Connections between TMD and collinear factorization for transverse single-spin asymmetries

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TMD Topical Collaboration

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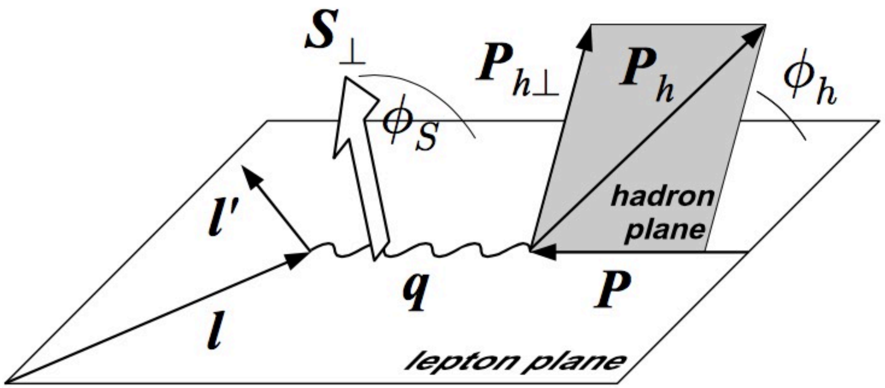
Outline

- Background
  - Transverse single-spin asymmetries
  - TMD and collinear twist-3 (CT3) functions
- TMD and CT3 observables
  - Sivers and Collins effects
  - A_N in $pp \rightarrow \{\gamma\ , \pi\}\ X$
- Relations between TMD and CT3 functions
  - Parton model
  - QCD (CSS formalism)
- Towards a global analysis of TMD and CT3 observables
- Summary

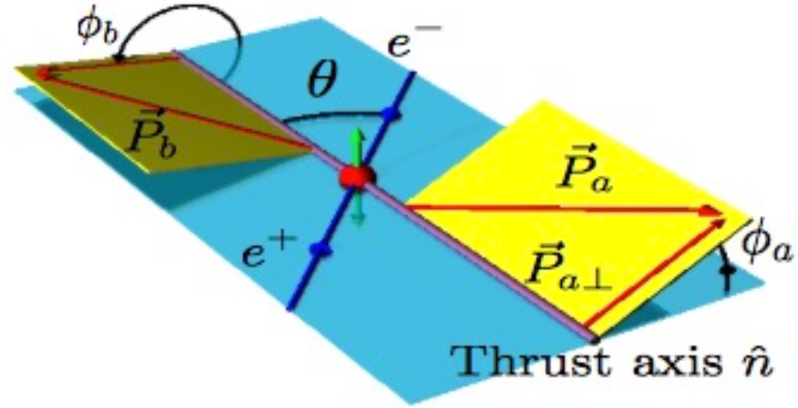
Background

$$e\,N \rightarrow e'\,h\,X$$



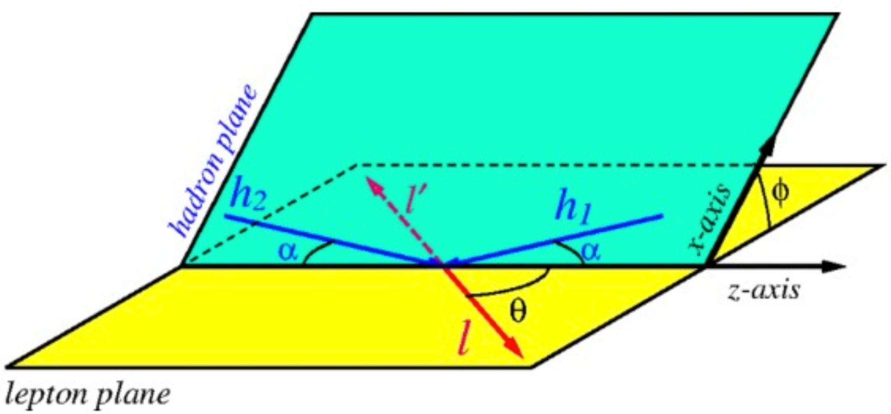
Sivers ~ $\sin(\phi_h - \phi_s)$, Collins ~ $\sin(\phi_h + \phi_s)$, ...

$$e^+e^- \rightarrow h_a h_b X$$



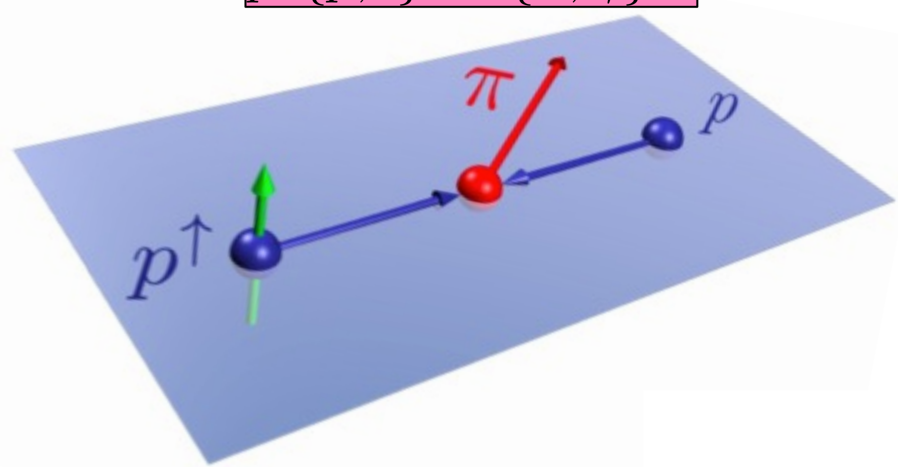
Collins ~ $\cos(\phi_a + \phi_b)$, ...

$$p^\uparrow\{p,\pi\} \rightarrow \{l^+l^-, W/Z\}X$$



Sivers ~ $\sin(\phi_s)$ (lepton pair) / Sivers ~ $\cos(\phi_{W/Z})$ (boson)

$$p^\uparrow\{p,l\} \rightarrow \{\pi,\gamma\}X$$



$A_N \sim d\sigma_L - d\sigma_R$

TMD PDFs (x, k_T)

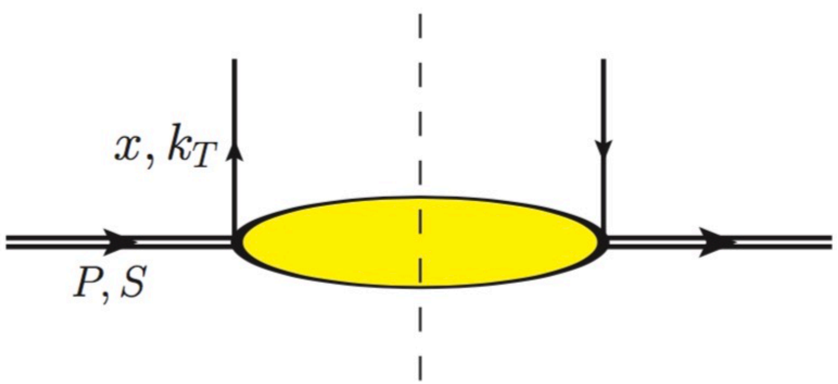
H pol. \ q pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

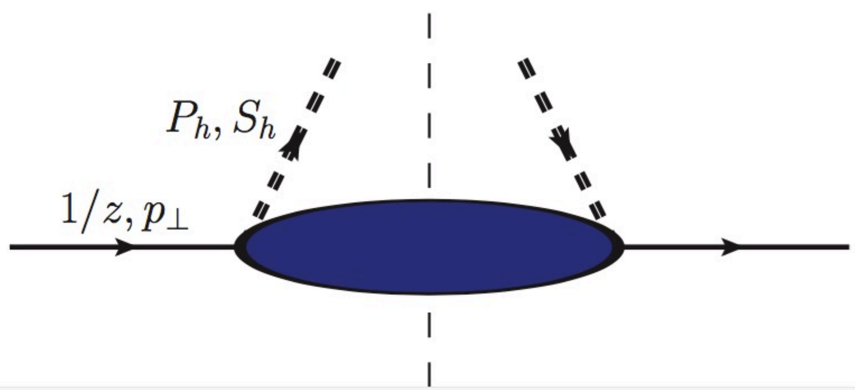
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

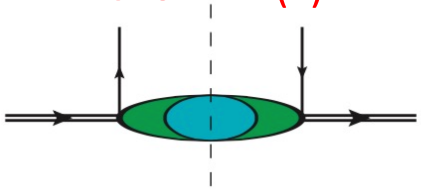
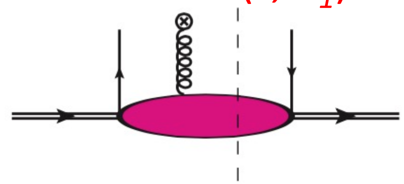
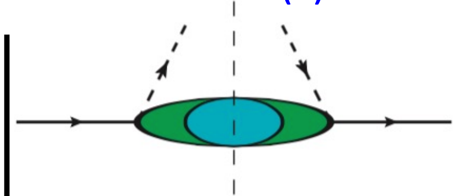
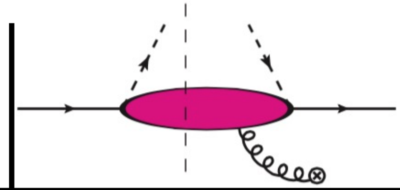
TMD FFs ($z, p_\perp$)

H pol. \ q pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))



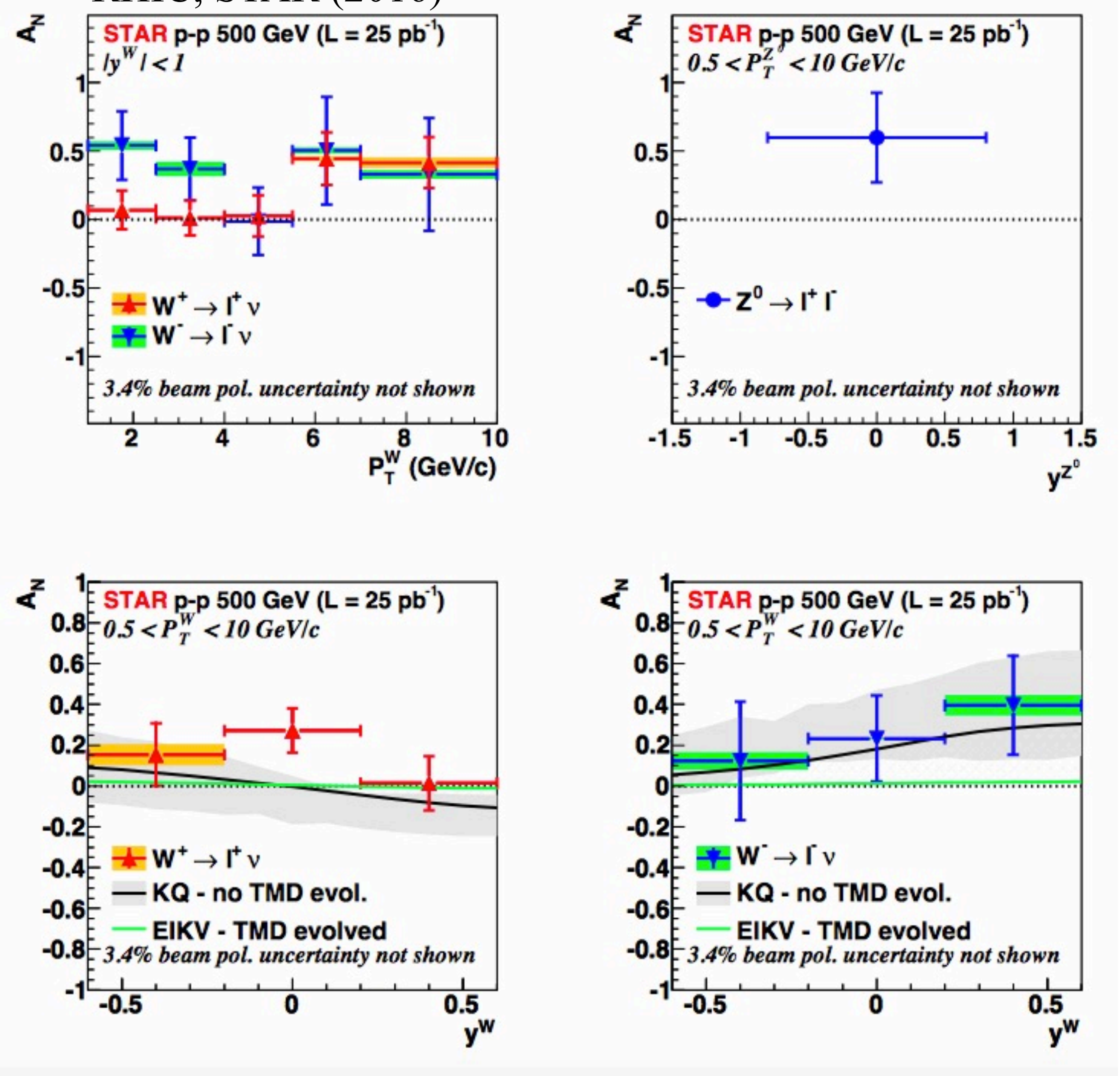


Hadron Pol.	CT3 PDF (x) 		CT3 PDF (x, x_1) 	CT3 FF (z) 		CT3 FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

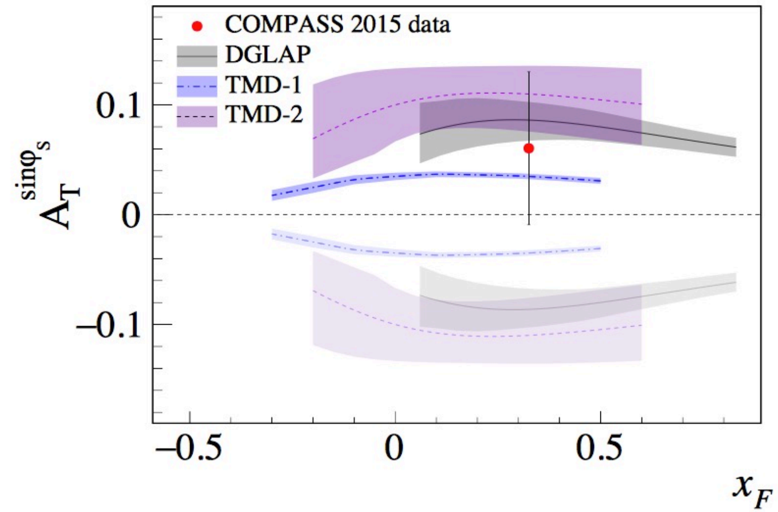
TMD and CT3 Observables

Drell-Yan Sivers effect

RHIC, STAR (2016)

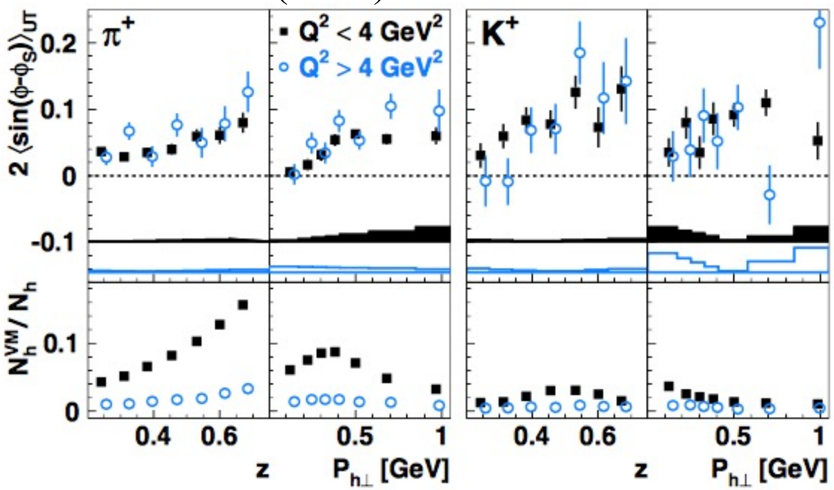


COMPASS (2017)

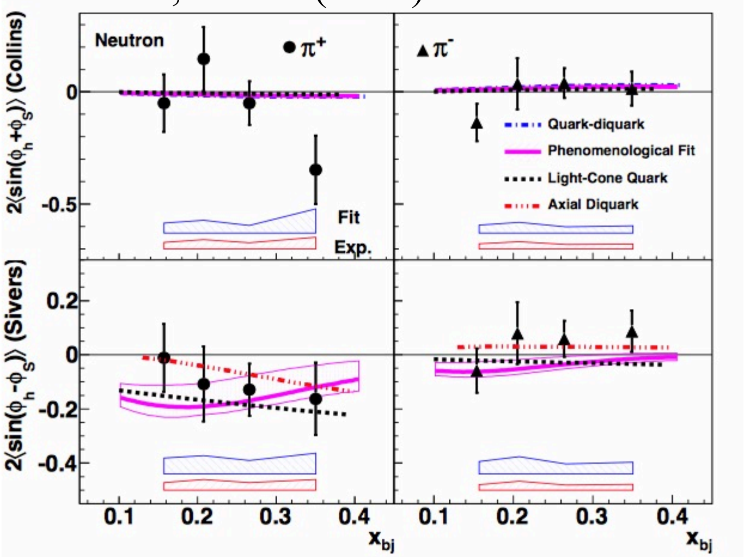


SIDIS Sivers effect ($sin(\phi_h - \phi_s)$)

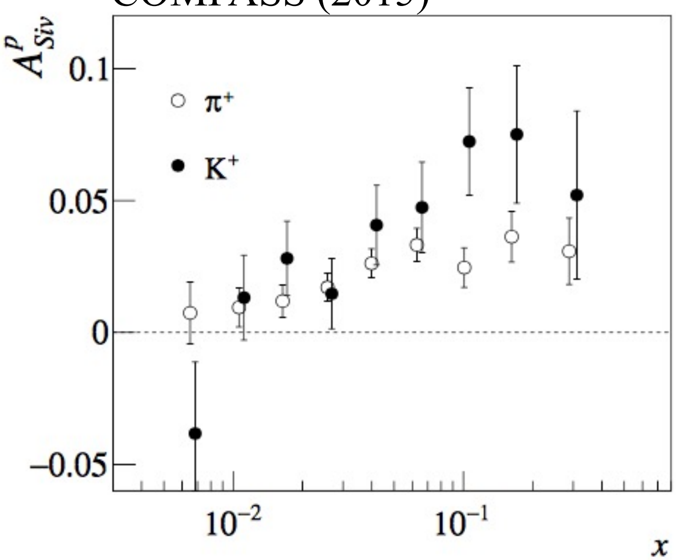
HERMES (2009)



JLab, Hall A (2011)

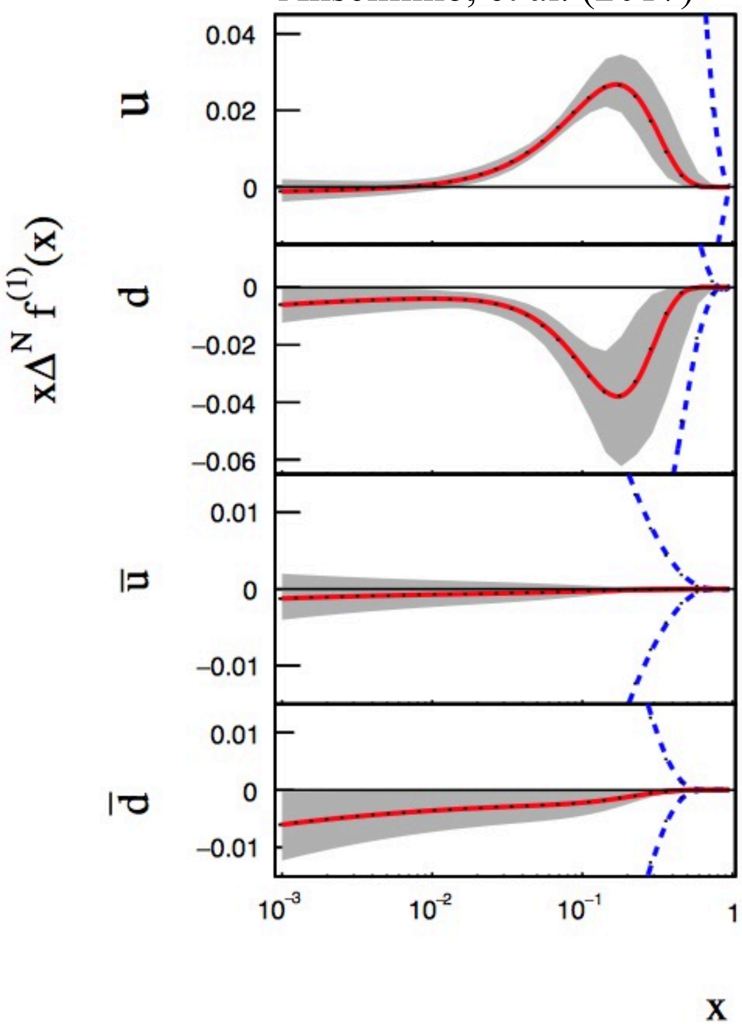


COMPASS (2015)

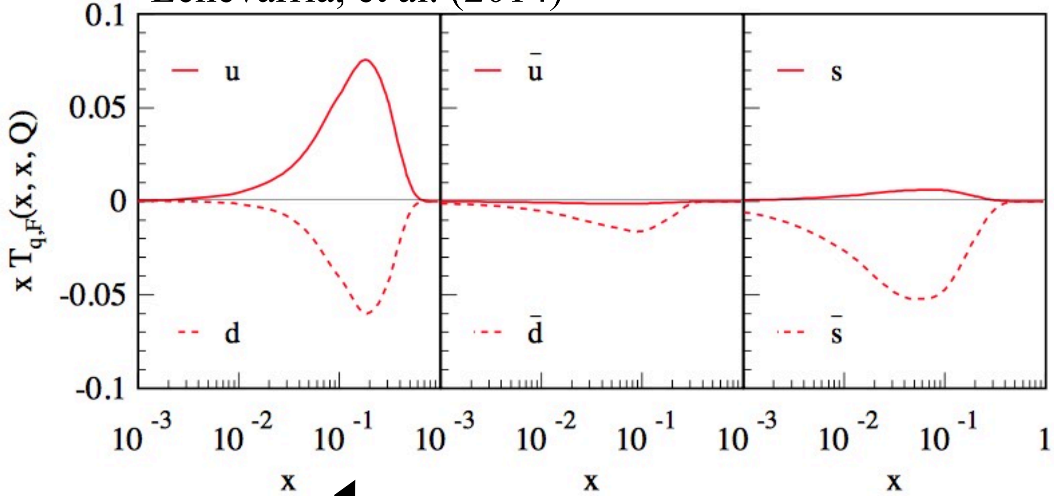


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M} f_{1T}^{\perp} D_1\right]$$

Anselmino, et al. (2017)



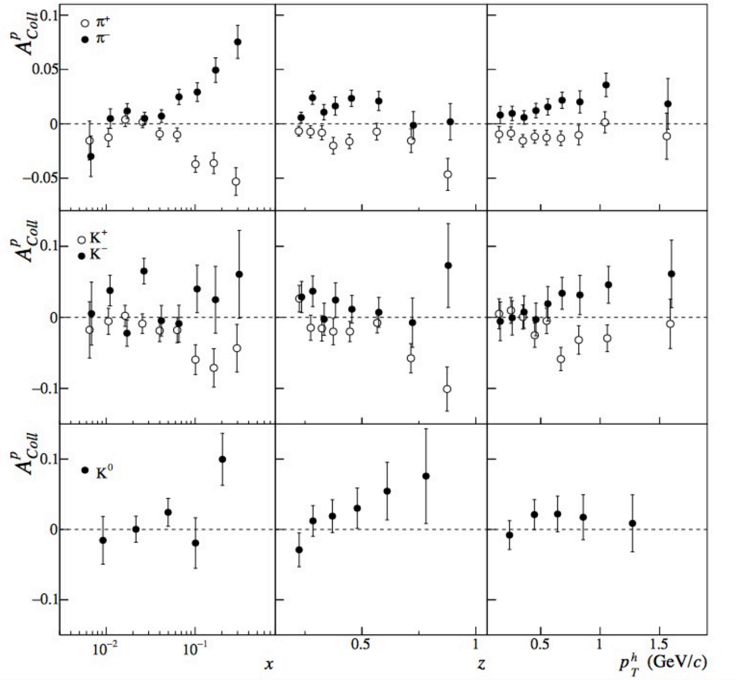
Echevarria, et al. (2014)



uses full TMD evolution

SIDIS Collins effect (*sin(ϕ_h + ϕ_s)*)

COMPASS (2015)

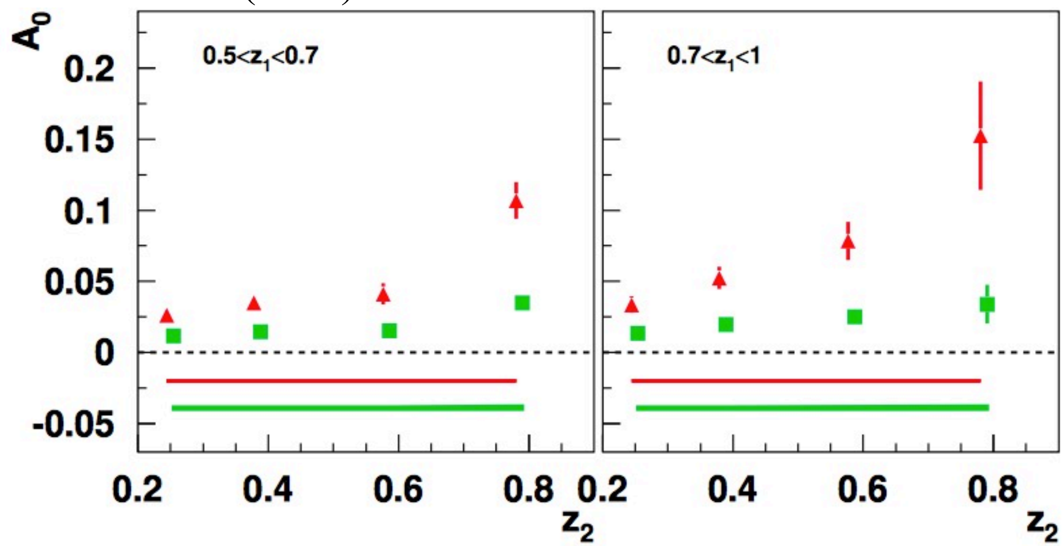


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\, h_1\, H_1^\perp\right]$$

e^+e^- Collins effect (*cos(2ϕ_0)*)

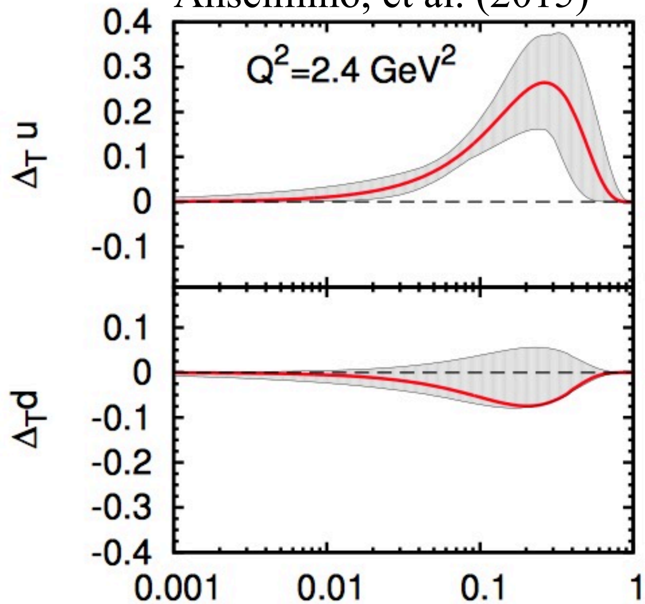
Belle (2008)

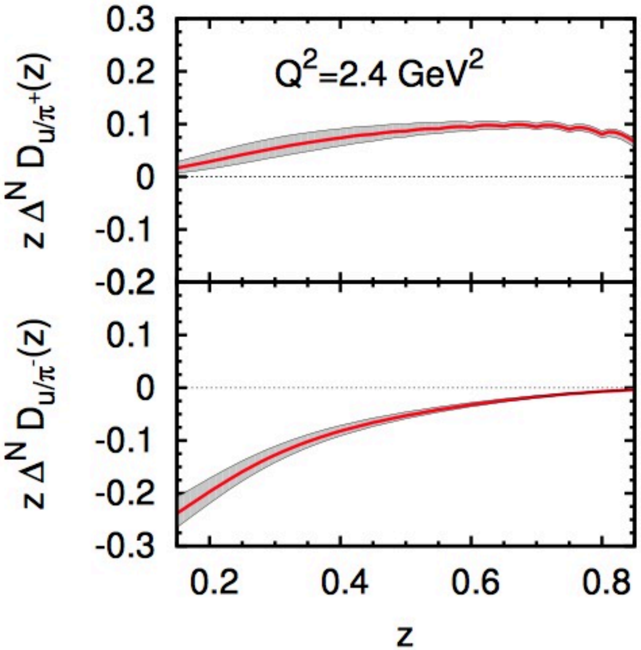


Also data from BaBar (2014) and BESIII (2016)

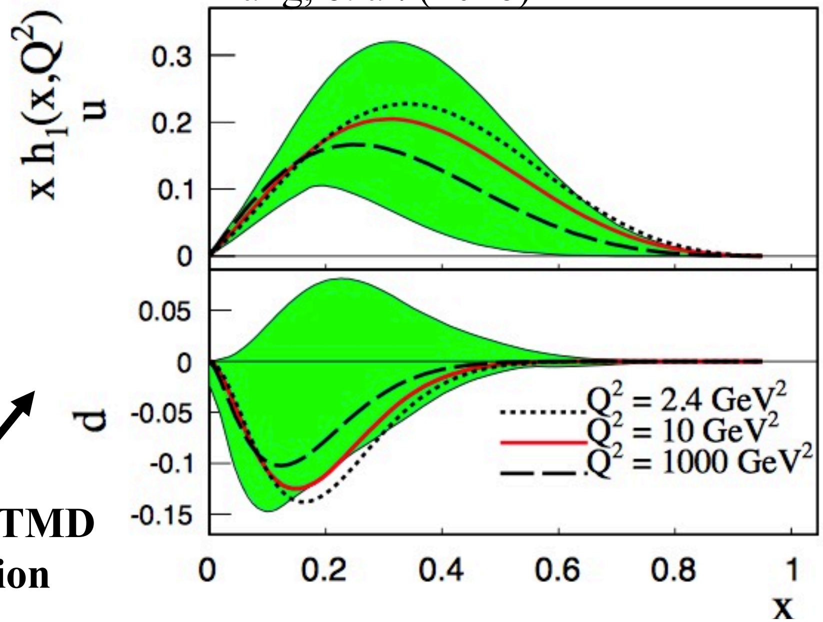
$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\;\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\, H_1^\perp\, \bar{H}_1^\perp\right]$$

Anselmino, et al. (2015)

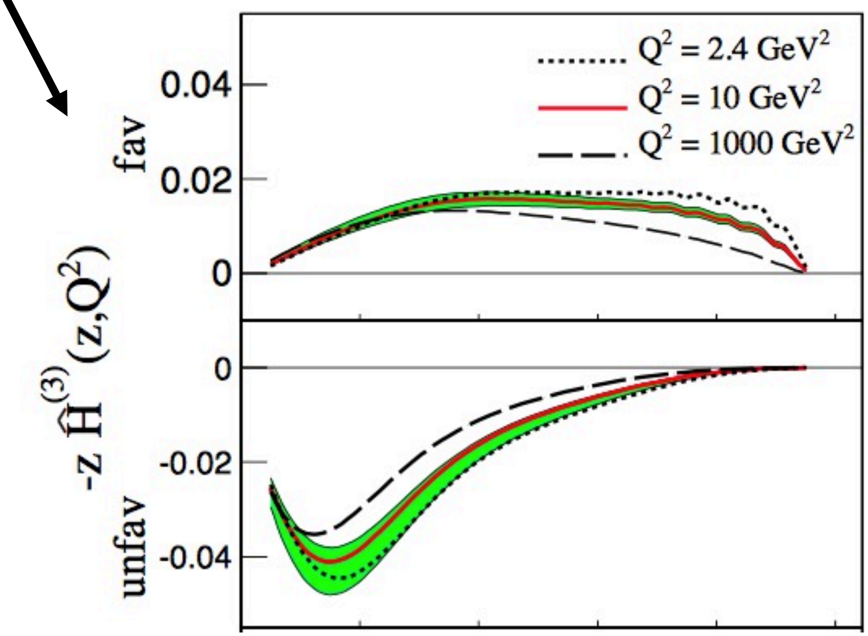


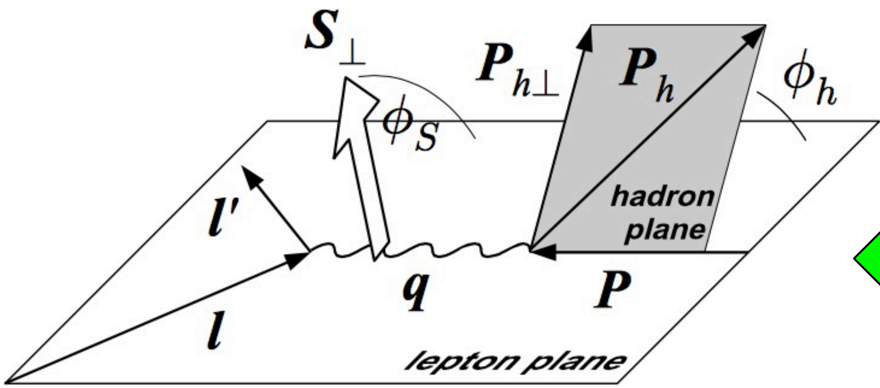


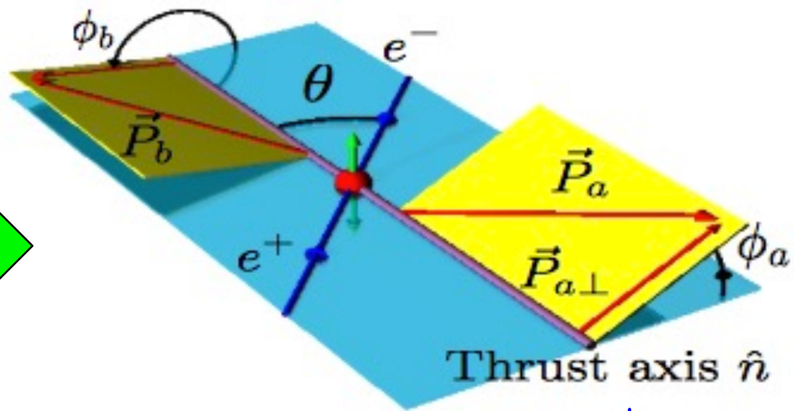
Kang, et al. (2016)



uses full TMD evolution

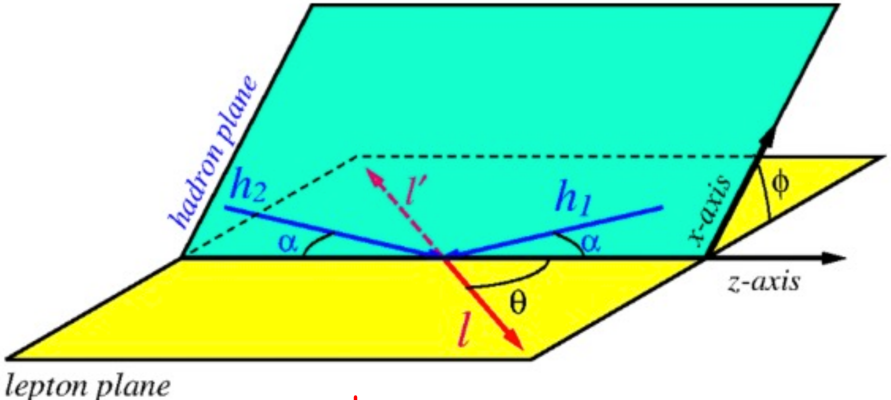






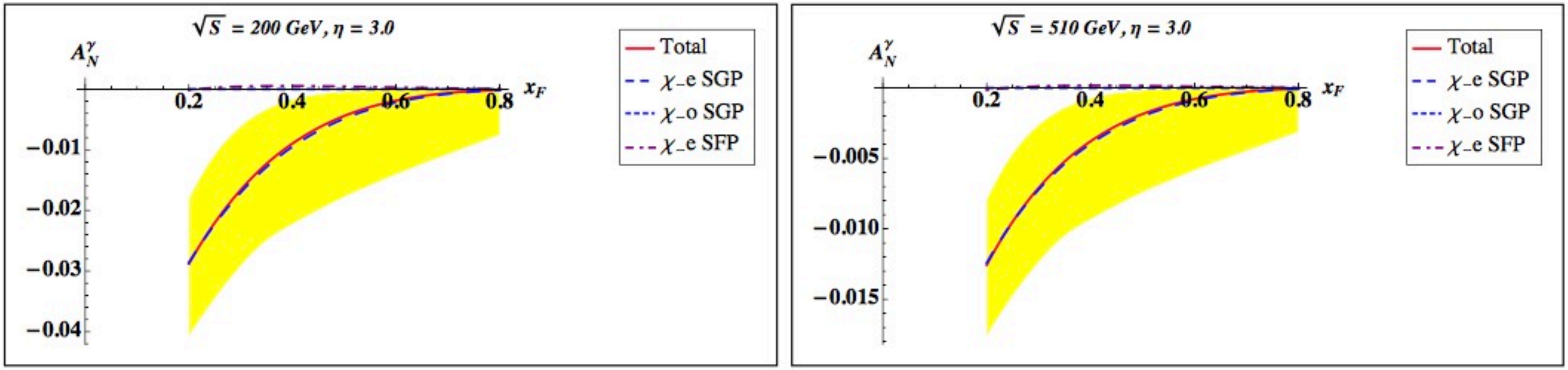
h_1, $f_{1T}^\perp$, $H_1^\perp$

$H_1^\perp$



$f_{1T}^\perp$

A_N in $pp \to \gamma X$



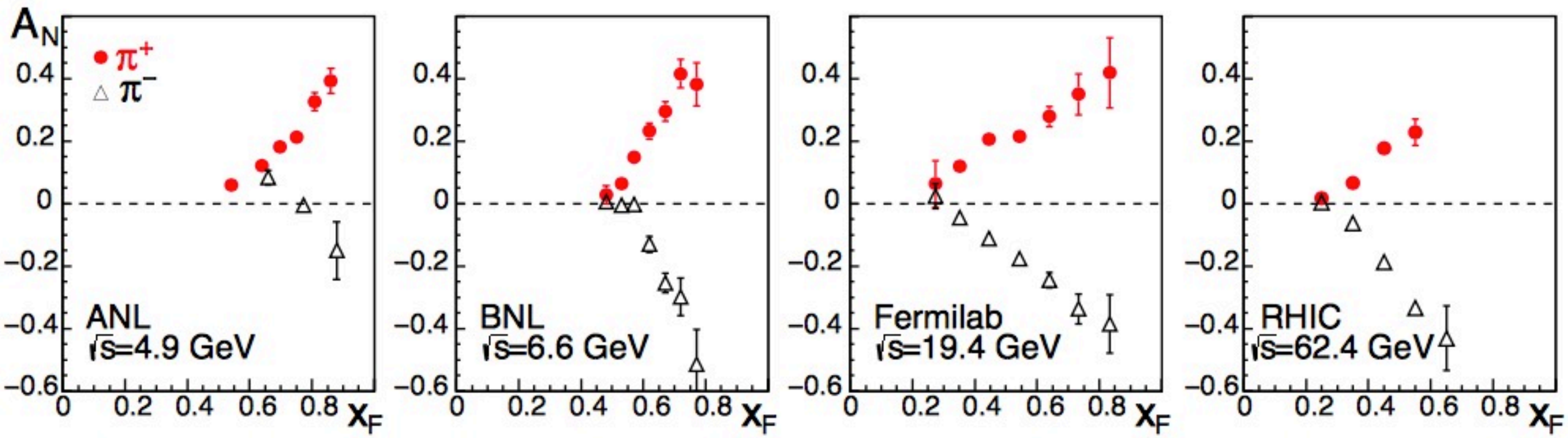
(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
(See also Gamberg, Kang, Prokudin (2013))

Qiu-Sterman term is the main cause of A_N in $pp \to \gamma X$

$$d\Delta\sigma^\gamma \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

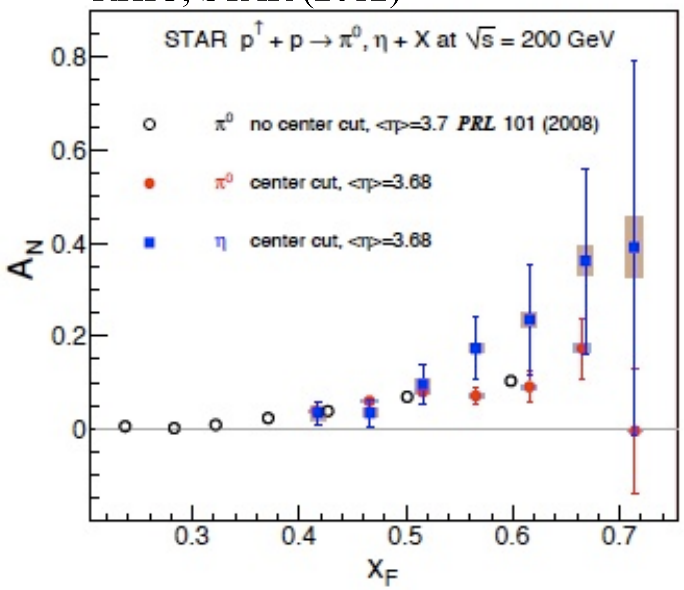
Qiu-Sterman function

A_N in $pp \to \pi X$ – PUZZLE FOR 40+ YEARS!

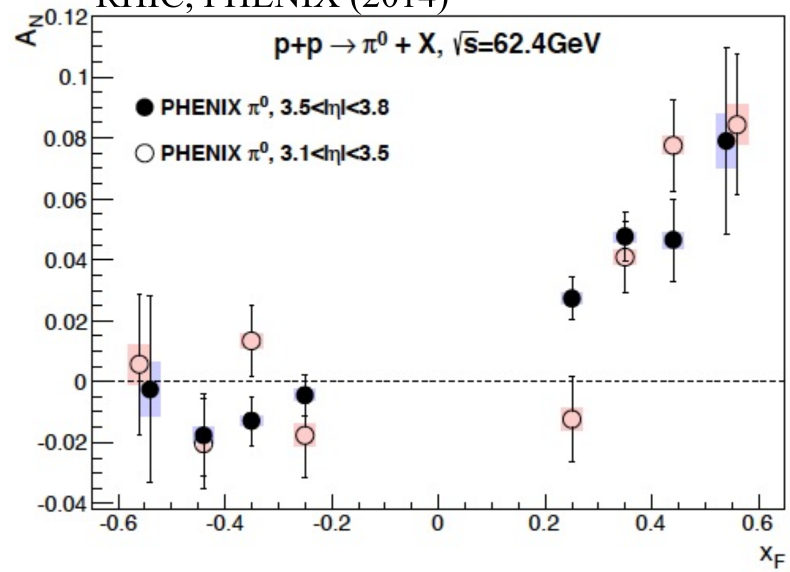


1976 →

RHIC, STAR (2012)



RHIC, PHENIX (2014)



$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

(Metz and DP - PLB **723** (2013))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

(Metz and DP - PLB **723** (2013))

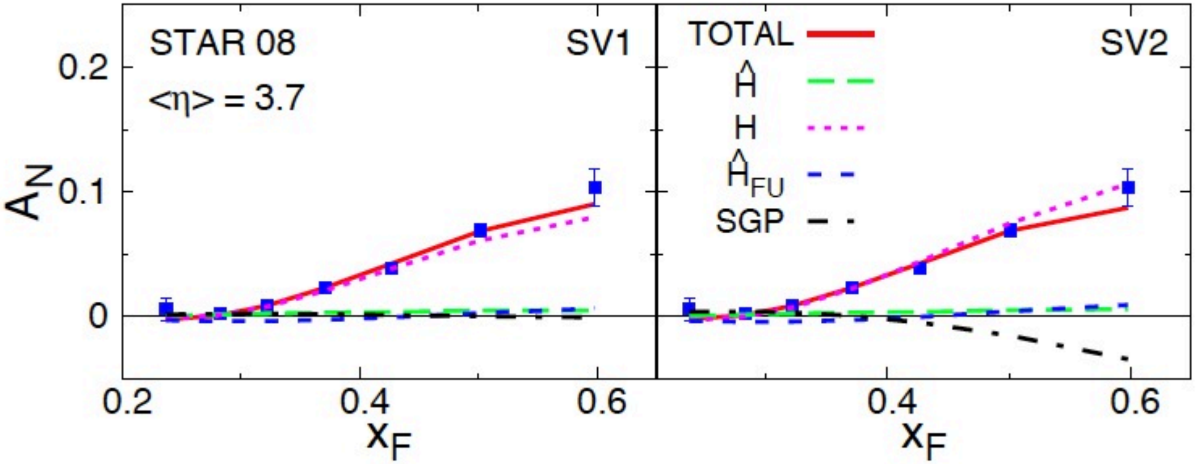
$$H^q(z) \;=\; -2z\, H_1^{\perp(1),q}(z) + \boxed{2z \int_z^{\infty} \frac{dz_1}{z_1^2} \, \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)} \equiv \tilde{H}^q(z)$$

QCD e.o.m. relation (EOMR)

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(H_1^{\perp(1)}, H, \int \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(H_1^{\perp(1)}, \tilde{H}, \int \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

Also included the Qiu-Sterman term $F_{FT}(x,x) \propto f_{1T}^{\perp(1)}(x)$



(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

Fragmentation term is the main cause of A_N in *pp -> π X*

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

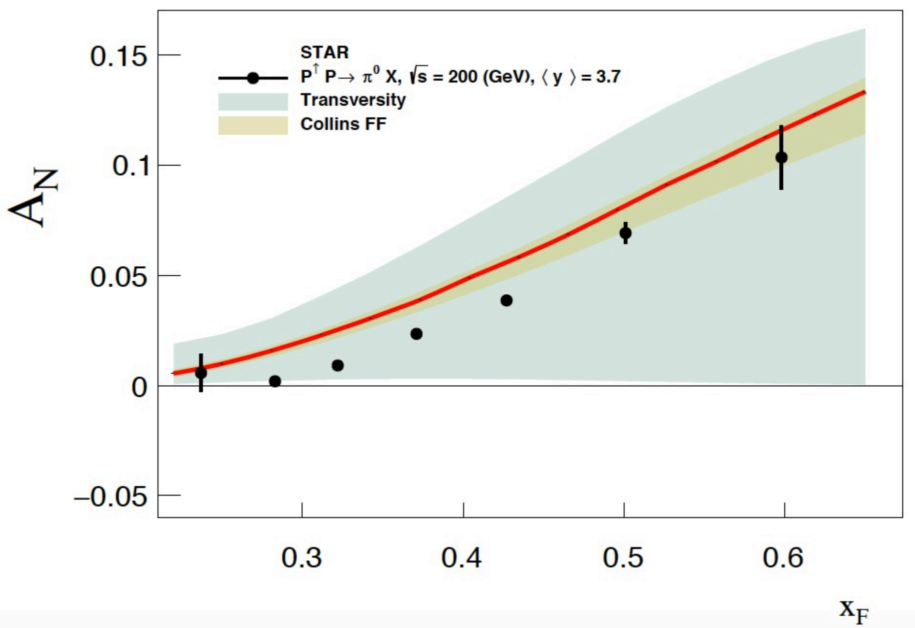
$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(H_1^{\perp(1)}, \tilde{H}, \int \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

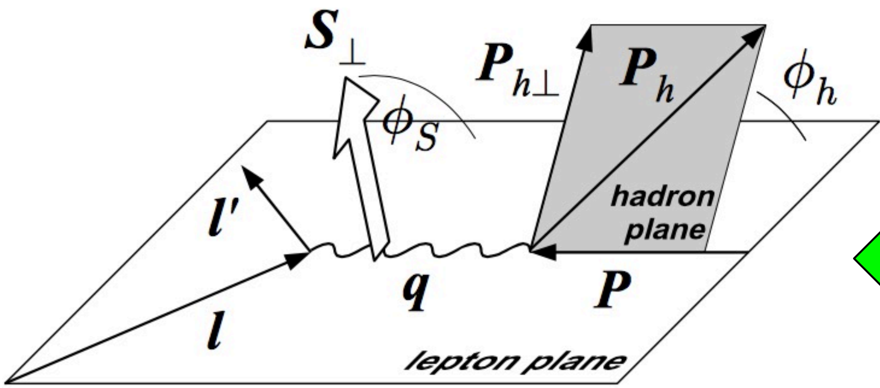
$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \left(H_1^{\perp(1)}, \tilde{H} \right)$$



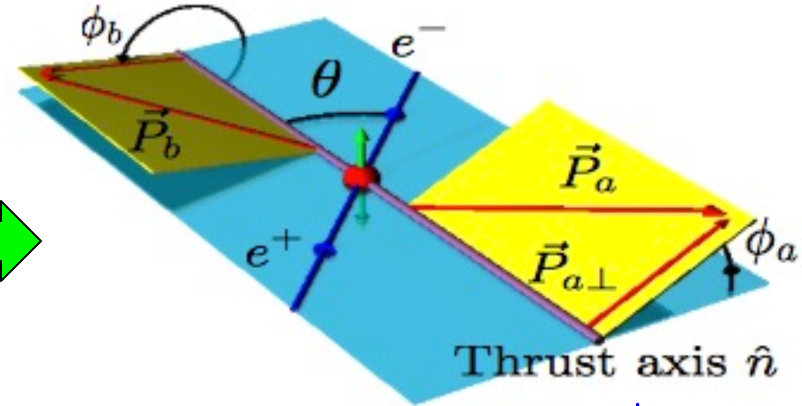
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

Fragmentation term is the main cause of A_N in *pp -> π X*

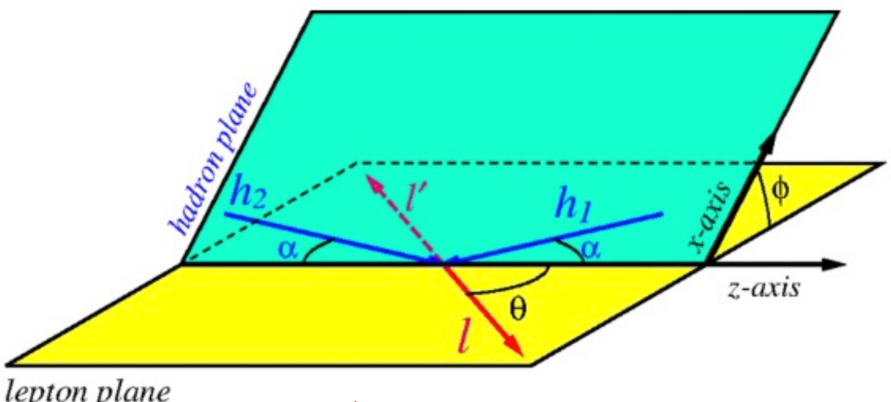
We can constrain ***transversity at large x*** with A_N data from RHIC!



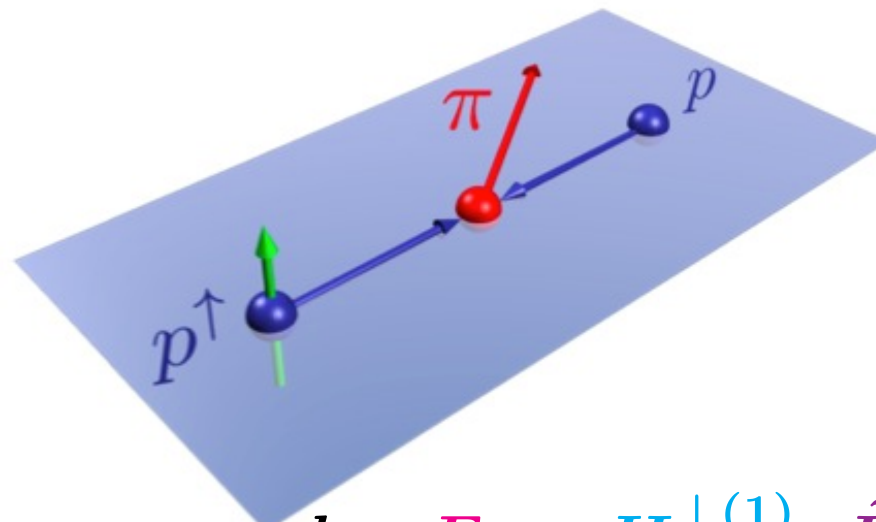
$$h_1,\ f_{1T}^{\perp},\ H_1^{\perp}$$



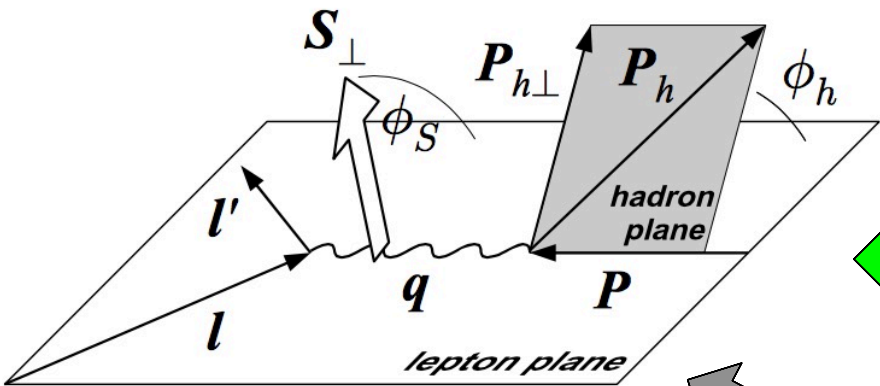
$$H_1^{\perp}$$



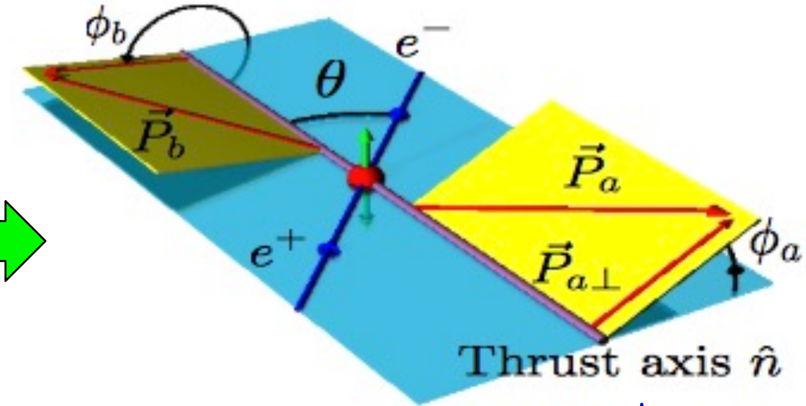
$$f_{1T}^{\perp}$$



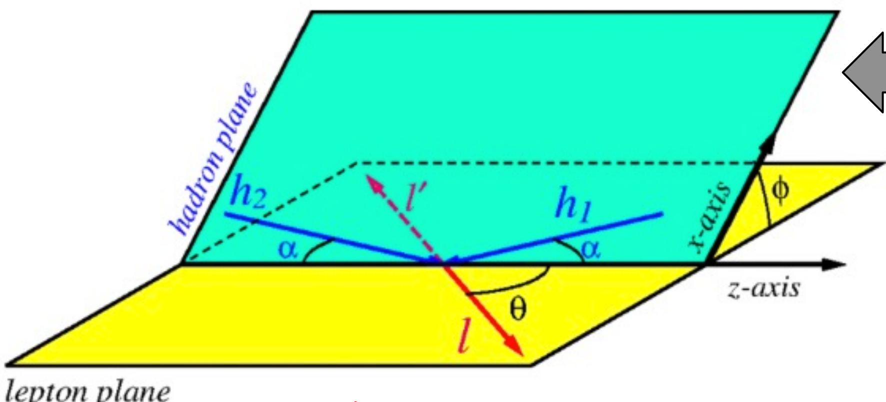
$$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$$



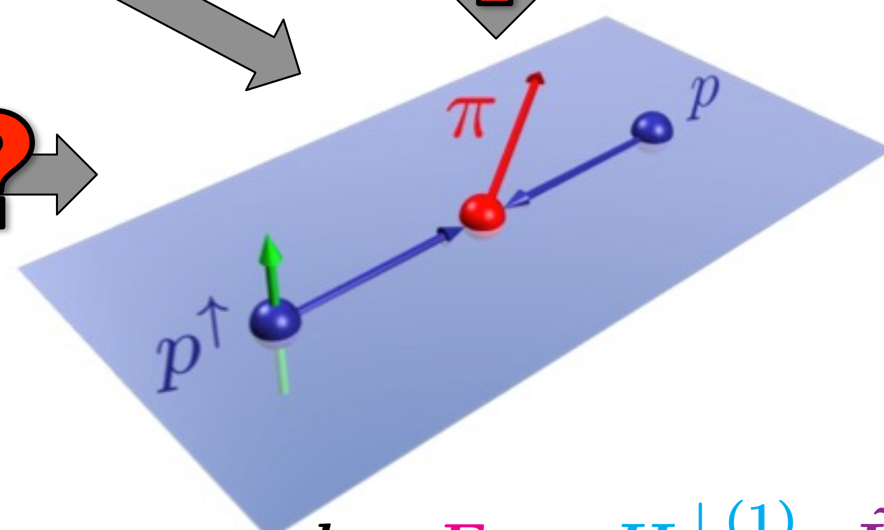
h_1, $f_{1T}^\perp$, $H_1^\perp$



$H_1^\perp$



$f_{1T}^\perp$



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Relations between TMD and CT3 Functions

(and other relevant issues...)

Ongoing work with L. Gamberg, Z. Kang, A. Metz, A. Prokudin, T. Rogers, N. Sato, ...

Parton Model

TMD — CT2

$$\int d^2\vec{k}_T \quad \boldsymbol{f_1(x, \vec{k}_T^2)} \quad = \quad \boldsymbol{f_1(x)}$$

$$\vdots$$

TMD — CT2

$$z^2 \int d^2\vec{p}_\perp \quad \boldsymbol{D_1(z, z^2\vec{p}_\perp^{\,2})} \quad = \quad \boldsymbol{D_1(z)}$$

$$\vdots$$

TMD — kinematical CT3 — dynamical CT3

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \quad \boldsymbol{f_{1T}^{\perp}(x, \vec{k}_T^2)}\Big|_{\rm SIDIS} \quad = \quad \boldsymbol{f_{1T}^{\perp(1)}(x)}\Big|_{\rm SIDIS} \quad = \quad \pi \boldsymbol{F_{FT}(x, x)}$$

Boer, Mulder, Pijlman (2003); Meissner (2009); ...

$$\vdots$$

TMD — kinematical CT3

$$\int d^2\vec{p}_\perp \, \frac{z^2\vec{p}_\perp^{\,2}}{2M_h^2} \quad \boldsymbol{H_1^{\perp}(z, z^2\vec{p}_\perp^{\,2})} \quad = \quad \boldsymbol{H_1^{\perp(1)}(z)}$$

Yuan and Zhou (2009)

$$\vdots$$

QCD – Original CSS

(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\mathcal{FT}_{b_T}\left[\boldsymbol{f_1(x,\vec{k}_T^2;Q^2,\mu_Q)}\right] \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right)$$

$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_1}(b_T,Q)\right]$$

QCD – Original CSS

(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\mathcal{FT}_{b_T}\Big[\boldsymbol{f_1(x,\vec{k}_T^2;Q^2,\mu_Q)}\Big] \sim \Big(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\Big)$$
$$\times\ \exp\Big[\underbrace{-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)}_{\text{perturbative Sudakov factor}} - \underbrace{S_{NP}^{f_1}(b_T,Q)}_{\text{non-perturbative Sudakov factor}}\Big]$$

<u>perturbative Sudakov factor</u>

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*,\mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(g(\mu');1)-\gamma_K(g(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

<u>non-perturbative Sudakov factor</u>

$$\underbrace{g_{f_1}(x,b_T)}_{\text{different for each TMD}} + \underbrace{g_K(b_T)}_{\text{universal}}\ln(Q/Q_0)$$

QCD – Original CSS

(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\mathcal{F}\mathcal{T}_{b_T}\left[\boldsymbol{f_1(x,\vec{k}_T^2;Q^2,\mu_Q)}\right] \sim \left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right)$$

$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_1}(b_T,Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*,\mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(g(\mu');1)-\gamma_K(g(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x,b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1+b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*$$

Note: $b_*(0) = 0 \text{ and } (\mu_{b_*})_{b_*\to 0} = \infty$

QCD – Original CSS

(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\mathcal{FT}_{b_T}\left[\boldsymbol{f_1(x,\vec{k}_T^2;Q^2,\mu_Q)}\right] \sim \left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right) \times \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_1}(b_T,Q)\right]$$

$$\mathcal{FT}_{b_T}\left[\boldsymbol{D_1(z,z^2\vec{p}_\perp^{\,2};Q^2,\mu_Q)}\right] \sim \left(\tilde{C}^{D_1}(z/\hat{z},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{D_1(\hat{z};\mu_{b_*})}\right) \times \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{D_1}(b_T,Q)\right]$$

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(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

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$$\vdots$$

$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{FT}_{b_T}\left[\boldsymbol{f_{1T}^\perp(x,\vec{k}_T^2;Q^2,\mu_Q)}\right] \sim \left(\tilde{C}^{f_{1T}^\perp}(\hat{x}_1,\hat{x}_2,b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{F_{FT}(\hat{x}_1,\hat{x}_2;\mu_{b_*})}\right) \times \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_{1T}^\perp}(b_T,Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

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$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{D_1}(b_T,Q)\right]$$

$$\vdots$$

$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{FT}_{b_T}\left[\boldsymbol{f_{1T}^\perp(x,\vec{k}_T^2;Q^2,\mu_Q)}\right] \sim \left(\tilde{C}^{f_{1T}^\perp}(\hat{x}_1,\hat{x}_2,b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{F_{FT}(\hat{x}_1,\hat{x}_2;\mu_{b_*})}\right)$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_{1T}^\perp}(b_T,Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{FT}_{b_T}\left[\boldsymbol{H_1^\perp(z,z^2\vec{p}_\perp^{\,2};Q^2,\mu_Q)}\right] \sim \left(\tilde{C}^{H_1^\perp}(z/\hat{z},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes \boldsymbol{H_1^{\perp(1)}(\hat{z};\mu_{b_*})}\right)$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{H_1^\perp}(b_T,Q)\right]$$

Echevarria, Idilbi, Scimemi (2014); Kang, Prokudin, Sun, Yuan (2015); ...

QCD – Original CSS

(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

For TSSAs (LO in C-factors)...

$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{F}\mathcal{T}_{b_T}\left[\boldsymbol{f_{1T}^{\perp}(x,\vec{k}_T^2;Q^2,\mu_Q)}\right] \quad \sim \quad \boldsymbol{F_{FT}(x,x;\mu_{b_*})}\,\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_{1T}^{\perp}}(b_T,Q)\right]$$

$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{F}\mathcal{T}_{b_T}\left[\boldsymbol{H_1^{\perp}(z,z^2\vec{p}_\perp^{\,2};Q^2,\mu_Q)}\right] \quad \sim \quad \boldsymbol{H_1^{\perp(1)}(z;\mu_{b_*})}\,\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{H_1^{\perp}}(b_T,Q)\right]$$

QCD – Original CSS

(Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

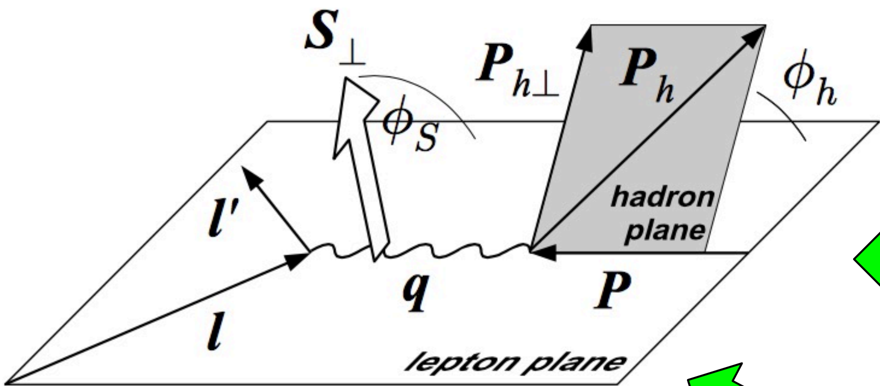
For TSSAs (LO in C-factors)...

$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{FT}_{b_T}\Big[f_{1T}^{\perp}(x,\vec{k}_T^2;Q^2,\mu_Q)\Big] \sim \boxed{F_{FT}(x,x;\mu_{b_*})}\exp\Big[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_{1T}^{\perp}}(b_T,Q)\Big]$$

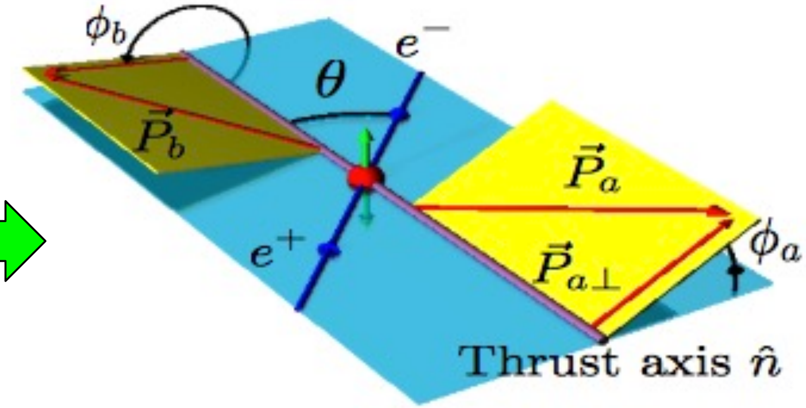
$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{FT}_{b_T}\big[H_1^{\perp}(z,z^2\vec{p}_\perp^{\,2};Q^2,\mu_Q)\big] \sim \boxed{H_1^{\perp(1)}(z;\mu_{b_*})}\exp\Big[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{H_1^{\perp}}(b_T,Q)\Big]$$

The **CT3 functions** are what get extracted in analyses of TSSAs in ***TMD processes*** that use full TMD (CSS) evolution!

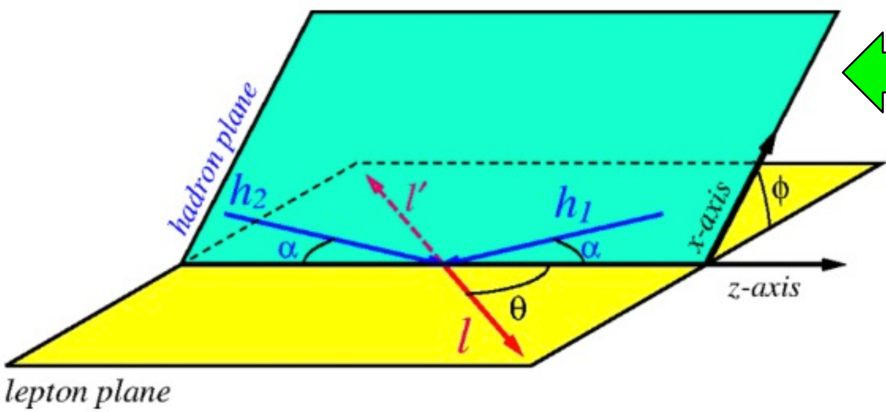
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



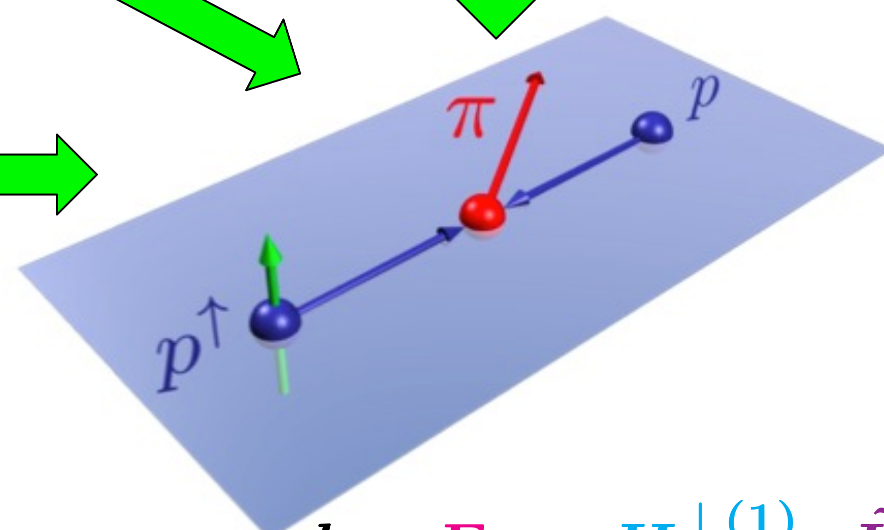
h_1, F_{FT}, $H_1^{\perp(1)}$



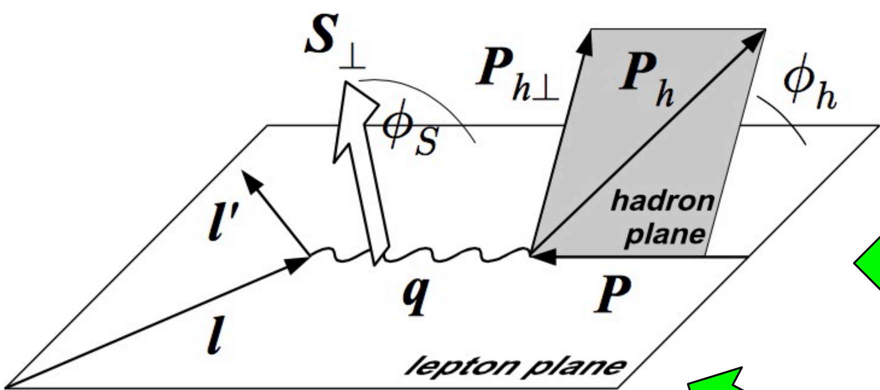
$H_1^{\perp(1)}$



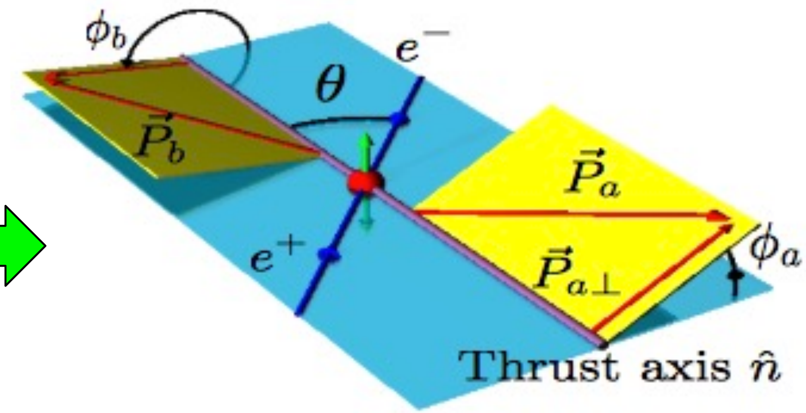
F_{FT}



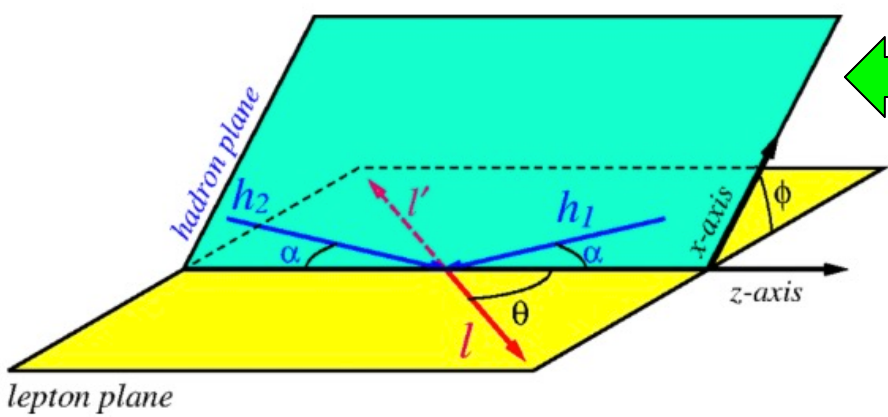
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



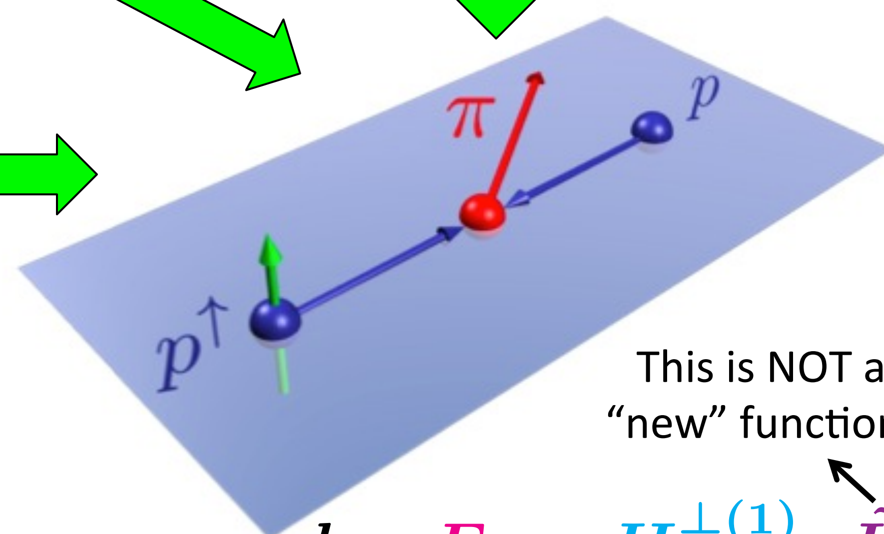
$h_1,\ F_{FT},\ H_1^{\perp(1)}$



$H_1^{\perp(1)}$



F_{FT}



This is NOT a "new" function!

$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T (Mulders, Tangerman (1996); Bacchetta, et al. (2007))

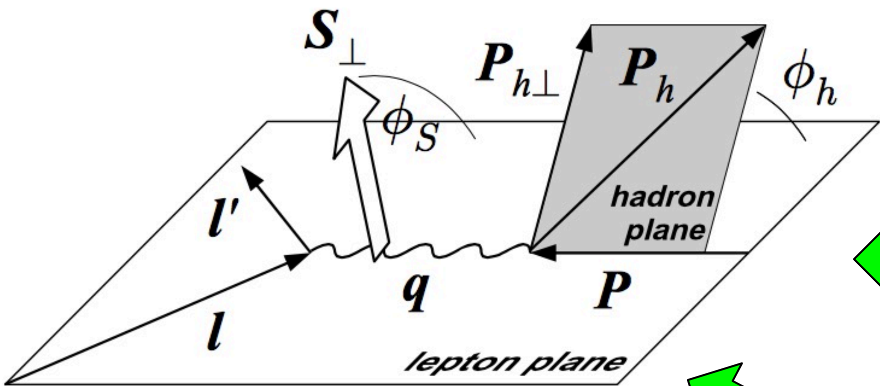
$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}^a(z)}{z}$$

$A_{UT}^{\sin\phi_S}$ in $e^+e^- \to h_1 h_2 X$ integrated over q_T (Boer, Jakob, Mulders (1997))

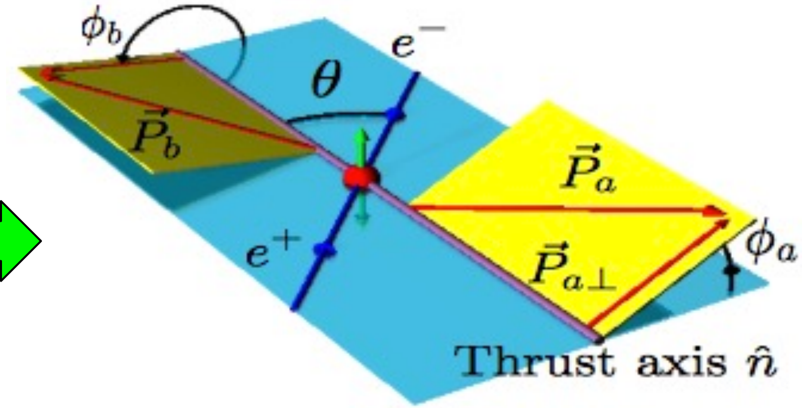
$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}(z_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

And also the TMD version of these (and other) observables (but with many more terms)

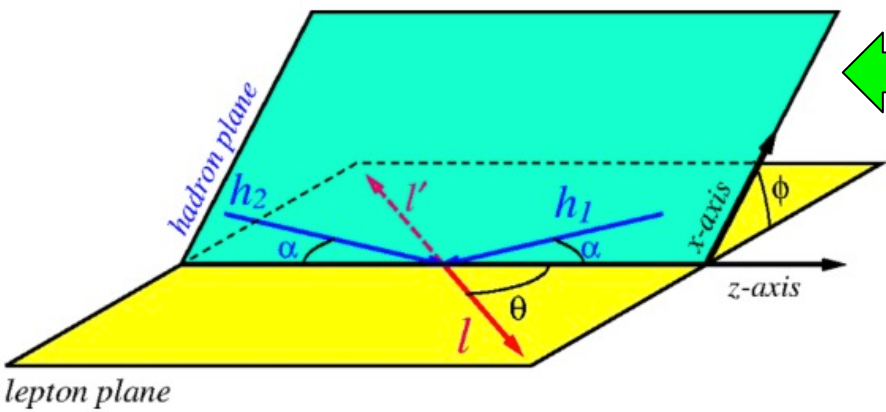
-Note: data from COMPASS, HERMES, and Belle show nonzero effects for the unintegrated version of the above asymmetries



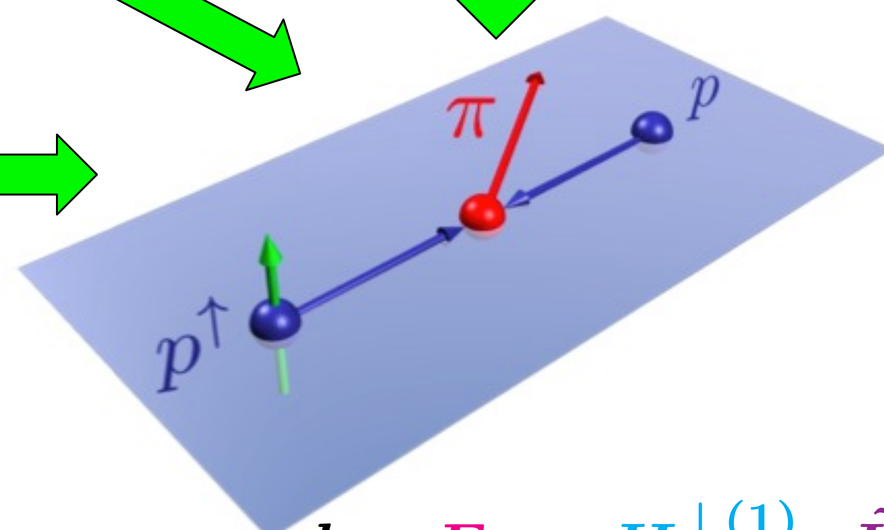
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

We've seen that the naïve relations in the parton model between TMDs and collinear functions become much more complicated in full QCD. *Nevertheless, we would expect at LO to recover parton model results from the full QCD ones.*

Let's analyze the differential cross section in SIDIS to see if this occurs.

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\Gamma(q_T,Q) \equiv \frac{d\sigma}{dx\,dy\,d\phi_s\,dz\,d\phi_h\,(z^2 dq_T^2)} = W(q_T,Q) + Y(q_T,Q) + O((m/Q)^c)\,\Gamma(q_T,Q)$$

$q_T \ll Q$ region

$q_T \sim Q$ region

error term

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\Gamma(q_T,Q) \equiv \frac{d\sigma}{dx\,dy\,d\phi_s\,dz\,d\phi_h\,(z^2 dq_T^2)} = \underbrace{W(q_T,Q)}_{q_T \ll Q \text{ region}} + \underbrace{Y(q_T,Q)}_{q_T \sim Q \text{ region}} + \underbrace{O((m/Q)^c)\,\Gamma(q_T,Q)}_{\text{error term}}$$

A note on the "error term" in the region $m \ll q_T \ll Q$:

$m \sim 1/\lambda\,,\quad q_T \sim 1\,,\quad Q \sim \lambda \quad \Rightarrow \quad \text{error} \sim (1/\lambda^2)^c$

unpolarized: W & Y are both $O(1)$

Sivers ("twist-3"): W & Y are both $O(1/\lambda)$ (see, e.g., Bacchetta, et al. (2008))

➔ *W+Y* method gives an error term that is suppressed in both cases

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\underbrace{\Gamma(q_T, Q) \equiv \frac{d\sigma}{dx dy d\phi_s dz d\phi_h (z^2 dq_T^2)} = W(q_T, Q)}_{q_T \ll Q \text{ region}} + \underbrace{Y(q_T, Q)}_{q_T \sim Q \text{ region}} + \underbrace{O((m/Q)^c)\,\Gamma(q_T, Q)}_{\text{error term}}$$

$$W(q_T, Q) = \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}_T}\, \tilde{W}(b_T, Q)$$

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\Gamma(q_T, Q) \equiv \frac{d\sigma}{dx dy d\phi_s dz d\phi_h (z^2 dq_T^2)} = W(q_T, Q) + Y(q_T, Q) + O((m/Q)^c)\,\Gamma(q_T, Q)$$

q_T << Q region $\qquad$ q_T ~ Q region $\qquad$ error term

$$W(q_T, Q) = \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}_T}\, \tilde{W}(b_T, Q)$$

$$\tilde{W}(b_T, Q) = \sum_j H_j(\mu_Q, Q)\left[\tilde{F}_{UU} + |\vec{S}_\perp|\sin(\phi_h - \phi_s)\,\tilde{F}_{UT}^{\sin(\phi_h-\phi_S)} + \ldots\right]$$

$$\tilde{F}_{UU} = \tilde{f}_1^{j/p}(x, b_T; Q^2, \mu_Q)\,\tilde{D}_1^{h/j}(z, b_T; Q^2, \mu_Q)$$

$$\tilde{F}_{UT}^{\sin(\phi_h-\phi_S)} = \frac{i\vec{b}_T\cdot\hat{h}}{M}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp,j/p}(x, b_T; Q^2, \mu_Q)\right)\tilde{D}_1^{h/j}(z, b_T; Q^2, \mu_Q)$$

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

Just a reminder...

$$\tilde{f}_1(x,b_T;Q^2,\mu_Q) \equiv \mathcal{FT}_{b_T}\Big[f_1(x,\vec{k}_T^2;Q^2,\mu_Q)\Big] \sim \Big(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes f_1(\hat{x};\mu_{b_*})\Big) \times\,\exp\Big[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_1}(b_T,Q)\Big]$$

$$\tilde{D}_1(z,b_T;Q^2,\mu_Q) \equiv \mathcal{FT}_{b_T}[D_1(z,z^2\vec{p}_\perp^{\,2};Q^2,\mu_Q)] \sim \Big(\tilde{C}^{D_1}(z/\hat{z},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes D_1(\hat{z};\mu_{b_*})\Big) \times\,\exp\Big[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{D_1}(b_T,Q)\Big]$$

$$\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T;Q^2,\mu_Q) \equiv \frac{1}{b_T}\frac{\partial}{\partial b_T}\mathcal{FT}_{b_T}\Big[f_{1T}^{\perp}(x,\vec{k}_T^2;Q^2,\mu_Q)\Big]$$

$$\sim \Big(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1,\hat{x}_2,b_*(b_T);\mu_{b_*}^2,\mu_{b_*},g(\mu_{b_*}))\otimes F_{FT}(\hat{x}_1,\hat{x}_2;\mu_{b_*})\Big) \times\,\exp\Big[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q)-S_{NP}^{f_{1T}^{\perp}}(b_T,Q)\Big]$$

W+Y **Method – Original CSS**

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\Gamma(q_T, Q) \equiv \frac{d\sigma}{dx dy d\phi_s dz d\phi_h (z^2 dq_T^2)} = W(q_T, Q) + Y(q_T, Q) + O((m/Q)^c)\,\Gamma(q_T, Q)$$

q$_T$ << *Q* region

q$_T$ ~ *Q* region

error term

$$W(q_T, Q) = \int \frac{d^2\vec{b}_T}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}_T}\, \tilde{W}(b_T, Q)$$

$$\tilde{W}(b_T, Q) = \sum_j H_j(\mu_Q, Q)\left[\tilde{F}_{UU} + |\vec{S}_\perp|\sin(\phi_h - \phi_s)\,\tilde{F}_{UT}^{\sin(\phi_h - \phi_S)} + \ldots\right]$$

$$\tilde{F}_{UU} = \tilde{f}_1^{j/p}(x, b_T; Q^2, \mu_Q)\,\tilde{D}_1^{h/j}(z, b_T; Q^2, \mu_Q)$$

$$\tilde{F}_{UT}^{\sin(\phi_h - \phi_S)} = \frac{i\vec{b}_T\cdot\hat{h}}{M}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp,j/p}(x, b_T; Q^2, \mu_Q)\right)\tilde{D}_1^{h/j}(z, b_T; Q^2, \mu_Q)$$

NOT associated with the scale evolution

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

TMD (q_T << Q)

$$\Gamma^{\rm unp}(q_T,Q) \sim H\left\{\mathcal{FT}_{q_T}\Big[\tilde{f}_1(x,b_T)\tilde{D}_1(z,b_T)\Big]\right\}$$

LO Collinear

$$\int d^2\vec{q}_T\,\Gamma(q_T,Q)\big|_{LO} \sim H_{LO}\,\left[f_1(x)D_1(z)\right]$$

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

TMD (q_T << Q)

$$\Gamma^{\rm unp}(q_T, Q) \sim H \left\{ \mathcal{FT}_{q_T}\Big[\tilde{f}_1(x, b_T)\tilde{D}_1(z, b_T)\Big]\right\}$$

LO Collinear

$$\int d^2\vec{q}_T\, \Gamma(q_T, Q)\Big|_{LO} \sim H_{LO}\; [f_1(x) D_1(z)]$$

But one finds...

$$\int d^2\vec{q}_T\, \Gamma(q_T, Q) \sim H\;\; \Big[\tilde{f}_1(x, b_T)\tilde{D}_1(z, b_T)\Big]\Big|_{b_T\to 0} \sim b_T^a \times (\log\text{ corrections}) \;\; \mathbf{= \;\; 0!}$$

Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016)

The q_T-integrated cross section does NOT reduce to the expected LO collinear result!

W+Y Method – Original CSS

(Collins, Soper Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

TMD (q_T << Q)

$$\Gamma^{\rm unp}(q_T,Q) \sim H\left\{\mathcal{FT}_{q_T}\Big[\tilde{f}_1(x,b_T)\tilde{D}_1(z,b_T)\Big]\right\}$$

LO Collinear

$$\int d^2\vec{q}_T\,\Gamma(q_T,Q)\big|_{LO} \sim H_{LO}\,\left[f_1(x)D_1(z)\right]$$

But one finds...

$$\int d^2\vec{q}_T\,\Gamma(q_T,Q) \sim H\,\left[\tilde{f}_1(x,b_T)\tilde{D}_1(z,b_T)\right]\Big|_{b_T\to 0} \sim b_T^a \times (\text{log corrections}) \;\; \mathbf{= \;\; 0!}$$

Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016)

The q_T-integrated cross section does NOT reduce to the expected LO collinear result!

And in fact...

$$\tilde{f}_1(x,b_T\to 0) = \int d^2\vec{k}_T\, f_1(x,\vec{k}_T^2) \;\; \mathbf{= \;\; 0!}$$

$$\tilde{D}_1(z,b_T\to 0) = \int d^2\vec{p}_\perp\, D_1(z,z^2\vec{p}_\perp^{\,2}) \;\; \mathbf{= \;\; 0!}$$

W+Y Method – Improved CSS

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Source of this issue: S_{pert} has large logs because, in the original CSS b_*-prescription, $b_*(0) = 0$

$$S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) = -\ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*(b_T);\mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]$$

W+Y Method – Improved CSS

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

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Resolution: Place a lower cut-off on b_T. Also, explicitly cut off W at large q_T.

$$W(q_T,Q) \to W_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int\frac{d^2\vec{b}_T}{(2\pi)^2}\,e^{i\vec{q}_T\cdot\vec{b}_T}\,\tilde{W}(b_c(b_T),Q)$$

$$\text{where } b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5Q)^2} \longrightarrow \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1C_5Q}{b_0}$$

Note: This also leads to a new *Y*-term.

W+Y Method – Improved CSS

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Source of this issue: S_{pert} has large logs because, in the original CSS b_*-prescription, $b_*(0) = 0$

$$S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) = -\ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*(b_T);\mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]$$

Resolution: Place a lower cut-off on b_T. Also, explicitly cut off W at large q_T.

$$W(q_T,Q) \to W_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int\frac{d^2\vec{b}_T}{(2\pi)^2}\,e^{i\vec{q}_T\cdot\vec{b}_T}\,\tilde{W}(b_c(b_T),Q)$$

$$\text{where}\;\; b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5Q)^2} \longrightarrow \mu_{b_*}\;\text{is cut off at}\;\mu_c \approx \frac{C_1C_5Q}{b_0}$$

Note: This also leads to a new Y-term.

With these modifications, one now finds...

$$\int d^2\vec{q}_T\,\Gamma(q_T,Q) = H_{j,LO}\,f_1^{j/p}(x;\mu_c)\,D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

W+Y Method – Improved CSS

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Source of this issue: S_{pert} has large logs because, in the original CSS b_*-prescription, $b_*(0) = 0$

$$S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) = -\ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*(b_T);\mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]$$

Resolution: Place a lower cut-off on b_T. Also, explicitly cut off W at large q_T.

$$W(q_T,Q) \to W_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int\frac{d^2\vec{b}_T}{(2\pi)^2}\,e^{i\vec{q}_T\cdot\vec{b}_T}\,\tilde{W}(b_c(b_T),Q)$$

$$\text{where } b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2} \longrightarrow \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

Note: This also leads to a new *Y*-term.

With these modifications, one now finds...

$$\int d^2\vec{q}_T\,\Gamma(q_T,Q) = H_{j,LO}\,f_1^{j/p}(x;\mu_c)\,D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

Do the same issues arise for TSSAs and can the improved CSS formalism resolve them?

TMD ($q_T << Q$)

$$\Gamma^{\rm siv}(q_T,Q) \sim H\left\{\mathcal{FT}_{q_T}\left[\frac{i\vec{b}_T\cdot\hat{h}}{M}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T)\right)\tilde{D}_1(z,b_T)\right]\right\}$$

LO Collinear (Twist-3)

$$\int d^2\vec{q}_T\,\epsilon^{q_T S_\perp}\,\Gamma(q_T,Q)\big|_{LO} \sim H_{LO}\,\left[F_{FT}(x,x)D_1(z)\right]$$

Boer, Mulders (1997);

NLO $\big\{$ Kang, Vitev, Xing (2013); Yoshida (2016)

TMD (q_T << Q)

$$\Gamma^{\rm siv}(q_T,Q) \sim H \left\{ \mathcal{FT}_{q_T}\left[\frac{i\vec{b}_T\cdot\hat{h}}{M}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T)\right)\tilde{D}_1(z,b_T)\right]\right\}$$

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Boer, Mulders (1997);

NLO $\{$ Kang, Vitev, Xing (2013); Yoshida (2016)

Original CSS...

$$\int d^2\vec{q}_T\, \epsilon^{q_T S_\perp}\,\Gamma(q_T,Q) \sim H\,\left[\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T)\right)\tilde{D}_1(z,b_T)\right]\Bigg|_{b_T\to 0} \sim \underbrace{b_T^a \times (\text{log corrections})}_{S_{pert}\text{ is same for unpol. and pol.}} \;\mathbf{= 0}$$

TMD (q_T << Q)

LO Collinear (Twist-3)

$$\Gamma^{\rm siv}(q_T,Q) \sim H\left\{\mathcal{FT}_{q_T}\left[\frac{i\vec{b}_T\cdot\hat{h}}{M}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T)\right)\tilde{D}_1(z,b_T)\right]\right\} \quad \Bigg| \quad \int d^2\vec{q}_T\,\epsilon^{q_T S_\perp}\,\Gamma(q_T,Q)\big|_{LO} \sim H_{LO}\,[F_{FT}(x,x)D_1(z)]$$

Boer, Mulders (1997);

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$$-\frac{1}{M^2}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T)\right)\Bigg|_{b_T\to 0} = f_{1T}^{\perp(1)}(x) \quad \mathbf{= 0}$$

TMD (q_T << Q) | LO Collinear (Twist-3)

$$\Gamma^{\rm siv}(q_T,Q) \sim H\left\{\mathcal{FT}_{q_T}\left[\frac{i\vec{b}_T\cdot\hat{h}}{M}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T)\right)\tilde{D}_1(z,b_T)\right]\right\} \quad \Bigg| \quad \int d^2\vec{q}_T\, \epsilon^{q_T S_\perp}\,\Gamma(q_T,Q)\big|_{LO} \sim H_{LO}\,\left[F_{FT}(x,x)D_1(z)\right]$$

Boer, Mulders (1997);
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Improved CSS...

$$W^{\rm siv}(q_T,Q) \to W^{\rm siv}_{\rm New}(q_T,Q;\eta,C_5) \equiv \Xi\left(\frac{q_T}{Q},\eta\right)\int\frac{d^2\vec{b}_T}{(2\pi)^2}\,e^{i\vec{q}_T\cdot\vec{b}_T}\,\tilde{W}^{\rm siv}(b_c(b_T),Q)$$

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$$\tilde{F}_{UT}^{\sin(\phi_h-\phi_S)}\to\tilde{F}_{UT,\,\rm New}^{\sin(\phi_h-\phi_S)}=\frac{i\vec{b}_T\cdot\hat{h}}{M}\underbrace{\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp,j/p}(x,b_T;Q^2,\mu_Q)\right)}\underbrace{\tilde{D}_1^{h/j}(z,b_T;Q^2,\mu_Q)}$$

NOT replaced with $b_c(b_T)$

$b_*(b_T)$ now replaced by $b_*(b_c(b_T))$

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$$\Gamma^{\rm siv}(q_T,Q) \sim H\left\{\mathcal{FT}_{q_T}\left[\frac{i\vec{b}_T\cdot\hat{h}}{M}\left(\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T)\right)\tilde{D}_1(z,b_T)\right]\right\}$$

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Boer, Mulders (1997);

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$b_*(b_T)$ now replaced by $b_*(b_c(b_T))$

$$\int d^2\vec{q}_T\,\epsilon^{q_T S_\perp}\,\Gamma(q_T,Q) = H_{j,LO}\left[\left(\pi\,M\,F_{FT}^{j/p}(x,x;\mu_c)\right)D_1^{h/j}(z;\mu_c)\right] + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

The expected LO result is recovered for the Sivers (“twist-3”) effect

Towards a Global Analysis of TMD and CT3 Observables

$$H^q(z) \;=\; -2z\,H_1^{\perp(1),q}(z) + 2z\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{q,\Im}(z,z_1)$$

QCD e.o.m. relation (EOMR)

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right)H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

QCD e.o.m. relation (EOMR)

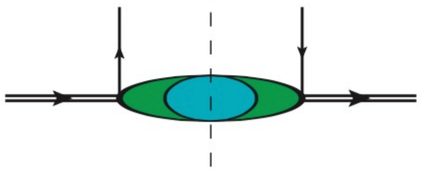
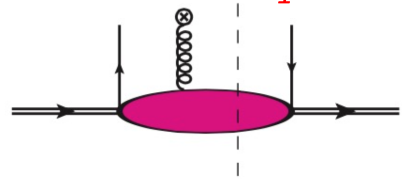
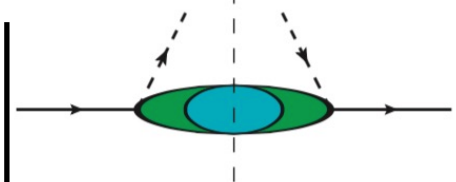
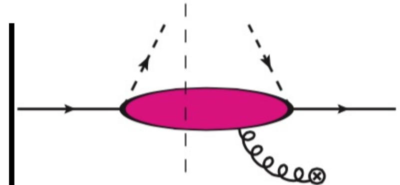
$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

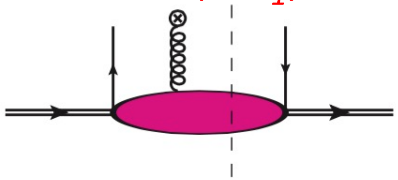
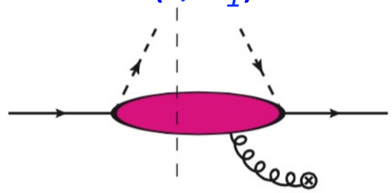
$$\boldsymbol{H(z)} = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\, 2 \left[\frac{\left(2(\frac{2}{z_1} - \frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \boldsymbol{\hat{H}_{FU}^{\Im}(z_1, z_2)} \right]$$

$$\boldsymbol{H_1^{\perp(1)}(z)} = -\frac{2}{z} \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} \frac{\left(\frac{2}{z_1} - \frac{1}{z_2}\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \boldsymbol{\hat{H}_{FU}^{\Im}(z_1, z_2)}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

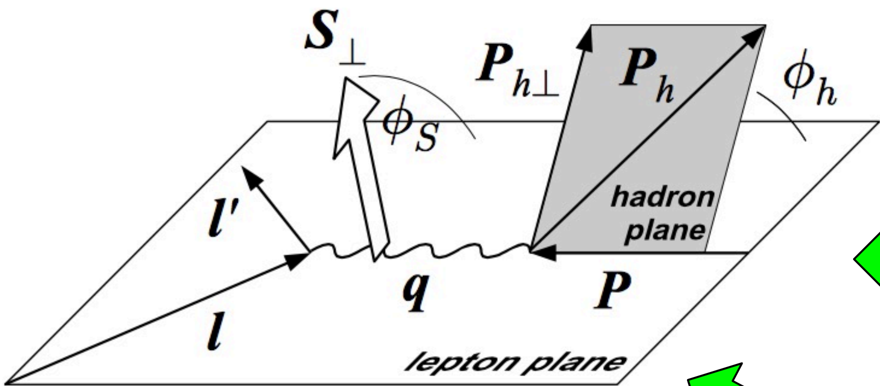
Hadron Pol.	PDF (x) 		PDF (x, x_1) 	FF (z) 		FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	$\cancel{e}$	$\cancel{h_1^{\perp(1)}}$	H_{FU}	$\cancel{E}, \cancel{H}$	$\cancel{H_1^{\perp(1)}}$	$\hat{H}_{FU}^{\Re,\Im}$
L	$\cancel{h_L}$	$\cancel{h_{1L}^{\perp(1)}}$	H_{FL}	$\cancel{E_L}, \cancel{H_L}$	$\cancel{H_{1L}^{\perp(1)}}$	$\hat{H}_{FL}^{\Re,\Im}$
T	$\cancel{g_T}$	$\cancel{f_{1T}^{\perp(1)}}$, $\cancel{g_{1T}^{(1)}}$	F_{FT}, G_{FT}	$\cancel{D_T}, \cancel{G_T}$	$\cancel{D_{1T}^{\perp(1)}}$, $\cancel{G_{1T}^{(1)}}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

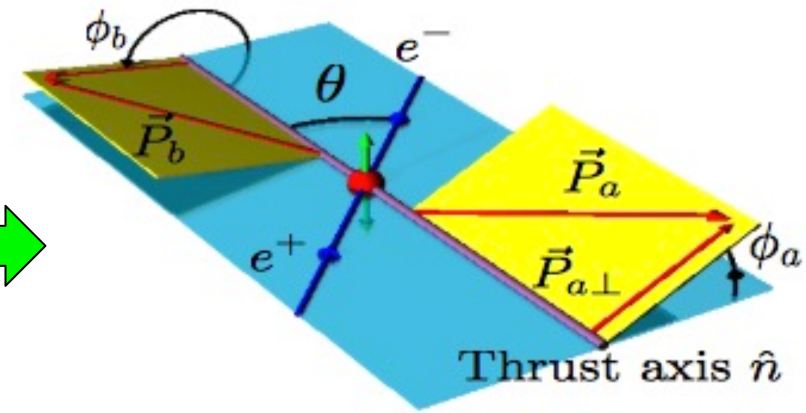
Hadron Pol.	PDF (x, x_1) 	FF (z, z_1) 
U	dynamical H_{FU}	dynamical $\hat{H}_{FU}^{\Re,\Im}$
L	H_{FL}	$\hat{H}_{FL}^{\Re,\Im}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

ALL *transverse spin observables* are driven by *multi-parton correlations*

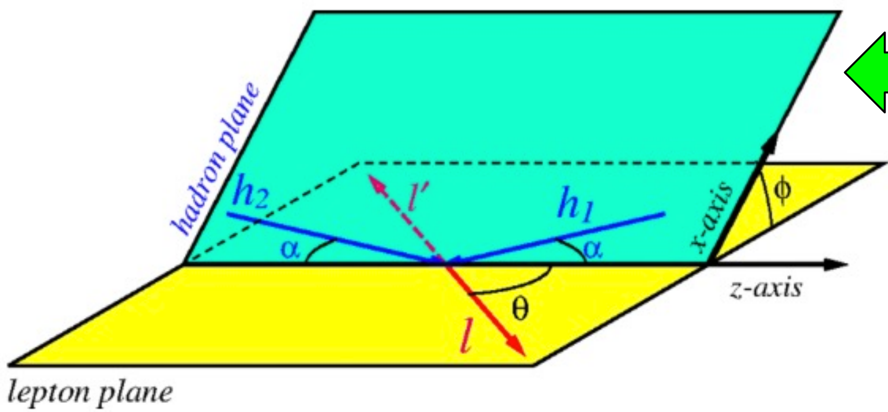
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))



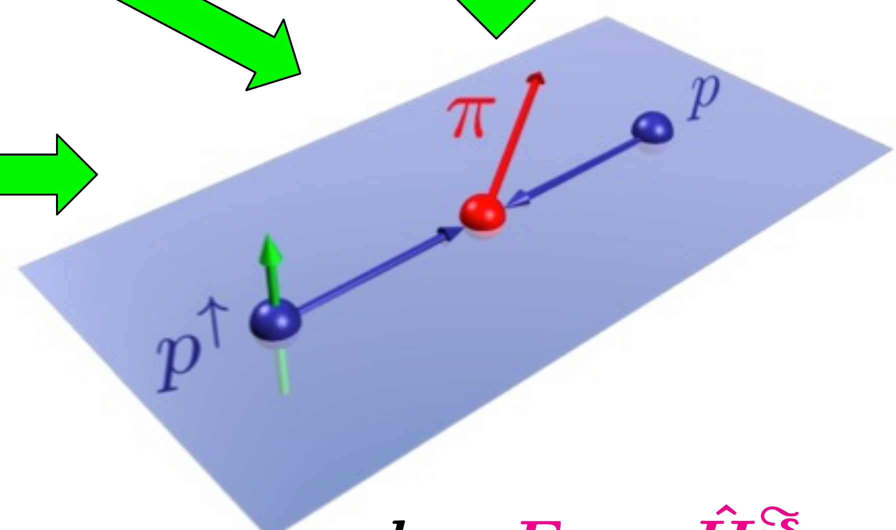
h_1, F_{FT}, $\hat{H}_{FU}^{\mathfrak{S}}$



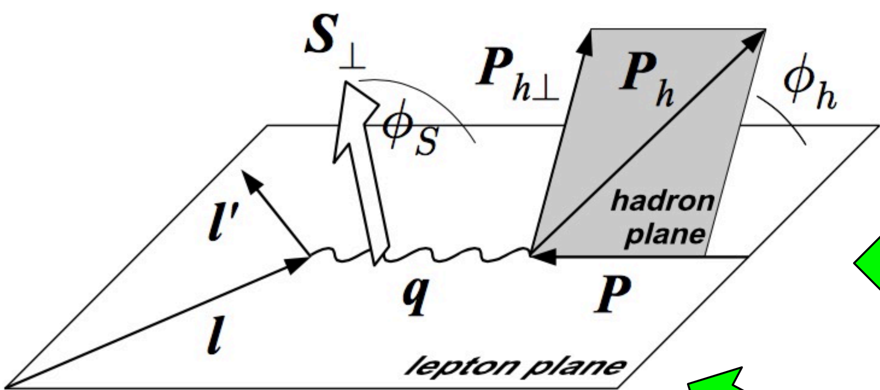
$\hat{H}_{FU}^{\mathfrak{S}}$



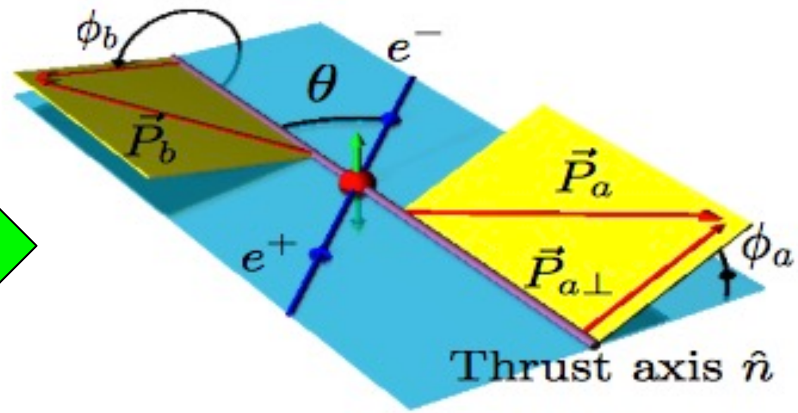
F_{FT}



h_1, F_{FT}, $\hat{H}_{FU}^{\mathfrak{S}}$

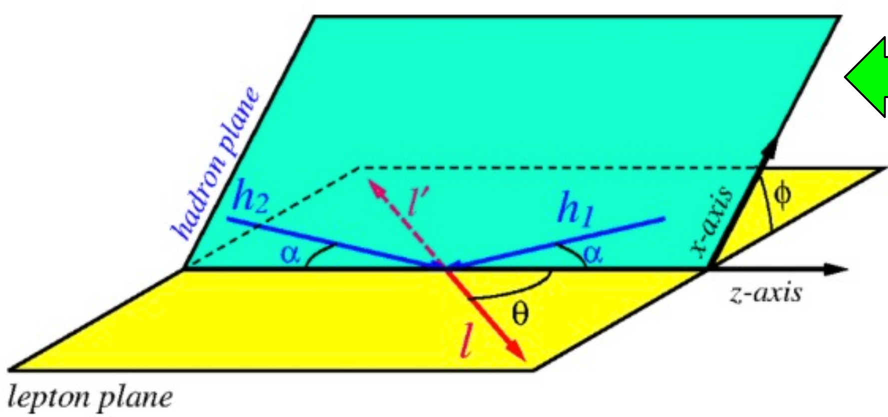


$h_1, \; F_{FT}, \; H_1^{\perp(1)}, \; \tilde{H}$

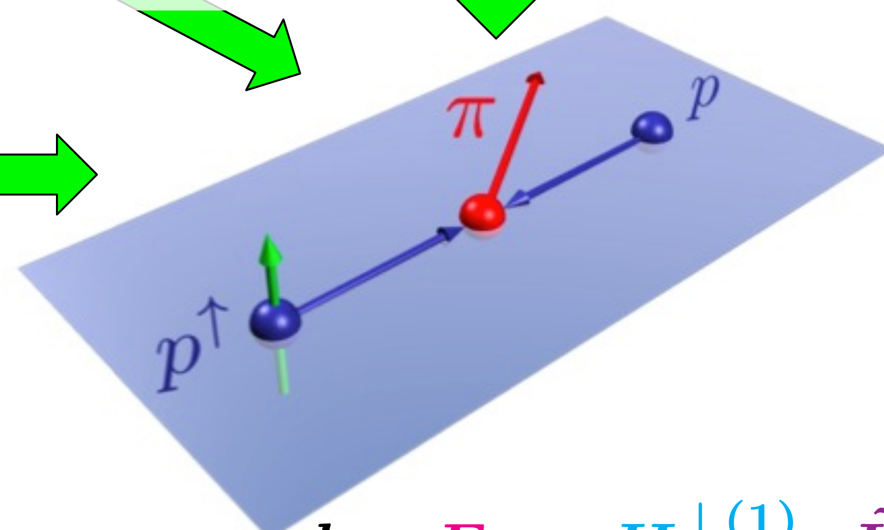


$H_1^{\perp(1)}, \; \tilde{H}$

Need to perform a “global” analysis!



F_{FT}



$h_1, \; F_{FT}, \; H_1^{\perp(1)}, \; \tilde{H}$

Summary

- TSSAs have been studied in both TMD processes (SIDIS, e^+e^-, DY) and collinear processes (A_N in pp & lp collisions)
- The current TMD formalism using improved CSS (iCSS) allows one to rigorously connect these two different types of observables
- (LIRs + EOMRs + iCSS) = ***ALL*** transverse spin observables are driven by 3-parton (dynamical) functions
- A global analysis of TMD *AND* collinear twist-3 transverse-spin observables is now possible

Back-up Slides

-Use Qiu-Sterman function extracted by Echevarria, Idilbi, Kang, Vitev (2014) from the Sivers asymmetry in SIDIS using full TMD evolution ➔ negligible (and opposite sign to the data)

$$\boxed{f^{\perp q(\alpha)}_{1T,\text{SIDIS}}(x,b;Q)} = \left(\frac{ib^\alpha}{2}\right)\boxed{T_{q,F}(x,x,c/b_*)}\exp\left\{-\int_{c/b_*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}\exp\left\{-b^2\left(g_1^{\text{sivers}}+\frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

(b-space) Sivers function

Qiu-Sterman function

-Use first k_T-moment of Collins function and **transversity PDF** extracted by Kang, Prokudin, Sun, Yuan (2016) from the Collins asymmetry in SIDIS/e^+e^- using full TMD evolution

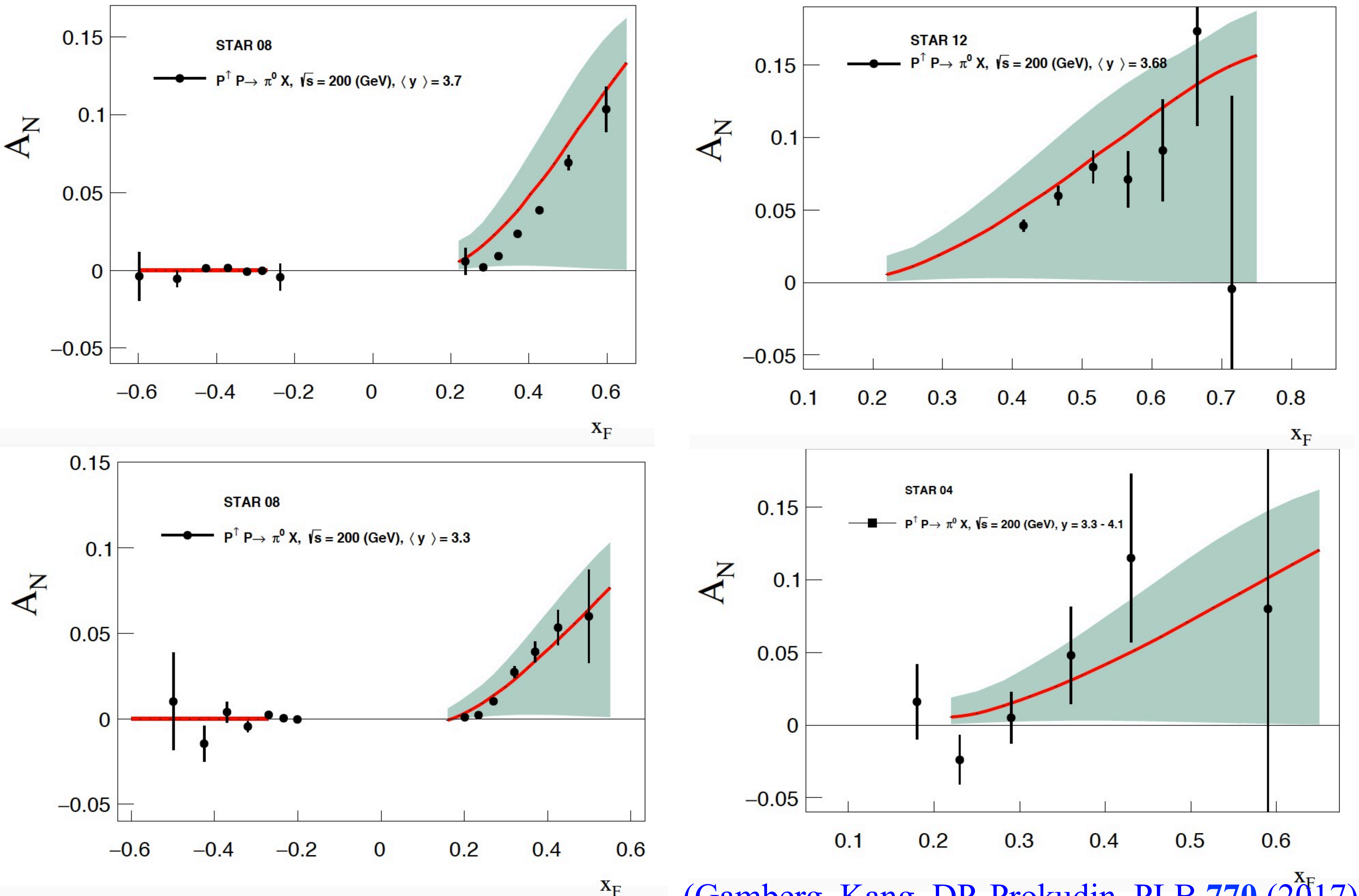
$$\boxed{\widetilde{H}^{(sub)\perp\alpha}_{1\,h/q}(z_h,b,\rho;Q^2,Q)} = \left(\frac{-ib^\alpha}{2z_h}\right)e^{-\frac{1}{2}S_{\text{pert}}(Q,b_*)-S_{\text{NP}}^{D_1}(Q,b)}\,\widetilde{\mathcal{H}}_c(\alpha_s(Q))\,\delta\hat{C}_{q'\leftarrow q}\otimes\boxed{\hat{H}^{(3)}_{h/q'}(z_h,\mu_b)}$$

(b-space) Collins function

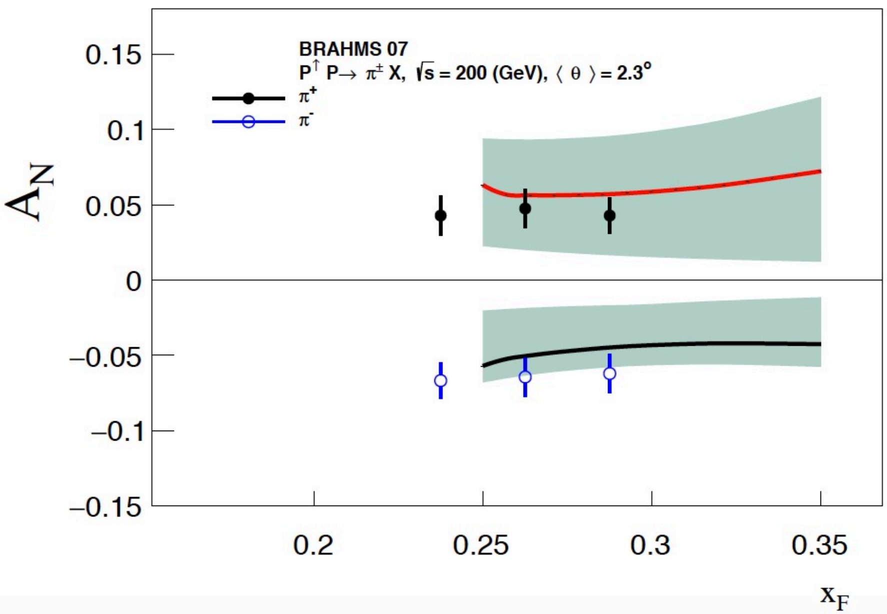
first k_T-moment of Collins function

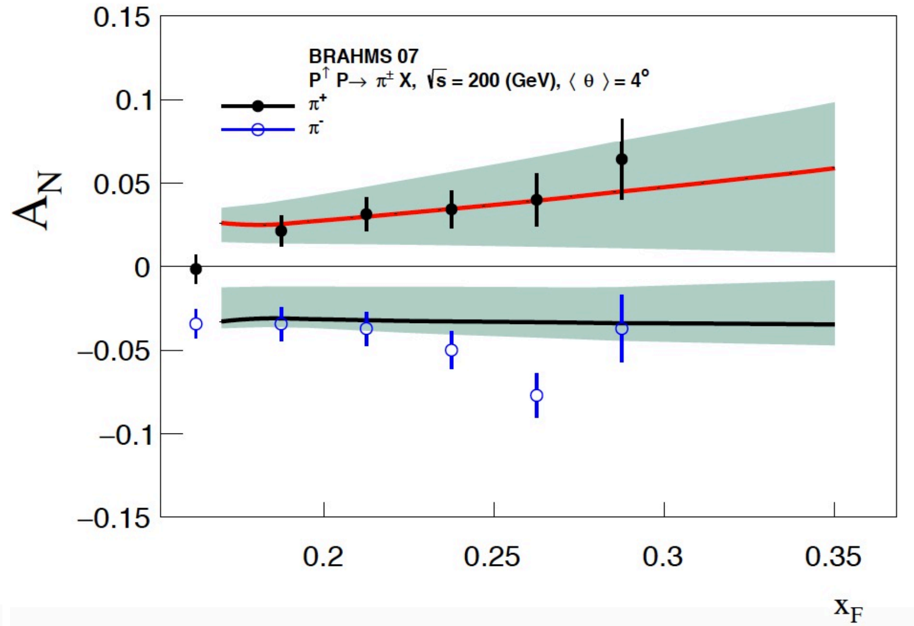
-Do not consider piece involving $\tilde{H}$ (BUT IT CANNOT BE ZERO)

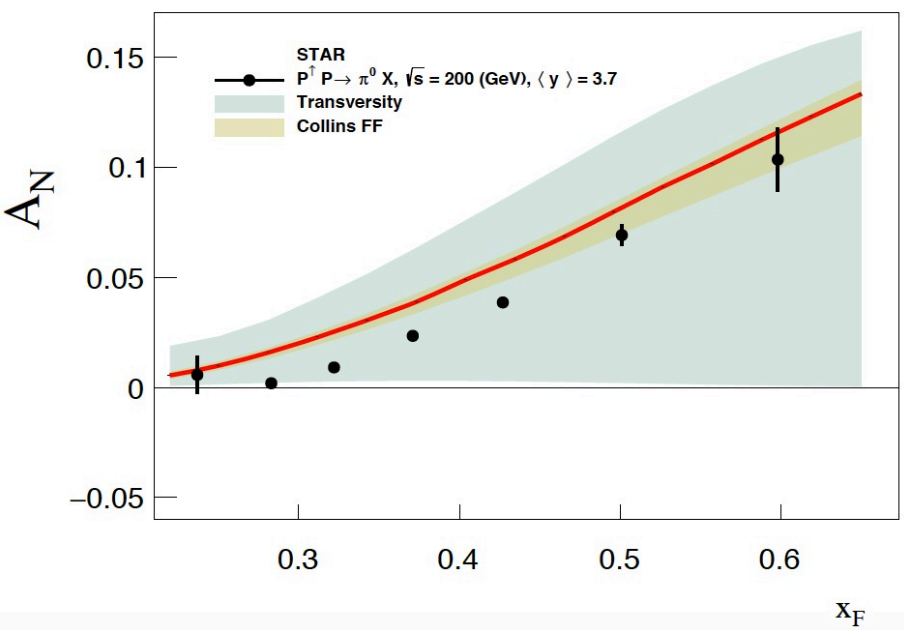
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))







-Confirms the original work of Kanazawa, Koike, DP, Metz (2014) that the twist-3 fragmentation term can dominate A_N

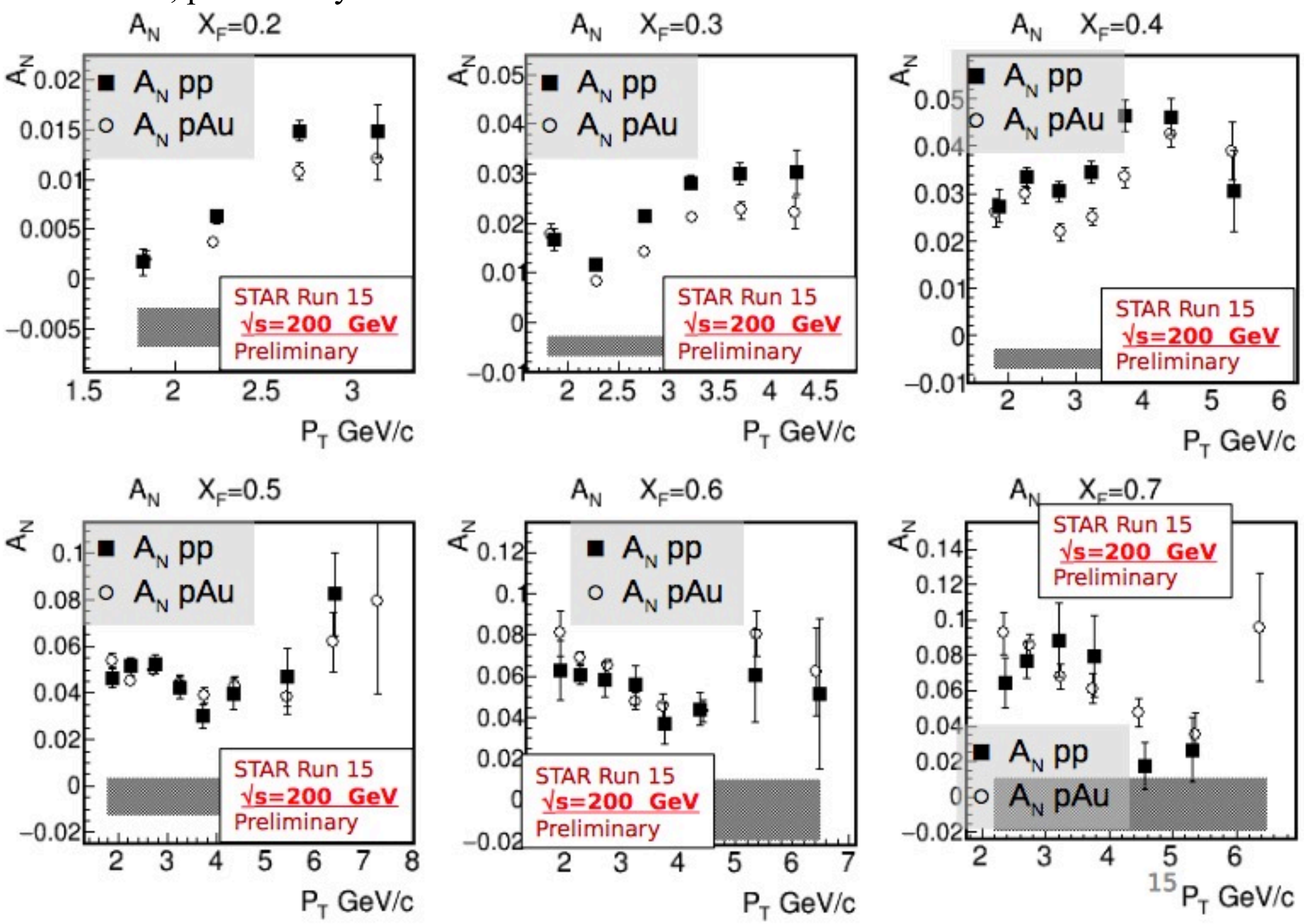
-Encouraged that we will be able to fully describe A_N through this mechanism even with the additional constraint from the LIR – still need to fit $\tilde{H}(z)$

-Large error due to uncertainty in transversity at large x – can be extracted from A_N data at RHIC!

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

➢ TSSAs in *pA* collisions

STAR, preliminary



No *A* dependence observed up to $x_F = 0.7$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i \sum_{a,b,c}\int_0^1 \frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

$$\left. + \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}\right\}$$

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\boxed{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]} S^i_{H_1^\perp} + \boxed{\frac{1}{z}H^c(z)\,S^i_H}\right.$$

$$\left. + \boxed{\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

~ $A^{-1/3}$

~ $A^{-1/3}$

~ A^{0}

Include saturation corrections to calculate *pA* TSSA

(Hatta, Xiao, Yoshida, Yuan (2017))

2013 expression from Metz and DP

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$

$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$

$$\left. + \boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

~ A^0

EOMR + LIR ➔

$$\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z) + z\frac{dH_1^{\perp(1),c}(z)}{dz} - \frac{1}{z}\tilde{H}^c(z)$$

2013 expression from Metz and DP

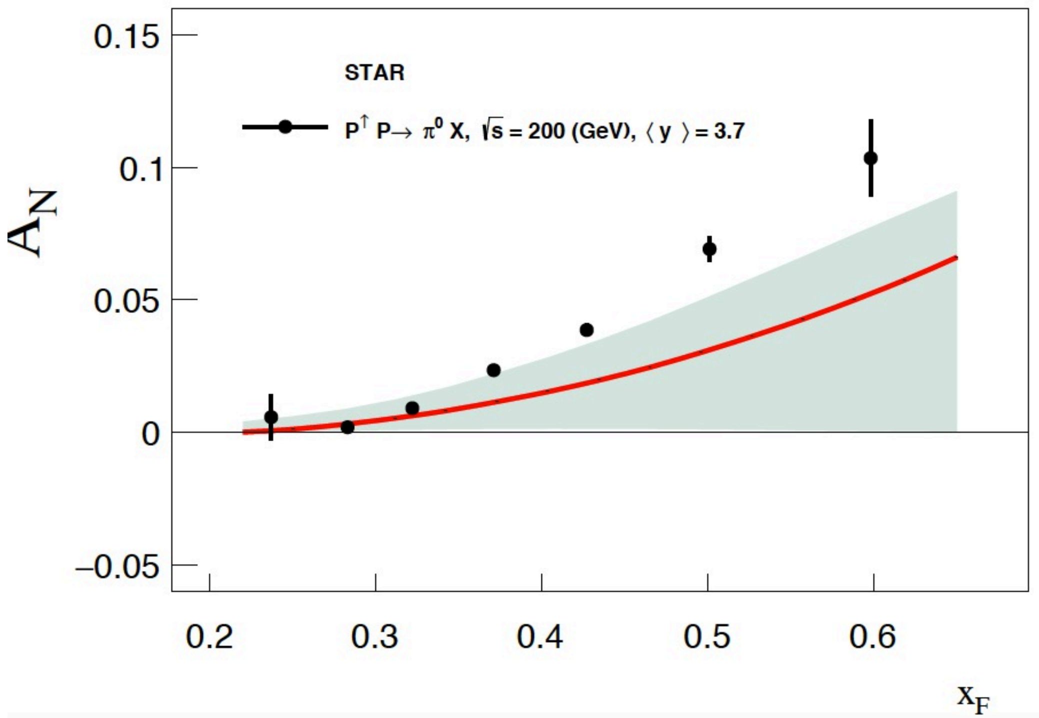
$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\;\delta(\hat{s}+\hat{t}+\hat{u})\,\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$
$$\times\; h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^c(z)\,S^i_H\right.$$
$$\left. + \boxed{\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{c,\Im}(z,z_1)\,S^i_{\hat{H}_{FU}}}\right\}$$

~ A^0

EOMR + LIR ➔

$$\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{\pi/c,\Im}(z,z_1) = H_1^{\perp(1),c}(z) + z\frac{dH_1^{\perp(1),c}(z)}{dz} - \frac{1}{z}\tilde{H}^c(z)$$

Calculate pieces involving the (first k_T-moment of the) Collins function to get an updated estimate for the term in blue



Fragmentation term as the cause of A_N in *pp* collisions is <u>not</u> ruled out by the STAR *pA* TSSA data

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))