Nuclear Physics & RIKEN Theory Seminar
Physics Department, Brookhaven National Laboratory, May 1, 2009

Spin – How Much Do We Know?

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Based on work with Collins, Ji, Kang, Kouvaris, Sterman, Vogelsang, and Yuan

Particle's Intrinsic Physical Properties

❑ Mass:

- ❖ Einstein: $M = E/c^2$ (fundamental constant *c*)
- ❖ Not sure its origin (beyond the Standard Model?)

❑ Charge:

- ❖ Interaction and interaction strength

❑ Spin:

- ❖ Pauli (1924): two-valued quantum degree of freedom of electron
- ❖ Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant $\hbar$)
- ❖ Composite particle:
  - o Total angular momentum when it is at rest
  - o Interaction dynamics

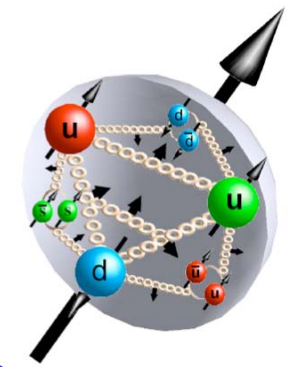
Spin of a composite particle

❑ Spin of a nucleus:

- ❖ Nuclear binding: 8 MeV/nucleon << mass of nucleon
- ❖ Nucleon number is fixed inside a given nucleus
- ❖ Spin of a nucleus = sum of the valence nucleon spin

❑ Spin of a nucleon – Naïve Quark Model:

- ❖ If the probing energy << mass of constituent quark
- ❖ Nucleon is made of three constituent (valence) quark
- ❖ Spin of a nucleon = sum of the constituent quark spin



❑ Spin of a nucleon – QCD:

- ❖ Current quark mass << energy exchange of the collision
- ❖ Number of quarks and gluons depends on the probing energy

Proton spin in QCD

❑ **Angular momentum of a proton at rest:**

$$S = \sum_f \langle P, S_z = 1/2 |\, \hat{J}_f^z \,| P, S_z = 1/2 \rangle = \frac{1}{2}$$

❑ **QCD Angular momentum operator:**

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x \; M^{0jk}_{\rm QCD} \quad \Longleftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

Angular momentum density

❖ Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q \right]$$

❖ Gluon angular momentum operator:

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B}) \right]$$

Need matrix elements of these partonic operators

Sum rule for proton spin

- **Partonic contribution to the proton spin:**

If $\vec{P} = 0$, $\langle P, S|\vec{J}_{q,g}(\mu^2)|P, S\rangle \propto \vec{S}$

$$\Longrightarrow \langle P, S|\vec{J}_{q,g}(\mu^2)|P, S\rangle \equiv J_{q,g}(\mu^2)\, 2\vec{S}$$

Quark contribution: $J_q(\mu^2)$ Gluon contribution: $J_g(\mu^2)$

- **Ji's sum rule:**

$$\frac{1}{2} = J_q(\mu^2) + J_g(\mu^2) = \left[\frac{1}{2}\Sigma(\mu^2) + L_q(\mu^2)\right] + J_g(\mu^2)$$

Quark helicity: $\Sigma(\mu^2) = \int_0^1 dx \sum_f \left[\Delta q_f(x, \mu^2) + \Delta\bar{q}_f(x, \mu^2)\right]$

- **Calculation of these matrix elements:**

  - Proton wave function in terms of quarks and gluons – unknown
  - Lattice QCD: non-local operators

Question

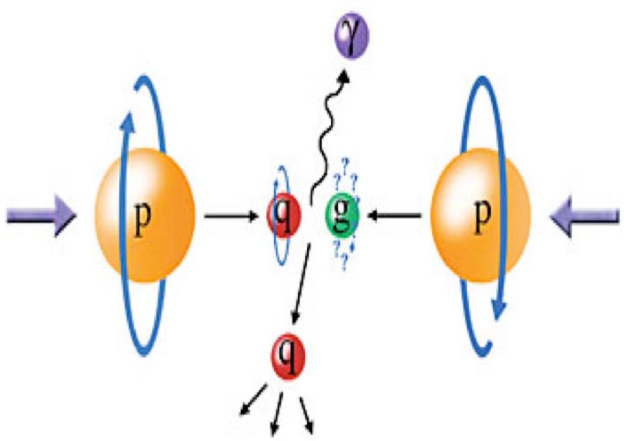
- Can we measure hadronic matrix elements of simple quark or gluon operators?

  Experiments measure hadronic cross sections

  Many parton could participate in the hadronic collisions

- Approximation:

  High energy scattering is dominated by single parton collision

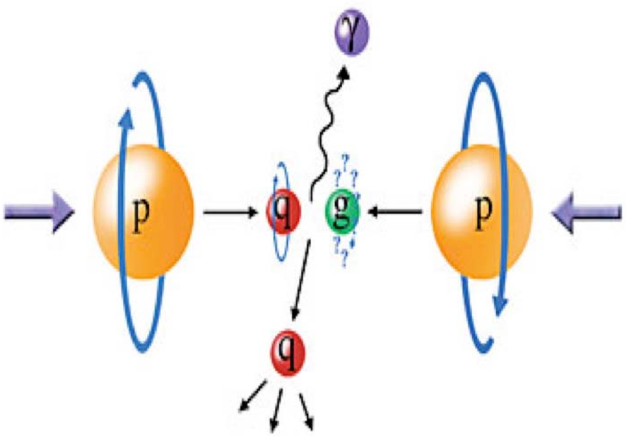


$\Longrightarrow$ Hadronic matrix elements at $p \neq 0$

OK for matrix elements independent of p

Connecting the partons to the hadrons

❑ Factorization – approximation:

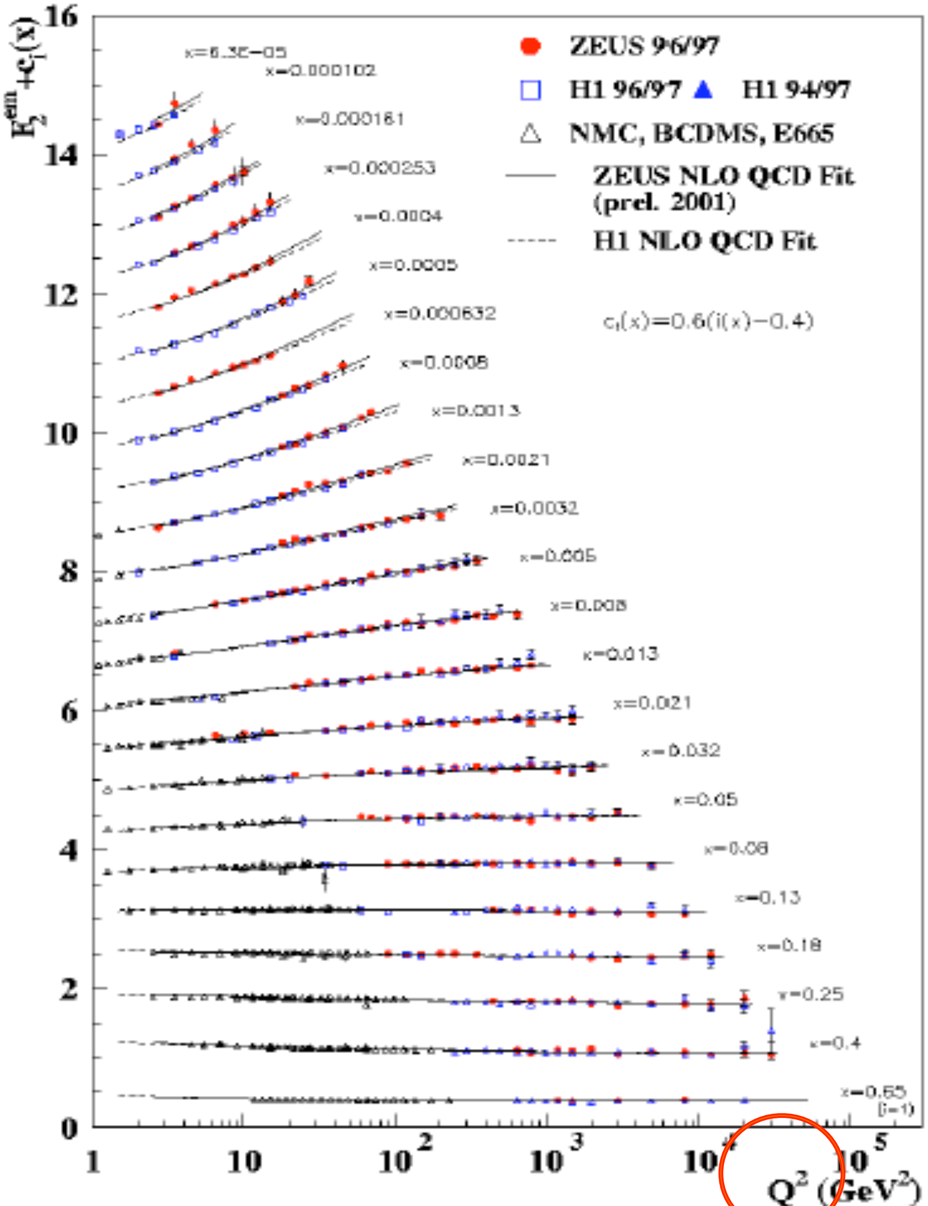


$$\frac{d\sigma}{dy\,dp_T^2} = \int \frac{dx}{x} q(x) \int \frac{dx'}{x'} g(x') \frac{d\hat{\sigma}_{qg\to\gamma q}}{dy\,dp_T^2} + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{1}{p_T^\alpha}\right)$$

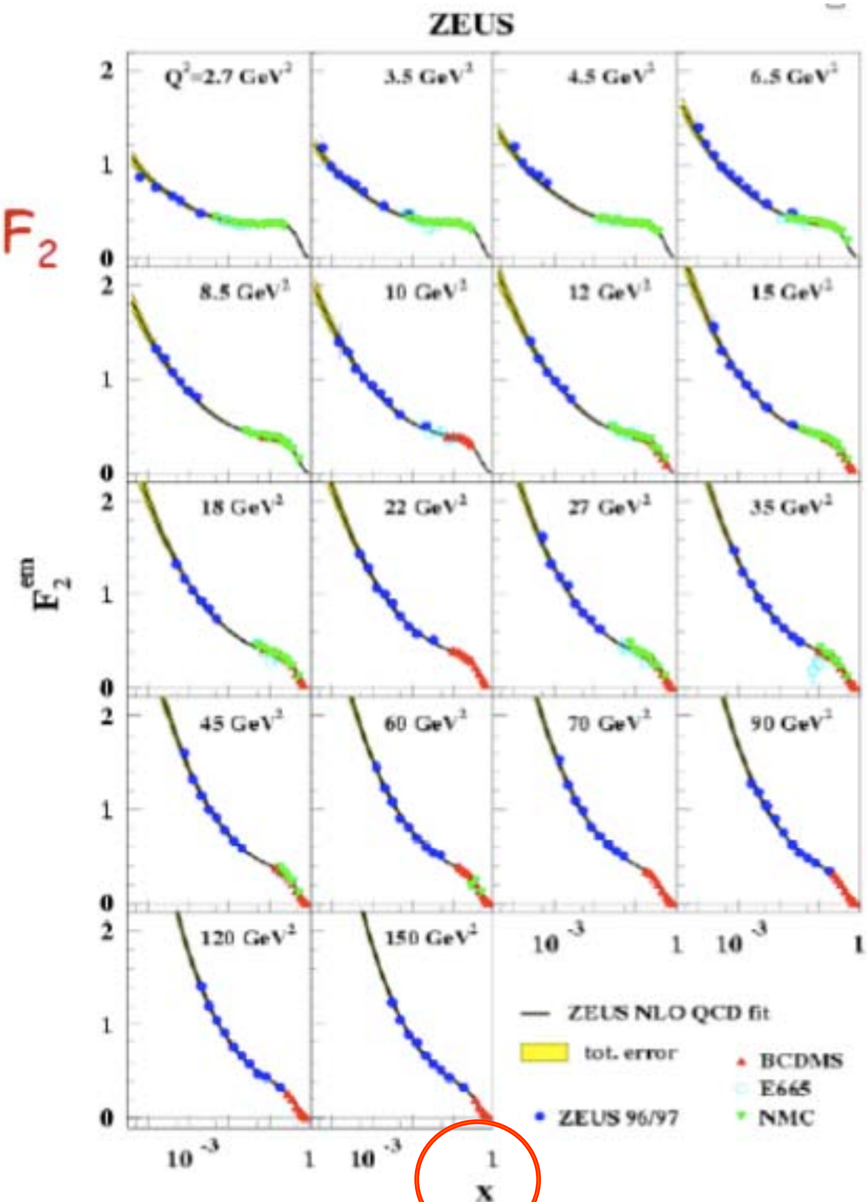
❑ Predictive power:

- ❖ short-distance and long-distance are separately gauge invariant
- ❖ short-distance part is Infrared-Safe, and calculable
- ❖ long-distance part can be defined to be Universal

Unpolarized inclusive DIS

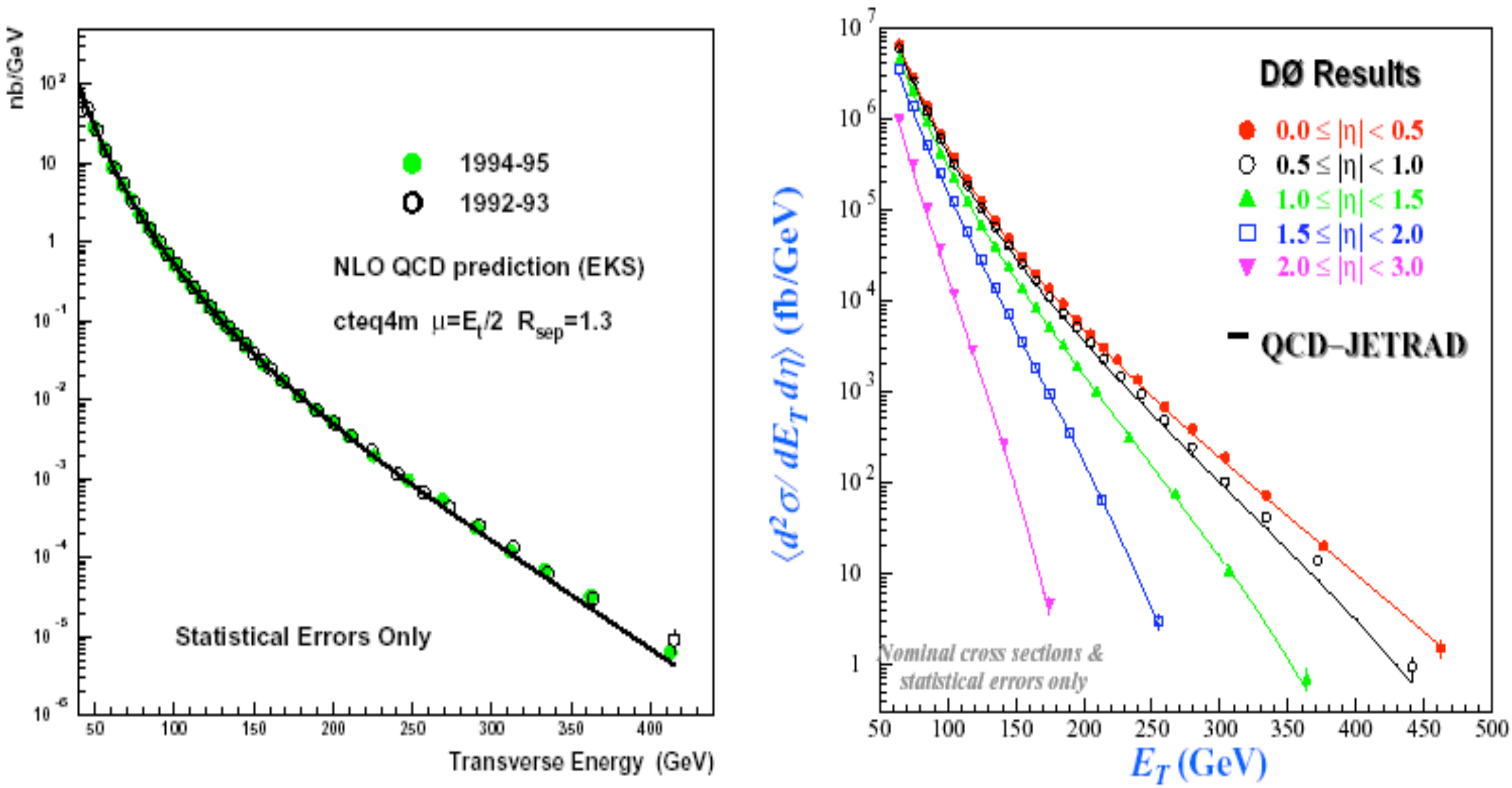


F2



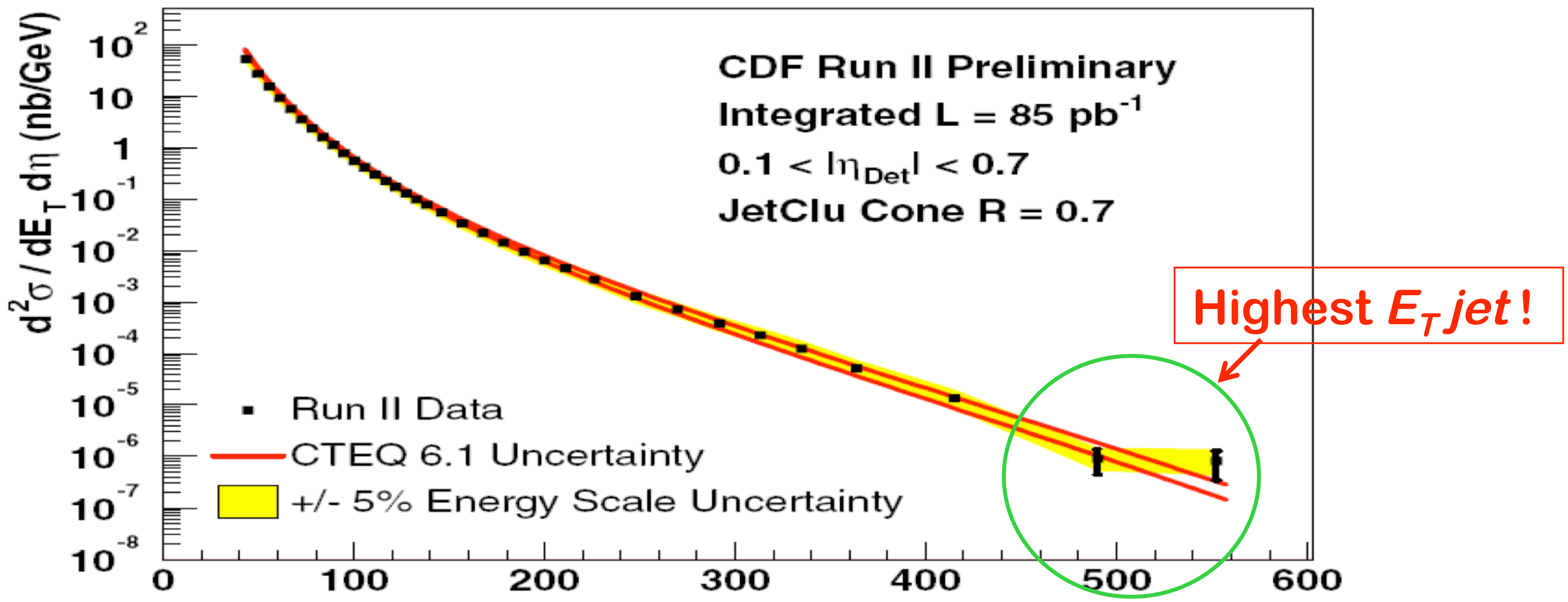
Jet production in hadronic collisions

Inclusive Jet cross section at Tevatron: Run – 1b results

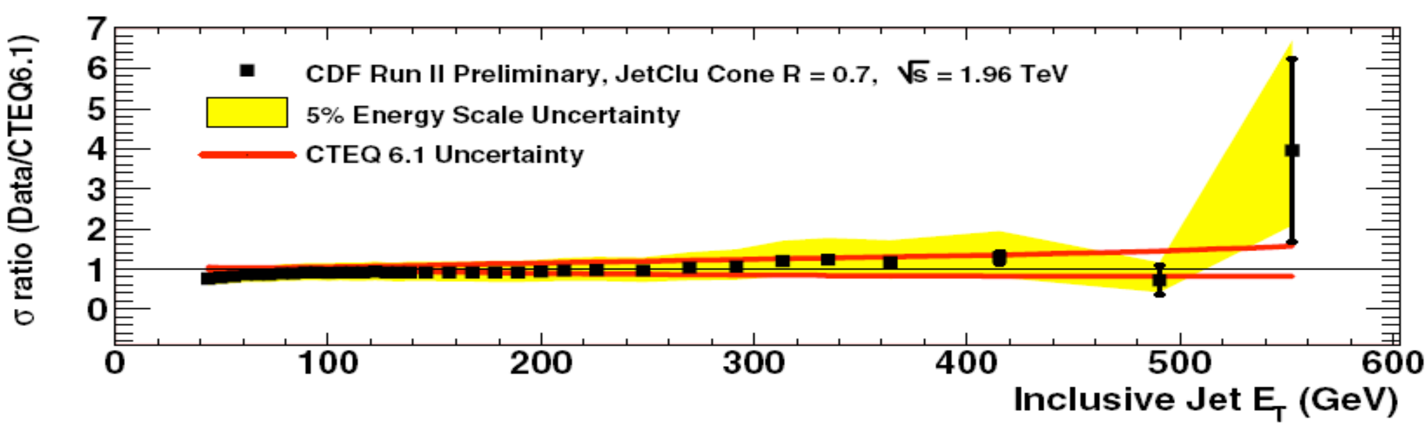


Data and Predictions span 7 orders of magnitude!

Prediction vs CDF Run-II data

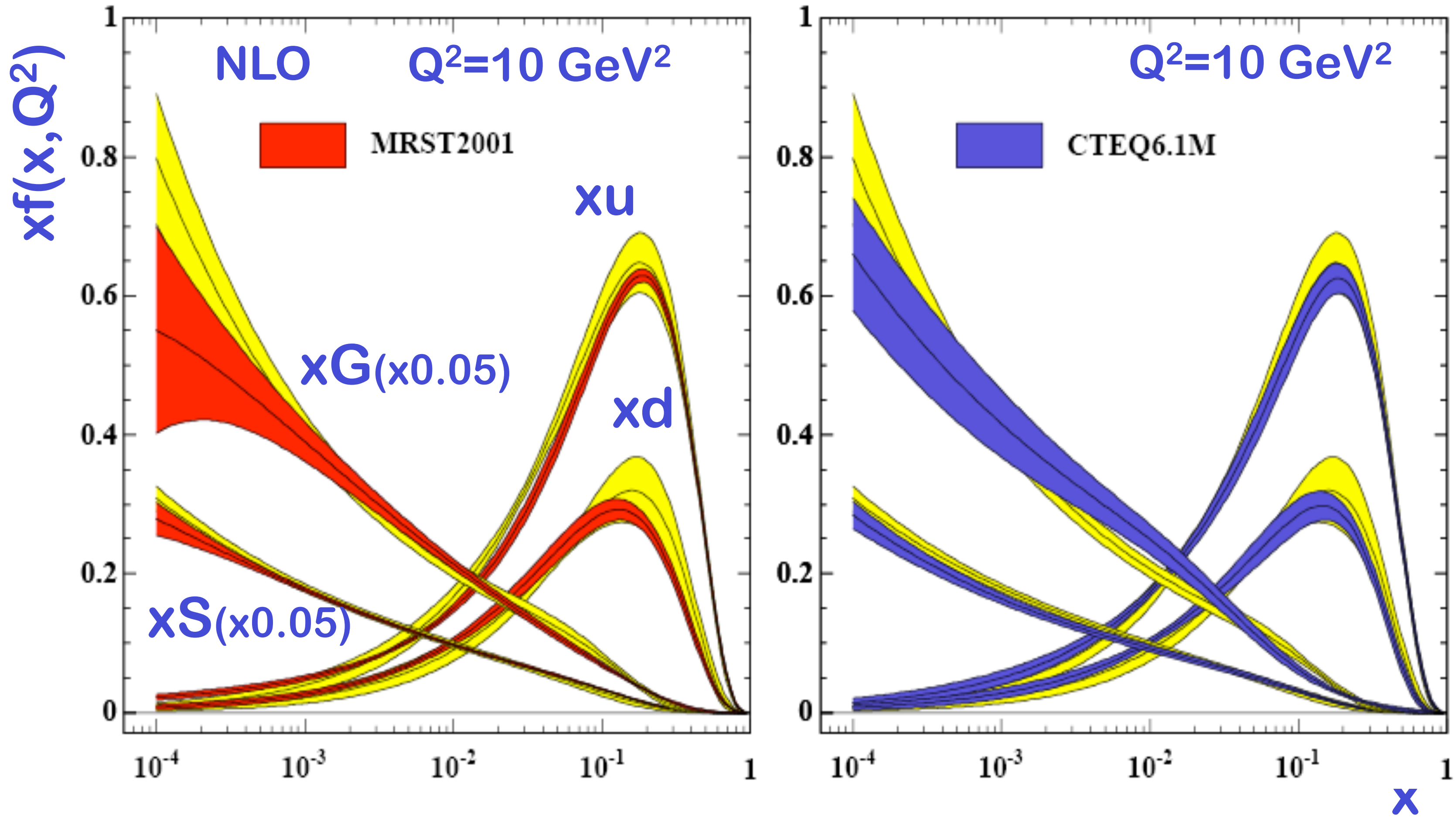


- 8 orders of magnitude!



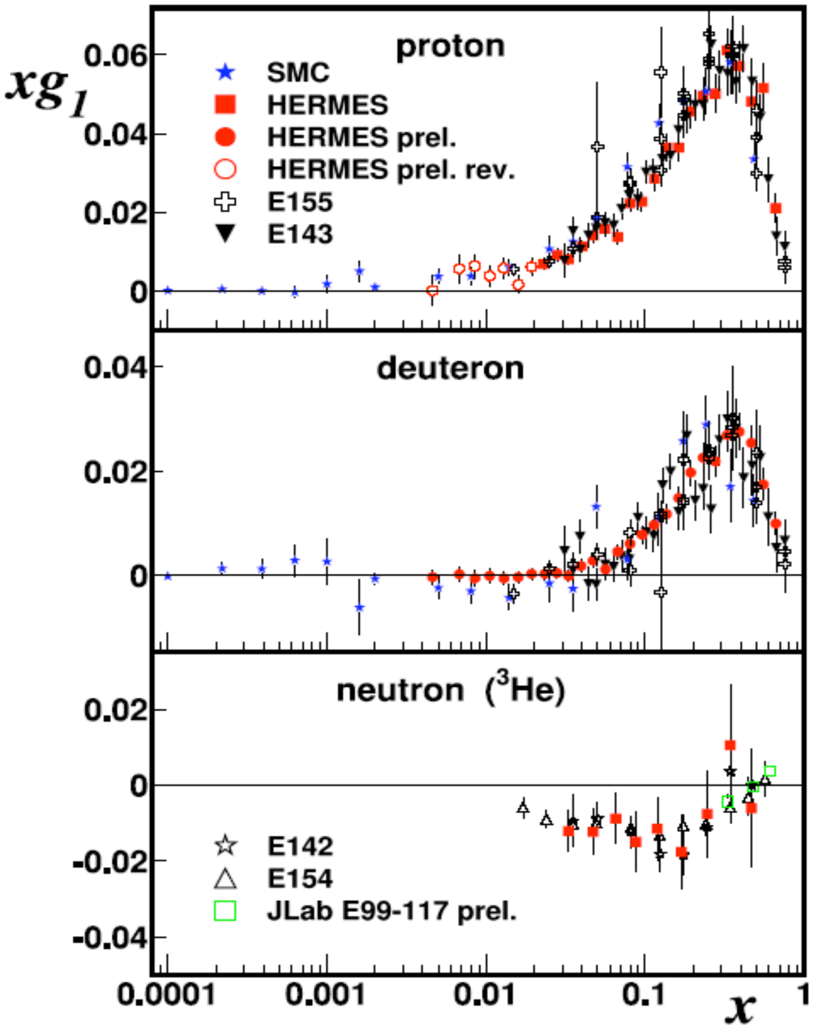
Universal parton distributions

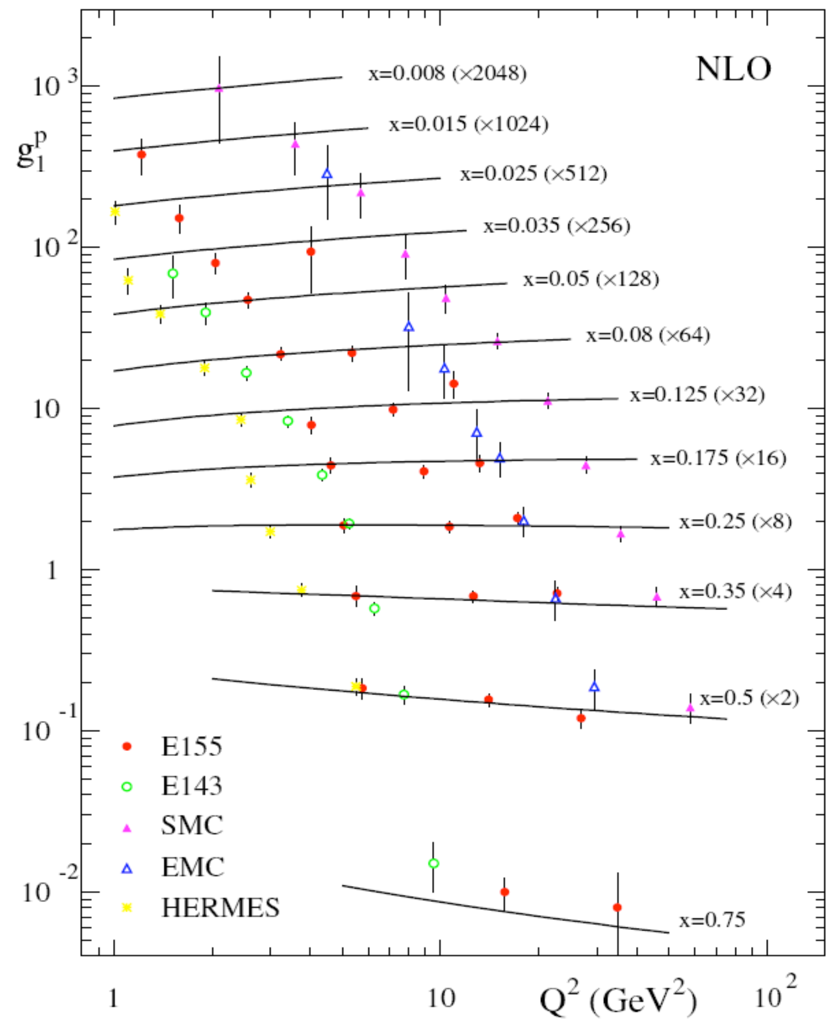
❑ Modern sets of PDFs with uncertainties:



Consistently fit almost all data with Q > 2GeV

Polarized inclusive DIS

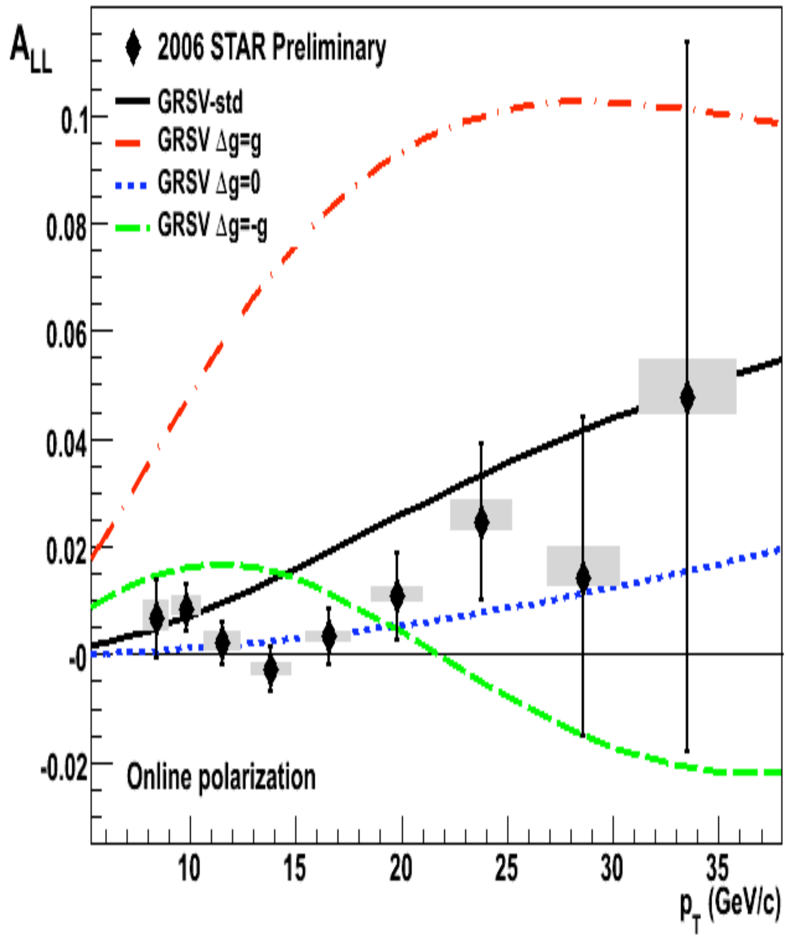




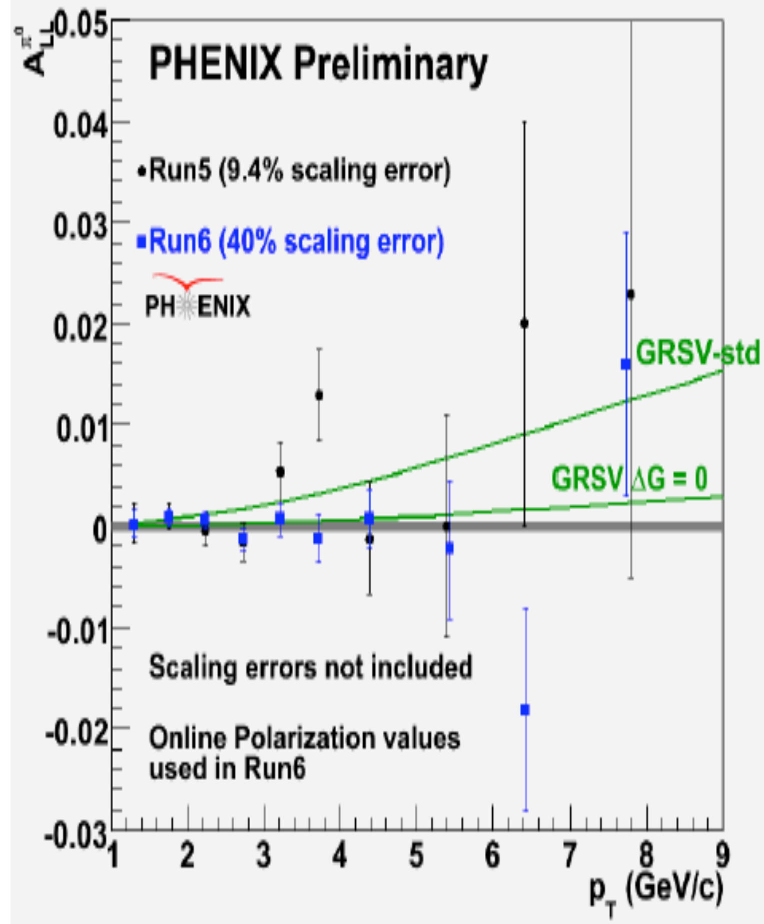
NLO QCD factorization is consistent with the data

Polarized hadronic collisions

Star jet



Phenix π^0



Small asymmetry leads to small gluon "helicity" distribution

Quark "helicity" to proton spin

- **Extracted by the leading power QCD:**

$$\Delta q = \int_0^1 dx\, \Delta q(x) = \langle P, s_{\|} | \overline{\psi}_q(0) \gamma^+ \gamma_5\, \psi_q(0) | P, s_{\|} \rangle$$

  - Integrated over "ALL" momentum components of active parton
  - Collinear factorization:
    - parton entering the hard part has only collinear momentum
    - parton in the distribution has all components

- **NLO QCD global fit - DSSV:**

$$\Delta u + \Delta \bar{u} = 0.813 \qquad \Delta d + \Delta \bar{d} = -0.458 \qquad \Delta \bar{s} = -0.057$$

$$\Sigma = 0.242 \approx 24\% \text{ proton spin}$$

de Florian, Sassot, Stratmann, and Vogelsang
Phys. Rev. Lett. 2008

- **From Ji's definition:**

$$J_q = \frac{1}{2} \int d^3x \, \langle \vec{P} = 0, \vec{S} |\, \psi_q^\dagger(x) \vec{\gamma} \cdot \vec{S} \gamma_5\, \psi_q(x)\, | \vec{P} = 0, \vec{S} \rangle + L_q$$

Gluon "helicity" to proton spin

- **Extracted by the leading power QCD:**

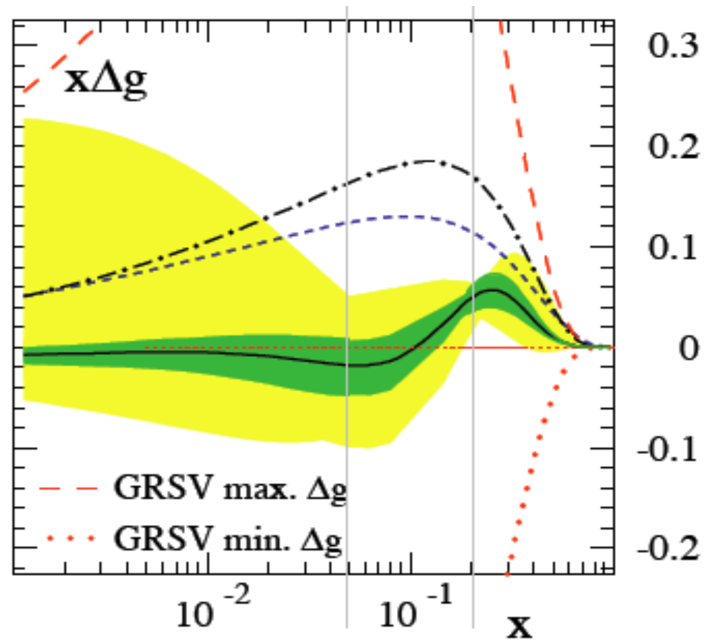
$$\Delta g = \int_0^1 dx\, \Delta g(x) = \langle P, s_{\parallel} | F^{+\mu}(0) F^{+\nu}(0) | P, s_{\parallel} \rangle (-i\epsilon_{\mu\nu})$$

Integrated over "ALL" momentum components of active gluon

- **NLO QCD global fit - DSSV:**

$$\Delta g = -0.084$$ arXiv:0804.0422

  - $\Delta g(x)$ change sign in RHIC region
  - Effectively, no contribution to proton spin



- **From Ji's definition:**

$$J_g = \frac{1}{2} \int d^3x \, \langle \vec{P}=0, \vec{S} | \, [\vec{x} \times \vec{E}(x) \times \vec{B}(x)] \cdot \vec{S} \, | \vec{P}=0, \vec{S} \rangle$$

Questions

How to go beyond
the probability distributions?

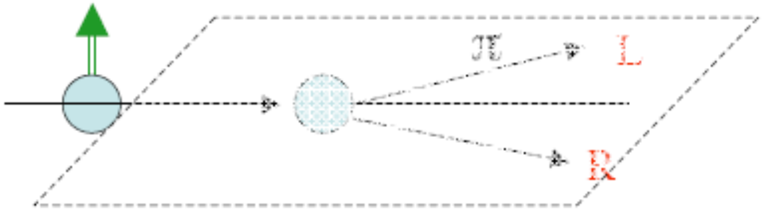
How to probe
parton's transverse motion?

Single Transverse-Spin Asymmetry (SSA)

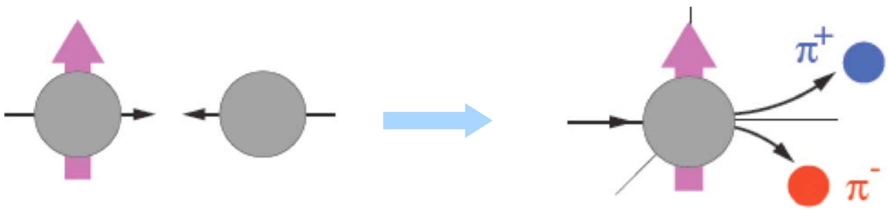
$$A(\ell,\vec{s}) \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s})-\sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s})+\sigma(\ell,-\vec{s})}$$

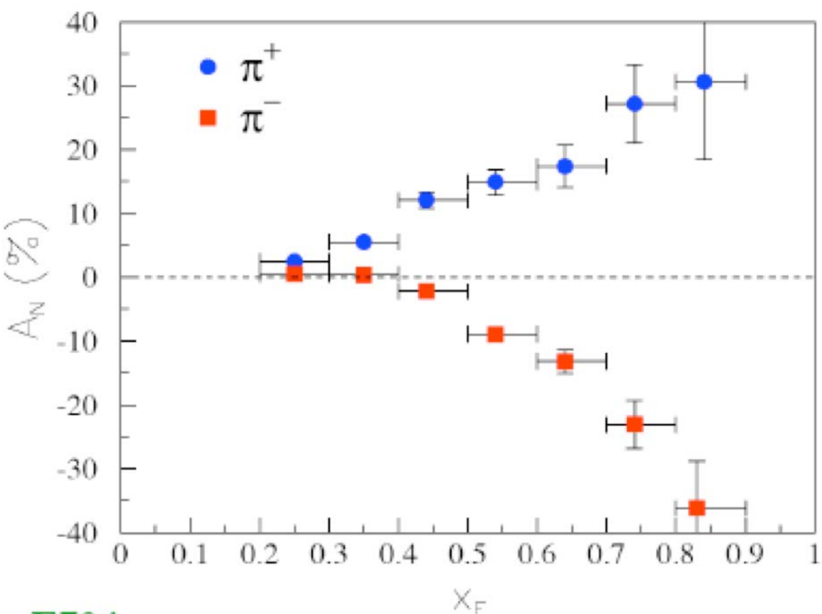
SSA in hadronic collisions

- **Hadronic** $p\uparrow + p \to \pi(l) X$:



$$A_N = \frac{1}{P_{\rm beam}} \frac{N^{\pi}_{\rm left} - N^{\pi}_{\rm right}}{N^{\pi}_{\rm left} + N^{\pi}_{\rm right}}$$





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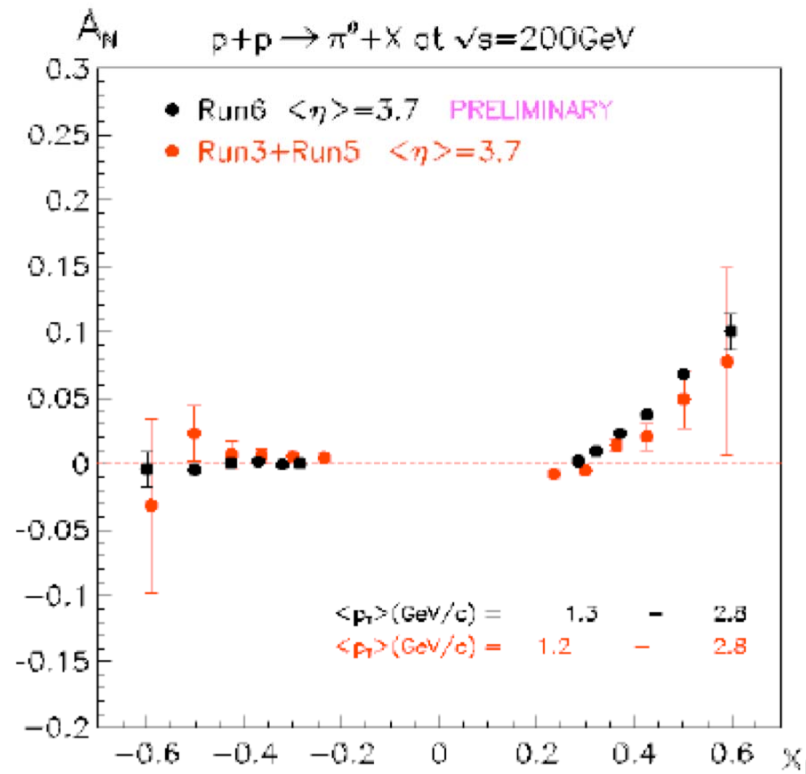


STAR (BRAHMS, too)

Role of fundamental symmetries

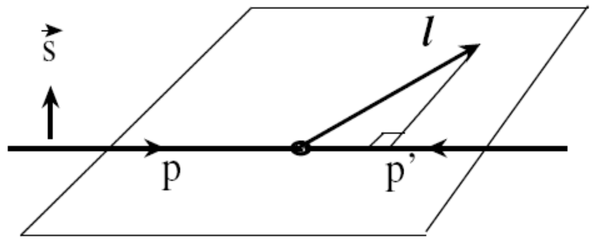
- **Fundamental symmetry and vanishing asymmetry:**
  - A_L=0 (longitudinal) for Parity conserved interactions
  - A_N =0 (transverse) for inclusive DIS – Time-reversal invariance

    – proposed to test T-invariance by Christ and Lee (1966)

  Even though the cross section is finite!

- **SSA corresponds to a T-odd triple product**

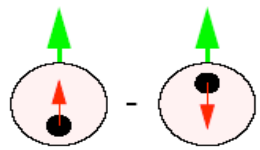
$$A_N \propto i\vec{s}_p \cdot \left(\vec{p} \times \vec{\ell}\right) \Rightarrow i\varepsilon^{\mu\nu\alpha\beta} p_\mu s_\nu \ell_\alpha p'_\beta$$



Novanishing A_N requires a phase, a spin flip, and enough vectors to fix a scattering plan

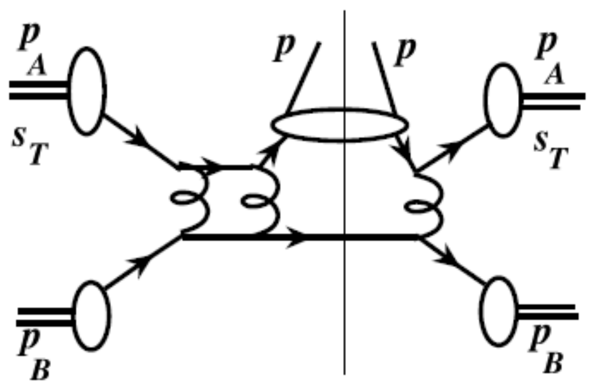
SSA in parton model

- The spin flip at leading twist – transversity:

$\delta q(x) =$  $\propto \langle P, \vec{S}_\perp | \overline{\psi}_q \left[\gamma^+ \gamma \cdot \vec{S}_\perp \right] \psi_q | P, \vec{S}_\perp \rangle$

Chiral-odd helicity-flip density

- the operator for δq has even γ's $\Longrightarrow$ quark mass term
- the phase requires an imaginary part $\Longrightarrow$ loop diagram



$\sim m_q \rightarrow 0$

$\Longrightarrow$ SSA vanishes in the parton model

connects to parton's transverse motion

Single transverse spin asymmetries for cross sections with ONE large measured momentum transfer

- Collinear factorization approach is more relevant

$$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$$ – Expansion

- SSA – difference of two cross sections with spin flip is power suppressed compared to the cross section

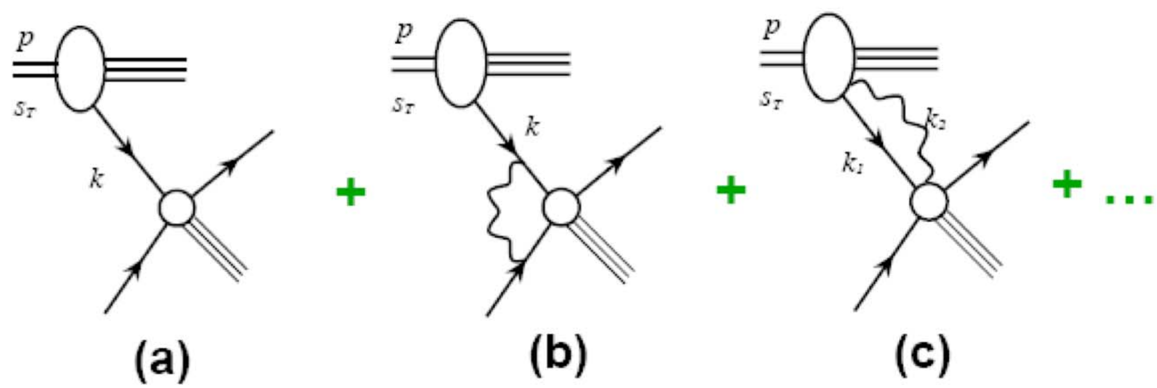
Sensitive to integrated information on parton's transverse motion

Asymmetry in collinear factorization

Efremov, Teryaev, 1982, Qiu, Sterman, 1991

- One large momentum transfer Q >> Λ_{QCD}:

$\sigma(s_T)$ ~ $\Big|$ (a) + (b) + (c) + … $\Big|^2$

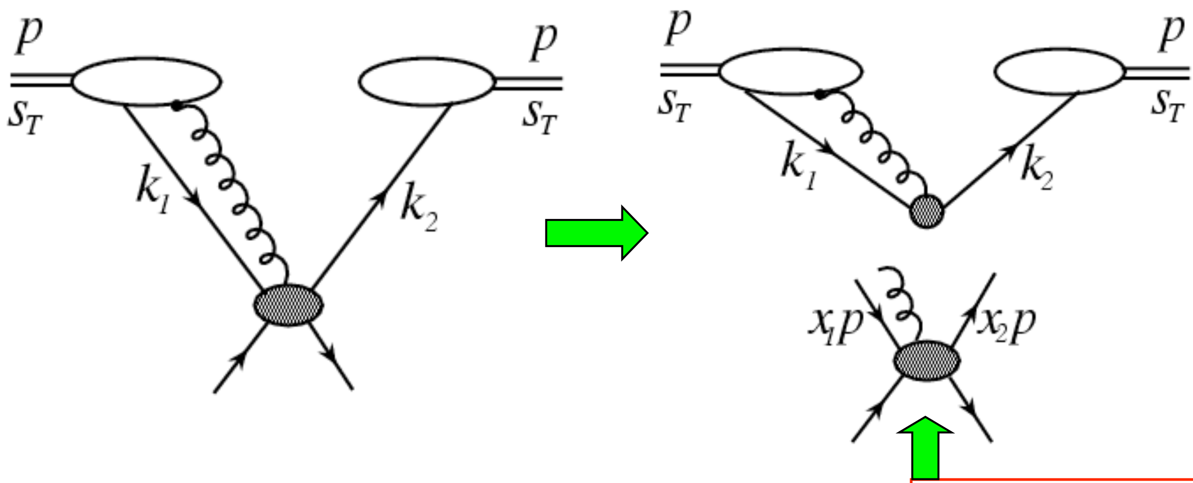


- Leading spin dependent part of the cross section
  - ➡ Interference between amplitudes (a) and (b) or (c)
- The hadronic phase – the "i"
  - ➡ $\mathrm{Re}[(a)]$ interferes with $\mathrm{Im}[(b)]$ or $\mathrm{Im}[(c)]$
- $\mathrm{Re}[(a)] \times \mathrm{Im}[(b)] \propto m_Q \; \delta q(s_\perp)$

A_N from polarized twist-3 correlations

Qiu, Sterman, 1991, 1999

- **Factorization:**



$$T_F(x_1,x_2) \propto \langle \overline{\psi}\gamma^+ F^{+\perp}\psi \rangle$$

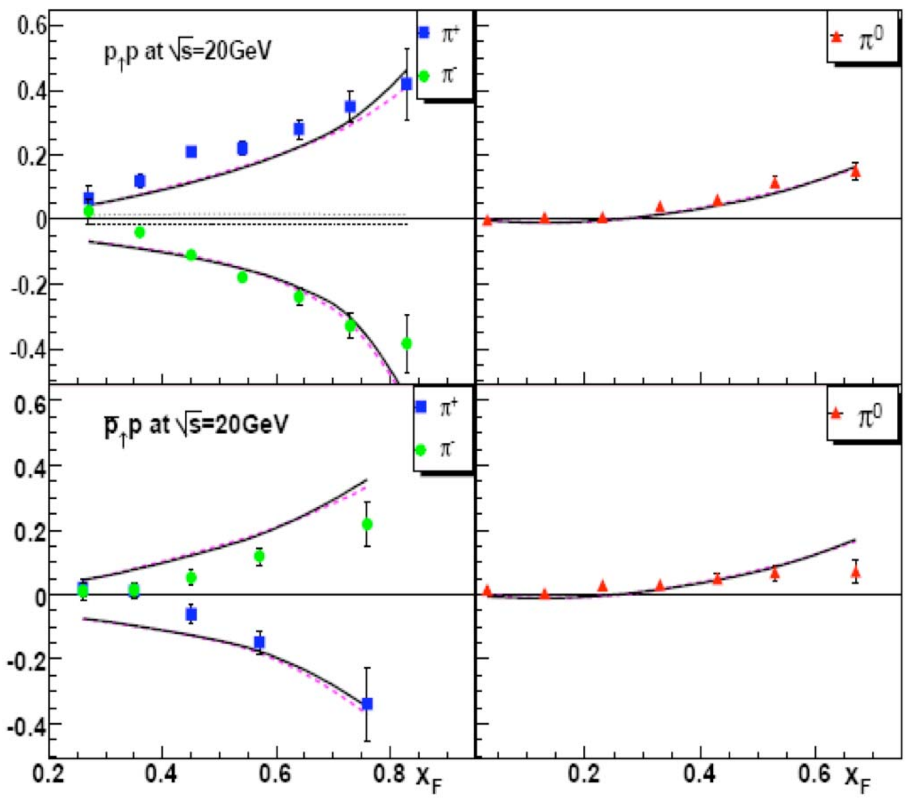
$$T_D(x_1,x_2) \propto \langle \overline{\psi}\gamma^+ D_\perp \psi \rangle$$

Normal twist-2 distributions

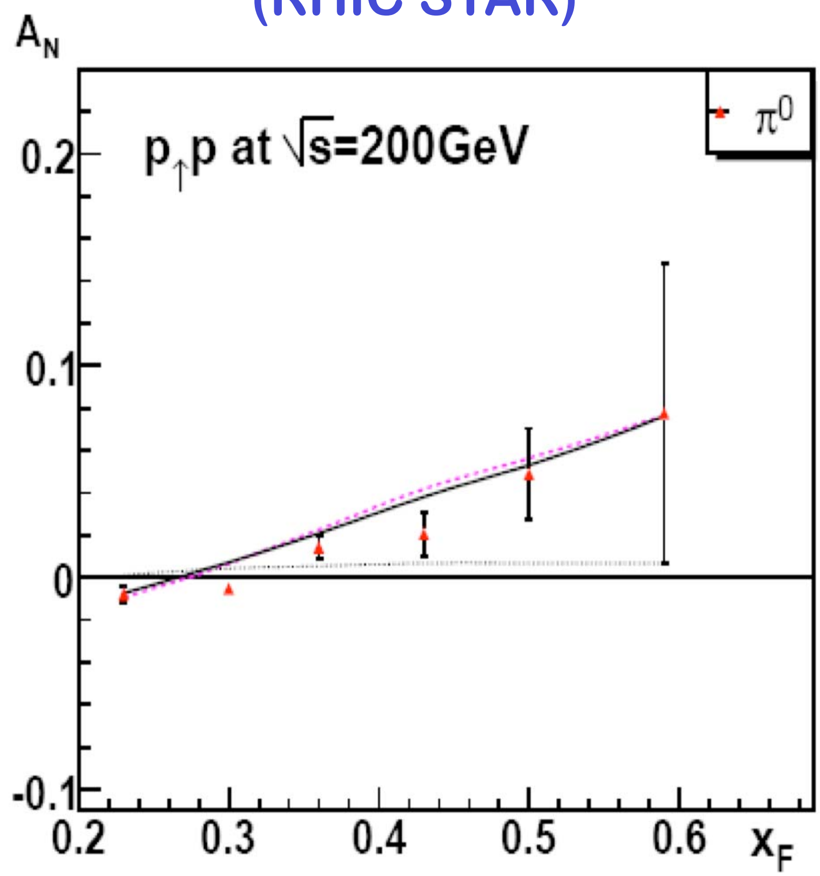
- **New twist-3 quark-gluon correlation functions:**
  - $T_F(x_1,x_2)$ and $T_D(x_1,x_2)$ have different properties under the P and T transformation
  - $T_D(x_1,x_2)$ does not contribute to the A_N
  - $T_F(x_1,x_2)$ is universal, $x_1=x_2$ for A_N due to the pole

Asymmetries from the $T_F(x,x)$

(FermiLab E704)



(RHIC STAR)



Kouvaris,Qiu,Vogelsang,Yuan, 2006

Nonvanish twist-3 function $\Longrightarrow$ Nonvanish transverse motion

What is the $T_F(x,x)$?

- Twist-3 correlation $T_F(x,x)$:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \times \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

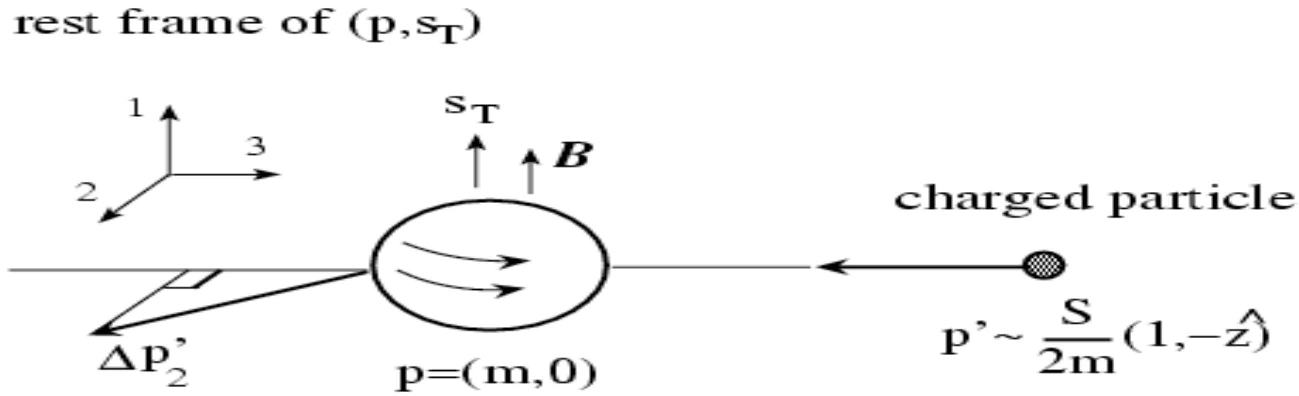
- Twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0) \gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

T_F **Represents a fundamental quantum correlation between quark and gluon inside a hadron**

What the $T_F(x,x)$ tries to tells us?

- Consider a classical (Abelian) situation:



– change of transverse momentum

$$\frac{d}{dt}p'_2 = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

– in the c.m. frame

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

$$\Longrightarrow \frac{d}{dt}p'_2 = e\, \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}$$

– total change: $\boxed{\Delta p'_2 = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}(y^-)}$

$T_F(x,x)$ probes a net asymmetry in parton transverse momentum caused by a color Lorentz force inside a spinning proton

Multi-gluon correlation functions

Ji, PLB289 (1992)

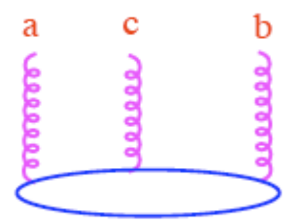
❑ **Diagonal tri-gluon correlations:**

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-)\right] F^{\alpha +}(y_1^-) | P, s_\perp \rangle$$

❑ **Two tri-gluon correlation functions – color contraction:**

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$



Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

❑ **D-meson production at EIC:**

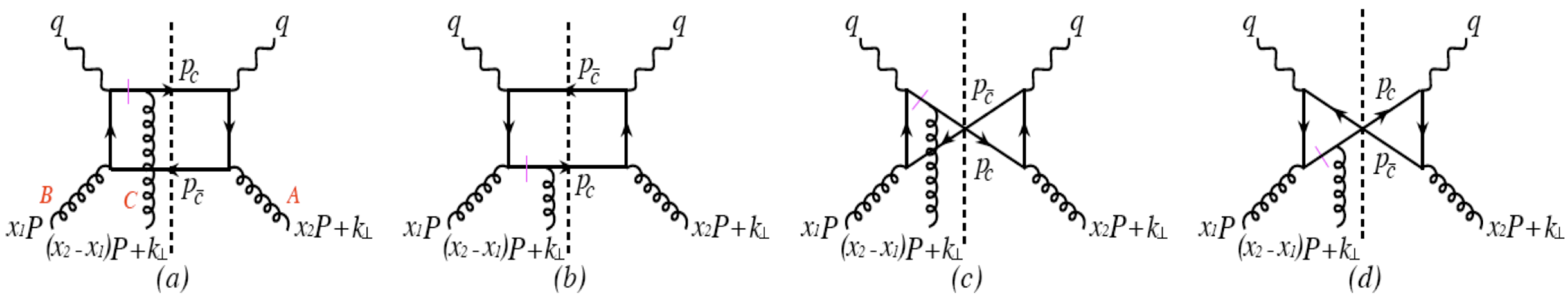
❖ Clean probe for gluonic twist-3 correlation functions

❖ $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

D-meson production at EIC

Kang, Qiu, PRD, 2008

❑ Dominated by the tri-gluon subprocess:



- ❖ Active parton momentum fraction cannot be too large
- ❖ Intrinsic charm contribution is not important
- ❖ Sufficient production rate

❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

SSA is directly proportional to tri-gluon correlation functions

Features of the SSA in D-production at EIC

- **Dependence on tri-gluon correlation functions:**

$$D - \text{meson} \propto T_G^{(f)} + T_G^{(d)} \qquad \overline{D} - \text{meson} \propto T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- **Model for tri-gluon correlation functions:**

$$T_G^{(f,d)}(x,x) = \lambda_{f,d} G(x) \qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07\text{GeV}$$

- **Kinematic constraints:**

$$x_{min} = \begin{cases} x_B \left[1 + \frac{P_{h\perp}^2 + m_c^2}{z_h(1-z_h)Q^2}\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \geq 1 \\ x_B \left[1 + \frac{2m_c^2}{Q^2}\left(1 + \sqrt{1 + \frac{P_{h\perp}^2}{z_h^2 m_c^2}}\right)\right], & \text{if } z_h + \sqrt{z_h^2 + \frac{P_{h\perp}^2}{m_c^2}} \leq 1 \end{cases}$$

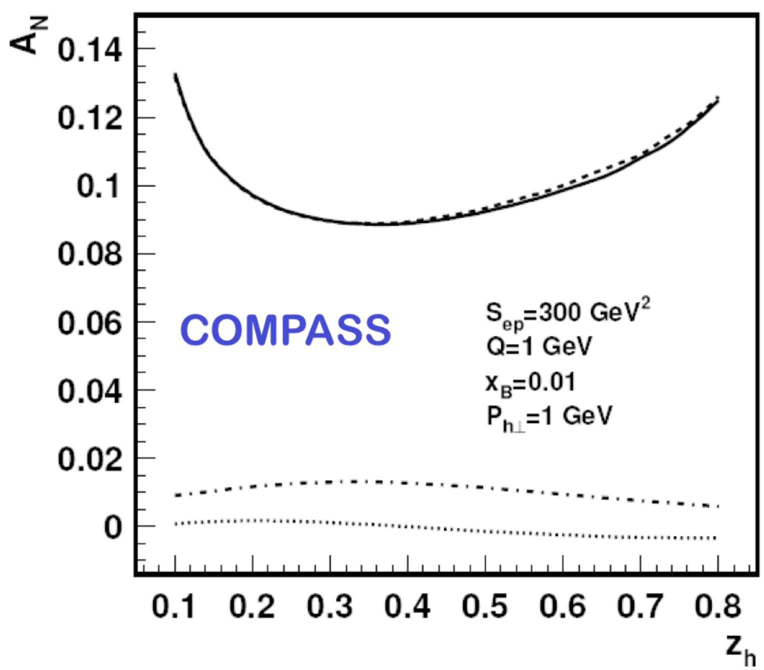
Note: The $z_h(1-z_h)$ has a maximum

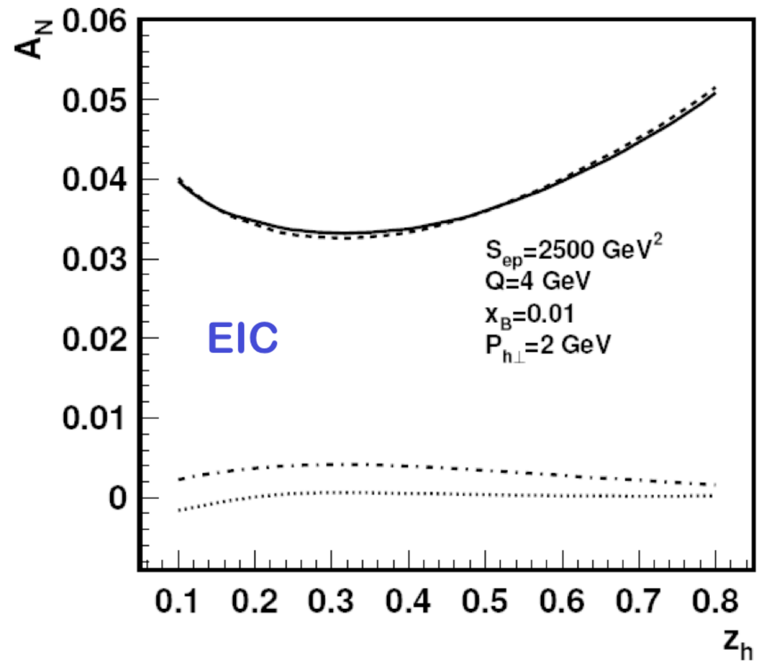
SSA should have a minimum if the derivative term dominates

Minimum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

❑ SSA for D^0 production (λ_f only):



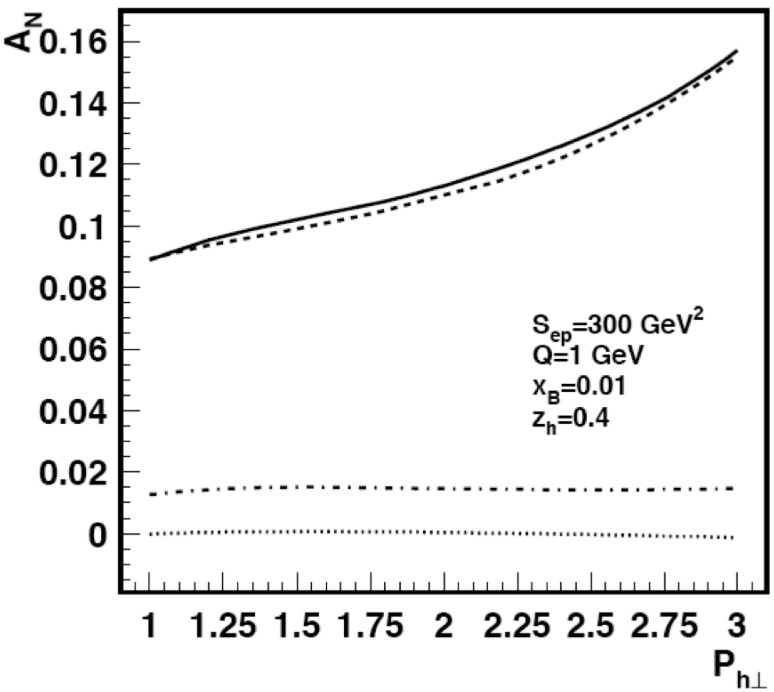


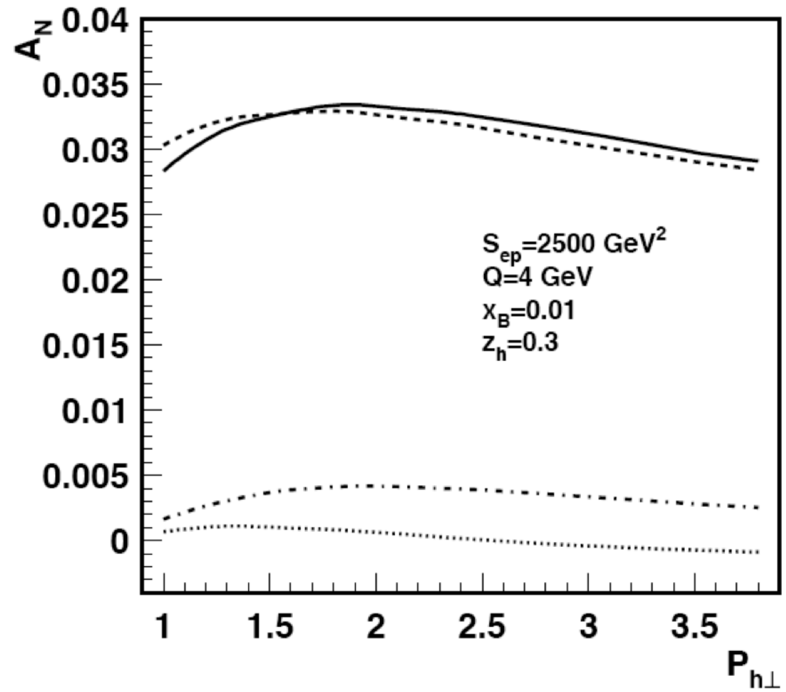
- ❖ Derivative term dominates, and small φ dependence
- ❖ Asymmetry is twice if $T_G^{(f)} = +T_G^{(d)}$, or zero if $T_G^{(f)} = -T_G^{(d)}$
- ❖ Opposite for the $\bar{D}$ meson
- ❖ Asymmetry has a minimum ~ z_h ~ 0.5

Maximum in the SSA of D-production at EIC

Kang, Qiu, PRD, 2008

❑ SSA for D^0 production (λ_f only):





- ❖ The SSA is a twist-3 effect, it should fall off as $1/P_T$ when $P_T \gg m_c$
- ❖ For the region, $P_T \sim m_c$,

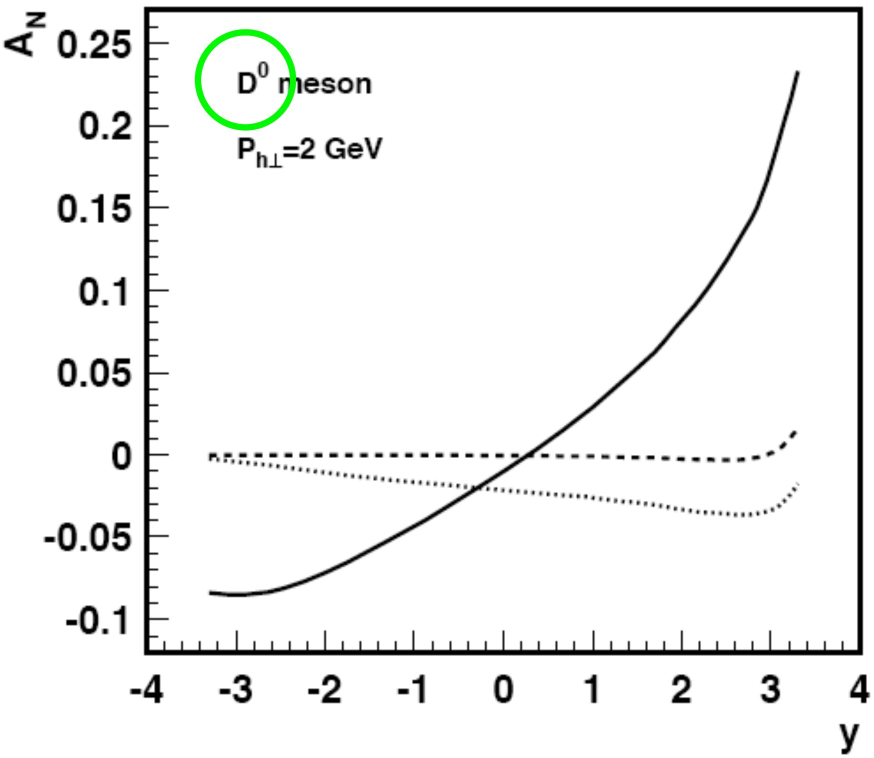
$$A_N \propto \epsilon^{P_h s_\perp n \bar{n}} \frac{1}{\tilde{t}} = -\sin\phi_s \frac{P_{h\perp}}{\tilde{t}}$$

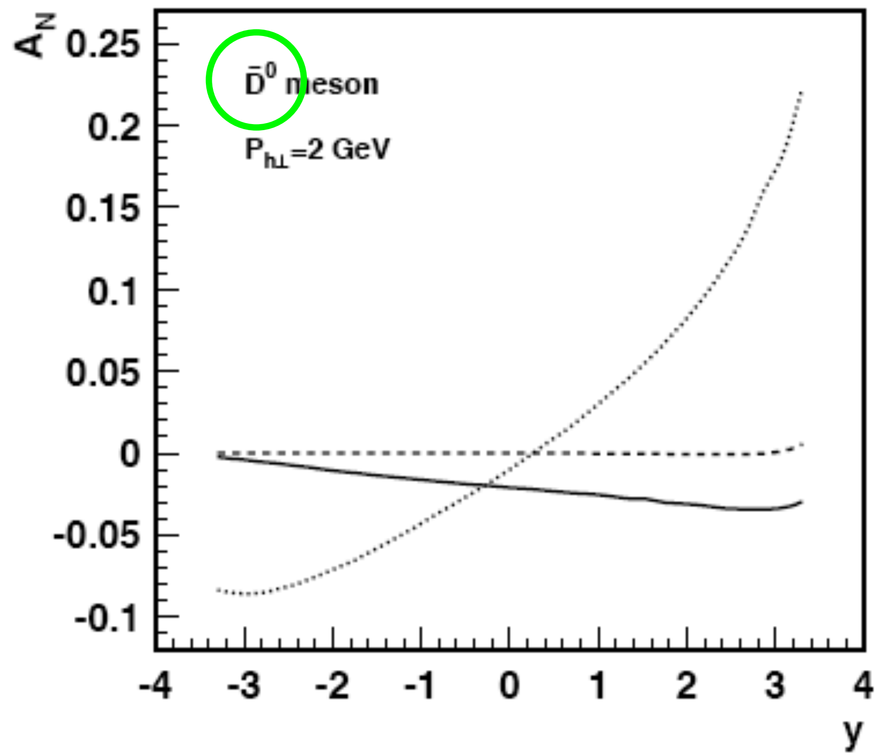
$$\tilde{t} = (p_c - q)^2 - m_c^2 = -\frac{1-\hat{z}}{\hat{x}} Q^2$$

$$\hat{z} = z_h/z, \quad \hat{x} = x_B/x$$

SSA of D-meson production at RHIC

- **Rapidity:** $\sqrt{s} = 200\ \text{GeV} \quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3\ \text{GeV}$





Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = T_G^{(d)}$

Dashed: $(2)\ \lambda_f = \lambda_d = 0 \qquad T_G^{(f)} = T_G^{(d)} = 0$

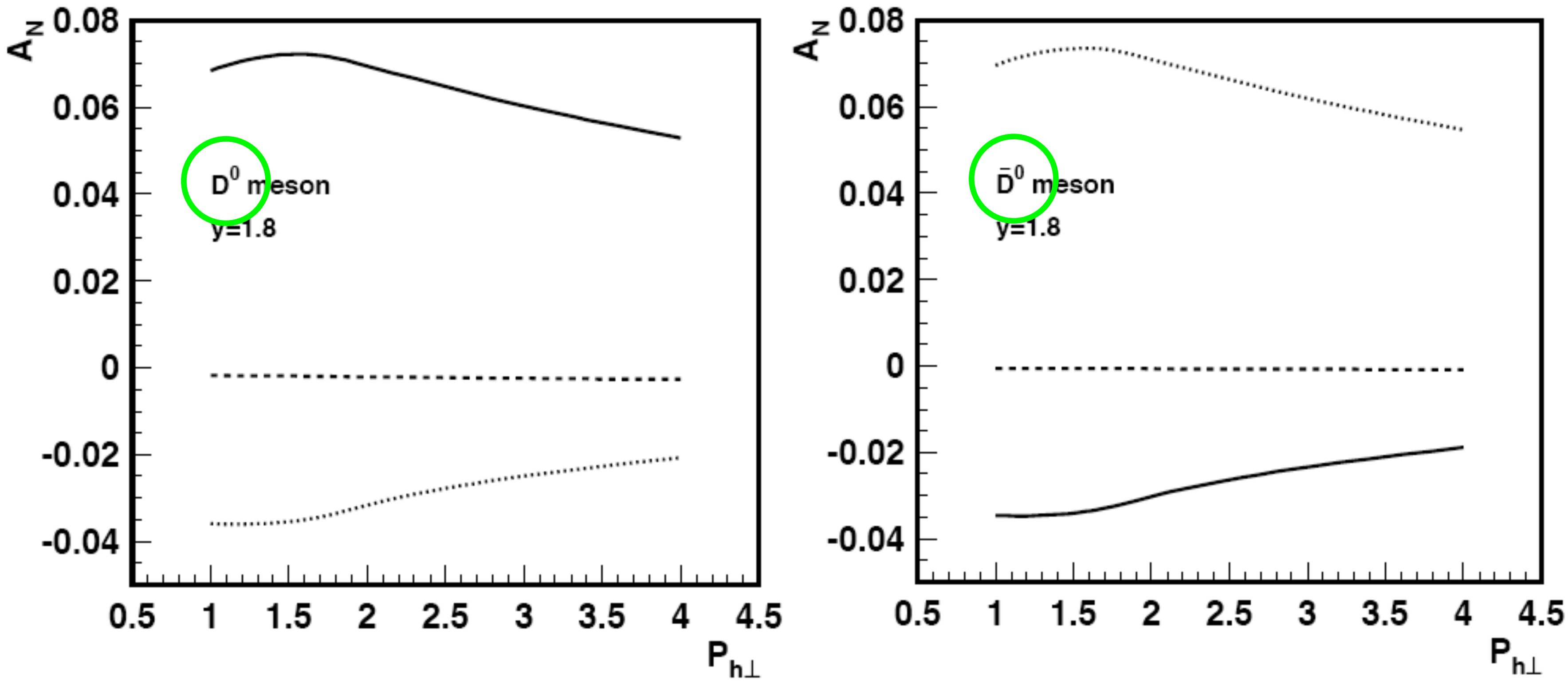
Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = -T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

SSA of D-meson production at RHIC

❑ **P_T dependence:** $\sqrt{s} = 200$ GeV $\quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3$ GeV



Solid: (1) $\lambda_f = \lambda_d = 0.07$ GeV $\qquad T_G^{(f)} = T_G^{(d)}$

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Dotted: (3) $\lambda_f = -\lambda_d = 0.07$ GeV $\qquad T_G^{(f)} = -T_G^{(d)}$

No intrinsic Charm included

Kang, Qiu, Vogelsang, Yuan, 2008

Scale dependence of SSA

- Almost all existing calculations of SSA are at LO:
  - Strong dependence on renormalization and factorization scales
  - Artifact of the lowest order calculation
- Improve QCD predictions:
  - Complete set of twist-3 correlation functions relevant to SSA
  - LO evolution for the universal twist-3 correlation functions
  - NLO partonic hard parts for various observables
  - NLO evolution for the correlation functions, …
- Current status:
  - Two sets of twist-3 correlation functions
  - LO evolution kernel for $T_{q,F}(x,x)$ and $T_{G,F}^{(f,d)}(x,x)$ Kang, Qiu, 2009
  - NLO hard part for SSA of p_T weighted Drell-Yan Vogelsang, Yuan, 2009

Two sets of twist-3 correlation functions

❑ Twist-2 distributions:

❖ Unpolarized PDFs:

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

❖ Polarized PDFs:

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:

Kang, Qiu, PRD, 2009

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+}{2}\, \left[\epsilon^{s_T\sigma n\bar{n}}\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

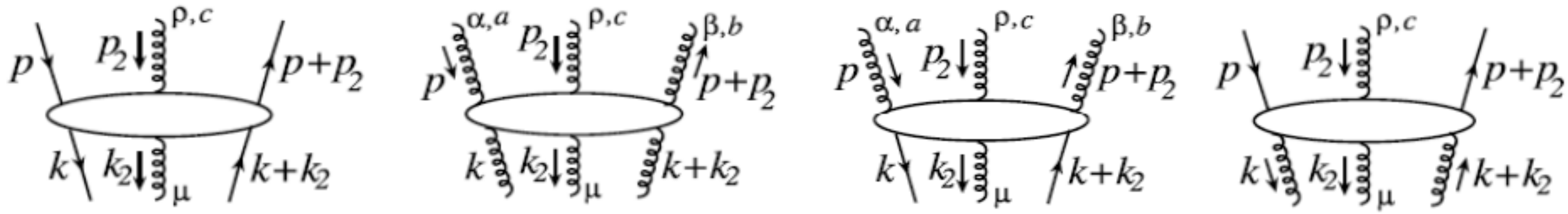
$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+\gamma^5}{2}\, \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P, s_T\rangle\, (i\epsilon_{\perp\rho\lambda})$$

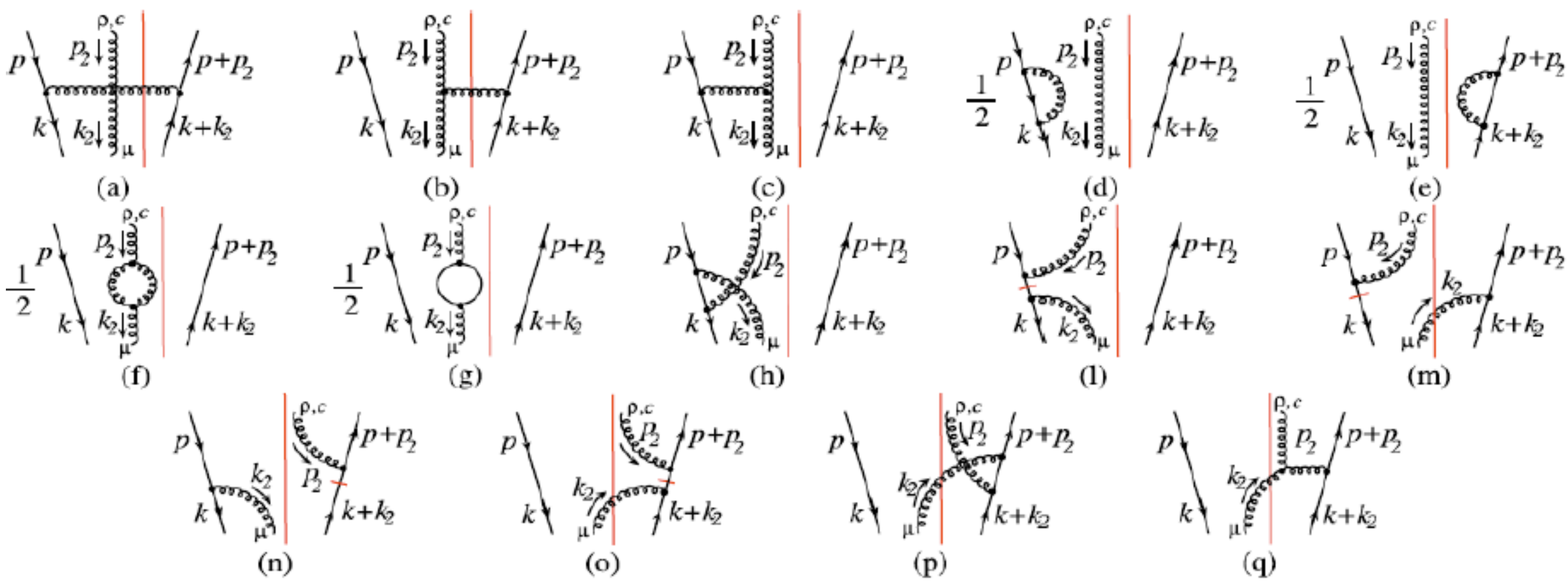
Evolution Kernels

Kang, Qiu, PRD, 2009

- Feynman diagrams:



- LO for flavor non-singlet channel:



Leading order evolution equations - I

Kang, Qiu, PRD, 2009

❑ Quark:

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\boxed{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu_F)}\right.$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F)-T_{q,F}(\xi,\xi,\mu_F)\right]+z\,T_{q,F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{qg}(z)\left(\frac{1}{2}\right)\boxed{\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)+T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]}\right\}$$

❑ Antiquark:

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\boxed{P_{qq}(z)\,T_{\bar{q},F}(\xi,\xi,\mu_F)}\right.$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F)-T_{\bar{q},F}(\xi,\xi,\mu_F)\right]+z\,T_{\bar{q},F}(\xi,x,\mu_F)\right]+\frac{C_A}{2}\left[T_{\Delta \bar{q},F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{qg}(z)\left(\frac{1}{2}\right)\boxed{\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)-T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]}\right\}$$

- ❖ All kernels are infrared safe
- ❖ Diagonal contribution is the same as that of DGLAP
- ❖ Quark and antiquark evolve differently – caused by tri-gluon

Leading order evolution equations - II

Kang, Qiu, PRD, 2009

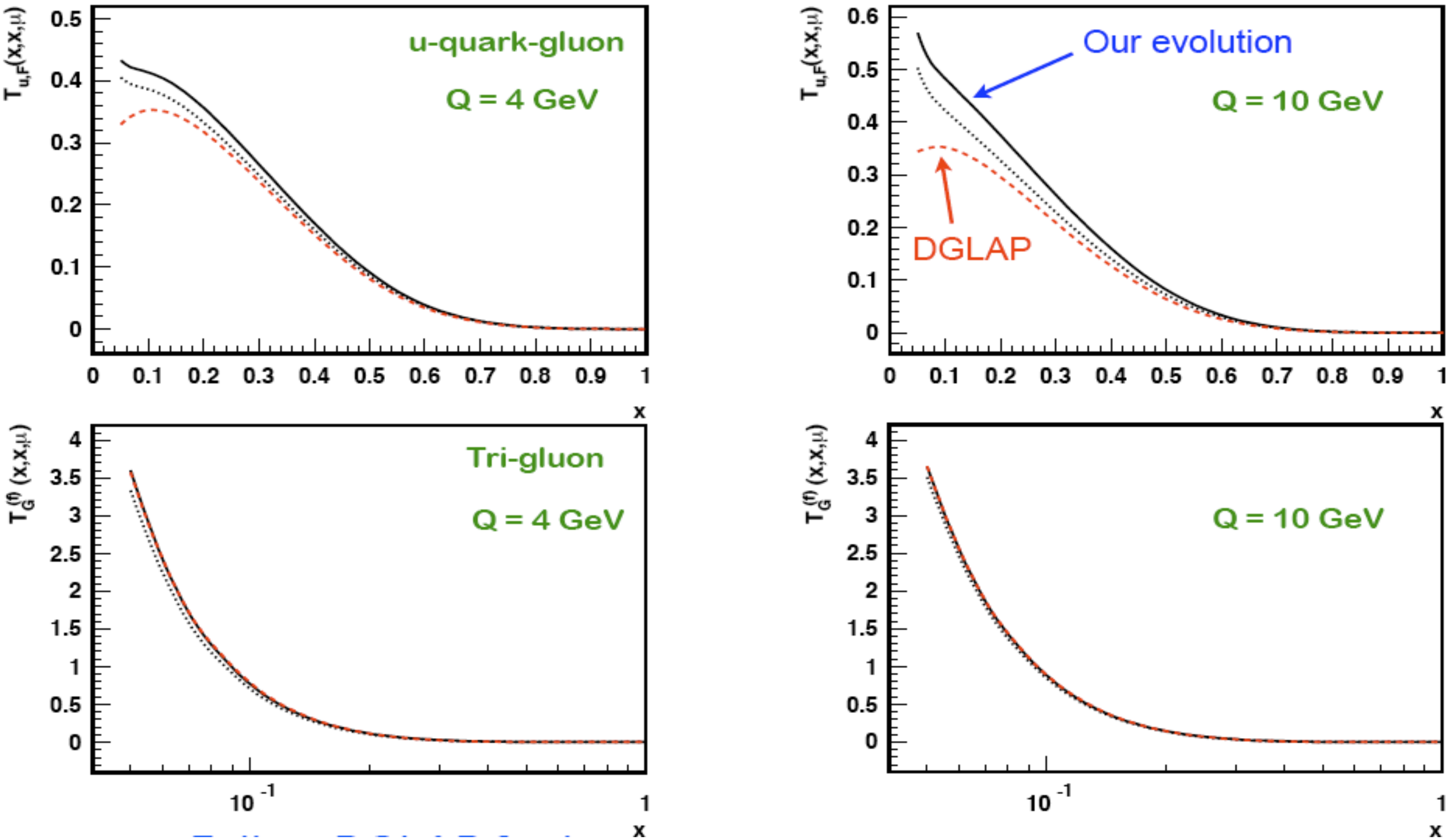
❑ Gluons:

$$\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\boxed{P_{gg}(z)\, T^{(d)}_{G,F}(\xi,\xi,\mu_F)}\right.$$
$$+\frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right.$$
$$\left.+2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)+(1+z)\,T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\boxed{\sum_q\left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar{q},F}(\xi,\xi,\mu_F)\right]}\right\}$$

Similar expression for $T^{(f)}_{G,F}(x,x,\mu_F)$

- ❖ Kernels are also infrared safe
- ❖ diagonal contribution is the same as that of DGLAP
- ❖ Two tri-gluon distributions evolve slightly different
- ❖ $T^{(d)}_{G,F}$ has no connection to TMD distribution
- ❖ Evolution can generate $T^{(d)}_{G,F}$ as long as $\sum_q [T_{q,F}+T_{\bar{q},F}] \neq 0$

Scale dependence of twist-3 correlations



- ❖ Follow DGLAP at large x
- ❖ Large deviation at low x (stronger correlation)

Kang, Qiu, PRD, 2009

Questions

How to probe
parton's transverse motion
at
a given transverse momentum ?

Transverse momentum dependent (TMD)
parton distribution functions (PDFs)

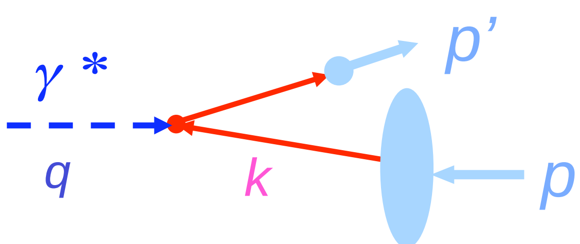
Can SSA be factorized in terms of TMD PDFs ?

TMD factorization

- Need processes with two observed momentum scales:

$$Q_1 \gg Q_2 \begin{cases} Q_1 & \text{necessary for pQCD factorization to have a chance} \\ Q_2 & \text{sensitive to parton's transverse motion} \end{cases}$$

- Example – semi-inclusive DIS:



- Both p and p' are observed
- p'_T probes the parton's k_T
- Effect of k_T is not suppressed by Q

- Very limited processes with valid TMD factorization
  - Drell-Yan transverse momentum distribution: Q, q_T
    - o quark Sivers function
    - o low rate
  - Semi-inclusive DIS for light hadrons: Q, p_T
    - o mixture of quark Sivers and Collins function

Collins, Qiu, 2007
Vogelsang, Yuan, 2007

TMD parton distributions

- **SIDIS:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

Gauge links:

$$\Phi_n(\{\infty,y^-\},\mathbf{y}_\perp) \equiv \mathcal{P}e^{-ig\int_{y^-}^{\infty} dy_1^- n^\mu A_\mu(y_1^-,\mathbf{y}_\perp)}$$
$$\Phi_{\mathbf{n}_\perp}(\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\}) \equiv \mathcal{P}e^{-ig\int_{\mathbf{0}_\perp}^{\mathbf{y}_\perp} d\mathbf{y}'_\perp \mathbf{n}_\perp^\mu A_\mu(\infty,\mathbf{y}'_\perp)}$$

- **Drell-Yan:**

$$f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2\mathbf{y}_\perp}{(2\pi)^3}\, e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\Phi_n^\dagger(\{-\infty,0\},\mathbf{0}_\perp)$$
$$\times\, \Phi_{\mathbf{n}_\perp}^\dagger(-\infty,\{\mathbf{y}_\perp,\mathbf{0}_\perp\})\frac{\gamma^+}{2}\Phi_n(\{-\infty,y^-\},\mathbf{y}_\perp)\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$

- **PT invariance:**

$$f_{q/h^\uparrow}^{\rm SIDIS}(x,\mathbf{k}_\perp,\vec{S}) = f_{q/h^\uparrow}^{\rm DY}(x,\mathbf{k}_\perp,-\vec{S})$$

- **Sivers function:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)\,\vec{S}\cdot(\hat{p}\times\hat{\mathbf{k}}_\perp)$$

$$\Longrightarrow\quad f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm SIDIS} = -f_{q/h^\uparrow}^{\rm Sivers}(x,k_\perp)^{\rm DY} \quad\Longleftarrow$$

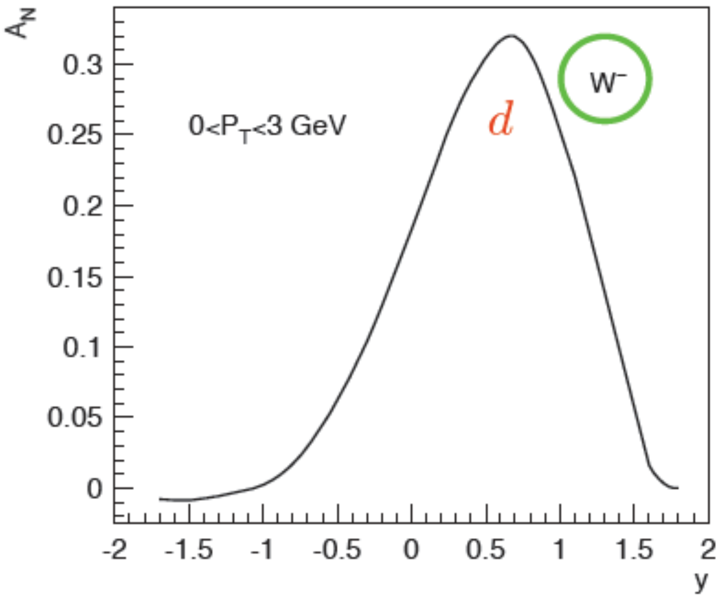
Modified Universality

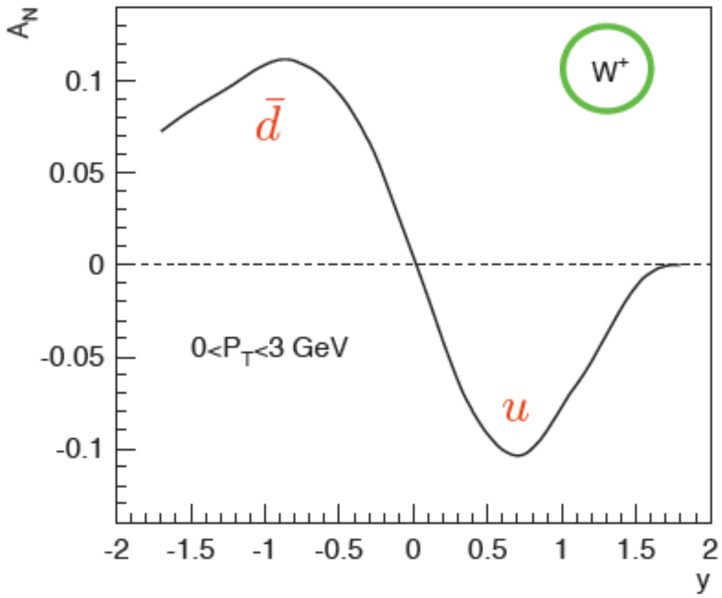
Test of the modified universality

- SSA of W-production at RHIC :

Kang, Qiu, 2009

Sivers function same as DY, different from SIDIS by a sign





- flavor separation
- large asymmetry: should be able to see sign change

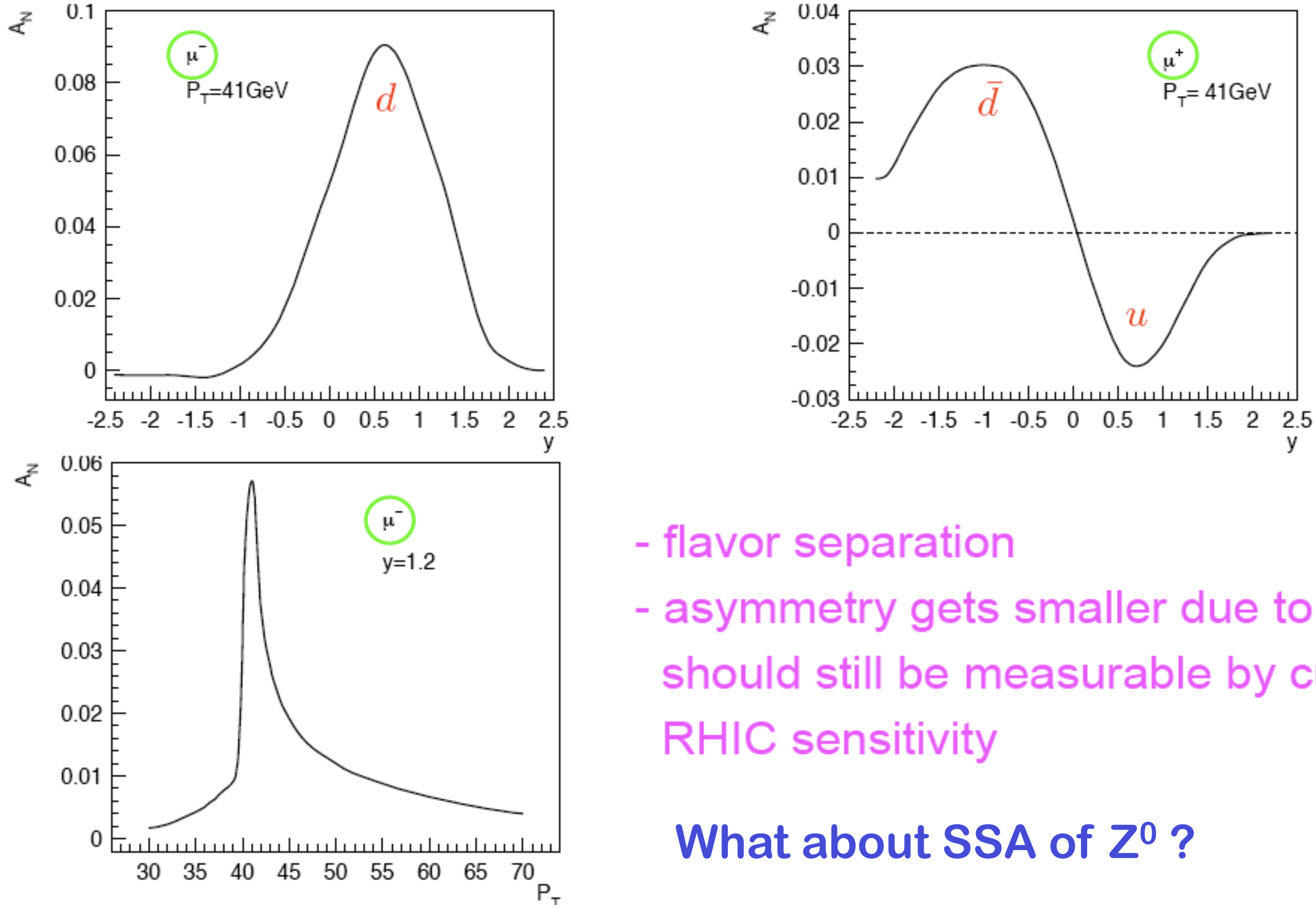
But, the detectors at RHIC cannot reconstruct the W's

The Sivers functions from Anselmino et al 2009

SSA of lepton from W-decay

❑ Lepton SSA is diluted from the decay:

Kang, Qiu, 2009



- flavor separation
- asymmetry gets smaller due to dilution
  should still be measurable by current
  RHIC sensitivity

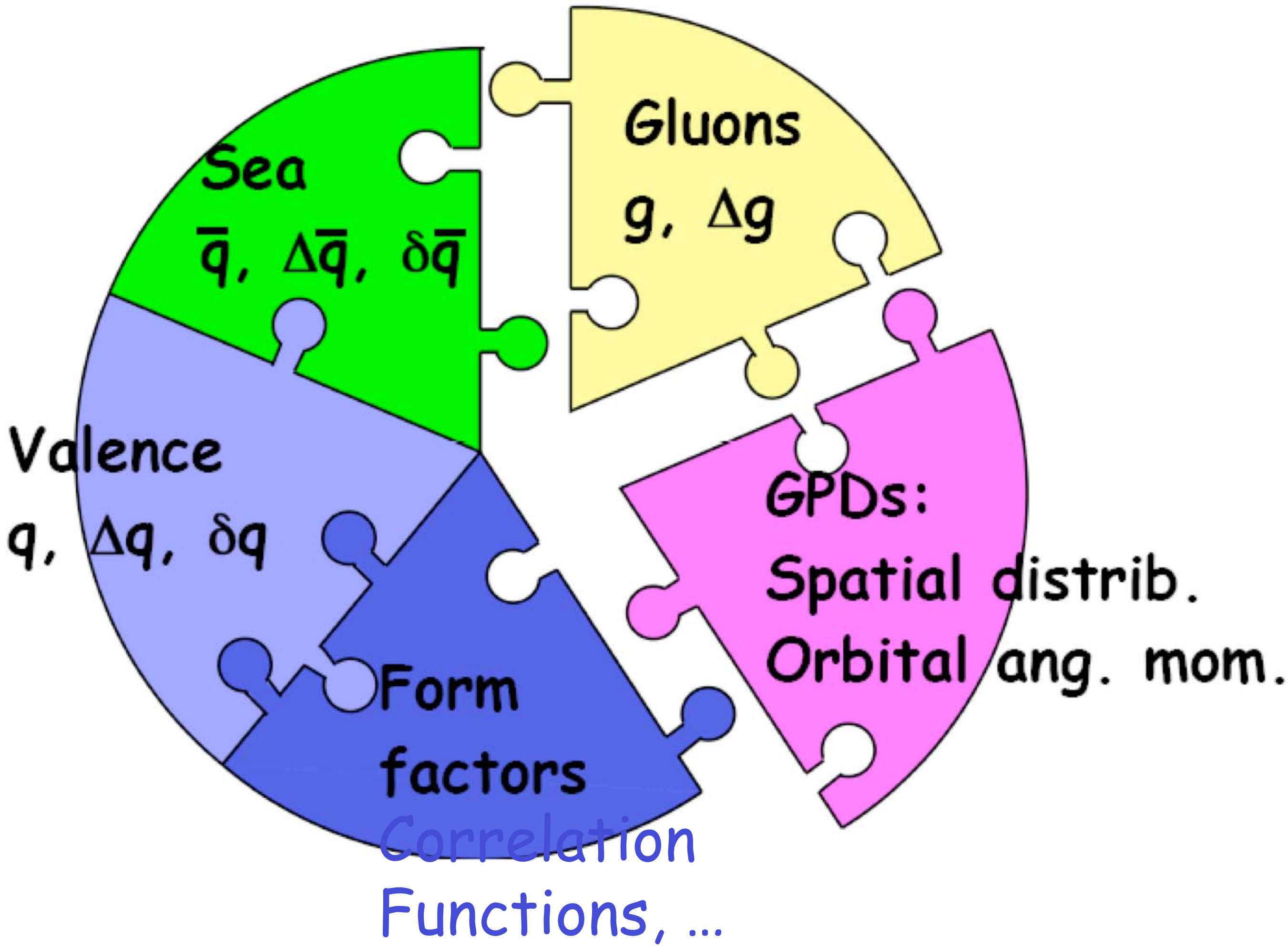
What about SSA of Z^0 ?

Summary and outlook

- ❑ Spin is a low energy property of the proton
- ❑ It is difficult to measure the low energy matrix elements in high energy experiments – unless they are independent of P
- ❑ It seems likely that quark and gluon helicity alone is not sufficient to make up the proton's spin
- ❑ SSA is directly connected to the parton's transverse motion
- ❑ Collinear factorization and the TMD factorization cover different kinematic regimes – they are consistent when they are overlap
- ❑ SSA of lepton from decay of W-bosons is ideal for testing time-reversal modified universality of TMD distributions
- ❑ Spin program opens a whole new meaning to test QCD dynamics!

Thank you!

Challenge: Map out the nucleon



RHIC spin and future EIC spin will play a key role!

Backup transparencies