3 Parton production in DIS at small x

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IN COLLABORATION WITH

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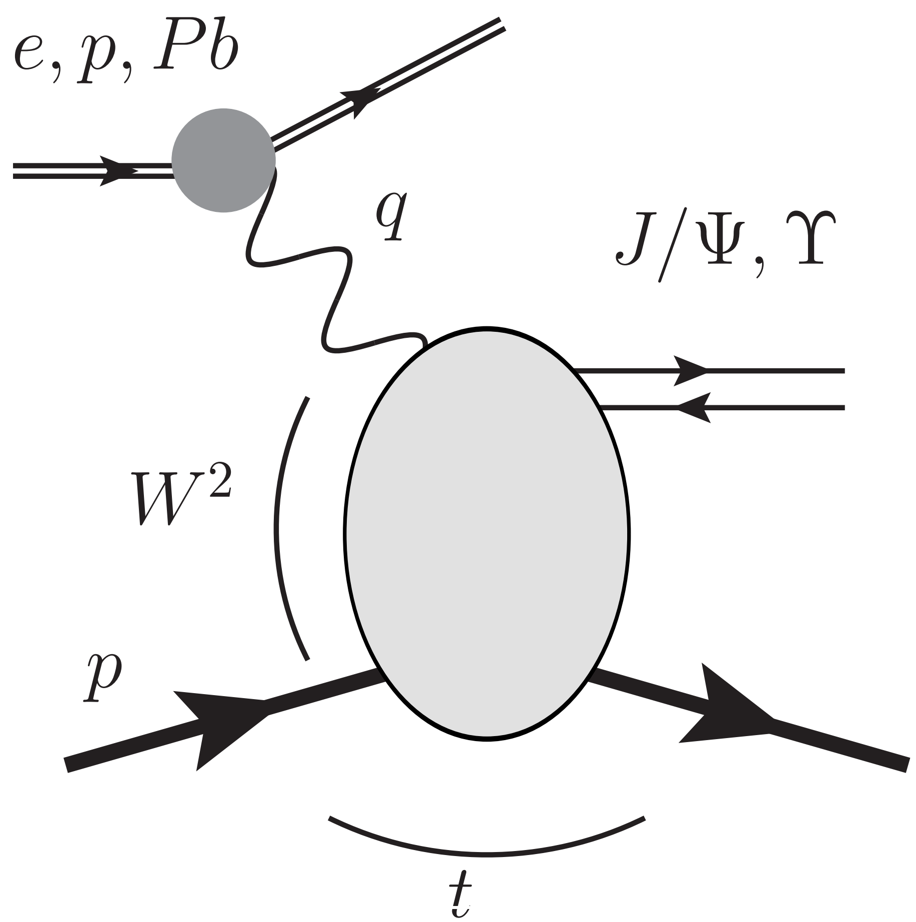
arXiv:1701.07143/Nucl. Phys. B 920, 232 (2017)
arXiv:1604.08526/Phys. Lett. B 761, 229 (2016)

RBRC workshop "*Synergies of pp and pA collisions with an Electron-Ion Collider*", June 26-28, 2017, BNL

photo-production of J/Ψ and Υ: explore proton at ultra-small x

[Bautista, Ferandez-Tellez, MH; 1607.05203]

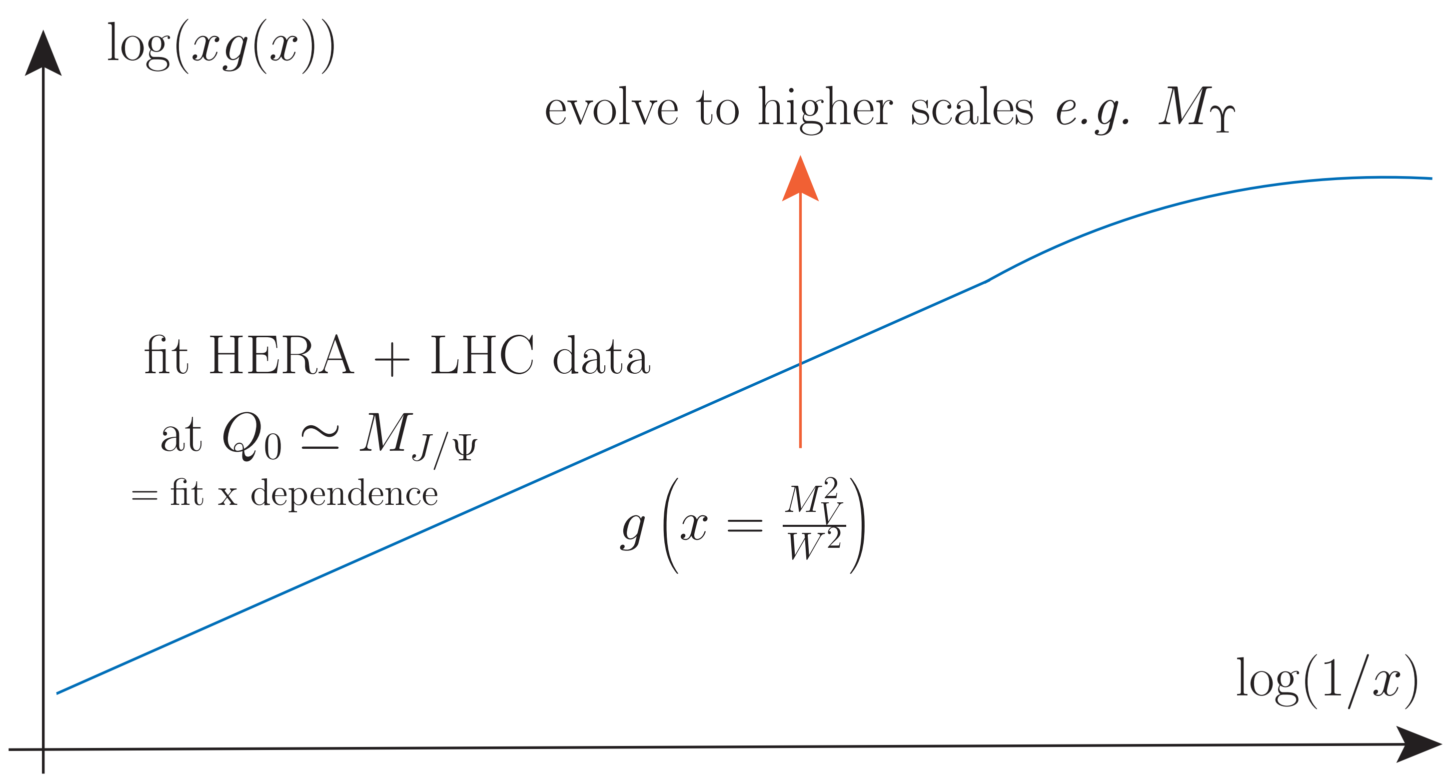


- measured at HERA (ep) and LHC (pp, ultra-peripheral pPb)
- charm and bottom mass provide hard scale → pQCD
- exclusive process, but allows to relate to inclusive gluon

reach values down to $x = 4 \times 10^{-6} \rightarrow$ (unique ?) opportunity to explore the low x gluon

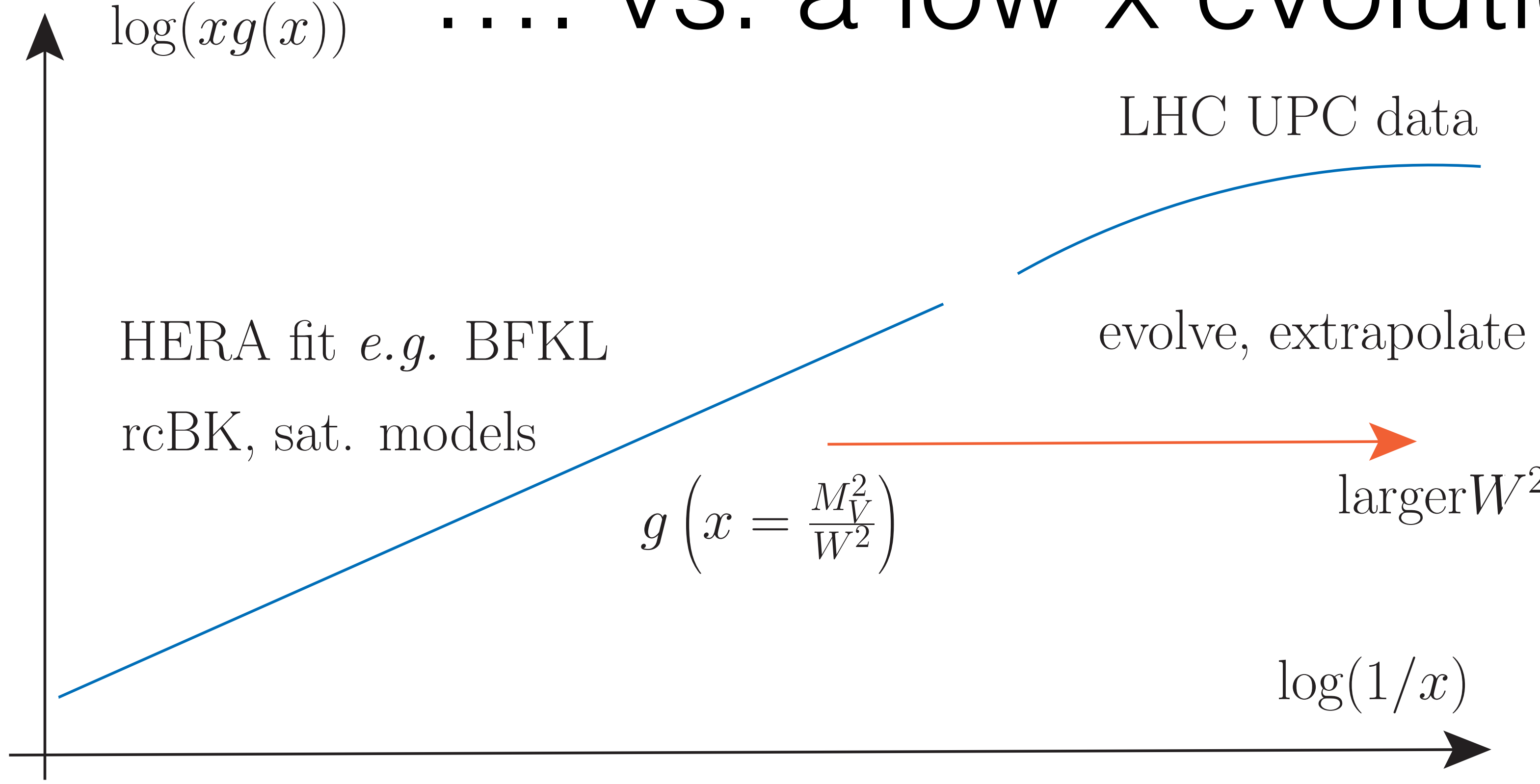
saturation effects?

What can be learned from a DGLAP study … ?

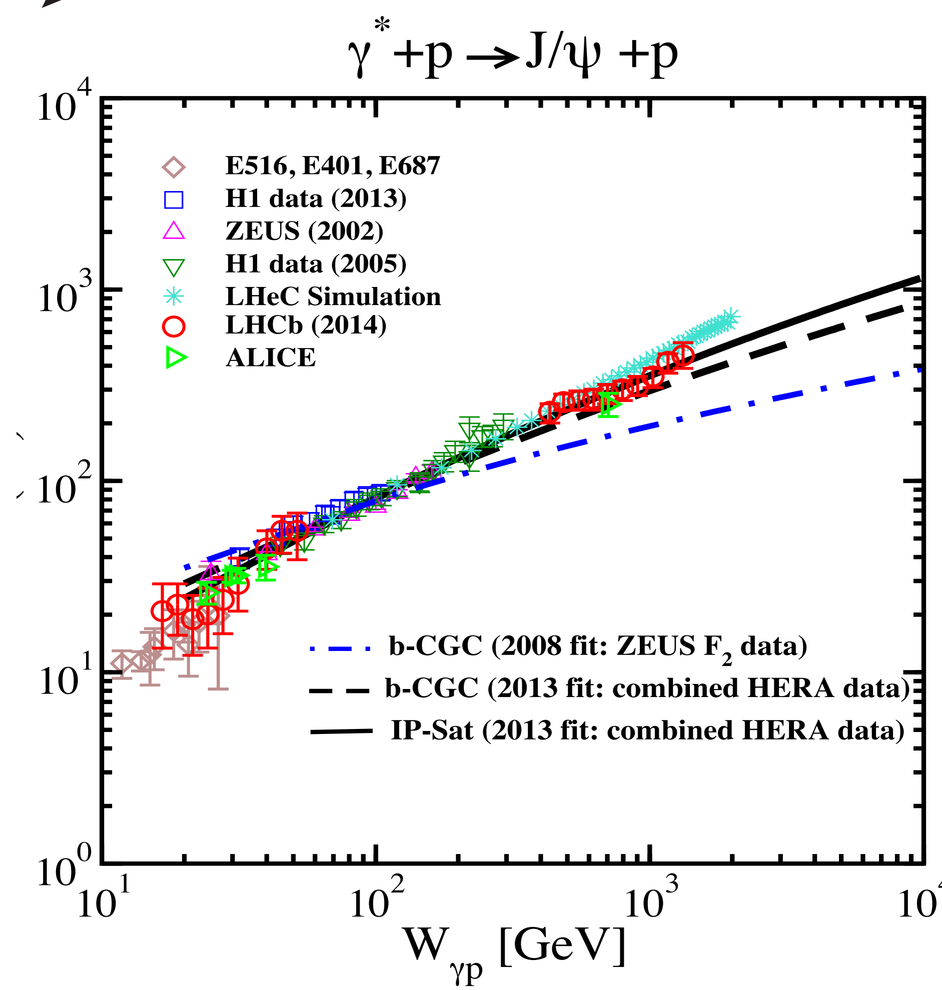


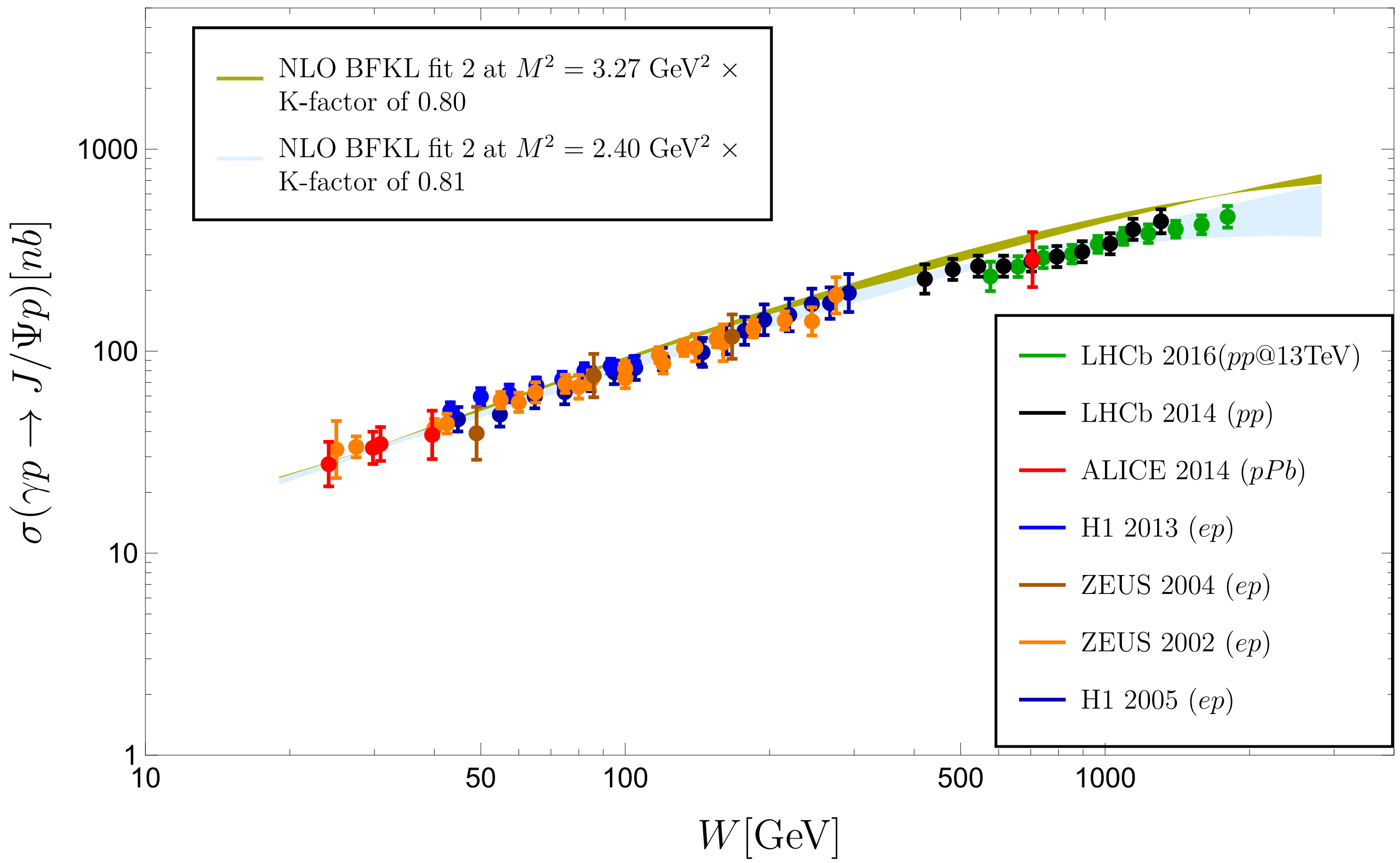
- convinced: pdf studies highly valuable → constrain pdfs at ultra-small x
- but DGLAP unstable at such low x + evolution in hard scale limited (J/Ψ → Y)
- don't really verify validity of DGLAP evolution

…. vs. a low x evolution study ?



- linear (BFKL) vs. non-linear (BK/JIMWLK) vs. models
- observation: saturation models work pretty well
  [Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1405.6977]





but *linear* NLO BFKL describes so far data as well → no need for non-linear effects?

[Bautista, Fernandez-Tellez, MH; 1607.05203]

2 potential explanations:

a) saturation still far away
b) BFKL can mimic effects in "transition region"
→ how can this happen?

BFKL gluon density ~ dipole amplitude

$$\mathcal{N}(\boldsymbol{r}, \boldsymbol{b}) = \frac{1}{N_c} \mathrm{Tr}\left(1 - V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right) \quad \sim \quad G^{\mathsf{BFKL}}(x, \boldsymbol{k})$$

evolution differs, but essentially same quantity

- at an EIC: not so much BFKL, but nuclear shadowing might mimic non-linear low x evolution

- unambiguous (?) identification of non-linear effects needs a) lots of phase space or b) observables directly sensitive to non-linearities *e.g.* observables sensitive to the quadrupole

$$\mathcal{N}^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4) = \frac{1}{N_c} \mathrm{Tr}\left(1 - V(\boldsymbol{x}_1) V^\dagger(\boldsymbol{x}_2) V(\boldsymbol{x}_3) V^\dagger(\boldsymbol{x}_4)\right)$$

$$\sim \quad G + \# G^2 + \# G^4 + \ldots$$

most prominent example process:
di-hadron production beyond dilute approximation
[Dominguez, Marquet, Xiao,Yuan; 1101.0715]

Our proposal:

di-hadron is a great (most useful ?) observable, but worthwhile to go a step beyond (→ extra constrains on so far little studied quadrupole)

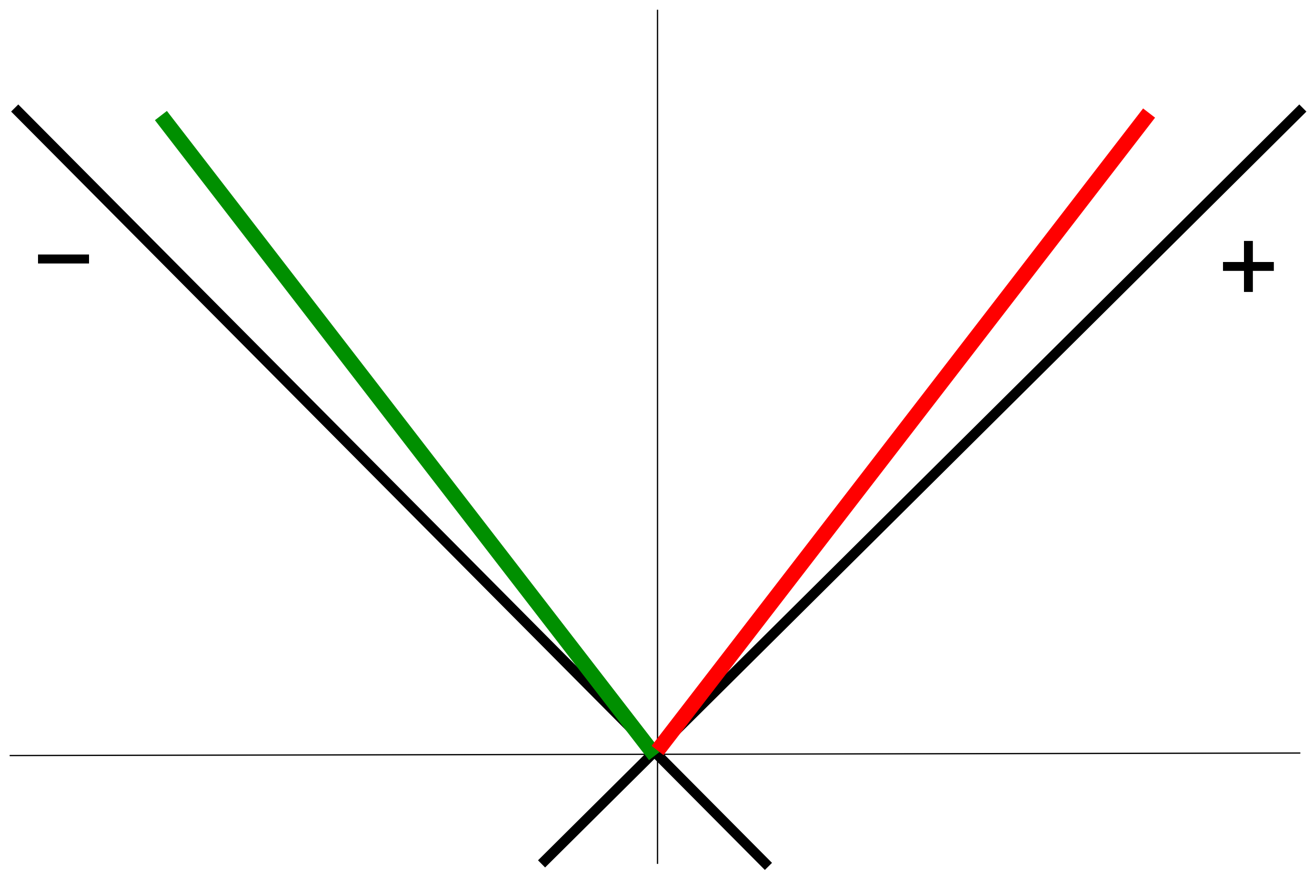
this project: calculate

a) inclusive tri-parton production at LO (real part of NLO corrections to di-partons)
b) question: how to organise calculation in effective way; develop techniques for complex calculation?
c) related calculation for diffraction (includes already virtual)
[Boussarie, Grabovsky, Szymanowski, Wallon; 1405.7676, 1606.00419]

NEW: re-vive idea of momentum space calculations

twofold interest: explore new (potential) EIC/LHeC/… observable + re-fine/develop further our techniques for calculations in the presence of high gluon densities

basic setup: high energy factorization



- scattering objects close to opposite sides of the light cone
- separated by large boost factors
- Lorentz contraction & time dilation → separation into "slow" & "fast" fields

$k^{\pm} = (k^0 \pm k^3)/\sqrt{2}$

light cone coordinates/ momenta

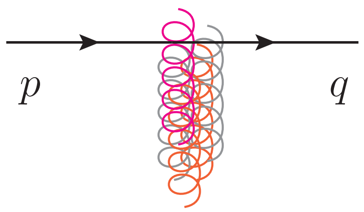
$\boxed{\begin{array}{c} \mathbf{k}^{+} \gg \mathbf{k}^{+}, \\ \mathbf{k}^{-} \gg \mathbf{k}^{-} \end{array}}$

+ a dense gluon field $A \sim 1/g$

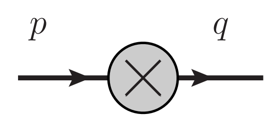
Theory: Propagators in background field

use light-cone gauge, with $k^- = n^+ \cdot k$, $(n^+)^2 = 0$, $n^+ \sim$ target momentum

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

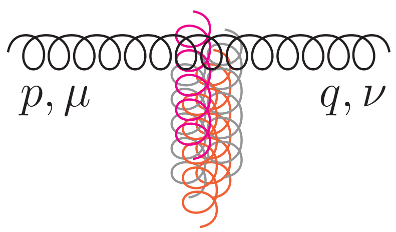


$$= (2\pi)^d \delta^{(d)}(p-q) \tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$

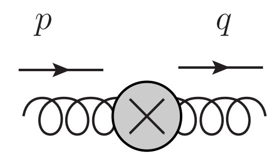


$$\tilde{S}_F^{(0)}(q)$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$



$$= (2\pi)^d \delta^{(d)}(p-q) \tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



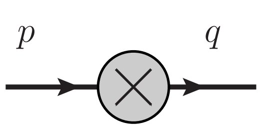
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

interaction with the background field:

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P} \exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

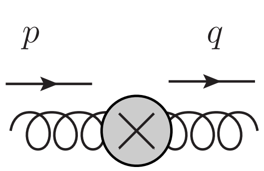
$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P} \exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$



$$= \tau_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+) \not{n}$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{ \theta(p^+)\left[V_{ij}(\boldsymbol{z}) - 1_{ij}\right] - \theta(-p^+)\left[V_{ij}^\dagger(\boldsymbol{z}) - 1_{ij}\right] \right\}$$

strong background field resummed into path ordered exponentials (Wilson lines)

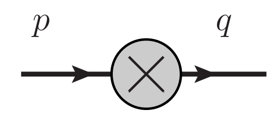


$$= \tau_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\ (-2p^+)$$

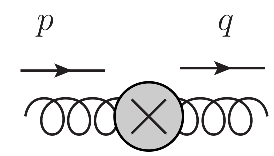
$$A^-(x^+, x_t) = \delta(x^+)\alpha(x_t)$$

$$\times \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{ \theta(p^+)\left[U^{ab}(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left(U^{ab}\right)^\dagger(\boldsymbol{z}) - 1\right] \right\}$$

Coordinate dependence in vertices → Calculation in configuration space



$$= 2\pi\delta(p^- - q^-)\not{n}^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \cdot \left\{\theta(p^-)[V(\boldsymbol{z})-1] - \theta(-p^-)[V^\dagger(\boldsymbol{z})-1]\right\}$$

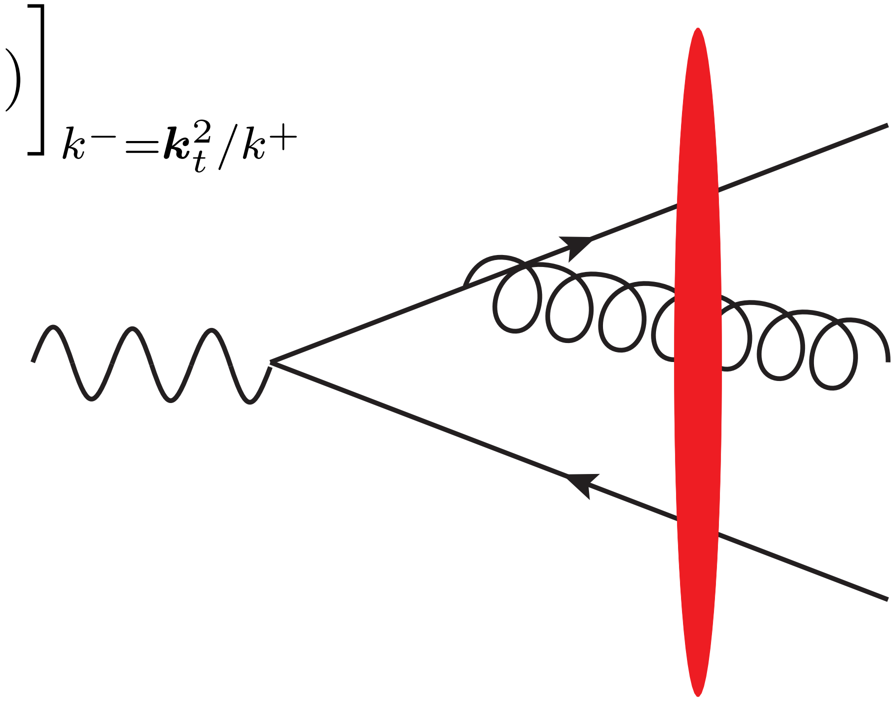


$$= -2\pi\delta(p^- - q^-)2p^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \cdot \left\{\theta(p^-)[U(\boldsymbol{z})-1] - \theta(-p^-)[U^\dagger(\boldsymbol{z})-1]\right\}$$

$$\Delta_F(x) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{ik\cdot x}}{k^2+i0}$$

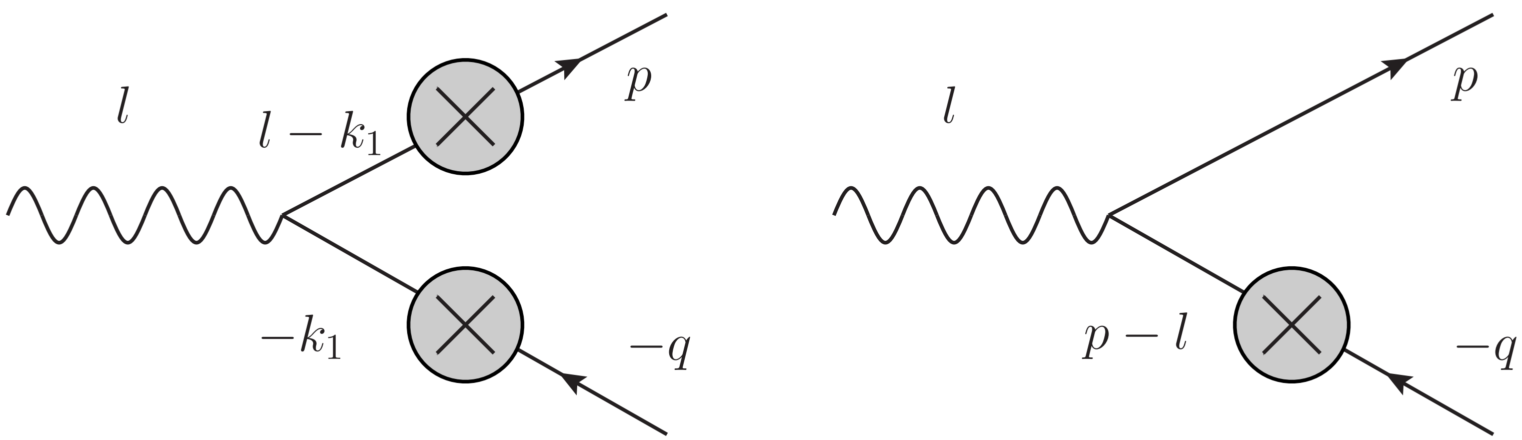
$$= \int \frac{dk^+}{2k^+}\int \frac{d^2\boldsymbol{k}_t}{(2\pi)^3} e^{ik\cdot x}\Big[\theta(k^+)\theta(x^+) + \theta(-k^+)\theta(-x^+)\Big]_{k^- = \boldsymbol{k}_t^2/k^+}$$

- popular: time ordered/old fashioned/ light-front/perturbation theory (also calc. completely in config. space)
  recent examples: **[Beuf, 1606.00777]; [Balitsky, Chirilli; 1009.4729]; [Boussarie, Grabovsky, Szymanowski, Wallon; 1405.7676, 1606.00419]**



- simplification due to Lorentz contraction of background field immediately useful

Calculation in momentum space



- also possible *e.g.* **[Gelis, Jalilian-Marian; hep-ph/0211363]**
- background field → "loop"-integrals for tree-level diagrams [k_1 is to be integrated over …]
- at first: all possible diagrams to be considered + keep track of Fourier factors

momentum vs. configuration space

	conventional pQCD (use known techniques)	inclusion of finite masses (charm mass!)	intuition: interaction at t=0 with Lorentz contracted target
momentum space	well explored	complication, but doable	lose intuitive picture at first -> large # of cancelations
configuration space	poorly explored	very difficult	many diagrams automatically zero

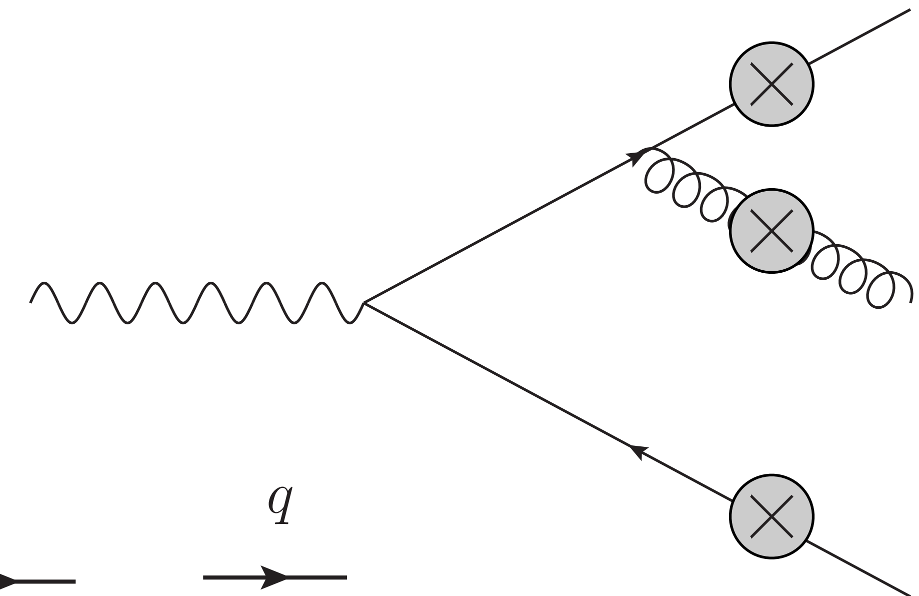
our approach:
work in momentum space + exploit configuration space to set a large fraction of all diagrams to zero

How to do that?

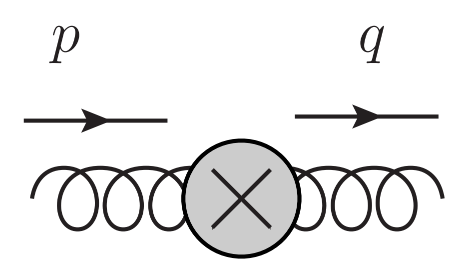
Essentially: re-install configuration space rules at the level of a single diagram

- for each Feynman-diagram, at each (standard) vertex

$$\delta(\{p_{\text{in}}^-\} - \{p_{\text{out}}^-\}) = \int \frac{dx^+}{2\pi} e^{-ix^+\cdot(\{p_{\text{in}}^-\}-\{p_{\text{out}}^-\})}$$



- at each special vertex a factor 1= exp[0 $\cdot$ (p_{in}^- - p_{out}^-)]



- propagators:

$$\tilde{S}_{F,kl}(x_{ij}^+; p^+, \boldsymbol{p}) = \int \frac{dp^-}{2\pi} e^{-ip^- x_{ij}^+} S_{F,kl}(p) =$$

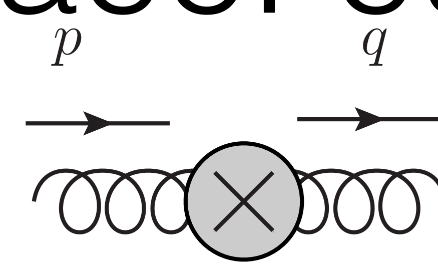
$$= \delta_{kl} \frac{e^{-ip^- x_{ij}^+}}{2p^+} \left[\left(\theta(p^+)\theta(x_{ij}^+) + \theta(-p^+)\theta(-x_{ij}^+) \right) (\not{p} + m) + \delta(x_{ij}^+)\not{n} \right]_{p^- = \frac{\boldsymbol{p}^2+m^2}{2p^+}}$$

$$\tilde{G}_{\mu\nu}^{(0),ab}(x_{ij}^+; p^+, \boldsymbol{p}) = \int \frac{dp^-}{2\pi} e^{-ip^- x_{ij}^+} G_{\mu\nu}^{(0),ab}(p) =$$

$$= \delta_{ab} \frac{e^{-ip^- x_{ij}^+}}{2p^+} \left[\left(\theta(p^+)\theta(x_{ij}^+) + \theta(-p^+)\theta(-x_{ij}^+) \right) \cdot d_{\mu\nu}(p) + 2\delta(x_{ij}^+) \frac{n_\mu n_\nu}{p \cdot n} \right]_{p^- = \frac{\boldsymbol{p}^2+m^2}{2p^+}} .$$

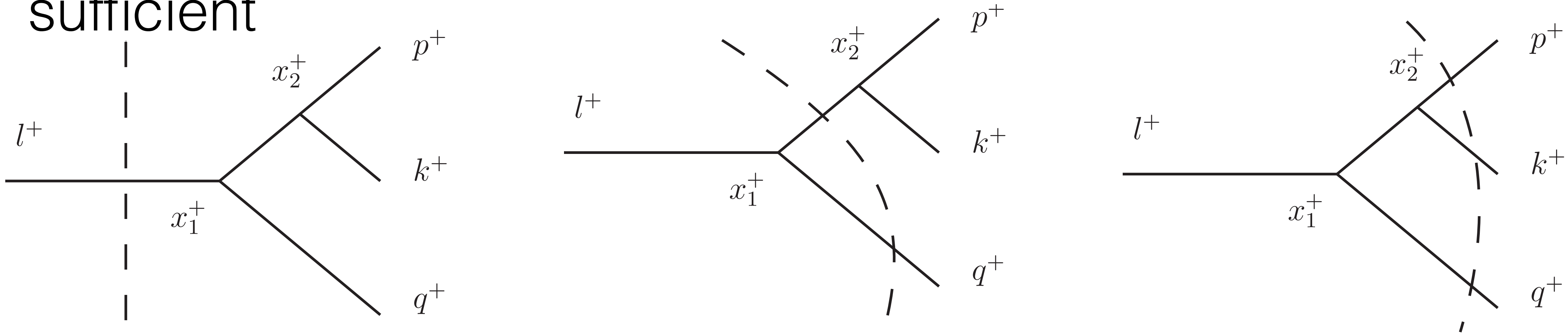
Configuration space: cuts at x^+=0

- start without special vertices



- divide x_i^+ integral $\int_{-\infty}^{\infty} dx^+ \to \int_{-\infty}^{0} dx^+ + \int_{0}^{\infty} dx^+$ + theta functions in plus momenta & coordinates → each of our diagrams cut by a line separating positive & negative light-cone time (left: negative; right: positive)

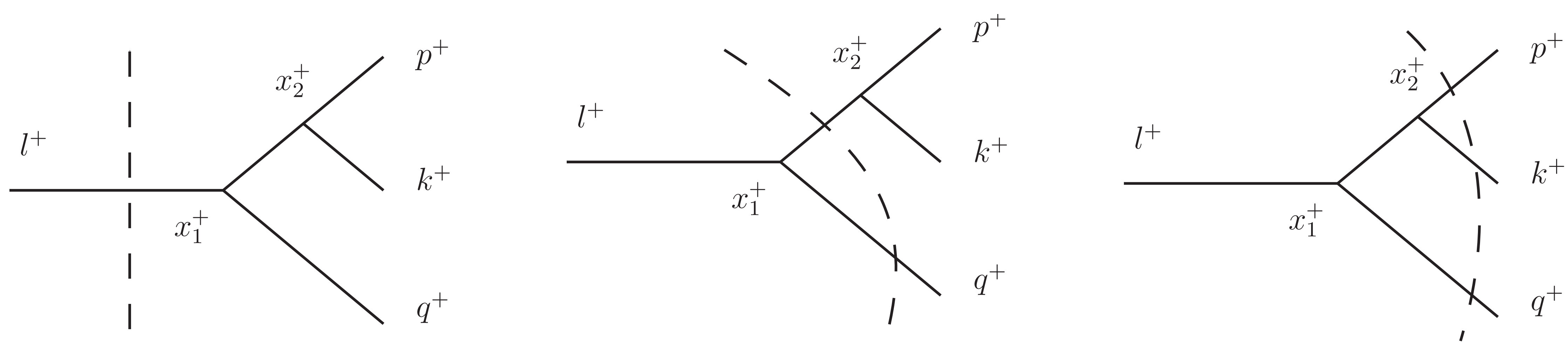
- only plus coordinates & momenta → skeleton diagrams sufficient



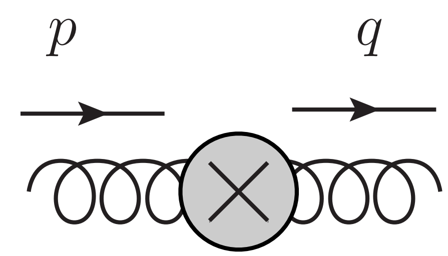
- a "cut" propagator crosses light-cone time x^+=0

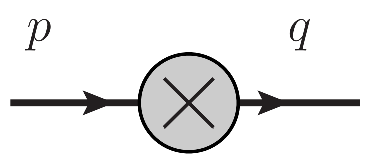
Which cuts are possible?

- in general: any line through the diagram
- fix kinematics to s-channel kinematics [l⁺=p⁺ +q⁺ +k⁺, all plus momenta positive always]
  → only s-channel type cuts possible (~vertical cuts)



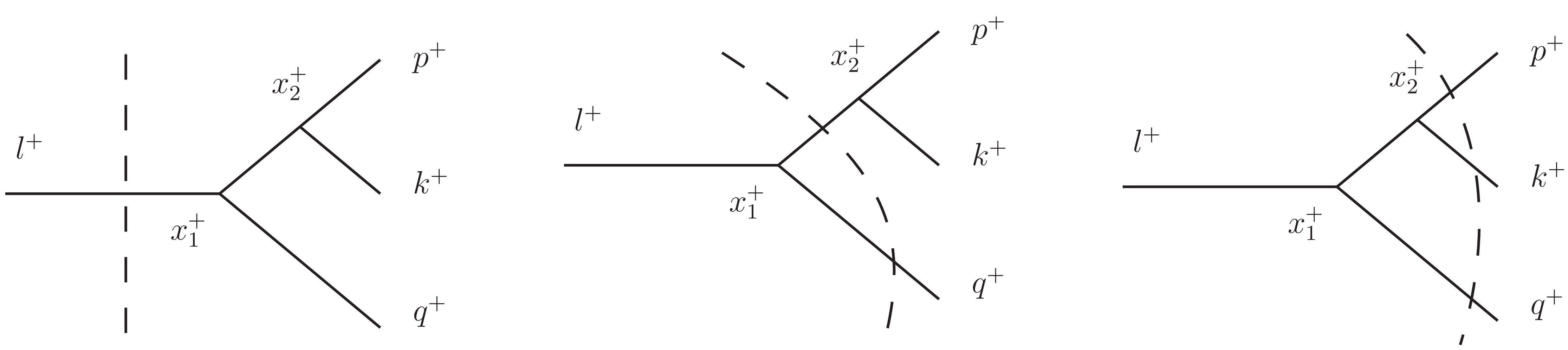
- for this topology, these are the only possible cuts

- NEXT: add special vertices 

- recall: 

$\sim \delta(p^+ - q^+)$ plus momentum flow not altered + placed at z⁺=0 ⇒ by default on the cut

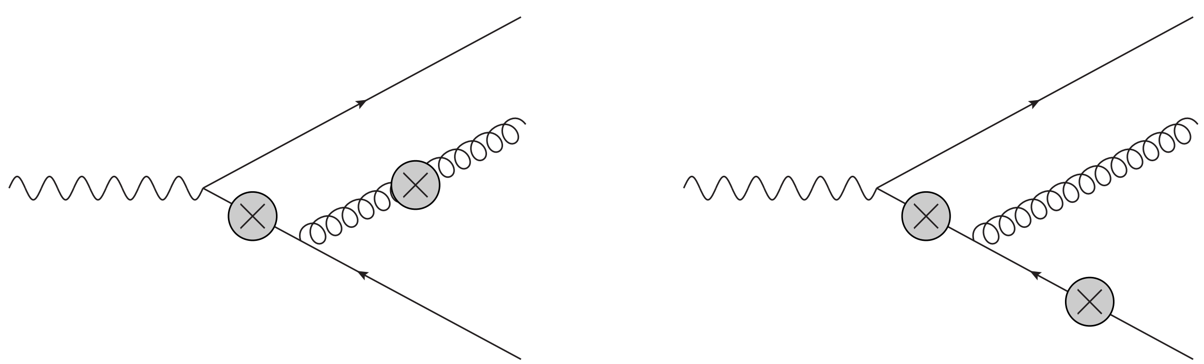
- go back to momentum space: special vertices still must be aligned along the cut



- at a cut: “propagator ⊗ special vertex ⊗ propagator” or “propagator” only; no special vertex anywhere else

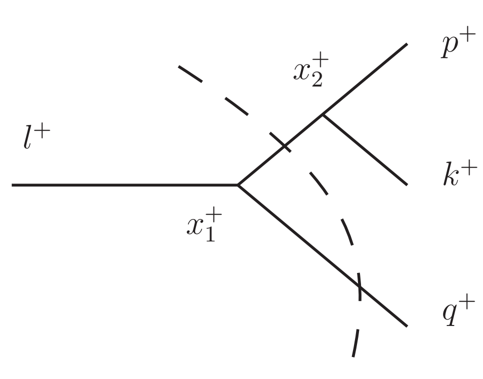
How does it help?

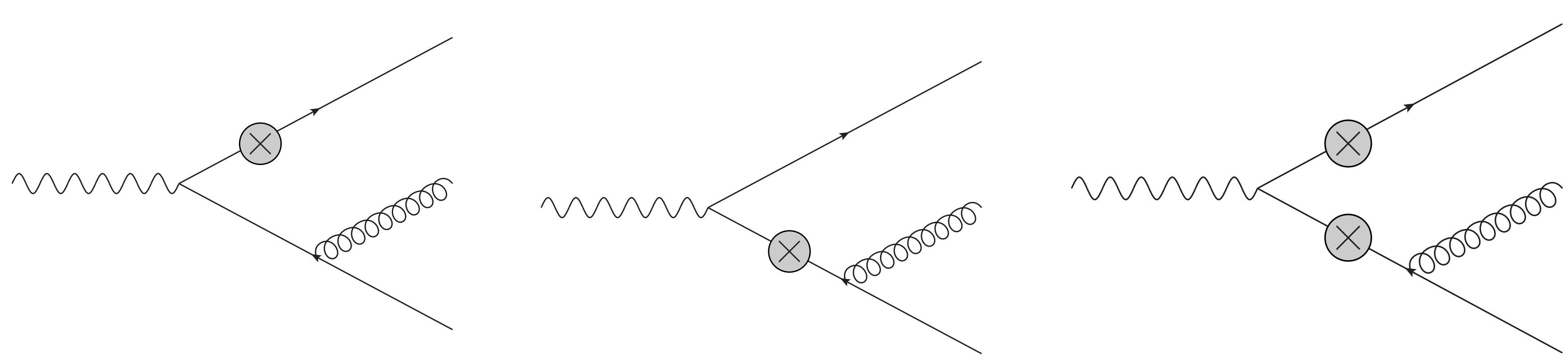
- evaluates 50% of possible momentum diagrams to zero



not possible for s-channel kinematics

- ….. but each cut contains still several diagrams



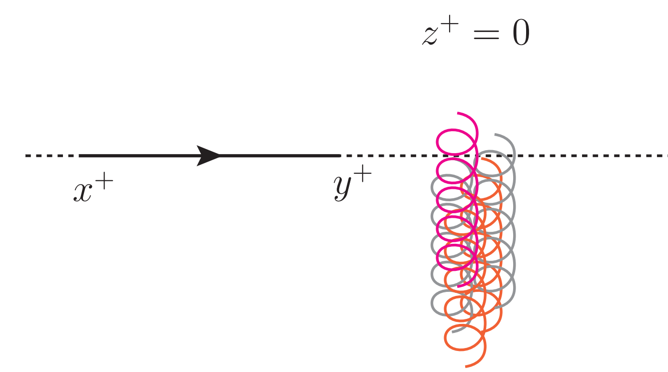


Configuration space knows more …
(partial) Fourier transform for complete propagator

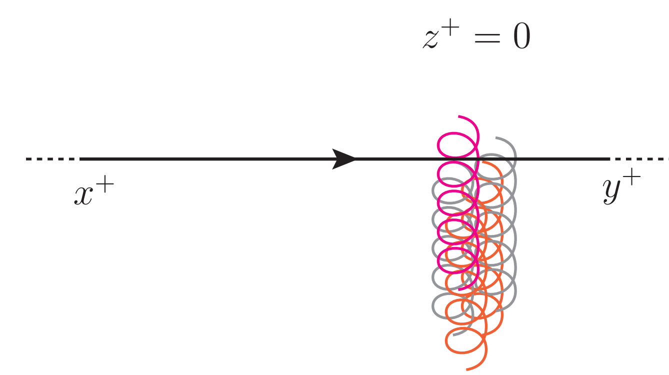
$$\int \frac{dp^-}{2\pi} \int \frac{dq^-}{2\pi} e^{-ip^-x^+} e^{iq^-y^+} \left[S^{(0)}_{F,il}(p)(2\pi)^4\delta^{(4)}(p-q) + S^{(0)}_{F,ij}(p) \cdot \tau_{F,jk}(p,q) \cdot S^{(0)}_{kl}(q) \right]$$

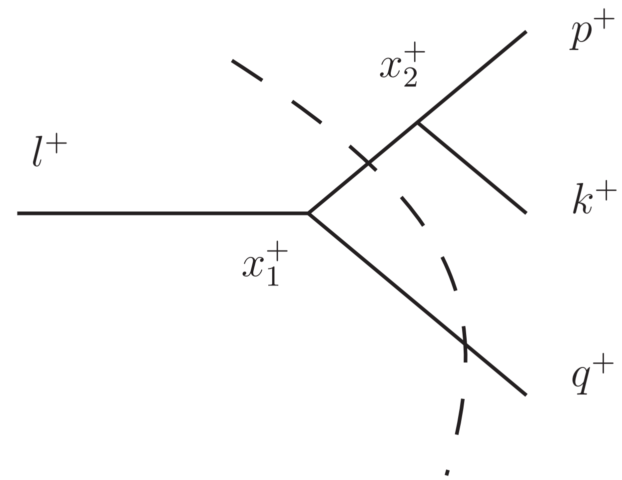
obtain free propagation for

- $x^+, y^+ < 0$ ("before interaction")
- $x^+, y^+ > 0$ ("after interaction")

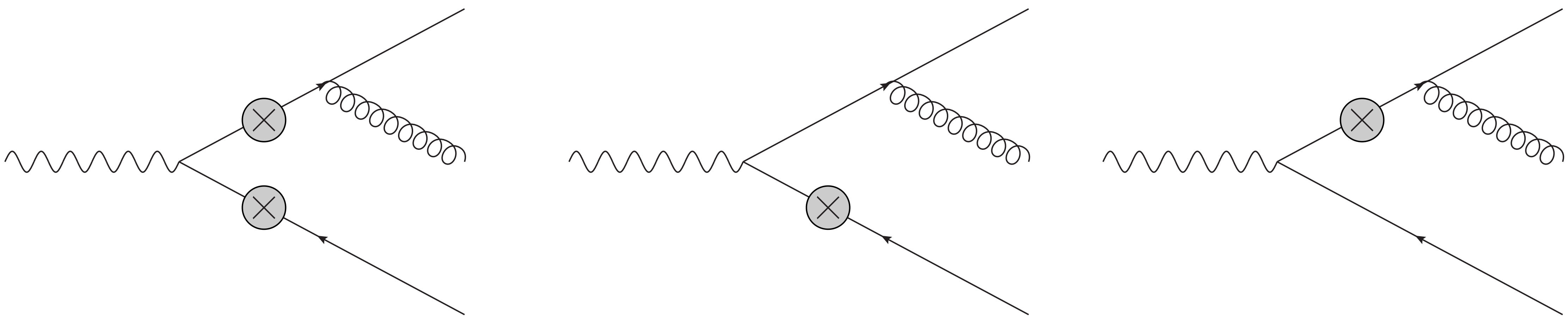


propagator proportional to complete Wilson line V (fermion) or U (gluon) if we cross light-cone time $z^+=0$
→ must pass through the cuts

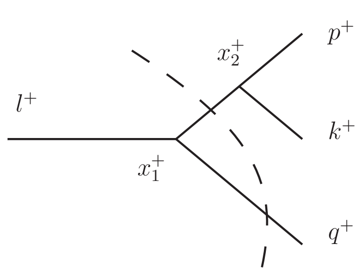


- for a single cut: 

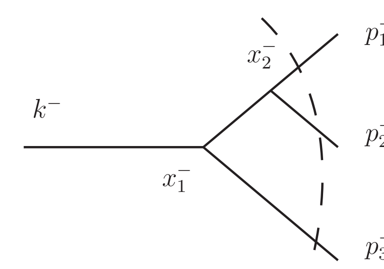
effectively adds up



- reality: more complicated due to mixing of different cuts



vs.



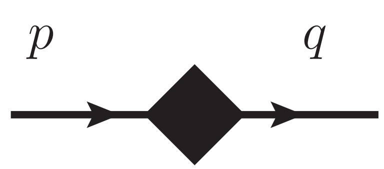
- crucial: positive plus momenta in all lines for tree diagrams

- allows to formulate a new set of effective ”Feynman rules”

New rules

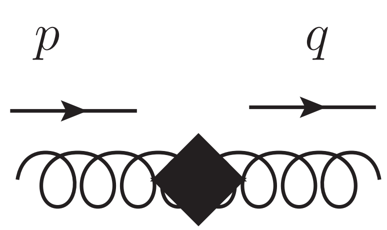
A. Determine zero light-cone time cuts of a given diagram

B. Place new vertices at these cuts



$$= \bar{\tau}_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\cdot \not{n}$$

$$\cdot \int d^2\boldsymbol{z}\, e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)V_{ij}(\boldsymbol{z}) - \theta(-p^+)V^\dagger_{ij}(\boldsymbol{z})\right\}$$



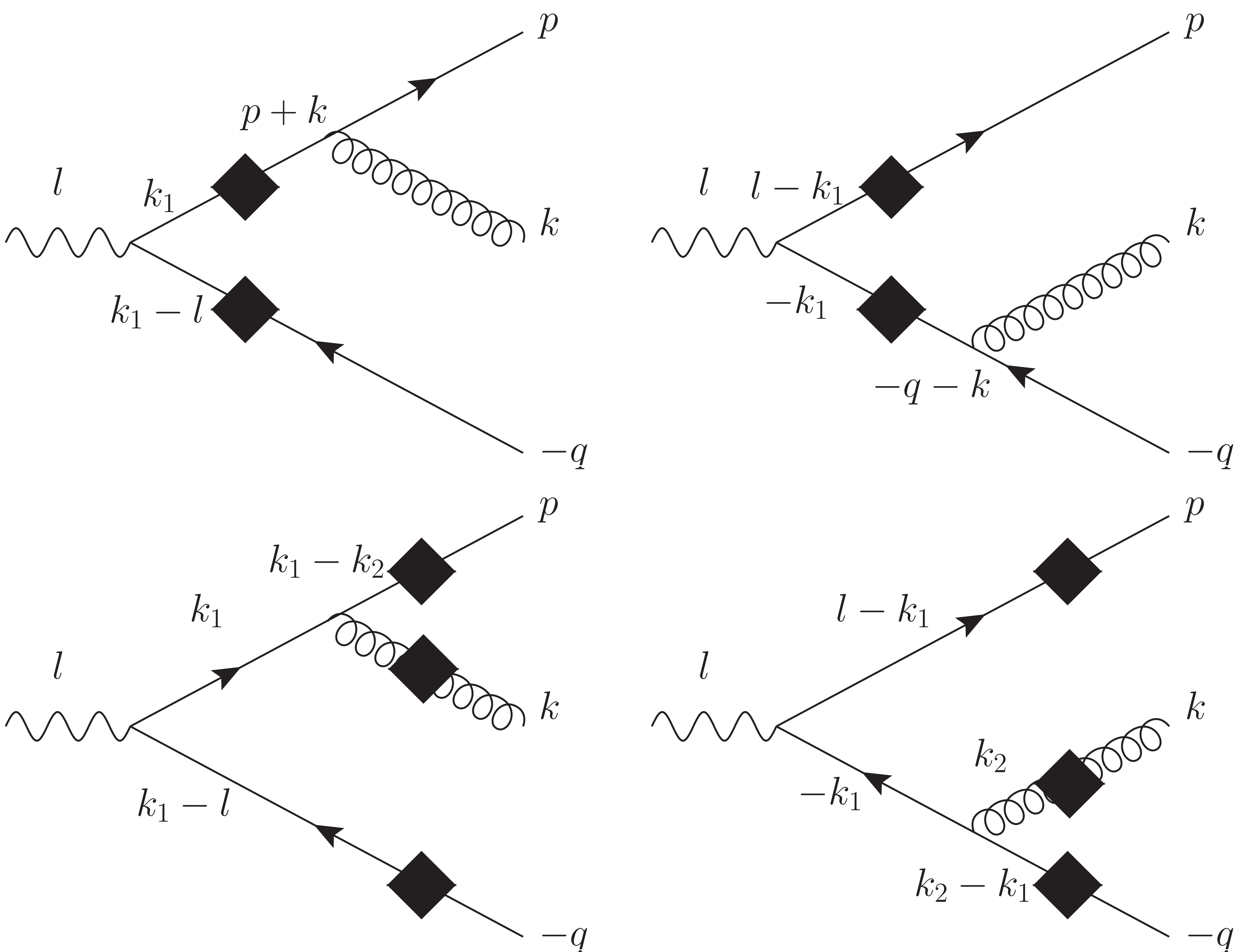
$$= \bar{\tau}_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\cdot(-2p^+)$$

$$\cdot \int d^2\boldsymbol{z}\, e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)U^{ab}(\boldsymbol{z}) - \theta(-p^+)\left(U^{ab}\right)^\dagger(\boldsymbol{z})\right\}$$

verified by explicit calculation for tree level diagrams; in general also extendable to loop diagrams …

First result: minimal set of amplitudes

(nothing new if you're used to work in coordinate space)



What do we win with new momentum space rules?

can use techniques explored in (conventional) Feynman diagram calculations

- loop integrals (d-dimensional, covariant) → won't talk about this today …. in general: complication due to Fourier factors remain

- **spinor helicity techniques** (calculate amplitudes not Xsec. + exploit helicity conservation in mass-less QCD) → compact expressions (→ for a different application to h.e.f. see **[van Hameren, Kotko, Kutak, 1211.0961]**)

A reminder from before we realised that …

Dirac traces from Computer Algebra Codes

- possible to express elements of Dirac trace in terms of scalar, vector and rank 2 tensor integrals
- Evaluation requires use of computer algebra codes;
  use 2 implementations:
  FORM [Vermaseren, math-ph/0010025] &
  Mathematica packages FeynCalc and FormLink
- result (3 partons) as coefficients of “basis”-functions $f_{(a)}$ and $h_{(a,b)}$;
  result lengthy (~100kB), but manageable
- currently working on further simplification through integration by parts relation between basis function (work in progress)

Spinor-helicity formalism

see *e.g.* **[Mangano, Parke; Phys. Rept. 200, 301 (1991)]** ,**[Dixon; hep-ph/9601359]**

$$u_\pm(k) = \frac{1 \pm \gamma_5}{2} u(p) \qquad v_\mp(k) = \frac{1 \pm \gamma_5}{2} v(p)$$

$$\bar{u}_\pm(k) = \bar{u}(k) \frac{1 \mp \gamma_5}{2} \qquad \bar{v}_\pm(k) = \bar{v}(k) \frac{1 \pm \gamma_5}{2} \,.$$

define spinors of **massless** momenta of definite helicity

$$|i^\pm\rangle \equiv |k_i^\pm\rangle \equiv u_\pm(k_i) = v_\mp(k_i)$$

$$\langle i^\pm| \equiv \langle k_i^\pm| \equiv \bar{u}_\pm(k_i) = \bar{v}_\mp(k_i)$$

appear a lot → short-hand notation

Essential IDs

$$\not{p} = \sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) \qquad \Rightarrow \qquad \not{p} = |p^+\rangle\langle p^+| + |p^-\rangle\langle p^-|$$

$$\begin{aligned}\bar{u}_+(k_i)u_+(k_j) &= \bar{u}(1-\gamma_5)(1+\gamma_5)u \qquad \Rightarrow \qquad \langle k_i^\pm|k_j^\pm\rangle = 0 \\ &= \bar{u}(1-\underbrace{\gamma_5^2}_{1})u = 0\end{aligned}$$

$$\bar{u}_\lambda(p)u_{-\lambda}(p) = \underbrace{2m}_{=0} = 0 \qquad \Rightarrow \qquad \langle k_i^\pm|k_i^\mp\rangle = 0$$

- chose spinor representation → evaluate brackets in terms of light-cone & transverse momenta

$$\langle k_i k_j \rangle = \sqrt{2k_i^- k_j^+} e^{i\phi_{k_i}} - \sqrt{2k_j^- k_i^+} e^{i\phi_{k_j}} = \sqrt{2k_i^+ k_j^+} \left(\frac{\boldsymbol{k}_i \cdot \boldsymbol{\epsilon}}{k_i^+} - \frac{\boldsymbol{k}_j \cdot \boldsymbol{\epsilon}}{k_j^+} \right)$$

$$= (k_i^+ k_j^+)^{-\frac{1}{2}} \left(k_j^+ |\boldsymbol{k}_i| e^{i\phi_{k_i}} - k_i^+ |\boldsymbol{k}_j| e^{i\phi_{k_j}} \right)$$

- usually: cumbersome → results in terms of brackets; evaluate numerically

- high energy factorisation: light-cone + transverse components reflect directly symmetry of problem

Gluons & photons …

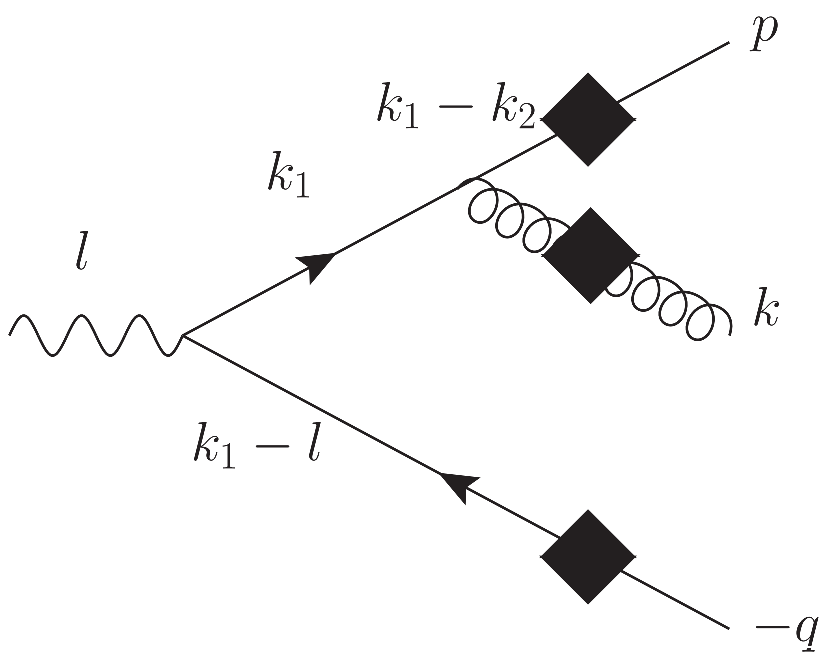
physical Hilbert space of massless vector boson up to Z_2 transform isomorphic to Hilbert space of massless spinor

$$\epsilon_\mu^{(\lambda=+)}(k,n) \equiv +\frac{\langle k^+|\gamma_\mu|n^+\rangle}{\sqrt{2}\langle n^-|k^+\rangle} = \left(\epsilon_\mu^{(\lambda=-)}(k,n)\right)^*$$

$$\epsilon_\mu^{(\lambda=-)}(k,n) \equiv -\frac{\langle k^-|\gamma_\mu|n^-\rangle}{\sqrt{2}\langle n^+|k^-\rangle} = \left(\epsilon_\mu^{(\lambda=+)}(k,n)\right)^*$$

polarisation sum = axial gauge; n arbitrary massless reference vector → identify with gauge vector A•n= 0

$$\sum_{\lambda=\pm} \epsilon_\mu^{(\lambda)}(k,n)\left(\epsilon_\mu^{(\lambda)}(k,n)\right)^* = -g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{k\cdot n}$$



Internal off-shell momenta

$$k^{\mu}_{1,2} = \bar{k}^{\mu}_{1,2} + \frac{k^2_{1,2}}{2k^+_{1,2}} n^{\mu}$$

$$\bar{k}^{\mu}_{1,2} = k^+_{1,2} \bar{n}^{\mu} + \frac{\boldsymbol{k}^2_{1,2}}{2k^+_{1,2}} n^{\mu} + k^{\mu}_{1,2;t}$$

- use: plus momenta always conserved & decompose into on-shell term + off-shell term ~ light-cone vector n

- n appears a lot & in most cases can use $n^2=0$ also $\langle np \rangle = \sqrt{2p^+}$

Calculation …

- can be done in principle by hand …

- implementation in `Mathematica` relatively easy (using own code based on `FeynCalc` ideas …)

- advantage: expressions directly available for numerical evaluation + avoid silly mistakes

$$\frac{d\sigma^{T,L}}{d^2\boldsymbol{p}\,d^2\boldsymbol{k}\,d^2\boldsymbol{q}\,dz_1 dz_2} = \frac{\alpha_s\alpha_{em}e_f^2 N_c^2}{z_1 z_2 z_3 (2\pi)^2}\prod_{i=1}^{3}\prod_{j=1}^{3}\int\frac{d^2\boldsymbol{x}_i}{(2\pi)^2}\int\frac{d^2\boldsymbol{x}'_j}{(2\pi)^2}e^{i\boldsymbol{p}(\boldsymbol{x}_1-\boldsymbol{x}'_1)+i\boldsymbol{q}(\boldsymbol{x}_2-\boldsymbol{x}'_2)+i\boldsymbol{k}(\boldsymbol{x}_3-\boldsymbol{x}'_3)}$$

$$\Big\langle (2\pi)^4\Big[\Big(\delta^{(2)}(\boldsymbol{x}_{13})\delta^{(2)}(\boldsymbol{x}_{1'3'})\sum_{h,g}\psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12})\psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\}\leftrightarrow\{2,2'\}\Big)N^{(4)}(\boldsymbol{x}_1,\boldsymbol{x}'_1,\boldsymbol{x}'_2,\boldsymbol{x}_2)$$

$$+\Big(\delta^{(2)}(\boldsymbol{x}_{23})\delta^{(2)}(\boldsymbol{x}_{1'3'})\sum_{h,g}\psi^{T,L}_{2;h,g}(\boldsymbol{x}_{12})\psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\}\leftrightarrow\{2,2'\}\Big)N^{(22)}(\boldsymbol{x}_1,\boldsymbol{x}'_1|\boldsymbol{x}'_2,\boldsymbol{x}_2)\Big]$$

$$+(2\pi)^2\Big[\delta^{(2)}(\boldsymbol{x}_{13})\sum_{h,g}\psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12})\psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'},\boldsymbol{x}_{2'3'})N^{(24)}(\boldsymbol{x}_{3'},\boldsymbol{x}_{1'}|\boldsymbol{x}_{2'},\boldsymbol{x}_2,\boldsymbol{x}_1,\boldsymbol{x}_{3'}) + \{1\}\leftrightarrow\{2\}$$

$$+\delta^{(2)}(\boldsymbol{x}_{1'3'})\sum_{h,g}\psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13},\boldsymbol{x}_{23})\psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'})N^{(24)}(\boldsymbol{x}_1,\boldsymbol{x}_3|\boldsymbol{x}_{2'},\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_{1'}) + \{1'\}\leftrightarrow\{2'\}\Big]$$

$$+\sum_{h,g}\psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13},\boldsymbol{x}_{23})\psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'},\boldsymbol{x}_{2'3'})N^{(44)}(\boldsymbol{x}_1,\boldsymbol{x}_{1'},\boldsymbol{x}_{3'},\boldsymbol{x}_3|\boldsymbol{x}_3,\boldsymbol{x}_{3'},\boldsymbol{x}_{2'},\boldsymbol{x}_2)\Big\rangle_{A^+},$$

$$N^{(4)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4)\equiv$$
$$\equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} - S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)},$$

$$N^{(22)}(\boldsymbol{x}_1,\boldsymbol{x}_2|\boldsymbol{x}_3,\boldsymbol{x}_4)\equiv$$
$$\equiv\Big[S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)}-1\Big]\Big[S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)}-1\Big]$$

$$N^{(24)}(\boldsymbol{x}_1,\boldsymbol{x}_2|\boldsymbol{x}_3,\boldsymbol{x}_4,\boldsymbol{x}_5,\boldsymbol{x}_6)\equiv$$
$$1 + S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)}S^{(4)}_{(\boldsymbol{x}_3\boldsymbol{x}_4\boldsymbol{x}_5\boldsymbol{x}_6)}$$
$$- S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)}S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_6)} - S^{(2)}_{(\boldsymbol{x}_4\boldsymbol{x}_5)},$$

$$N^{(44)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4|\boldsymbol{x}_5,\boldsymbol{x}_6,\boldsymbol{x}_7,\boldsymbol{x}_8)\equiv$$
$$\equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)}S^{(4)}_{(\boldsymbol{x}_5\boldsymbol{x}_6\boldsymbol{x}_7\boldsymbol{x}_8)}$$
$$- S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_4)}S^{(2)}_{(\boldsymbol{x}_5\boldsymbol{x}_8)} - S^{(2)}_{(\boldsymbol{x}_2\boldsymbol{x}_3)}S^{(2)}_{(\boldsymbol{x}_6\boldsymbol{x}_7)}$$

full (large N_c) result in terms of these wave functions + target correlators

$$S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} \equiv \frac{1}{N_c}\mathrm{tr}\Big[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)\Big]$$

$$S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} \equiv \frac{1}{N_c}\mathrm{tr}\Big[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)V(\boldsymbol{x}_3)V^\dagger(\boldsymbol{x}_4)\Big]$$

wave functions & amplitudes

$$\psi^L_{j,hg} = -2\sqrt{2}QK_0\left(QX_j\right)\cdot a^{(L)}_{j,hg}, \qquad j=1,2$$

$$\psi^T_{j,hg} = 2ie^{\mp i\phi_{\boldsymbol{x}_{12}}}\sqrt{(1-z_3-z_j)(z_j+z_3)}QK_1\left(QX_j\right)\cdot a^{\pm}_{j,hg} \qquad j=1,2$$

$$\psi^L_{3,hg} = 4\pi iQ\sqrt{2z_1z_2}K_0\left(QX_3\right)\left(a^{(L)}_{3,hg}+a^{(L)}_{4,hg}\right),$$

$$\psi^T_{3,hg} = -4\pi Q\sqrt{z_1z_2}\frac{K_1\left(QX_3\right)}{X_3}\left(a^{\pm}_{3,hg}+a^{\pm}_{4,hg}\right).$$

symmetry relation between amplitudes

$$a^{T,L}_{k+1,hg} = -a^{T,L}_{k,-hg}(\{p,\boldsymbol{x}_1\}\leftrightarrow\{q,\boldsymbol{x}_2\}), \qquad k = \boxed{1,3}$$

$$a^{T,L}_{j,hg} = a^{(-T,L)*}_{j,-h-g}, \quad j=1,\ldots,4.$$

longitudinal photon

$$a_{1,++}^{(L)} = -\frac{(z_1 z_2)^{3/2}\,(z_1+z_3)}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a_{1,-+}^{(L)} = -\frac{\sqrt{z_1}\, z_2^{3/2}\,(z_1+z_3)^{\,2}}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a_{3,++}^{(L)} = \frac{z_1 z_2}{|\boldsymbol{x}_{13}| e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a_{3,-+}^{(L)} = \frac{z_2(1-z_2)}{|\boldsymbol{x}_{13}| e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

transverse photon

for precise def. see paper

take away message:

very compact expressions

$$a^{(+)}_{1,++} = -\frac{(z_1 z_2)^{3/2}}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{1,+-} = \frac{\sqrt{z_1}(z_2)^{\frac{3}{2}}(z_1 + z_3)}{z_1 e^{i\theta_k}|\boldsymbol{k}| - z_3 e^{i\theta_p}|\boldsymbol{p}|},$$

$$a^{(+)}_{1,-+} = \frac{\sqrt{z_1 z_2}(z_1 + z_3)^2}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{1,--} = \frac{z_1^{3/2}\sqrt{z_2}(z_1 + z_3)}{z_3 e^{i\theta_p}|\boldsymbol{p}| - z_1 e^{i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{3,++} = \frac{z_1 z_2(z_2 z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}} + z_3|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}} - z_1 z_2|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}})}{(z_1 + z_3)|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,+-} = \frac{z_2^2(z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}} - z_1|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}})}{|\boldsymbol{x}_{13}|e^{i\phi_{\boldsymbol{x}_{13}}}},$$

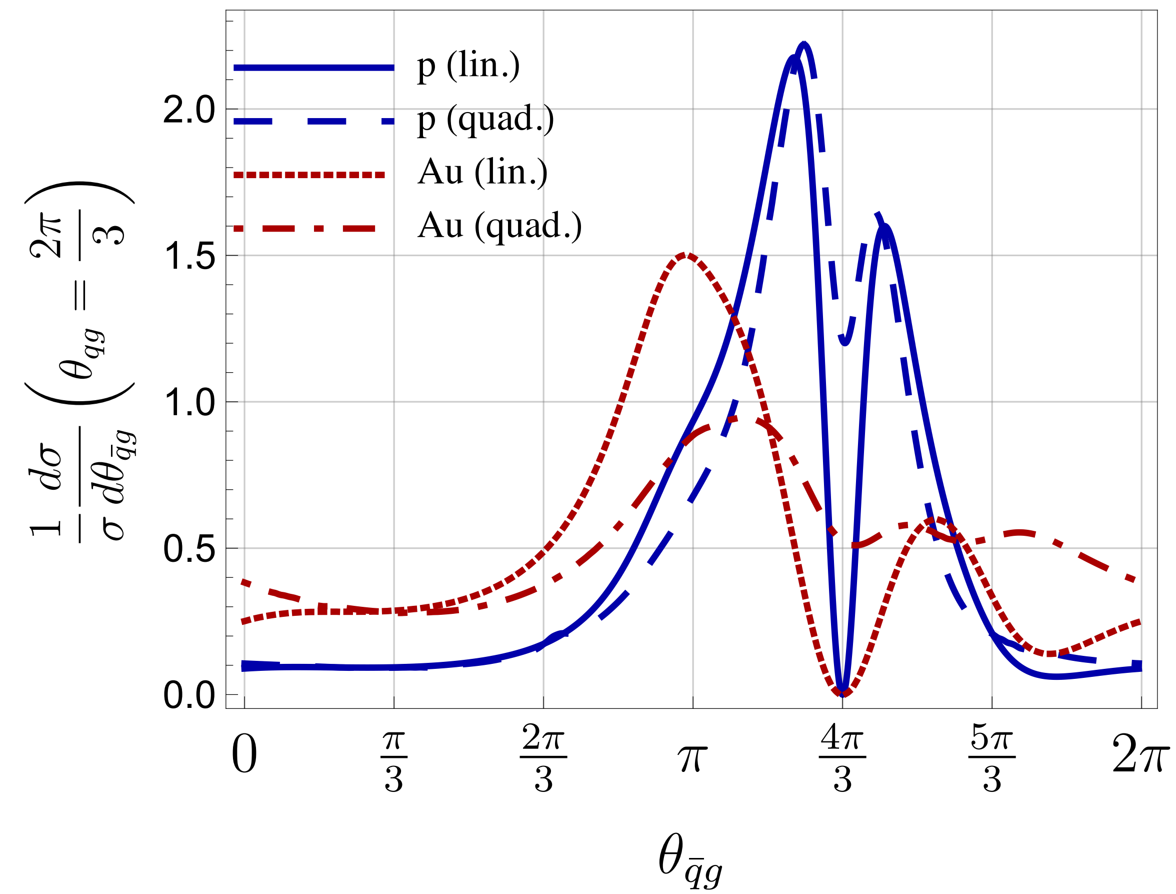
$$a^{(+)}_{3,-+} = -\frac{z_2(z_1 + z_3)(z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}} - z_1|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}})}{|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,--} = \frac{z_1 z_2(z_1|\boldsymbol{x}_{12}|e^{-i\phi_{\boldsymbol{x}_{12}}} - z_3|\boldsymbol{x}_{23}|e^{-i\phi_{\boldsymbol{x}_{23}}})}{|\boldsymbol{x}_{13}|e^{i\phi_{\boldsymbol{x}_{13}}}}.$$

What is it good for?

First phenomenology on partonic level: angular decorrelation

- here: density effects through large nucleus (Au)
- fix one (transverse) angle at $\theta_{qg} = \frac{2\pi}{3}$, vary θ_{qg}



- no p_T from the nucleus/proton $Q_S \to 0$ $\Rightarrow$ Merceces-Benz-star configuration dominant
- higher correlators: Gaussian-approximation expanded to quadratic order in 2-point correlator (model)

- 3 partons → extra handle on quadrupole in addition to 2 partons (linear+ quadratic; more angles to modify)

- phase space: 3 forward hadrons might be difficult at EIC (maybe LHeC?)→ still lack detailed numerical study at hadronic level

- a different perspective: do we need a matrix element generator for high parton densities? (*e.g.* something like **`Madgraph`**) → capability to calculate tree-level amplitudes effectively would be an essential step in this direction

Outlook: energy loss

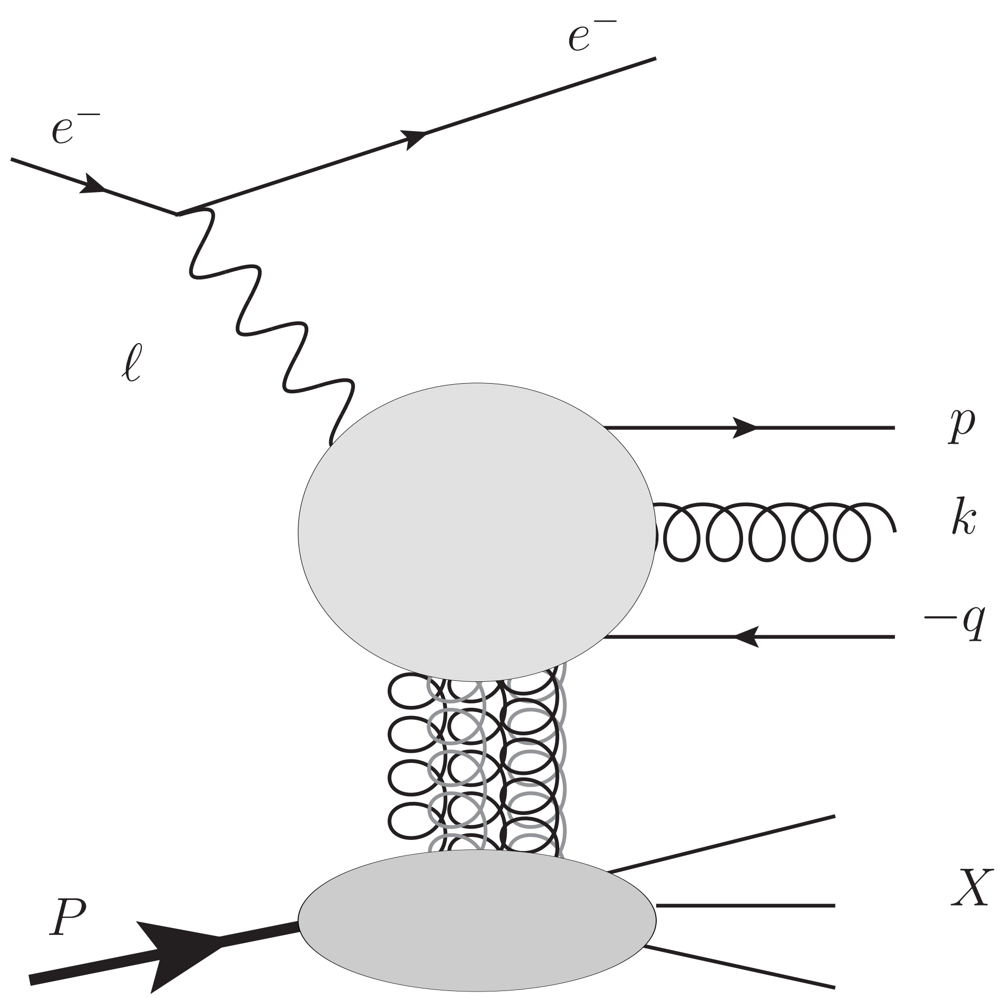
$$z\frac{dI}{dz} \equiv \frac{\frac{d\sigma^{a+A\to a+g+X}}{dy\,dy'\,d^2p_t}}{\frac{d\sigma^{a+A\to a+X}}{dy\,d^2p_t}}$$

- how much energy does the photon lose while interacting with a dense nucleus?

in pA collisions: similar mechanism used to understand energy loss in cold nuclear matter (in contrast to Quark Gluon Plasma)

[Neufeld,Vitev,Zhang;1010.3708]; [Liou, Mueller; 1402.1647], [Munier, Peigne, Petreska, 1603.01028]

Outlook: energy loss



- DIS: Allow to make these ideas more precise
- Can determine energy of initial photon
- energy loss = energy carried away by unobserved hadrons in *e.g.* di-hadron event

Summary:

- to detect high gluon density effects, observables directly sensitive to such effects should help ("evolution only" might require too much phase space)

- possible to transfer obvious simplifications in configuration space to momentum space calculations → access to momentum space techniques

- helicity spinor formalism can greatly simplify calculations within high energy factorisation

Gracias!