

TMDs from parton branching method at LO and NLO and first applications

Hannes Jung (DESY)

- Why TMDs are needed
 - TMDs for hadron-hadron collisions
- New developments (together with F. Hautmann, A. Lelek, V. Radescu, R. Zlebcik)
 - parton branching algorithm to solve evolution equations
 - benchmark tests
 - advantages for integrated PDFs
 - determination of TMD densities at LO and NLO with xFitter
- Applications: DY production using TMDs

TMDs – what is it ?

- TMDs (Transverse Momentum Dependent parton distribution)
 - at very small transverse momenta
 - typically for very small q_t in DY production, or semi-inclusive DIS
 - at very small x – un-integrated PDFs
 - essentially only gluon densities (CCFM, BFKL etc)
 - new approach to cover all transverse momenta from small k_t to large k_t as well as to cover all x and all μ^2
 - parton branching method (described here)
 - for an overview of different approaches and state-of-the-art discussion see
R. Angeles-Martinez et al
Transverse momentum dependent (TMD) parton distribution functions: status and prospects. Acta Phys. Polon., B46(12):2501–2534, 07 2015, arXiv 1507.05267

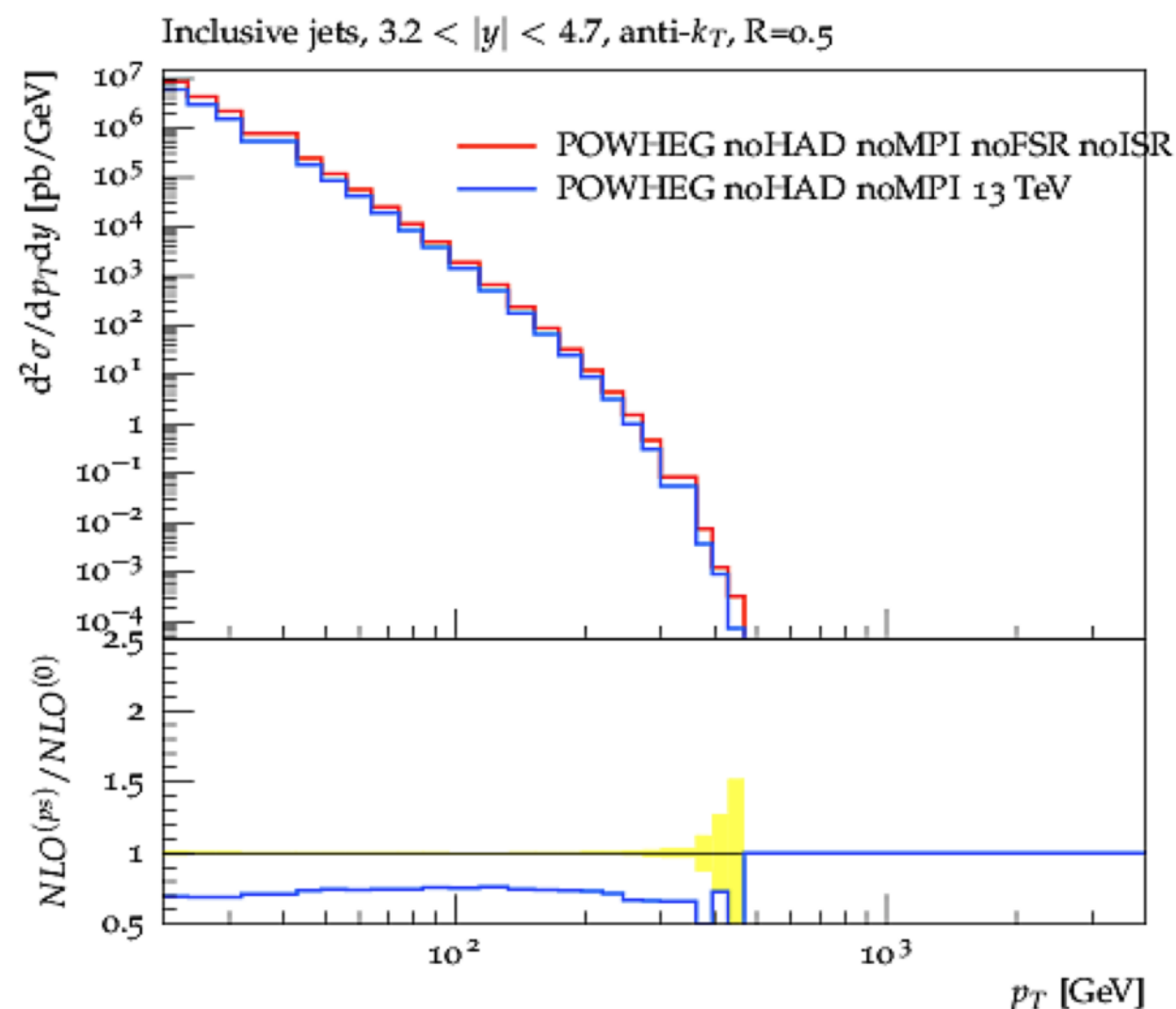
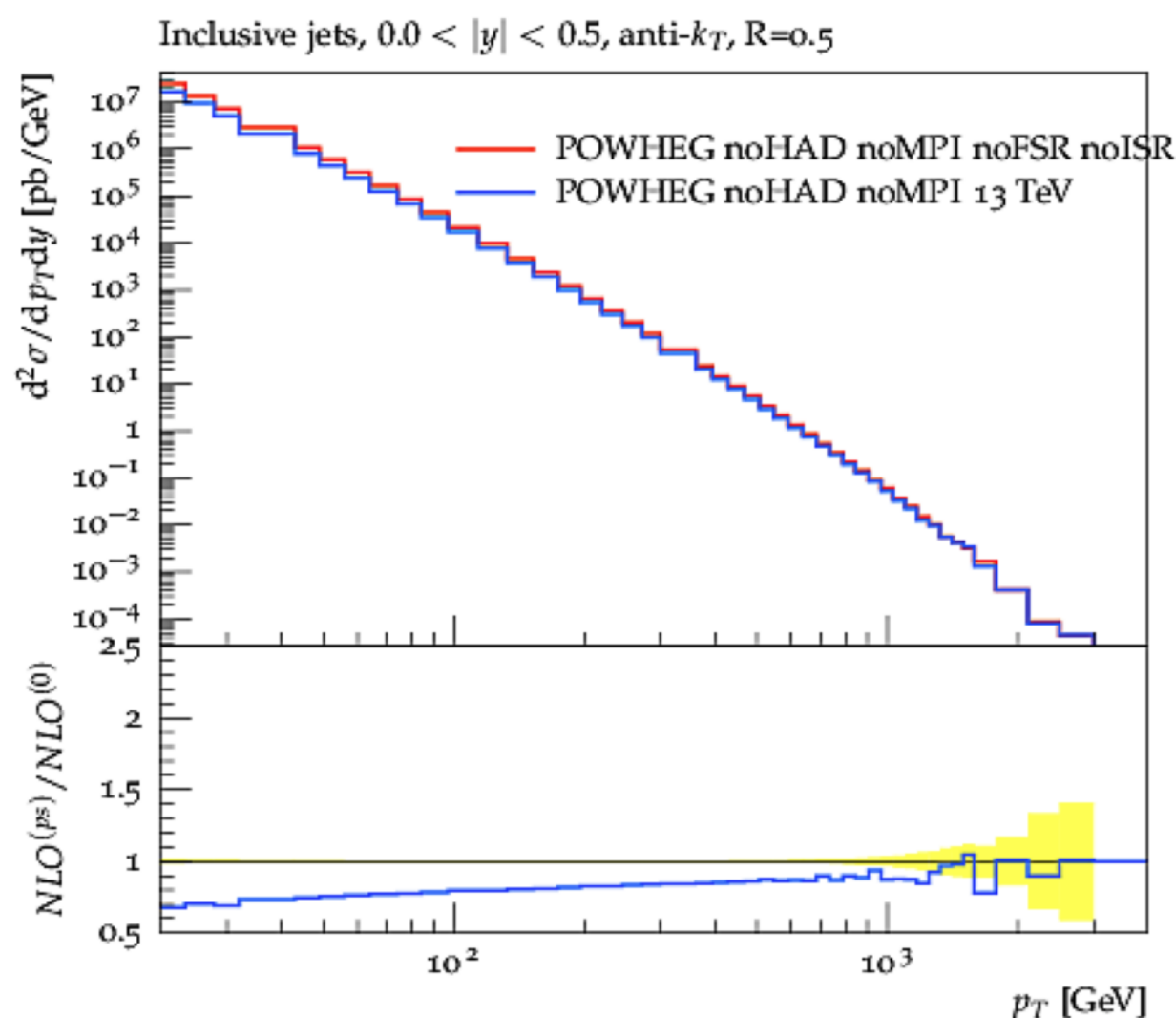
Why TMDs are needed even at large scales ?

- use NLO+PS to calculate:

Approach described in: S. Dooling et al
Phys.Rev., D87:094009, 2013.

$$K^{PS} = \frac{N_{NLO-MC}^{(ps)}}{N_{NLO-MC}^{(0)}}$$

- Corrections to be applied to fixed order NLO calculations:
 - kinematic effects: TMDs !
 - radiation outside of jet-cone



TMDs – how to determine ?

- Transverse momentum effects are naturally coming from intrinsic k_t and parton showers
- TMD effects can be significant in all distributions, even (for inclusive or semi-inclusive distributions) at large p_t
- **New: parton branching method**
 - perform evolution using a parton branching method
 - determine integrated PDF from parton branching solution of evolution equation
 - check consistency with standard evolution on integrated PDFs
 - at LO, NLO and NNLO
 - determine TMD:
 - since each branching is generated explicitly, energy-momentum conservation is fulfilled and transverse momentum distributions can be obtained

for similar approaches see also:

W. Placzek, K. J. Golec-Biernat, S. Jadach, M. Skrzypek. Acta Phys. Polon., B38:2357–2368, 2007.

H. Tanaka. Prog. Theor. Phys., 110:963–973, 2003.

How to obtain TMDs – the parton branching method

- F. Hautmann, H. Jung, A. Lelek, V. Radescu, R. Zlebcik.
Soft-gluon resolution scale in QCD evolution equations.
arXiv 1704.01757, DESY 16-174.

DGLAP evolution – solution with parton branching method

- differential form:
$$\mu^2 \frac{\partial}{\partial \mu^2} f(x, \mu^2) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z}, \mu^2\right)$$

$$\Delta_s(\mu^2) = \exp \left(- \int^{\mu^2} dz \int_{\mu_0^2}^{\mu'^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P^{(R)}(z) \right)$$

- differential form using f/Δ_s with

$$\mu^2 \frac{\partial}{\partial \mu^2} \frac{f(x, \mu^2)}{\Delta_s(\mu^2)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{P^{(R)}(z)}{\Delta_s(\mu^2)} f\left(\frac{x}{z}, \mu^2\right)$$

- integral form

$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$


 no – branching probability from μ_0^2 to μ^2

DGLAP evolution – solution with parton branching method

$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

- solve integral equation via iteration:

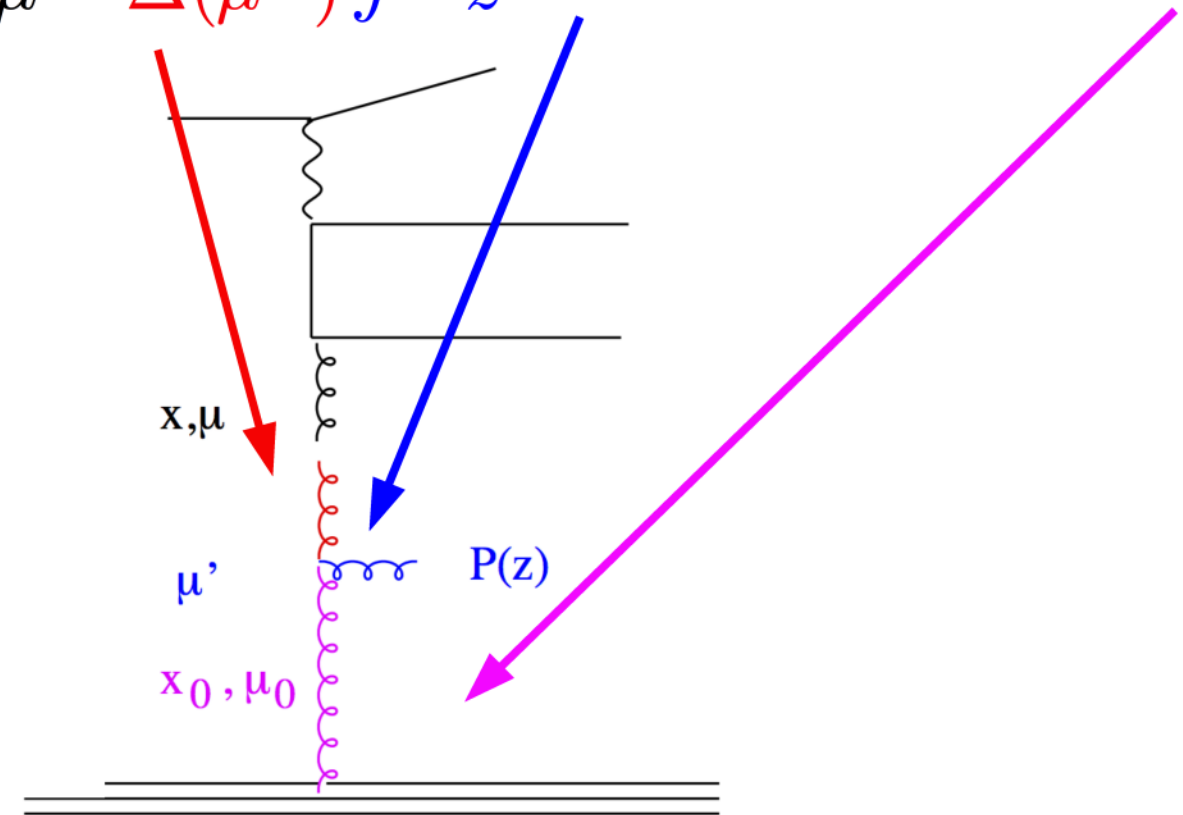
$$f_0(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2)$$

from t' to t
w/o branching

branching at t'

from t_0 to t'
w/o branching

$$f_1(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int \frac{dz}{z} P^{(R)}(z) f(x/z, \mu_0^2) \Delta(\mu'^2)$$



Evolution equation and parton branching method

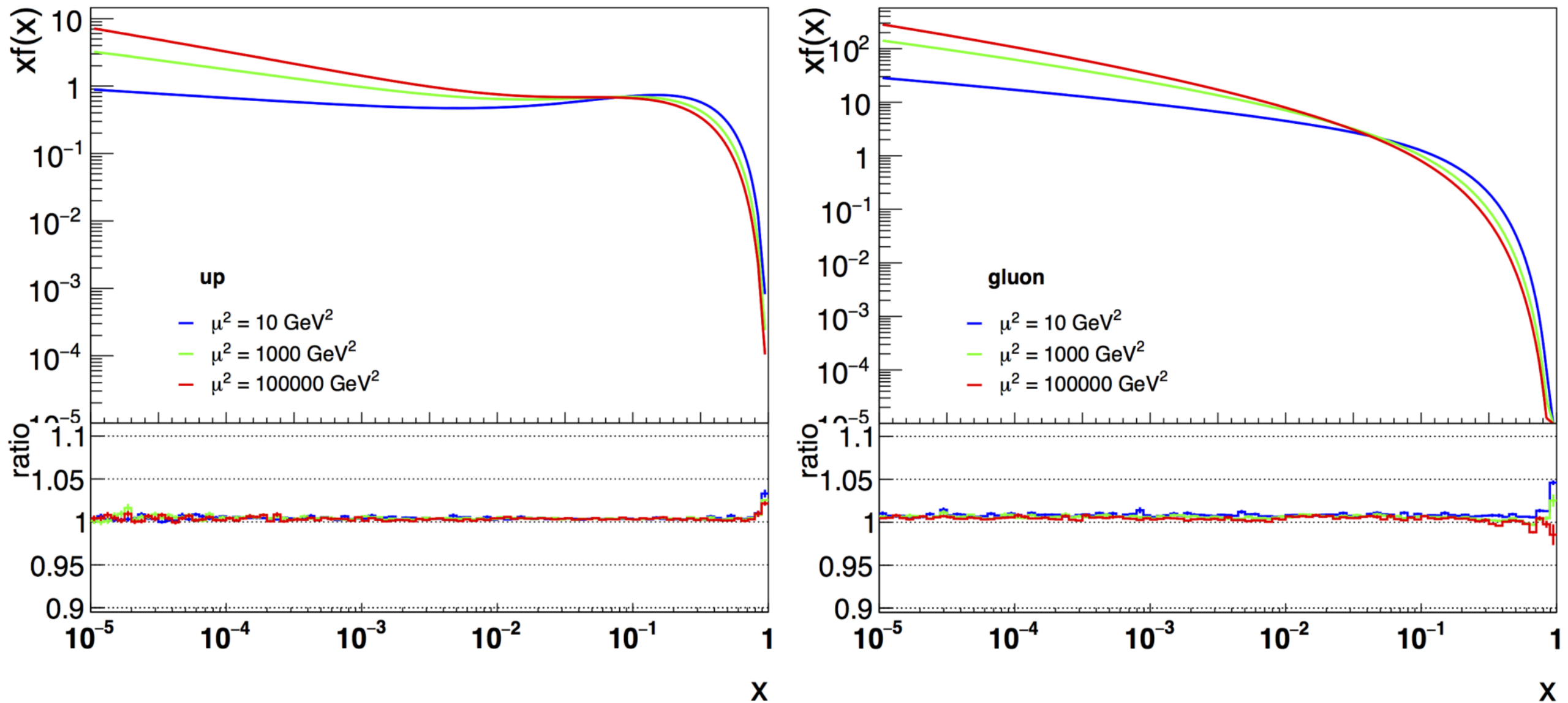
- use momentum-weighted PDFs: $xf(x,t)$

$$xf_a(x, \mu^2) = \Delta_a(\mu^2) xf_a(x, \mu_0^2) + \sum_b \int_{\mu_0}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_s, z) \frac{x}{z} f_b\left(\frac{x}{z}, \mu'^2\right)$$

- with $P_{ab}^{(R)}(\alpha_s(t'), z)$ real emission probability (without virtual terms)
 - z_M introduced to separate real from virtual and non-emission probability
- make use of momentum sum rule to treat virtual corrections
 - use Sudakov form factor to treat non-resolvable and virtual corrections

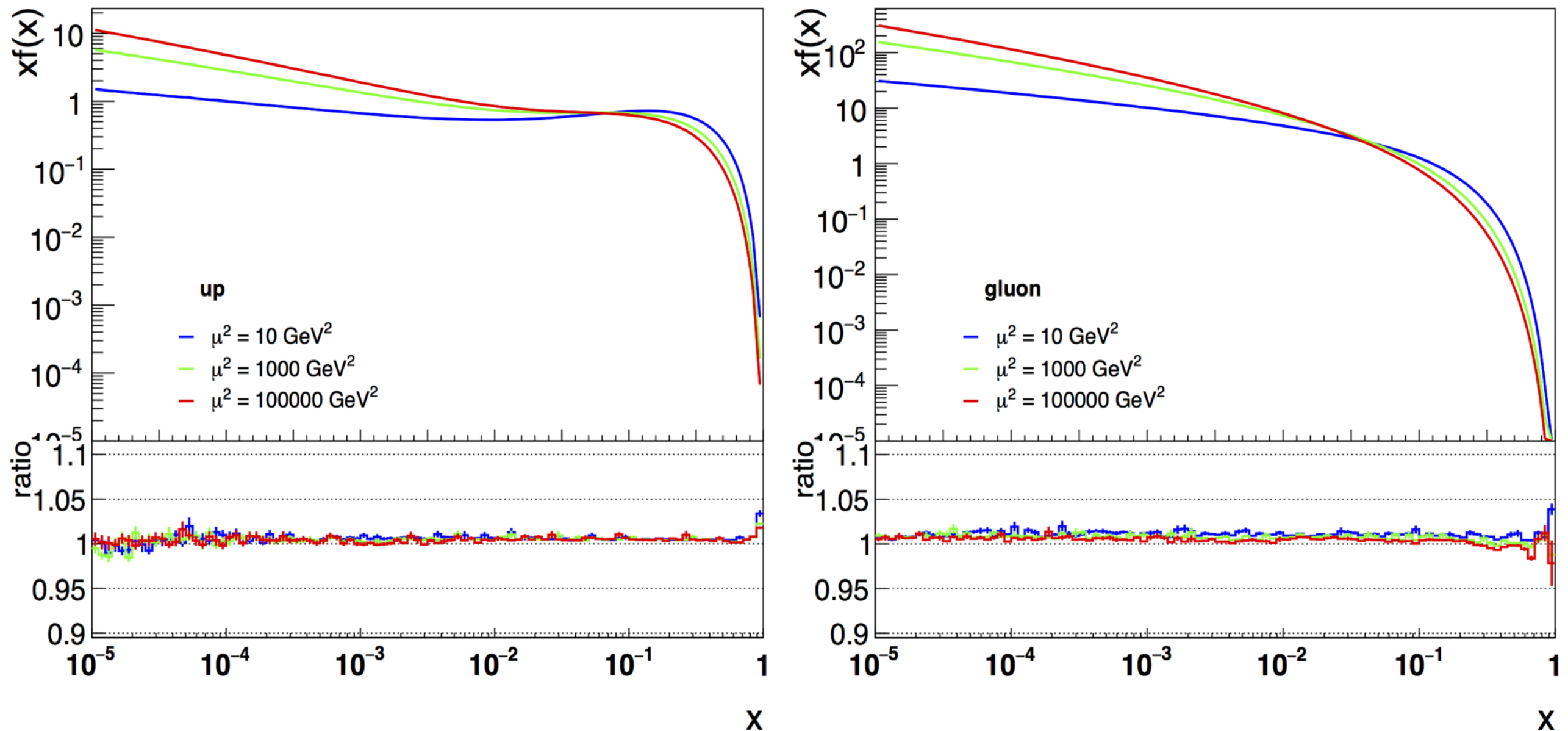
$$\Delta_a(z_M, \mu^2, \mu_0^2) = \exp \left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz z P_{ba}^{(R)}(\alpha_s), z \right)$$

Validation of method with semi-analytic result at LO



- Very good agreement with LO – semi-analytic method (QCDnum) over all x and μ^2

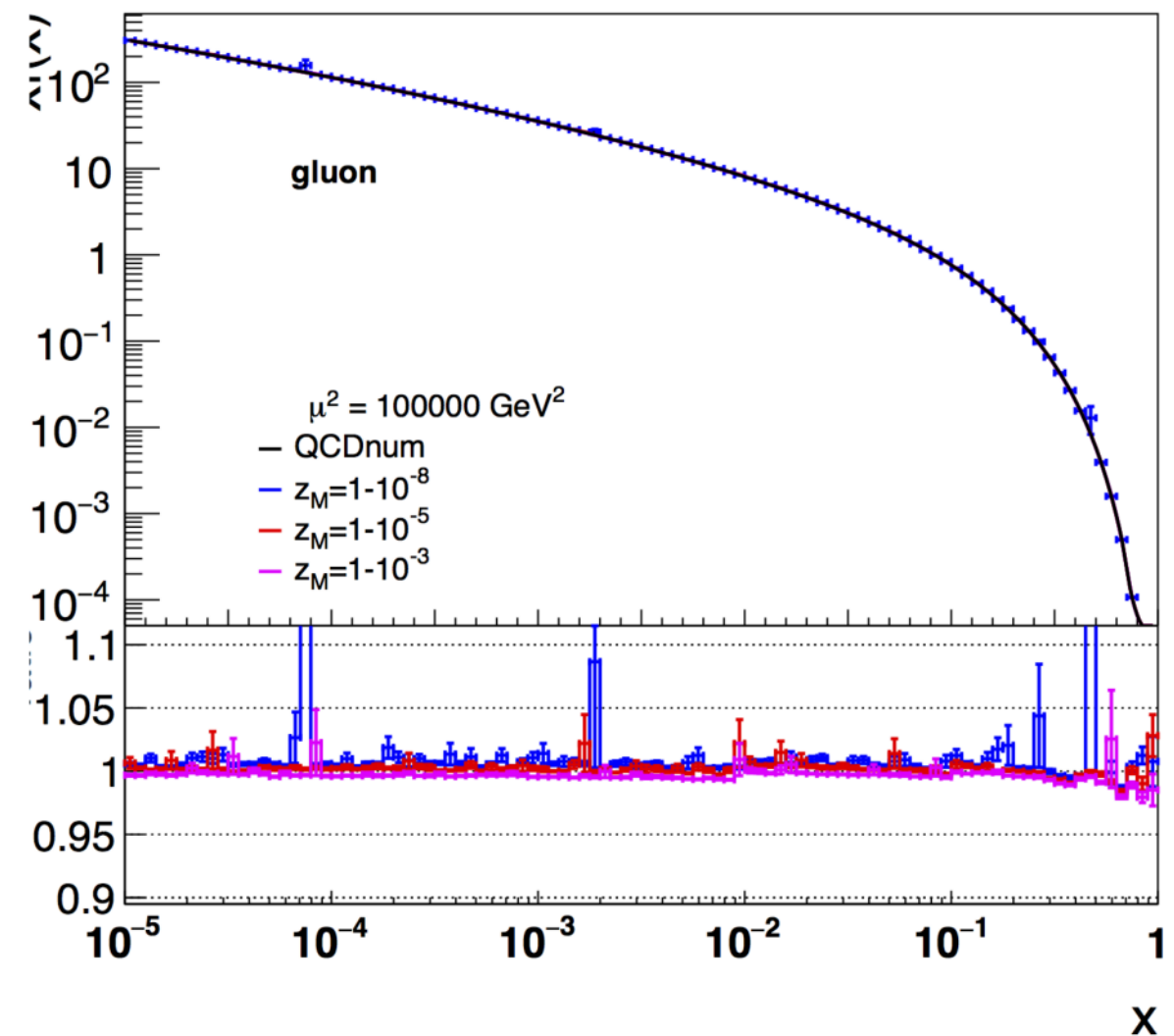
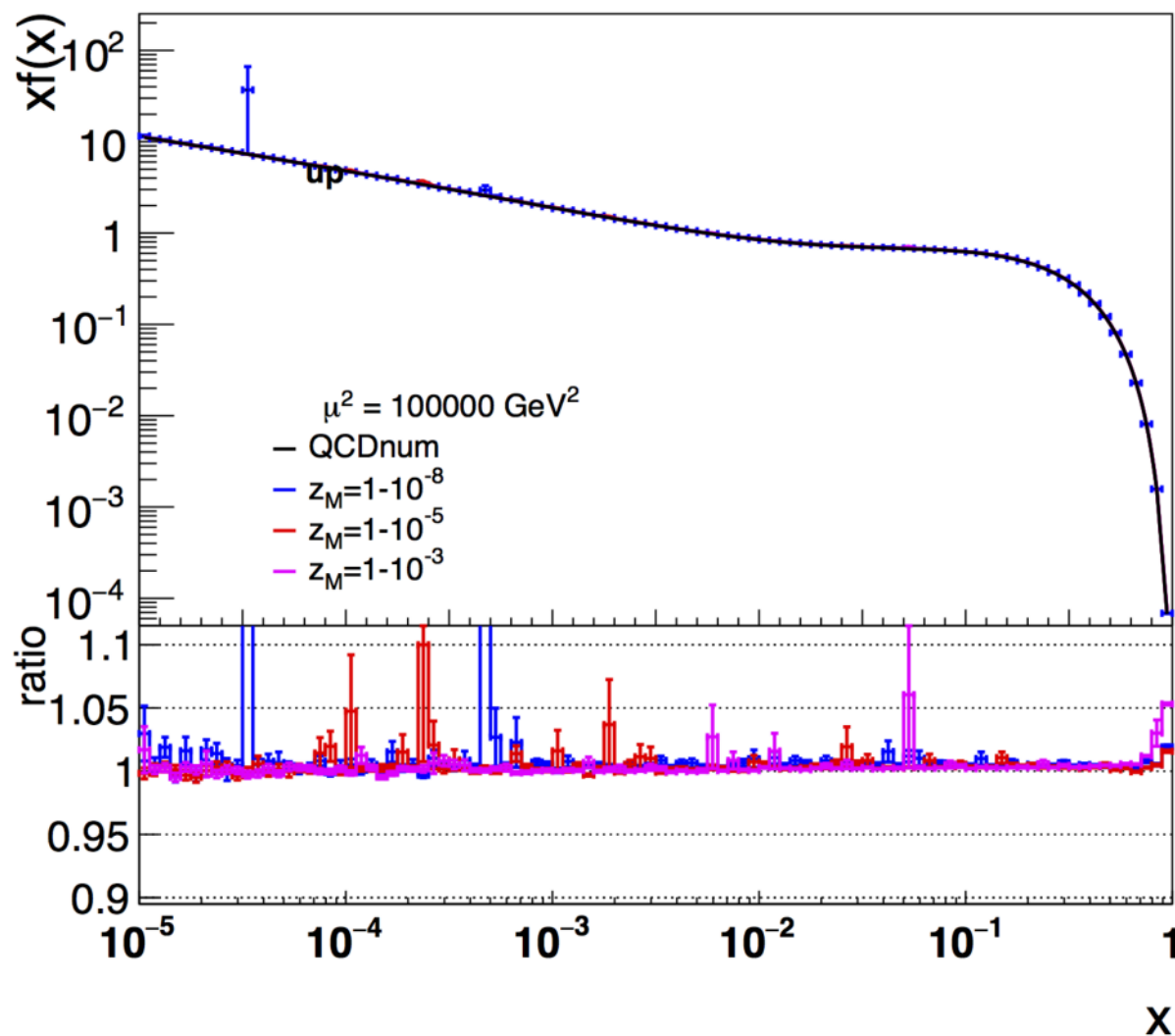
Validation of method with semi-analytic result at **NLO**



- Very good agreement with **NLO** - QCDnum over all x and μ^2
 - the same approach work also at **NNLO** !

Resolvable branching – at LO and NLO

- Investigate dependence on z_M : separate resolvable from virtual and non-resolvable branchings



- for large enough z_M : results are stable, both at LO and NLO (shown)
- Sudakov treats non-resolvable and virtual branchings to all orders !

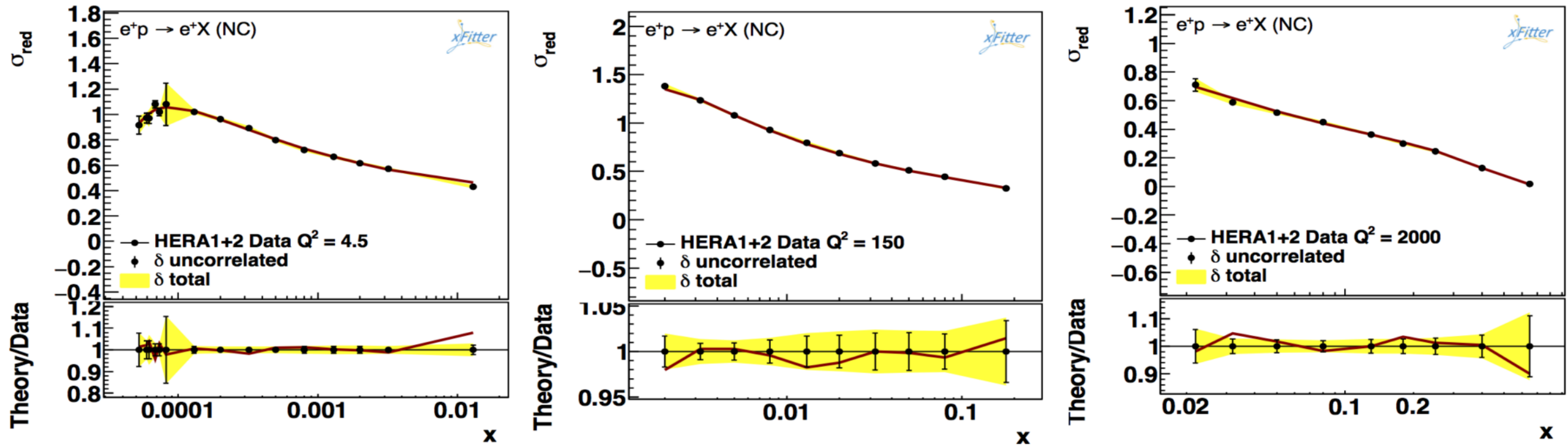
Parton branching method in xFitter

- Determine starting distribution

A. Lelek et al REF 2016

$$\begin{aligned}
 x f_a(x, \mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b(x'', \mu^2) \delta(x'x'' - x) \\
 &= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'}, \mu^2\right)
 \end{aligned}$$

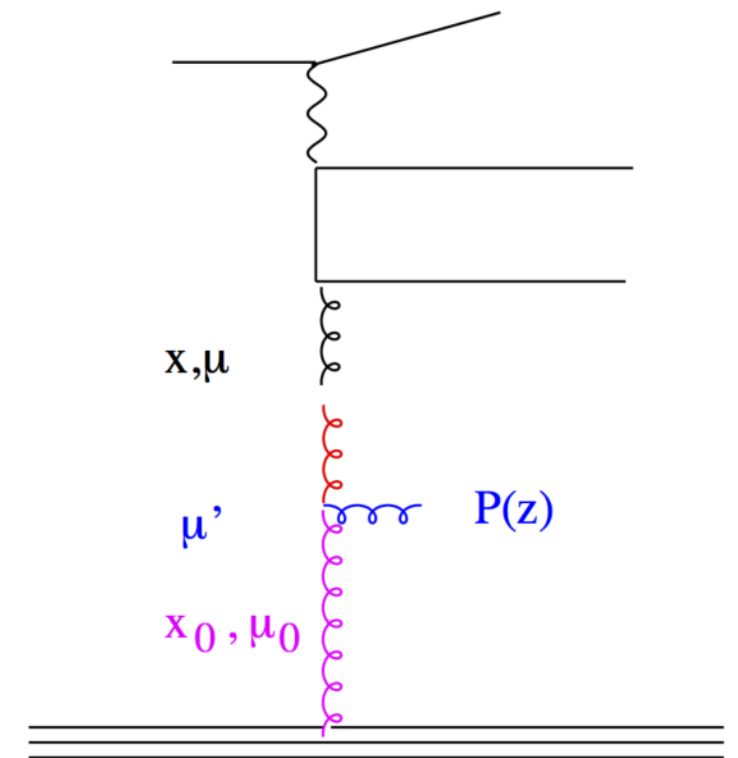
- fit to HERA data (using xFitter) with $Q^2 \geq 3.5 \text{ GeV}^2$ gives $\chi^2/ndf \sim 1.2$



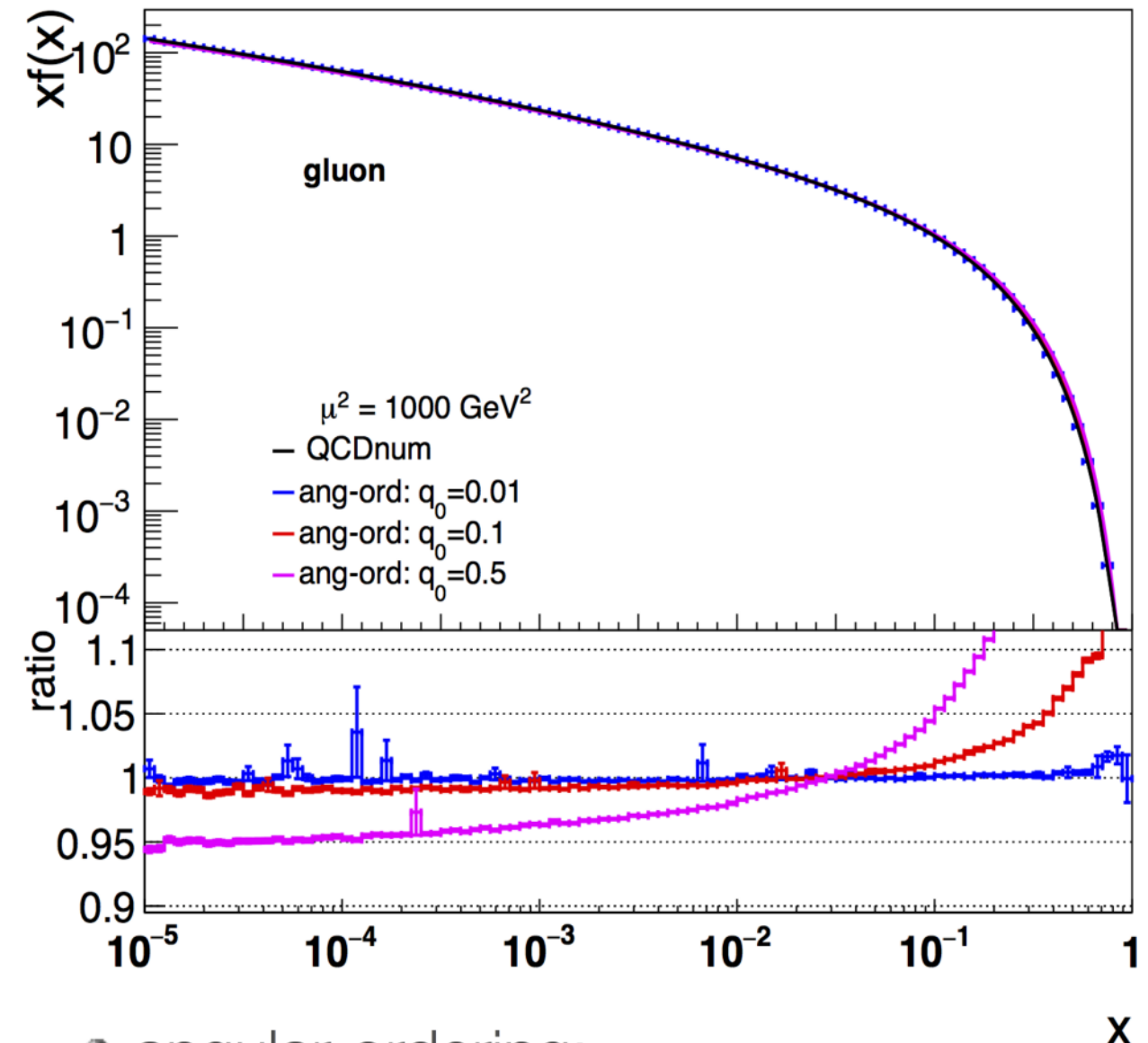
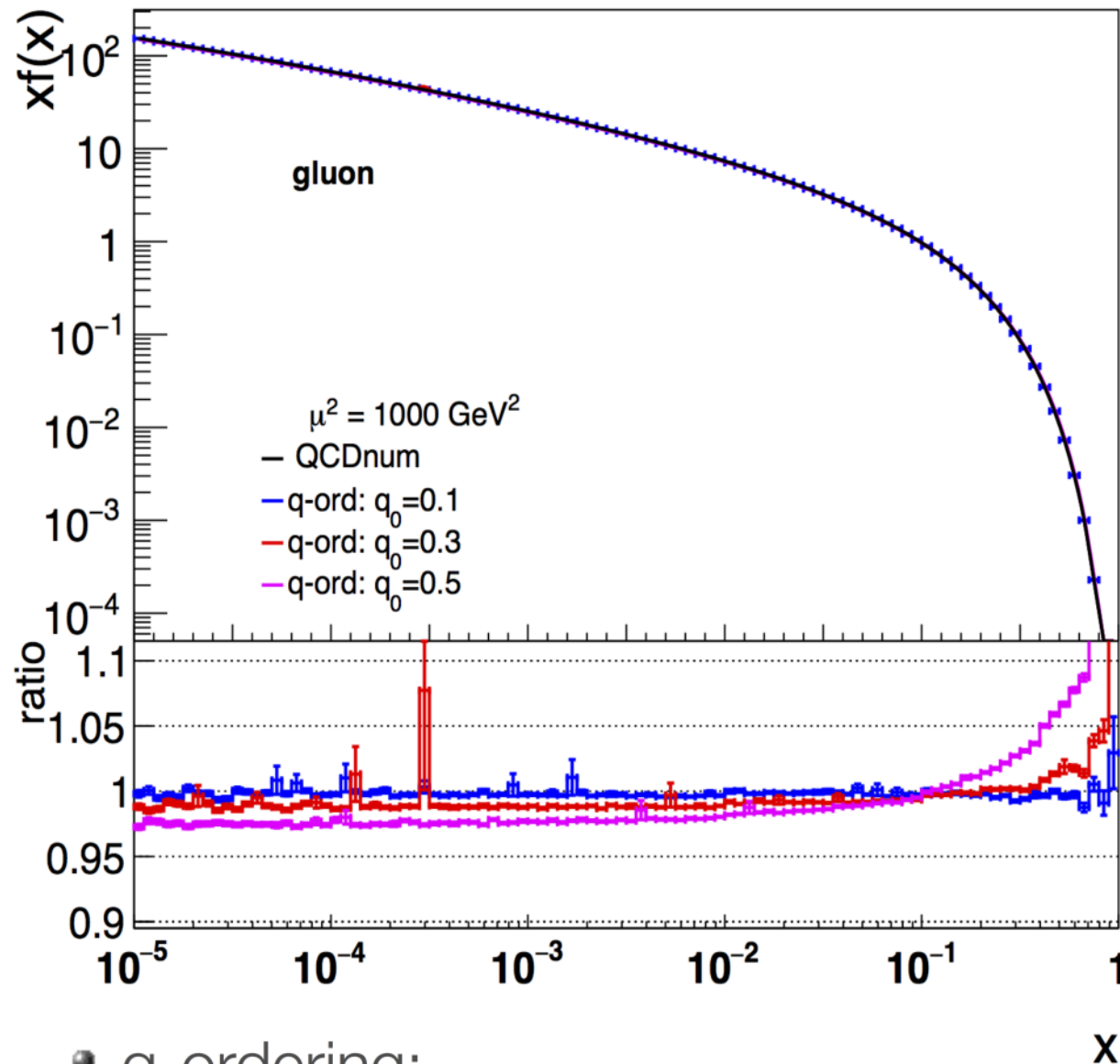
- procedure to fit initial distribution is working and producing results as expected

Advantages of parton branching method

- Consistency checks with QCDnum show agreement for inclusive distributions at the 0.1% level
- Advantages of parton branching method for collinear PDFs:
 - studies of different ordering conditions possible for the first time
 - resolvable branchings as defined by (q is evolution variable):
 - angular-ordering with varying $z_M = 1 - q_0/\mu'$
 - q^2 - ordering with varying $z_M = 1 - q_0^2/\mu'^2$



parton branching: different ordering conditions



- q-ordering:
 - dependence on parameter for resolvable branching $z_M = 1 - q_0^2/\mu'^2$

- angular-ordering:
 - strong dependence on parameter for resolvable branching $z_M = 1 - q_0/\mu'$

- large x resummation effects in parton densities: resolvable branchings !

Advantages of parton branching method

- Consistency checks with QCDnum show agreement for inclusive distributions at the 0.1% level
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 - studies of different ordering conditions possible for the first time
 - resolvable branchings as defined by:
 - angular ordering with varying $z_M = 1 - q_0/\mu'$
 - Q^2 ordering with varying $z_M = 1 - q_0^2/\mu'^2$
 - different choices of scales in $\alpha_s(\mu)$ possible
 - any investigation which involves details of parton branching kinematics
- further advantages – determination of TMD parton densities
 - since parton branching kinematics are known, transverse momenta of propagating partons can be calculated – determine TMD

Determination of TMD distribution

$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

- solve integral equation via iteration:

$$f_0(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2)$$

from t' to t
w/o branching

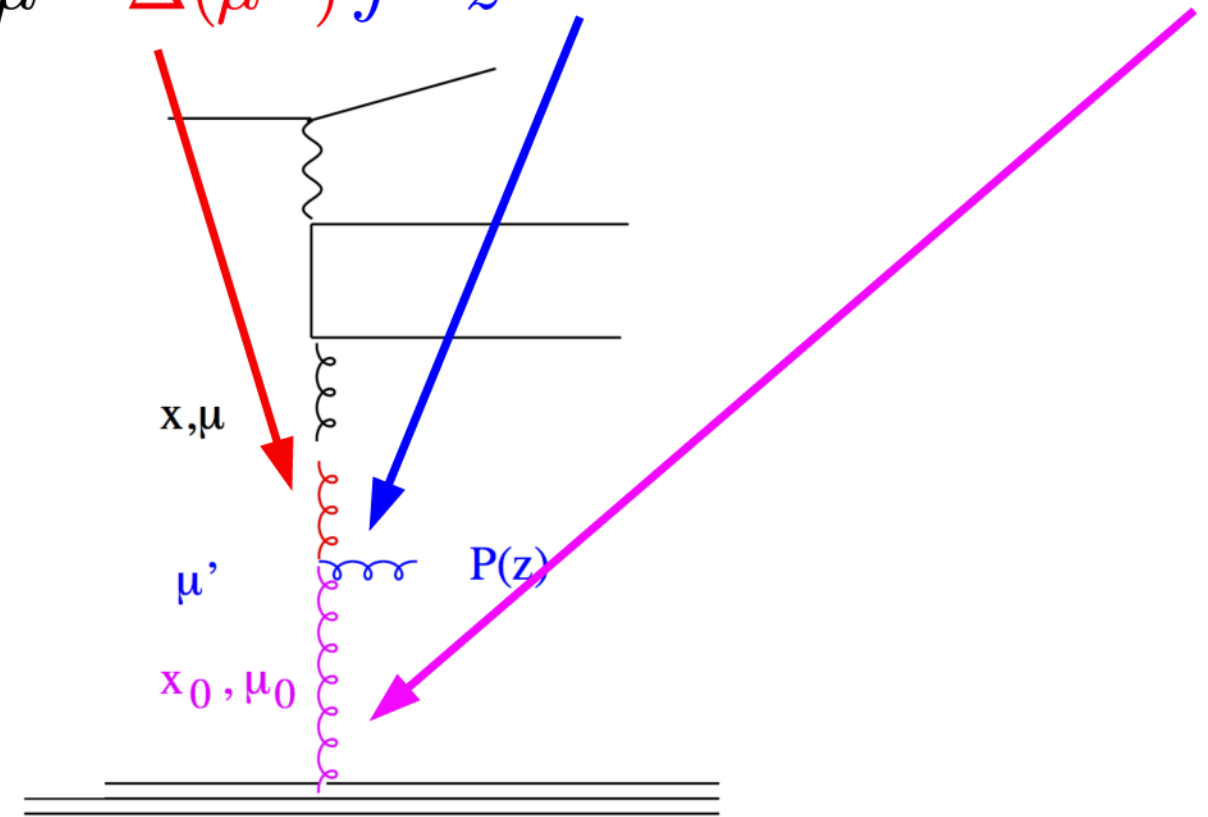
branching at t'

from t_0 to t'
w/o branching

$$f_1(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int \frac{dz}{z} P^{(R)}(z) f(x/z, \mu_0^2) \Delta(\mu'^2)$$

- in every step, kinematics are known:

- calculate k_t of propagator



Determination of TMD distribution

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- in every step, kinematics are known:

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- need correspondence:

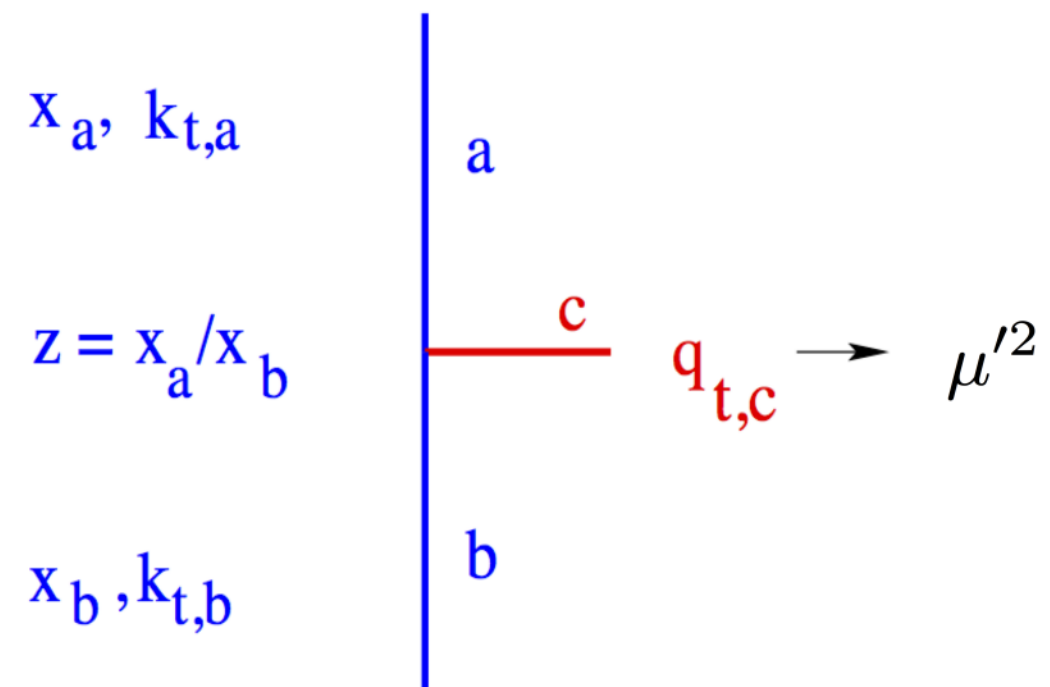
- $q_t^2 = \mu'^2$ with q_t emitted parton

OR

- $q_t^2 = (1-z) \mu'^2$, q^2 - ordering

OR

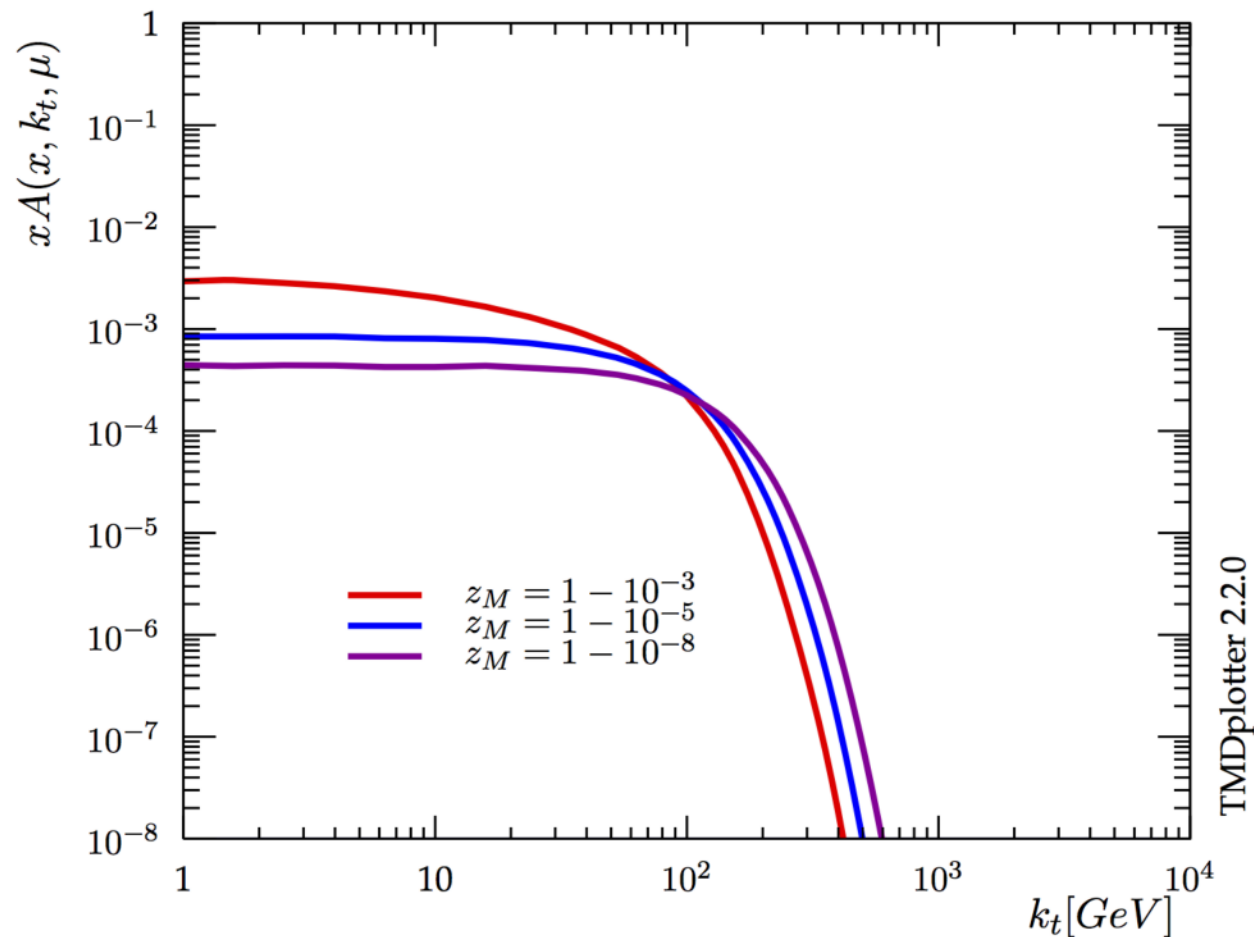
- $q_t^2 = (1-z) \mu'$ angular - ordering



Determination of TMD distribution

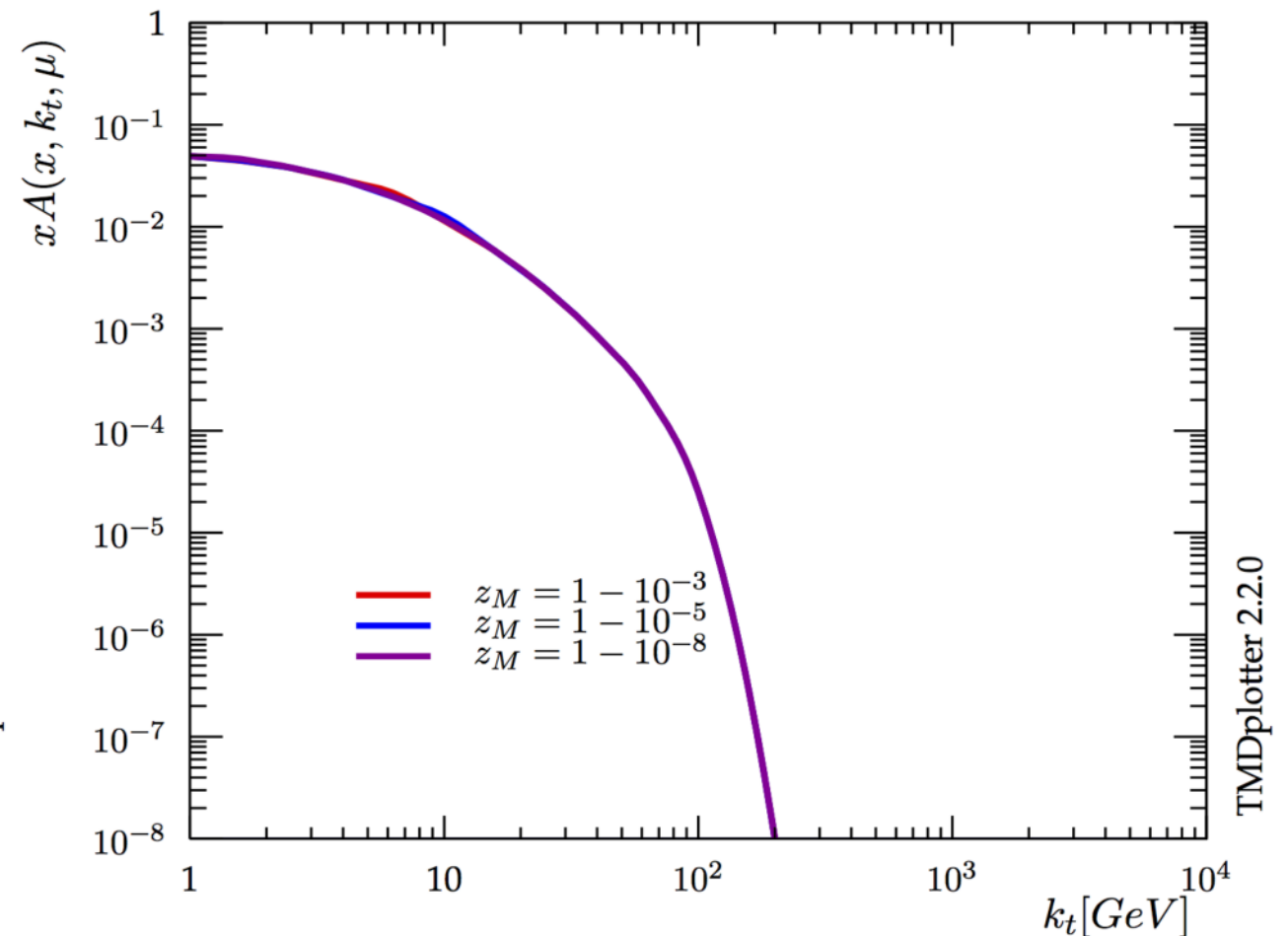
- naïve q_t - ordering
 - $q_t^2 = \mu^2$ with q_t emitted parton
 - $k_t = k'_t + q_t$

gluon, $x = 0.01, \mu = 100 \text{ GeV}$



- q^2 - ordering
 - $q_t^2 = (1 - z) \mu'^2$
 - $k_t = k'_t + q_t \sqrt{(1 - z)}$

gluon, $x = 0.01, \mu = 100 \text{ GeV}$

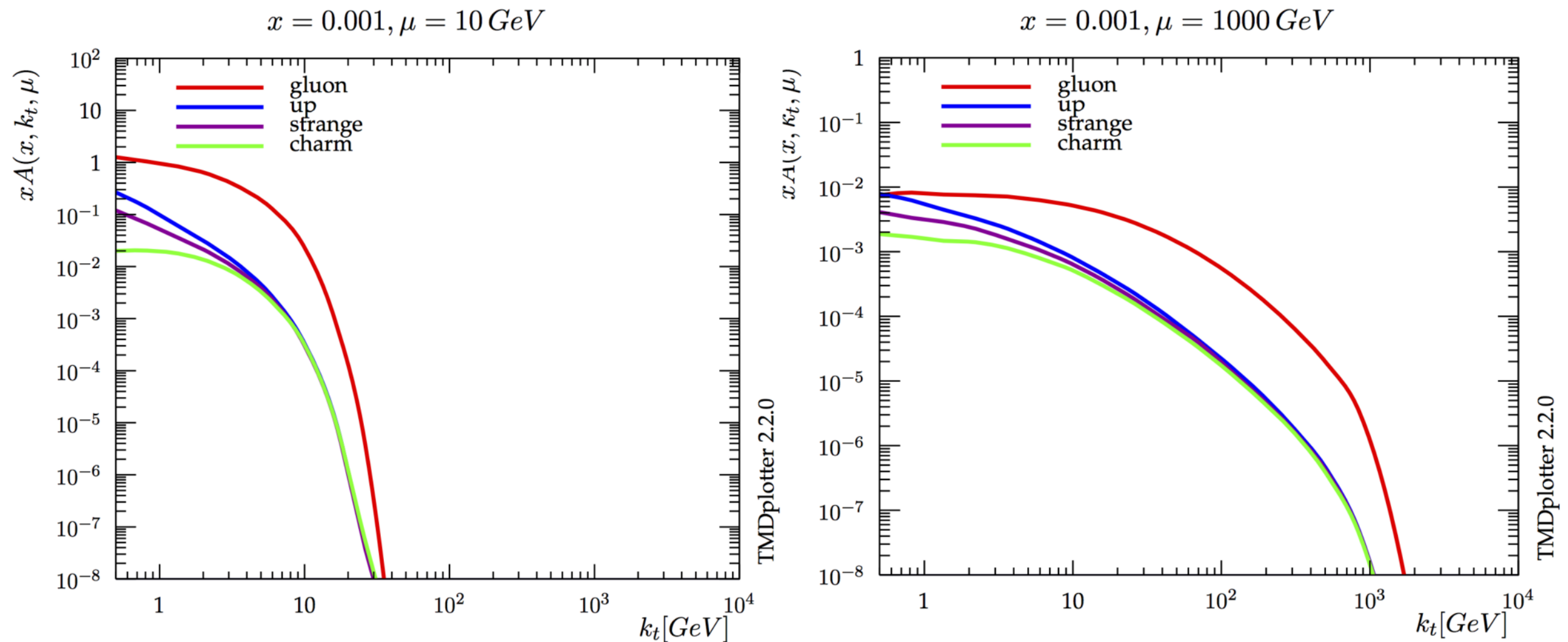


- Huge z_{max} dependence due to many soft gluons ($z_{max} \rightarrow 1$)

- Due to q^2 -ordering, soft gluons are suppressed $\rightarrow$ stable results

TMD distributions for different flavors

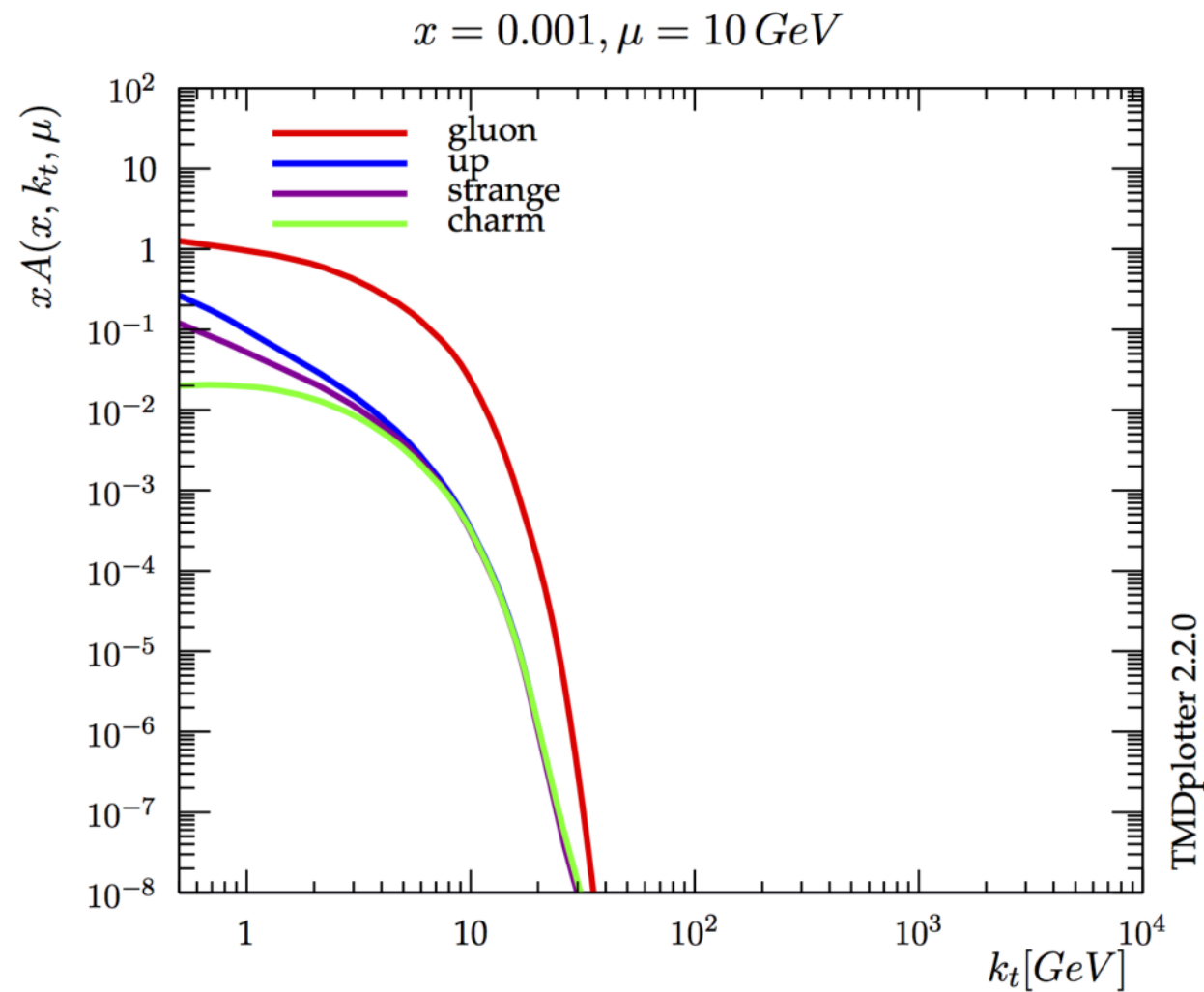
- with parton-branching method, TMD distribution for all flavors can be determined.



- at small k_t intrinsic (gauss) distribution is used $\rightarrow$ subject to fit at small k_t
- at $k_t \geq Q_0$, k_t – distribution comes entirely from evolution,
 - **no free parameters**, **except** association of evolution scale with q_t

TMD distributions for different flavors

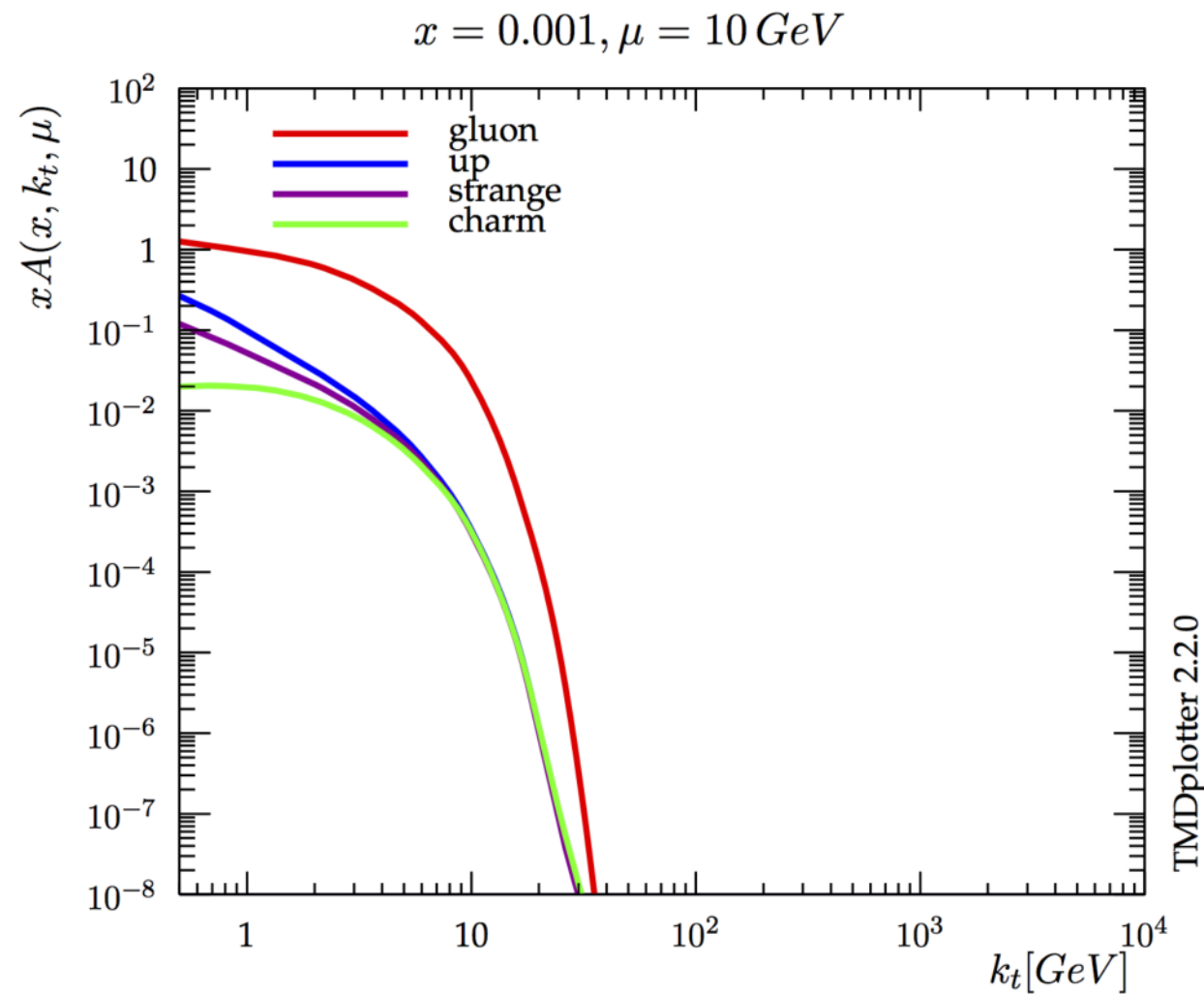
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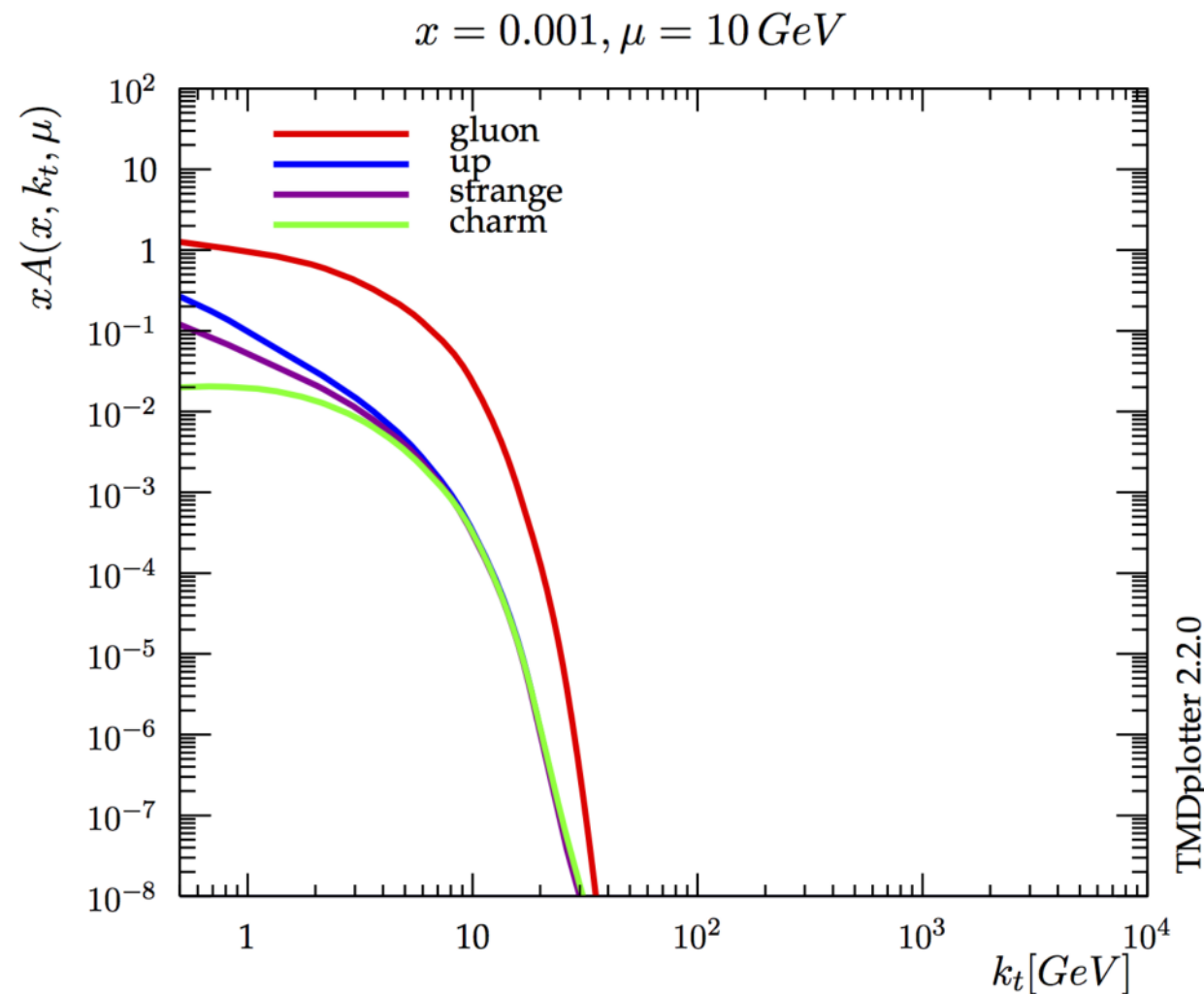
- small k_t : distributions are different

$$xf_a(x, \mu^2) = \Delta_a(\mu^2) xf_a(x, \mu_0^2) + \dots$$

➔ depends on distribution at starting scale

TMD distributions for different flavors

- with parton-branching method, TMD distribution for all flavors can be determined.



- small k_t : distributions are different

$$x\mathcal{A}(x, k_{\perp}^2, \mu^2) = \Delta_a(\mu^2) x\mathcal{A}(x, k_{\perp}^2, \mu_0^2) + \dots$$

→ depends on distribution at starting scale

- large k_t : quarks have similar shape

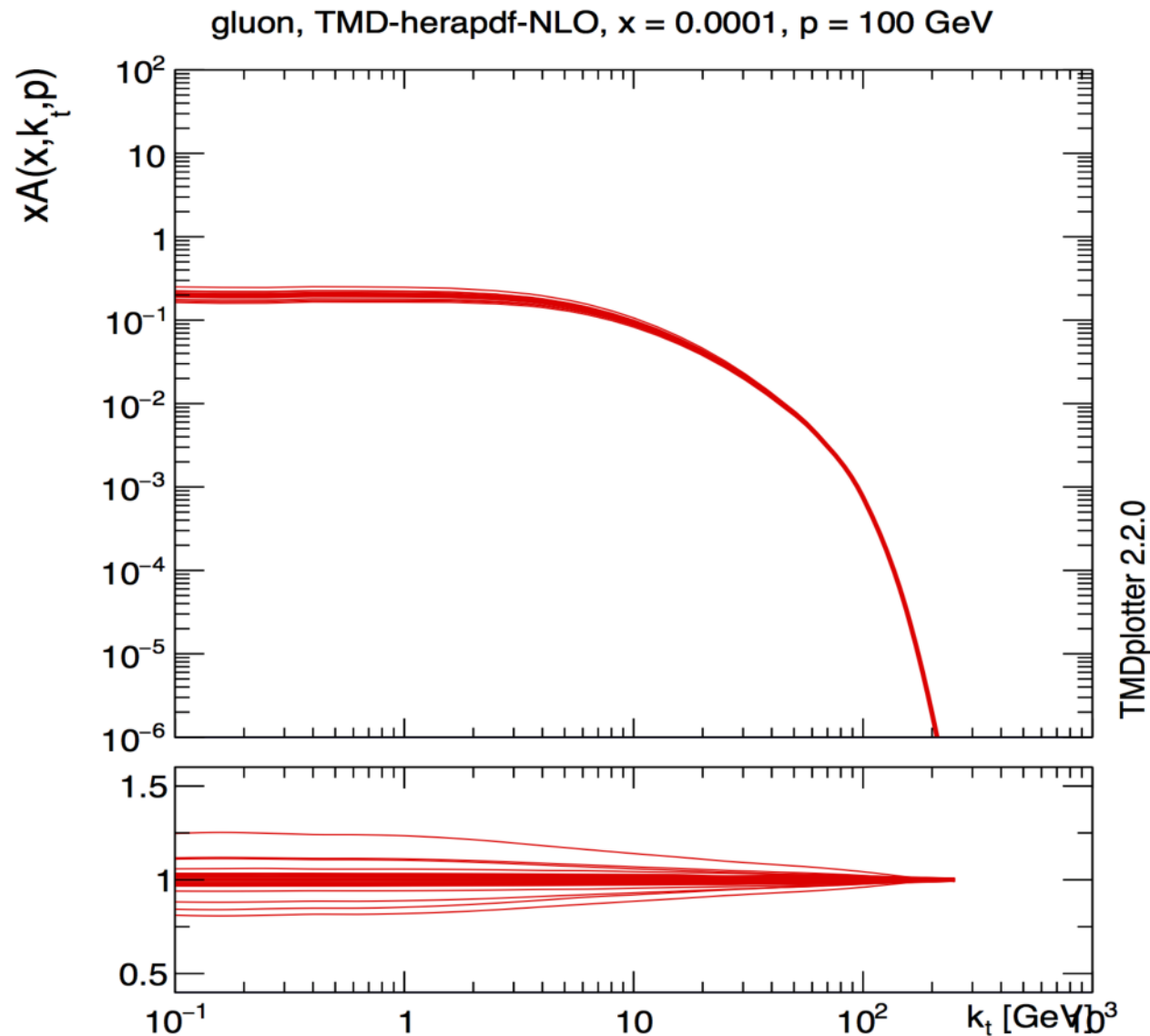
$$x\mathcal{A}(x, k_{\perp}^2, \mu^2) = \dots +$$

$$\sum_b \int_{\mu_0}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_x^{z_M} dz P_{ab}^{(R)} \frac{x}{z} \mathcal{A}\left(\frac{x}{z}, k_{\perp}^{\prime 2}, \mu'^2\right)$$

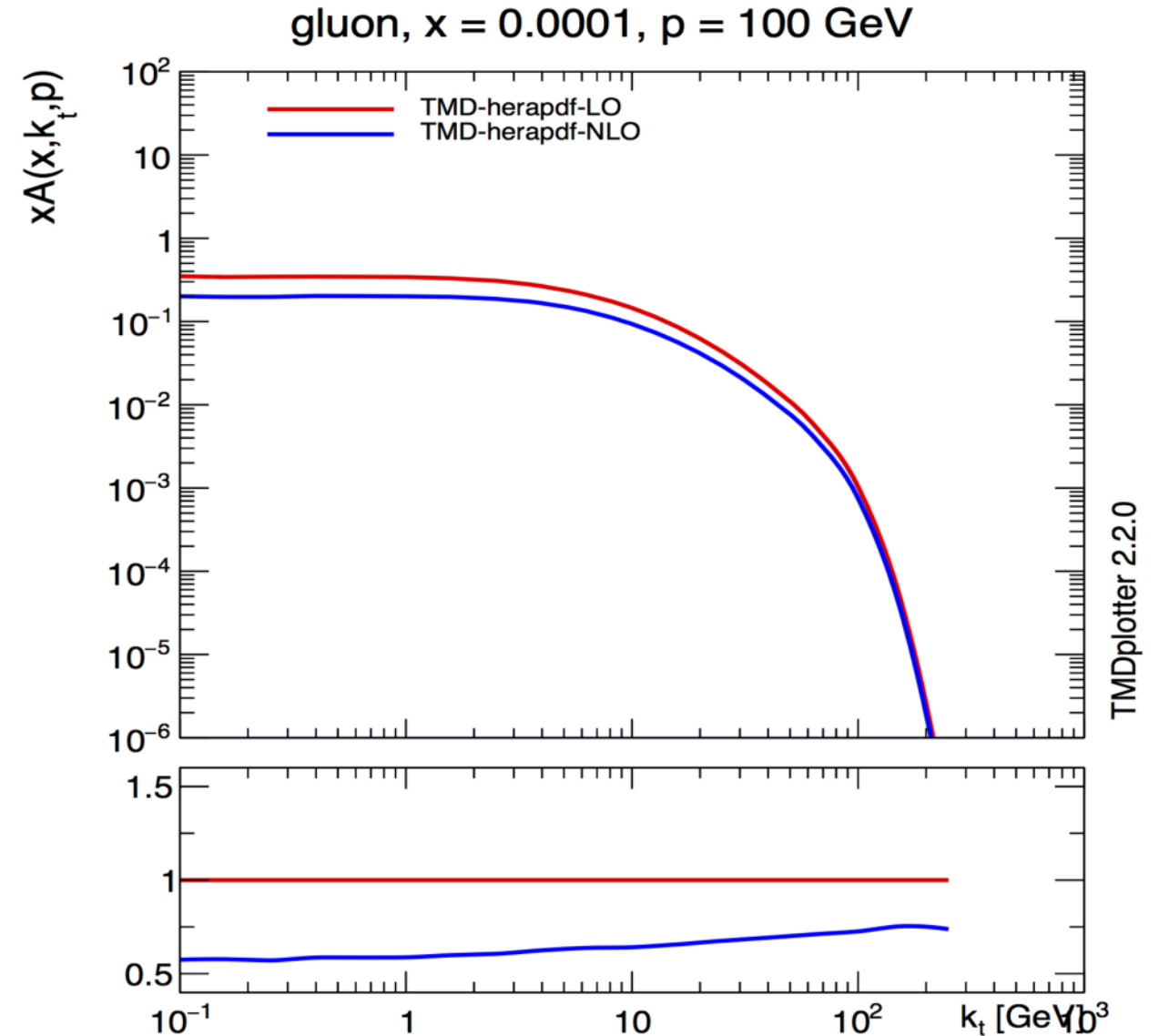
→ distributions similar due to parton evolution

- k_t -distributions at large scales depend on initial and evolved distributions

TMD distributions from xFitter: herapdf-type



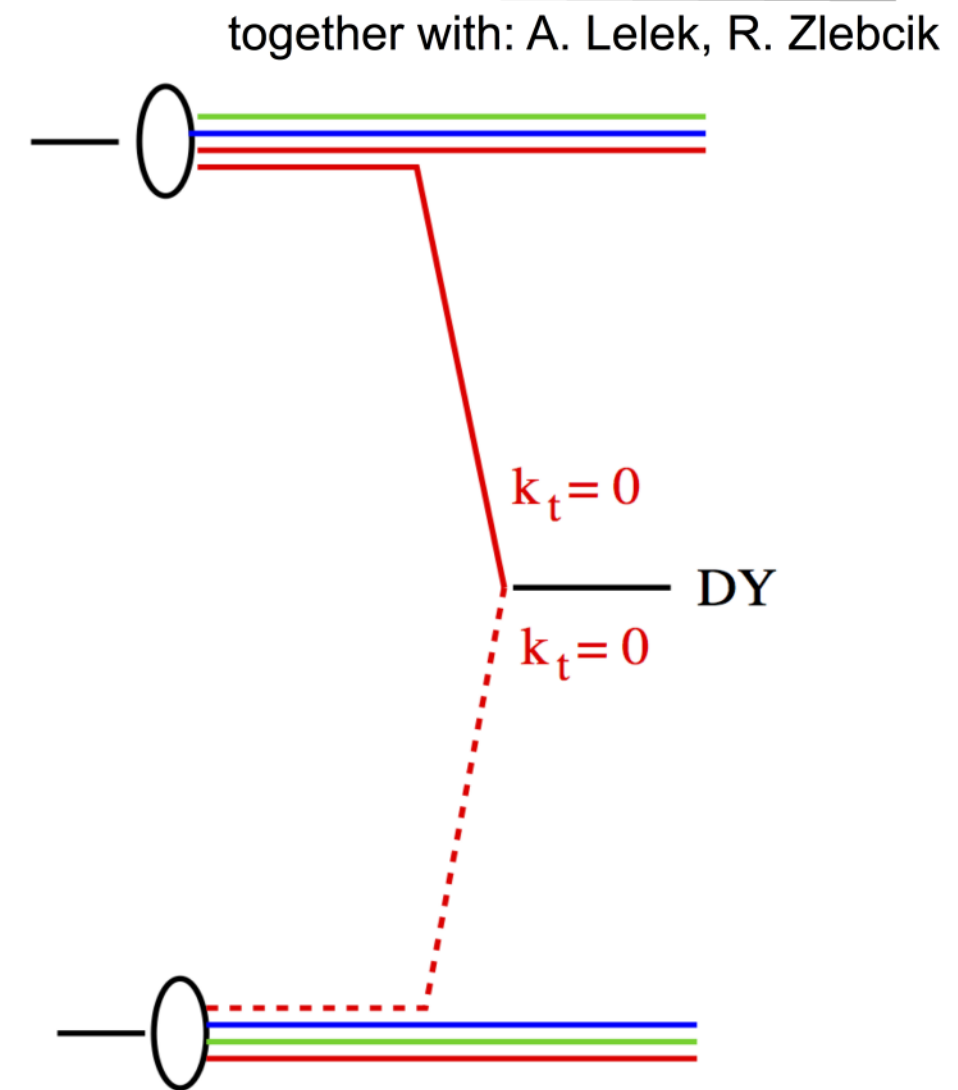
- Transverse momentum distributions including uncertainties from herapdf fit
 - only experimental uncertainties



- TMD distribution is different for NLO and LO
 - LO and NLO have different number of branchings !

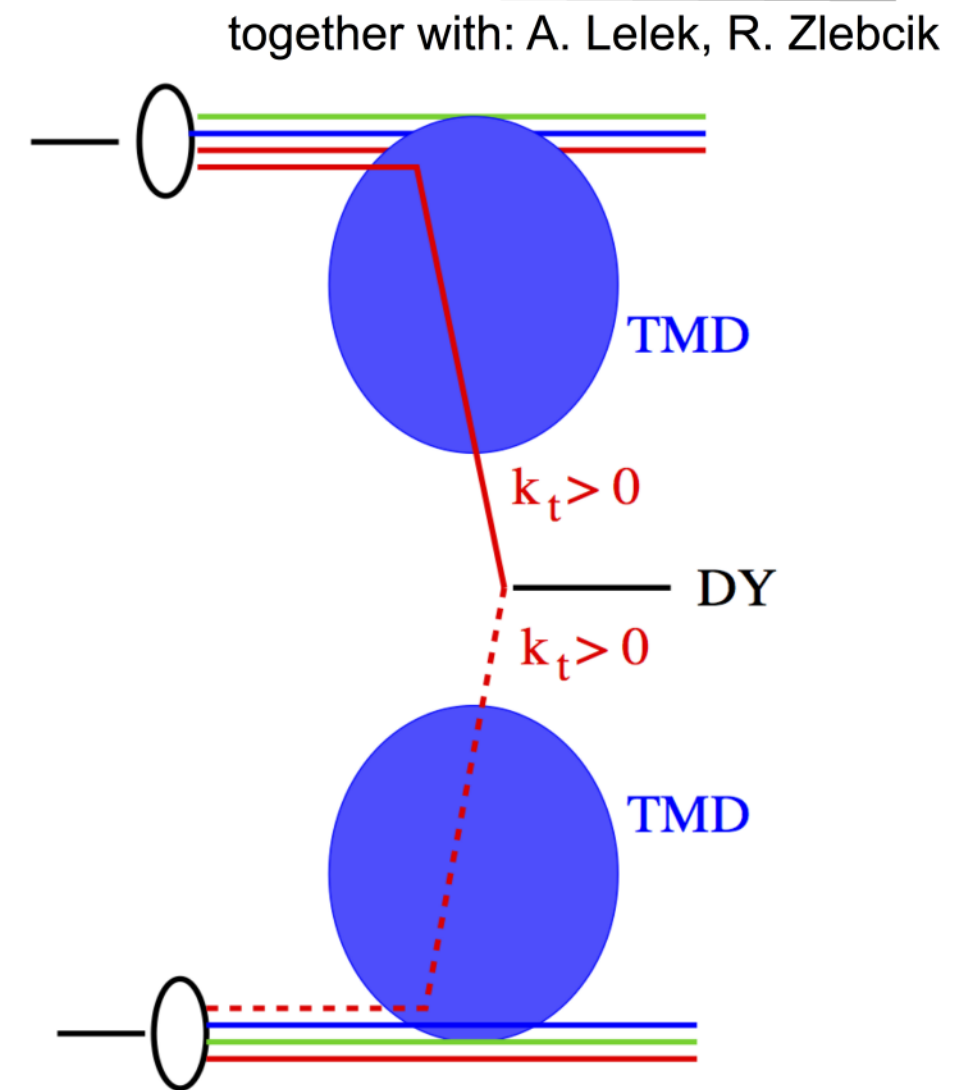
Application: DY production using TMDs

- Use LO DY production
 - $q\bar{q} \rightarrow Z_0$



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- Use LO DY production
 - $q\bar{q} \rightarrow Z_0$
- add k_t for each parton as function of x and μ according to TMD
- keep final state mass fixed:
 - x_1 and x_2 (light-cone fraction) are different after adding k_t

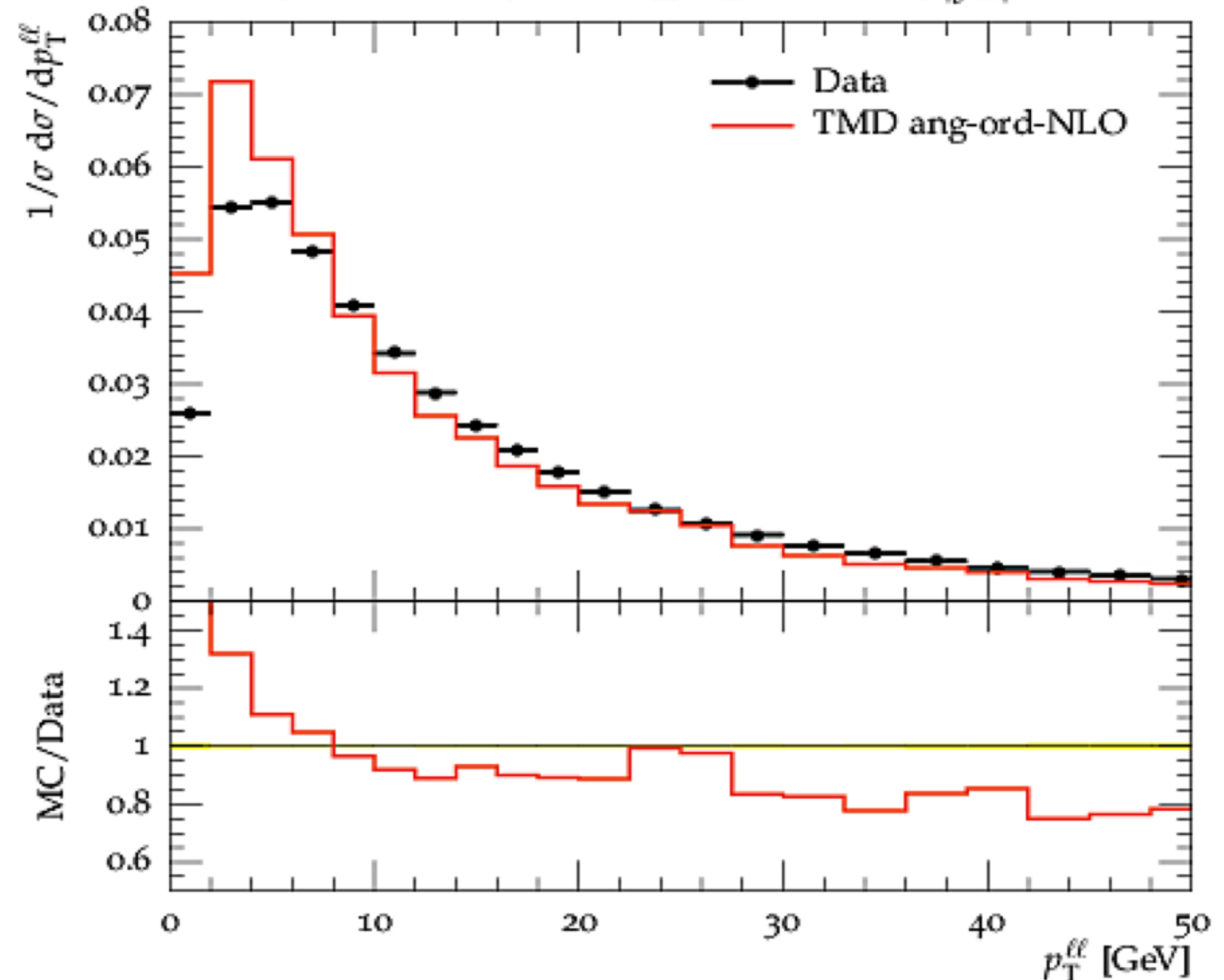


Application: DY production using TMDs

together with: A. Lelek, R. Zlebcik

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- Use TMD with angular ordering

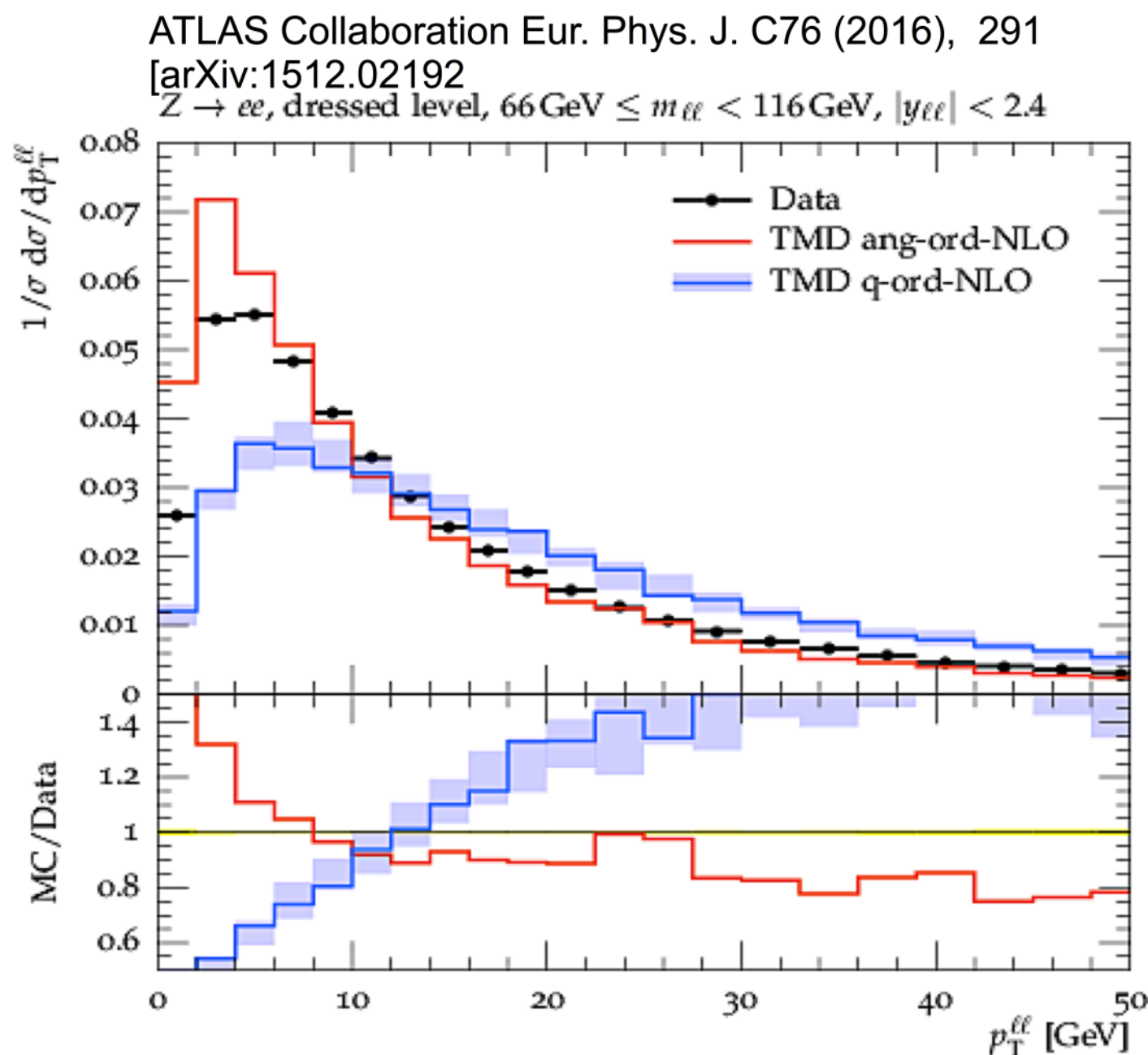
ATLAS Collaboration Eur. Phys. J. C76 (2016), 291
[arXiv:1512.02192
 $Z \rightarrow ee$, dressed level, $66 \text{ GeV} \leq m_{ee} < 116 \text{ GeV}$, $|y_{ee}| < 2.4$



Application: DY production using TMDs

together with: A. Lelek, R. Zlebcik

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- TMD with q^2 -ordering gives different distribution (including experimental uncertainties from herapdf fit)



Application: DY production using TMDs

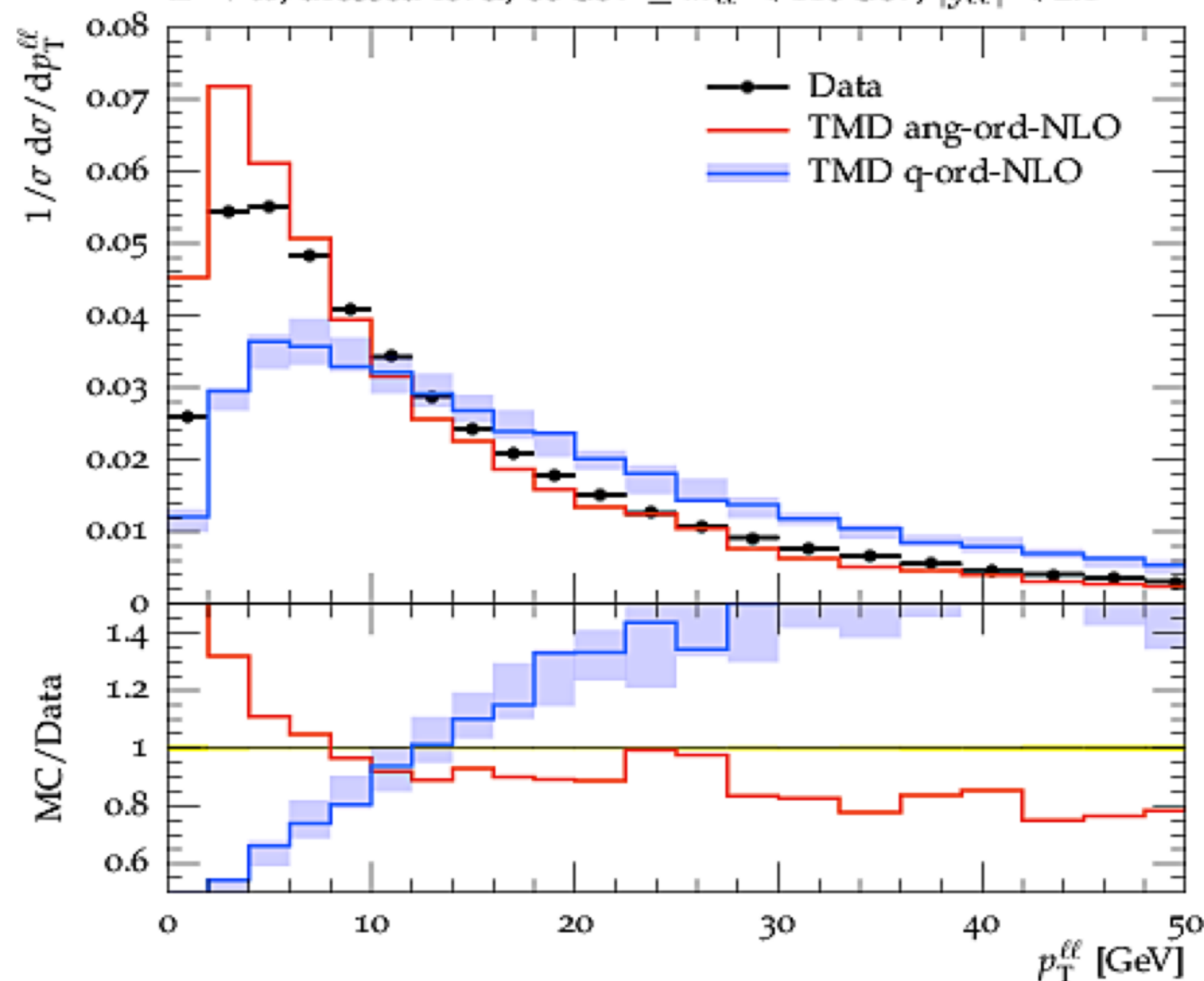
together with: A. Lelek, R. Zlebcik

- Use LO DY production
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 - add k_t for each parton as fct of x and μ according to TMD
 - keep final state mass fixed:
 - x_1 and x_2 (light-cone fraction) are different after adding k_t
- no free parameters, once TMD is determined
 - depends on intrinsic k_t as used in parton evolution (here gauss with mean=0)
 - can be determined in fit!

ATLAS Collaboration Eur. Phys. J. C76 (2016), 291

[arXiv:1512.02192

$Z \rightarrow ee$, dressed level, $66 \text{ GeV} \leq m_{ee} < 116 \text{ GeV}$, $|y_{ee}| < 2.4$



Where to find TMDs ? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of REF workshop and developed since
- combine and collect different ansaetze and approaches:

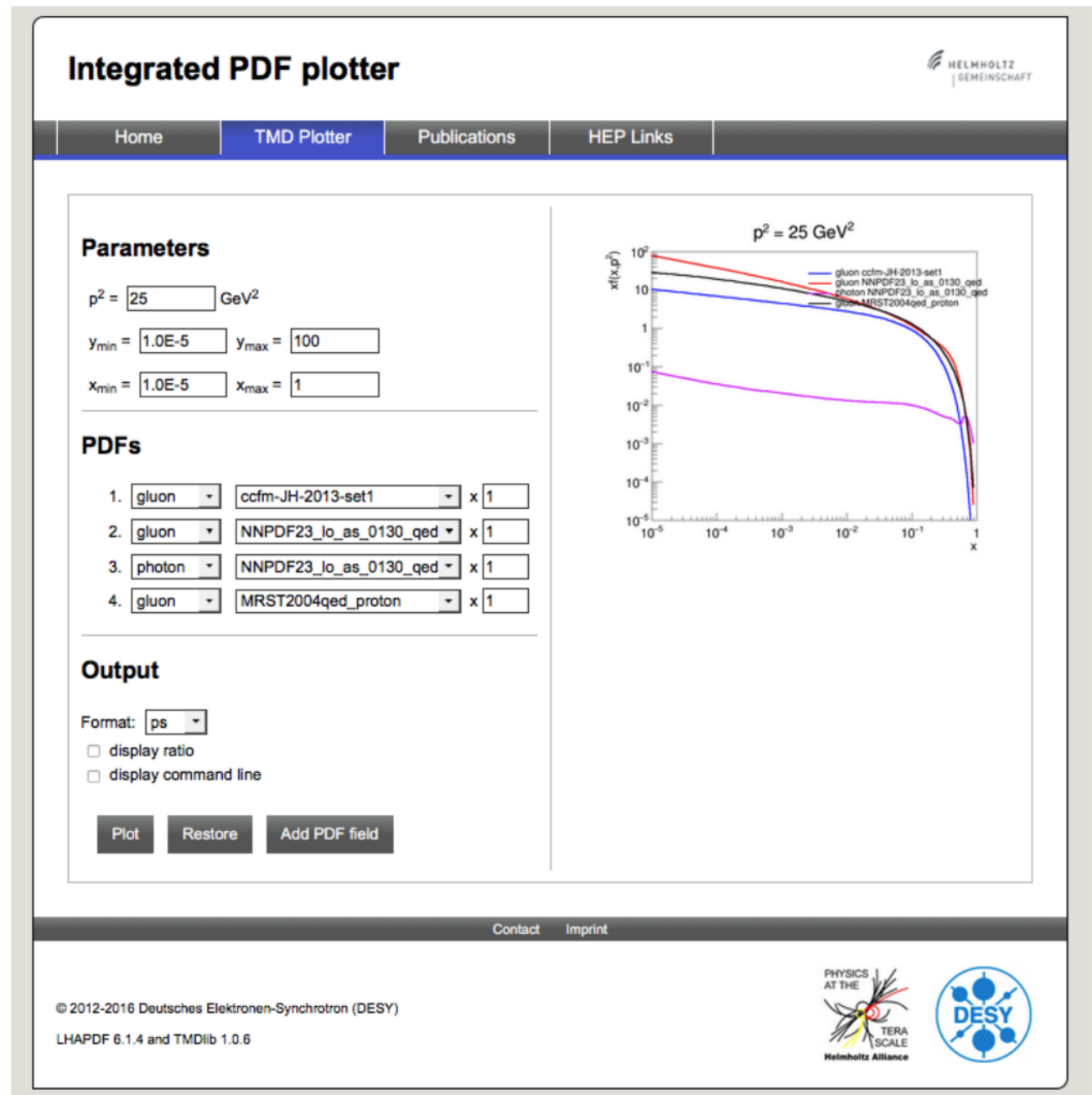
<http://tmd.hepforge.org/> and
<http://tmdplotter.desy.de>

- TMDlib: a library of parametrization of different TMDs and uPDFs (similar to LHApdf)

TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions, *F. Hautmann et al.* arXiv 1408.3015, *Eur. Phys. J.*, C 74(12):3220, 2014.

- Also integrated pdfs (including photon pdf are available via LHAPDF)

- Feedback and comments from community is needed – just use it !



Resummation, Evolution, Factorization 2017

<http://jacobi.fis.ucm.es/REF2017/index.html>



» Main Menu

- Presentation
- Scientific Program
- Registration
- Important Dates
- Participants
- Accommodation
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» Presentation

Dates: 13/11/2017 (AFTERNOON)-16/11/2017

REF 2017 is the 6th workshop in the series of workshops on **Resummation, Evolution, Factorization**.
Previous discussion meetings and workshops were

7-10 November 2016 Antwerp (Belgium)
2-5 November 2015 DESY Hamburg (Germany)
1-3 June 2015 Amsterdam (The Netherlands)
8-11 December 2014 Antwerp (Belgium)
23-24 June 2014 Antwerp (Belgium)

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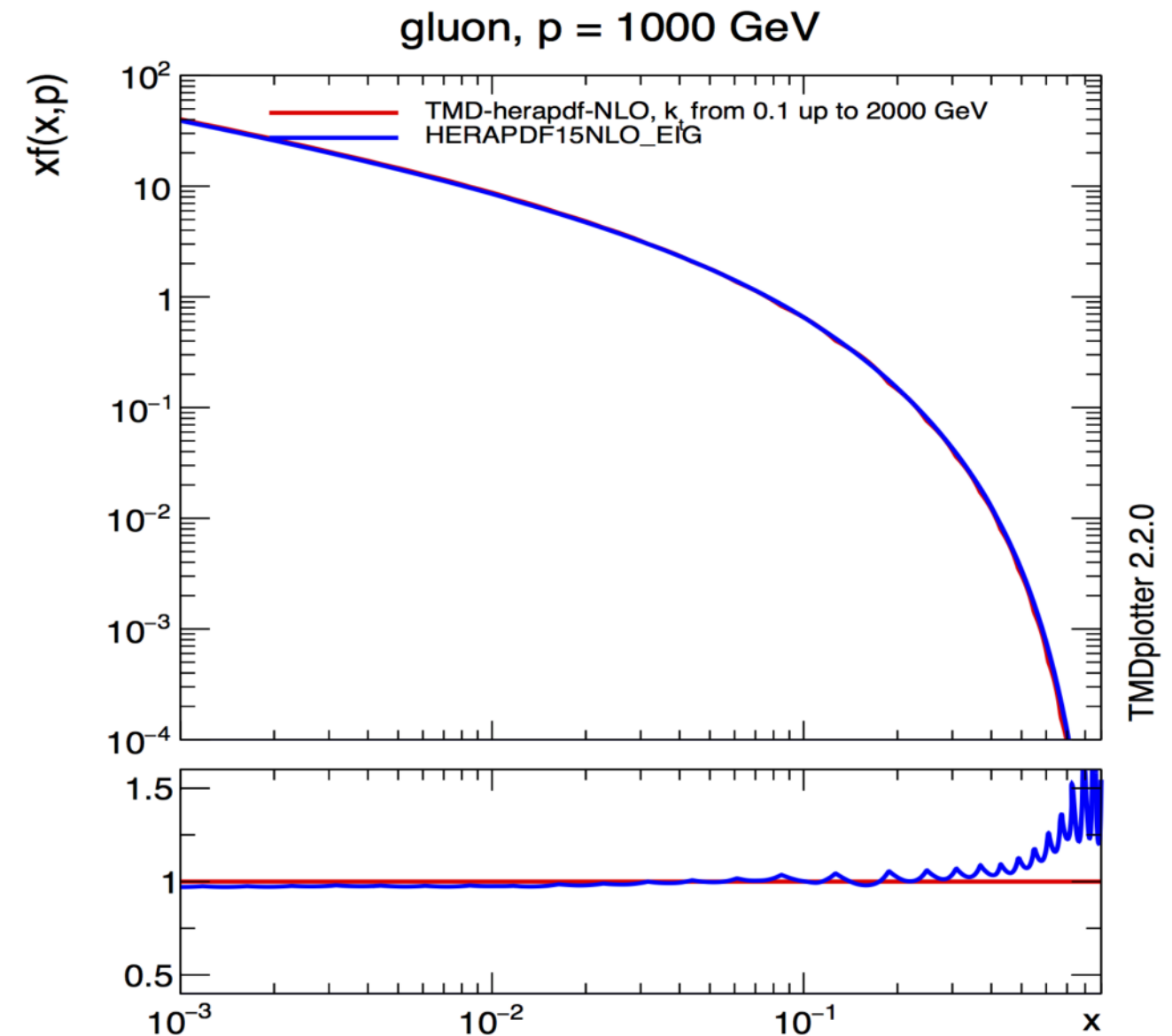
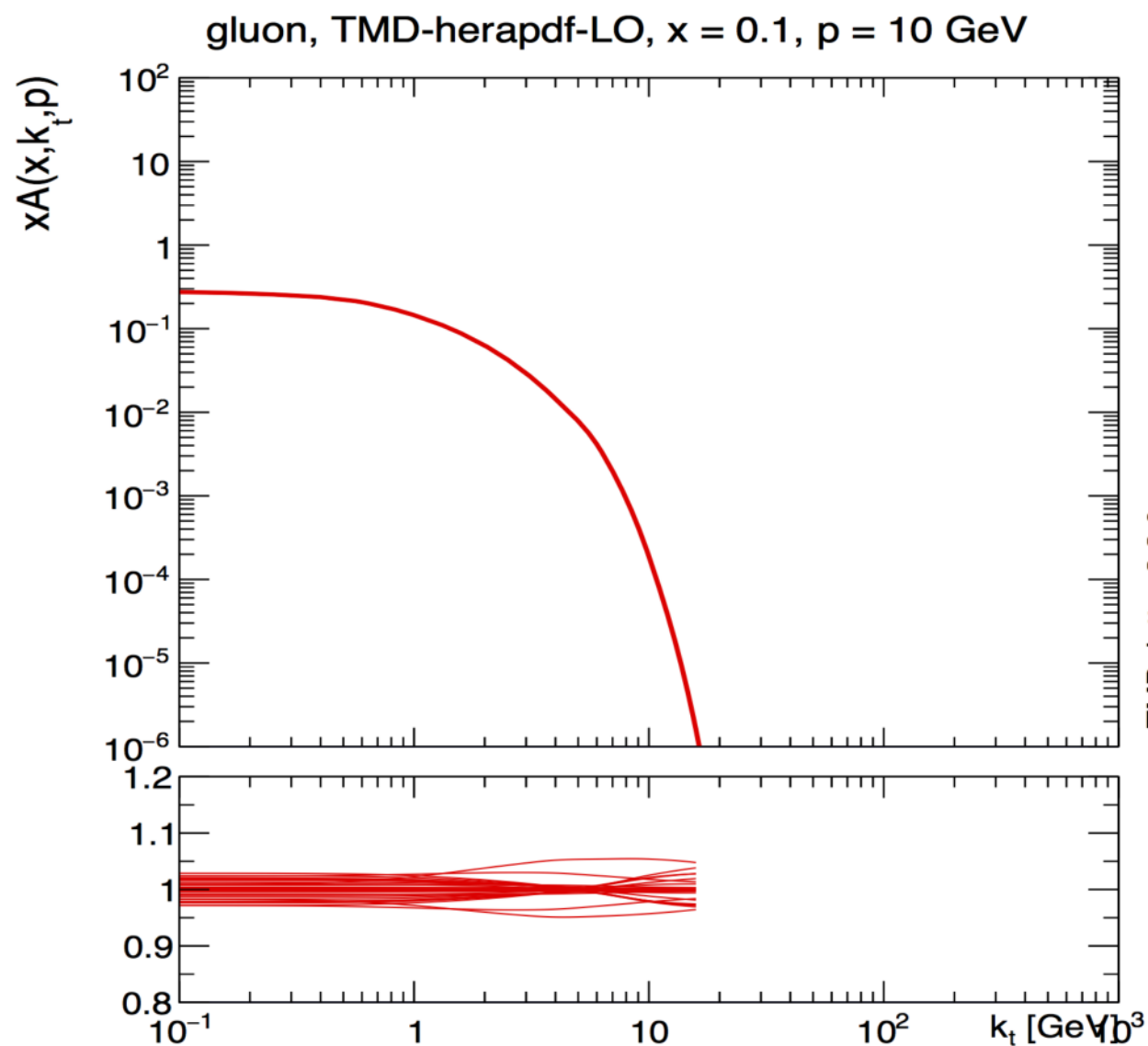
23-24 June 2014 Antwerp (Belgium)

Conclusion

- transverse momenta of interaction partons can be important for precision physics
 - ➔ need for TMDs
- Parton Branching method developed for solving DGLAP equation at LO, NLO and NNLO
 - ➔ consistence for collinear (integrated) PDFs shown
- method directly applicable to determine k_t distribution (as would be done in PS)
 - ➔ TMD distributions for all flavors determined at LO and NLO, without free parameters
 - ➔ TMD evolution implemented in xFitter – applicable for DIS processes
- First application to DY production
 - ➔ using TMD distributions gives reasonable q_t spectrum of DY
 - ➔ no free parameters
- Parton branching method for PDFs and TMD successful !

Appendix

TMD distributions from xFitter: herapdf-type



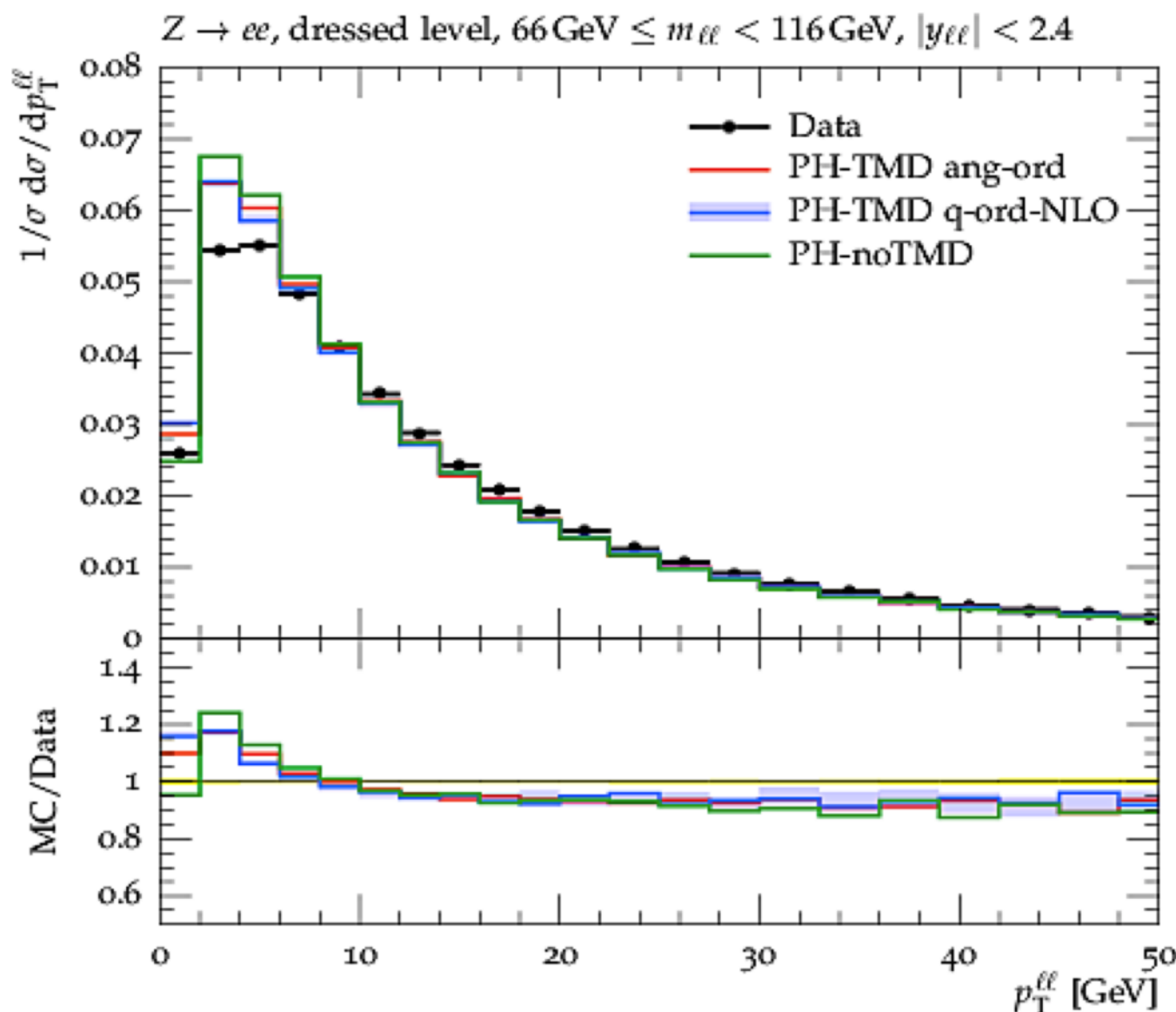
- Transverse momentum distributions including uncertainties from herapdf fit
 - essentially constant at fixed x and μ^2
- iTMD distribution: (integrated TMD) compared to herapdf

Application: DY production using TMDs

together with: A. Lelek, R. Zlebcik

- Use LO DY production
 - $q\bar{q} \rightarrow Z_0$
 - add k_t for each parton as fct of x and μ according to TMD
 - keep final state mass fixed:
 - x_1 and x_2 (light-cone fraction) are different after adding k_t
- Use TMD with angular ordering
- TMD with q^2 -ordering gives different distribution (including experimental uncertainties from herapdf fit)
- Using POWHEG, leaves very little room for TMD contribution:
 - Sudakov term is already included in POWHEG.

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[arXiv:1512.02192]



Application: DY production using TMDs

together with: A. Lelek, R. Zlebick

- Use LO DY production
 - $q\bar{q} \rightarrow Z_0$
- comparison to NLO calculations

