$$\mathrm{d}\Delta\hat{\sigma} = \mathrm{d}\hat{\sigma}^{\ell q^{\uparrow} \to \ell q^{\uparrow}} - \mathrm{d}\hat{\sigma}^{\ell q^{\uparrow} \to \ell q^{\downarrow}}$$