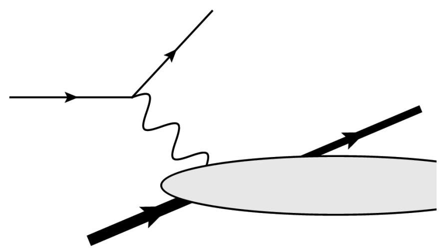
Conclusions

~~Conclusions~~

My wish-list for transverse PP data

Complementarity of SIDIS, e+e- and Drell-Yan, and hadron-hadron

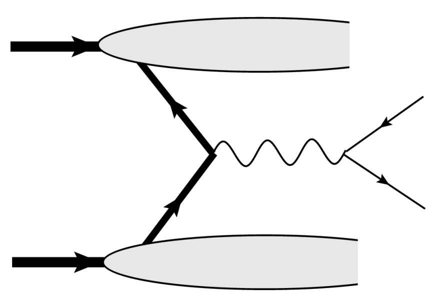
Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them



$$f(x) \otimes D(z)$$

Semi Inclusive DIS –
convolution of distribution functions and fragmentation functions

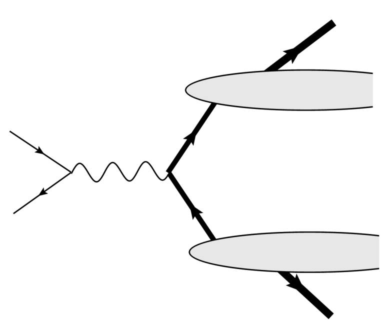
$$\ell + P \to \ell' + h + X$$



$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

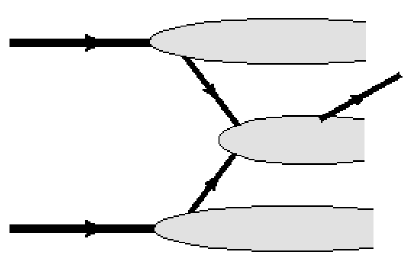
$$P_1 + P_2 \to \bar{\ell}\ell + X$$



$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \to h_1 + h_2 + X$$

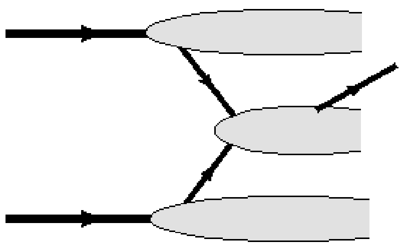


$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and fragmentation functions

$$h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

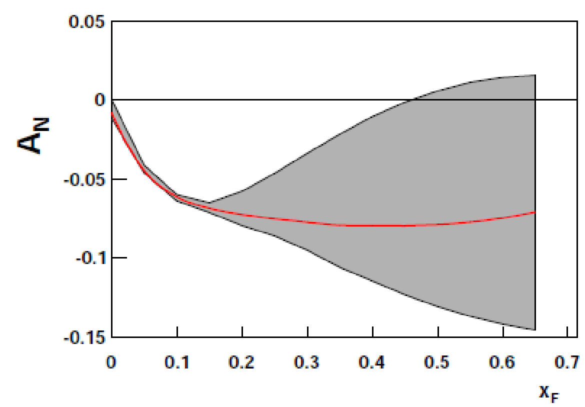
Combining measurements from all above is important



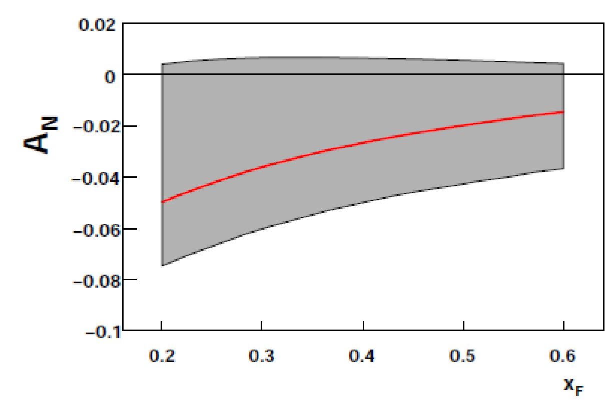
$$f(x_1) \otimes f(x_2) \otimes D(z) \qquad h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

What I (personally) wish to see from transverse spin measurements at RHIC:

Drell-Yan Measurement

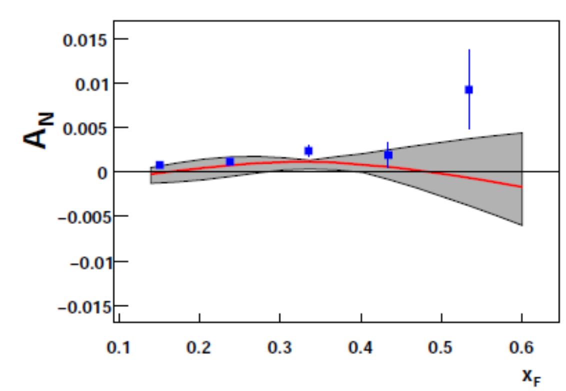


Direct photon measurement



W/Z measurement (arXiv today!)

Pion in jet measurement



Jet asymmetry measurement

Etc, etc, etc

THANK YOU!