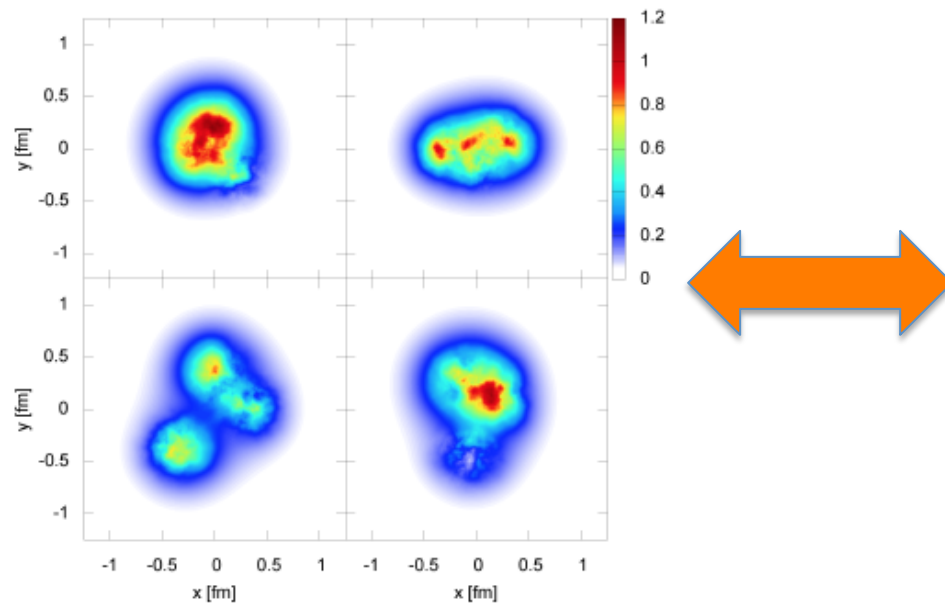
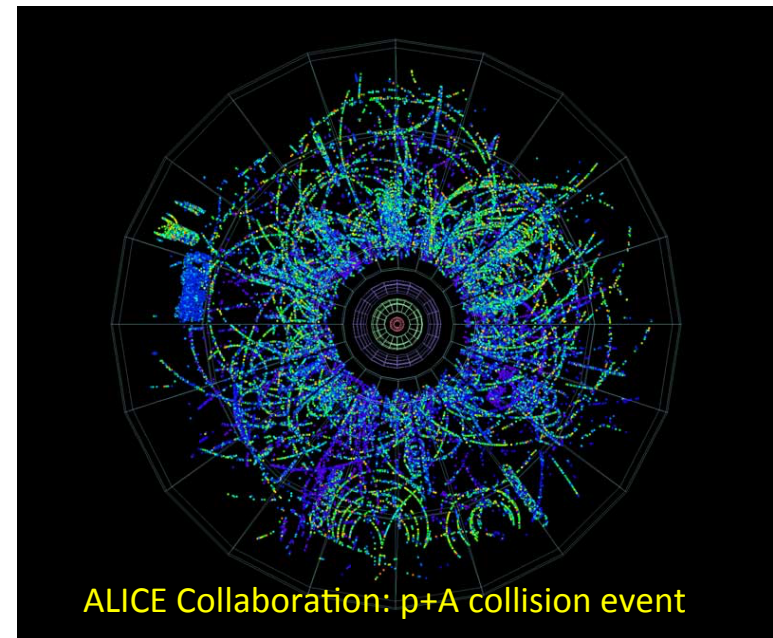


Parton model description of multiparticle azimuthal cumulants in p+A collisions

Raju Venugopalan
Brookhaven National Laboratory

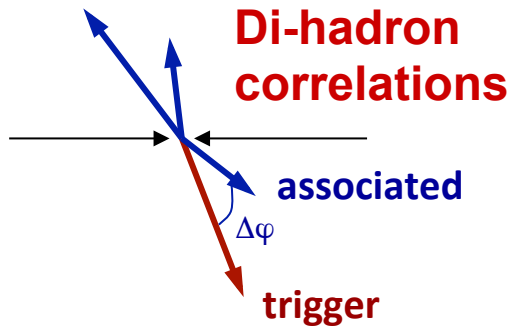


Mantysaari, Schenke, arXiv:1603.043049

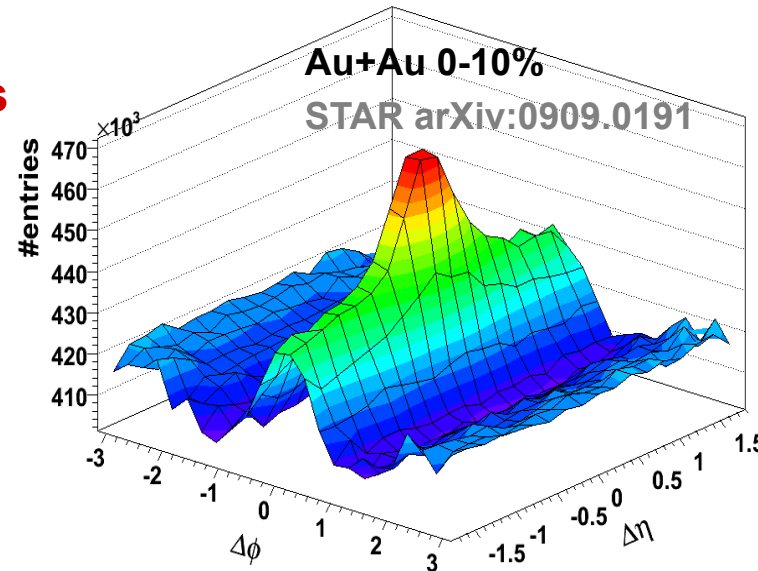


RBRC pA workshop, June 26-28, 2017

Azimuthal correlations in A+A collisions: the ridge



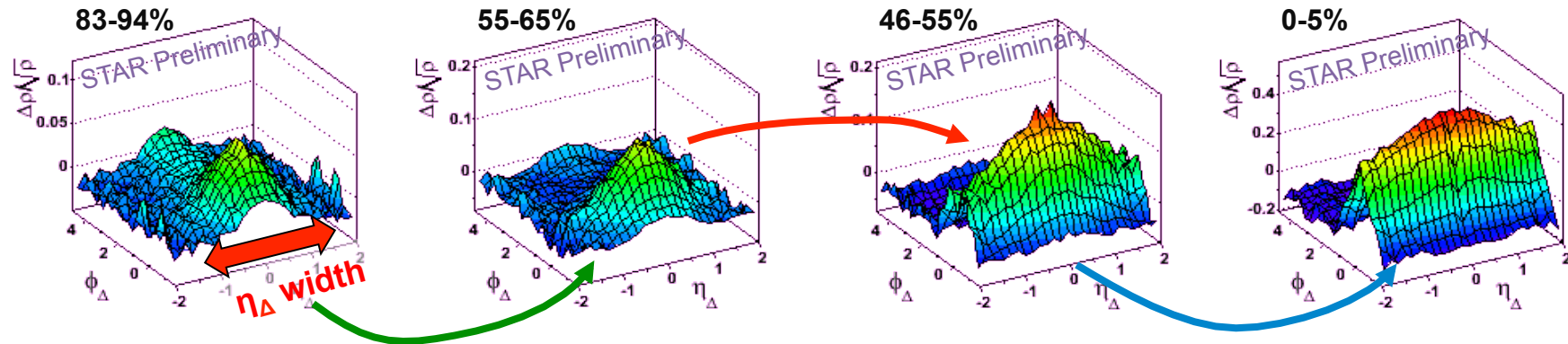
$$3 < p_{t,trigger} < 4 \text{ GeV}$$
$$p_{t,assoc.} > 2 \text{ GeV}$$



Novel long range “near-side” ($\Delta\phi \approx 0$) collimation

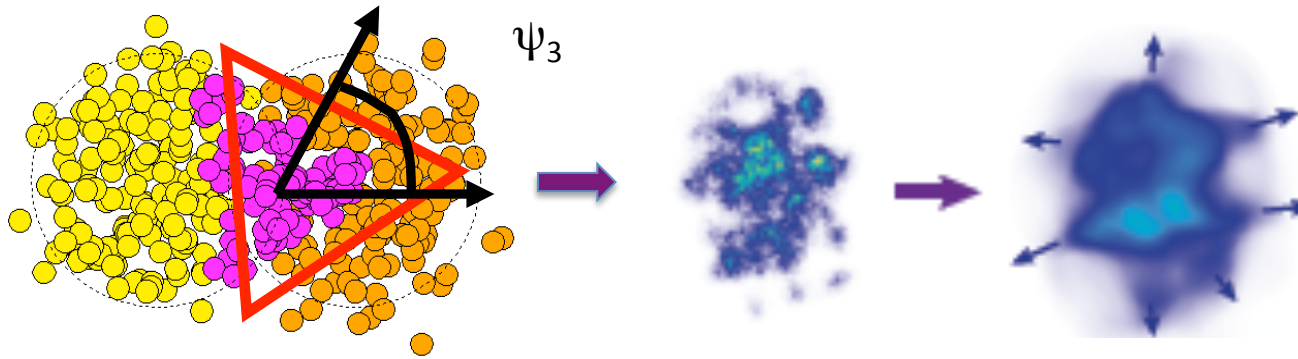
Long range “away-side” ($\Delta\phi \approx \pi$) correlations suppressed...jet quenching

Also seen in untriggered correlations



First seen by RHIC Au+Au experiments: STAR, PHOBOS, PHENIX

The A+A ridge and “collectivity”

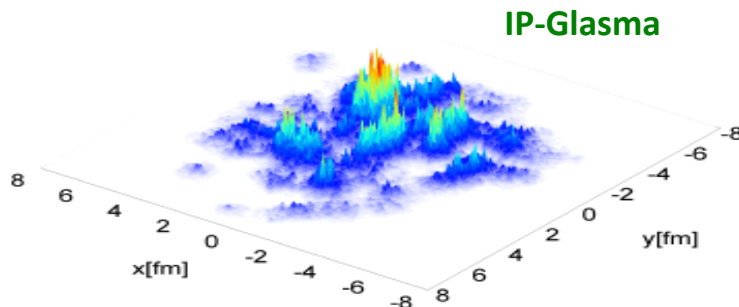


Alver, Roland, PRC81(2010)054905
Alver, Gombeaud, Luzum, Ollitrault,
PRC82 (2010) 03491

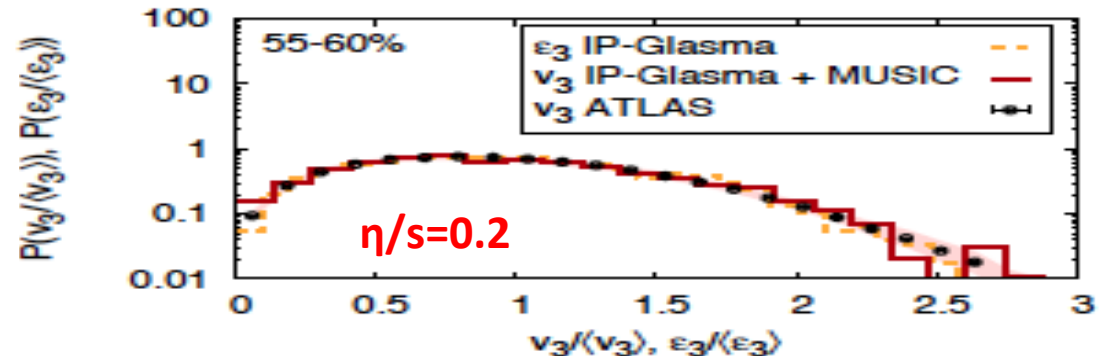
Structure of ridge correlations: hydrodynamic flow driven by event-by-event “eccentricity” fluctuations in nucleon positions

In hydro: multi-particle correlations factorize into product of single particle distributions that are commonly correlated to an “event plane”

$$\frac{1}{N_{\text{trig}} N_{\text{assoc.}}} \frac{d^2 N}{d\Delta\Phi} = 1 + V_1 \cos(\Delta\Phi) + V_2 \cos(2\Delta\Phi) + \dots$$



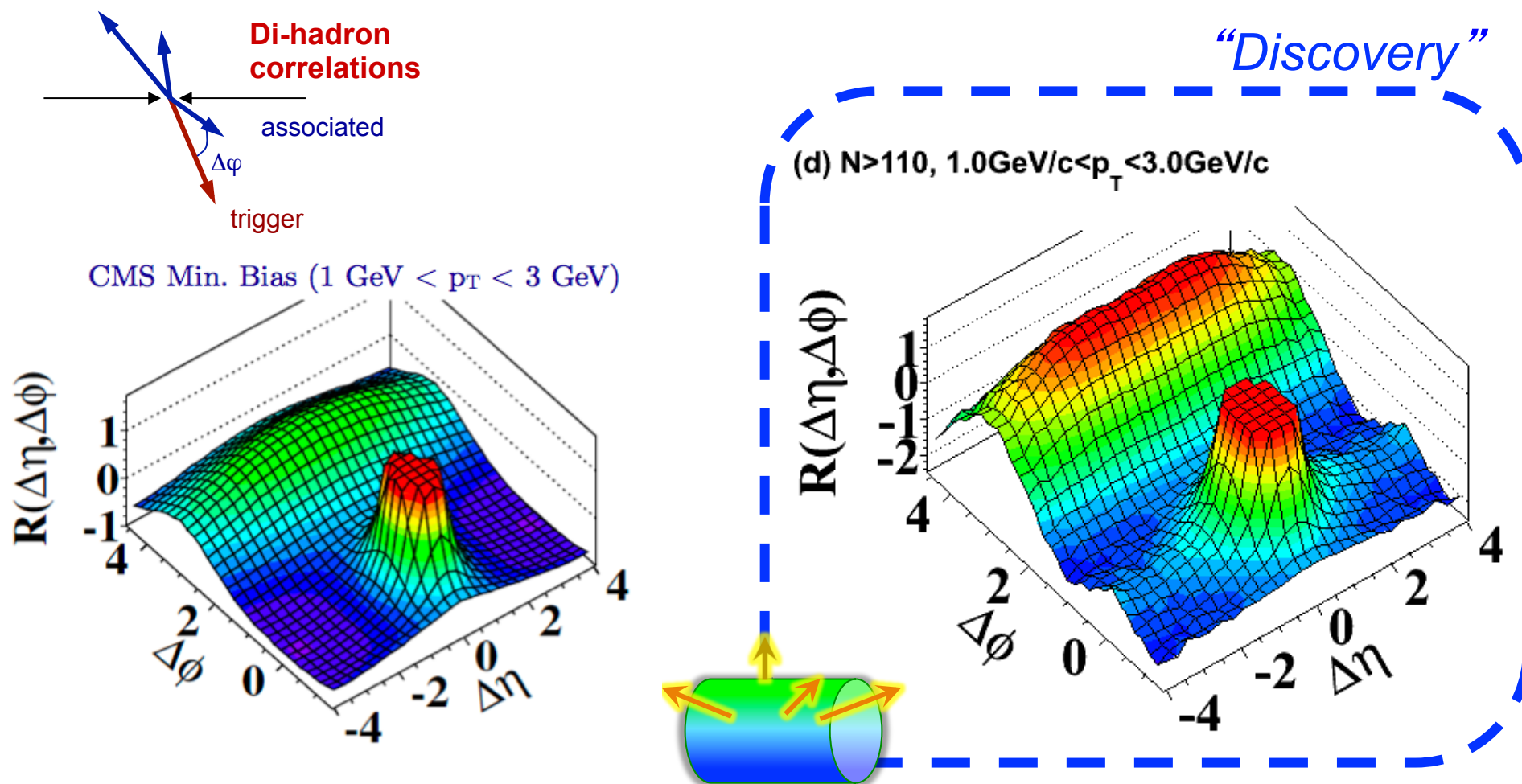
IP-Glasma



Some evidence of sensitivity of data to sub-nucleon scale fluctuations

Gale, Jeon, Schenke, Tribedy, Venugopalan, PRL110 (2013) 012302

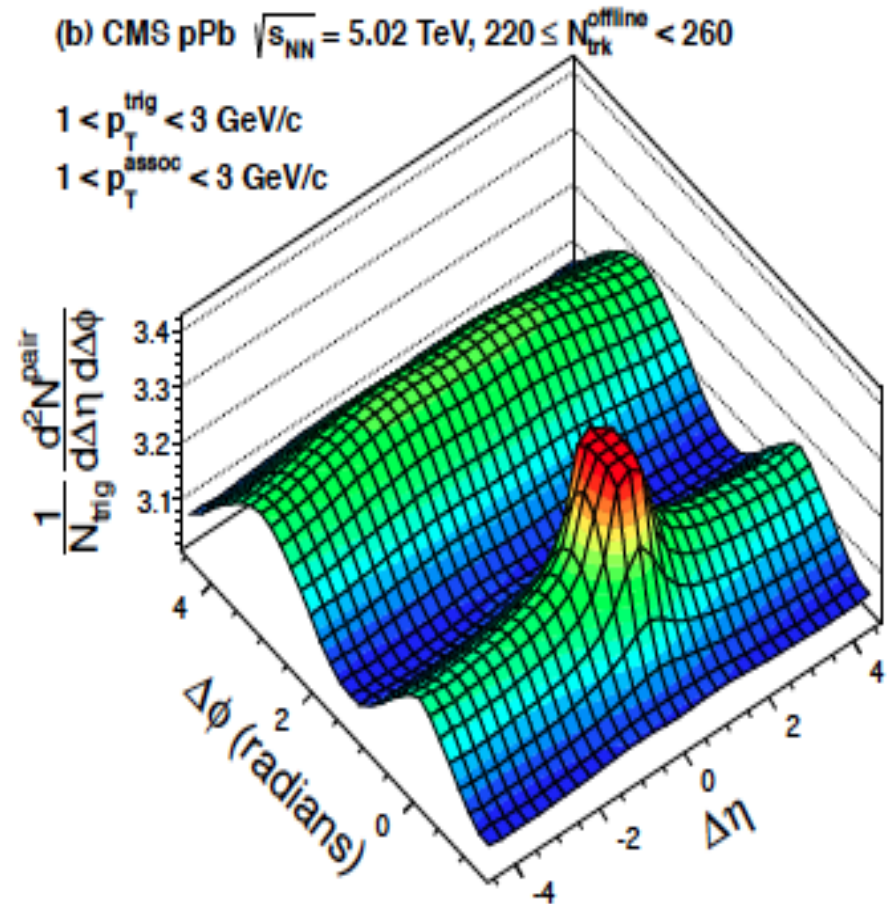
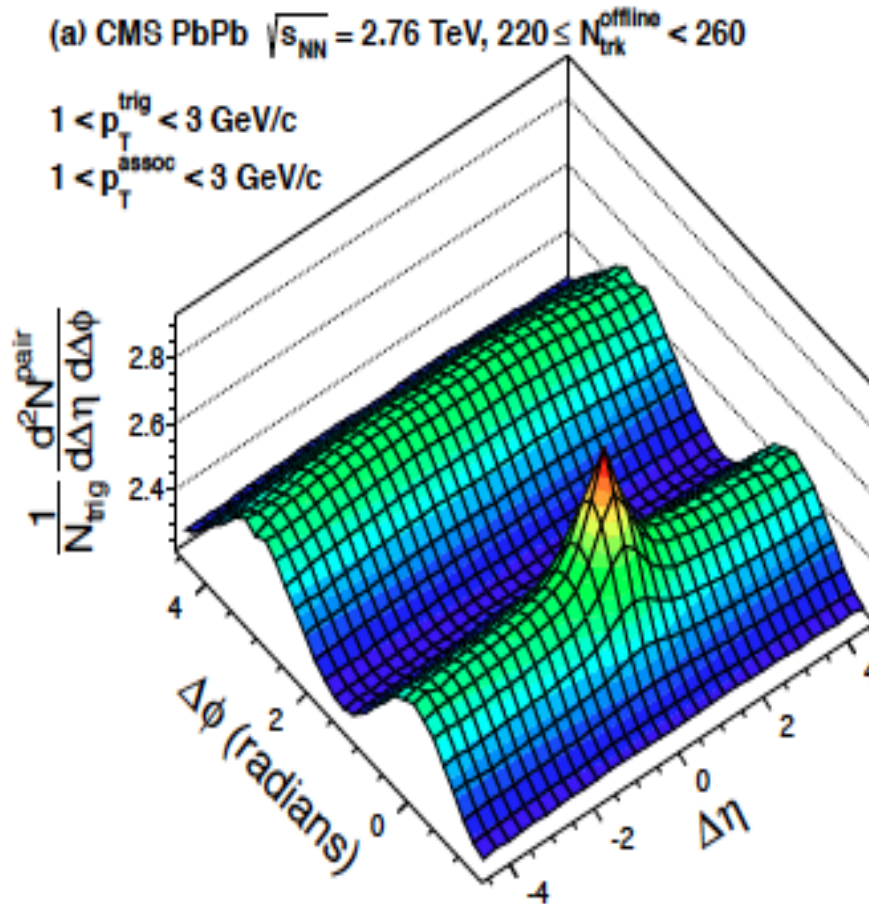
Two particle correlations: CMS results



Observation of Long-Range Near-Side Angular Correlations in Proton-Proton Collisions at the LHC [CMS Collaboration \(Vardan Khachatryan \(Yerevan Phys. Inst.\) et al.\)](#). JHEP 1009 (2010) 091
[Cited by 566 records](#)

5th most cited CMS physics paper to date!

Striking results from LHC p+A collisions



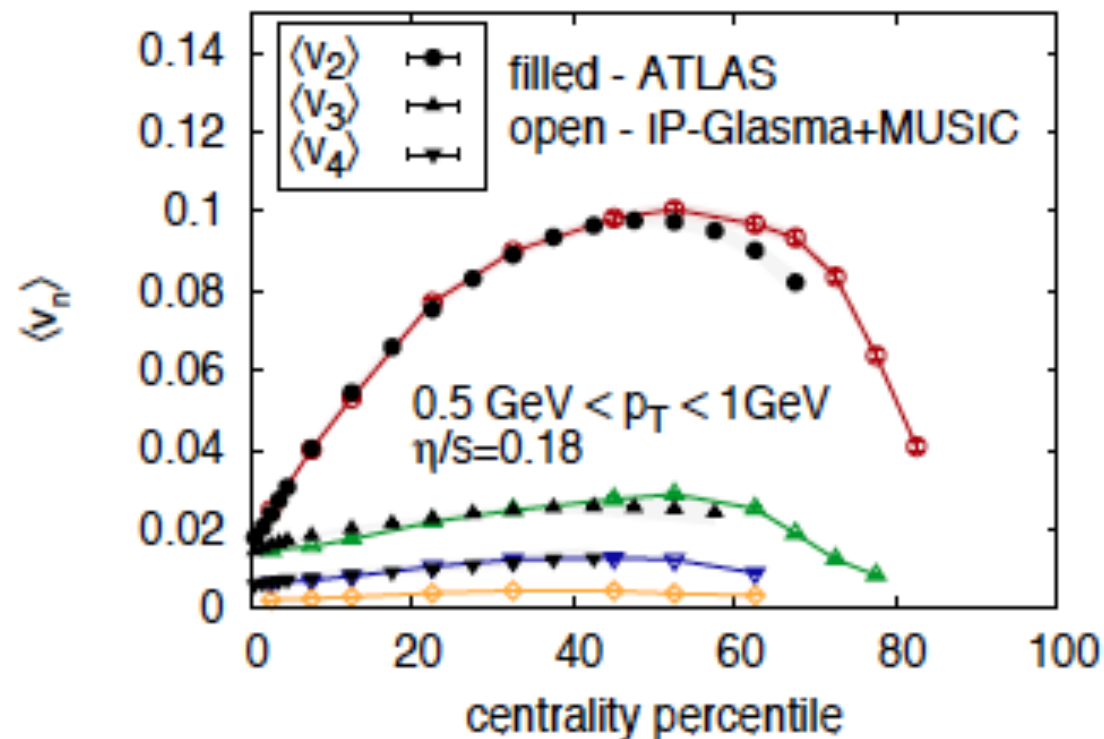
**p+A ridge much larger than p+p at same multiplicity
and nearly as large as that in peripheral Pb+Pb collisions**

What's the smallest sized QGP droplet?

IP-Glasma: CGC based framework employing Yang-Mills evolution of two lumpy light cone sources

MUSIC: Event-by-event relativistic viscous hydrodynamics

Schenke, Venugopalan, PRL 113 (2014) 102301



Where does the hydro paradigm break down?

Fundamental feature of QCD: Yang-Mills Theory



PHYSICAL REVIEW

VOLUME 96, NUMBER 1

OCTOBER 1, 1954

Conservation of Isotopic Spin and Isotopic Gauge Invariance*

C. N. YANG † AND R. L. MILLS

Brookhaven National Laboratory, Upton, New York

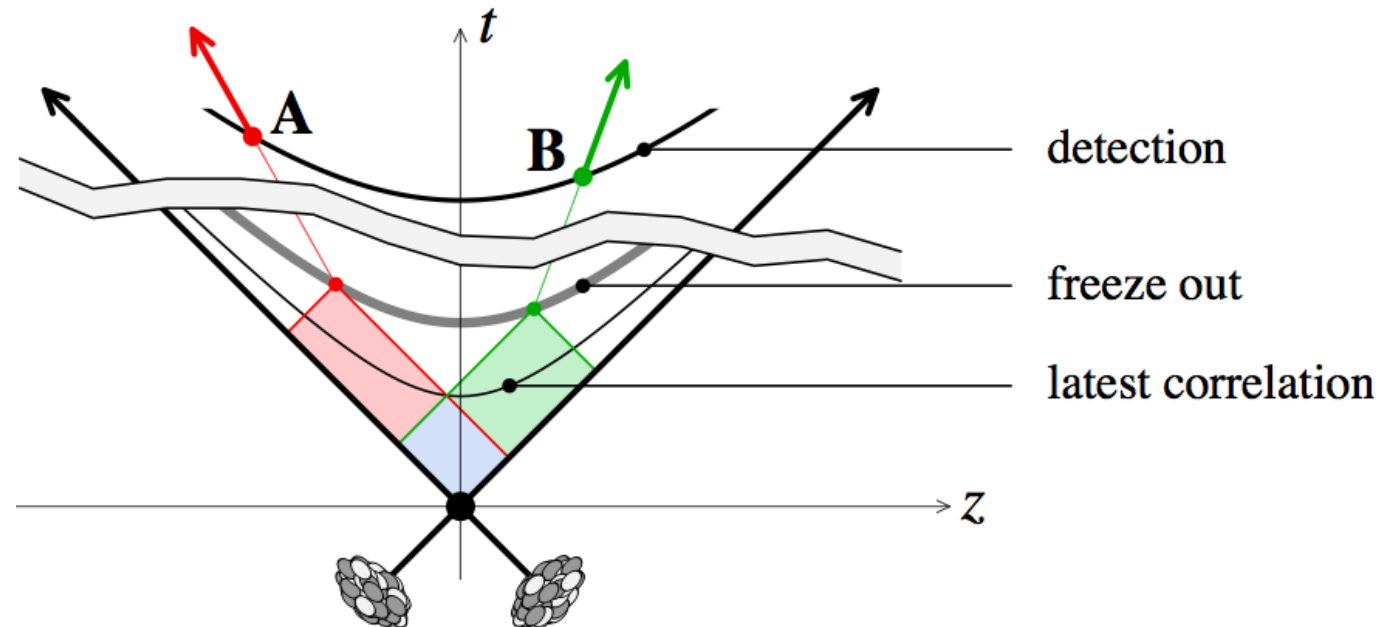
(Received June 28, 1954)

It is pointed out that the usual principle of invariance under isotopic spin rotation is not consistent with the concept of localized fields. The possibility is explored of having invariance under local isotopic spin rotations. This leads to formulating a principle of isotopic gauge invariance and the existence of a $\mathbf{b}$ field which has the same relation to the isotopic spin that the electromagnetic field has to the electric charge. The $\mathbf{b}$ field satisfies nonlinear differential equations. The quanta of the $\mathbf{b}$ field are particles with spin unity, isotopic spin unity, and electric charge $\pm e$ or zero.

Possibly the most important paper from BNL...yet not mentioned in the APS plaque on BNL's historic achievements ... outside in the Lounge

The J (aka J/ψ) discovery--November revolution, 1974--didn't make it in either...

Long range rapidity correlations as a chronometer



$$\tau \leq \tau_{\text{frz-out}} \exp \left(-\frac{1}{2} \underbrace{|y_A - y_B|} \right)$$

- ❖ Long range correlations sensitive to very early time (fractions of a femtometer $\sim 10^{-24}$ seconds) dynamics in collisions

What's the underlying QCD dynamics?

- ◆ Is it final state collective flow of the world's smallest droplets?
- ◆ Is it the initial state dynamics arising from rare configurations in the hadron wavefunctions?
- ◆ Or, is it some combination, where there is a smooth transition from one description to the other?

Option 1 stretches to the limit -- the applicability of thermodynamic and hydrodynamic concepts in high energy physics

Option 2 stretches to the limit -- our understanding of the quark-gluon sub-structure of hadrons

Sensitive measure of collectivity: azimuthal cumulants

2m-particle azimuthal cumulants

Borghini,Dinh,Ollitrault, nucl-th/0105040

$$c_n \{2m\} = \langle \langle e^{in(\phi_1 + \dots + \phi_m - \phi_{m+1} - \dots - \phi_{2m})} \rangle \rangle$$

If cumulants factorize into product of correlations relative to a reaction plane, define flow coefficients:

$$v_n \{2\}^2 \equiv c_n \{2\} \quad v_n \{4\}^4 \equiv -c_n \{4\} \quad v_n \{6\}^6 \equiv c_n \{6\} / 4$$

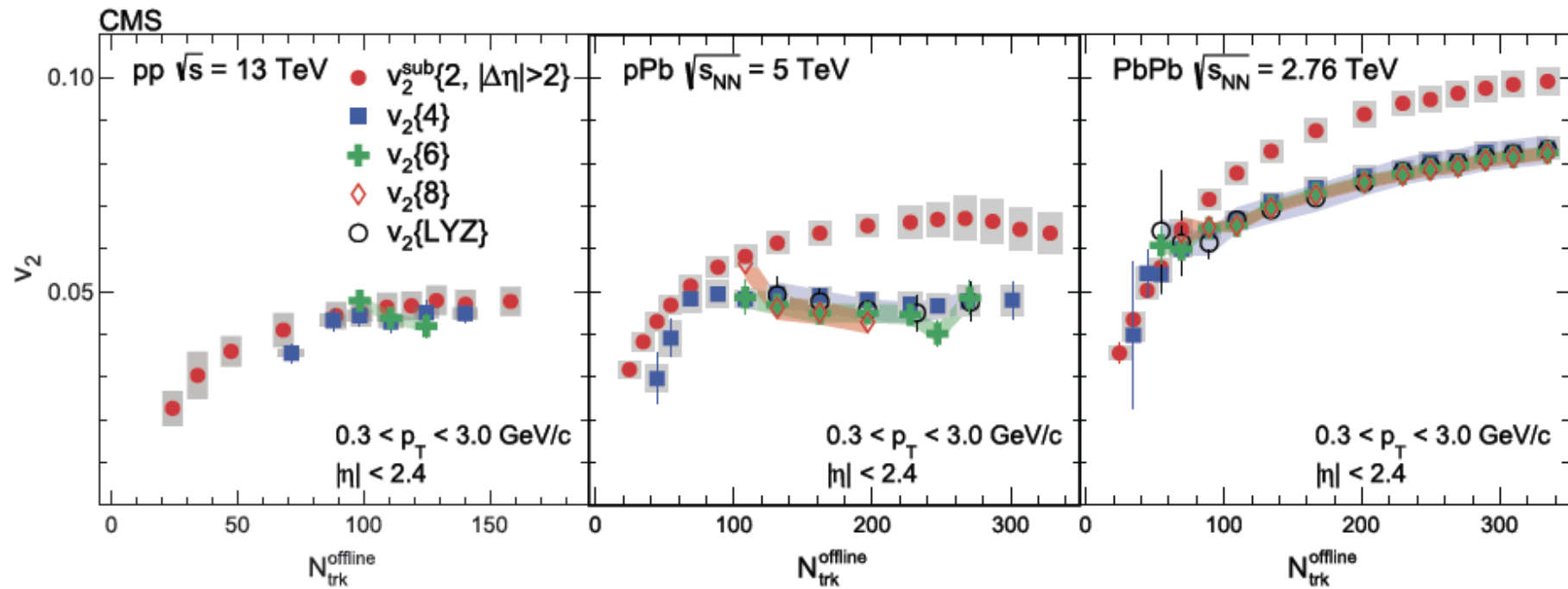
Spatial eccentricities: $\epsilon_n = \frac{1}{\langle r_\perp^n \rangle} \int d^2 r_\perp e^{in\phi_r} r_\perp^n \frac{dN}{dy d^2 r_\perp}$

A number of simple “Gaussian” models give $\epsilon_n \{2\} > \epsilon \{4\} = \epsilon \{6\} = \dots$

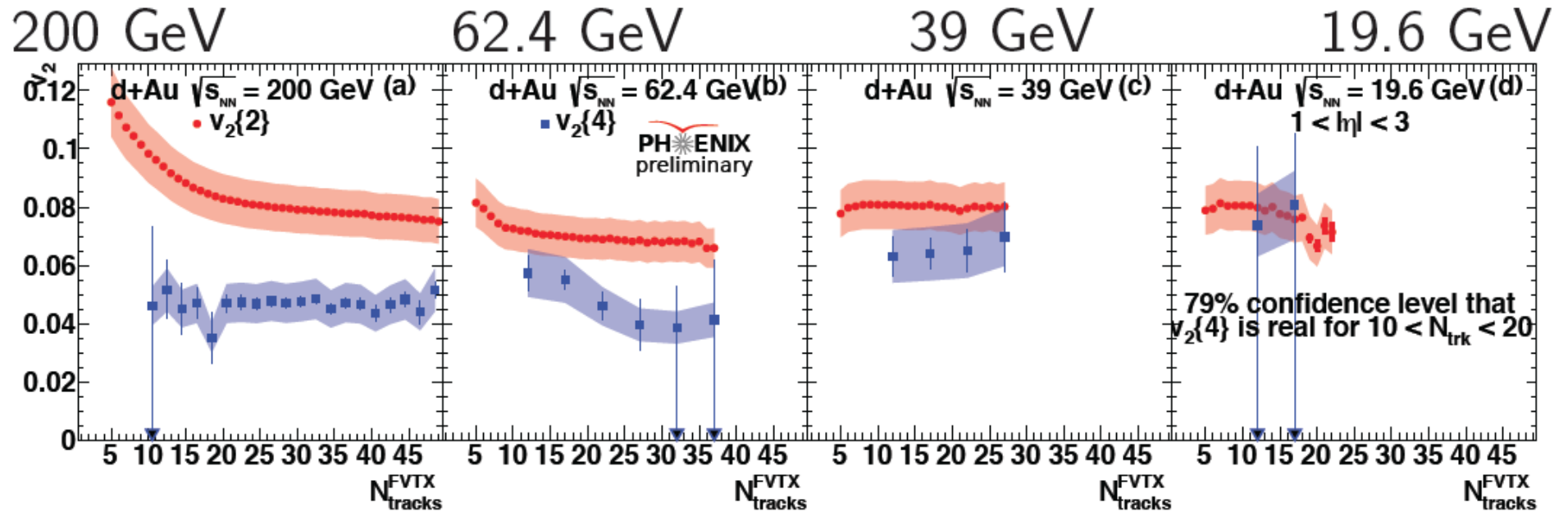
Hydro linear response: $v_n \{m\} \approx c_n \epsilon_n \{m\}$

Gardim,Grassi,Luzum,Ollitrault, PRC (2012)024908; Niemi,Denicol,Holopainen,Huovinen, PRC87 (2013)054901
Bzdak,Bozek,McLerran, arXiv:1311.7325, Bzdak, Skokov, arXiv: 1312.7349
Yan, Ollitrault, arXiv:1312.6555, Basar,Teaney, arXiv:1312.6770

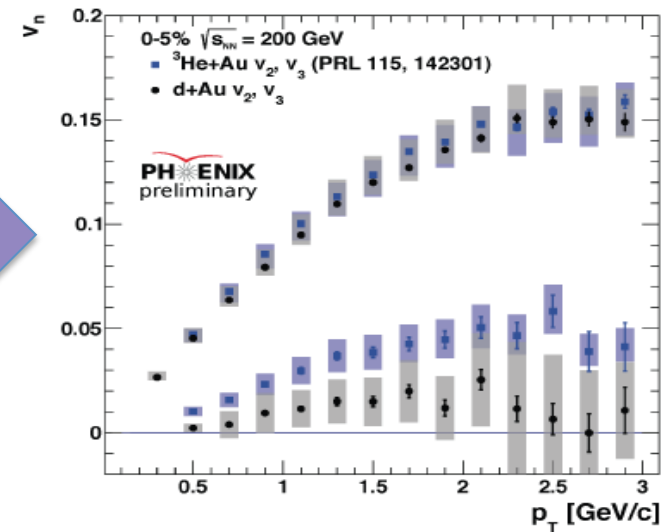
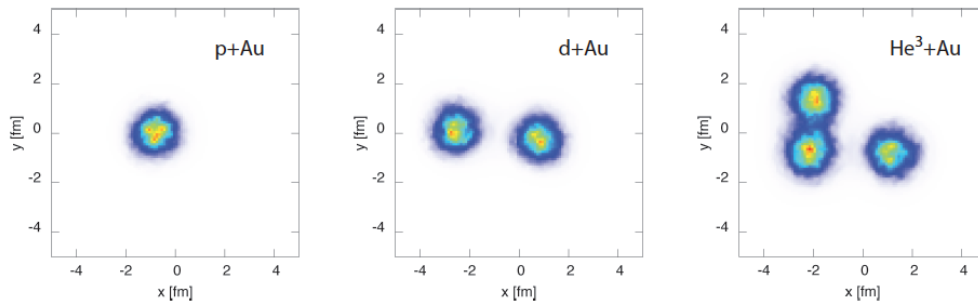
Collectivity across system size



Collectivity across wide energy scales



Schenke, RV:1407.7557



Panta Rhei?



Heraclitus of Ephesus
535-475 BC

Natural in hydro – yet, very few ab initio hydro computations of 4-particle cumulants for p+A -- none for p+p

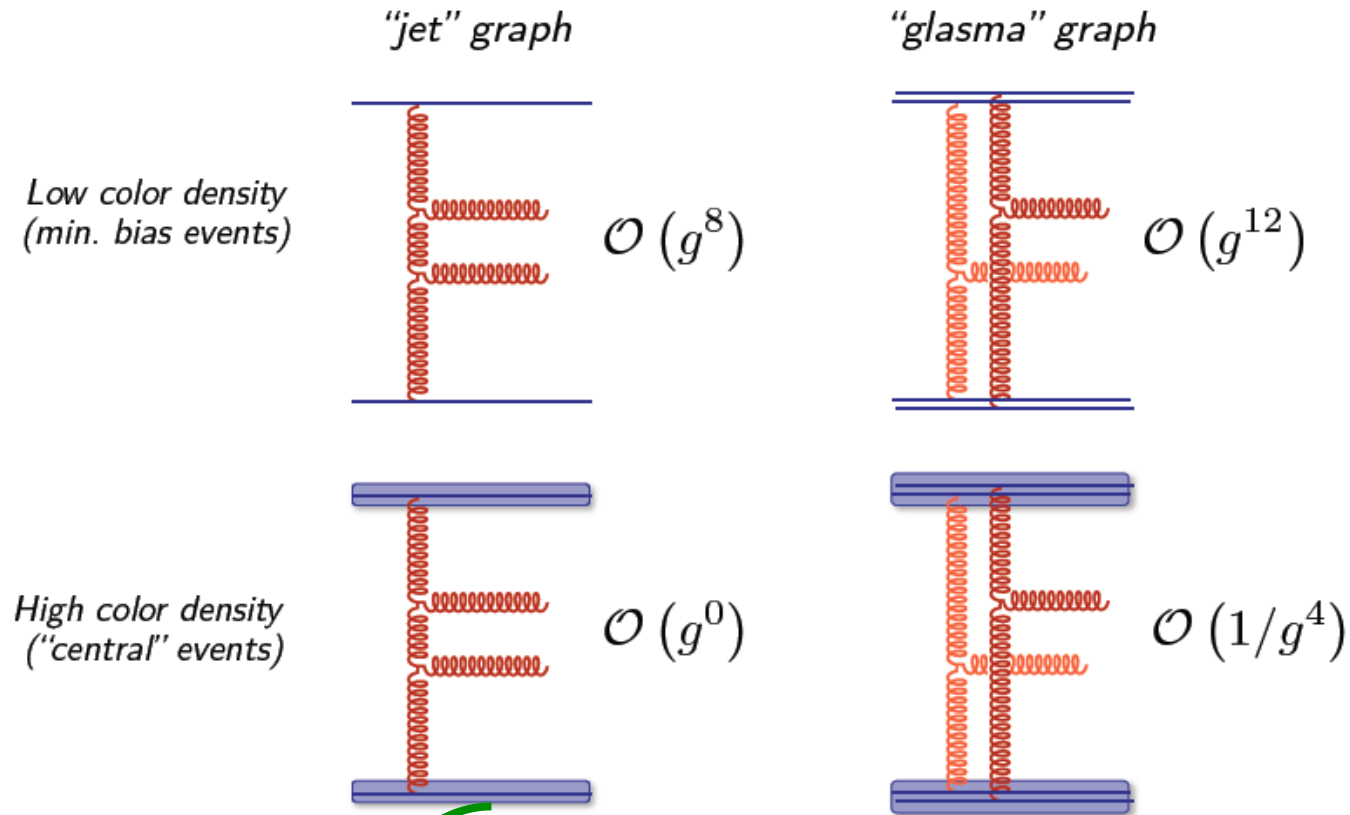
Problems with hydro interpretation for small systems:

- i) Absence of “jet” quenching**
- ii) Lack of convergence of hydro expansion (large Knudsen numbers)**
- iii) Effects seen for small multiplicity and high p_T (9 GeV)**

**Can we understand multiparticle correlations
in an *ab initio* approach ?**

Review: Dusling,Li,Schenke, arXiv:1509.07939

Glasma graph approximation: power counting

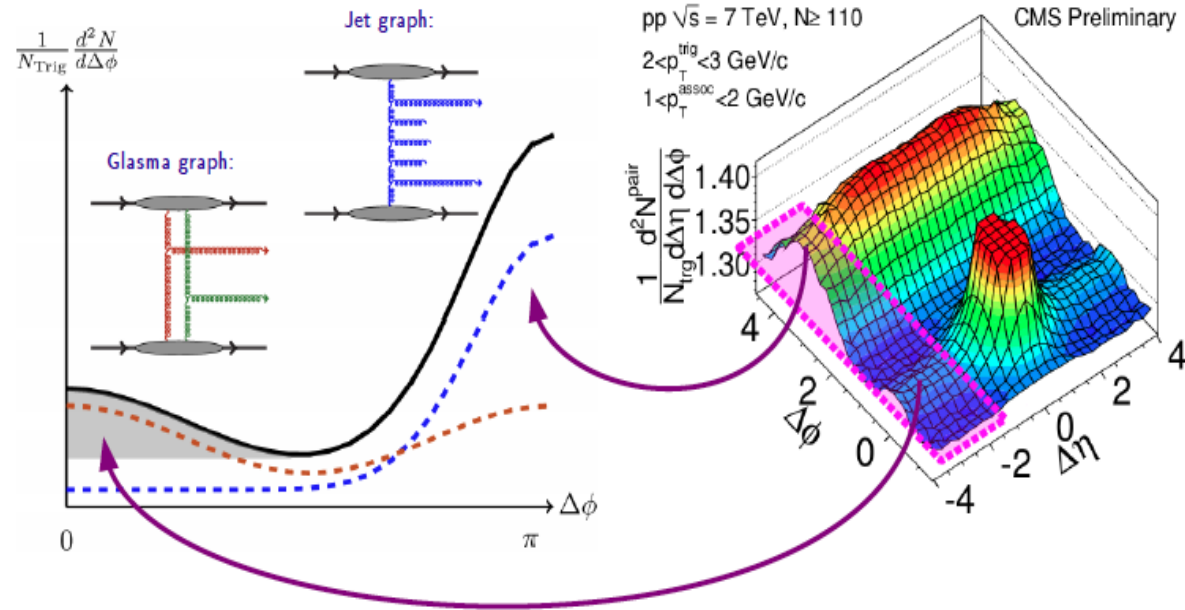


Gluons with $k_T \sim Q_s$ resolve
 $n \sim 1/g^2$ color sources
 Effective coupling: $g*n \sim 1/g$

Huge enhancement of Glasma graphs for
 high parton densities...

Dumitru, Dusling, Gelis, Jalilian-Marian,
 Lappi, Venugopalan, PLB697 (2011)21
 Dusling, Venugopalan, PRL108 (2012)262001

Anatomy of long range collimations



RG evolution of Glasma graphs:

$$C(\mathbf{p}, \mathbf{q}) \propto \frac{g^4}{\mathbf{p}_\perp^2 \mathbf{q}_\perp^2} \int d^2 \mathbf{k}_{1\perp} \Phi_{A_1}^2(y_p, \mathbf{k}_{1\perp}) \Phi_{A_2}(y_p, \mathbf{p}_\perp - \mathbf{k}_{1\perp}) \Phi_{A_2}(y_q, \mathbf{q}_\perp - \mathbf{k}_{1\perp}) + \text{permutations}$$

RG evolution of the mini-jets: $C_{\text{dijet}}(\mathbf{p}, \mathbf{q}) \propto \Phi_A \otimes \Phi_B \otimes G_{\text{BFKL}}$

Good agreement with data for $p_T > Q_s$

However no odd harmonics v_3, v_5 for **gluons**

because $C(\mathbf{p}, \mathbf{q}) = C(\mathbf{p}, -\mathbf{q})$

Dusling, RV, PRD 87, 051502 (R) (2013); PRD87 (2013) 094034
Dusling, Tribedy, RV, PRD93 (2016) 014034

Tracing azimuthal initial state correlations

Simple ab initio initial state model:

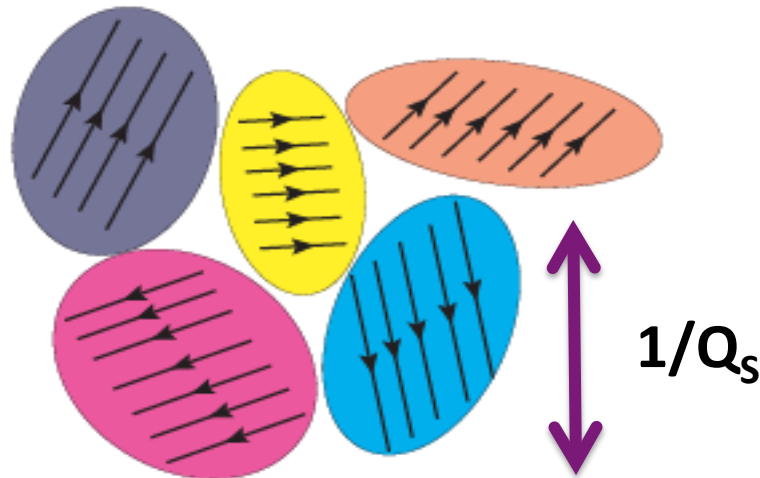
Multi-particle correlations from Eikonal scattering of partons

off color domains in a nuclear target

Lappi, arXiv:1501.05505

Lappi,Schenke,Schlichting,RV, arXiv:1509.03499

Dusling,Mace,RV, arXiv:1705.00745



Color domain model:

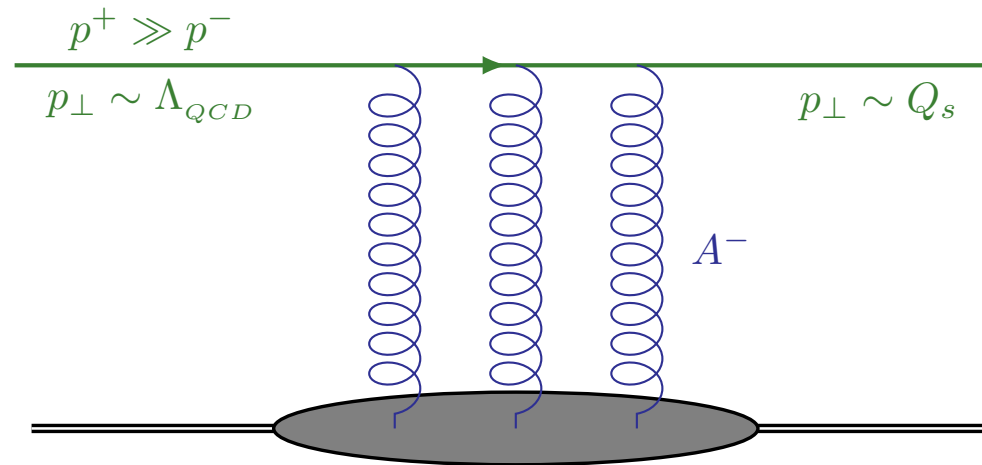
Kovner,Lublinsky,arXiv:1012.3398,1109.0347

Dumitru,Gianini, arXiv:1406.5781

Dumitru,Skokov,arXiv:1411.6630,

Dumitru,McLerran,Skokov,arXiv:1410.4844

Eikonal scattering: the parton model



Color rotation of parton in external field by a lightlike Wilson line

$$W[A](x) = \mathcal{P} \exp \left[ig \int dz^+ A_a^-(z^+, x) \right]$$

Parton distribution after coherent multiple scattering off nucleus:

$$\frac{dN_q}{d^2p} = \frac{1}{\pi^2} \int d^2b \int \frac{d^2k}{(2\pi)^2} \int d^2r \underbrace{e^{-b^2/B} e^{-k^2 B}}_{\text{Wigner function}} \left\langle \underbrace{D(b + r/2, b - r/2)}_{\text{Dipole correlator}} \right\rangle$$

$$D(x, \bar{x}) = \frac{1}{N_c} \text{Tr} [W(x) W^\dagger(\bar{x})]$$

B is transverse area of proton

Bjorken, Kogut, Soper, Phys. Rev., D3:1382, (1971)

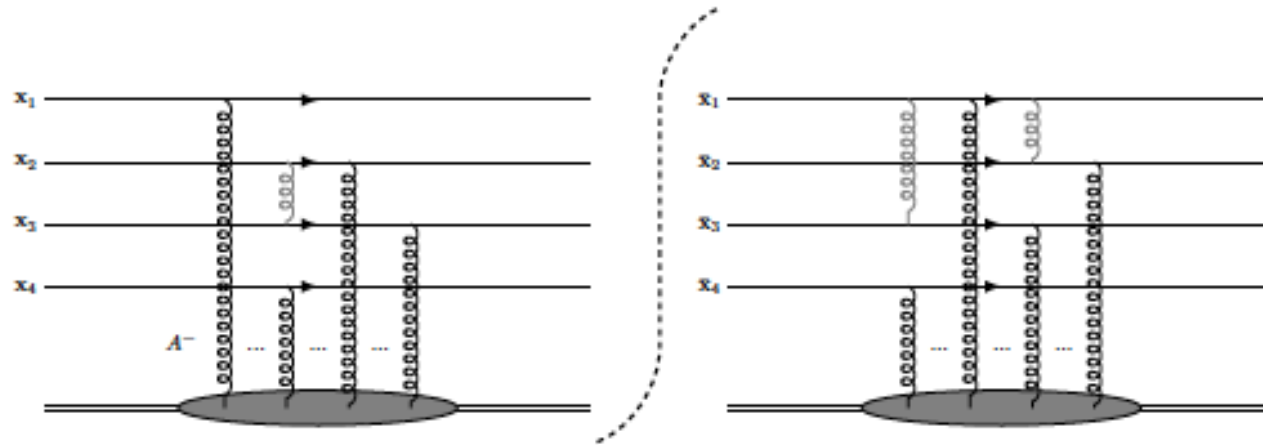
Dumitru, Jalilian-Marian, Phys. Rev. Lett., 89:022301, (2002)

Multiparton Eikonal scattering

Two partons:

$$\frac{d^2 N}{d^2 p_1 d^2 p_2} \simeq \frac{1}{(\pi B_p)^2} \int_{x\bar{x}y\bar{y}} e^{-(x^2+\bar{x}^2)/2B_p} e^{-(y^2+\bar{y}^2)/2B_p} e^{ip_1 \cdot (x-\bar{x})} e^{ip_2 \cdot (y-\bar{y})} \\ \times \left\langle \frac{1}{N_c} \text{Tr} [W(x)W^\dagger(\bar{x})] \frac{1}{N_c} \text{Tr} [W(y)W^\dagger(\bar{y})] \right\rangle \propto \langle D D \rangle$$

Four parton



$$d^4 N \sim \int \langle \text{Tr} [W(w)W^\dagger(\bar{w})] \text{Tr} [W(x)W^\dagger(\bar{x})] \text{Tr} [W(y)W^\dagger(\bar{y})] \text{Tr} [W(z)W^\dagger(\bar{z})] \rangle \\ \propto \langle D D D D \rangle \text{ and so on } \dots$$

Oversimplification: n-particle Wigner distributions factorize

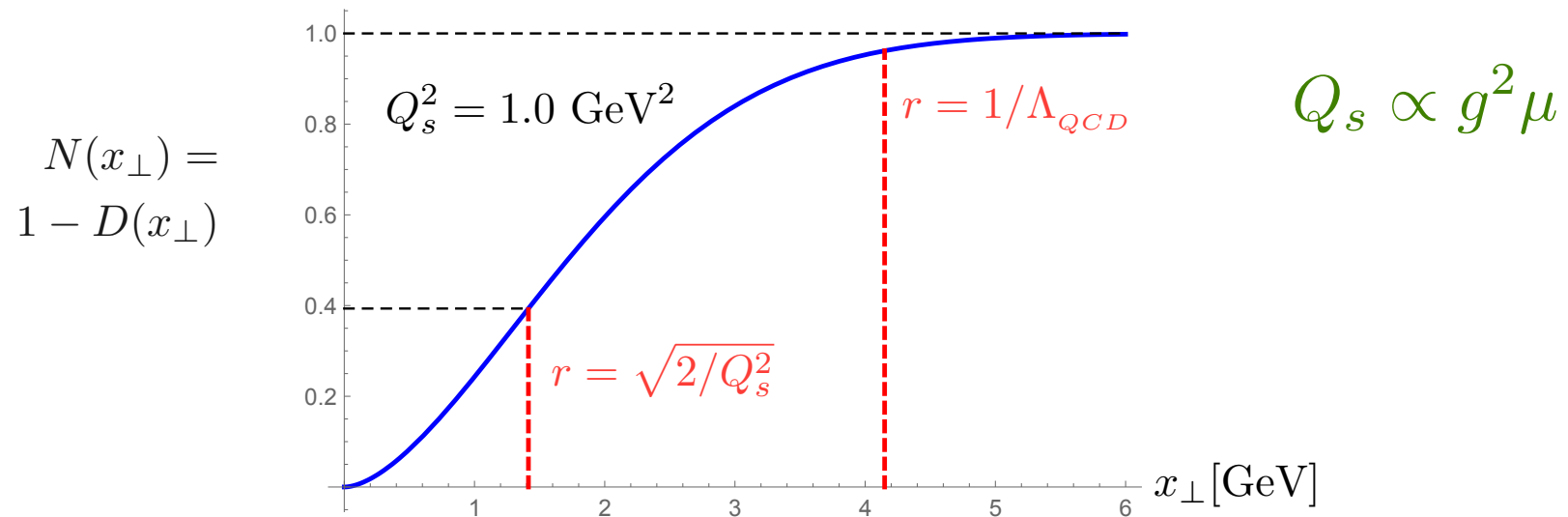
$$W_{q^n}(b_1, k_1, \dots, b_n, k_n) = W_q(b_1, k_1) \cdot \dots \cdot W_q(b_n, k_n)$$

Averaging over color representations in the target

Dipole correlator is evaluated in the MV model where color correlations in the target are Gaussian (random walk in color)

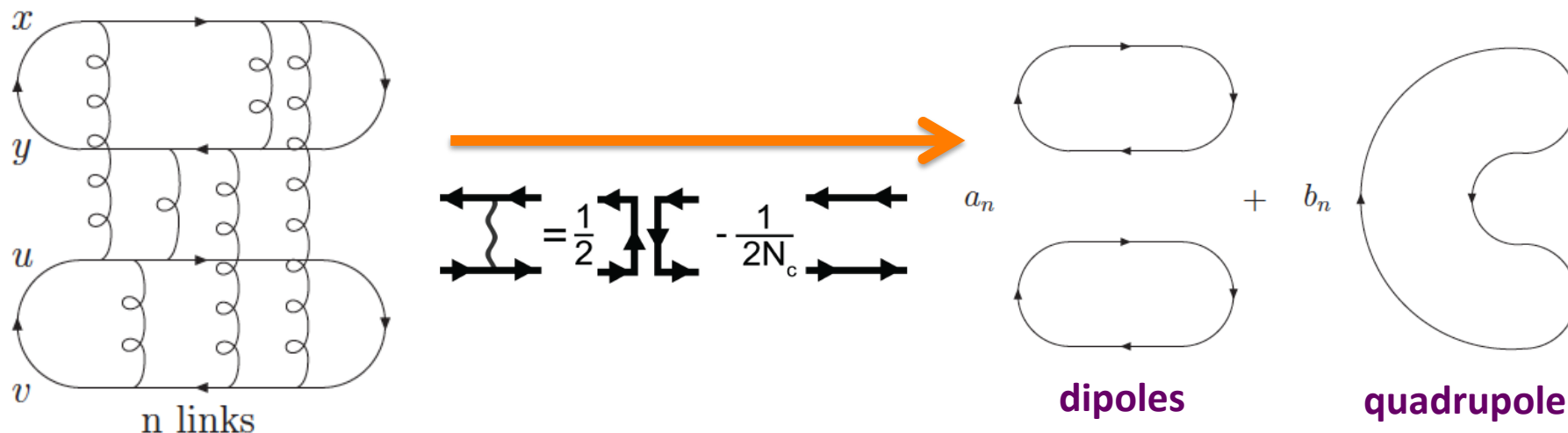
$$g^2 \langle A_a^-(x) A_b^-(y) \rangle = \delta^{ab} L_{xy} \quad L_{xy} = -\frac{g^4 \mu^2}{16\pi} |x - y|^2 \ln \frac{1}{\Lambda |x - y|}$$

gives $D(x, \bar{x}) = \frac{1}{N_c} \text{Tr} [W(x) W^\dagger(\bar{x})] = \exp(C_F L_{x\bar{x}})$



Averaging over multi-point dipole correlators

To compute the 2-dipole correlator:



Use $W(x) \equiv \mathcal{P} \exp \left[ig \int dz^+ A_a^-(z^+, x) \right] \simeq V(x) [1 + ig A_a^-(\xi, x) T^a + \dots]$

This gives $\langle D_{x\bar{x}} D_{y\bar{y}} \rangle_W \simeq \alpha_{x\bar{x}y\bar{y}} \langle D_{x\bar{x}} D_{y\bar{y}} \rangle_V + \beta_{xy\bar{x}\bar{y}} \langle Q_{x\bar{y}y\bar{x}} \rangle_V$

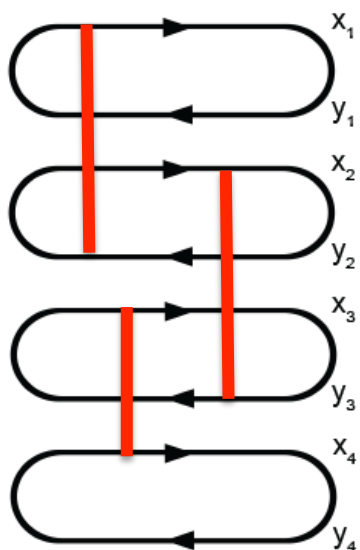
Equivalently,
$$\begin{pmatrix} \langle D_{x\bar{x}} D_{y\bar{y}} \rangle \\ \langle Q_{x\bar{y}y\bar{x}} \rangle \end{pmatrix}_W = \begin{pmatrix} \alpha_{x\bar{x}y\bar{y}} & \beta_{xy\bar{x}\bar{y}} \\ \beta_{xy\bar{y}\bar{x}} & \alpha_{x\bar{y}y\bar{x}} \end{pmatrix} \begin{pmatrix} \langle D_{x\bar{x}} D_{y\bar{y}} \rangle \\ \langle Q_{x\bar{y}y\bar{x}} \rangle \end{pmatrix}_V$$

Kovner, Wiedemann, Phys. Rev., D64:114002, (2001)
 Fujii, Nucl. Phys., A709:236 (2002).
 Blaizot, Gelis, Venugopalan. Nucl. Phys., A743:57, (2004)
 Dominguez, Marquet, Wu, Nucl. Phys., A823:99, (2009)

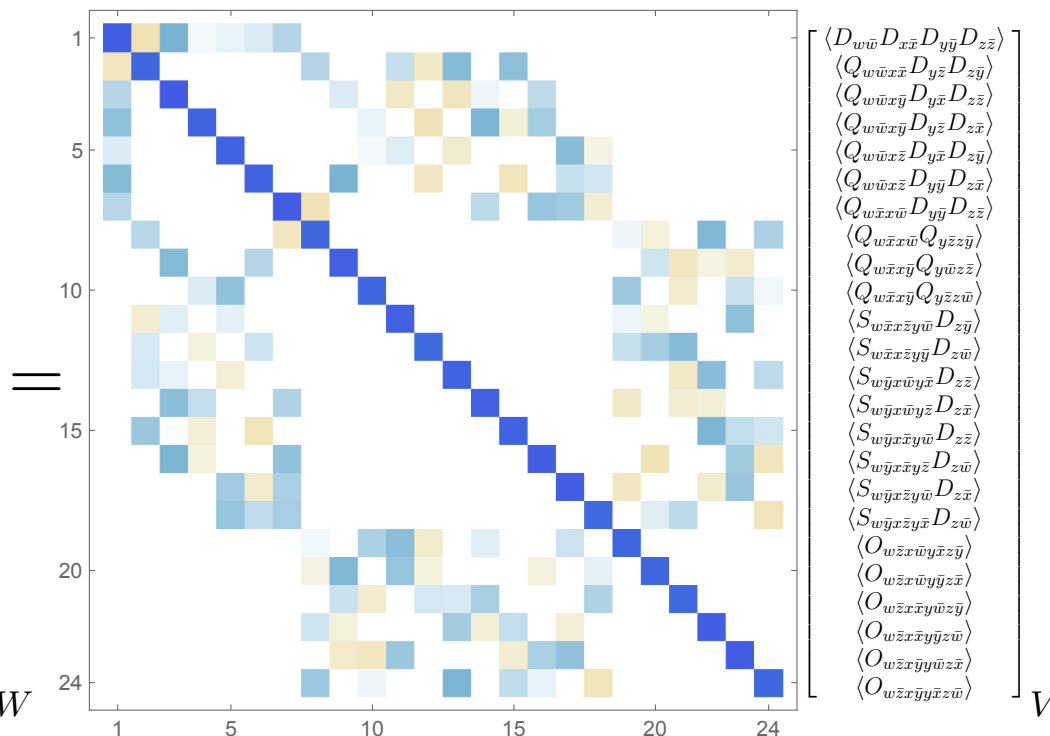
**Iterate, diagonalize, exponentiate,
to compute correlator**

Averaging over multi-point dipole correlators

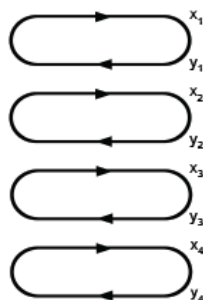
4-dipole correlator:



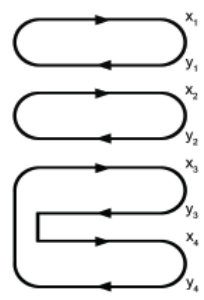
$$\begin{bmatrix} \langle D_{ww} D_{xx} D_{yy} D_{zz} \rangle \\ \langle Q_{w\bar{w}x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{x}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{z}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{y}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{x}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} Q_{y\bar{z}z\bar{y}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{w}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{y}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{y}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{y}y\bar{w}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{y}y\bar{z}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{z}} D_{z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{z}z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{z}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{z}z\bar{w}} \rangle \end{bmatrix}$$



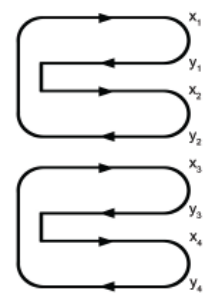
To compute n-gluon exchange, diagonalize 24×24 matrix and exponentiate



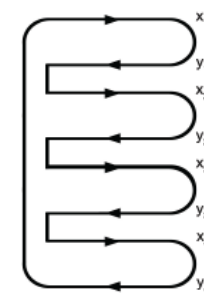
dipoles



quadrupoles



sextupole



octupole

Results for azimuthal anisotropies from multiparton eikonal scattering

Dusling, Mace, Venugopalan, arXiv:1705.00745
Dusling, Mace, Venugopalan, arXiv:1706.06260



My hard working collaborators...

Objects to be computed

N-particle distributions:

$$\frac{d^n N}{d^2 p \dots} \sim \int \overbrace{e^{-(x^2 + \bar{x}^2)/2B_p} \dots}^{B_p = 4 \text{ GeV}^{-2}} \left\langle \overbrace{\frac{1}{N_c} \text{Tr} [W(x) W^\dagger(\bar{x})] \dots}^{Q_s^2 \sim 1 - 3 \text{ GeV}^2} \right\rangle e^{ip \cdot (x - \bar{x}) \dots}$$

Two-particle cumulants:

$$c_n\{2\} = \frac{\kappa_n\{2\}}{\kappa_0\{2\}}, \quad v_n\{2\} = \sqrt{c_n\{2\}}$$

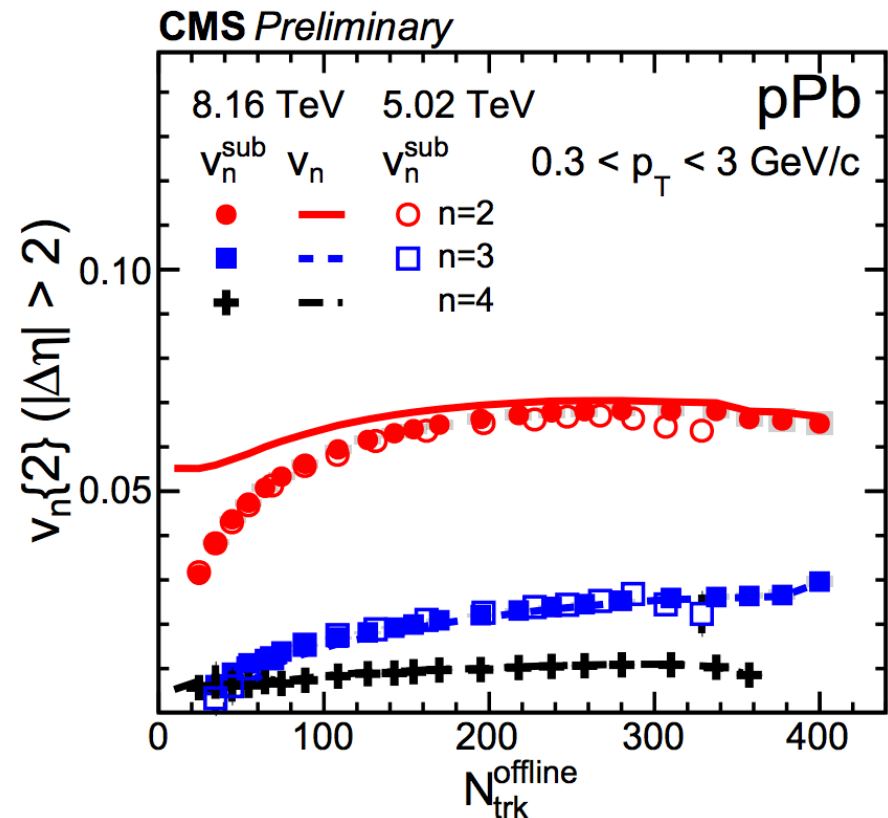
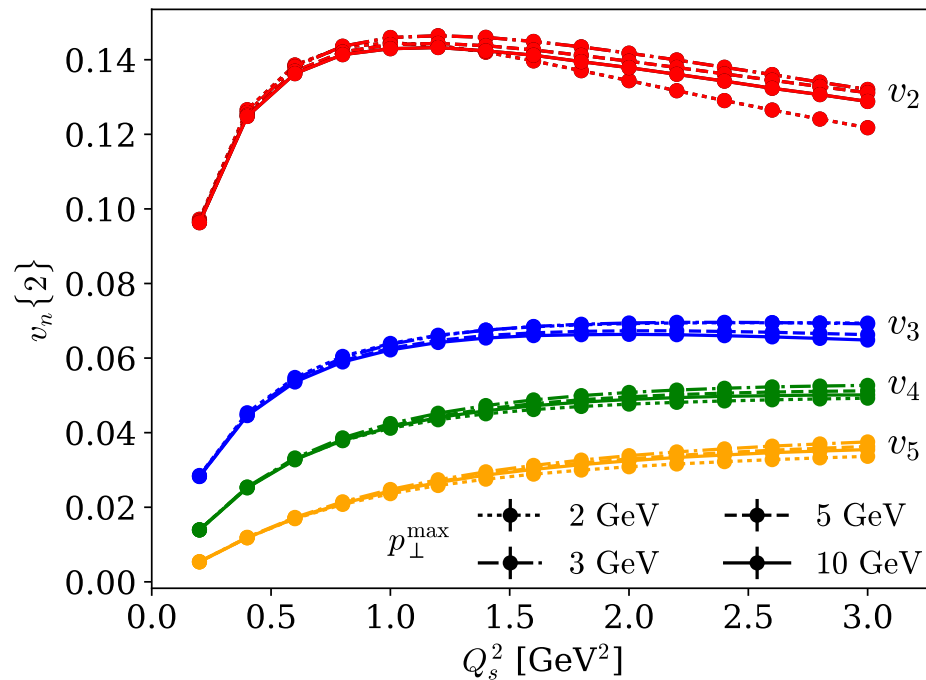
$$\kappa_n\{2\} = \int d^2 p_1 d^2 p_2 \cos[n(\phi_{p1} - \phi_{p2})] \frac{d^2 N}{d^2 p_1 d^2 p_2}$$

Four-particle cumulants:

$$c_n\{4\} = \frac{\kappa_n\{4\}}{\kappa_0\{4\}} - 2 \left(\frac{\kappa_n\{2\}}{\kappa_0\{0\}} \right)^2, \quad v_n\{4\} = (-c_n\{4\})^{1/4}$$

$$\kappa_n\{4\} = \int d^2 p_1 d^2 p_2 d^2 p_3 d^2 p_4 \cos[n(\phi_{p1} + \phi_{p2} - \phi_{p3} - \phi_{p4})] \frac{d^4 N}{d^2 p_1 d^2 p_2 d^2 p_3 d^2 p_4}$$

Integrated anisotropy coefficients



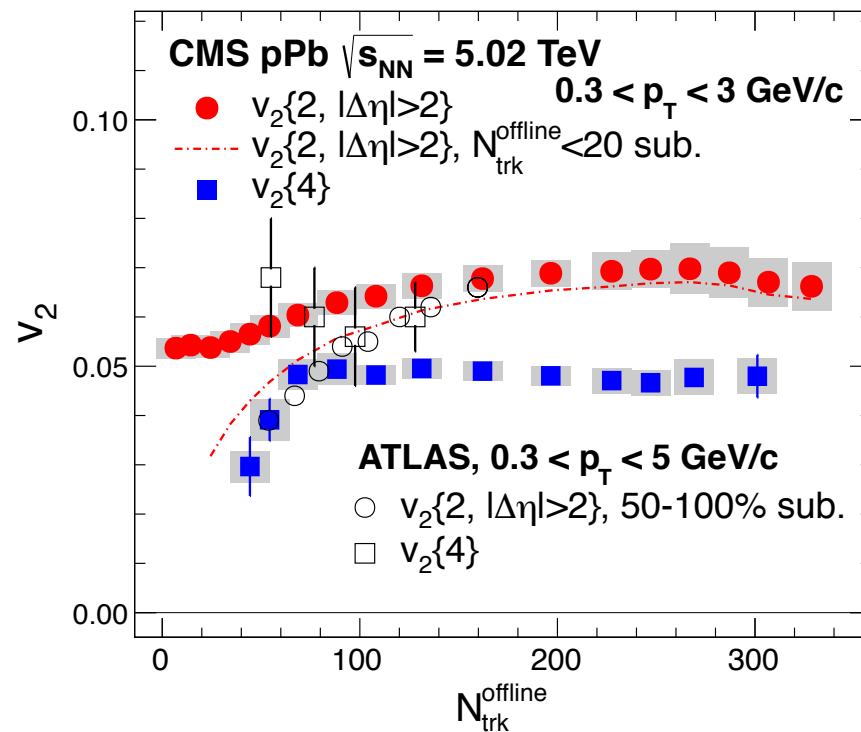
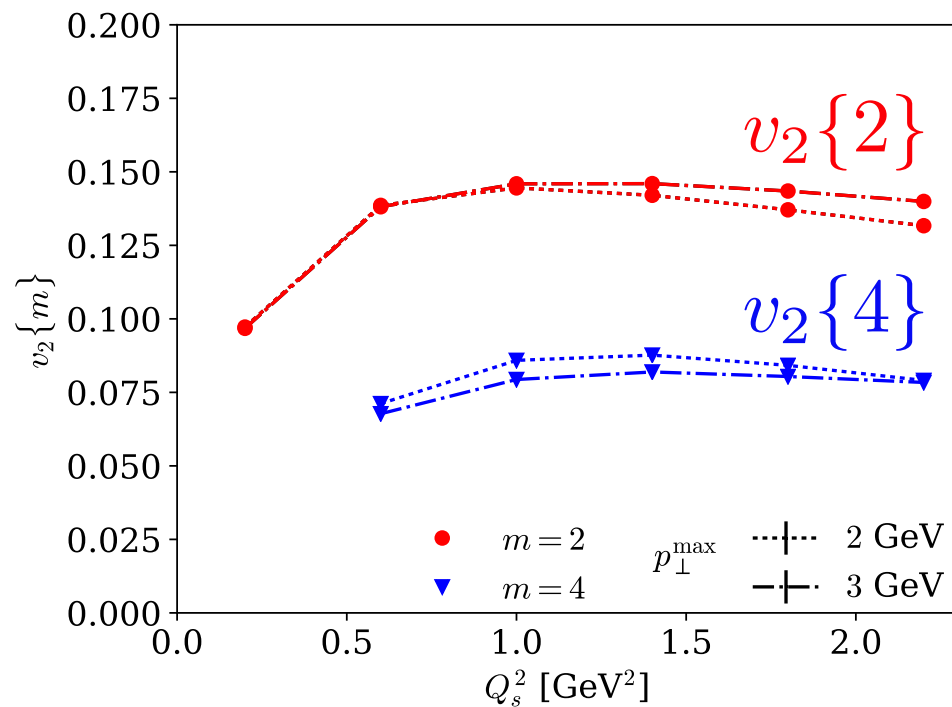
Similar ordering of “Flow” coefficients as seen in the data

Important caveat: No simple map between theory and experiment
Theory results are for quarks, Q_s^2 is the saturation scale in the target

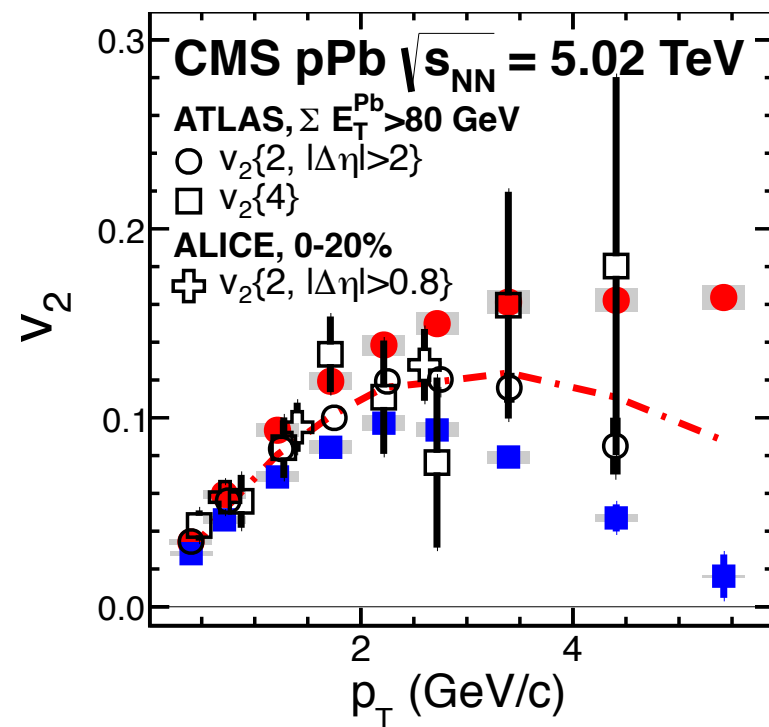
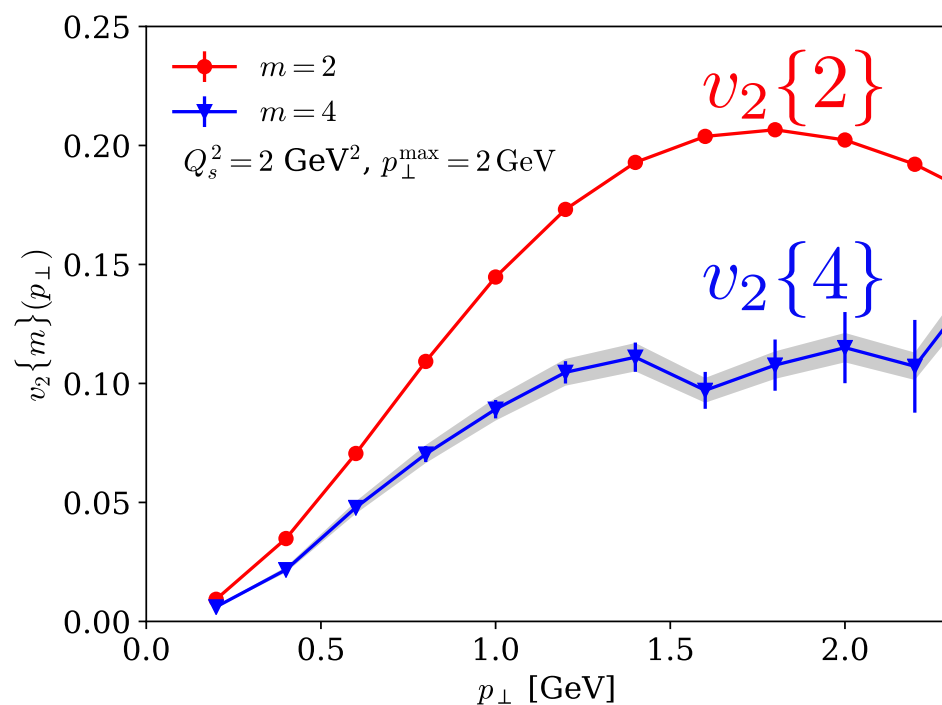
Lappi, Phys. Lett., B744:315, (2015)

Lappi, Schenke, Schlichting, Venugopalan. JHEP, 01:061, (2016)

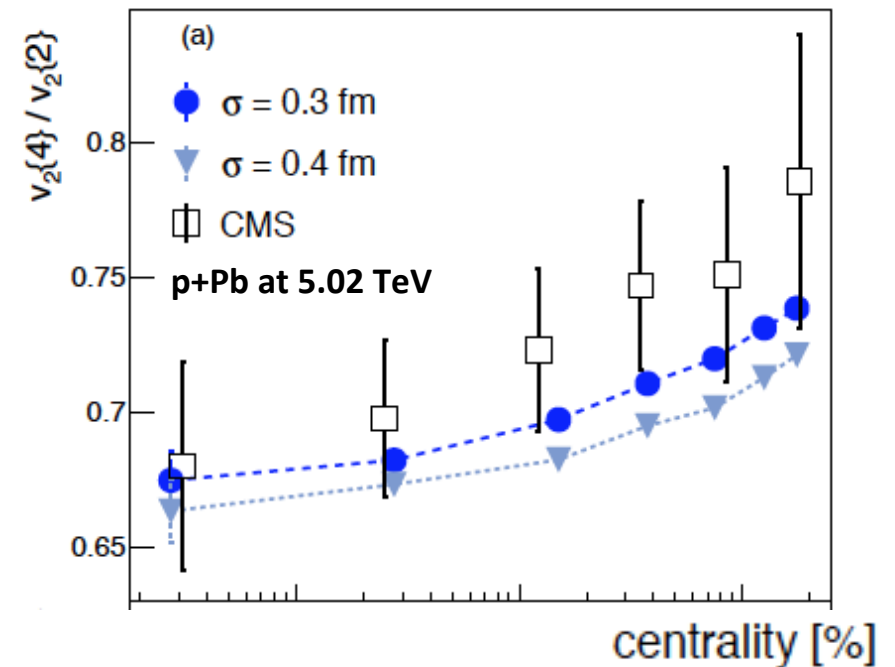
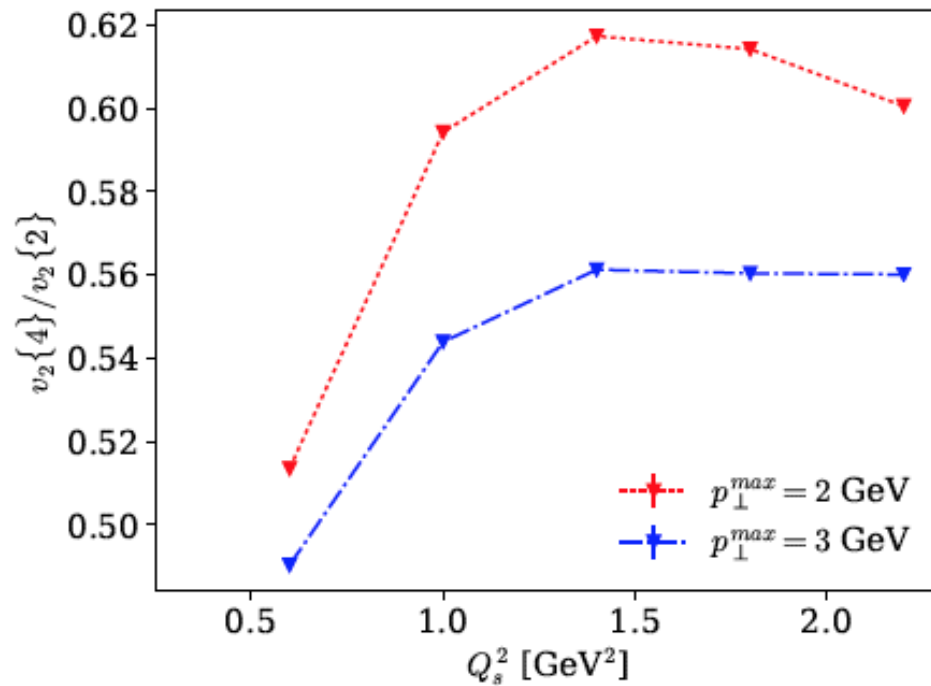
Integrated anisotropy coefficients



p_T dependence of anisotropy coefficients



Ratio v_4/v_2 of Fourier harmonics



From Giacalone, Noronha-Hostler, Ollitrault, arXiv:1702.01730

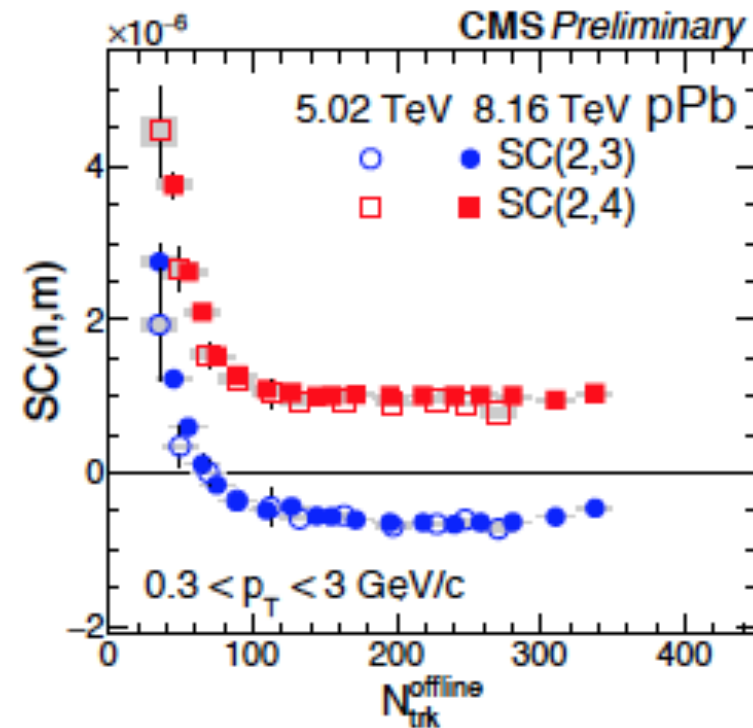
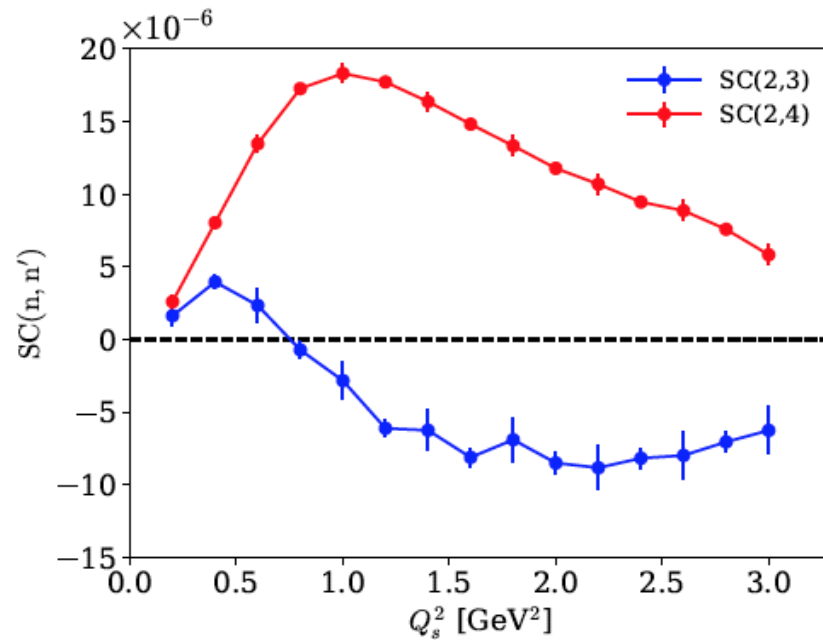
Overall magnitude in ball-park of CMS data

Symmetric cumulants

Mixed cumulants of Fourier harmonics

Bilandzic et al., PRC 89 (2014) 064904

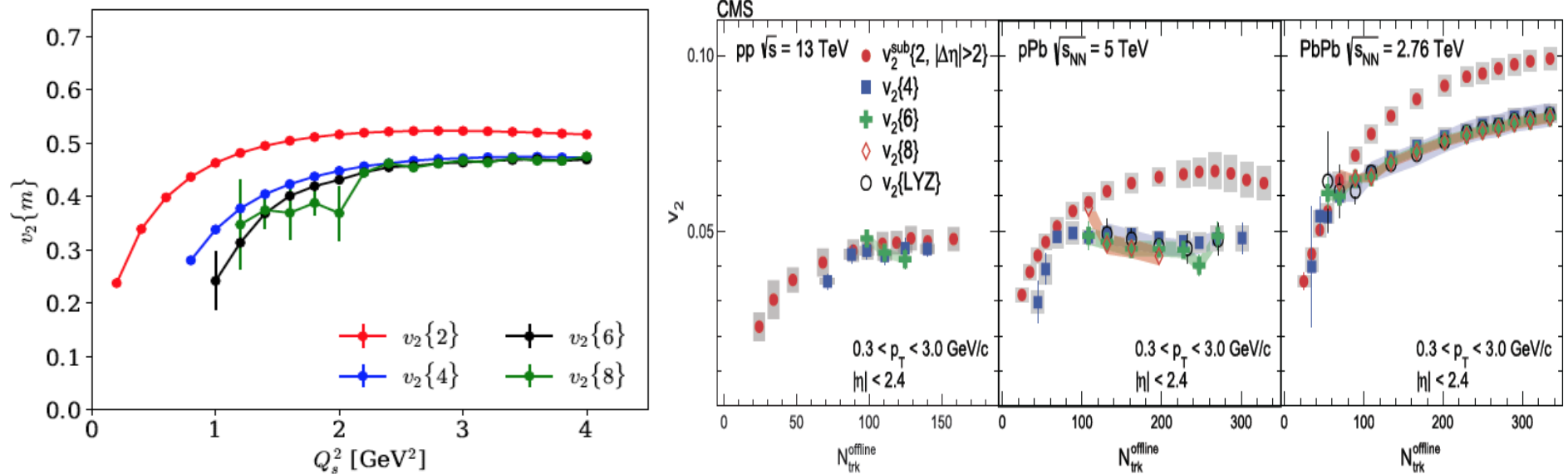
$$SC(n, n') = \langle e^{i(n(\phi_1 - \phi_3) - n'(\phi_2 - \phi_4))} \rangle - \langle e^{in(\phi_1 - \phi_3)} \rangle \langle e^{in'(\phi_2 - \phi_4)} \rangle$$



In hydrodynamics, linear response of flow to correlations/anti-correlations of spatial eccentricities

For e.g., Qian, Heinz, arXiv:1607.01732

Higher cumulants from scattering off coherent **Abelian** fields



Replace $N_c \times N_c$ trace with simple path ordered exponentials ($N_c=1$)

2m-particle collectivity reproduced in this simple parton model...

Exegesis

Power counting

Three dimensionful scales:

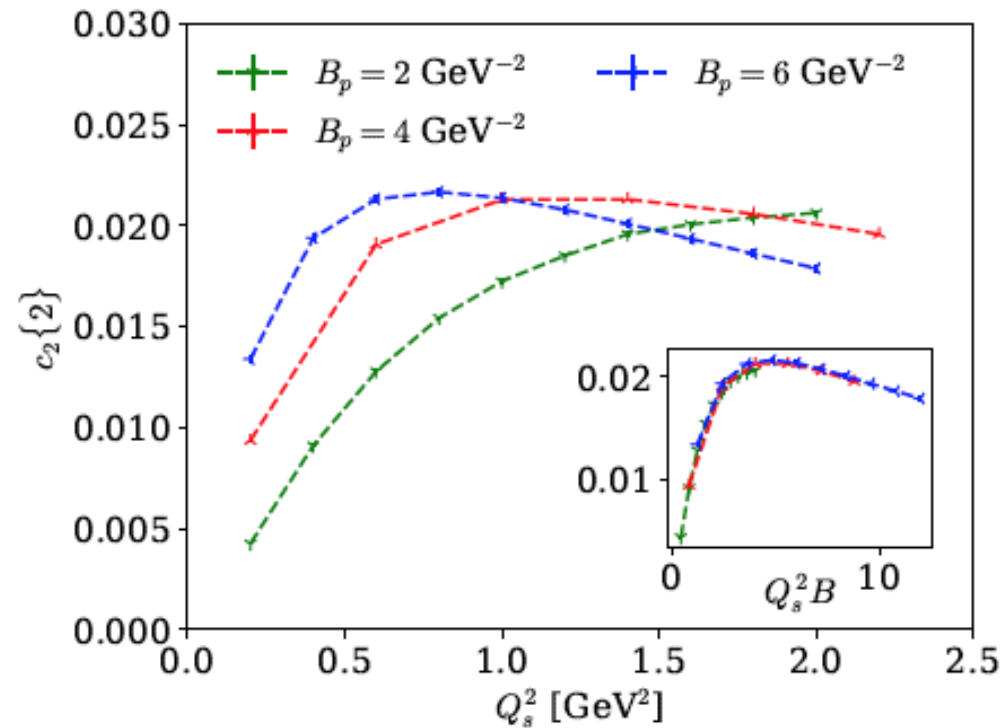
- i) Saturation scale in the target (inverse domain size): Q_s
- ii) Transverse overlap area: B_p
- iii) Maximal momentum kick to projectile: $p_T^{\max}$

Dimensionless ratios are

of color domains, $Q_s^2 B_p$

Resolving power of projectile, $Q_s^2 / (p_T^{\max})^2$

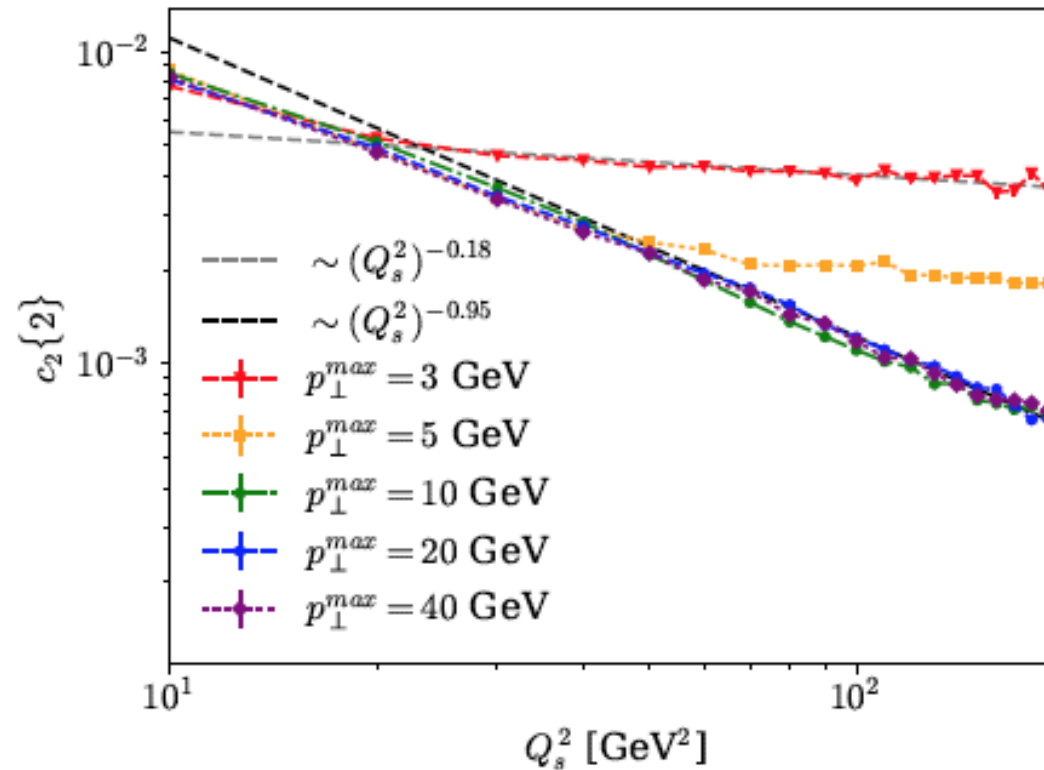
Scaling with # of color domains



“Independent cluster” model: fluctuation driven v_n ’s scale with # of clusters

Basar, Teaney, arXiv:1312.6770

Scaling with # of color domains



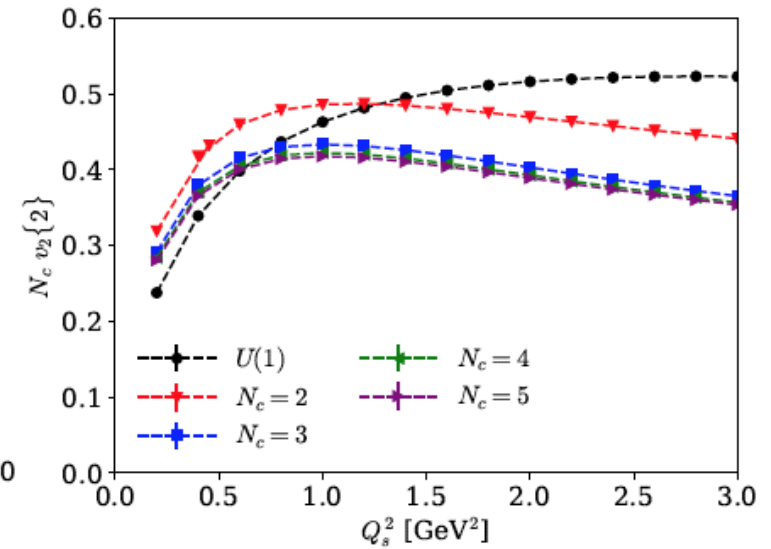
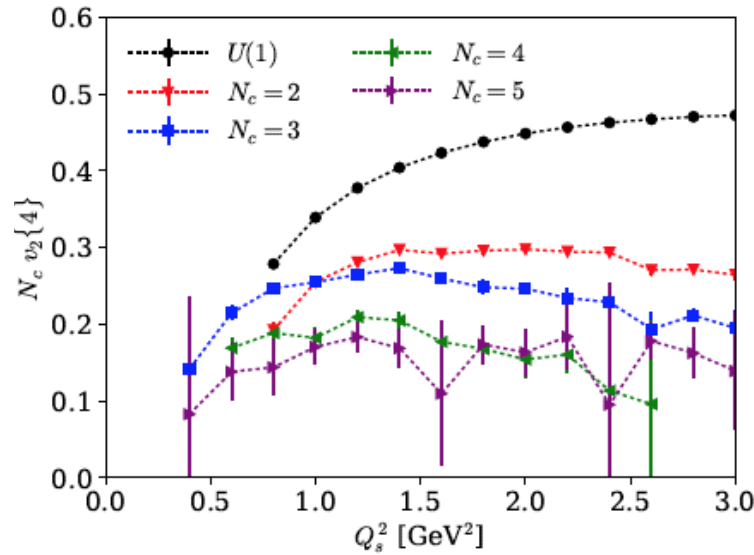
For $(p_T^{\max})^2 \gg Q_s^2$, projectile partons resolve individual domains, cumulant scales as $\sim 1/(Q_s^2 B_p)$

For $(p_T^{\max})^2 \ll Q_s^2$, partons see fewer domains, weak dependence of cumulant on # of domains.

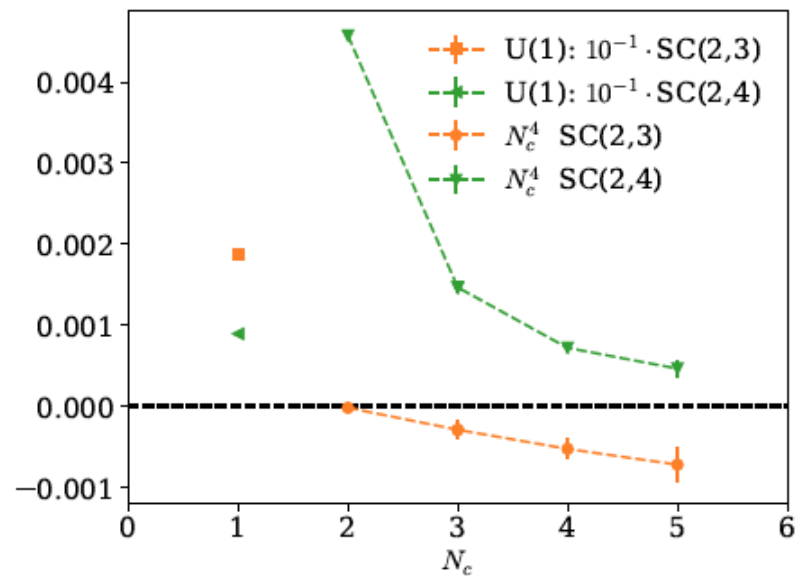
Moral: scaling analyses/conclusions sensitive to p_T window probed in experiments

Scaling with N_c

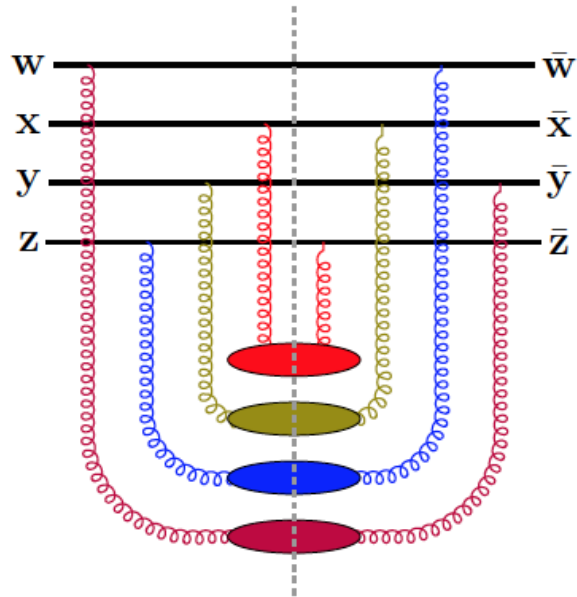
$v_2\{2\}, v_2\{4\}$
scale as $1/N_c$



Symmetric cumulants
show correlations/anti-correlations
seen in experiment only for $N_c \geq 3$

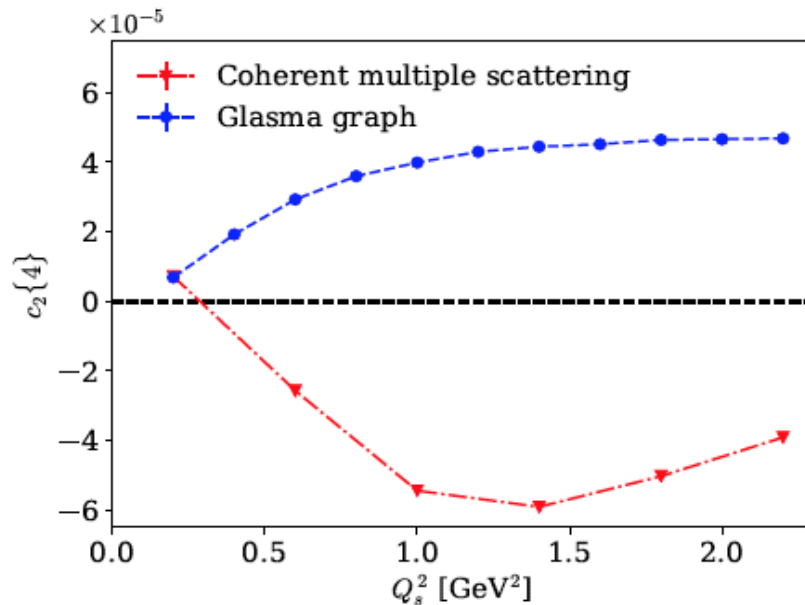


Back to glasma graphs



Glasma graph (single scattering) correlations are very strong – the n -particle distribution is close to a *Bose distribution* – as in a laser

Gelis,Lappi,McLerran, arXiv:0905.3234



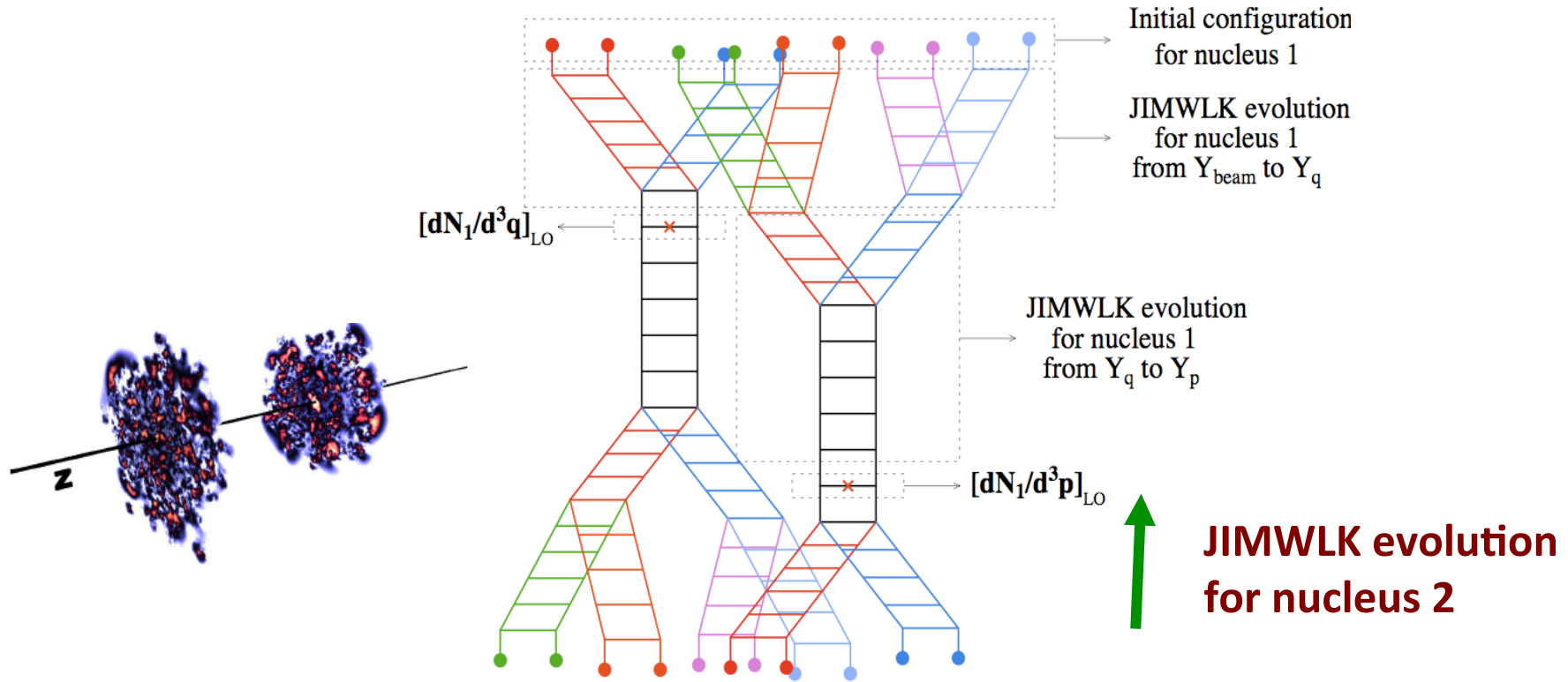
But $v_2\{4\}$ is imaginary...

For real $v_2\{4\}$, must have dominance of first two moments of distribution
-- achieved by coherent multiple scattering...

Dusling,Mace,Venugopalan, 1706.06260

Towards a more realistic initial state description

Two-parton azimuthal correlations in the CGC



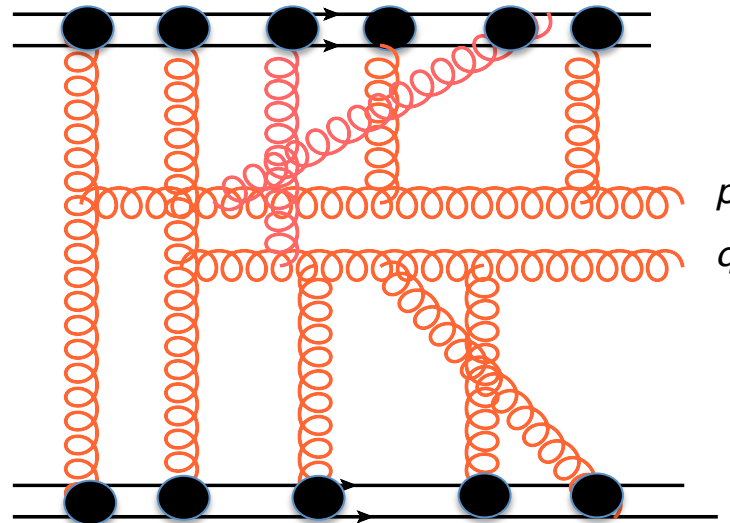
$$\langle \frac{dN_2}{d^3p d^3q} \rangle_{\text{LLogs}} = \int [d\rho_1][d\rho_2] W_{Y_1}[\rho_1] W_{Y_2}[\rho_2] \frac{dN}{d^3p}|_{\text{LO}} \frac{dN}{d^3q}|_{\text{LO}}$$

JIMWLK evolution

Solution of Yang-Mills for dense-dense systems (as in A+A)

Dumitru, Gelis, McLerran, Venugopalan: 0804.3858
Gelis, Lappi, Venugopalan, arXiv: 0807.1306
Dusling, Gelis, Lappi, Venugopalan, arXiv: 0911.2720

Beyond glasma graphs



Coherent multiple scattering is of the same order in the coupling:
power suppressed for $p_T \gg Q_s$, important for $p_T < Q_s$

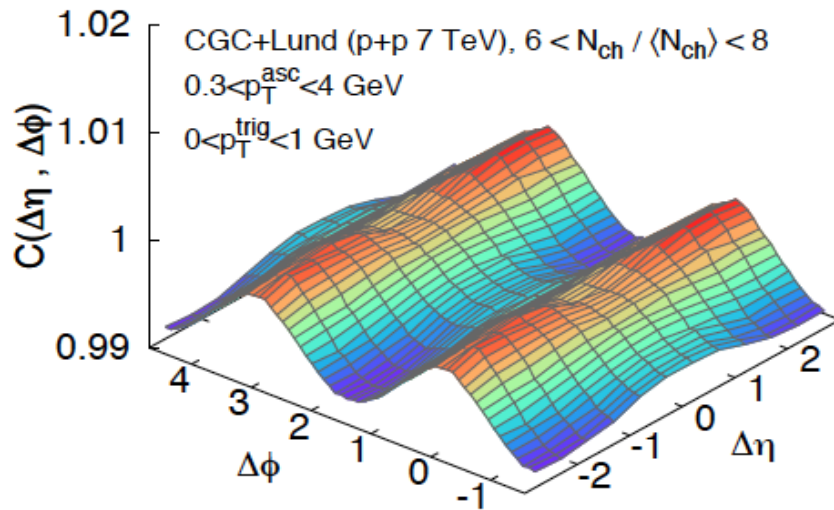
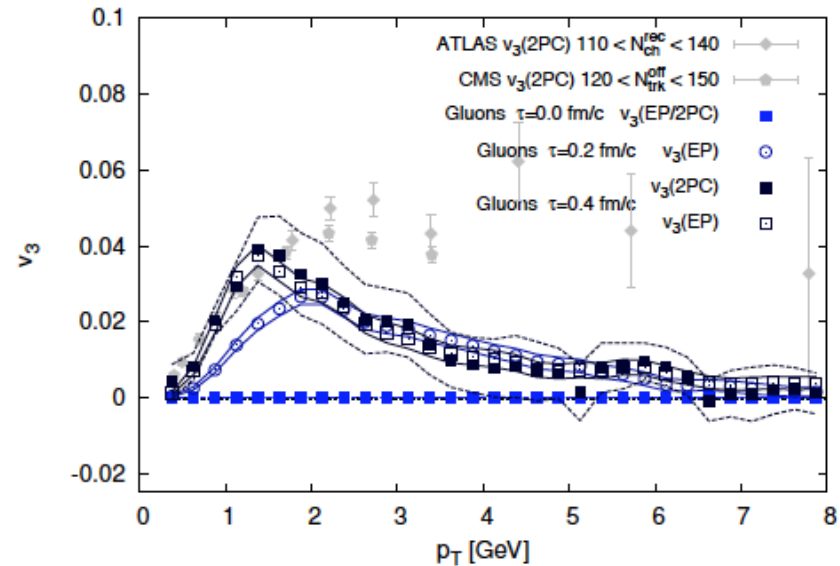
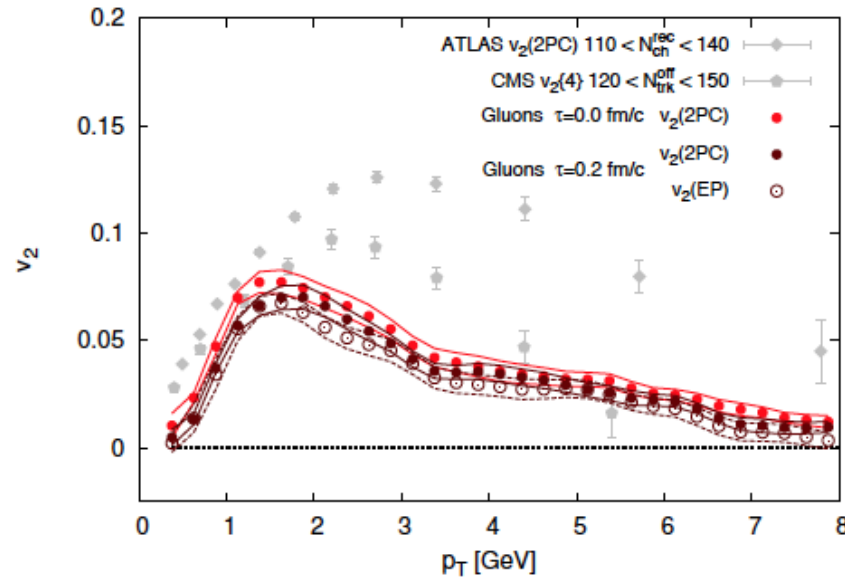
Compute (numerically) by solving Yang-Mills equations in presence of two
light cone sources: IP-Glasma model [Schenke,Triedy,Venugopalan, arXiv:1202.6646, 1206.6805](#)

$$\left\langle \frac{d^2 N}{d^2 p_T d^2 q_T} \right\rangle = \int D\rho_A D\rho_B e^{-\int d^2 x_T \rho_A^2 / Q_{s,A}^2} e^{-\int d^2 x_T \rho_B^2 / Q_{s,B}^2} \frac{dN}{d^2 p_T}[\rho_A, \rho_B] \frac{dN}{d^2 q_T}[\rho_A, \rho_B]$$

$C(p,q) \neq C(p,-q)$ -- all harmonics contribute [Lappi,Srednyak,Venugopalan, arXiv:0911.2068](#)

Azimuthal anisotropy from Yang-Mills dynamics

Schenke, Schlichting, RV, PLB747(2015)76



Recent analytical work in dilute-dense approx:

Kovchegov, Wertepny, NPA906 (2013)50

McLerran, Skokov arXiv:1611.09870

Kovner, Lublinsky, Skokov, arXiv:1612.07790

Combining gluon distributions from CGC with PYTHIA reproduces ridge distributions

However, 4-particle collectivity is computationally challenging at present...

Schenke, Schlichting, Tribedy, Venugopalan, PRL117 (2016)162301

Summary-I

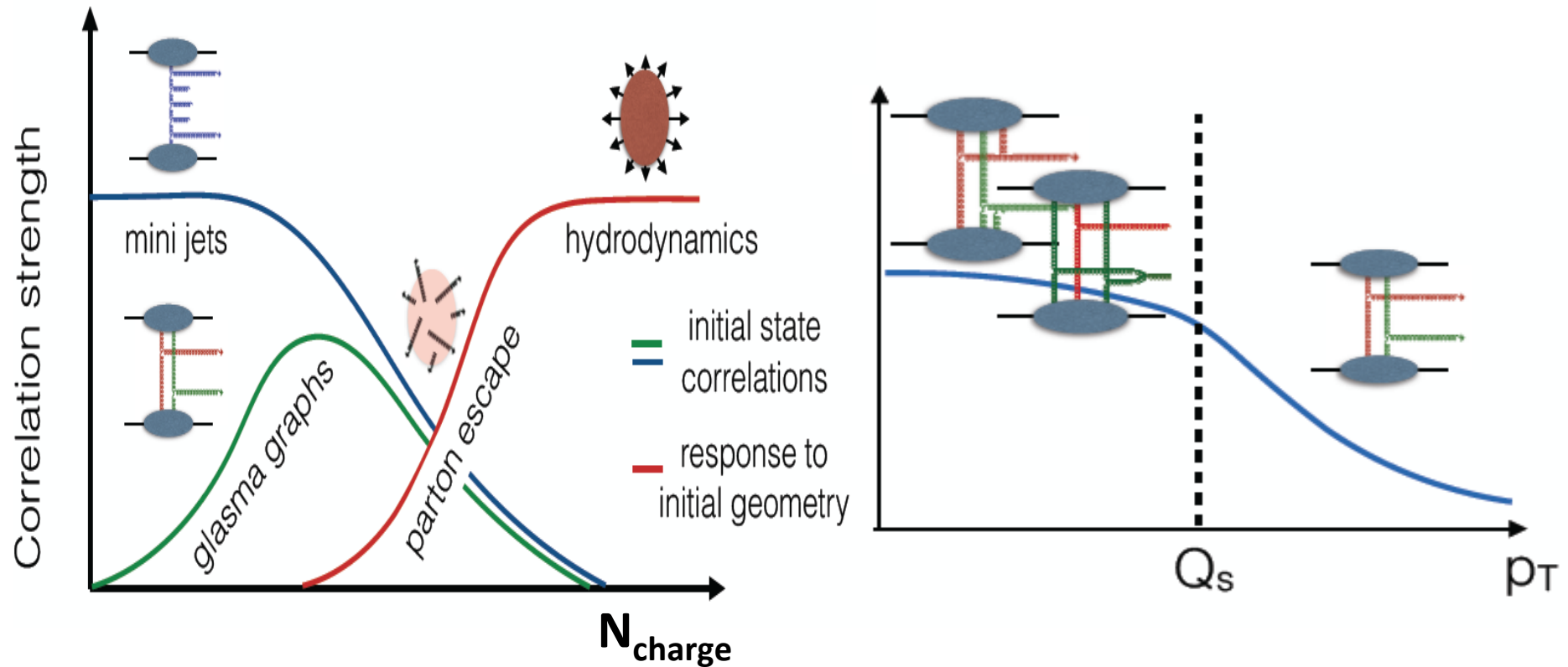
Hydrodynamic paradigm appears to describe multi-particle correlations even in the smallest systems

There are however puzzling features of the data, questions about the the validity of hydro, fine tuning of initial conditions (requiring implicitly strong initial state correlations), absence of jet quenching, ... and explanation of anisotropies for $p_T > \text{few GeV}$

Initial state QCD frameworks now also able to explain many features of the data but systematic treatments are still in their infancy

Despite much progress, no completely satisfactory explanation of the data -- the problem is still wide open

Summary-II

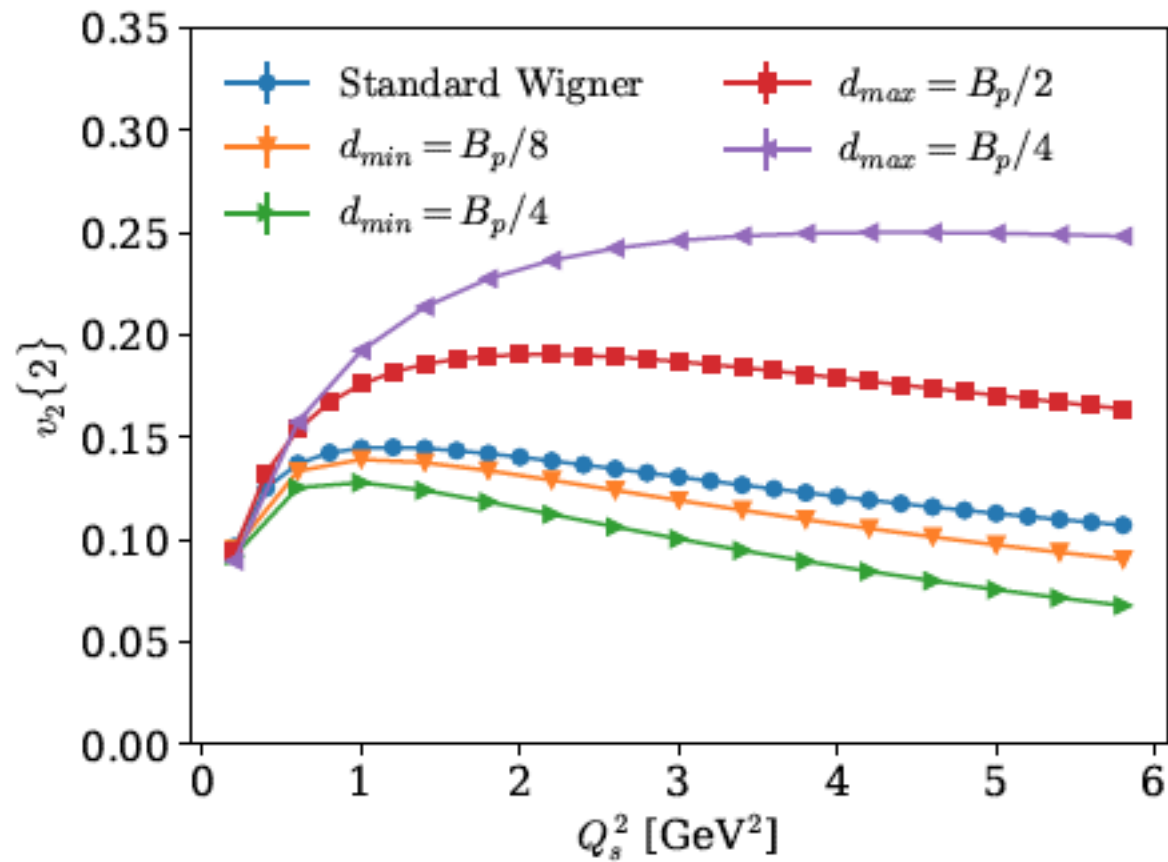


**Event engineering across system sizes, energies, and varieties of probes:
Offers exciting possibility of exploring dynamical evolution of strongly correlated
quark-gluon matter from high occupancy, out of equilibrium, dynamics...
to hydrodynamics**

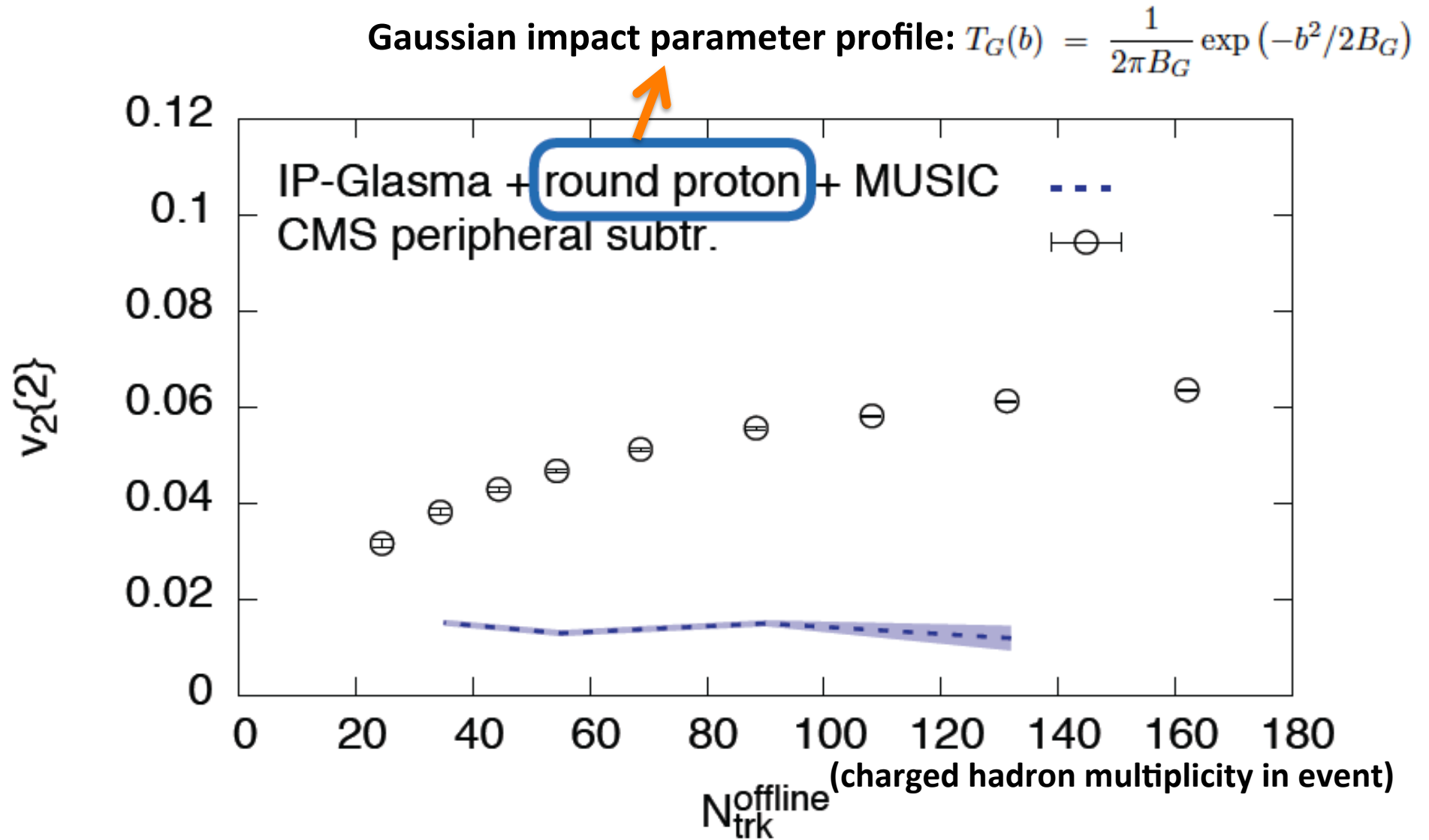
Figures: S. Schlichting at Quark Matter 2015

Thanks for listening!

Effect of correlations amongst projectile partons

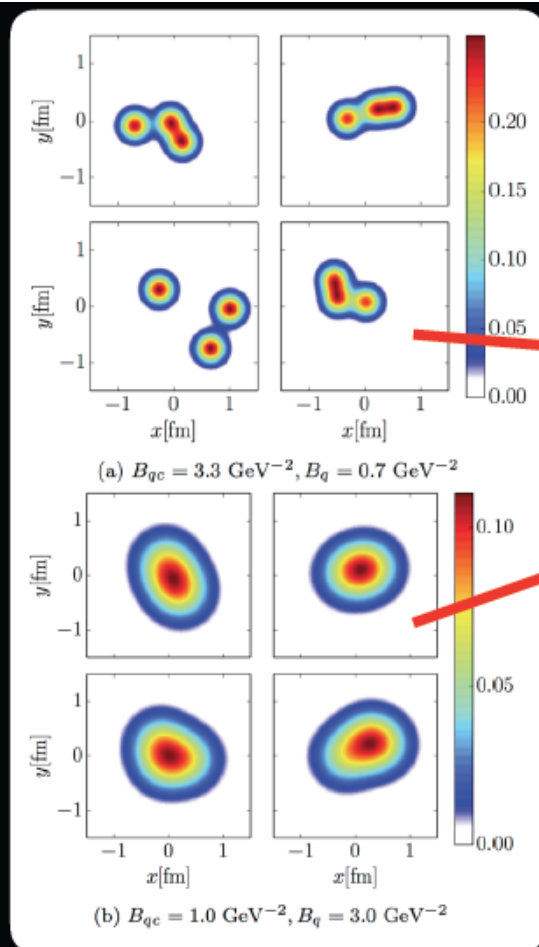


Shape fluctuations essential to generate flow-I

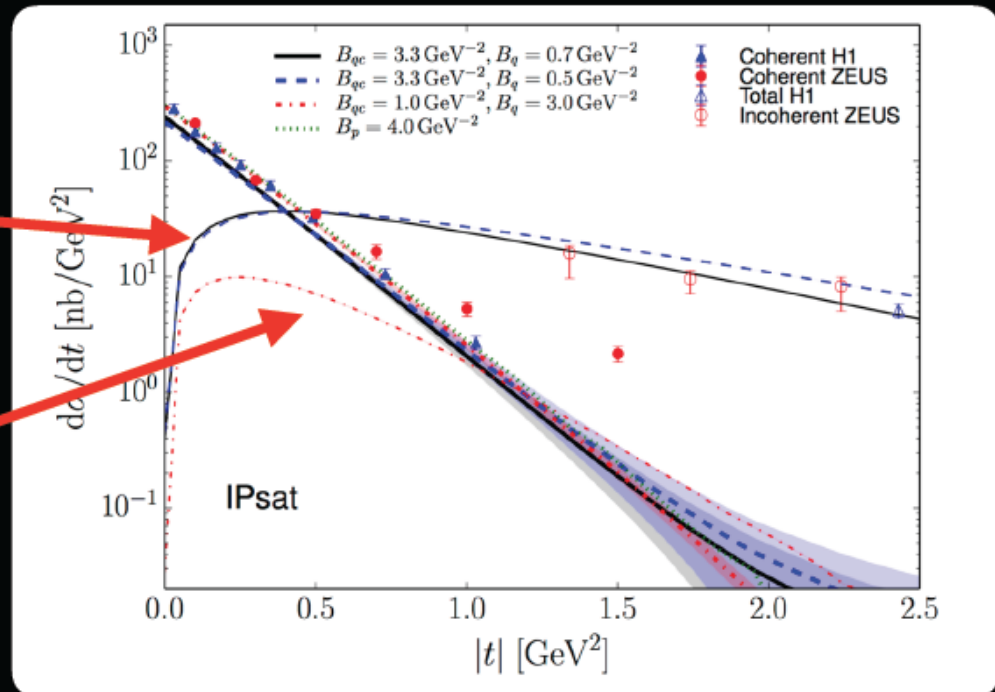


Shape fluctuations essential to generate flow-I

Incoherent exclusive vector meson (J/ψ) production is sensitive to fluctuations in transverse spatial profile of proton – compute in dipole picture



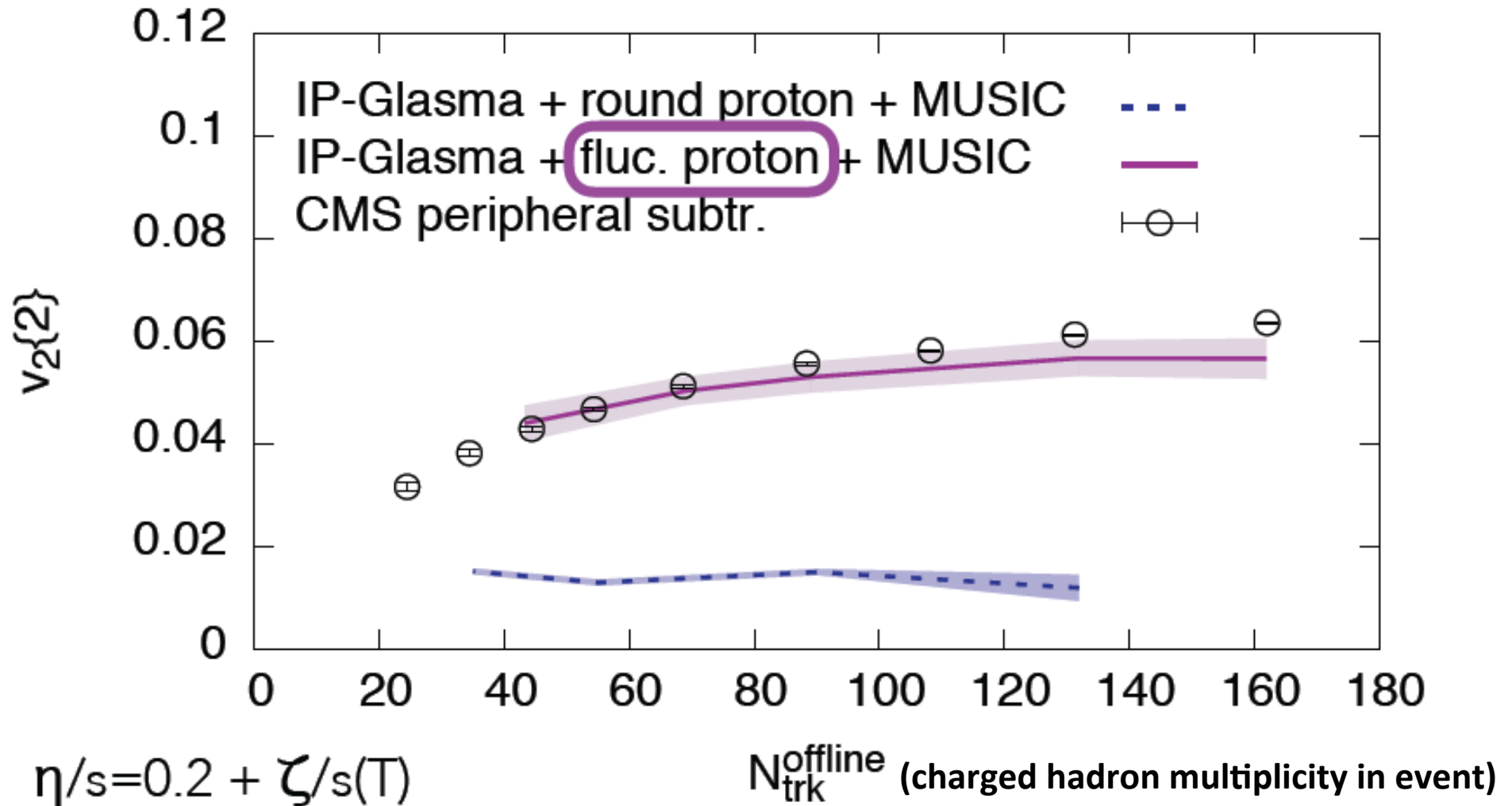
H. Mäntysaari, B. Schenke, Phys. Rev. Lett. 117 (2016) 052301
Phys.Rev. D94 (2016) 034042



H1 collaboration, Eur. Phys. J. C46 (2006) 585,
Phys. Lett. B568 (2003) 205
ZEUS collaboration, Eur. Phys. J. C24 (2002) 345
Eur. Phys. J. C26 (2003) 389

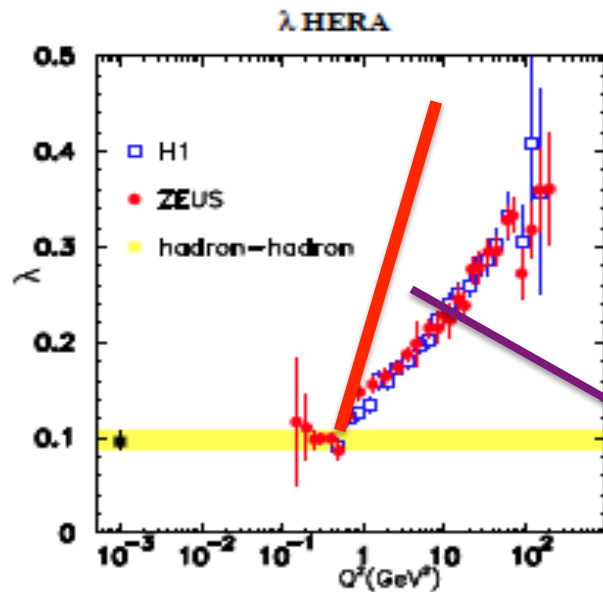
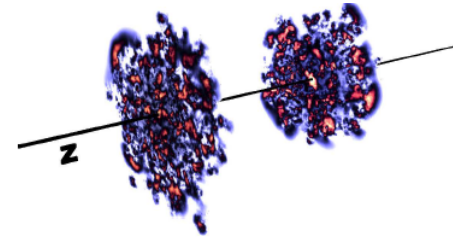
Björn Schenke, BNL

Shape fluctuations essential to generate flow-III



High multiplicities + small systems = gluon saturation

What does it take to produce ~ 150 hadrons per 5 units of rapidity in a single p+p event ?



For $\lambda=0.14$, get about 13 gluons produced in 5 units ~ min.bias hadron multiplicity

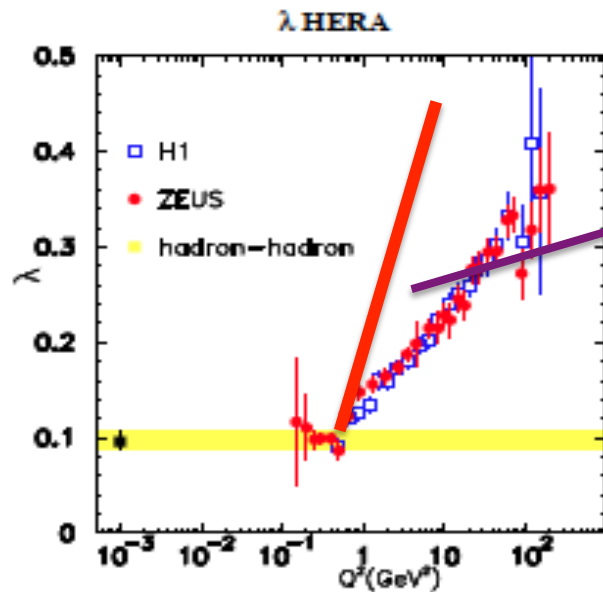
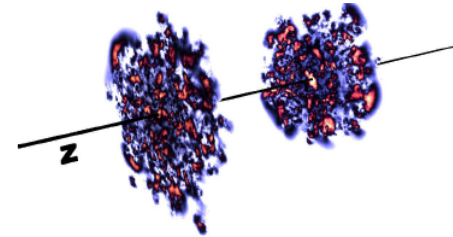
$\lambda=0.3$: ~45 gluons in 5 units,

$\lambda=0.4$: ~90 gluons in 5 units, in ball park...

Very rapid growth of gluon dist. in such events...

High multiplicities + small systems = gluon saturation

What does it take to produce ~ 150 hadrons per 5 units of rapidity in a single p+p event ?

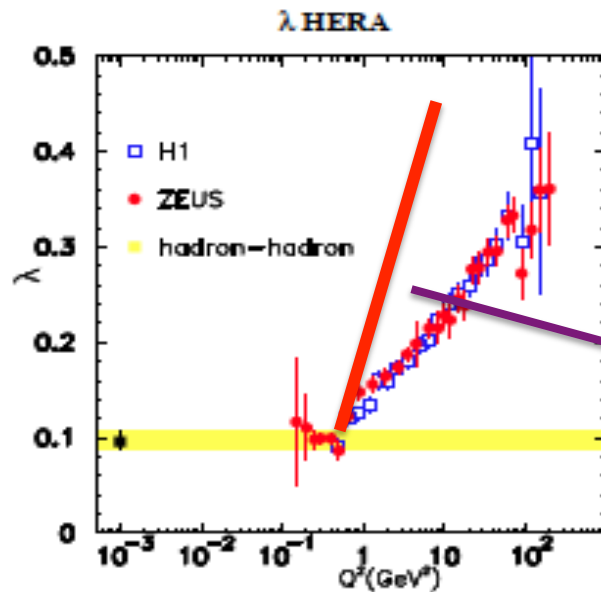
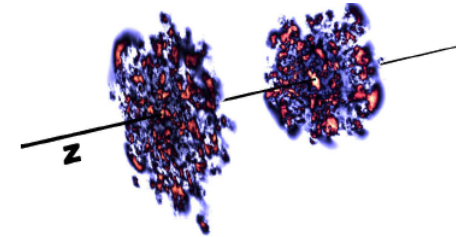


$$\frac{4\pi}{Q^2} * N_g(Q^2) = \pi R_{\text{glue}}^2$$

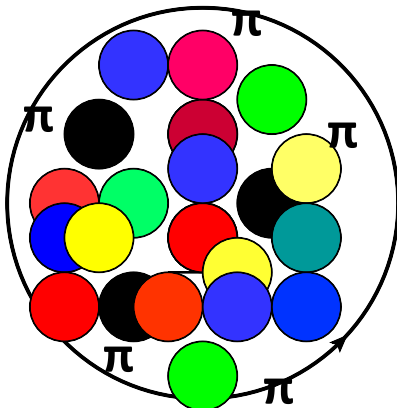
Such rapid growth, in an “independent parton” picture lead to very large gluon radii, $R_g > 1$ fm

High multiplicities + small systems = gluon saturation

What does it take to produce ~ 150 hadrons per 5 units of rapidity in a single p+p event ?



Saturation regulates this by adding increasingly “smaller” gluons of size $1/Q_s(x)$ with decreasing x (increasing energy)



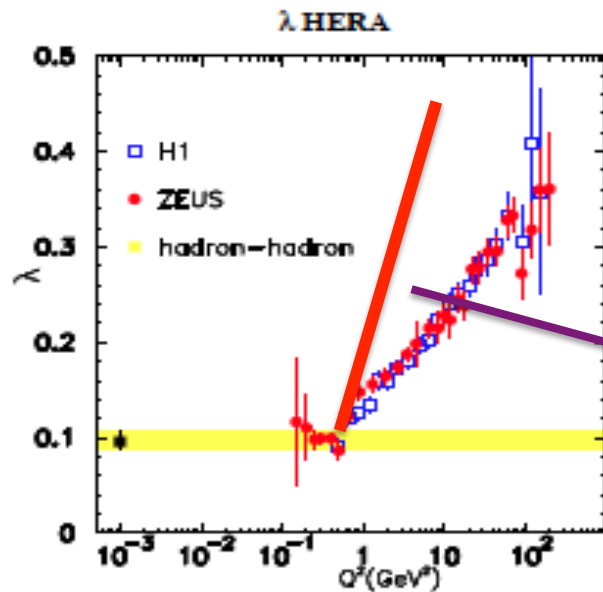
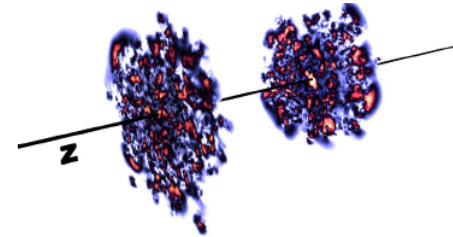
$$\frac{dN_g^{\text{prot.}}}{d\eta} \approx \frac{1.1C_F}{2\pi^2} \frac{S_{\perp} Q_{S,\text{prot.}}^2}{\alpha_S}$$

Lappi:
arXiv 0711.3039

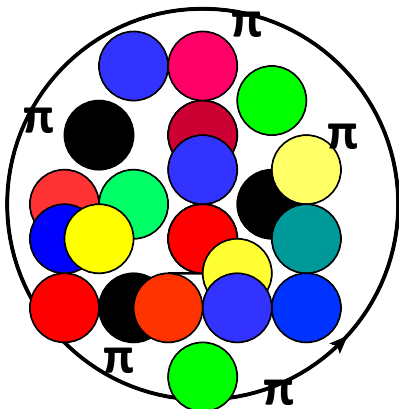
$N_g \sim 100$ in 5 units for $Q_s^2 \sim 2 \text{ GeV}^2$: a semi-hard scale !

High multiplicities + small systems = gluon saturation

What does it take to produce ~ 150 hadrons per 5 units of rapidity in a single p+p event ?



Saturation regulates this by adding increasingly "smaller" gluons of size $1/Q_s(x)$ with decreasing x (increasing energy)



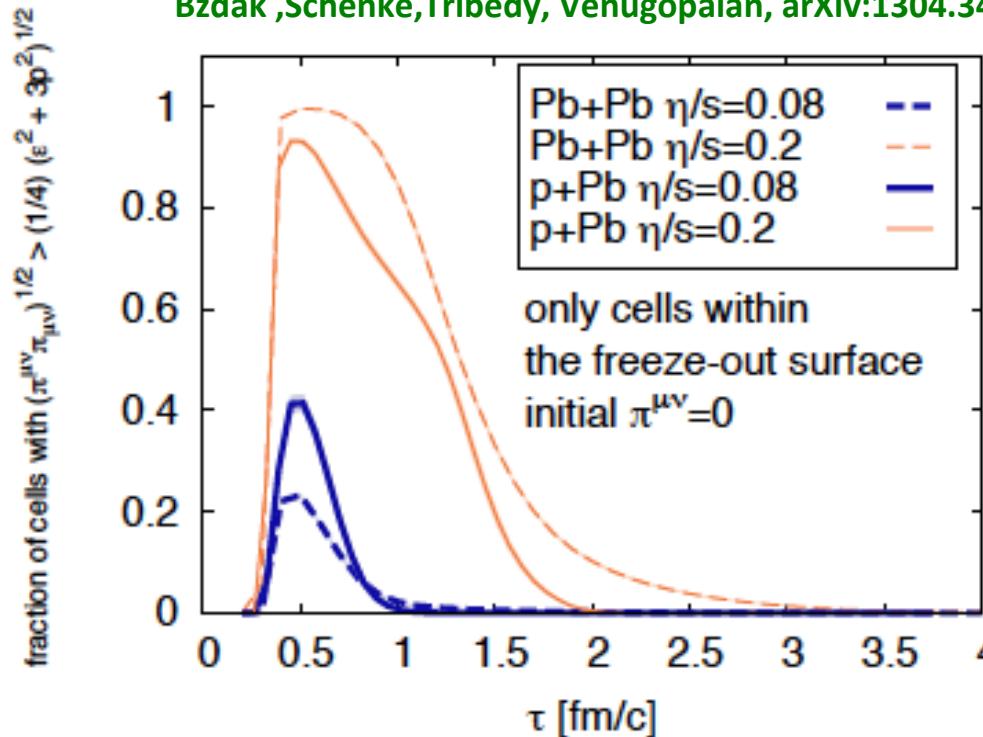
Event generators (such as EPOS) that describe data in high multiplicity events, build in a saturation scale ...

Issues with the hydrodynamic paradigm: I

Two frequently used measures: Reynolds # and Knudsen #

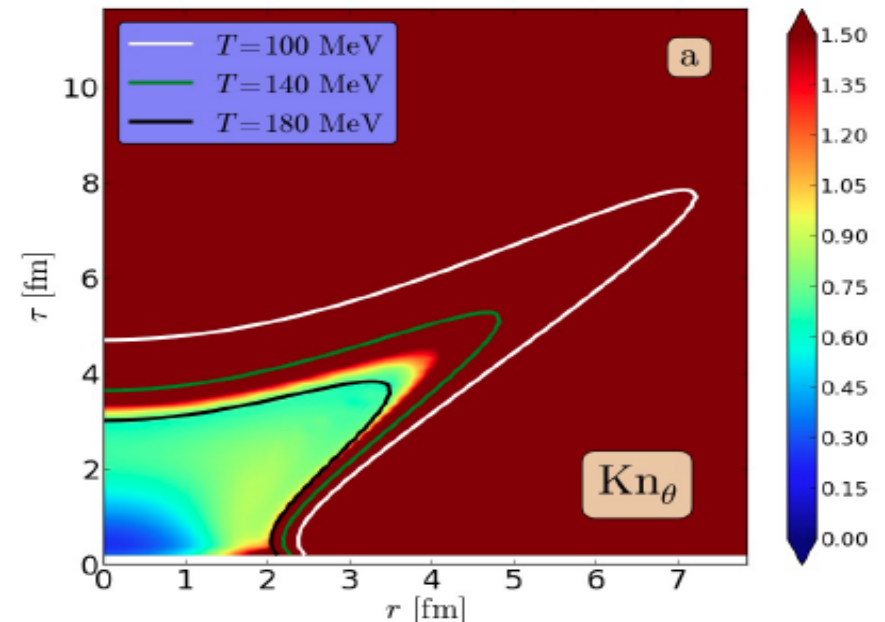
$$R^{-1} \propto (\Pi^{\mu\nu} \Pi_{\mu\nu})^{1/2} / (\epsilon^2 + 3P^2)^{1/2} \quad \text{Kn} = \frac{\tau_\pi}{L} \quad ; \quad \tau_\pi \propto \frac{\eta}{sT}$$

Bzdak ,Schenke,Tribedy, Venugopalan, arXiv:1304.3403



Dumitru,Molnar,Nara,arXiv:0706.2233

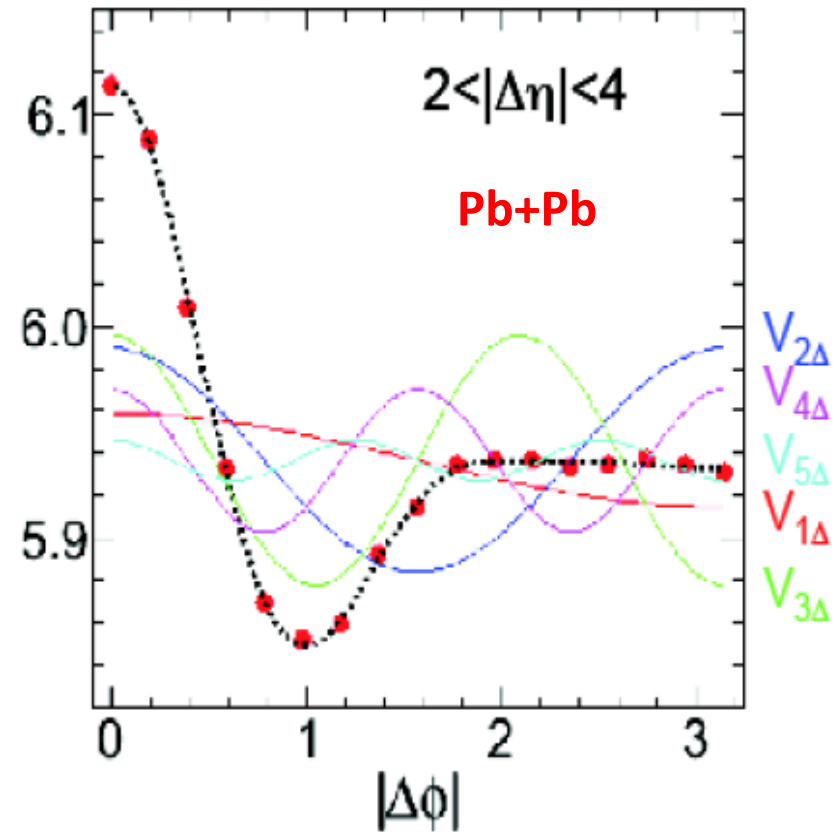
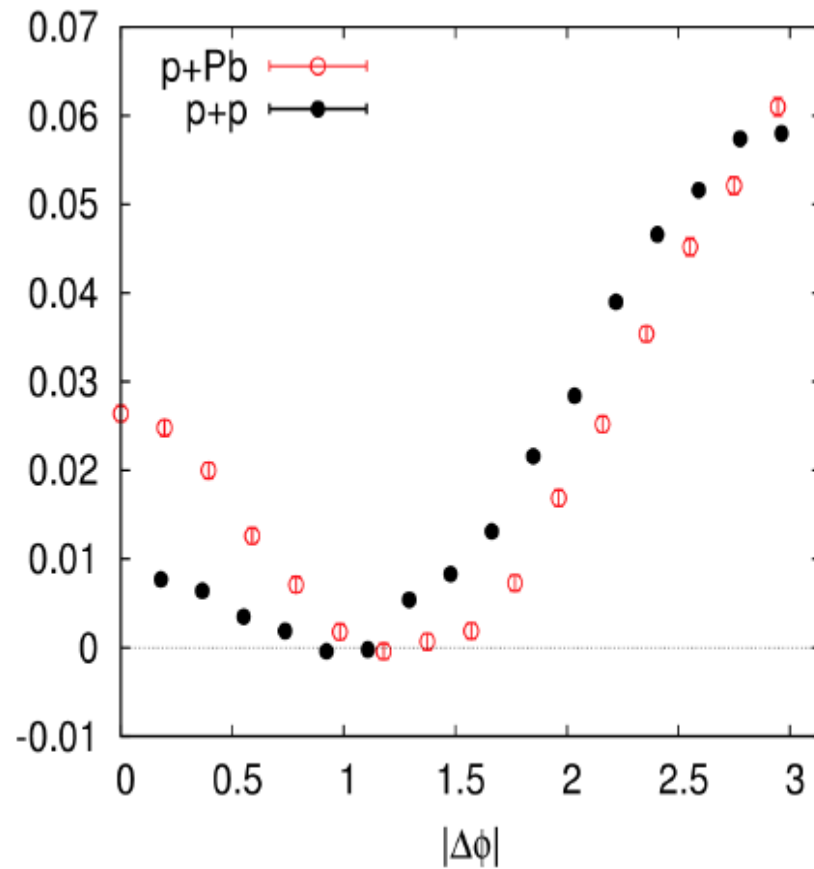
Denicol, Niemi, arXiv:1404.7327



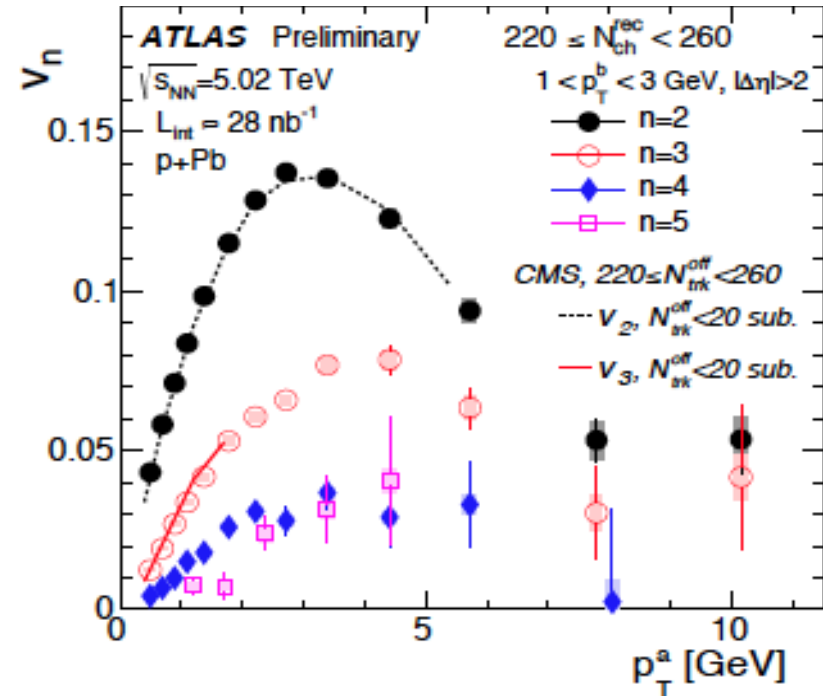
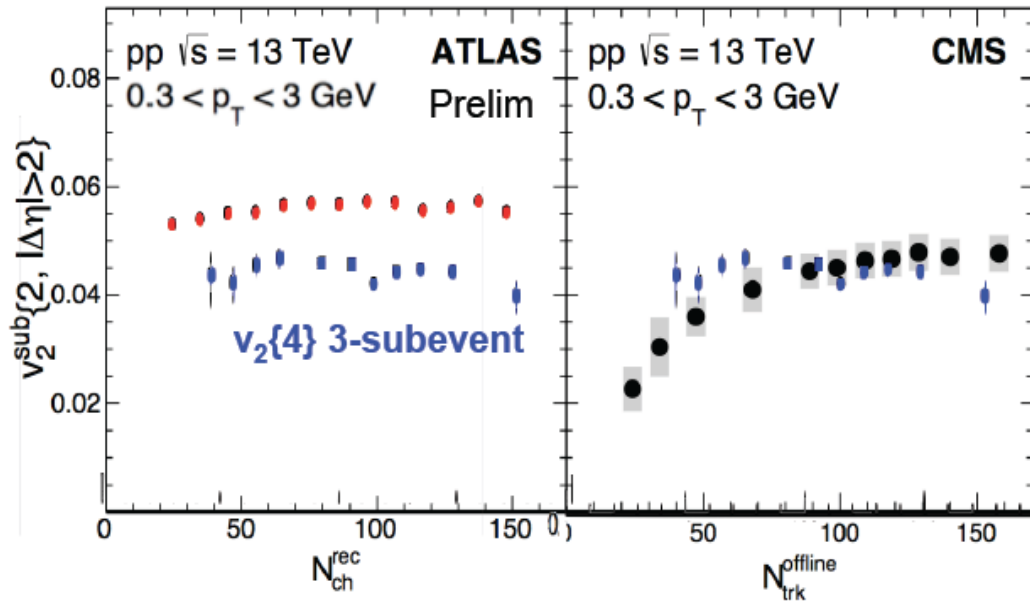
Hydro good for $\text{Kn} < 0.5$,
marginal for $\text{K} < 1$ transient regime;
 $\text{K} > 1$ free streaming

Issues with the hydrodynamic paradigm: II

No (mini-) jet quenching seen in the smaller systems



Issues with the hydrodynamic paradigm: III

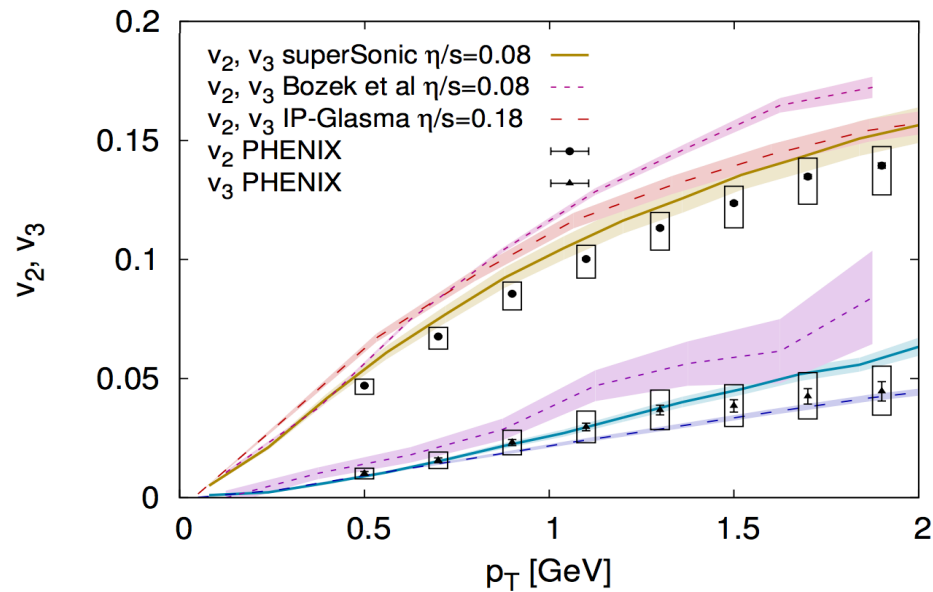


Large anisotropies at larger p_T and smaller N_{ch} than one might reconcile with a hydrodynamic description

Four-particle collectivity seen in minimum bias events...

Collectivity in 3He+Au collisions

A. Adare et al. (PHENIX Collaboration)
Phys. Rev. Lett. 115, 142301 (2015)

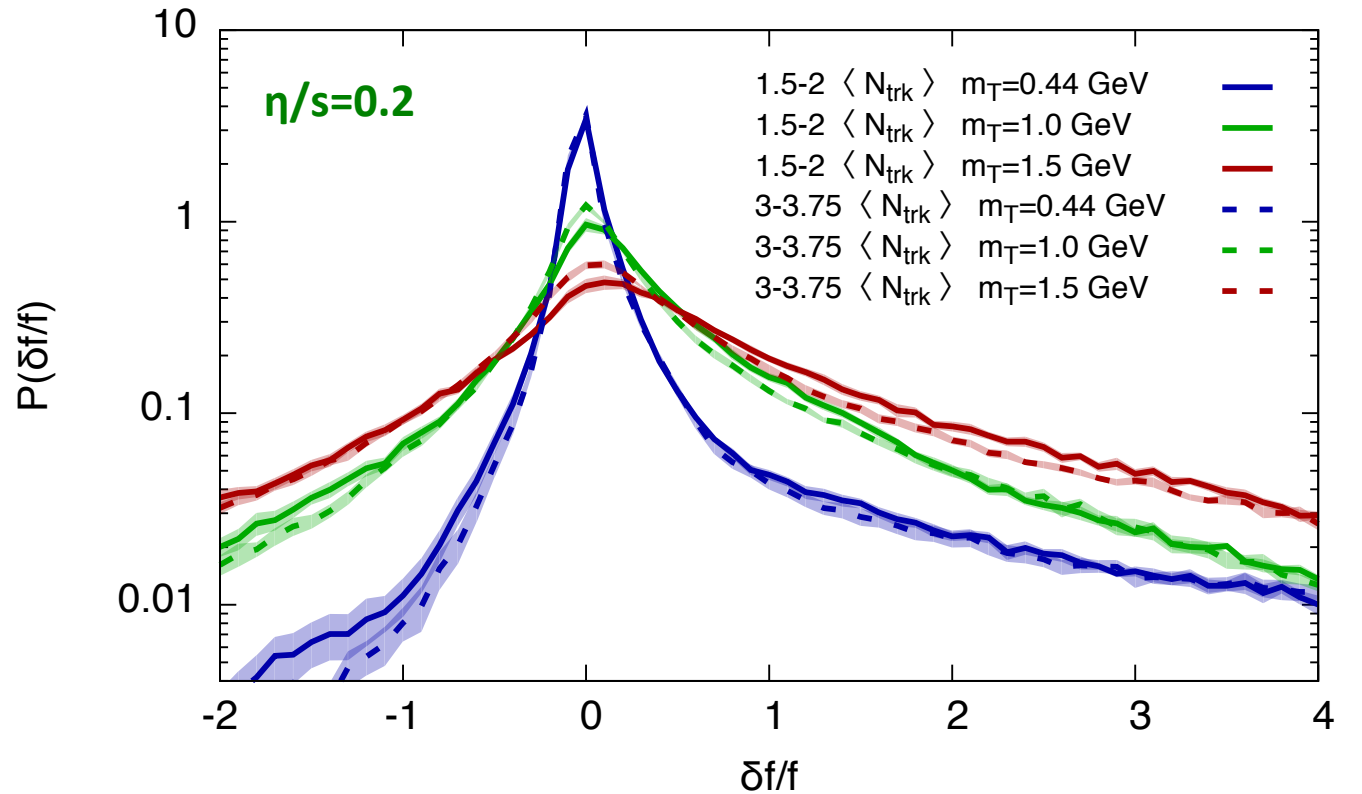


Schenke, Venugopalan
Nucl. Phys. A931 (2014) 1039-1044

J.L. Nagle, et al.
Phys. Rev. Lett. 113, 112301

Bożek, Broniowski
Physics Letters B 747 (2015) 135–138

Freeze-out corrections in p+Pb as function of p_T

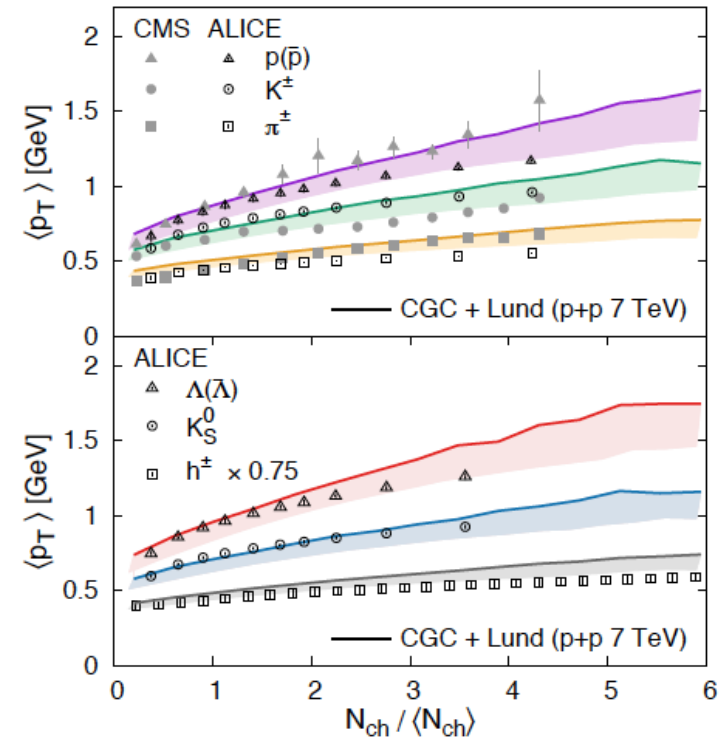
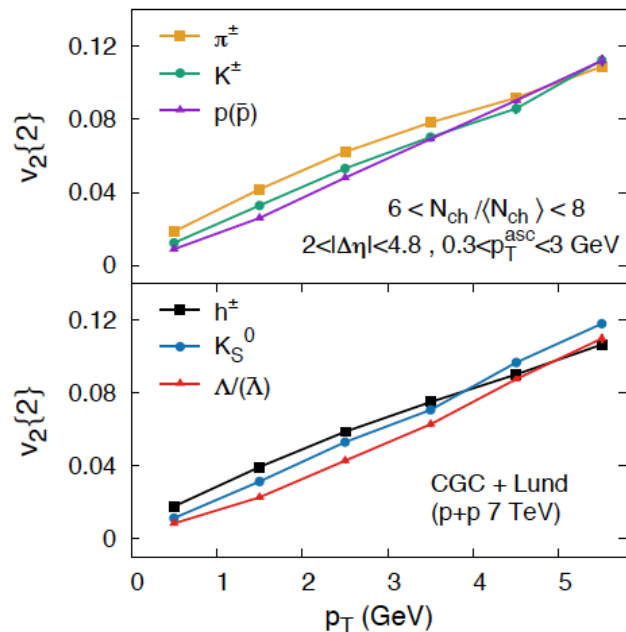
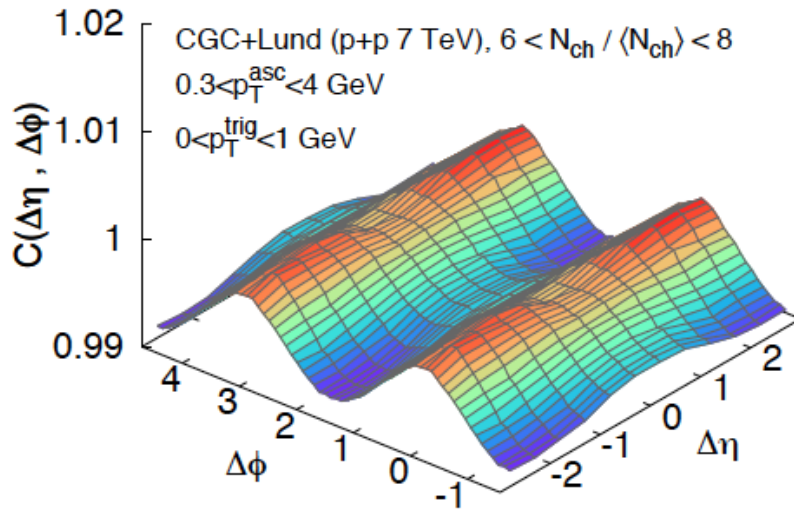


Plot by Bjoern Schenke

For $m_T = 1$ GeV, 26% of hydro cells have a 100% correction

For $m_T = 1.5$ GeV, 43% have a 100% correction

IP-Glasma+Lund fragmentation

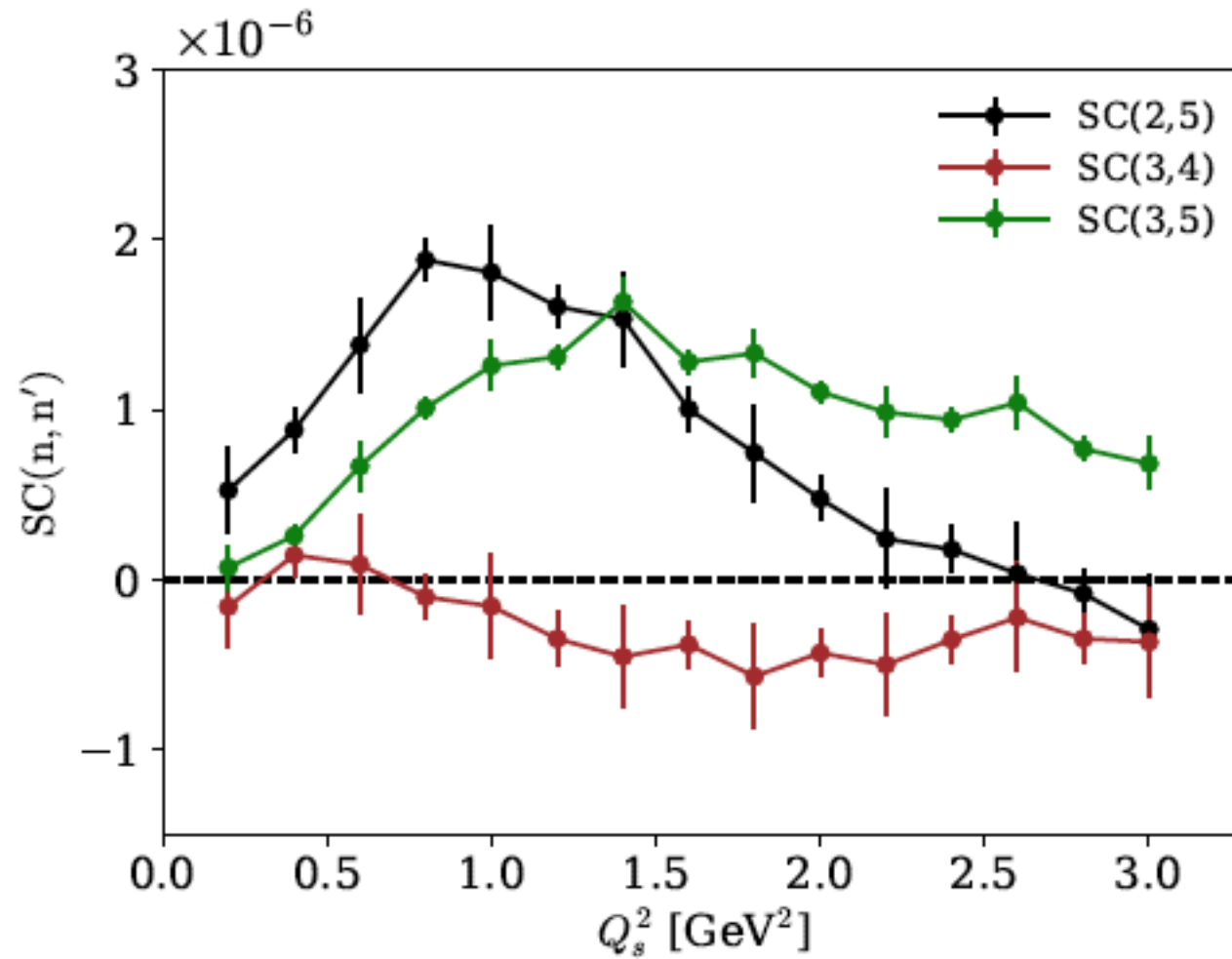


Pattern of mass splitting of $\langle p_T \rangle$ and v_2 seen in high multiplicity events is reproduced [Schenke,Schlichting,Tribedy,RV, PRL117\(2016\)162301](#)

What about 4-particle collectivity?
 Numerically very challenging-in progress

[Schenke,Schlichting,Tribedy,RV](#)

Predictions for p+A symmetric cumulants



Higher cumulants in the color domain model

Dumitru, McLerran, Skokov, 1410.4844

Color domain model: express intrinsic higher point correlators as correlators of produced particles with a target field in a color domain, averaged over all orientations of the field.

$$c_2\{2\} = \frac{1}{N_D} \left(\mathcal{A}^2 + \frac{1}{4(N_c^2 - 1)} \right) :$$
$$c_2\{4\} = -\frac{1}{N_D^3} \left(\mathcal{A}^4 - \frac{1}{4(N_c^2 - 1)^3} \right)$$

“A” term is the correlation induced between projectile particles due to color field orientation of target (more generically, non-Gaussian correlations)

The N_c term is the “connected Glasma graph” (Gaussian correlations)

N_D is # of color domains – few in p+A, several in A+A

