Many body QCD: from RHIC (& LHC) to the EIC

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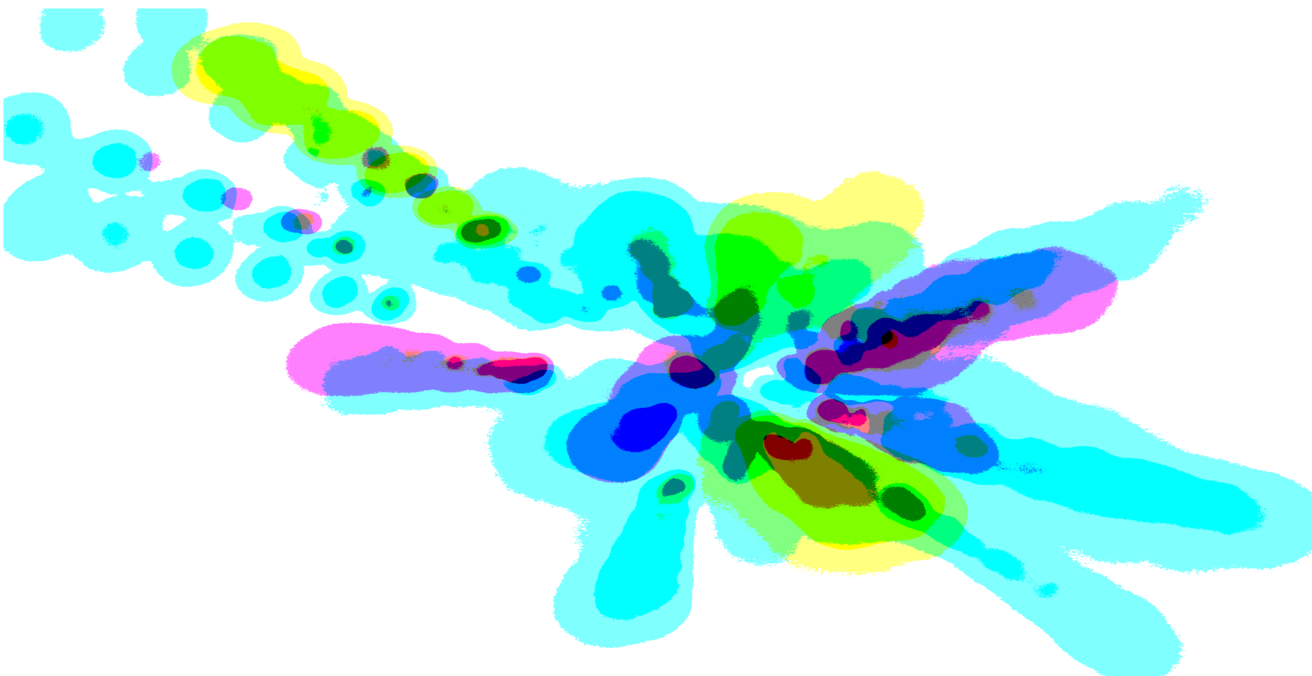


BNL Drell-Yan workshop, May 11-13, 2011

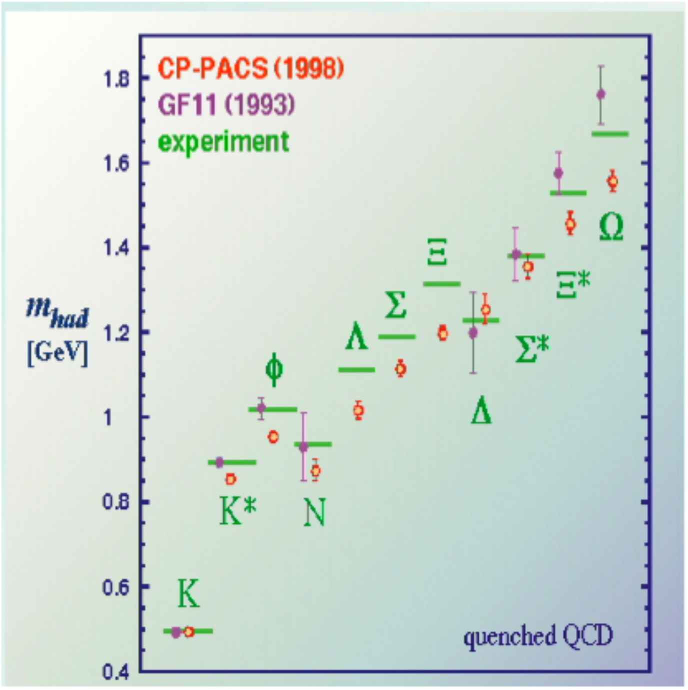
Many body QCD @ RHIC

- If QCD is the "perfect theory", a serious study of its many body features is of fundamental interest. Many body QED constitutes a large part of present day physics
- RHIC has ushered in a new era of studies of many body QCD: jet quenching, perfect fluidity, gluon saturation... unanticipated connections to other sub-fields in physics
- A quantitative understanding demands ultimately no less than understanding the high energy (many body) structure of hadrons
- A lot can be learned from Drell-Yan and other hadron-hadron final states. Isolating universal structure and precision studies of final states will require a high luminosity polarized electron-ion collider

The big picture

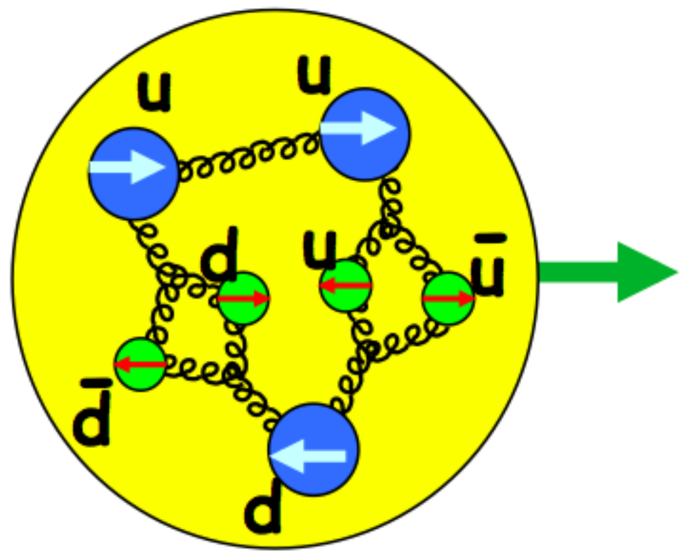
How do the fundamental constituents of the theory form matter (hadrons) ?

One answer: lattice QCD



Absolutely essential but also far from the full story…how can one connect this to light front dynamics of high energy scattering ?

The hadron as a many body system



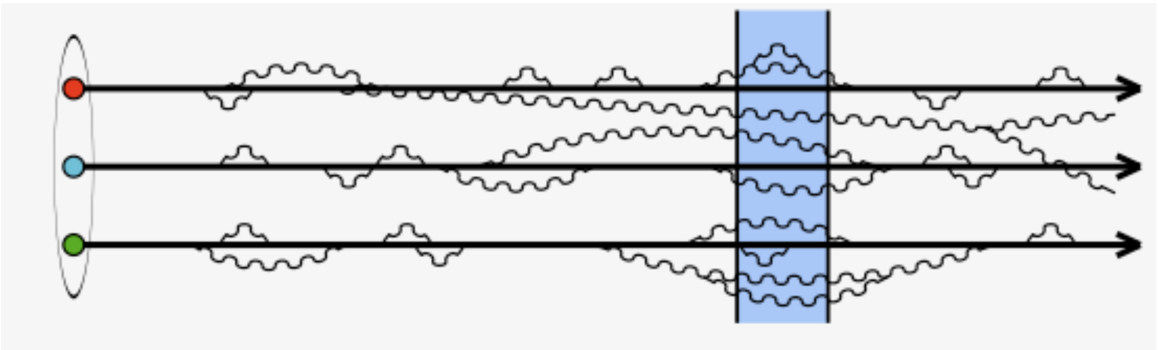
How do these many body quark and gluon fluctuations constitute the Mass, Spin, Flavor of the hadron ?

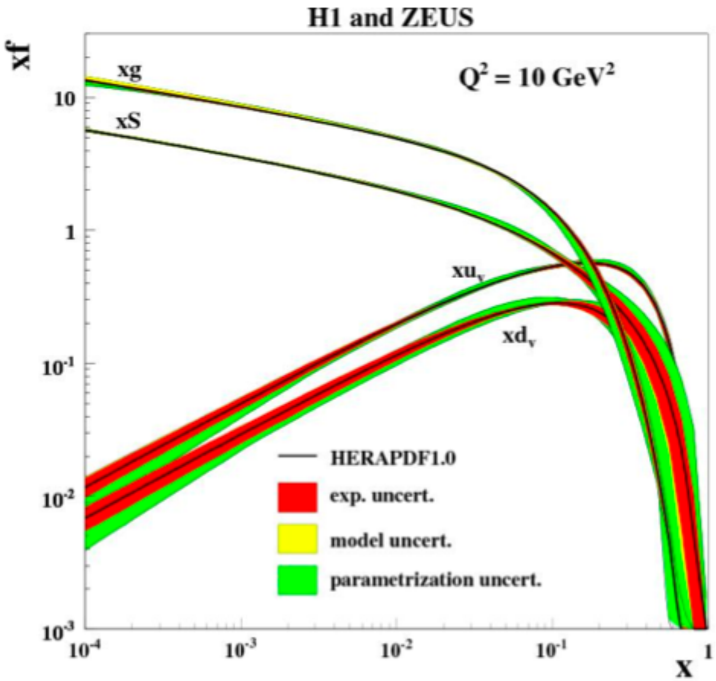
The additive quark model describes hadron spectroscopy but fails spectacularly for spin (Jaffe)

Also, is this picture invariant under boosts ?

The big picture

Taking snapshots of the hadron at short "time scales" helps "tease out" the underlying QCD dynamics





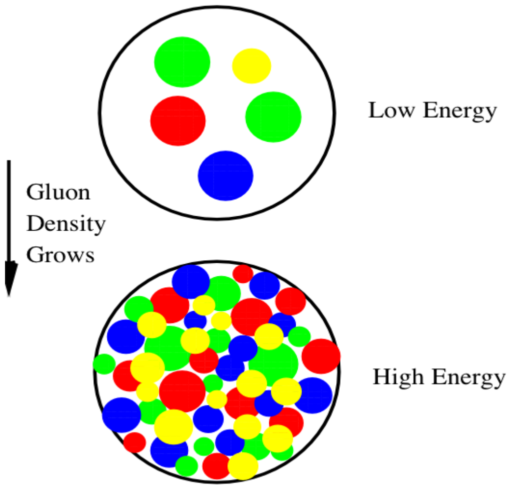
These configurations are no less "the proton"…
-they are the relevant configurations for high energy hadron/nuclear scattering

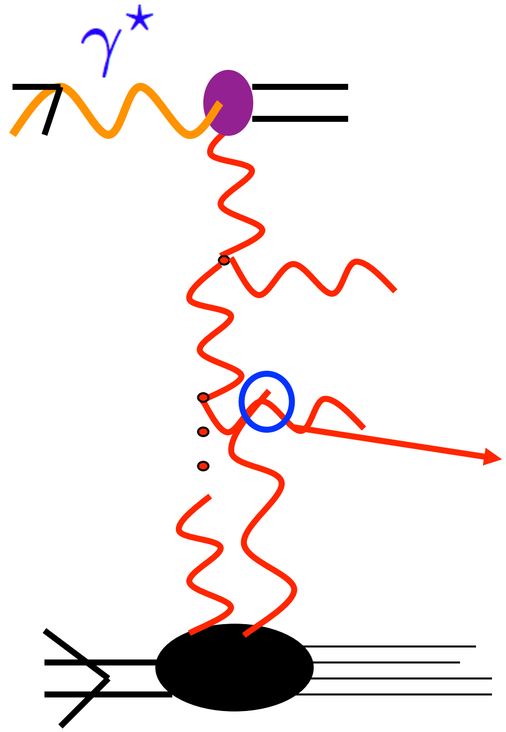
Can this approach to hadron structure provide more detailed insight into its many body, non-perturbative dynamics and help us ultimately construct a "boost invariant picture?

Gluon saturation: the Regge-Gribov limit

Gribov,Levin,Ryskin
Mueller,Qiu

$$x_{Bj} \to 0;\ s \to \infty;\ Q^2(>> \Lambda^2_{QCD}) = \text{fixed}$$





Large x - bremsstrahlung
linear evolution (DGLAP/BFKL)
-α_S ln(Q^2) / α_S ln(x) resummation

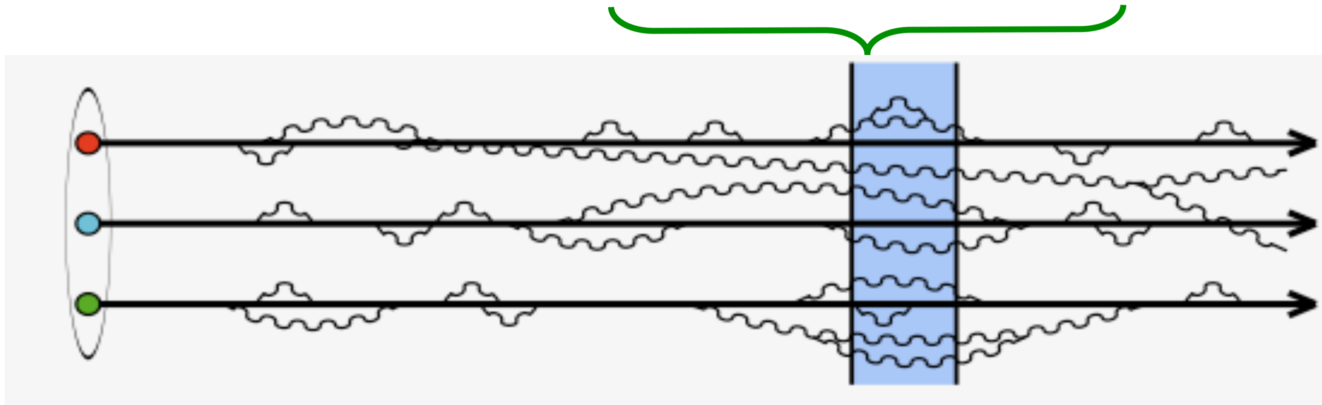
Small x -gluon recombination
non-linear evolution
(BK/JIMWLK)

Saturation scale $Q_S(x)$ - dynamical scale below which non-linear ("higher twist") QCD dynamics is dominant

In IMF, occupation # f = 1/α_S => hadron is a dense, many body system

EFT for the Regge-Gribov limit

$|A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q\,gg\ldots g\rangle$



What are the right degrees of freedom: classical fields+color sources, Dipoles+Multipoles, Pomerons+Reggeons, …. are these universal ?

What are their correlations- is there "long range order" ? Are there novel fixed points in the evolution ?

How does the weak coupling non-pert. dynamics of saturation match onto intrinsically non-pert. dynamics (Chiral symmetry breaking, Confinement)

I will address a small sub-set of these issues in the Color Glass Condensate framework of classical fields and color sources

Effective Field Theory on Light Front

Susskind
Bardacki-Halpern

Poincare group on LF $\longleftrightarrow$ Galilean sub-group of 2D Quantum Mechanics

isomorphism

Eg., LF dispersion relation

$$P^- = \frac{P_\perp^2}{2P^+}$$

Momentum

Mass

Energy

Large x (P^+) modes: static LF (color) sources ρ^a

Small x ($k^+ \ll P^+$) modes: dynamical fields A_μ^a

McLerran, RV

CGC: Coarse grained many body EFT on LF

$$< P|\mathcal{O}|P > \longrightarrow \int [d\rho^a][dA^{\mu,a}]\, W_{\Lambda^+}[\rho]\, e^{iS_{\Lambda^+}[\rho,A]}\, \mathcal{O}[\rho, A]$$

$W_{\Lambda^+}[\rho]$ non-pert. gauge invariant "density matrix" defined at initial scale Λ_0^+

RG equations describe evolution of W with x

JIMWLK, BK

Classical field of a large nucleus

$$< AA >_\rho = \int [d\rho] A_{\mathrm{cl.}}(\rho) A_{\mathrm{cl.}}(\rho) W_{\Lambda^+}[\rho]$$

For a large nucleus, A >>1,

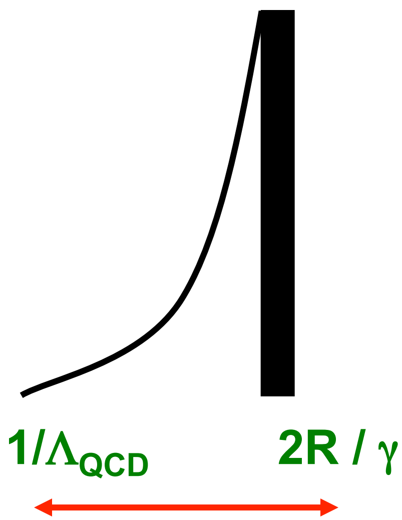
"Pomeron" excitations

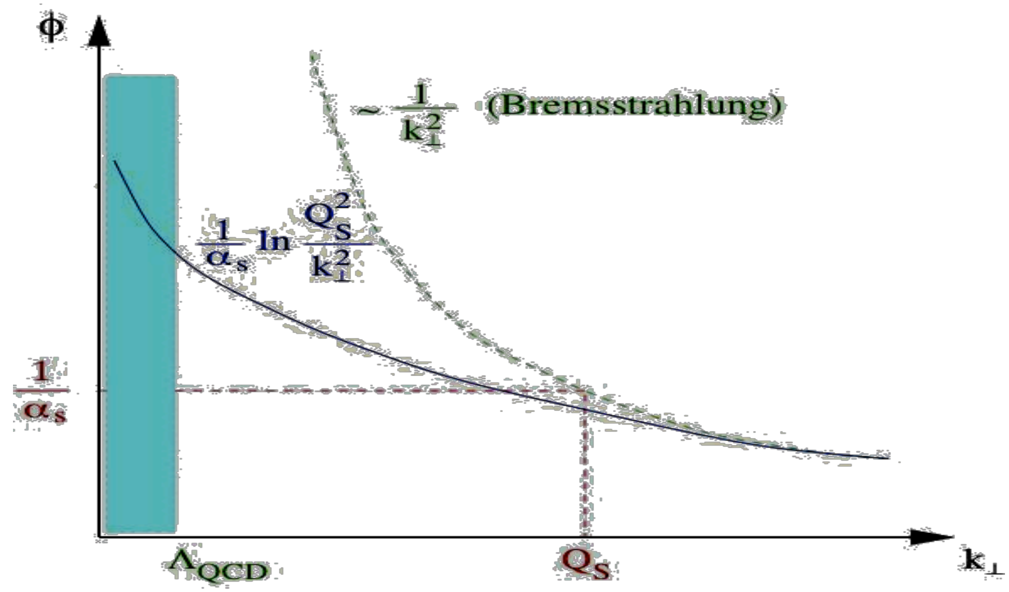
"Odderon" excitations

$$W_{\Lambda^+} = \exp\left(-\int d^2x_\perp \left[\frac{\rho^a \rho^a}{2\,\mu_A^2} - \frac{d_{abc}\,\rho^a \rho^b \rho^c}{\kappa_A}\right]\right)$$

McLerran,RV
Kovchegov
Jeon, RV

A_{cl} from $\longrightarrow$ $(D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu +}\,\delta(x^-)\,\rho^a(x_\perp)$



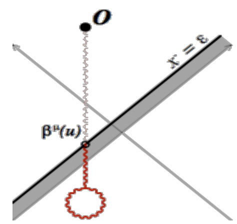


Wee parton dist. : $\frac{1}{\Lambda_{QCD}} e^{-\lambda \Delta Y/2}$

determined from RG

JIMWLK RG evolution for a single nucleus:

$$\mathcal{O}_{\rm NLO} = \Bigg($$



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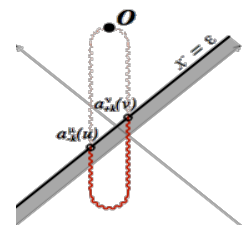


$\chi(x_\perp, y_\perp)$

$$\Bigg)\, \mathcal{O}_{\rm LO}$$

$$= \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H}\, \mathcal{O}_{\rm LO}$$

(keeping leading log divergences)

Gelis,Lappi,RV (2008)

$$\langle \mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \rangle = \int [d\tilde{\rho}] W[\tilde{\rho}] \left[\mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO}\right]$$

$$= \int [d\tilde{\rho}] \left\{ \left[1 + \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H} \right] W_{\Lambda^+} \right\} \mathcal{O}_{\rm LO}$$

LHS independent of Λ^+ **=>** $$\boxed{\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H}\, W[\tilde{\rho}]}$$

JIMWLK eqn.

Jalilian-Marian,Iancu,McLerran,Weigert,Leonidov,Kovner

Wee parton correlations

$$\langle O[\rho]\rangle_Y = \int [d\rho] O[\rho] W_Y[\rho]$$

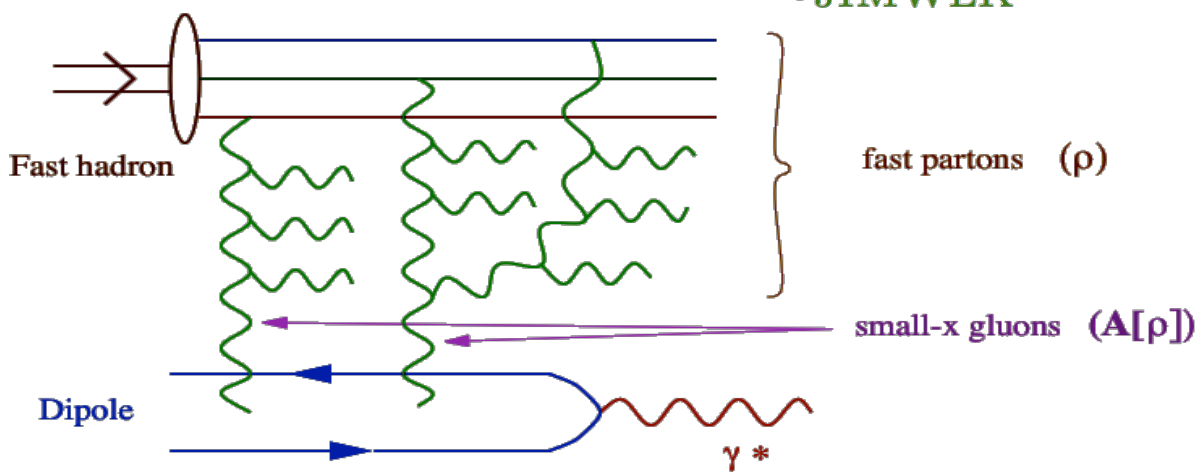
Fokker Planck eqn: Brownian motion in functional space

"diffusion coefficient"

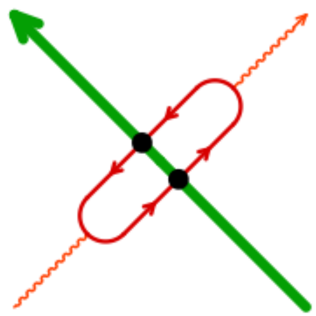
$$\frac{\partial W[\tilde\rho]}{\partial Y} = \mathcal{H}\, W[\tilde\rho] \quad => \frac{\partial}{\partial Y} < O[\alpha] >_Y = < \underbrace{\frac{1}{2}\int_{x,y} \frac{\delta}{\delta\alpha_Y^a(x)} \chi_{x,y}^{ab} \frac{\delta}{\delta\alpha_Y^b(y)} O[\alpha] >_Y}_{\mathcal{H}_{\mathrm{JIMWLK}}}$$

"time"

$$\alpha = \frac{\tilde\rho}{\nabla_\perp^2}$$



Inclusive DIS: dipole evolution

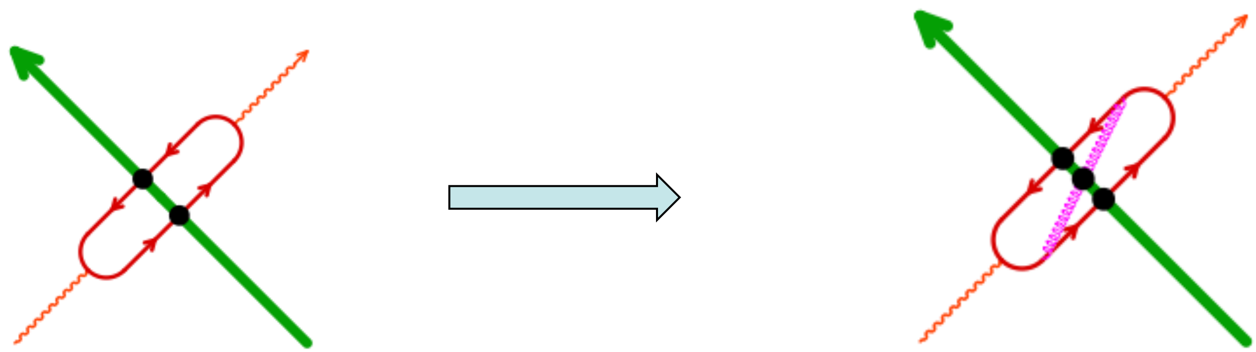


$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 r_\perp \, |\psi(z, r_\perp)|^2 \sigma_{\mathrm{dipole}}(x, r_\perp)$$

$$\sigma_{\mathrm{dipole}}(x, r_\perp) = 2 \int d^2 b \int [D\rho] \, W_{\Lambda^+}[\rho] \, T(b + \frac{r_\perp}{2}, b - \frac{r_\perp}{2})$$

$$1 - \frac{1}{N_c} \mathrm{Tr}\left(V\left(b + \frac{r_\perp}{2}\right) V^\dagger \left(b - \frac{r_\perp}{2}\right)\right)$$

Inclusive DIS: dipole evolution



B-JIMWLK eqn. for dipole correlator

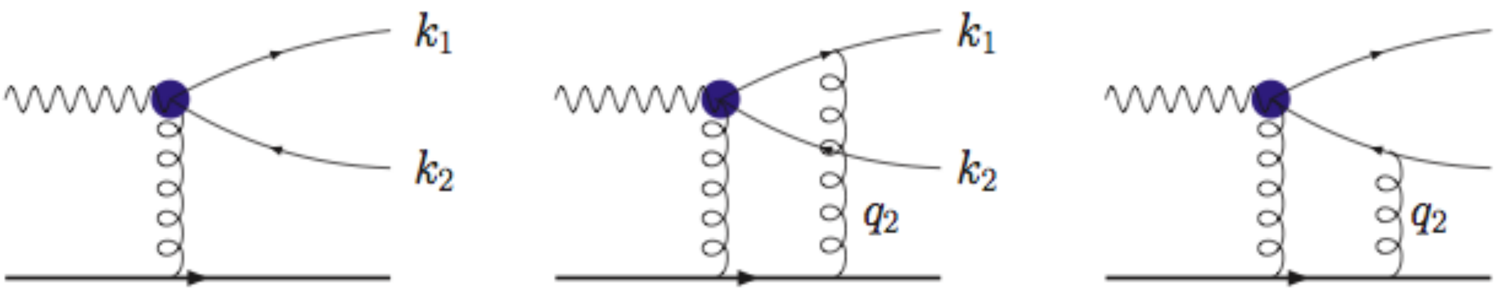
$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Dipole factorization:

$$\langle \mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \longrightarrow \langle \mathrm{Tr}(V_x V_z^\dagger)\rangle_Y \langle \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \qquad N_c \rightarrow \infty$$

Resulting closed form eqn. is the Balitsky-Kovchegov eqn.
Widely used in phenomenological applications

Semi-inclusive DIS: quadrupole evolution



Dominguez,Marquet,Xiao,Yuan (2011)

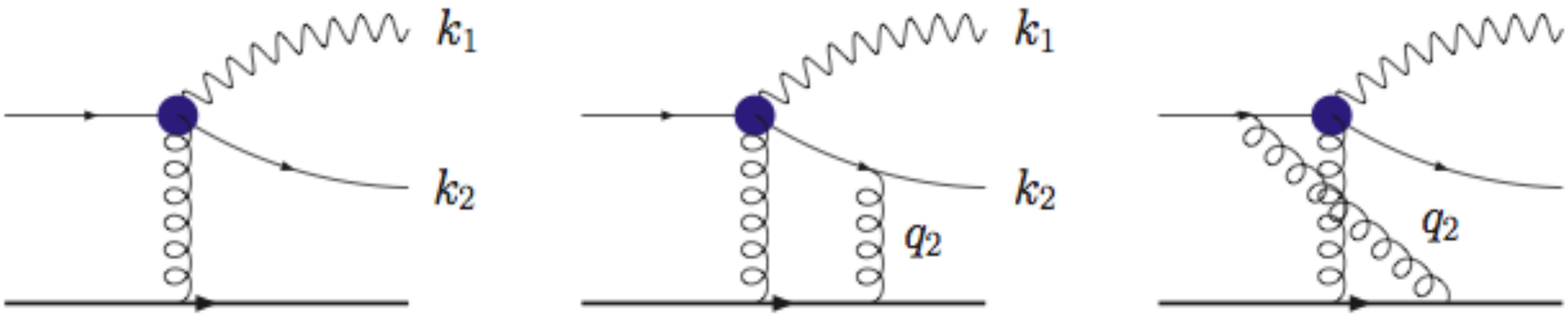
$$\frac{d\sigma^{\gamma^*_{\mathrm{T,L}}A\to q\bar{q}X}}{d^3k_1d^3k_2} \propto \int_{x,y,\bar{x}\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})}e^{ik_{2\perp}\cdot(y-\bar{y})}\ [1+Q(x,y;\bar{y},\bar{x})-D(x,y)-D(\bar{y},\bar{x})]$$

$$D(x,y)=\frac{1}{N_c}\langle\mathrm{Tr}(V_xV_y^\dagger)\rangle_Y \qquad Q(x,y;\bar{y},\bar{x})=\frac{1}{N_c}\langle\mathrm{Tr}(V_xV_{\bar{x}}^\dagger V_{\bar{y}}V_y^\dagger)\rangle_Y$$

Cannot be further simplified a priori even in the large N_c limit

(See talks by Mueller, Xiao, Jalilian-Marian)

Universality: Di-jets in p/d-A collisions



Jalilian-Marian, Kovchegov (2004)
Marquet (2007)
Dominguez,Marquet,Xiao,Yuan (2011)

$$\frac{d\sigma^{qA\to qgX}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x},\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})} e^{ik_{2\perp}\cdot(y-\bar{y})} \left[S_6(x,y,\bar{x},\bar{y}) - S_4(x,y,v) - \ldots\right]$$

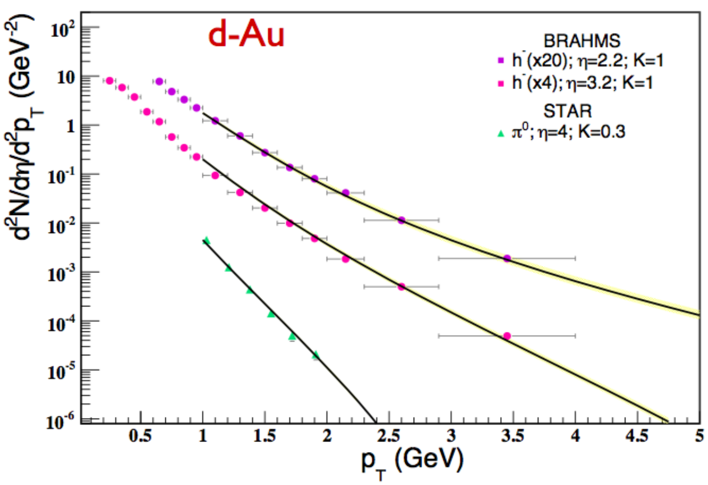
$$\frac{N_c}{2\,C_F}\left\langle Q(x,y\,\bar{y},\bar{x})D(y,\bar{y}) - \frac{D(x,\bar{x})}{N_c}\right\rangle \qquad \frac{N_c}{2\,C_F}\left\langle D(x,y)D(\bar{y},\bar{x}) - \frac{D(x,\bar{x})}{N_c}\right\rangle$$

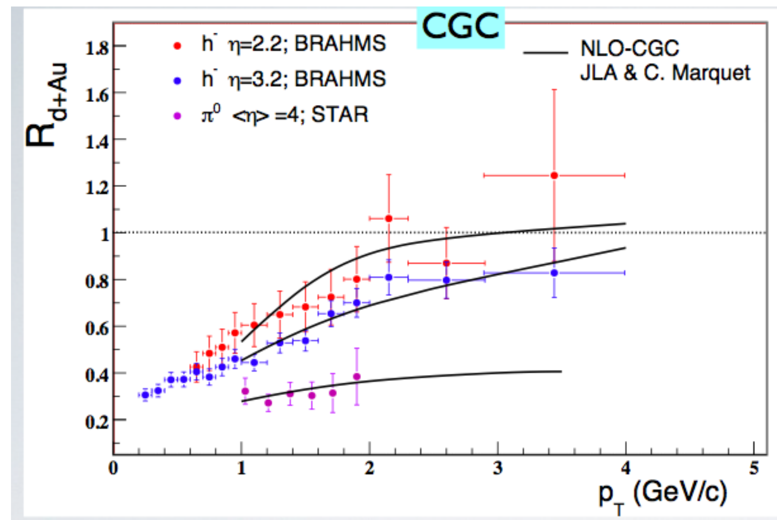
Fundamental ingredients are the universal dipoles and quadrupoles

CGC phenomenology

"State of the art": running coupling BK eqn

Albacete,Marquet (2010)





Parameters constrained by i) fits to HERA F2 data

ii) fits to NMC nuclear F2 data

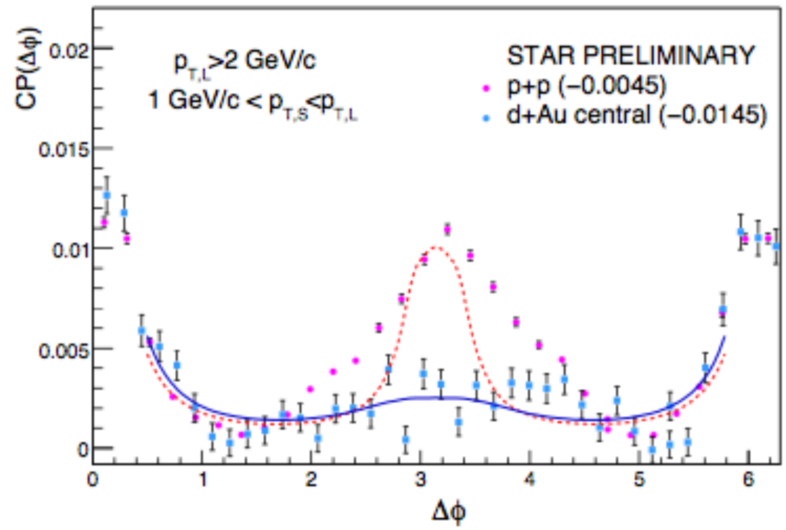
Albacete,Armesto,Milhano,Salgado
Dusling,Gelis,Lappi,RV

In the JIMWLK framework, relies on "dipole factorization"
Is this OK ?

CGC phenomenology

Albacete,Marquet (2010)

d+A: di-hadron data



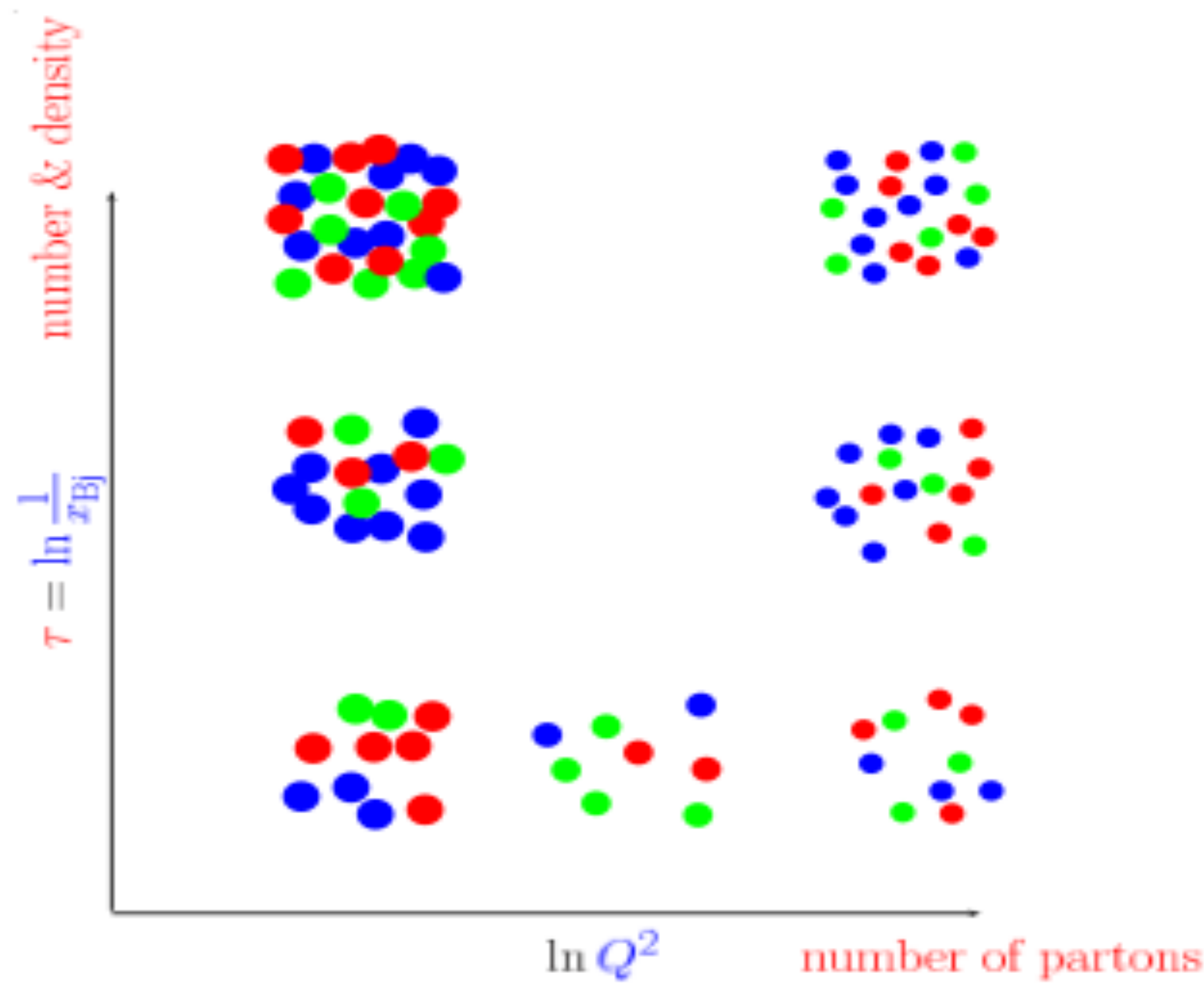
This assumes both Dipole and Quadrupole factorization:

The latter is a no-no (see Dumitru-Jalilian-Marian ; Dominguez et al.)

- **Need to do better for this and other exclusive final states**

(Talk by Bowen Xiao)

Solving the B-JIMWLK hierarchy



Solving the B-JIMWLK hierarchy

- JIMWLK includes all multiple scattering and leading log evolution in x

- Expectation values of Wilson line correlators at small x satisfy a Fokker-Planck eqn. in functional space

Weigert (2000)

- This translates into a hierarchy of equations for n-point Wilson line correlators

- As is generally the case, Fokker-Planck equations can be re-expressed as Langevin equations – in this case for Wilson lines

Blaizot,Iancu,Weigert
Rummukainen,Weigert

First numerical solutions exist: I will report on recent developments

B-JIMWLK hierarchy: Langevin realization

Numerical evaluation of Wilson line correlators on 2+1-D lattices:

$$\left\langle \mathcal{O}[U] \right\rangle_Y = \int D[U]\, W_Y[U]\, \mathcal{O}[U] \longrightarrow \frac{1}{N} \sum_{U \epsilon W} \mathcal{O}[U]$$

Langevin eqn:

Gaussian random variable

$$\partial_Y [V_x]_{ij} = [V_x i t^a]_{ij} \left[\int d^2y\, [\mathcal{E}^{ab}_{xy}]_k\, [\xi^b_y]_k + \sigma^a_x \right]$$

$$\mathcal{E}^{ab}_{xy} = \left(\frac{\alpha_S}{\pi^2}\right)^{1/2} \frac{(x-y)_k}{(x-y)^2} \left[1 - U^\dagger_x U_y\right]^{ab}$$

"square root" of JIMWLK kernel

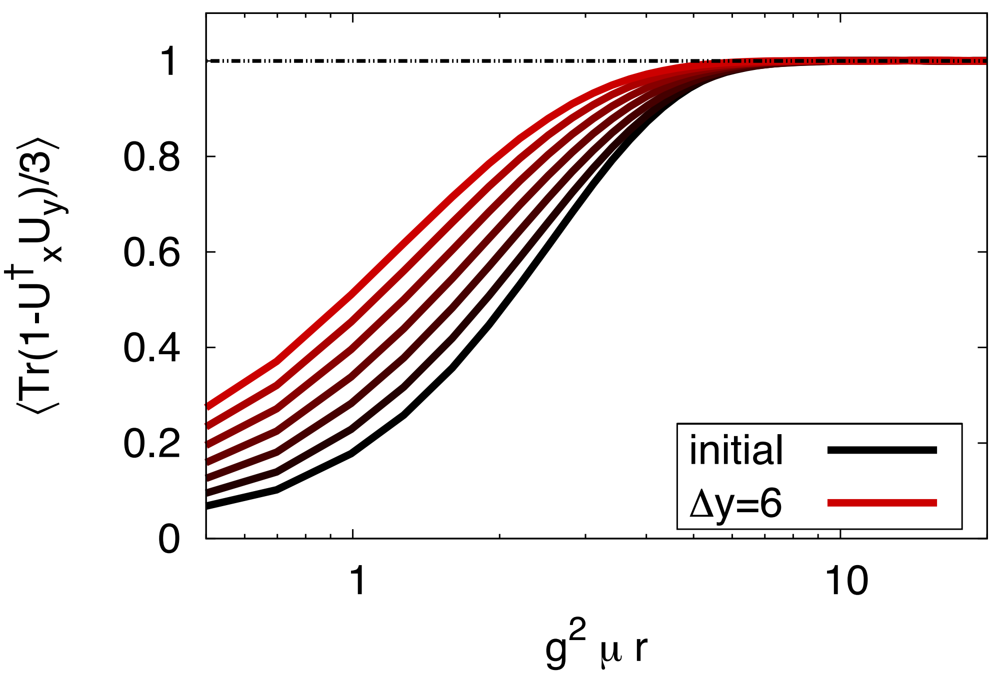
$$\sigma^a_x = -i \left(\frac{\alpha_S}{2\pi^2} \int d^2z \frac{1}{(x-z)^2} \mathrm{Tr}(T^a\, U^\dagger_x U_z) \right)$$

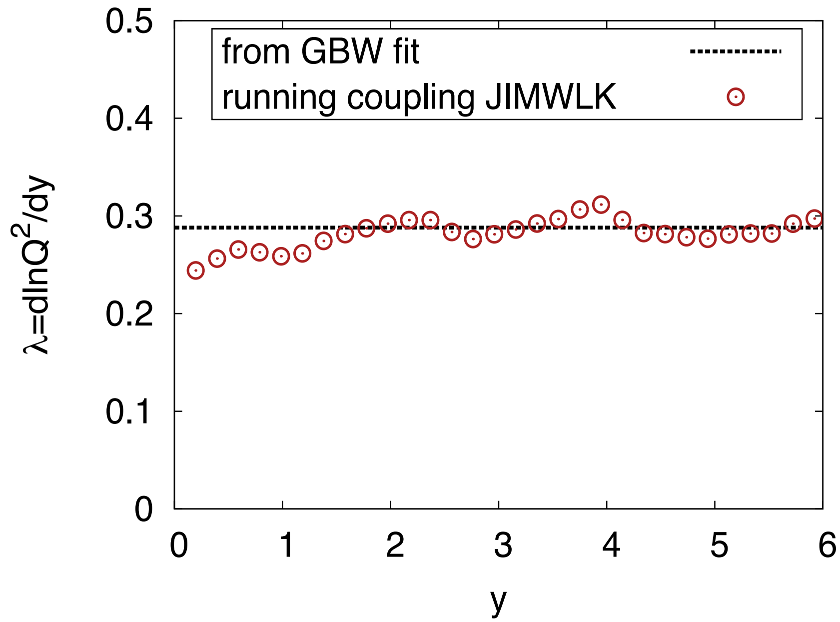
"drag"

- Initial conditions for V's from the MV model
- Daughter dipole prescription for running coupling

Numerical results-I

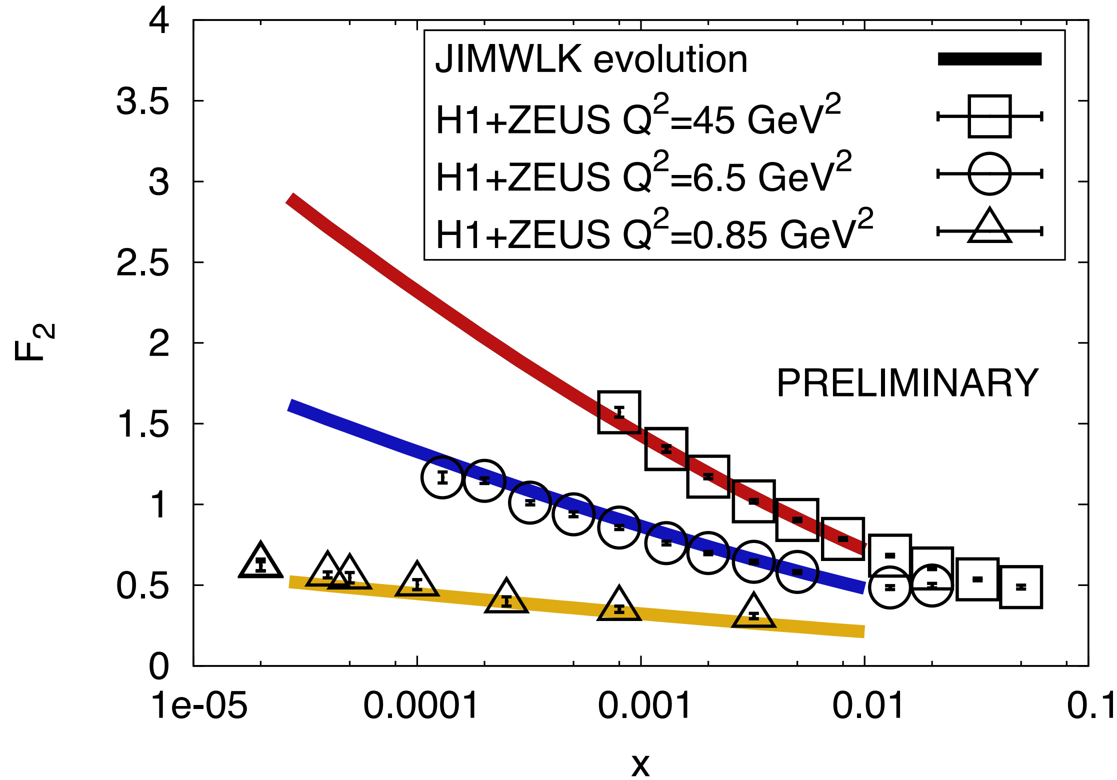
Lappi,Schenke,RV





- Gaussian (MV) initial condition with $g^2\mu$=0.5 GeV
- Parameters: running coupling frozen at scale μ_0 – results are rather insensitive
- Λ_{QCD} ~ 100 MeV
- Overall normalization ~ 25 mb (consistent with other studies)

Numerical results-II



- **Gaussian (MV) initial condition with $g^2\mu$=0.5 GeV**
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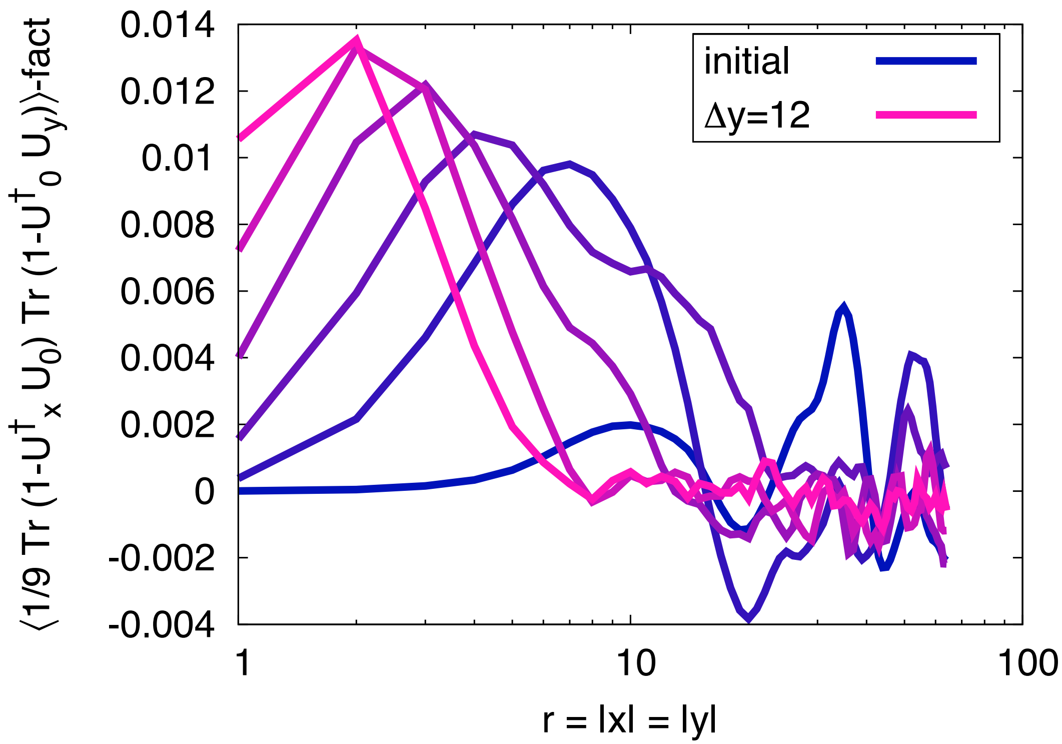
Numerical results-III

Lappi,Schenke,RV



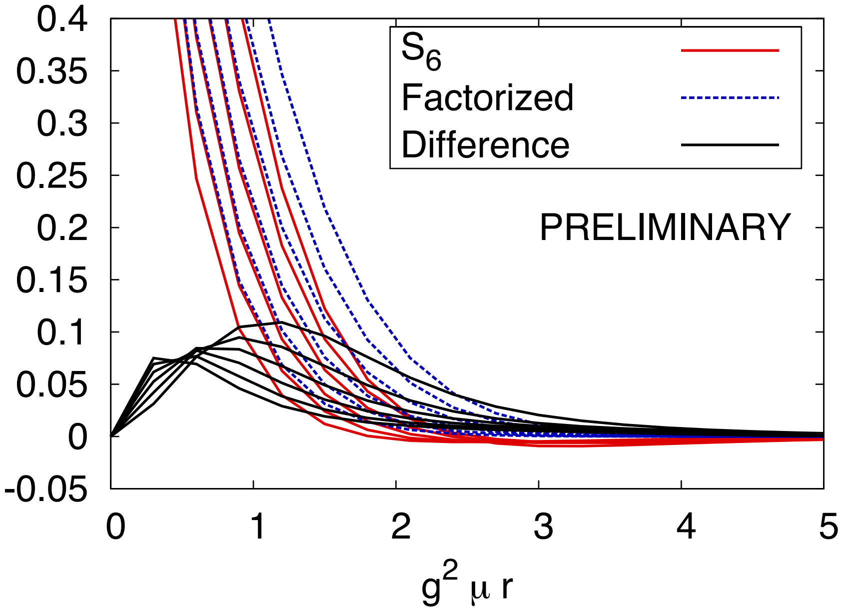
$$\langle \mathrm{Tr}(V_x V_z^\dagger)\ \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \longrightarrow \langle \mathrm{Tr}(V_x V_z^\dagger)\rangle_Y \langle \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

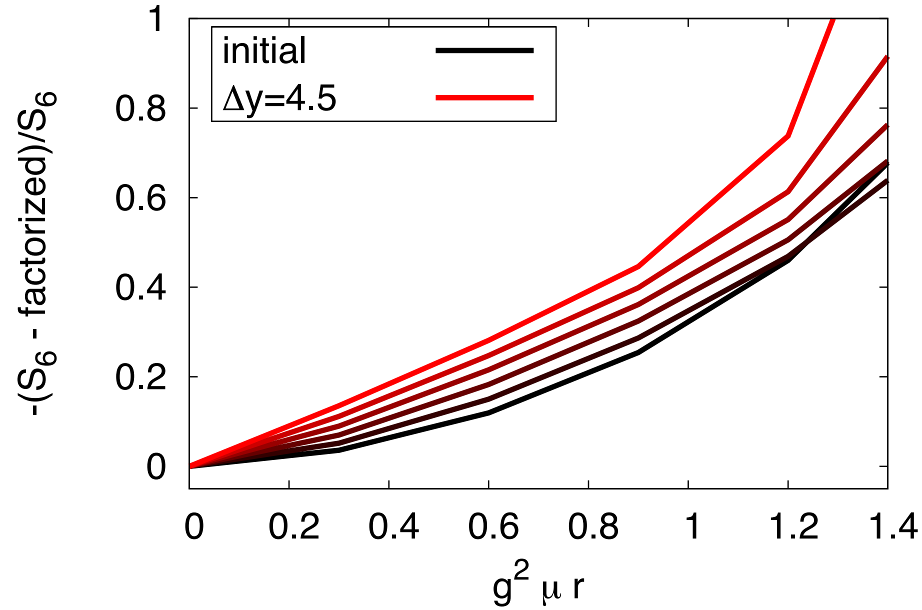
Dipole factorization quite a good approximation in line with Rummukainen-Weigert result for fixed coupling

Numerical results-IV

Lappi,Schenke,RV

How about the quantity S_6 containing quadrupoles that appear in di-hadron correlations ?





Violations large for large r and for large Y
(i.e., when saturation effects are important) – confirming analytical estimates

Outlook - I

❖ The JIMWLK hierarchy contains non-trivial "many body" correlations -these are now being explored using numerical and analytical techniques

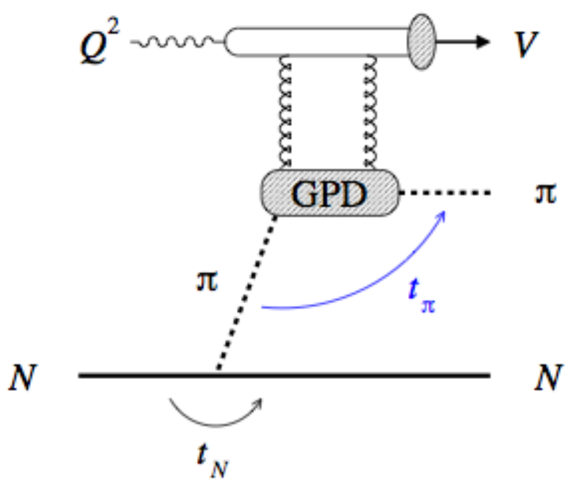
❖ It is likely that they could be inferred (given sufficient precision) from experiments thereby providing key insight into QCD many body dynamics in the Regge-Gribov limit

❖ There are many open questions that hopefully will be resolved in the next decade, such as i) NLL corrections, ii) matching to OPE based analyses at larger x and Q^2

Outlook - II

❖ A particularly compelling open issue is the treatment of impact parameter dependence in the EFT

❖ It provides the interface between non-perturbative weak coupling dynamics and fundamental features of the theory such as chiral symmetry breaking and confinement…eg., a dynamical understanding of the Froissart bound



❖ An EIC is powerful machine to address these issues by looking at a range of exclusive final states