Exclusive diffractive processes with saturation at NLO accuracy

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In collaboration with
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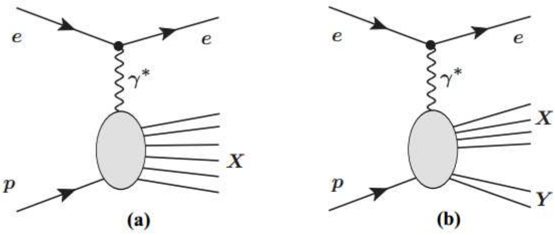
Diffractive DIS

Rapidity gap events at HERA

Experiments at HERA : about 10% of scattering events reveal a rapidity gap



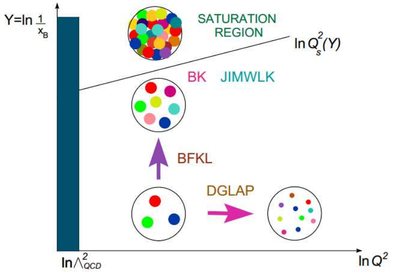
DIS events DDIS events

DIS : Deep Inelastic Scattering, DDIS : Diffractive DIS

Rapidity gap $\equiv$ Pomeron exchange

Diffractive DIS

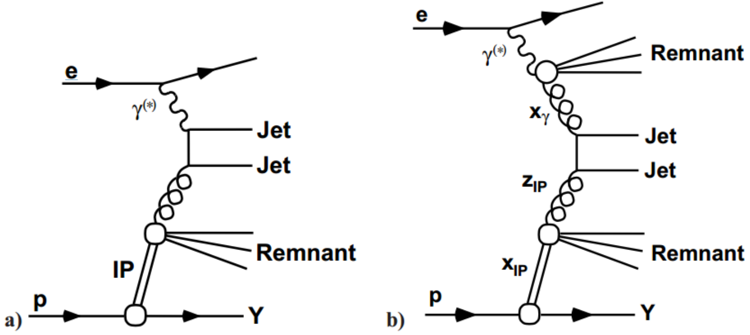
Theoretical approaches for DDIS using pQCD



- Collinear factorization approach
  - Relies on a QCD factorization theorem, using a hard scale such as the virtuality Q^2 of the incoming photon
  - One needs to introduce a diffractive distribution function for partons *within a pomeron*
- k_T factorization approach for two exchanged gluons
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron is described as a two-gluon color-singlet state

Theoretical approaches for DDIS using pQCD

Collinear factorization approach

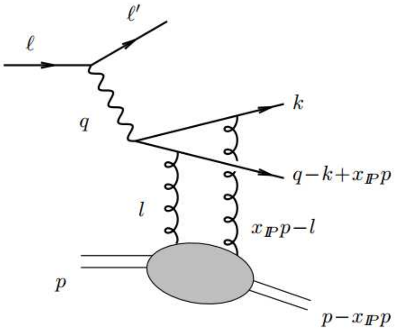


Direct

Resolved

Theoretical approaches for DDIS using pQCD

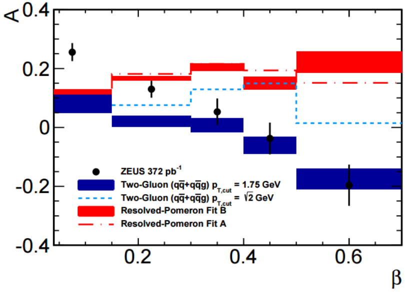
k_T-factorization approach : two gluon exchange



Bartels, Diehl, Ewerz, Ivanov, Jung, Lotter, Wüsthoff
Braun and Ivanov developed a similar model in collinear factorization

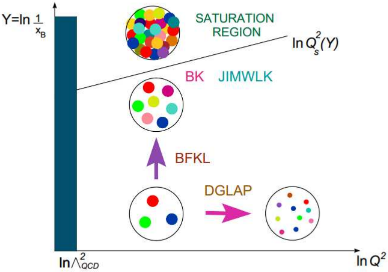
Theoretical approaches for DDIS using pQCD

Confrontation of the two approaches with HERA data

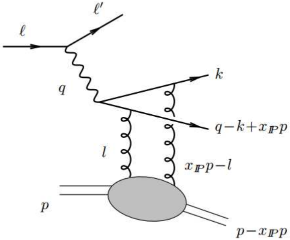


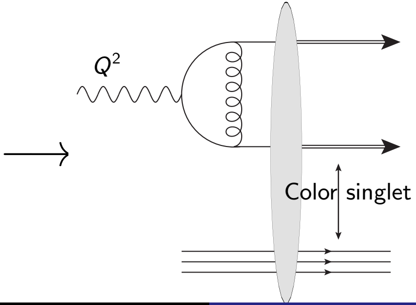
ZEUS collaboration, 2015

Diffractive DIS



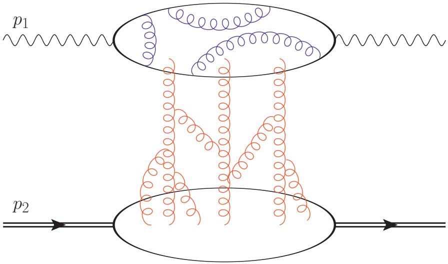
- Shockwave (CGC) approach
  - low-x QCD approach : $s \gg Q^2 \gg \Lambda_{QCD}$
  - The pomeron exchange is described as the action of a color singlet Wilson line operator on the target states





[Bartels, Wüsthoff]

Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2s} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone (Sudakov) vectors

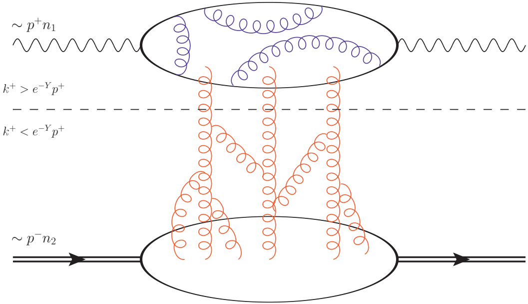
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \rightarrow (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation

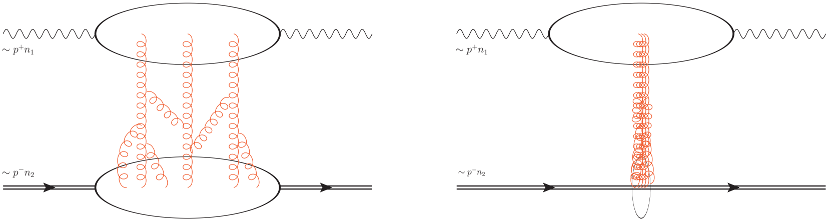


Let us split the gluonic field between "fast" and "slow" gluons

$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}) + b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k})$$

$e^\eta = e^{-Y} \ll 1$

Large longitudinal boost to the projectile frame



$$b^+(x^+, x^-, \vec{x}) \qquad \qquad \frac{1}{\Lambda} b^+\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

$$b^-(x^+, x^-, \vec{x}) \quad \longrightarrow \quad \Lambda b^-\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

$$b^k(x^+, x^-, \vec{x}) \qquad \Lambda \sim \sqrt{\frac{s}{m_t^2}} \qquad b^k\left(\Lambda x^+, \frac{x^-}{\Lambda}, \vec{x}\right)$$

$$b^\mu(x) \to b^-(x)\, n_2^\mu = \delta(x^+)\, \mathbf{B}(\vec{x})\, n_2^\mu + O\left(\sqrt{\frac{m_t^2}{s}}\right)$$

Shockwave approximation

Propagator through the external shockwave field

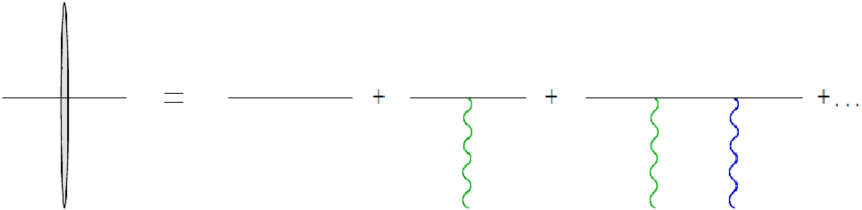
$$G(z_2,\, z_0) = -\int d^4 z_1 \theta\left(z_2^+\right) \delta\left(z_1^+\right) \theta\left(-z_0^+\right) G\left(z_2 - z_1\right) \gamma^+ G\left(z_1 - z_0\right) U_1$$

Wilson lines :

$$U_i^\eta = U_{\vec{z}_i}^\eta = P \exp\left[ig \int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) dz_i^+\right]$$

$$U_i^\eta = 1 + ig \int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_\eta^-\left(z_i^+, \vec{z}_i\right) b_\eta^-\left(z_j^+, \vec{z}_j\right) \theta(z_{ji}^+) dz_i^+ dz_j^+$$

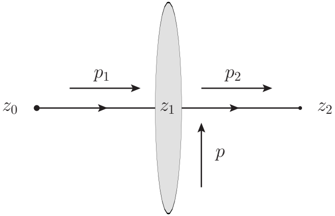
...



Quark propagator through the external field in momentum space

Fourier transform of a Wilson line

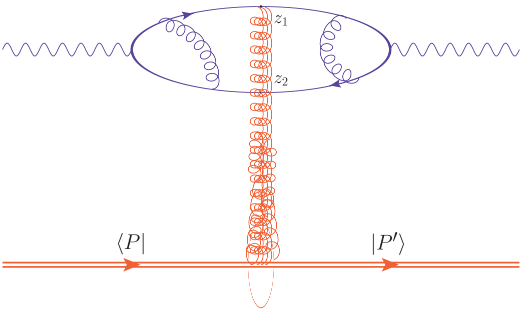
$$\tilde{U}^{\eta}(\vec{p}) = \int d^{D-2}\vec{z}\; e^{-i(\vec{p}\cdot\vec{z})}\, U^{\eta}_{\vec{z}}$$



$$G(p_2, p_1) \propto \theta(p_1^+) \int d^D p\; \delta(p^+)\, \delta(p + p_1 - p_2) G(p_2) \gamma^+ G(p_1) \tilde{U}^{\eta}_{\vec{p}}$$

Exchange in t-channel of an effective off-shell particle with 0 momentum along n_1

Factorized picture



Factorized amplitude

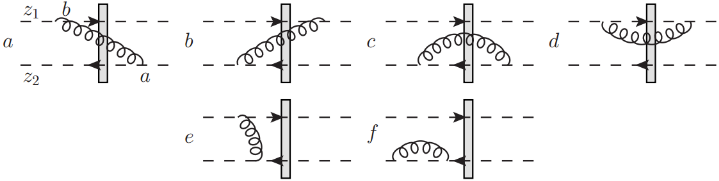
$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2 \, \Phi^\eta(\vec{z}_1, \vec{z}_2) \, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation!

Evolution for the dipole operator

$$\mathcal{U}_{12}^{\eta+\delta\eta} - \mathcal{U}_{12}^{\eta}$$



B-JIMWLK hierarchy of equations

[Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

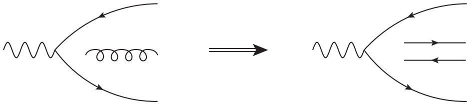
$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta} + \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}\right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}}{\partial \eta} = \dots$$

Evolves a dipole into a double dipole

The BK equation

Mean field approximation, or 't Hooft planar limit $N_c \to \infty$ in the dipole B-JIMWLK equation



[Mueller]

$\Rightarrow$ BK equation [Balitsky, 1995] [Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2} \vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle + \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

BFKL/BKP part — Triple pomeron vertex

Non-linear term : saturation

Equivalence with BFKL at NLL accuracy

Linear limit: usual k_t-factorization (BFKL framework)

s-channel discontinuity of $A + B \to A' + B'$ scattering amplitudes

$$\delta(p_{A'} + p_{B'} - p_A - p_B) Disc_s \mathcal{A}_{AB}^{A'B'} \propto \Phi(A', A) \otimes \mathcal{K} \otimes \Phi(B', B)$$

For any non-singular operator $\mathcal{O}$ this discontinuity is invariant under

$$\Phi(A', A) \to \Phi(A', A)\,\mathcal{O}, \quad \mathcal{K} \to \mathcal{O}^{-1}\mathcal{K}\mathcal{O}, \quad \Phi(B', B) \to \mathcal{O}^{-1}\Phi(B', B)$$

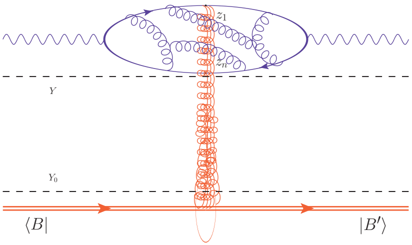
i.e. there is an ambiguity of distribution of corrections between the impact factors and the kernel. In the linear approximation of BK there exists an operator $\mathcal{O}$ such that

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi_{BK} = (\Phi_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL} \otimes \mathcal{O}) \otimes (\mathcal{O}^{-1} \otimes \Phi_{BFKL})$$

The expression for $\mathcal{O}$ to make the kernels explicitly equivalent at NLO accuracy under such a change of variables is known [Fadin, Fiore, Grabovsky, Papa]

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

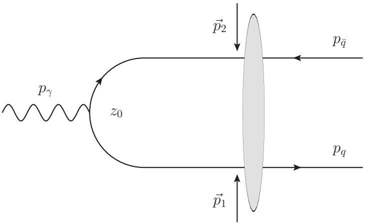
Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

First step: open parton production

- Regge-Gribov limit : $s \gg Q^2 \gg \Lambda_{QCD}$
- Otherwise completely general kinematics
- Shockwave (CGC) Wilson line approach
- Transverse dimensional regularization $d = 2 + 2\varepsilon$, longitudinal cutoff

$$|p_g^+| > \alpha p_\gamma^+$$

LO diagram



$$\mathcal{A} = \frac{\delta^{ik}}{\sqrt{N_c}} \int d^D z_0 [\bar{u}(p_q, z_0)]_{ij} (-ie_q) \hat{\varepsilon}_\gamma e^{-i(p_\gamma \cdot z_0)} [v(p_{\bar{q}}, z_0)]_{jk} \theta(-z_0^+)$$

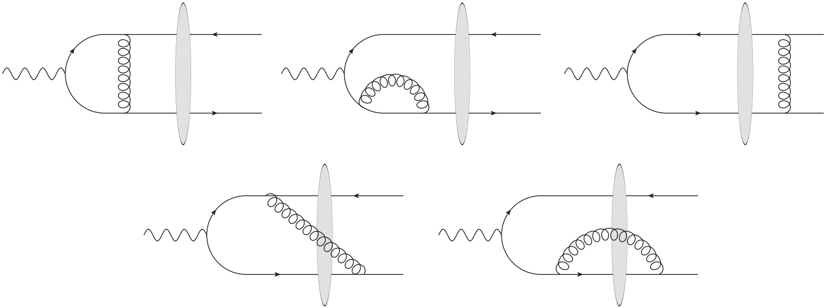
Color factor

$$\frac{\delta^{ik}}{\sqrt{N_c}} [(\tilde{U}^\alpha_{\vec{p}_1})_{ij} (\tilde{U}^\alpha_{-\vec{p}_2})_{jk} - \delta_{ij}\delta_{jk}] = \sqrt{N_c}\, \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2)$$

$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d \vec{z}_1 d^d \vec{z}_2\, e^{-i(\vec{p}_1 \cdot \vec{z}_1) - i(\vec{p}_2 \cdot \vec{z}_2)} [\tfrac{1}{N_c} \mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1]$$

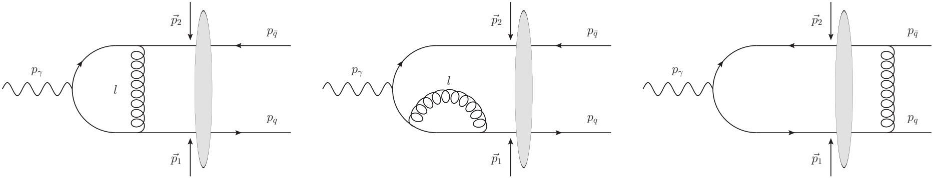
$p_{ij} = p_i - p_j$

NLO open $q\bar{q}$ production



Diagrams contributing to the NLO correction

First kind of virtual corrections



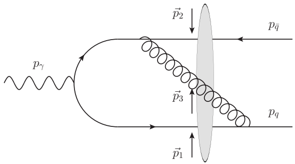
Color factor

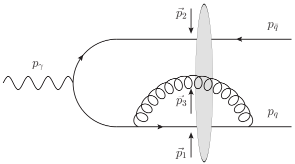
$$\frac{C_F}{\sqrt{N_c}}\tilde{\mathcal{U}}^{\alpha}(\vec{p}_1,\vec{p}_2)$$

Impact factor

$$\mathcal{A}^{(1)}_{NLO} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+)\int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\,\Phi_{V1}(\vec{p}_1,\vec{p}_2)$$
$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^{\alpha}(\vec{p}_1,\vec{p}_2) \middle| P \right\rangle$$

Second kind of virtual corrections





Color factor

$$\frac{\delta^{ik}}{\sqrt{N_c}}(t^a U_1 t^b U_2^\dagger)_{ik}(U_3)^{ab}$$

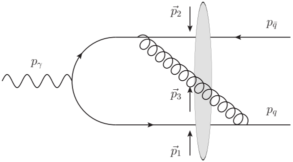
Action of the Wilson line in the adjoint representation

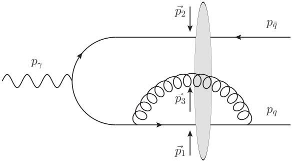
$$(U_3)^{ab} t^b = U_3 t^a U_3^\dagger \quad \Rightarrow \quad (U_3)^{ab} = 2\mathrm{Tr}(t^a U_3 t^b U_3^\dagger)$$

+ Fierz identity

$$C_F \mathcal{U}_{12} + \frac{1}{2}[\mathcal{U}_{13} + \mathcal{U}_{32} - \mathcal{U}_{12} + \mathcal{U}_{13}\mathcal{U}_{32}] \quad = C_F \mathcal{U}_{12} + \mathcal{W}_{123}$$

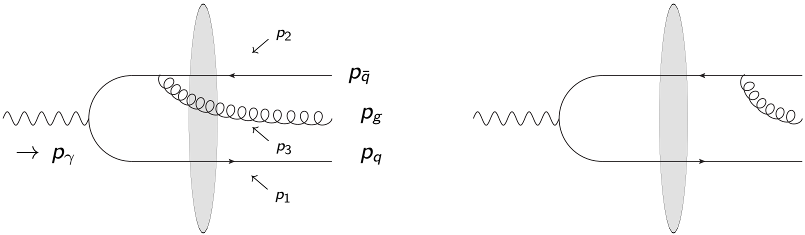
Second kind of virtual corrections





$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$
$$+ \Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \middle| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \middle| P \right\rangle]$$

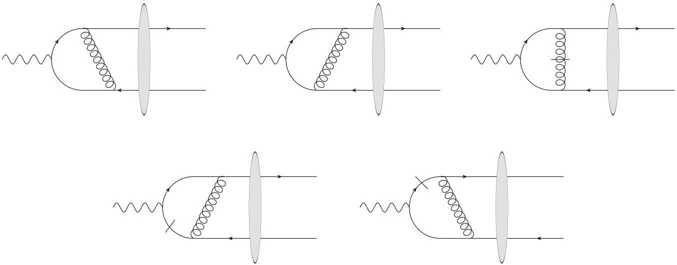
LO open $q\bar{q}g$ production



$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$
$$+\Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \middle| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \middle| P \right\rangle]$$

$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$
$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Generic computation method



- Perform the $k_\perp$ integration with the usual *d*-dimensional regularization methods
- Perform the k^+ integration with the longitudinal cutoff αp_γ^+ when possible, or isolate the divergent term by $+$ prescription

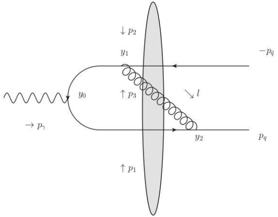
$$\int_{\alpha p_\gamma^+}^{p^+} dk^+ \frac{F(k^+)}{k^+} = \int_{\alpha p_\gamma^+}^{p^+} dk^+ \frac{F(0)}{k^+} + \int_0^{p^+} dk^+ \left[\frac{F(k^+)}{k^+}\right]_+$$

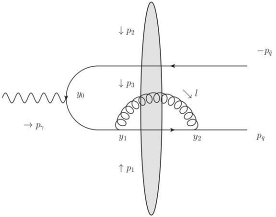
Divergences

Remaining divergences

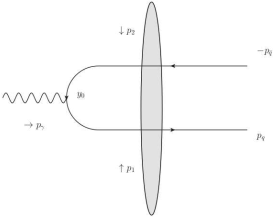
- Rapidity divergence $p_g^+ \to 0$ $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+}p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+}p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Rapidity divergence





Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Rapidity divergence

B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{\mathcal{U}}_{12}^{\alpha}}{\partial \log \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathcal{U}}_{13}^{\alpha} \tilde{\mathcal{U}}_{32}^{\alpha} + \tilde{\mathcal{U}}_{13}^{\alpha} + \tilde{\mathcal{U}}_{32}^{\alpha} - \tilde{\mathcal{U}}_{12}^{\alpha} \right)$$

$$\times \left[2 \frac{(\vec{k}_1 - \vec{p}_1) \cdot (\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2 \right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2 \right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{\mathcal{U}}_{12}^{\alpha} \to \Phi_0 \tilde{\mathcal{U}}_{12}^{\eta} + 2 \log \left(\frac{e^{\eta}}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \tilde{\mathcal{W}}_{123}$$

Rapidity divergence

Virtual contribution

$$(\Phi^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{\alpha^2}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

BK contribution

$$(\Phi^{\mu}_{BK})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{\alpha^2}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] \right\}$$

Sum : the α dependence cancels

$$(\Phi'^{\mu}_{V2})_{div} \propto \Phi^{\mu}_{0} \left\{ 4 \ln\left(\frac{x\bar{x}}{e^{2\eta}}\right) \left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon} \right\}$$

Rapidity divergence

Cancellation of the remaining $1/\epsilon$ divergence

Convolution

$$
\begin{aligned}
(\Phi'^{\mu}_{V2} \otimes \mathcal{W}) &= 2\int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \left\{ 4\ln\left(\frac{x\bar{x}}{e^{2\eta}}\right)\left[\frac{1}{\varepsilon} + \ln\left(\frac{\vec{p}_3^{\,2}}{\mu^2}\right)\right] - \frac{6}{\varepsilon}\right\} \\
&\times \quad \delta(\vec{p}_{q1} + \vec{p}_{\bar{q}2} - \vec{p}_3)\left[\tilde{\mathcal{U}}_{13} + \tilde{\mathcal{U}}_{32} - \tilde{\mathcal{U}}_{12} - \tilde{\mathcal{U}}_{13}\tilde{\mathcal{U}}_{32}\right]\Phi_0^{\mu}(\vec{p}_1, \vec{p}_2)
\end{aligned}
$$

Rq :

- $\Phi_0(\vec{p}_1, \vec{p}_2)$ only depends on one of the t-channel momenta.
- The double-dipole operators cancels when $\vec{z}_3 = \vec{z}_1$ or $\vec{z}_3 = \vec{z}_2$.

This permits one to show that the convolution cancels the remaining $\frac{1}{\varepsilon}$ divergence.

Then $\tilde{\mathcal{U}}^{\alpha}_{12}\Phi_0 + \Phi_{V2}$ is finite

Divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*,\ \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\qquad \Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\qquad \Phi_{R1}\Phi_{R1}^*$

UV divergence

Cancelling tadpole integrals in dimensional regularization

Split the phase space between a UV part and an IR part

$$\int \frac{d^D k}{(k^2+i0)^2} = i\, S_{D-1} \int_0^{+\infty} dk_E (k_E)^{D-5}$$
$$= \lim_{k_{IR}\to 0,\, k_{UV}\to+\infty} \left[\int_{k_{IR}}^{\Lambda} dk_E (k_E)^{D-5} + \int_{\Lambda}^{k_{UV}} dk_E (k_E)^{D-5} \right]$$

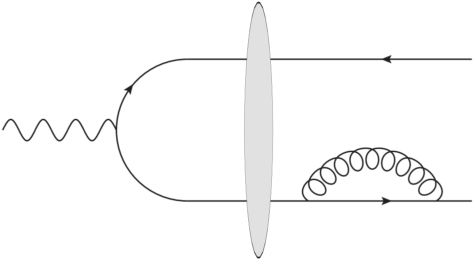
For $D = 4 + \epsilon$, the divergence is regulated by $\epsilon_{IR} > 0$ in the IR part and by $\epsilon_{UV} < 0$ in the UV part. The pole in the previous integral then reads

$$\int \frac{d^D k}{(k^2+i0)^2} = \frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}}$$

Then one can cancel the result in the analytic continuation for $\epsilon_{IR} = \epsilon_{UV} = \epsilon \simeq 0$.

UV divergence

Tadpole diagrams



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$

In the massless limit, renormalization of the external quark lines is absent in dimensional regularization.

Divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$, $\Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+}p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+}p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Constructing a finite cross section

Exclusive diffractive production of a forward dijet

From partons to jets

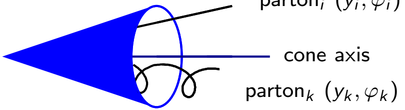
Soft and collinear divergence

Jet cone algorithm

We define a cone width for each pair of particles with momenta p_i and p_k, rapidity difference ΔY_{ik} and relative azimuthal angle $\Delta\varphi_{ik}$

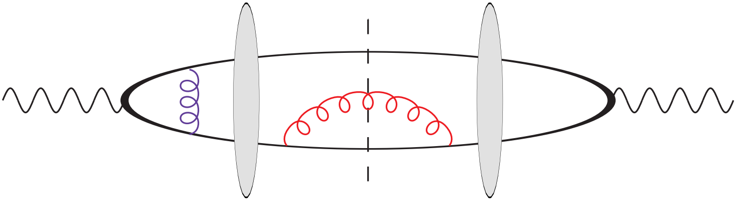
$$(\Delta Y_{ik})^2 + (\Delta\varphi_{ik})^2 = R_{ik}^2$$

If $R_{ik}^2 < R^2$, then the two particles together define a single jet of momentum $p_i + p_k$.



Applying this in the small R^2 limit cancels our soft and collinear divergence.

Remaining divergence



- Soft divergence $p_g \to 0$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$$

- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$

$$\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^* + \Phi_{R1}\Phi_{R1}^*$$

Remaining divergence

Soft real emission

$$\left(\Phi_{R1}\Phi_{R1}^{*}\right)_{soft} \propto \left(\Phi_0\Phi_0^{*}\right)\int_{\text{outside the cones}}\left|\frac{p_q^{\mu}}{(p_q.p_g)}-\frac{p_{\bar{q}}^{\mu}}{(p_{\bar{q}}.p_g)}\right|^2 \frac{dp_g^+}{p_g^+}\frac{d^d p_g}{(2\pi)^d}$$

Collinear real emission

$$\left(\Phi_{R1}\Phi_{R1}^{*}\right)_{col} \propto \left(\Phi_0\Phi_0^{*}\right)\left(\mathcal{N}_q+\mathcal{N}_{\bar{q}}\right)$$

Where $\mathcal{N}$ is the number of jets in the quark or the antiquark

$$\mathcal{N}_k = \frac{(4\pi)^{\frac{d}{2}}}{\Gamma(2-\frac{d}{2})}\int_{\alpha p_\gamma^+}^{p_{jet}^+}\frac{dp_g^+ dp_k^+}{2p_g^+ 2p_k^+}\int_{\text{in cone k}}\frac{d^d\vec{p}_g d^d\vec{p}_k}{(2\pi)^d\,\mu^{d-2}}\frac{\mathrm{Tr}\left(\hat{p}_k\gamma^{\mu}\hat{p}_{jet}\gamma^{\nu}\right)d_{\mu\nu}(p_g)}{2p_{jet}^+\left(p_k^- + p_g^- - p_{jet}^-\right)^2}$$

Those two contributions cancel exactly the virtual divergences

Cancellation of divergences

Total divergence

$$(d\sigma_1)_{div} = \alpha_s \frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}} \left(\frac{N_c^2-1}{2N_c}\right) (S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2}) d\sigma_0$$

Virtual contribution

$$\begin{aligned} S_V &= \left[2\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - 3\right]\left[\ln\left(\frac{x_j x_{\bar{j}}\mu^2}{(x_j \vec{p}_{\bar{j}} - x_{\bar{j}}\vec{p}_j)^2}\right) - \frac{1}{\epsilon}\right] \\ &+ 2i\pi\ln\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) + \ln^2\left(\frac{x_j x_{\bar{j}}}{\alpha^2}\right) - \frac{\pi^2}{3} + 6 \end{aligned}$$

Real contribution

$$\begin{aligned} S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} &= 2\left[\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)\right. \\ &+ 2\ln\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right)\left(\frac{1}{\epsilon} - \ln\left(\frac{x_{\bar{j}}x_j\mu^2}{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^2}\right)\right) - \ln^2\left(\frac{x_{\bar{j}}x_j}{\alpha^2}\right) \\ &+ \left.\frac{3}{2}\ln\left(\frac{16\mu^4}{R^4\vec{p}_j^{\,2}\vec{p}_{\bar{j}}^{\,2}}\right) - \ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) - \frac{3}{\epsilon} - \frac{2\pi^2}{3} + 7\right] \end{aligned}$$

Cancellation of divergences

Total "divergence"

$$
\begin{aligned}
div &= S_V + S_V^* + S_R + \mathcal{N}_{jet1} + \mathcal{N}_{jet2} \\
&= 4\left[\frac{1}{2}\ln\left(\frac{(x_{\bar{j}}\vec{p}_j - x_j\vec{p}_{\bar{j}})^4}{x_{\bar{j}}^2 x_j^2 R^4 \vec{p}_{\bar{j}}^{\,2}\vec{p}_j^{\,2}}\right)\left(\ln\left(\frac{4E^2}{x_{\bar{j}}x_j(p_\gamma^+)^2}\right)+\frac{3}{2}\right)\right. \\
&+ \left.\ln(8) - \frac{1}{2}\ln\left(\frac{x_j}{x_{\bar{j}}}\right)\ln\left(\frac{x_j\vec{p}_{\bar{j}}^{\,2}}{x_{\bar{j}}\vec{p}_j^{\,2}}\right) + \frac{13-\pi^2}{2}\right]
\end{aligned}
$$

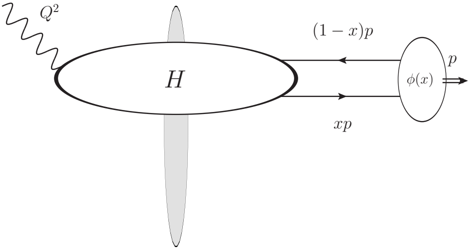
Our cross section is thus finite

Constructing a finite amplitude

Exclusive diffractive production of a light neutral vector meson

Towards an extension of [Munier, Staśto, Mueller] and [Ivanov, Kotsky, Papa]

Additional factorization



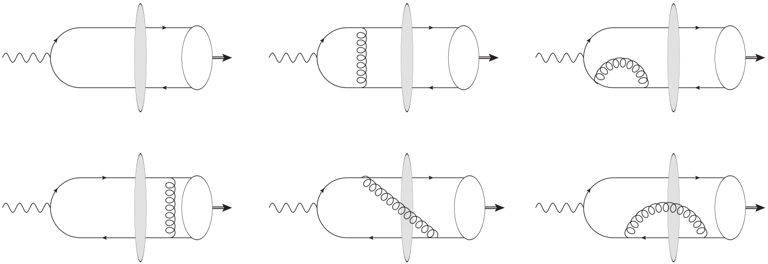
Once the amplitude is factorized in terms of impact factors, we perform an additional twist expansion in powers of a hard Björken scale (photon virtuality, Madelstam t..).

Thus we can factorize, in terms of collinear factorization, the bilocal matrix element

$$\langle V(p)|\bar{\psi}(z_{12})\gamma^{\mu}\psi(0)|0\rangle|_{z_{12}^2\to 0} = p_{\mu} f_V \int_0^1 dx\, e^{ix(p\cdot z_{12})}\phi_{\parallel}(x)$$

$\phi_{\parallel}(x)$ = meson Distribution Amplitude (DA)

Exclusive diffractive production of a light neutral vector meson



$$\mathcal{A}_0 = -\frac{e_V f_V \varepsilon_\beta}{N_c} \int_0^1 dx \varphi_\parallel (x) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d}$$
$$\times (2\pi)^{d+1} \delta \left(p_V^+ - p_\gamma^+\right) \delta \left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right)$$
$$\times \Phi_0^\beta \left(x, \vec{p}_1, \vec{p}_2\right) \tilde{\mathcal{U}}_{12}^\eta.$$

Leading twist for a longitudinally polarized meson

Otherwise general kinematics, including transverse virtual photon (twist 3) contributions, and the photoproduction limit (for large $t-$channel momentum transfer)

ERBL evolution equation

Efremov, Radyushkin, Brodsky, Lepage evolution equation for a DA

Renormalization of the bilocal operator

$$\bar{\psi}(z_{12})\gamma^{\mu}\psi(0)$$

$\Rightarrow$ Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial\ln\mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^{\epsilon}}\left(\frac{\mu_F^2}{\mu^2}\right)^{\epsilon}\int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

$\mathcal{K}$ = ERBL kernel

ERBL evolution equation

Evolution equation for the distribution amplitude in the $\overline{MS}$ scheme

$$\frac{\partial\varphi(x,\mu_F^2)}{\partial \ln \mu_F^2} = \frac{\alpha_s C_F}{2\pi}\frac{\Gamma(1-\epsilon)}{(4\pi)^\epsilon}\left(\frac{\mu_F^2}{\mu^2}\right)^\epsilon \int_0^1 dz\varphi(z,\mu_F^2)\mathcal{K}(x,z),$$

where we parameterize the ERBL kernel for consistency as

$$\begin{aligned}\mathcal{K}(x,z) &= \frac{x}{z}\left[1+\frac{1}{z-x}\right]\theta(z-x-\alpha)\\ &+ \frac{1-x}{1-z}\left[1+\frac{1}{x-z}\right]\theta(x-z-\alpha)\\ &+ \left[\frac{3}{2}-\ln\left(\frac{x(1-x)}{\alpha^2}\right)\right]\delta(z-x).\end{aligned}$$

It is equivalent to the usual ERBL kernel

Infrared finiteness

The amplitude we obtain is finite. For example the dipole $\gamma_L^* \to V_L$ contribution reads

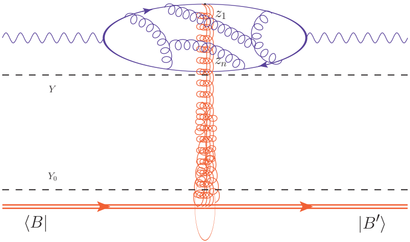
$$\begin{aligned}
\Phi_1^+(x) &= \int_0^x dz \left(\frac{x-z}{x}\right) \Phi_0^+(x-z) \\
&\times \left[1 + \left(1 + \left[\frac{1}{z}\right]_+\right) \ln\left(\frac{\left(((\bar{x}+z)\vec{p}_1 - (x-z)\vec{p}_2)^2 + (x-z)(\bar{x}+z)Q^2\right)^2}{\mu_F^2 (x-z)(\bar{x}+z)Q^2}\right)\right] \\
&+ \frac{1}{2}\Phi_0^+(x)\left[\frac{1}{2}\ln^2\left(\frac{\bar{x}}{x}\right) + 3 - \frac{\pi^2}{6} - \frac{3}{2}\ln\left(\frac{\left((\bar{x}\vec{p}_1 - x\vec{p}_2)^2 + x\bar{x}Q^2\right)^2}{x\bar{x}\mu_F^2 Q^2}\right)\right] \\
&+ \frac{\left(p_\gamma^+\right)^2}{2x\bar{x}} \int_0^x dz \left[(\phi_5)_{LL}|_{\vec{p}_3=\vec{0}} + (\phi_6)_{LL}|_{\vec{p}_3=\vec{0}}\right]_+ + (x \leftrightarrow \bar{x}, \vec{p}_1 \leftrightarrow \vec{p}_2).
\end{aligned}$$

No end point singularity, even for a transverse photon and even in the photoproduction limit.

Practical use of such results for phenomenology

Practical use of such results

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity $\eta = Y_0$
- Evaluate the solution at a typical projectile rapidity $\eta = Y$
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P' | U_{\vec{z}_1} ... U_{\vec{z}_n} | P \rangle$$

Residual parameter dependence

Required parameters

- Renormalization scale μ_R
- Factorization scale μ_F in the case of meson production (if assumed that $\mu_F \neq \mu_R$)
- Typical target rapidity Y_0
- Typical projectile rapidity Y

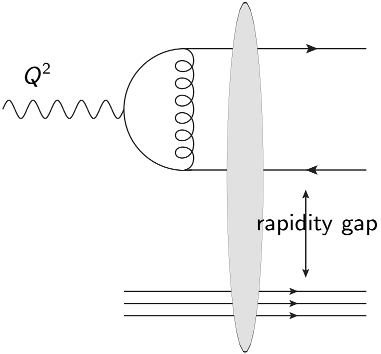
In the linear BFKL limit, the cross section only depends on $Y - Y_0$, so one only needs one arbirary parameter s_0 defined by

$$Y - Y_0 = \ln\left(\frac{s}{s_0}\right).$$

Modifying any of these parameter results in a higher order (NNLO) contribution

General amplitude

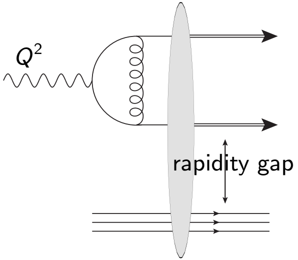
- Most general kinematics
- The hard scale can be Q^2, t, M_X^2...
- The target can be either a proton or an ion, or another impact factor.
- Finite results for $Q^2 = 0$
- One can study ultraperipheral collision by tagging the particle which emitted the photon, in the limit $Q^2 \to 0$.



The general amplitude

Phenomenological applications : exclusive dijet production at NLO accuracy

- HERA data for exclusive dijet production in diffractive DIS can be fitted with our results
- For $Q^2 = 0$ we can give predictions for ultraperipheral *pp* and *pA* collisions
- Our results are best suited for electron ion colliders for precision saturation physics



Amplitude for diffractive dijet production

Phenomenological applications : exclusive trijet production at LO accuracy

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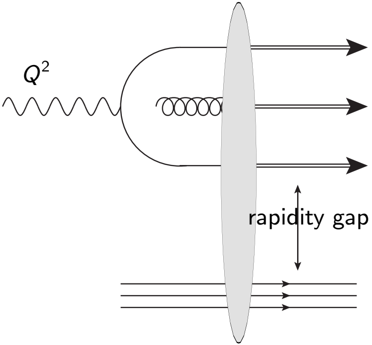
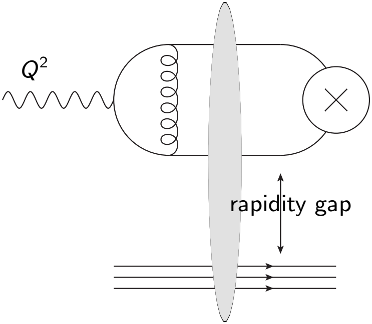


Amplitude for diffractive trijet production

[Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans]

Phenomenological applications

- HERA data can be fitted with our results
- For $Q^2 = 0$ we can give predictions for ultraperipheral pp and pA collisions at large t
- Our results are best suited for electron ion colliders for precision saturation physics



Amplitude for diffractive V production

Comparison with previous results [Work in progress]

- The $\gamma_L^* \to V_L$ contribution in the forward limit should coincide with previous results of Ivanov, Kotsky, Papa
- The comparison is non-trivial due to additional contributions from the formal BFKL/BK transition

$$\Phi_{BK} \otimes \mathcal{K}_{BK} \otimes \Phi'_{BK} = (\Phi_{BFKL} \otimes \mathcal{O})(\mathcal{O}^{-1} \otimes \mathcal{K}_{BFKL}\mathcal{O})(\mathcal{O}^{-1} \otimes \Phi'_{BFKL})$$

$\mathcal{O}$ was obtained to prove the kernel equivalence, but never checked on an impact factor

Conclusion

- We provided the full computation of the impact factor for the exclusive diffractive production of a forward dijet and of a light neutral vector meson with NLO accuracy in the shockwave approach
- It leads to an enormous number of possible phenomenological applications to test QCD in its Regge limit and towards saturation in past, present and future *ep*, *eA*, *pp* and *pA* colliders
- Our results open up possibilities for precision saturation physics with $b_\perp$ dependence in future *eA* colliders