Factorisation in colour singlet production

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Outline

- When (and why) factorisation holds (or doesn't) for production of colour singlet *V* at hadron-hadron colliders.
- Glauber gluons and why they are an obstacle to factorisation.
- Review CSS method for proving factorisation. Glauber cancellation for total cross section and p_T of *V*. Non-cancellation for certain other observables.
- Colour entanglement for azimuthal asymmetries in Drell-Yan?

Factorisation Formulae

Factorisation formulae are essential to make predictions at colliders involving p/A.

Separate out short distance interaction of interest from long-distance QCD-dominated interactions. Low-momentum part of long-distance piece will not be calculable perturbatively, but is (hopefully) universal.

Examples of factorisation formulae for pp collisions:

Collinear factorisation for pp → V + X inclusive total cross section, V colourless

PDFs (long distance physics, universal)

$$\sigma = \int dx_A dx_B \hat{\sigma}_{ij\to X}(\hat{s} = x_A x_B s) f_i(x_A) f_j(x_B) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^4}\right)$$

Parton-level cross section/coefficient function (short distance physics)

Corrections suppressed by Λ^2/Q^2

Factorisation Formulae

TMD factorisation for pp → V + X cross section differential in p_T, $p_T << Q$, V colourless

Hadronic tensor

Approximated momenta (transverse momenta → 0)

Rapidity regulator

$$W^{\mu\nu} = \frac{8\pi^2 s}{Q^2} \sum_f C_f^{\mu\nu}\left(\hat{k}_A, \hat{k}_B\right) \int d^2\mathbf{b}_T e^{i\mathbf{p}_T\cdot\mathbf{b}_T} \tilde{f}_f(x_A, \mathbf{b}_T; \zeta_A)\, \tilde{f}_{\bar{f}}(x_B, \mathbf{b}_T; \zeta_B) + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$$

Transverse momentum dependent PDFs (long distance physics)

Coefficient function (short distance physics)

Corrections suppressed by p_T^2/Q^2 (can be augmented to Λ^2/Q^2 by adding matching to fixed order)

Both formula proved to leading power by Collins, Soper and Sterman

Bodwin Phys. Rev. 31 (1985) 2616
Collins, Soper, Sterman Nucl. Phys. B261 (1985) 104, Nucl. Phys. B308 (1988) 833
Collins, pQCD book
See also Diehl, JG, Ostermeier, Plößl, Schäfer, JHEP 1601 (2016) 076

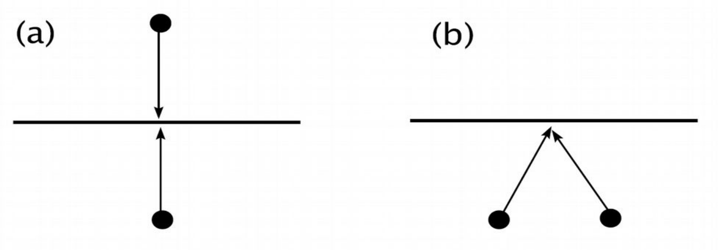
CSS Factorisation Analysis

How do we establish a leading power factorisation for a given observable?

I will review here the original Collins-Soper-Sterman (CSS) method

Want to identify IR leading power regions of Feynman graphs – i.e. small regions around the points at which certain particles go on shell, which despite being small are leading due to propagator denominators blowing up.

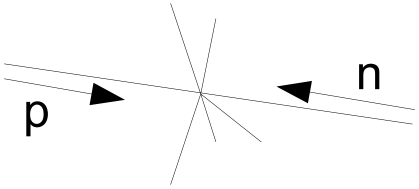
More precisely, need to find regions around pinch singularities – these are points where propagator denominators pinch the contour of the Feynman integral.



Pinched Non-pinched

Momentum regions

Once one has determined where the singularities are, need to determine their strength. Supplement pinch finding with a power counting analysis to determine if region around singularity gives a leading contribution, and what the shape of this region is.



In general, relevant regions in QCD are:

1) Hard region – momentum with large virtuality (order Q)

$$k \sim Q\,(1, 1, 1)$$

2) Collinear region – momentum close to some beam/jet direction

$$k \sim Q\left(1, \lambda^2, \lambda\right)$$ (for example)

$$\lambda \ll 1$$

3) (Central) soft region – all momentum components small and of same order

$$k \sim Q\,(\lambda^n, \lambda^n, \lambda^n)$$

AND...

Momentum regions

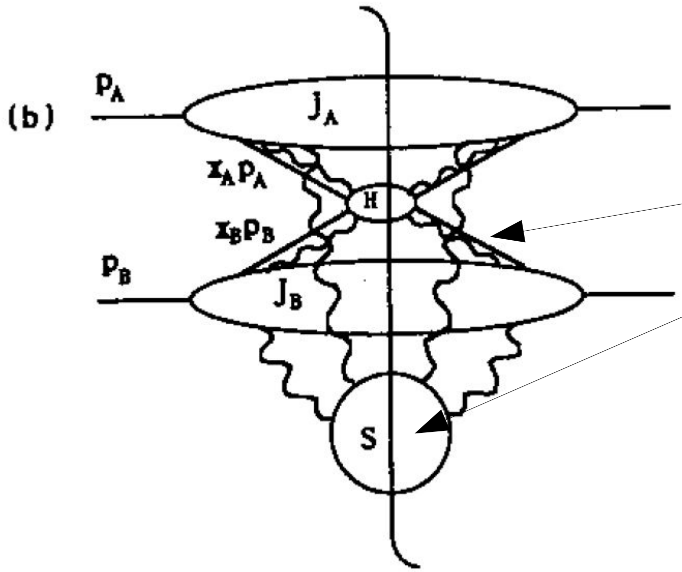
4) Glauber region – all momentum components small, but transverse components much larger than longitudinal ones

$$|k^+k^-| \ll \mathbf{k}_T^2 \ll Q^2$$

Canonical example: $k \sim Q\left(\lambda^2, \lambda^2, \lambda\right)$

Obtaining a factorisation formula

Leading region for pp → V +X:

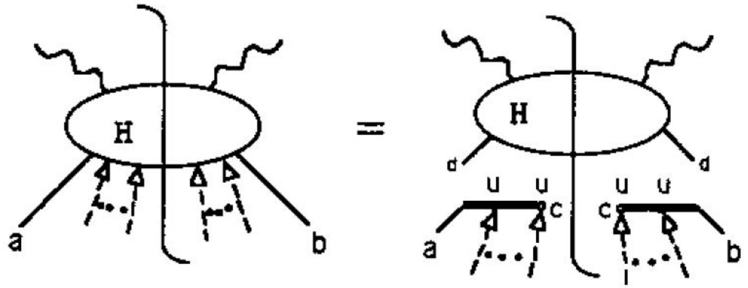


One physically polarised parton + multiple scalar polarised gluons

Soft + Glauber particles

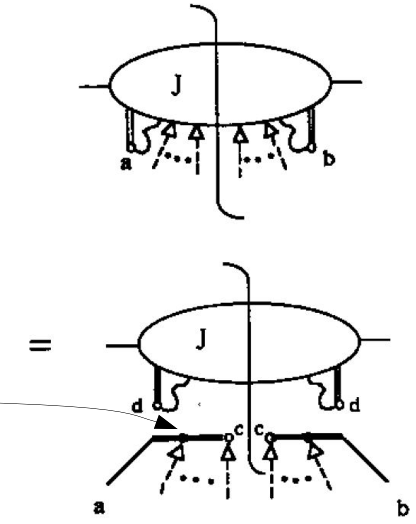
Already with this leading diagram we are close to some kind of a factorisation formula, but must separate different pieces – too many connections between H, J , S at the moment!

Collinear scalar polarised gluons can be stripped from hard by using Ward identities, physically polarised parton detached using some projector.



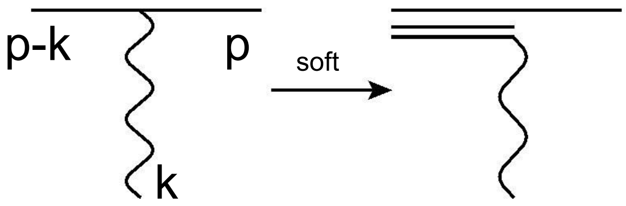
Obtaining a factorisation formula

If blob S only contained central soft, then we could strip soft attachments to collinear J blobs using Ward identities too.



Wilson line in direction of J

Simple example:



Propagator denominator:

$$(p-k)^2 = -2p\cdot k + k^2 \overset{\text{soft}}{\rightarrow} -2p\cdot k$$

Eikonal piece

Glauber Gluons and Factorisation

The manipulation used for soft gluons is NOT POSSIBLE for Glauber gluons

Propagator denominator:

$$(p-k)^2 = -2p\cdot k + k^2 \not\to -2p\cdot k$$

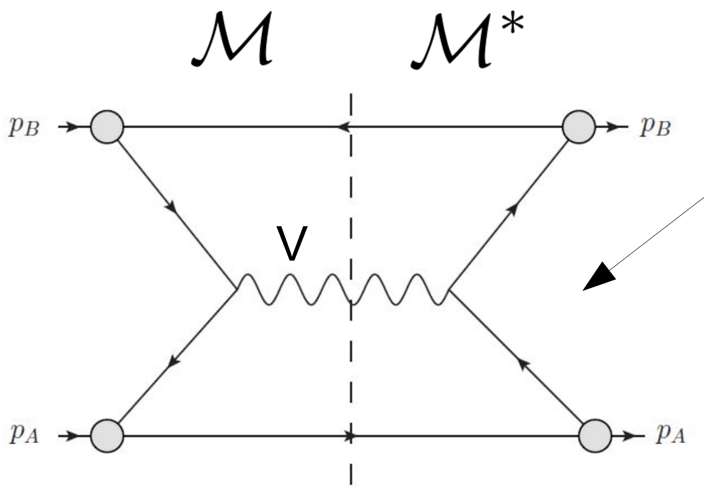
Two terms in denominator are of same order in Glauber region

How do we get around this problem?

What CSS did for total cross section and p_T of V: show that contribution from Glauber gluons cancels

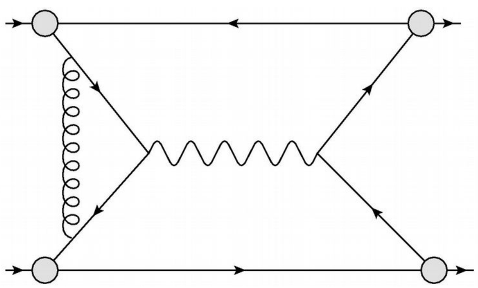
Let's review how and why this cancellation happens for one/two gluon exchange in a simple model

One-Glauber Exchange

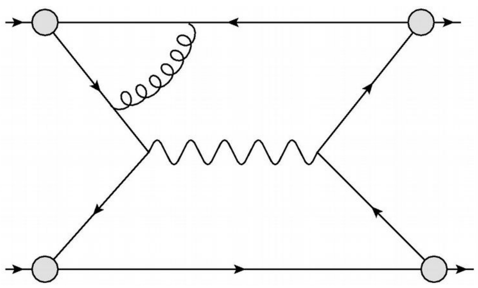


Lowest order 'parton model' process for p + p → V + X

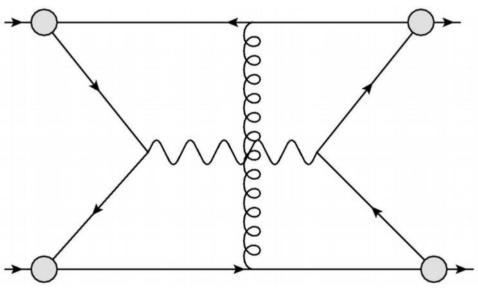
Consider one gluon corrections to this picture. Various possibilities:



Active-active interaction

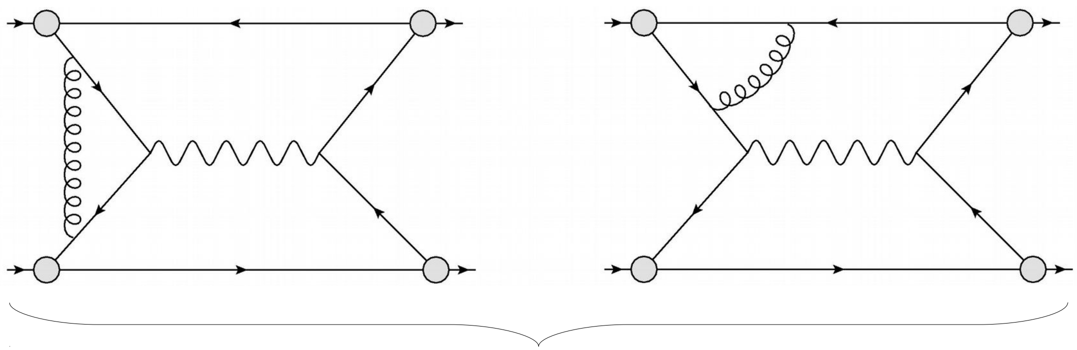


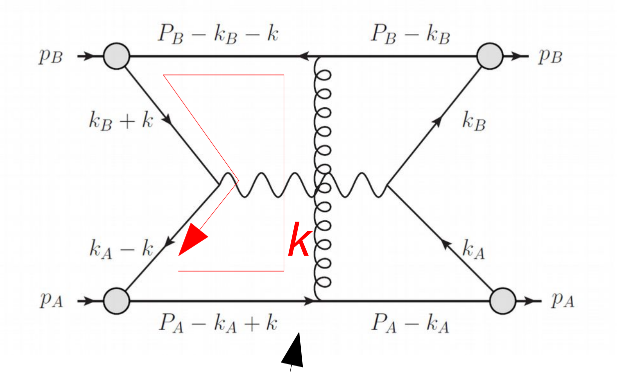
Active-spectator interaction



Spectator-spectator interaction

One-Glauber Exchange

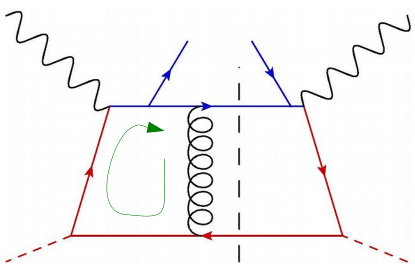




For these graphs, gluon momentum is not trapped in the Glauber region however → can deform loop integration contour and handle these using usual soft/collinear/hard regions.

Collins, Phys.Rev. D57 (1998) 3051–3056
Collins, Metz, Phys.Rev.Lett. 93 (2004) 252001

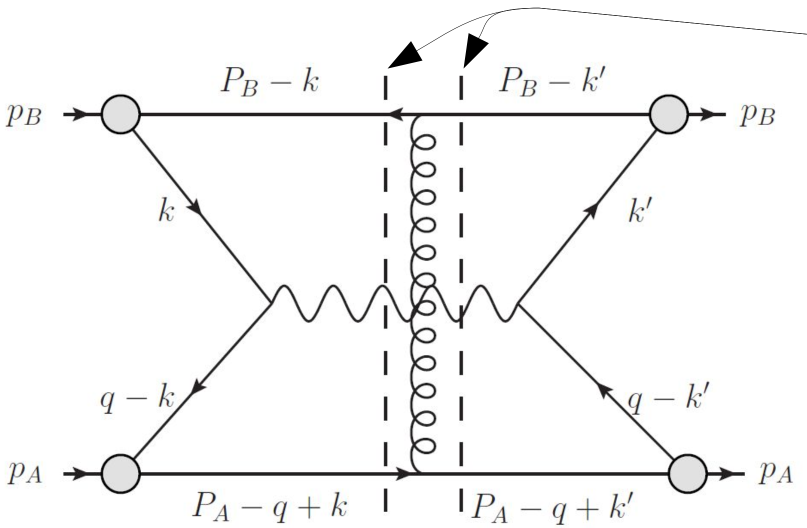
In this graph gluon is trapped in Glauber region



Note in SIDIS gluon momentum is always deformable out of Glauber

Collins Phys.Rev. D57 (1998) 3051-3056
Collins and Metz Phys.Rev.Lett. 93 (2004) 252001

One-Glauber Exchange



Two possible cuts of graph that leave gluon in Glauber region (cut through gluon forces it into central soft)

Consider case where cut is to the right, and consider k^+ integral.

In top half of graph can ignore q^--k^- compared to large components (q^-, P_B^--q^-)

In bottom half of graph can ignore k^+ compared to large components (q^+, P_A^+-q^+)

In gluon propagator can ignore lightcone components of k compared to transverse components

$$\int \frac{\mathrm{d}k^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} \frac{i}{(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)}$$

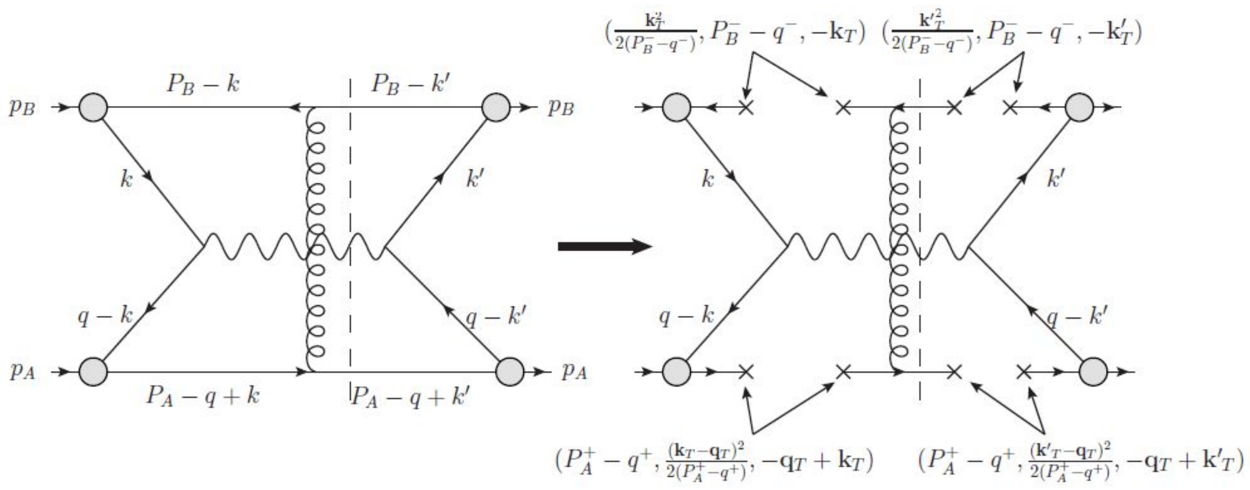
$$= \frac{i}{2(P_B^- - q^-)} \frac{i}{2q^- k^+_{\text{on-shell}} - \mathbf{k}_T^2 + i0}$$

$$= \int \frac{\mathrm{d}k^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} 2\pi\delta(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)$$

Net effect – set P_B-k line on shell!

One-Glauber Exchange

Repeat with k^-, k'^+, k'^- integrations:



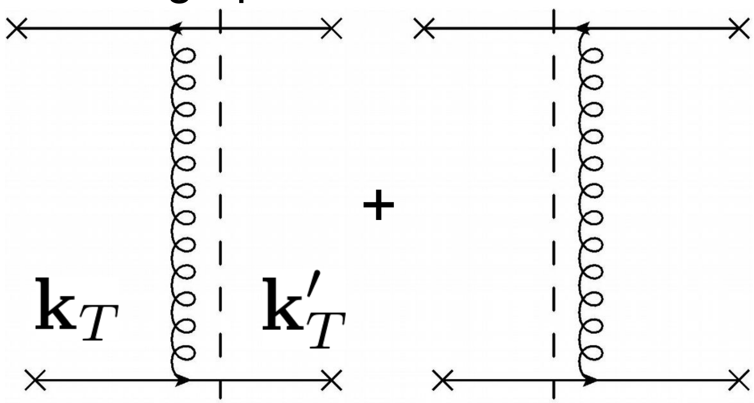
Factor out on-shell Glauber exchange graph!

Can do a similar procedure when cut is on left.

Unitarity cancellation for one-Glauber exchange

Consider case where we measure p_T of V. For given momenta in the two decomposed graphs, the value of the measurement is the same.

Therefore, we can factor out the parton model graph and measurement and add together the two Glauber subgraphs:



= 0

CUTKOSKY RULE

$\mathcal{M} = -iL$

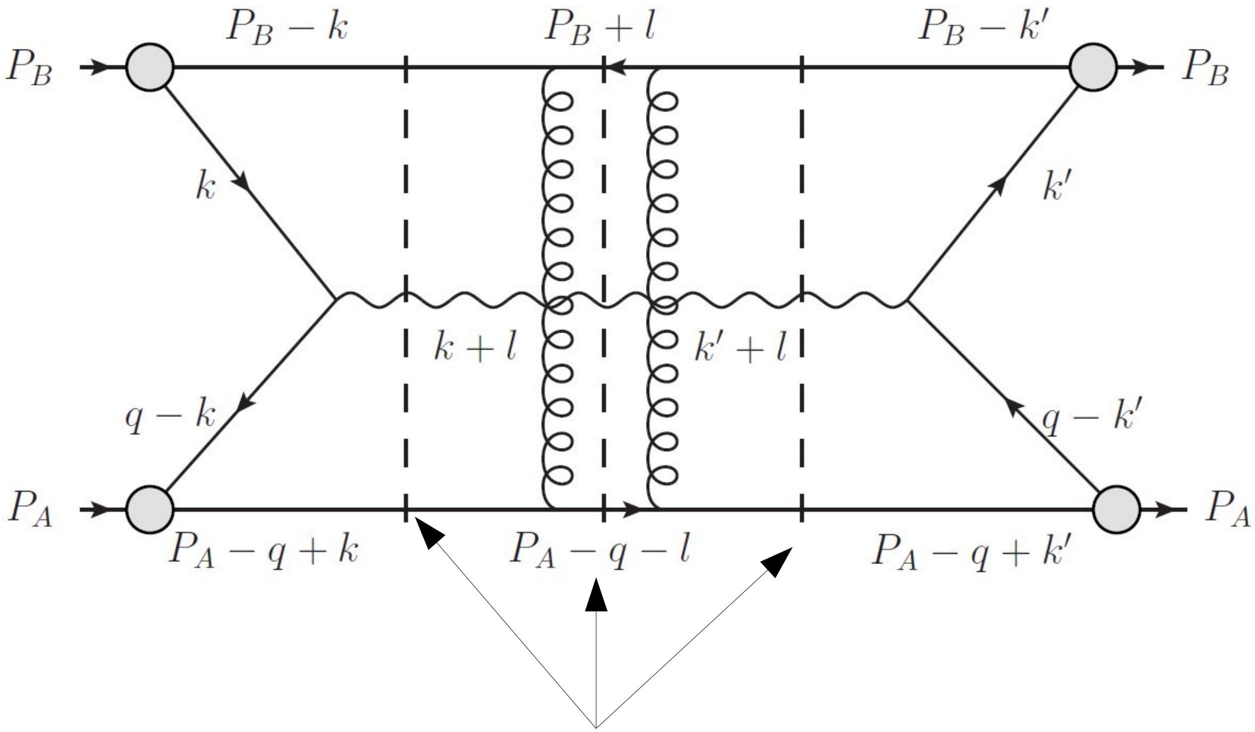
Graph level: $L(\mathbf{k}_T \to \mathbf{k}'_T) + L^*(\mathbf{k}'_T \to \mathbf{k}_T)$

Amplitude level: $i\mathcal{M}(\mathbf{k}_T \to \mathbf{k}'_T) - i\mathcal{M}^*(\mathbf{k}'_T \to \mathbf{k}_T)$

Actually: at this level the Glauber subgraph is pure imaginary. Since parton model skeleton is real, and in the end we must end up with a real cross section, the one gluon exchange must always cancel (for any observable).

Two-Glauber Exchange

Now add in one more (Glauber) gluon between spectators:

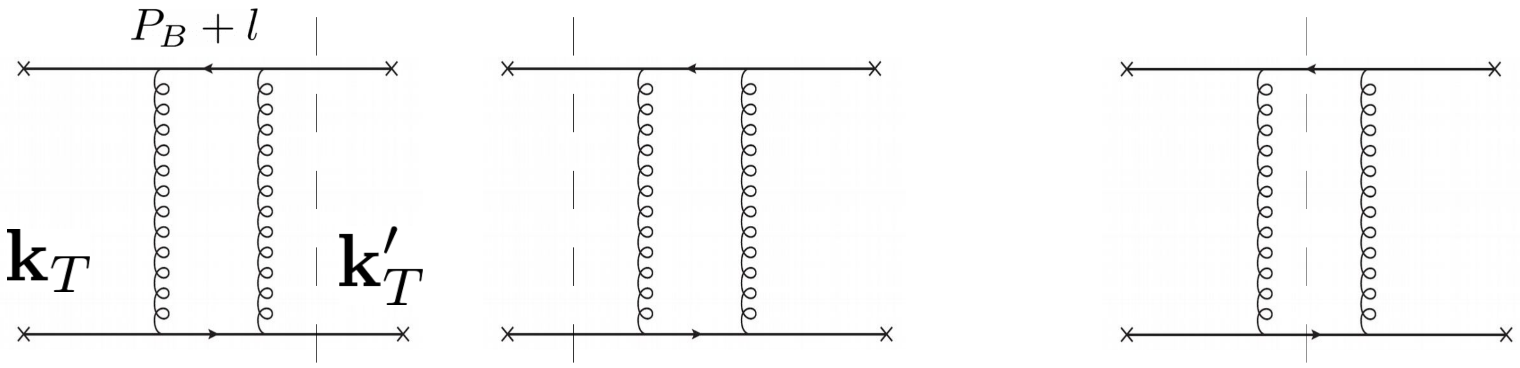


Now 3 cuts that leave both gluons in Glauber region

Factor out Glauber subgraphs as before.

Unitarity cancellation for two-Glauber exchange

Again, for given momenta in (decomposed) graphs, measurement (p_T) is the same – factor measurement and parton model graphs, and combine Glauber subgraphs:



$$L(\mathbf{k}_T \to \mathbf{k}'_T; l) + L^*(\mathbf{k}'_T \to \mathbf{k}_T; l) \;\; + \int \Phi_2 L(\mathbf{k}_T \to \mathbf{l}_T) L^*(\mathbf{k}'_T \to \mathbf{l}_T)$$

$$i\mathcal{M}(\mathbf{k}_T \to \mathbf{k}'_T; l) - i\mathcal{M}^*(\mathbf{k}'_T \to \mathbf{k}_T; l) \;\; + \int \Phi_2 \mathcal{M}(\mathbf{k}_T \to \mathbf{l}_T) \mathcal{M}^*(\mathbf{k}'_T \to \mathbf{l}_T)$$

-Imaginary Part

Sum over (1) cuts

=0 using Cutkosky rules

For all-order argument, more sophisticated techniques based on light-cone perturbation theory needed. However, basic principle behind cancellation is the same – UNITARITY.

Collins, Soper, Sterman Nucl. Phys. B308 (1988) 833
Collins, pQCD book
See also Diehl, JG, Ostermeier, Plößl, Schäfer, JHEP 1601 (2016) 076

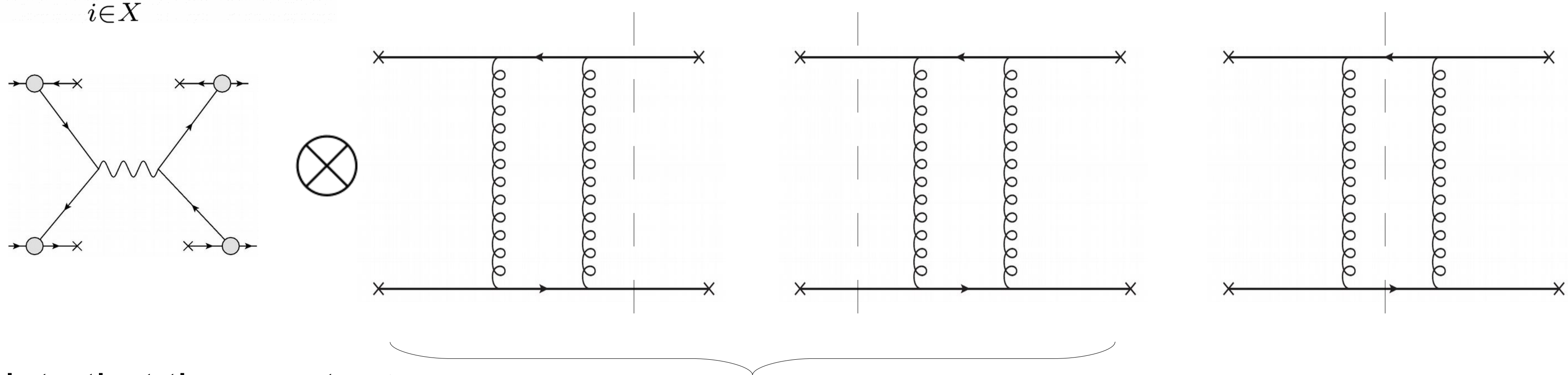
Failure of the unitarity cancellation for other observables

Note that if you measure an observable that is sensitive to the details of the final state other than the produced colourless boson V, the CSS unitarity cancellation argument for the Glauber gluons will fail:

Factorisation breaking!

e.g.

$E_T = \sum_{i \in X} \sqrt{\mathbf{p}_{Ti}^2 + m_i^2}$ $\quad |\mathbf{k}'_T| + |\mathbf{q}_T - \mathbf{k}'_T| \quad |\mathbf{k}_T| + |\mathbf{q}_T - \mathbf{k}_T| \quad |\mathbf{l}_T| + |\mathbf{q}_T + \mathbf{l}_T|$



Note that these cuts can be interpreted as:

Secondary low scale absorptive process

Secondary low scale scattering

Factorisation breaking effects are related to additional low-scale scatters, or multiple parton interactions (MPI). Glauber cancellation for the p_T observable works because this is insensitive to whether extra scatters occurred or not.

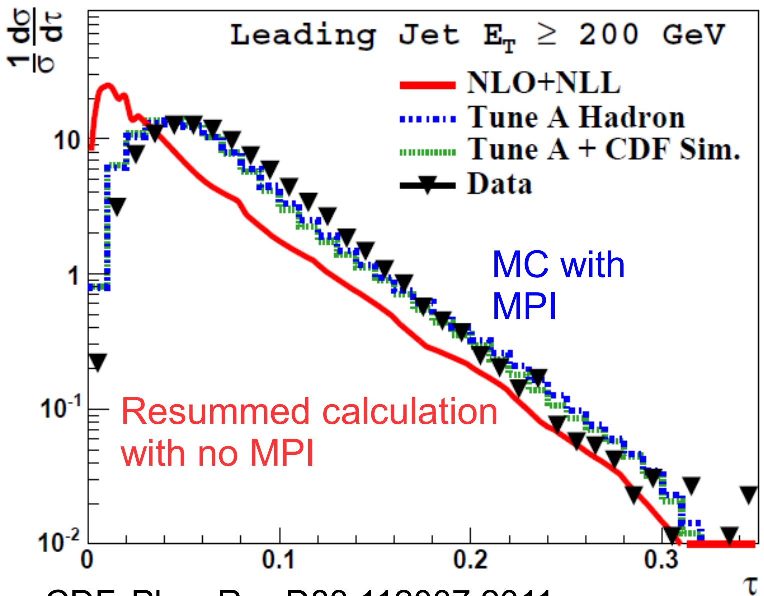
JG, JHEP 1407 (2014) 110

Failure of the unitarity cancellation for other observables

Can see the factorisation breaking experimentally – e.g. for transverse thrust:

$$T_\perp \equiv \max_{\vec{n}_T} \frac{\sum_{i=1}^{n} |\vec{q_{\perp,i}} \cdot \vec{n}_T|}{\sum_{i=1}^{n} |\vec{q_{\perp,i}}|} \qquad \tau \equiv 1 - T_\perp$$

Additional uncorrelated scatters make event more spherical and raise τ – observable sensitive to extra activity/MPI.



CDF, Phys.Rev.D83:112007,2011

Note similar observables in ep/eA collisions will not suffer from the same effects (no MPI for DIS)

Colour entanglement in the Drell-Yan process?

Factorisation prediction for DY with unpolarised hadrons, including angular dependence:

Boer, Brodsky, Huang, Phys. Rev. D67 (2003) 054003,
Boer, Phys. Rev. D60 (1999) 014012

$$\frac{d^6\sigma}{d\Omega\, dx_1 dx_2\, d^2\boldsymbol{q}} = \frac{\alpha^2}{N_c\, q^2} \sum_q e_q^2 \left\{ A(\theta)\, \mathcal{F}\left[f_1 \bar{f}_1\right] + B(\theta)\, \cos(2\phi)\, \mathcal{F}\left[w(\boldsymbol{k}_1, \boldsymbol{k_2})\, h_1^\perp \bar{h}_1^\perp\right] \right\},$$

Unpolarised TMD

Collins-Soper angles

Boer-Mulders TMD (measures correlation between quark transverse spin and transverse mtm)

$$\mathcal{F}\left[f_1 \bar{f}_1\right] \equiv \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, \delta^{(2)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{q})\, f_{1,q}(x_1, \boldsymbol{k}_1^2)\, \bar{f}_{1,q}(x_2, \boldsymbol{k}_2^2)$$

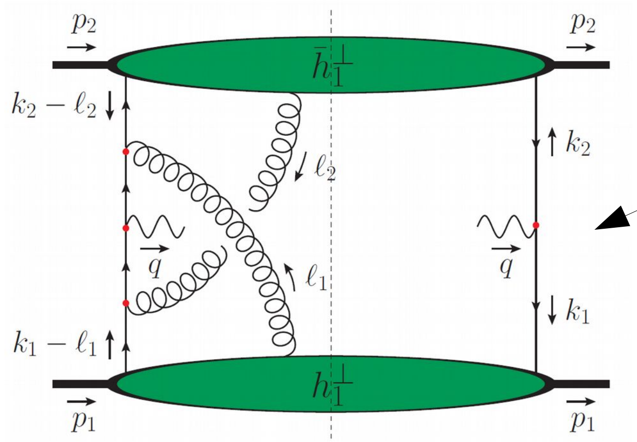
Note that the colour structure for the unpolarised and double BM piece is the same!

In PRL 112 (2014), 092002 (Buffing, Mulders), it was suggested that there was colour entanglement in the ϕ dependent piece: factorised prediction not correct for this.

$$\begin{aligned} d\sigma_{\mathrm{DY}} &= \mathrm{Tr}_c \Big[U_-^\dagger[p_2] \Phi(x_1, p_{1T}) U_-[p_2] \Gamma^* \\ &\qquad \times U_-^\dagger[p_1] \overline{\Phi}(x_2, p_{2T}) U_-[p_1] \Gamma \Big] \\ &\neq \frac{1}{N_c} \Phi^{[-]}(x_1, p_{1T}) \Gamma^* \overline{\Phi}^{[-^\dagger]}(x_2, p_{2T}) \Gamma, \end{aligned}$$

(also for double Sivers effect)

Colour entanglement in the Drell-Yan process?



"Factorisation breaking" effects associated with graphs in which we have gluon attachments between spectator system of one hadron and active quark from the other, on both sides ("crossed gluon" graph).

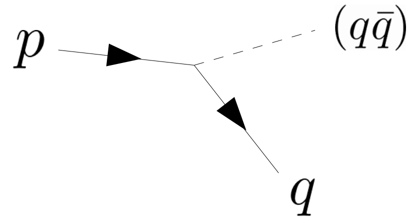
If true this would suggest some loophole in the CSS argument (for spin effects) – need to verify if it is true or not!

We check using a model calculation. Need to go to the two gluon exchange level to include effects mentioned above. No contour deformations – just compute graphs.

Boer, van Daal, JG, Kasemets, Mulders, in preparation

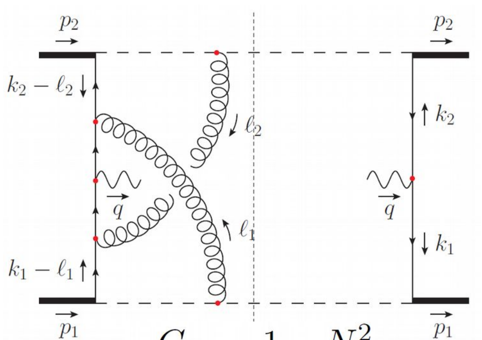
Checking colour entanglement

We use a model in which each hadron is a massive spin ½ particle than can split into a massless spin ½ quark and a massless scalar 'diquark' via Yukawa-type interaction.
Vector boson V produced via q$\bar{\text{q}}$ fusion.

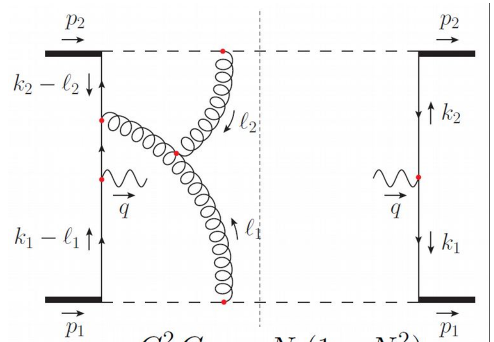


Readily verified that parton-level and one-gluon exchange diagrams do not give any contribution to the ϕ dependent piece.

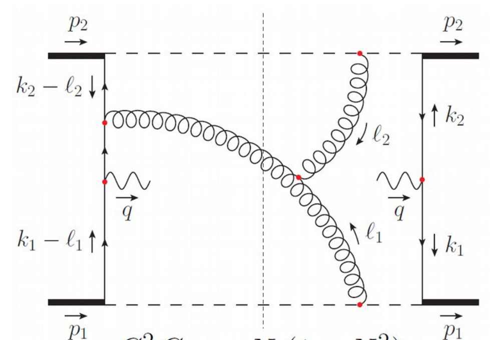
Important (colour entangled) diagrams for ϕ dependence at two-gluon exchange level:



$$-\frac{C_F}{2} = \frac{1-N_c^2}{4N_c}$$



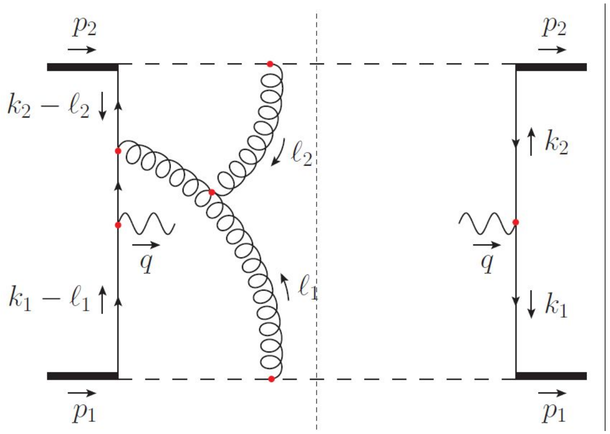
$$-\frac{C_A^2 C_F}{2} = \frac{N_c(1-N_c^2)}{4}$$



$$-\frac{C_A^2 C_F}{2} = \frac{N_c(1-N_c^2)}{4}$$

Remaining diagrams either give no leading-power ϕ dependence straight away, or cancel straightforwardly after the sum over cuts.

Checking colour entanglement

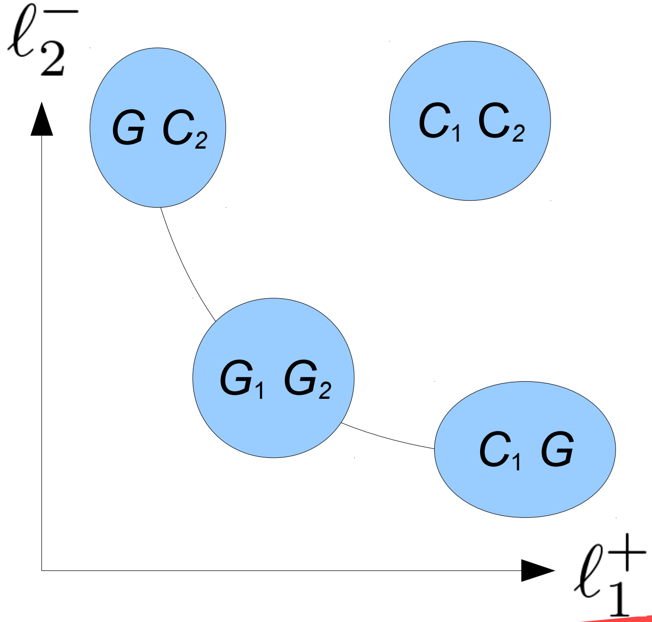


Very hard to directly compute integrals over gluon momenta ℓ_1, ℓ_2 - so instead we split calculation of each graph into leading momentum regions.

Apply approximations valid for those regions, and subtractions for smaller regions. Collins, pQCD book

Non-trivial leading regions for each graph are:

$$\begin{pmatrix} G : (\lambda^2, \lambda^2, \lambda)Q \\ G_1 : (\lambda, \lambda^2, \lambda)Q \\ G_2 : (\lambda^2, \lambda, \lambda)Q \end{pmatrix}$$



Colour dis-entanglement

The $C_1\ G\ /\ G_1\ G_2\ /\ G\ C_2$ regions smoothly join onto one another on a hyperbola in rapidity space → each piece is ill-defined without a rapidity regulator. We use the same rapidity regulator in each region, for simplicity: $|\ell_1^+/\nu|^{-\eta_1}|\ell_2^-/\nu|^{-\eta_2}$

Regulator along the lines of that proposed by Chiu, Jain, Neill, Rothstein, Phys.Rev.Lett. 108 (2012) 151601, JHEP 1205 (2012) 084 – also known as Caregie Mellon University/CMU regulator

Performing the computations, we see a region-by-region cancellation of the colour entanglement, and restoration of the colour factor anticipated by factorisation. The 3g vertex diagram is crucial to cancel the colour entanglement in the crossed gluon graph!

- Fundamental mechanisms behind this are unitarity cancellation + non-Abelian Ward identities.
- With the rapidity regulator used the contribution to the ϕ dependence at this order comes entirely from the $G_1\ G_2$ region.
- By changing the rapidity regulator employed one can shift contributions between regions and diagrams.

Summary

- Glauber gluons are soft gluons with much larger transverse than longitudinal components. They can cause problems for factorisation.
- For the production of a colourless boson *V*, if one measures an observable depending only on the properties of *V* (total cross section, transverse momentum of V), then the effects of the Glauber gluons cancel by a unitarity argument.
- On the other hand, if one measures some quantity that is sensitive to the remainder of the event, the unitarity argument fails, at the level of two-gluon exchange (examples: transverse thrust, hadronic transverse energy).
- An explicit model calculation with two gluon exchange indicates that the spin-dependent colour-singlet production cross sections at small p_T obey factorisation, just as the unpolarised cross section does.