Process dependence of TMDs and factorization (breaking)

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university of groningen

Color flow in high energy scattering processes

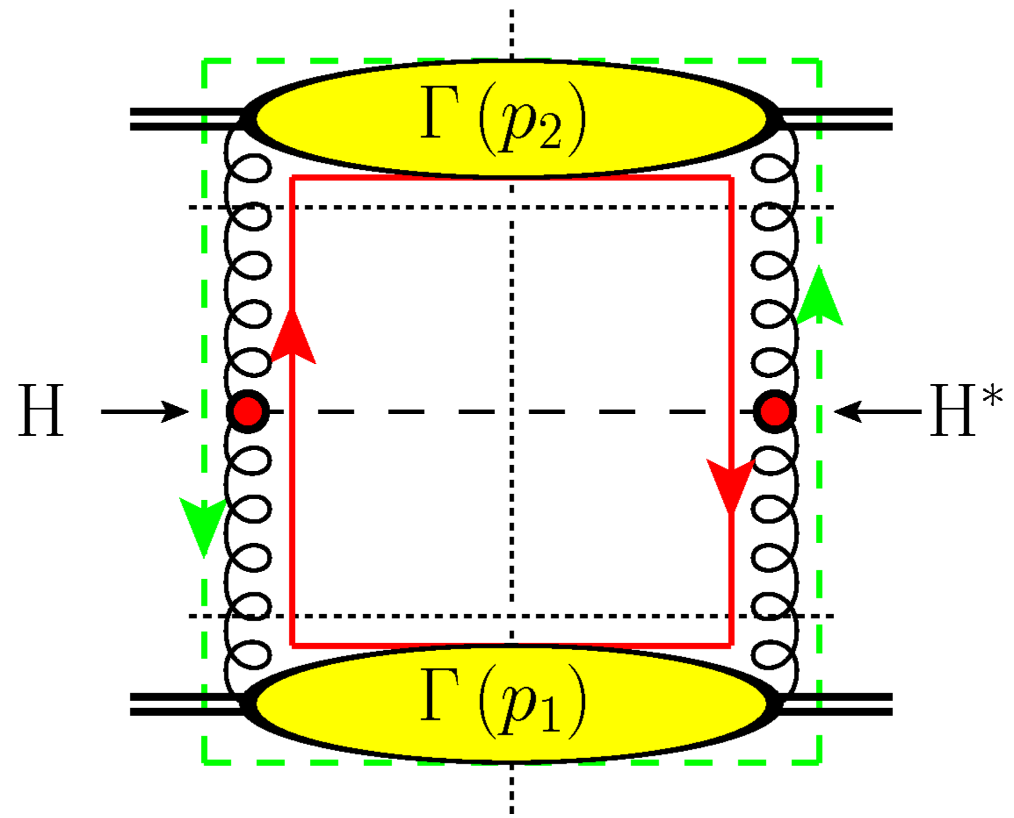
Factorization and color flow

The theoretical description of high-energy scattering cross sections is based on **factorization** in, on the one hand, the perturbative scattering of partons, and on the other hand, the nonperturbative parton distributions

Higgs production: $pp \to HX$

Color treatment is simple at high energies: separate traces, not dependent on kinematics

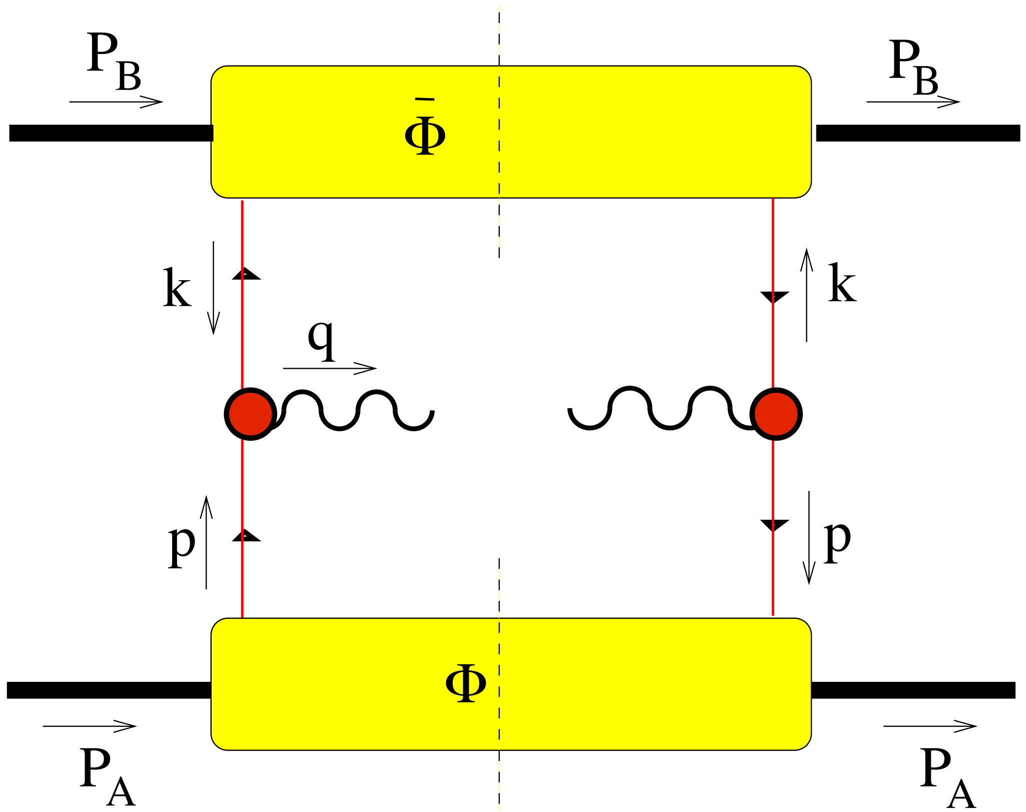
But in the actual process there are no colored final states
and there are many soft gluons exchanged to balance the color



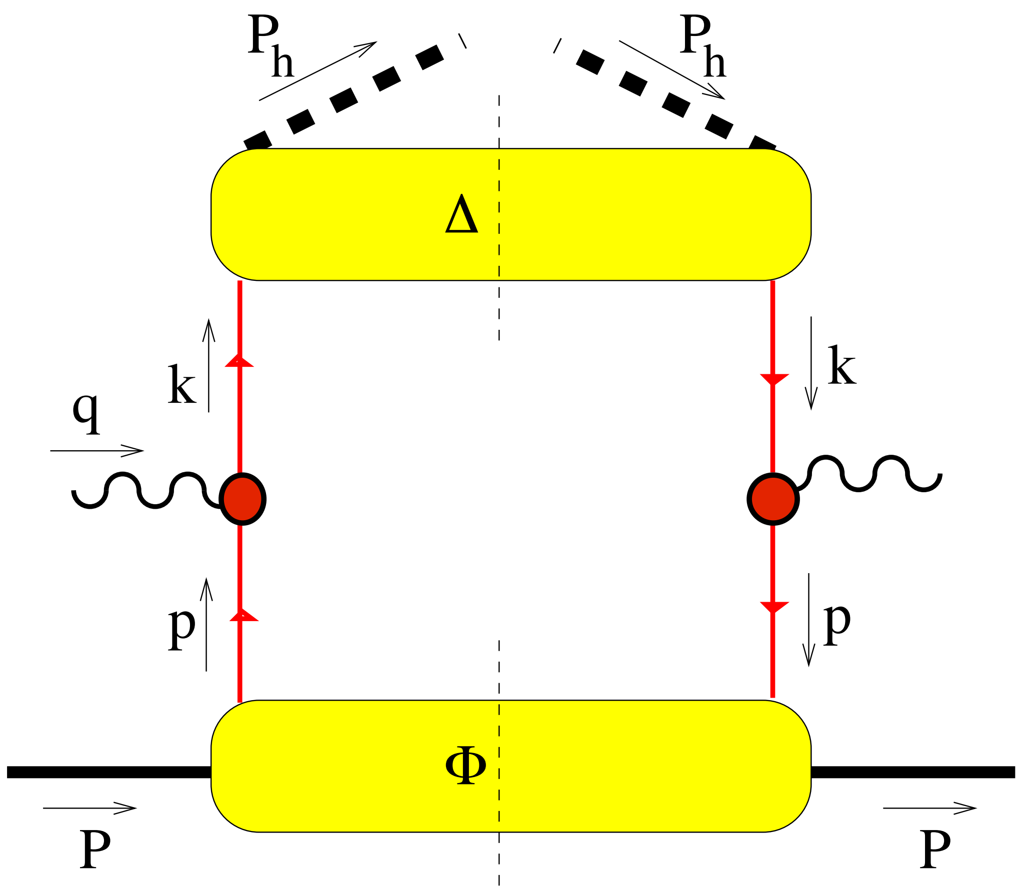
The cartoon version of the color flow works fine in most cases, when collinear factorization applies

Factorization in terms of correlators

Similarly, one would expect that the following two processes involve the same color trace and that the dynamics is unaffected by the color flow



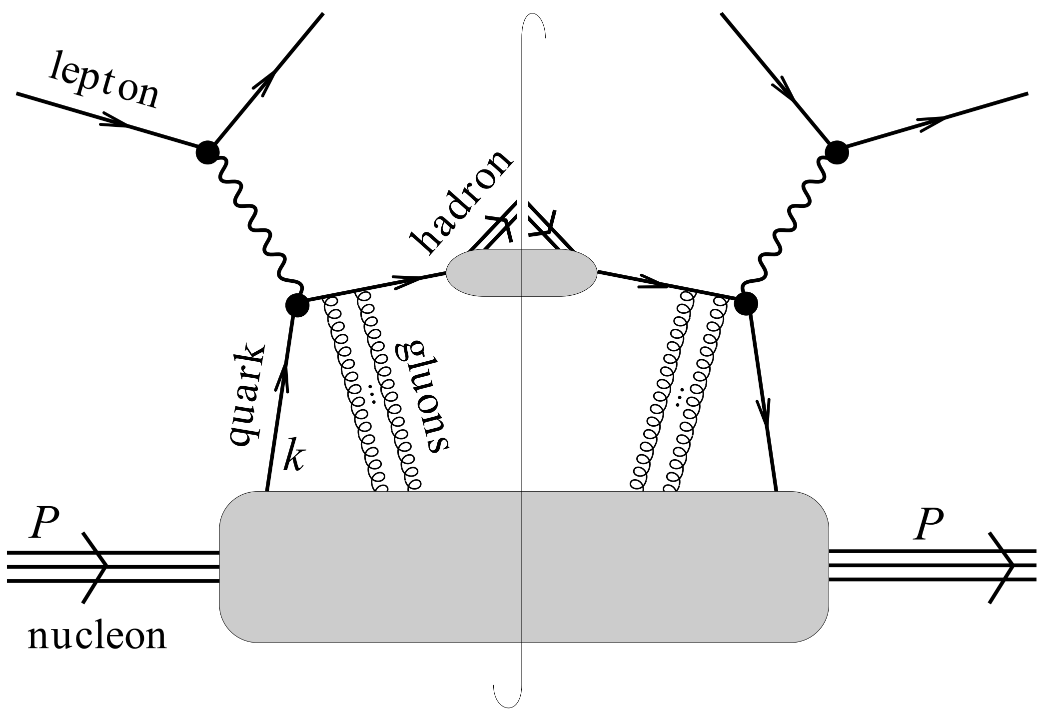
$pp \rightarrow \gamma^{*}X$ (Drell-Yan)



$\gamma^{*}p \rightarrow h\ X$ (SIDIS)

However, this is not always the case, e.g. for certain differential cross sections, that are sensitive to the transverse momentum of the partons

Gauge invariance of correlators



summation of all gluon exchanges leads to *path-ordered exponentials* in the correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

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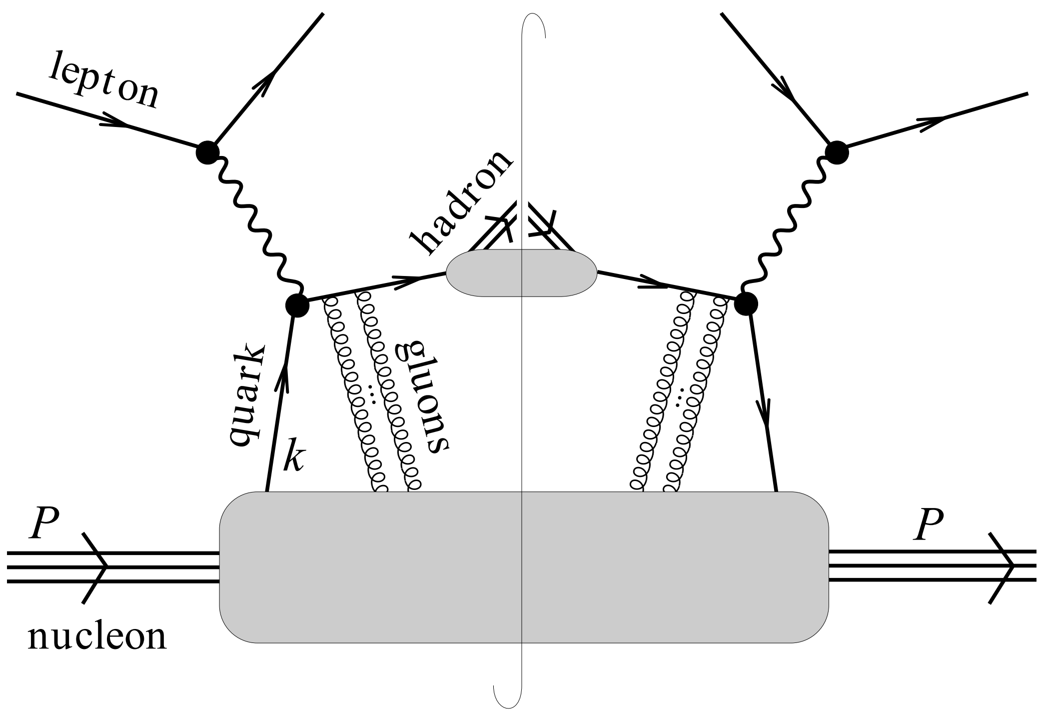

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The path C depends on whether the color interactions are with an incoming or outgoing color charge, yielding different paths for different processes

[Collins & Soper, 1983; Boer & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003; Boer, Mulders & Pijlman, 2003]

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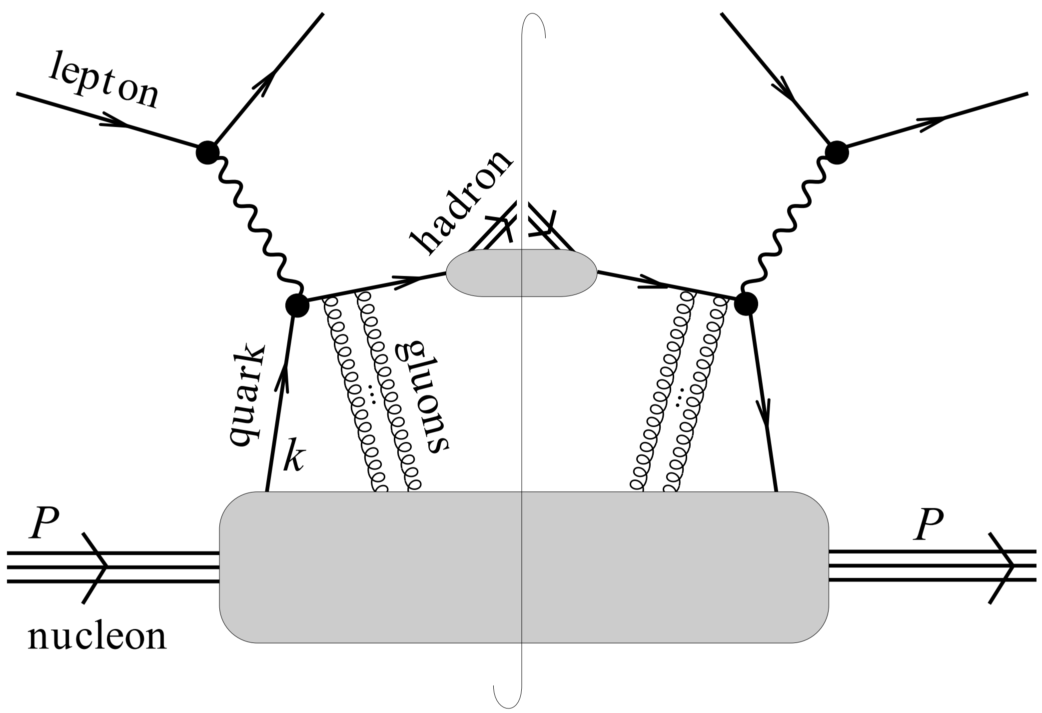

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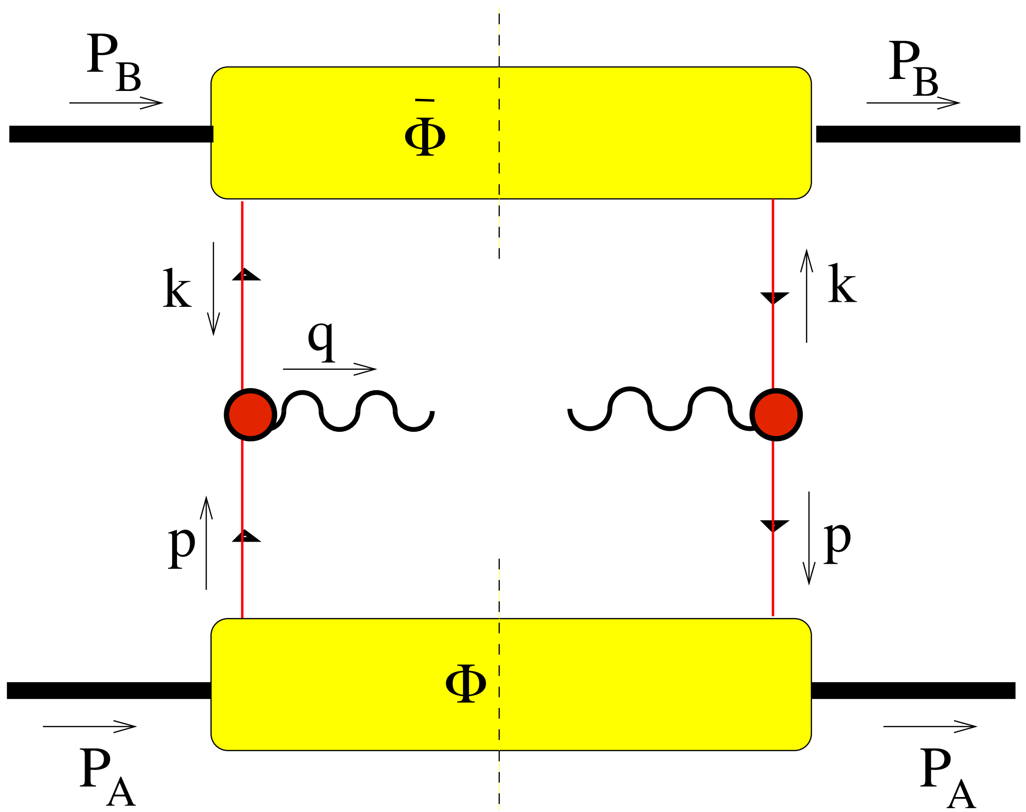
This does not automatically imply that these *gauge links* affect observables, but it turns out that they do in certain cases sensitive to the transverse momentum

In that case the gauge link path has extent ξ_T in the transverse direction (ξ_T is conjugate to k_T) which can be located at different places along the lightfront

Process dependence of gauge links

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing

[Belitsky, Ji & Yuan '03]



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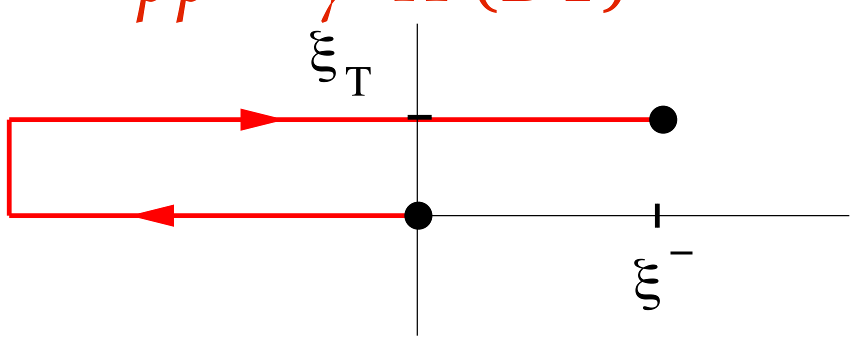

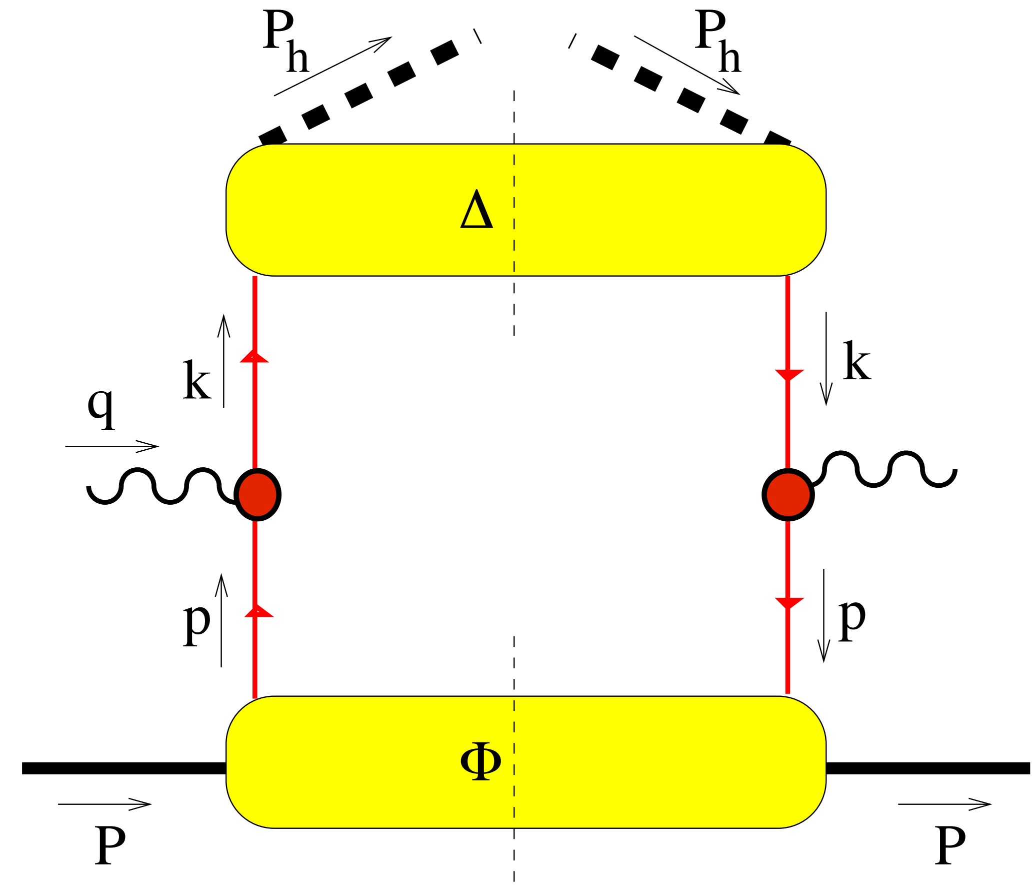

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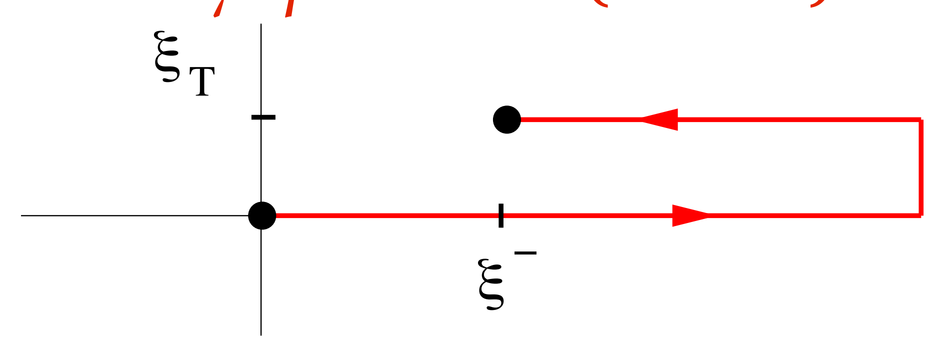

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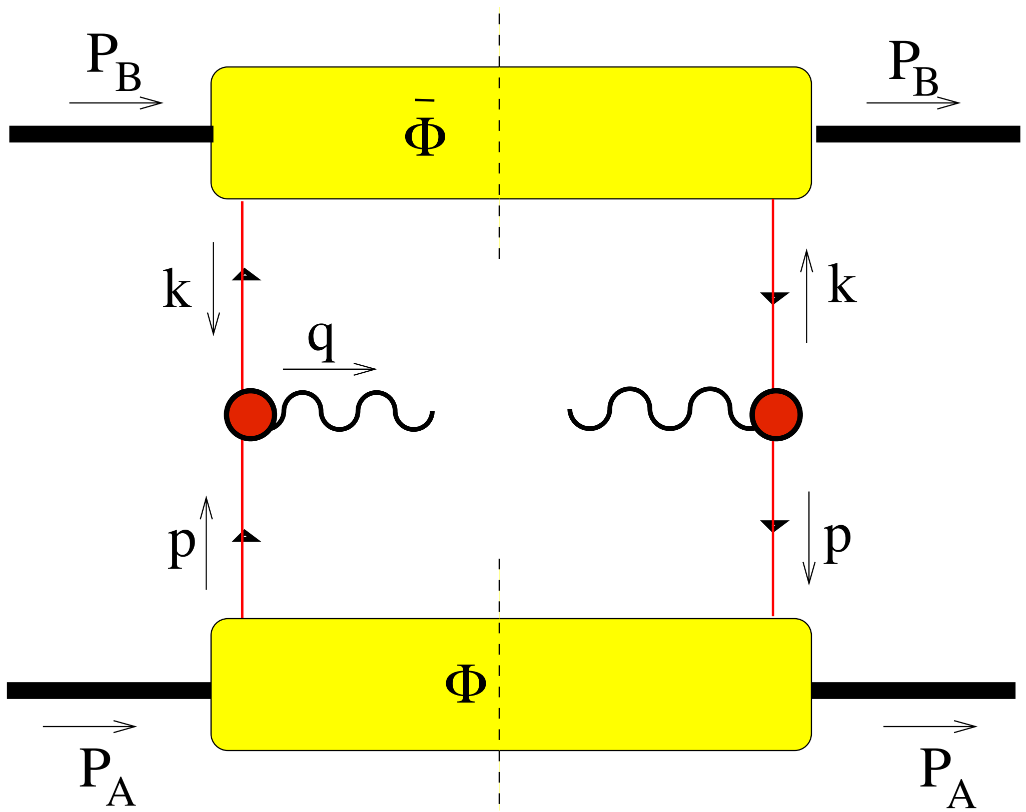

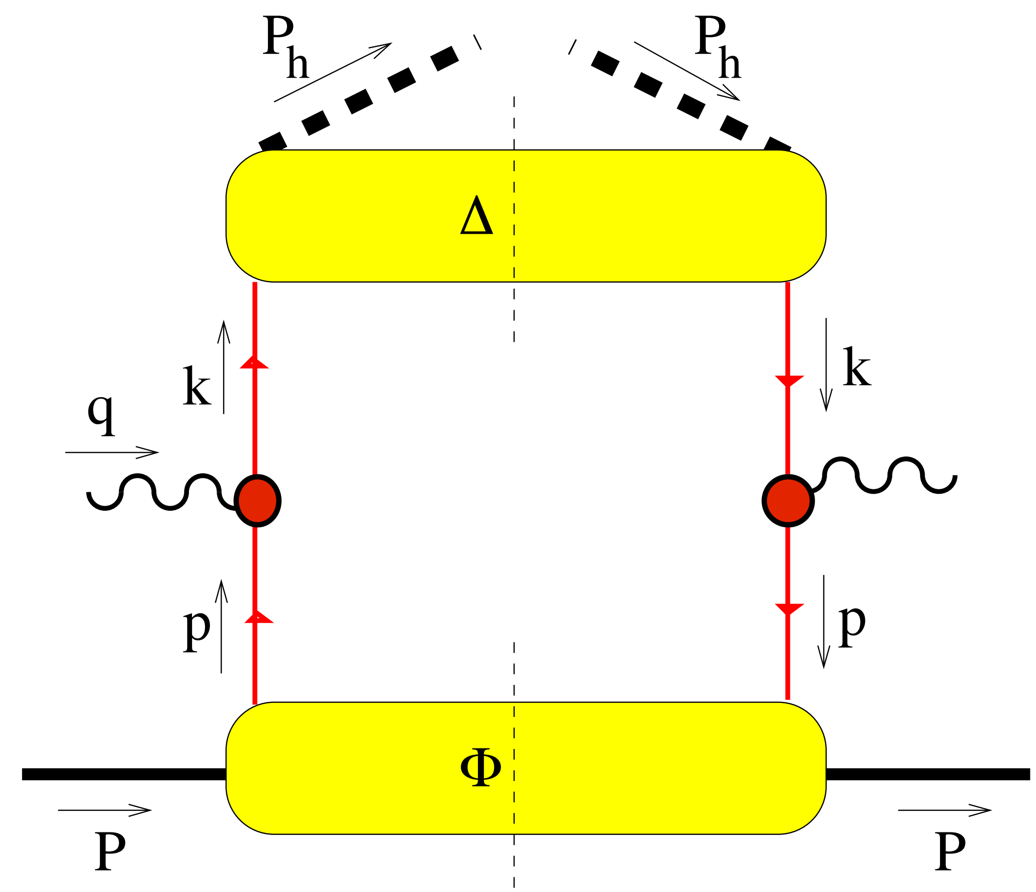

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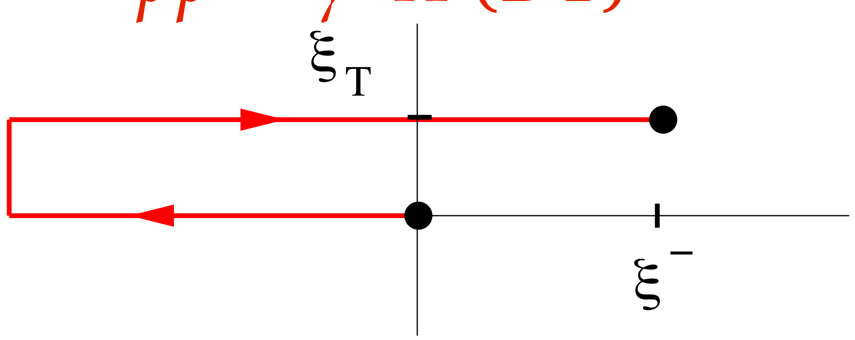
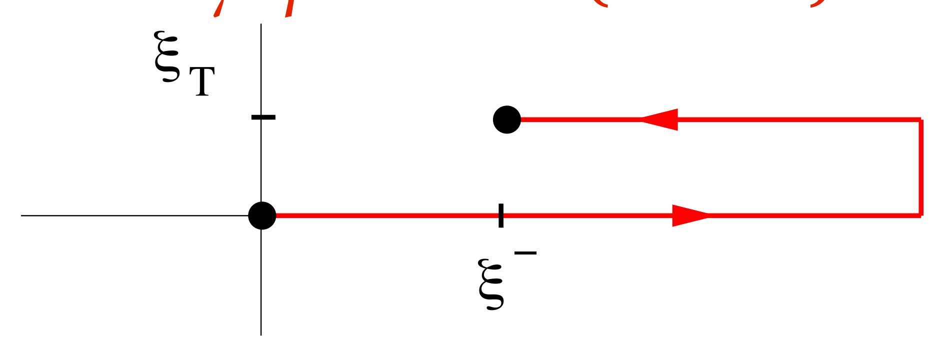
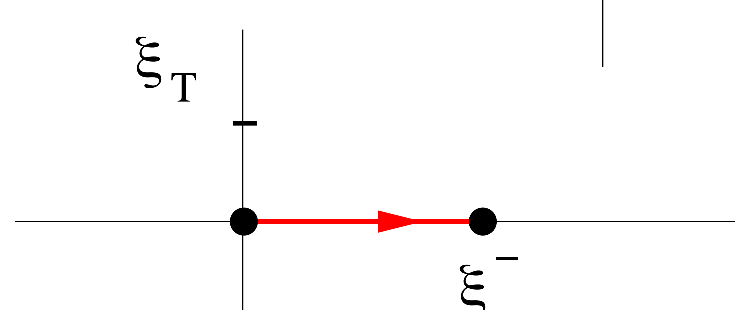




$$\int dk_T \longrightarrow \xi_T = 0 \longrightarrow$$



the same in both cases

Process dependence of TMDs for polarized protons

Transverse Momentum of Partons

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

The quark correlator is parametrized in terms of *transverse momentum dependent* parton distributions (TMDs)

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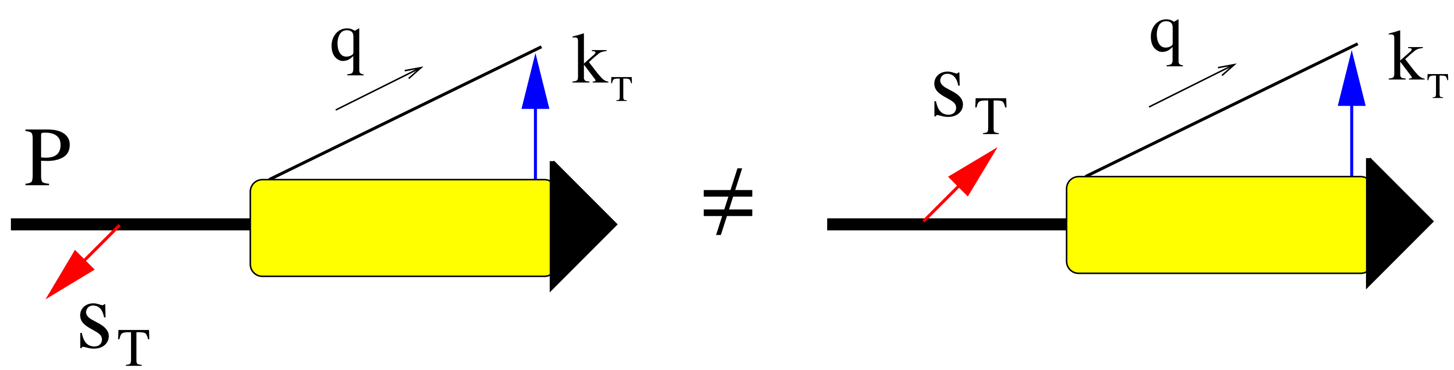
The transverse momentum dependence can be correlated with the spin, e.g.

D. Sivers ('90):



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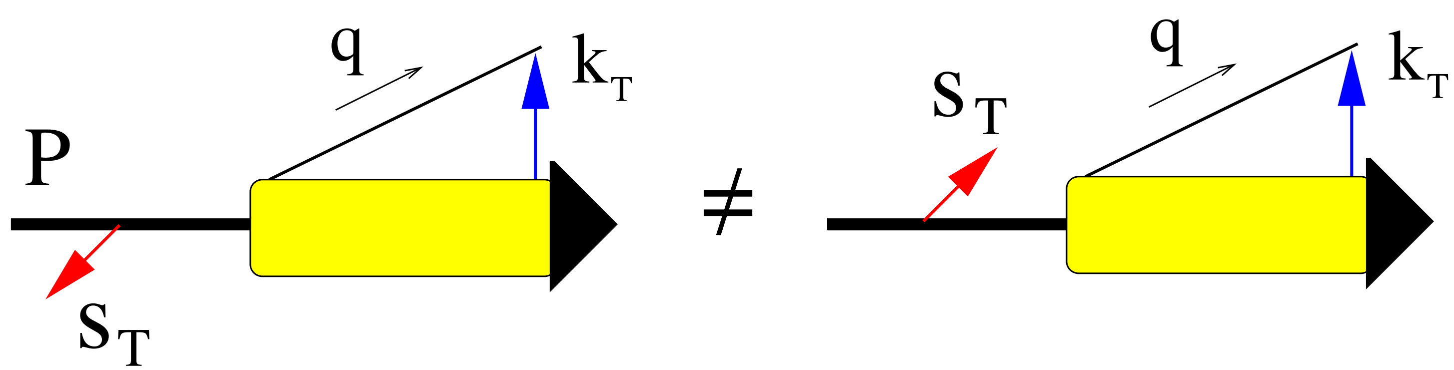

$k_T \times S_T$

Quark correlator:

Sivers function

$$\Phi(x,\boldsymbol{k}_T) = \frac{M}{2}\Big\{ f_1(x,\boldsymbol{k}_T^2)\,\frac{\not{P}}{M} + f_{1T}^{\perp}(x,\boldsymbol{k}_T^2)\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k_T^\rho S_T^\sigma}{M^2} + g_{1s}(x,\boldsymbol{k}_T^2)\,\frac{\gamma_5\,\not{P}}{M}$$

$$+h_{1T}(x,\boldsymbol{k}_T^2)\,\frac{\gamma_5\,\not{S}_T\,\not{P}}{M} + h_{1s}^{\perp}(x,\boldsymbol{k}_T^2)\,\frac{\gamma_5\,\not{k}_T\,\not{P}}{M^2} + h_1^{\perp}(x,\boldsymbol{k}_T^2)\,\frac{i\,\not{k}_T\,\not{P}}{M^2}\Big\}$$

[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98]

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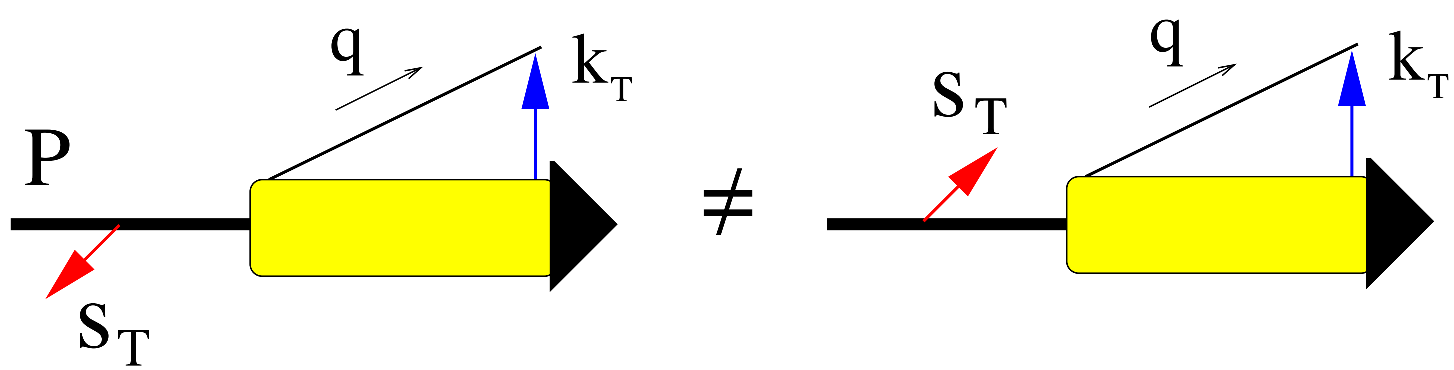

$$k_T \times S_T$$

k_T-odd

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Sivers TMD

The proper theoretical definition of the Sivers TMD is *not unique*

$$P\cdot(\mathbf{k}_T \times \mathbf{S}_T)\, f_{1T}^{\perp[\mathcal{C}]}(x,\mathbf{k}_T^2) \propto \ \mathrm{F.T.}\, \langle P, S_T|\bar{\psi}(0)\,\mathcal{L}_{\mathcal{C}[0,\xi]}\,\gamma^+\,\psi(\xi)|P,S_T\rangle\big|_{\xi=(\xi^-,0^+,\xi_T)}$$

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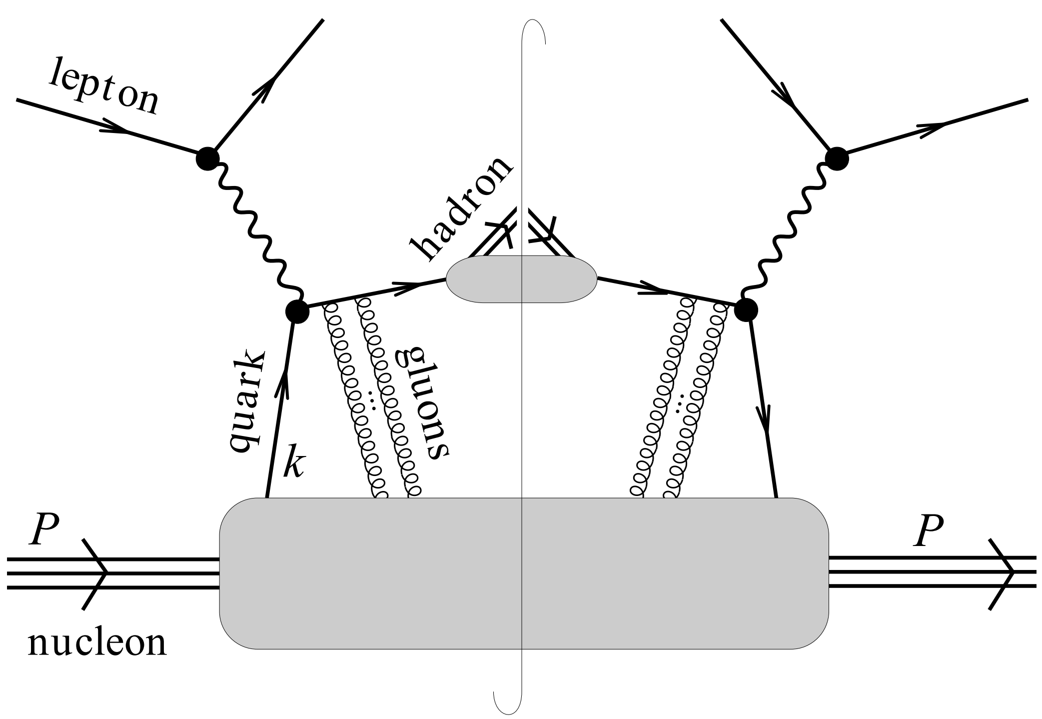
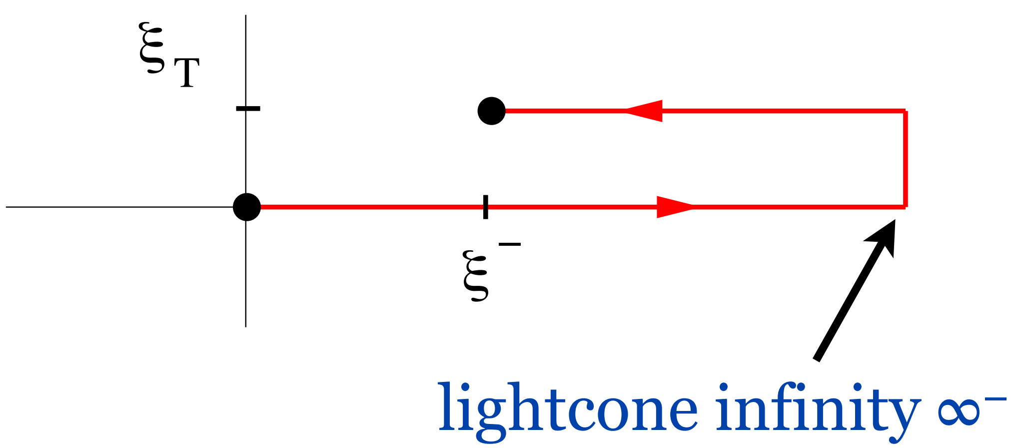
$$e\,p \to e'\,h\,X$$





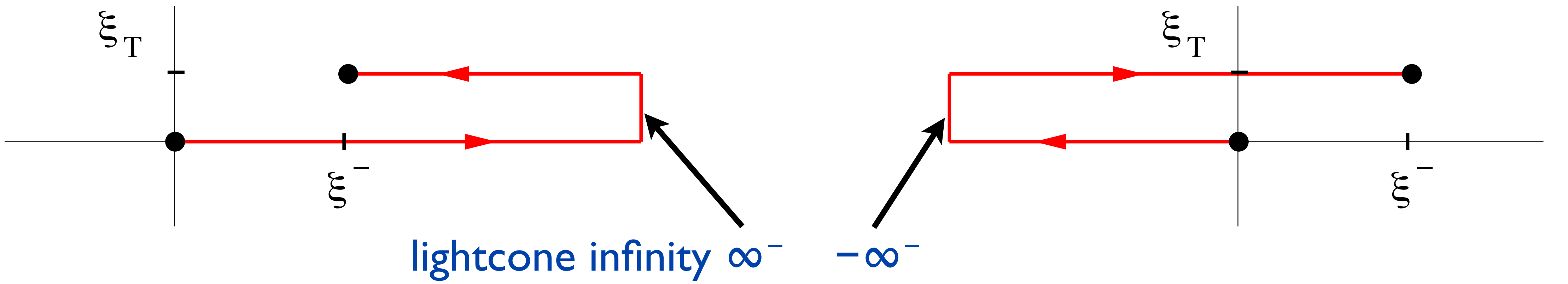
$$k \approx xP + k_T$$

$$P^\mu \approx P^+$$

Process dependence of Sivers TMDs

SIDIS DY

FSI lead to a future pointing Wilson line (+ link), whereas ISI to past pointing (− link)



Time reversal invariance and parity relate the Sivers functions of SIDIS and DY

$$f_{1T}^{\perp q[\mathrm{SIDIS}]}(x, k_T^2) = -f_{1T}^{\perp q[\mathrm{DY}]}(x, k_T^2)$$ [Collins '02]

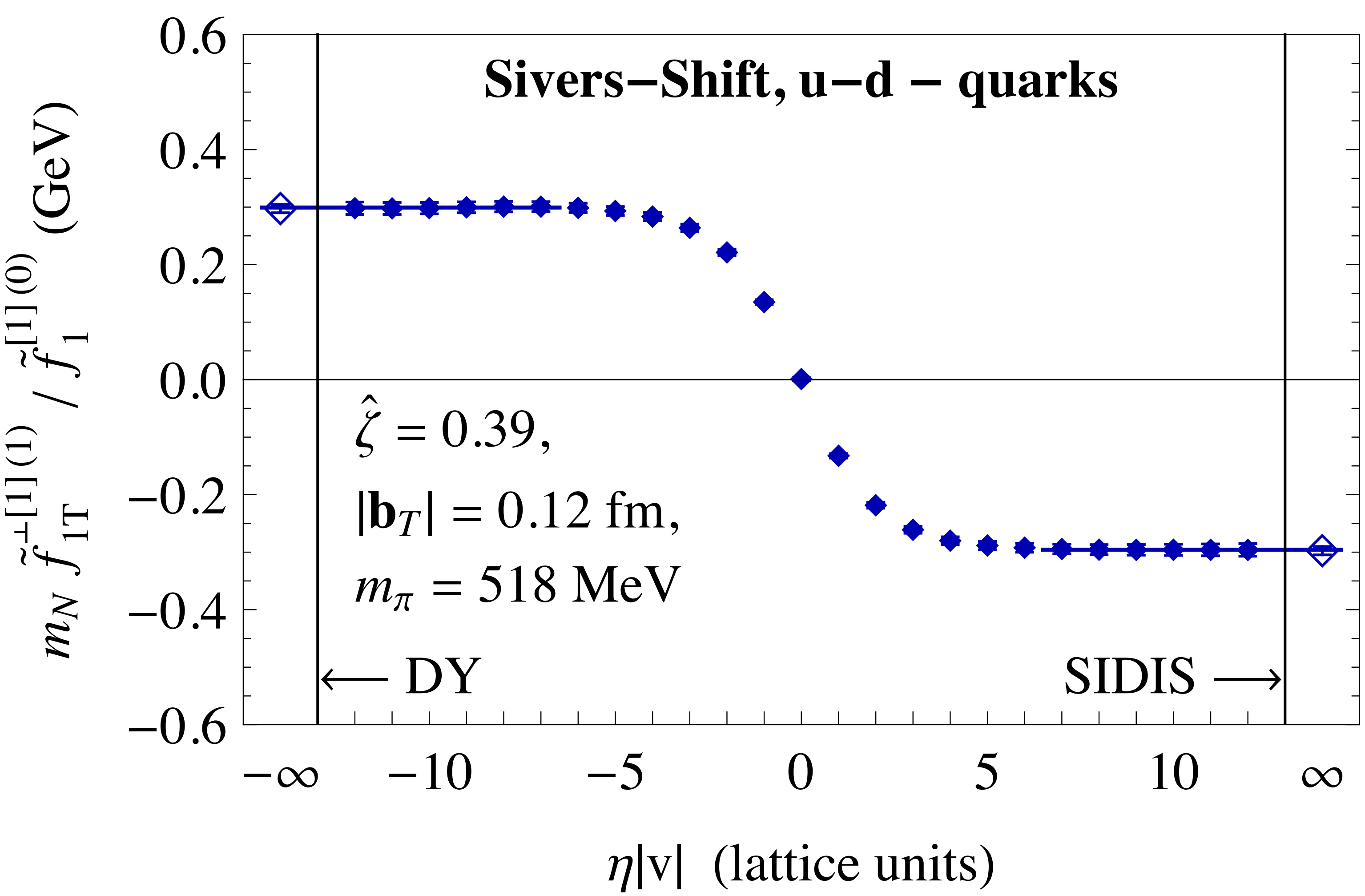
In more complicated processes, more complicated gauge links appear, not necessarily related by just a number to the SIDIS Sivers TMD

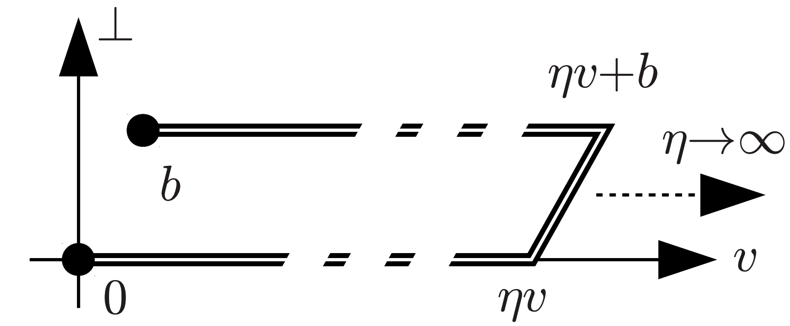
But the first transverse moment is always just a number times the one of SIDIS

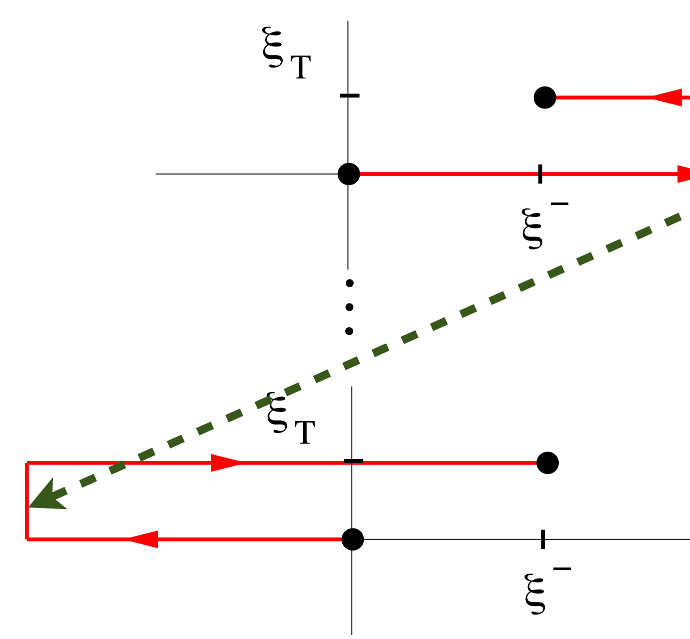
Sivers function on the lattice

By taking specific x and k_T integrals one can define the "Sivers shift" $\langle k_T \times S_T\rangle(n,b_T)$: the average transverse momentum shift orthogonal to transverse spin S_T
[Boer, Gamberg, Musch, Prokudin, 2011]

This well-defined quantity can be evaluated on the lattice
[Musch, Hägler, Engelhardt, Negele & Schäfer, 2012]







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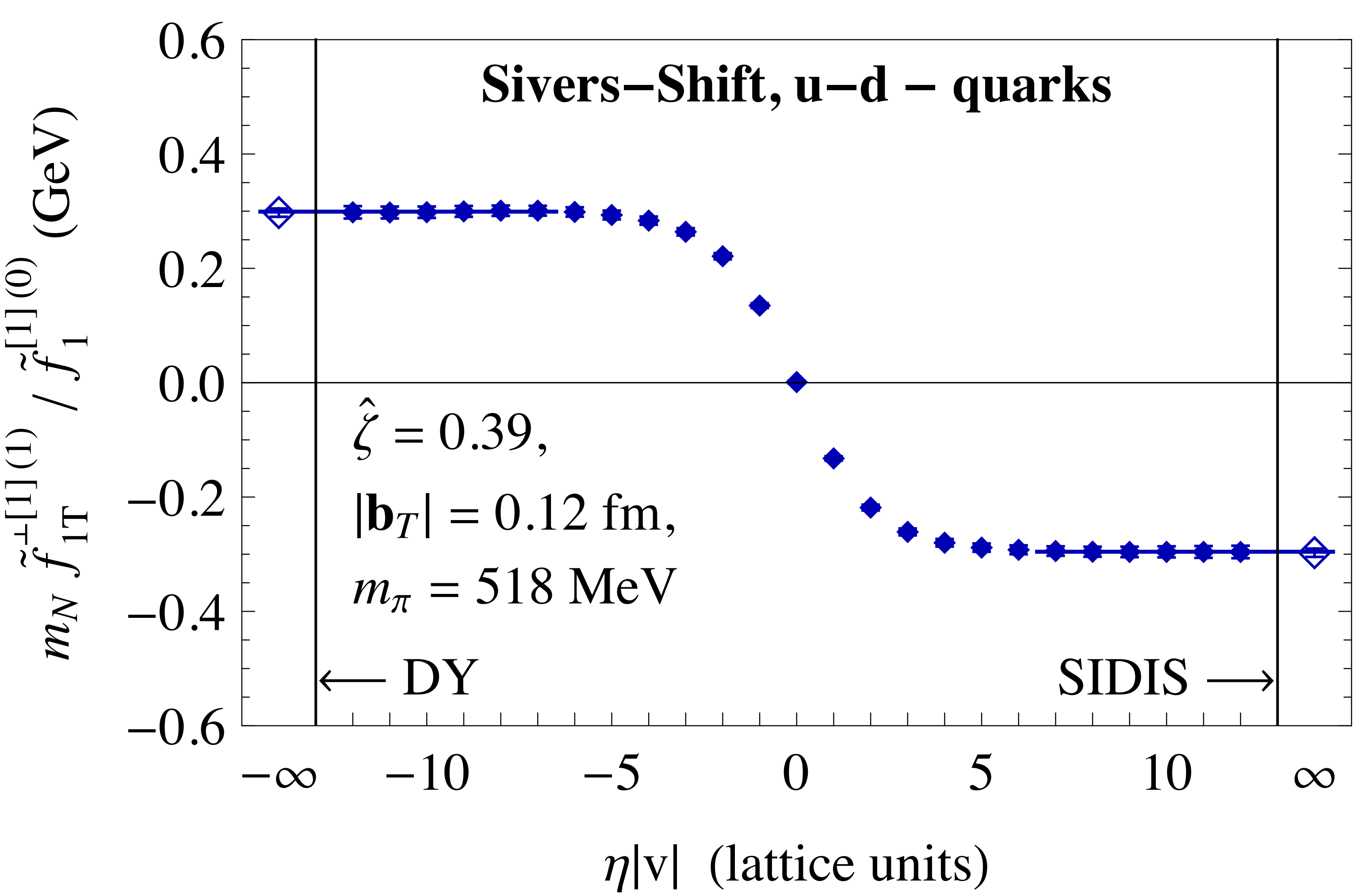
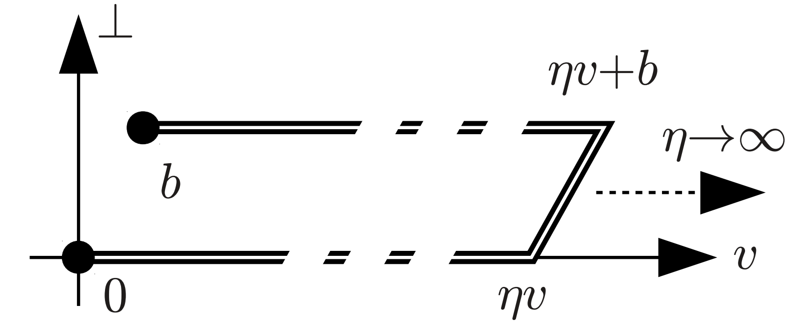
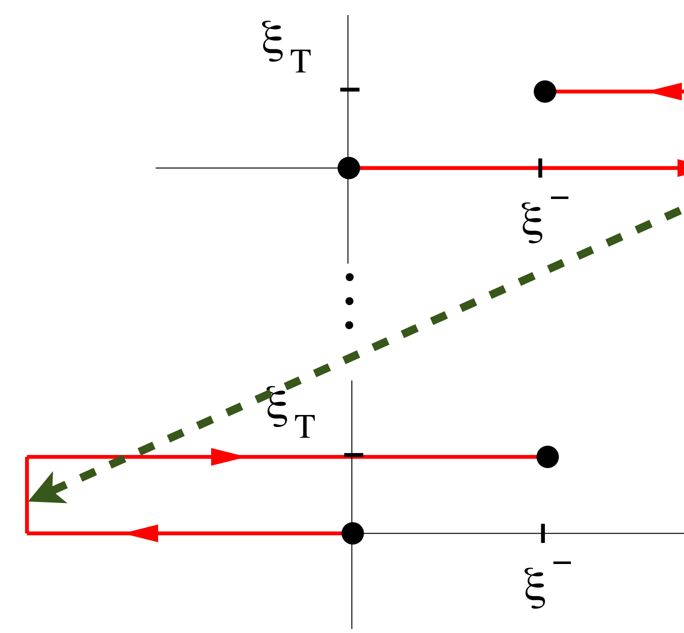






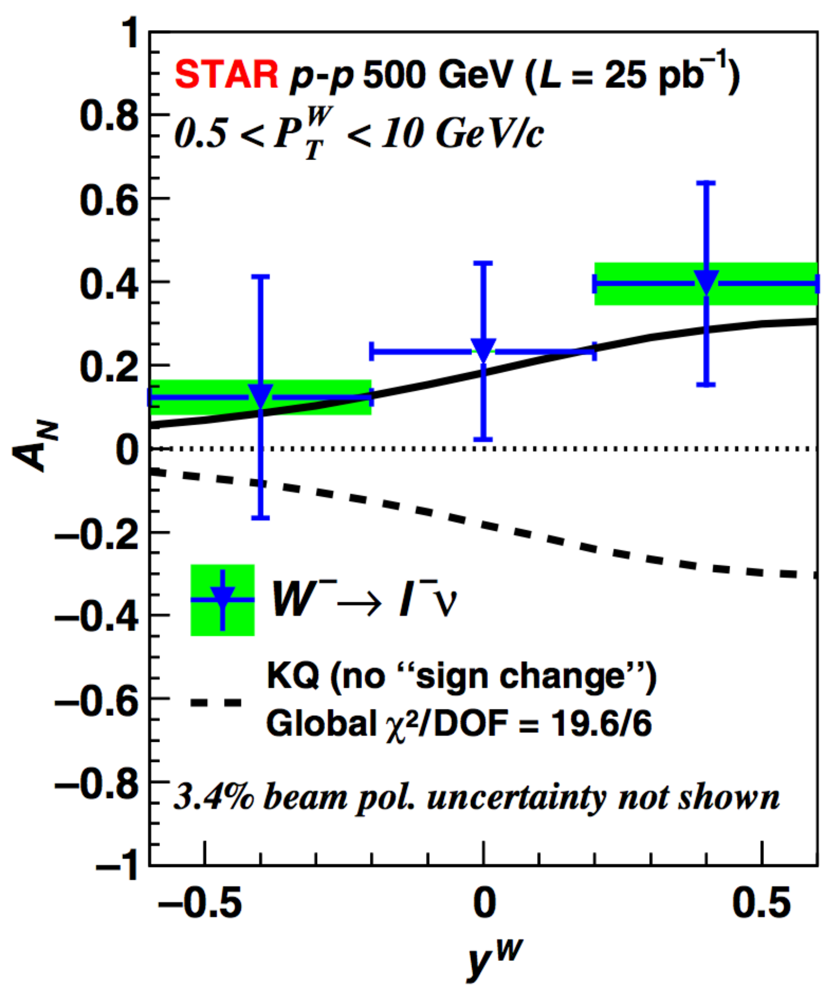
This is the first `first-principle' demonstration that the Sivers function is nonzero for staple-like links. It clearly corroborates the sign change relation (as it should)

Measurements of the Sivers TMD

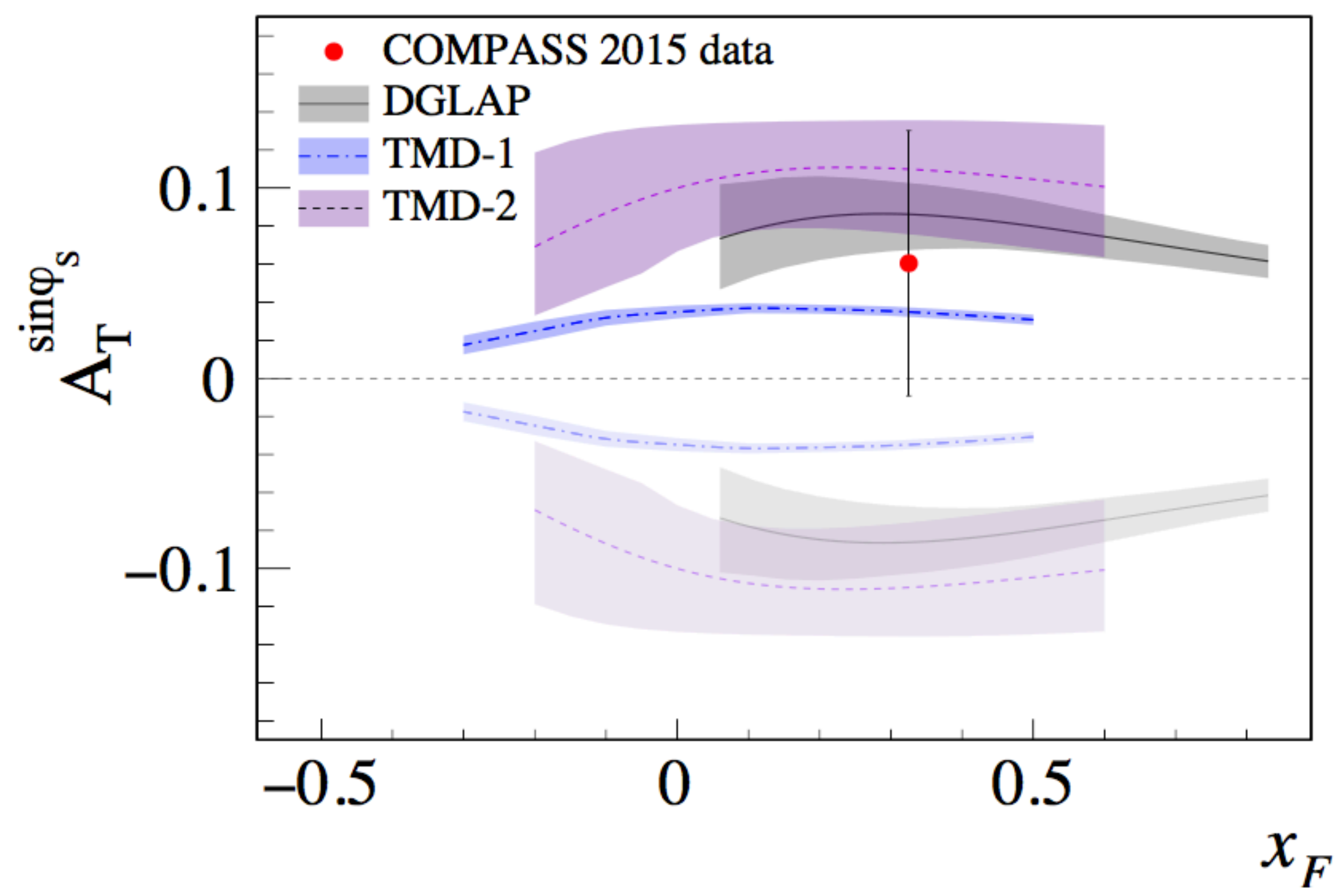
The Sivers effect in SIDIS has been clearly observed by HERMES at DESY (PRL 2009) & COMPASS at CERN (PLB 2010)

The corresponding DY experiments are investigated at CERN (COMPASS), Fermilab (SeaQuest) & RHIC (W-boson production rather) & planned at NICA (Dubna) & IHEP (Protvino)

The first data is compatible with the sign-change prediction of the TMD formalism



STAR, PRL 2016



COMPASS, arXiv:1704.00488

Gluon Sivers effect

There is also a Sivers effect for gluons

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv \mathrm{F.T.}\langle P|\mathrm{Tr}_c\left[F^{+\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]|P\rangle$$

Gluon TMDs depend on two path-dependent gauge links

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For transversely polarized protons:

gluon Sivers function

$$\Gamma_T^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{1}{2x}\left\{g_T^{\mu\nu}\,\frac{\epsilon_T^{\rho\sigma}\,p_{T\rho}\,S_{T\sigma}}{M_p}\,f_{1T}^{\perp\,g}(x, \boldsymbol{p}_T^2) + \ldots\right\}$$

[Mulders, Rodrigues '01]

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[Mulders, Rodrigues '01]

$e\,p^\uparrow \to e'\,Q\bar{Q}\,X$ probes a gluon correlator with two + links

$p^\uparrow\,p \to \gamma\,\gamma\,X$ probes a gluon correlator with two − links

$p^\uparrow\,p \to \gamma\,\mathrm{jet}\,X$ probes a gluon correlator with a + and − link

Sign change relation for **gluon** Sivers TMD

$e\, p^{\uparrow} \to e'\, Q\bar{Q}\, X$ $\qquad \gamma^{*}\, g \to Q\bar{Q}$ probes [+,+]

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Qiu, Schlegel, Vogelsang, 2011

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In the kinematic regime where pair rapidity is central, one effectively selects the subprocess:

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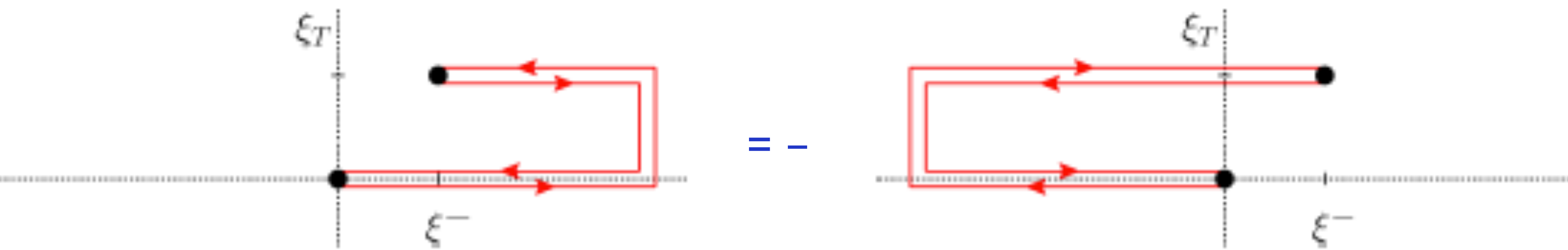

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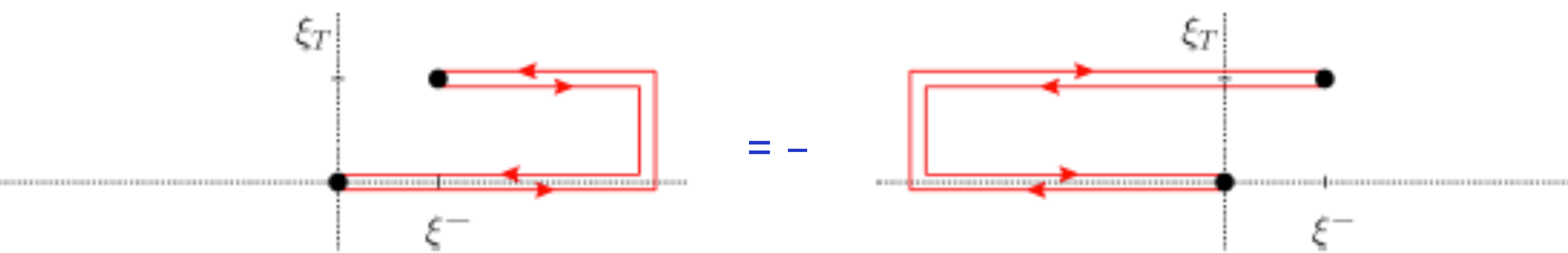

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D.B., Mulders, Pisano, Zhou, 2016

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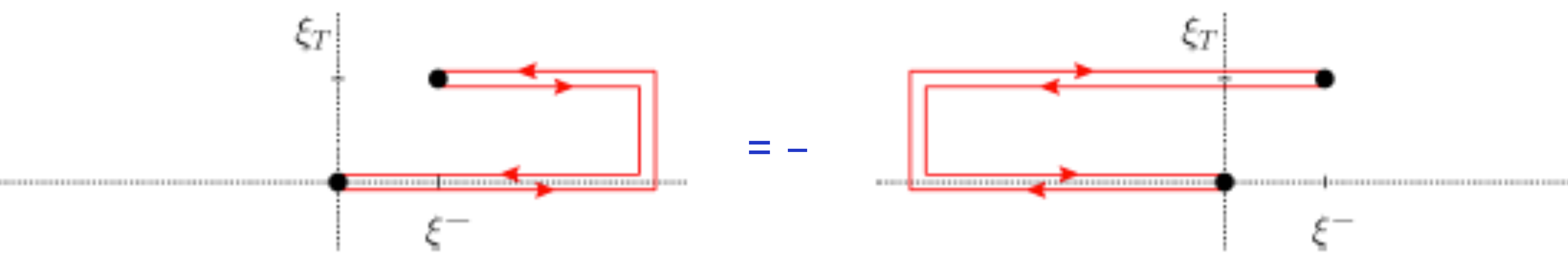

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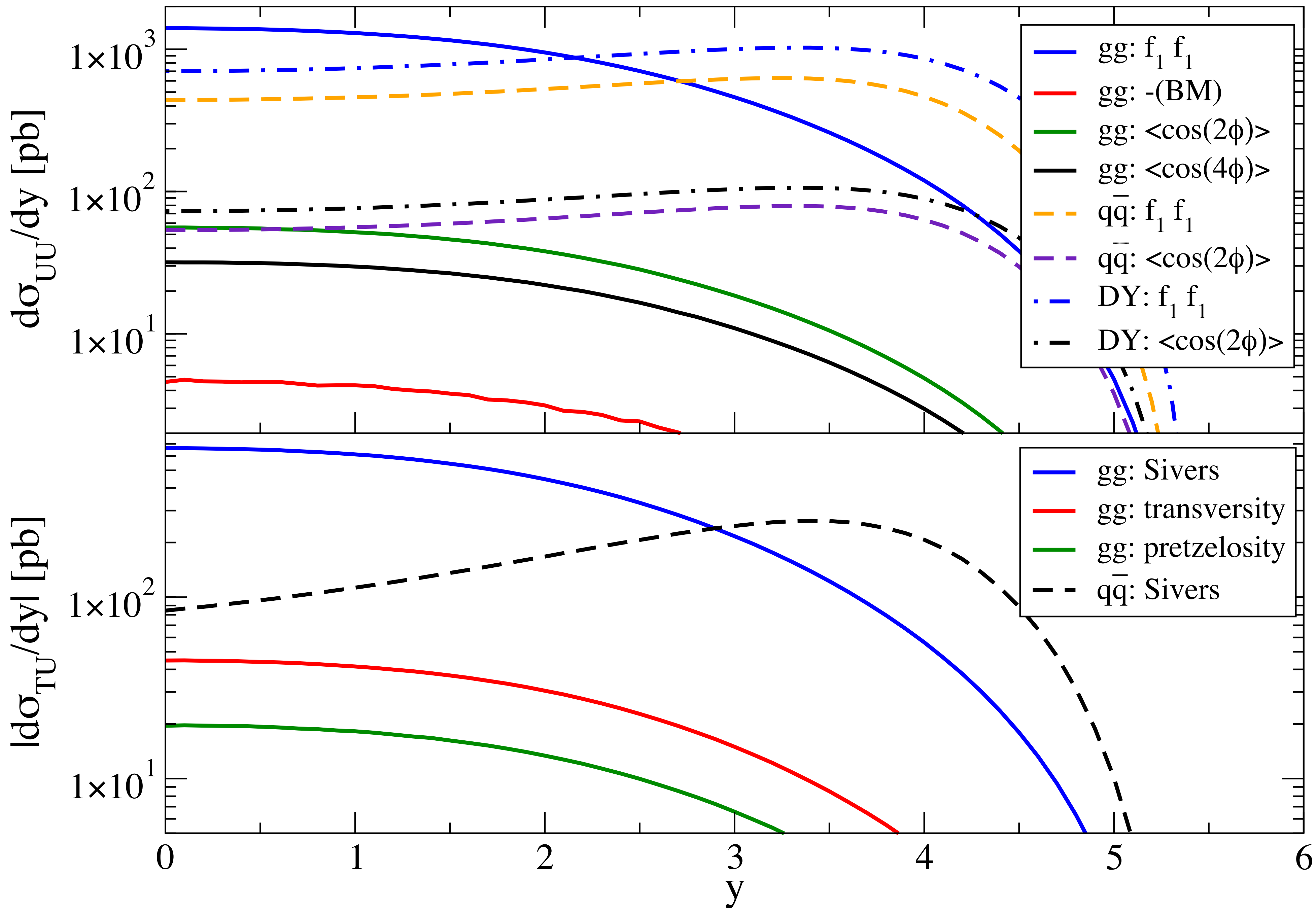
D.B., Mulders, Pisano, Zhou, 2016

Opportunity for RHIC and EIC

Photon pair production

$p p \to \gamma\gamma X$

$p^{\uparrow} p \to \gamma\gamma X$



$\sqrt{s}$=500 GeV, $p_T^{\gamma} \geq 1$ GeV, integrated over $4 < Q^2 < 30$ GeV^2, $0 \leq q_T \leq 1$ GeV

At photon pair rapidity $y < 3$ gluon Sivers dominates and max($d\sigma_{TU}/d\sigma_{UU}$) ~ 30-50%

[Qiu, Schlegel, Vogelsang, 2011]

f and d type gluon Sivers TMD

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In the kinematic regime where gluons in the polarized proton dominate, one effectively selects the subprocess: $g\,q \to \gamma\,q$ probes [+,−]

f and d type gluon Sivers TMD

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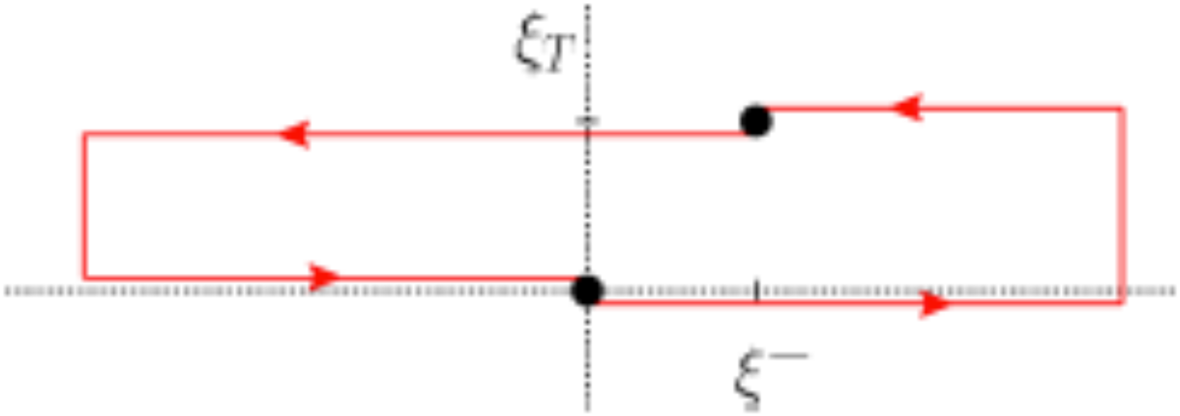

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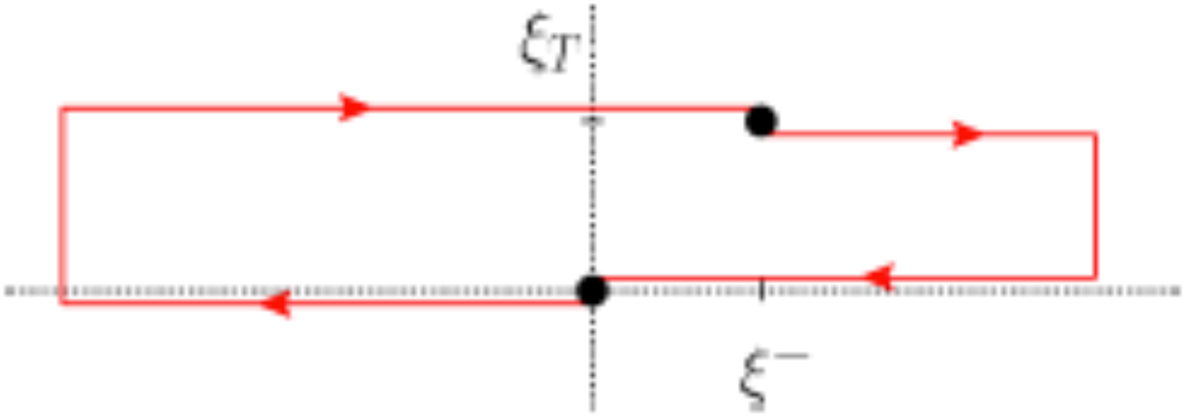

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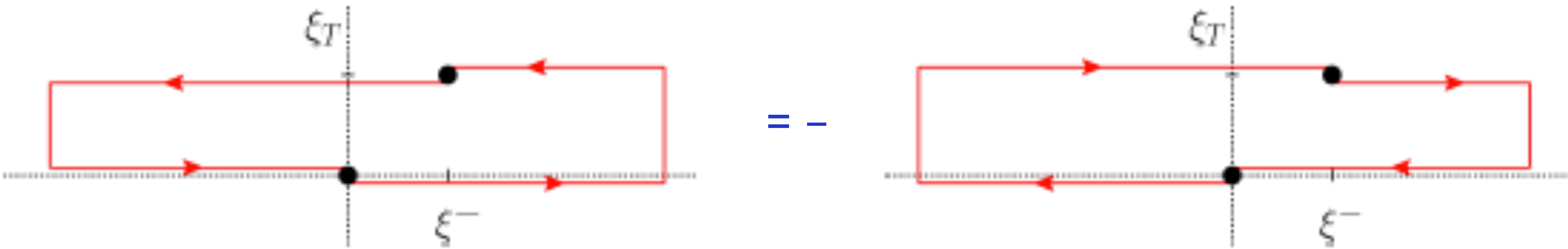


These processes probe 2 distinct, **independent** gluon Sivers functions

Related to antisymmetric (f^{abc}) and symmetric (d^{abc}) color structures

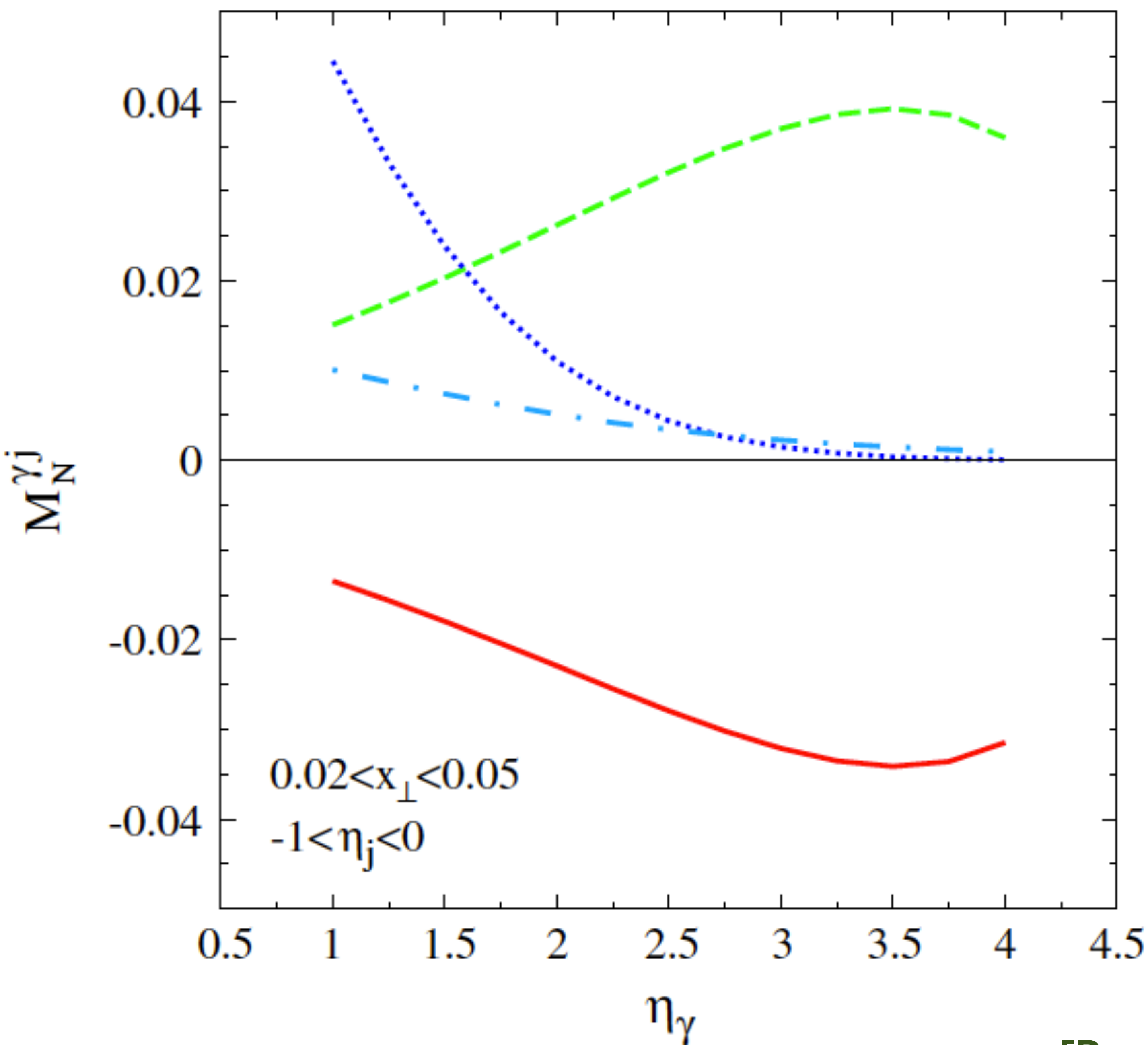
Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Conclusion: gluon Sivers TMD studies at EIC and at RHIC (or AFTER@LHC) can be related or complementary, depending on the processes considered

D.B., Lorcé, Pisano & Zhou, arXiv:1504.04332

Photon-jet production

$$M_N^{\gamma j}(\eta_\gamma, \eta_j, x_\perp) = \frac{\int d\phi_j\, d\phi_\gamma \frac{2|\boldsymbol{K}_{\gamma\perp}|}{M} \sin(\delta\phi) \cos(\phi_\gamma) \frac{d\sigma}{d\phi_j\, d\phi_\gamma}}{\int d\phi_j\, d\phi_\gamma \frac{d\sigma}{d\phi_j\, d\phi_\gamma}}$$



Prediction for the azimuthal moment at $\sqrt{s}$=200 GeV, $p_T^\gamma \geq 1$ GeV, integrated over $-1 \lesssim \eta_j \lesssim 0$, $0.02 \lesssim x_\perp \lesssim 0.05$

Dashed line: GPM

Solid line: using gluonic-pole cross sections

Dotted line: maximum contribution from the gluon Sivers function (absolute value)

Dot-dashed line: maximum contribution from the Boer-Mulders function (abs. value)

[Bacchetta, Bomhof, D'Alesio, Mulders, Murgia, 2007]

Gluon Sivers effect at small x

Selection of processes that probe the WW (f type) or DP (d type) Sivers gluon TMD:

	DY	SIDIS	$p^\uparrow A \to h\, X$	$p^\uparrow A \to \gamma^{(*)}\,\mathrm{jet}\, X$	$p^\uparrow p \to \gamma\,\gamma\, X$ $p^\uparrow p \to J/\psi\,\gamma\, X$ $p^\uparrow p \to J/\psi\, J/\psi\, X$	$e\, p^\uparrow \to e'\, Q\, \overline{Q}\, X$ $e\, p^\uparrow \to e'\, j_1\, j_2\, X$
$f_{1T}^{\perp\, g\,[+,+]}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$
$f_{1T}^{\perp\, g\,[+,-]}$ (DP)	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\times$

backward hadron production

At small x the [+,+] operator corresponds to what is called the Weizsäcker-Williams (WW) gluon operator and [+,−] operator to the dipole (DP) one

For the Sivers function the first transverse moments of the WW and DP cases involve the antisymmetric (f^{abc}) and symmetric (d^{abc}) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

At small x the WW Sivers function appears to be suppressed by a factor of x compared to the unpolarized gluon function, unlike the DP one

Dipole gluon Sivers function at small x

The DP-type Sivers function is not suppressed and can be probed in pA collisions

$$\Gamma^{[+,-]\,ij}(x,\boldsymbol{k}_T) \xrightarrow{x\to 0} \frac{k_T^i k_T^j}{2\pi L}\Gamma_0^{[\Box]}(\boldsymbol{k}_T)$$

D.B., Cotogno, Van Daal, Mulders, Signori, Ya-Jin Zhou, 2016

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D.B., Cotogno, Van Daal, Mulders, Signori, Ya-Jin Zhou, 2016

The DP-type Sivers function at small x is the ***spin-dependent odderon***

$$\Gamma_{(d)}^{(T-\mathrm{odd})} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \mathrm{F.T.}\,\langle P, S_T|\mathrm{Tr}\left[U^{[\Box]}(0_T,y_T) - U^{[\Box]\dagger}(0_T,y_T)\right]|P,S_T\rangle$$

D.B., Echevarria, Mulders, Jian Zhou, 2016

a single Wilson loop matrix element

$$U^{[\Box]} = U^{[+]}_{[0,y]}U^{[-]}_{[y,0]}$$

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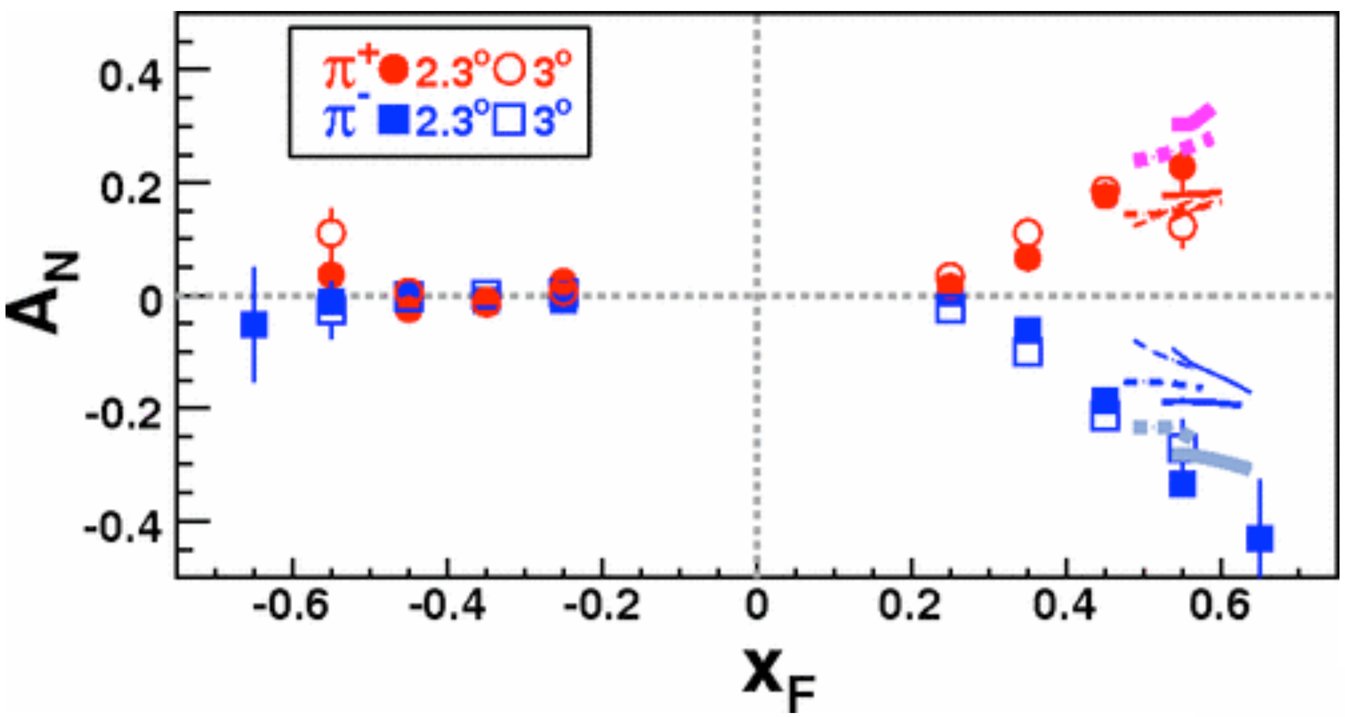
$$U^{[\Box]} = U^{[+]}_{[0,y]}U^{[-]}_{[y,0]}$$

The imaginary part of the Wilson loop determines the gluonic single spin asymmetry

It is the only relevant contribution in A_N at negative x_F, as opposed to the multiple contributions at positive x_F

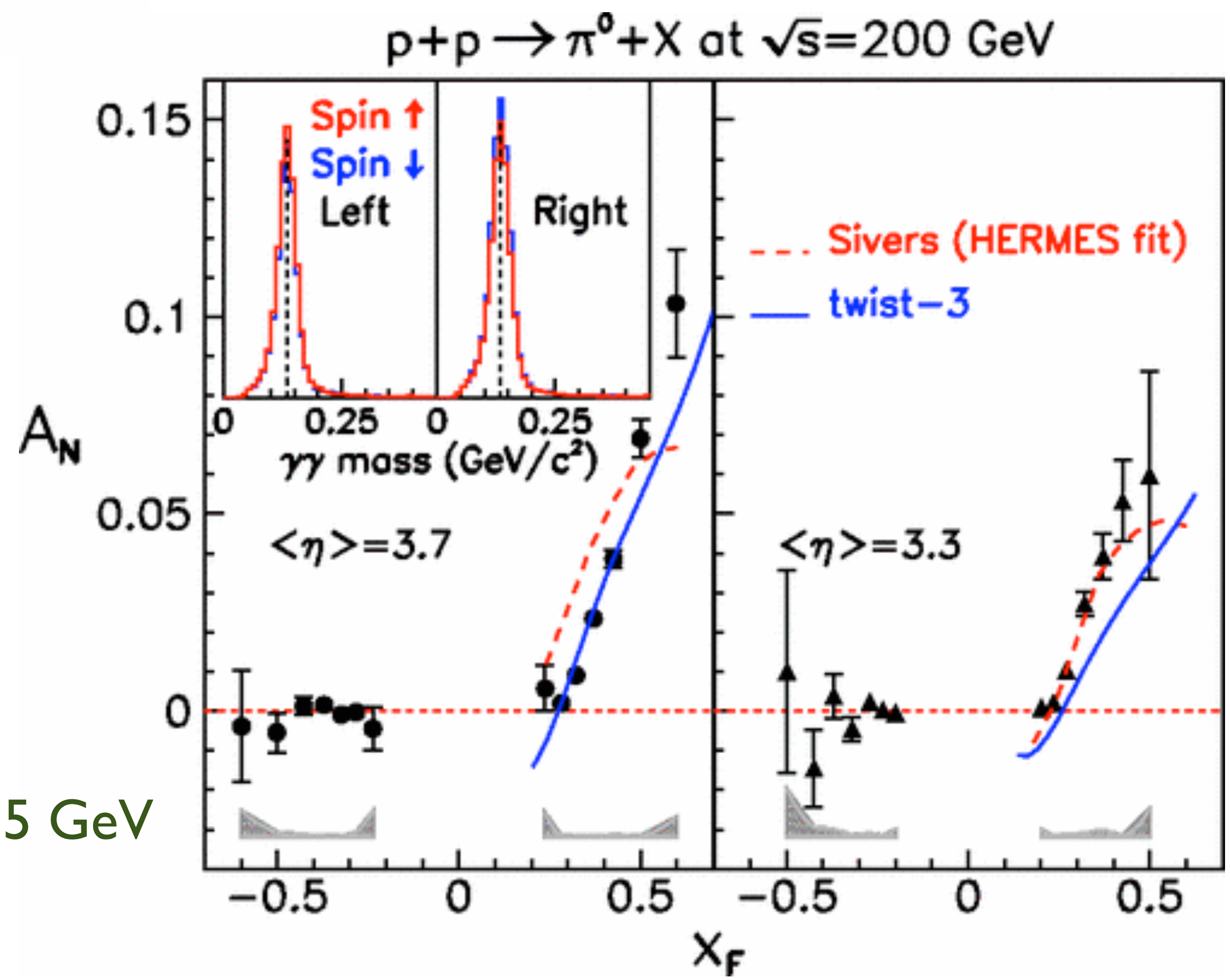
$p^{\uparrow}p \rightarrow h^{\pm} X$ at $x_F < 0$



BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

spin-dependent odderon is C-odd,
whereas gg in the CS state is C-even

expect smaller asymmetries
in neutral pion and jet production



STAR, 2008
$\sqrt{s}$ = 200 GeV
p_T between 1 and 3.5 GeV

Factorization breaking

In general single hadron production in pp or pA is not a TMD process

From that perspective it is best to study imbalance observables, like $\gamma\gamma$ production or γ^*-jet production that probe partonic transverse momenta (γ^*-jet probes the DP gluon Sivers function but its TMD factorization has not been (dis)proven yet)

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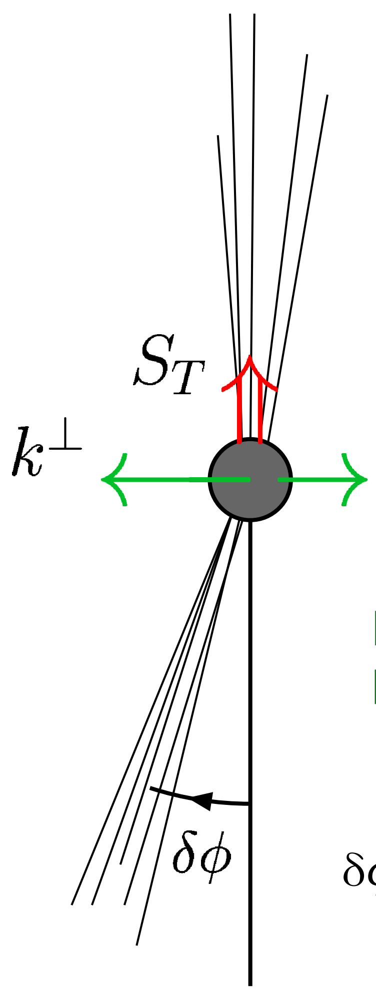
Asymmetric jet or hadron correlations in $p^\uparrow p \to h_1\, h_2\, X$ is expected to exhibit a Sivers asymmetry for the produced jet or hadron pair, but factorization breaking prevents trustworthy predictions



D.B. & Vogelsang 2004
Bacchetta *et al.*, 2005

$\delta\phi$ = dijet imbalance angle

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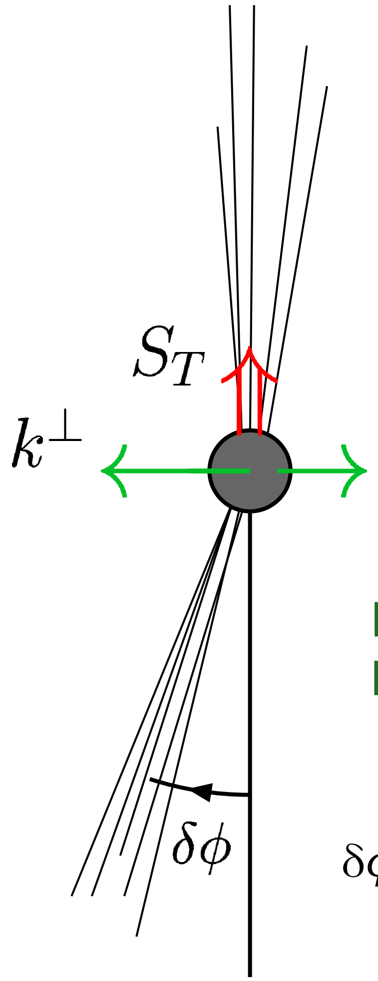


D.B. & Vogelsang 2004
Bacchetta *et al.*, 2005

$\delta\phi$ = dijet imbalance angle

When color flow is in too many directions: *factorization breaking*

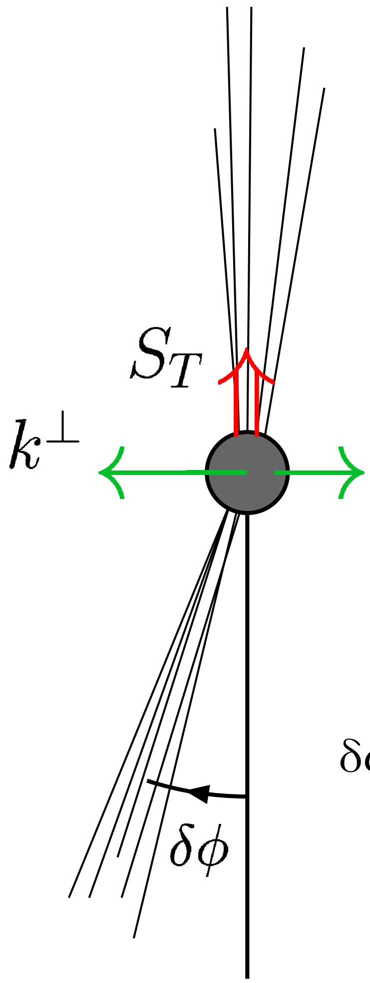
[Collins & J. Qiu '07; Collins '07; Rogers & Mulders '10]

Magnitude of factorization breaking is unknown

Factorization breaking

RHIC data on $p^\uparrow p \to j_1\, j_2\, X$ consistent with zero at the few percent level

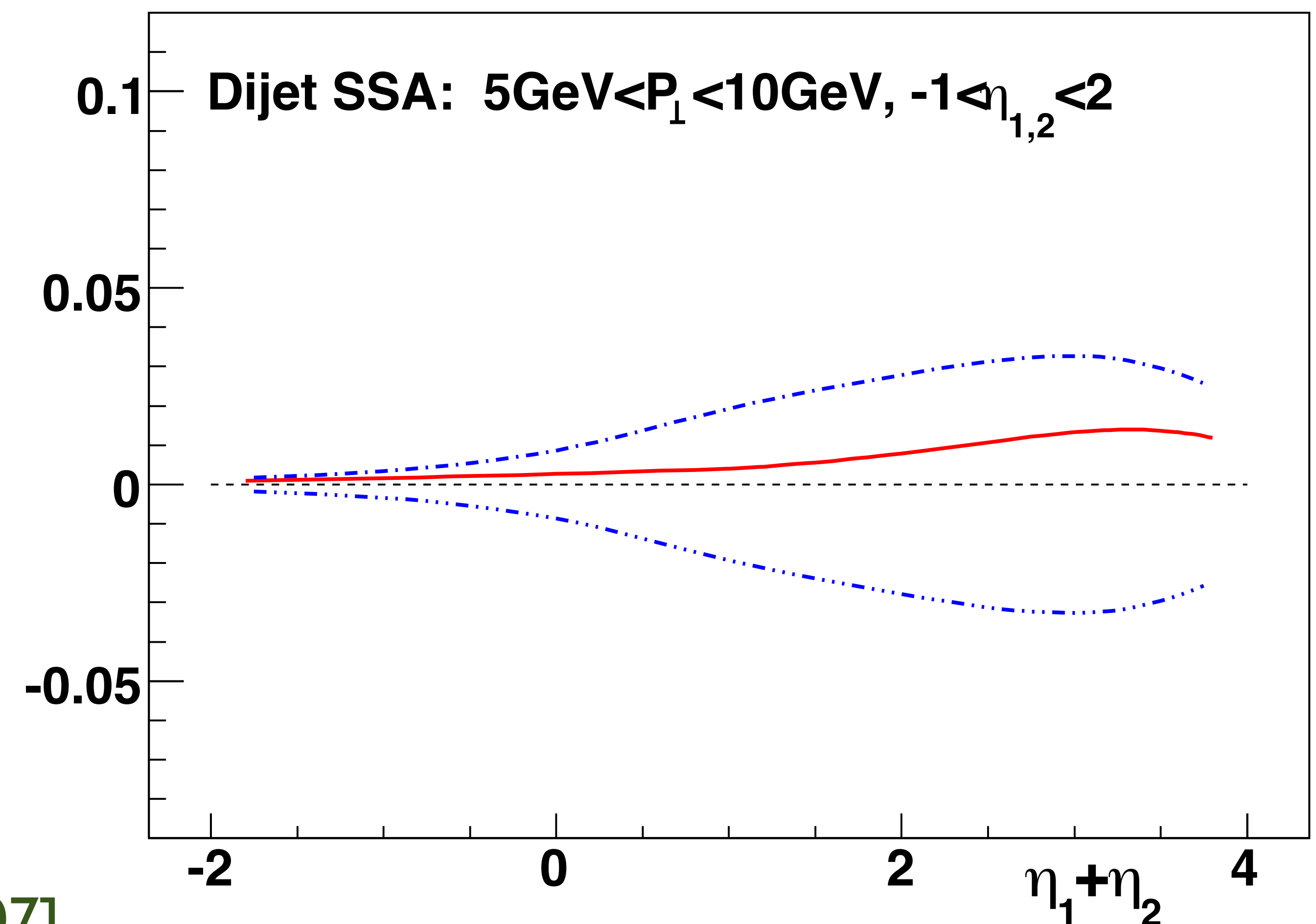
[STAR Collaboration, Abelev *et al.* PRL 2007]



$\delta\phi$ = dijet imbalance angle

$$\mathcal{U}_{qq'} = \frac{1}{N_c^2 - 1}\left[(N_c^2 + 1)\frac{\mathrm{Tr}(\mathcal{U}^{[\Box]})}{N_c}\,\mathcal{U}^{[+]} - 2\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right],$$

[Bomhof, Mulders, Vogelsang, Yuan, PRD 2007]



Should be measured more precisely (incl. the color factor of the $P_\perp \sin\delta\phi$ moment)

Unpolarized protons

Quark TMDs

$$f_1^{[+]}(x, p_T^2) = f_1^{[-]}(x, p_T^2)$$

$$f_1^{[\Box+]}(x, p_T^2) \neq f_1^{[+]}(x, p_T^2)$$

[D.B., Buffing, Mulders, JHEP 2015]

Irrespective of whether one can isolate the function with an additional loop from experiment, one can study particular Mellin-Bessel moments of it on the lattice:

$$\frac{\tilde{f}_1^{[1](1)[\Box+]}(\boldsymbol{b}_T^2; \mu, \zeta)}{\tilde{f}_1^{[1](1)[+]}(\boldsymbol{b}_T^2; \mu, \zeta)} = \frac{\langle P|\overline{\psi}(0, 0_T)\gamma^+ \, U_{[0,b]}^{[+]} U_{[b,0]}^{[-]} U_{[0,b]}^{[+]} \, \psi(0, b_T)|P\rangle}{\langle P|\overline{\psi}(0, 0_T)\gamma^+ \, U_{[0,b]}^{[+]} \, \psi(0, b_T)|P\rangle}$$

This will give us information on how important the flux of $F^{\mu\nu}$ through the loop is and hence how important the process dependence effects are or can be

The dipole ([+,−]) gluon Sivers TMD at small-x is entirely determined by the loop

In this sense, the SSA at small-x is to QCD what the Aharonov-Bohm effect in the double-slit experiment is to QED

Gluons TMDs

The gluon correlator:

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x, k_T) \equiv \mathrm{F.T.} \langle P | \mathrm{Tr}_c \left[F^{+\nu}(0)\, \mathcal{U}_{[0,\xi]}\, F^{+\mu}(\xi)\, \mathcal{U}'_{[\xi,0]} \right] | P \rangle$$

For unpolarized protons:

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu}\, f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\boldsymbol{p}_T^2}{2M_p^2} \right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right\}$$

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unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside *unpolarized* protons can be polarized!

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unpolarized gluon TMD

linearly polarized gluon TMD

Gluons inside *unpolarized* protons can be polarized!

The gauge links are process dependent, affecting even the unpolarized gluon TMDs as was first realized in a small-x context

Dominguez, Marquet, Xiao, Yuan, 2011

Explains Kharzeev, Kovchegov & Tuchin's "tale of two gluon distributions" (2003)

WW vs DP

For most processes of interest there are 2 relevant unpolarized gluon distributions

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \qquad [+,+]$$

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \qquad [+,-]$$

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For unpolarized gluons [+,+] = [−,−] and [+,−] = [−,+]

At small x the two correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different in magnitude and width:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g} \qquad \mathrm{WW}$$

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr} U(0)U^\dagger(r_\perp)\right\rangle_{x_g} \qquad \mathrm{DP}$$

WW vs DP

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Different processes probe one or the other or a mixture, so this can be tested

WW vs DP

Selection of processes that probe the WW or DP unpolarized gluon TMD:

	DIS	DY	SIDIS	$pA \to \gamma\,\mathrm{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$f_1^{g\,[+,+]}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$	$\surd$
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WW vs DP

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$f_1^{g\,[+,+]}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$	$\surd$
$f_1^{g\,[+,-]}$ (DP)	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\times$	$\times$

Dijet production in pA probes a combination of 6 distinct unpolarized gluon TMDs
In the large N_c limit it probes a combination of DP and WW functions

Akcakaya, Schäfer, Zhou, 2013; Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren, 2015

Dijet production in pA generally suffers from factorization breaking contributions

Collins, Qiu, 2007; Rogers, Mulders, 2010

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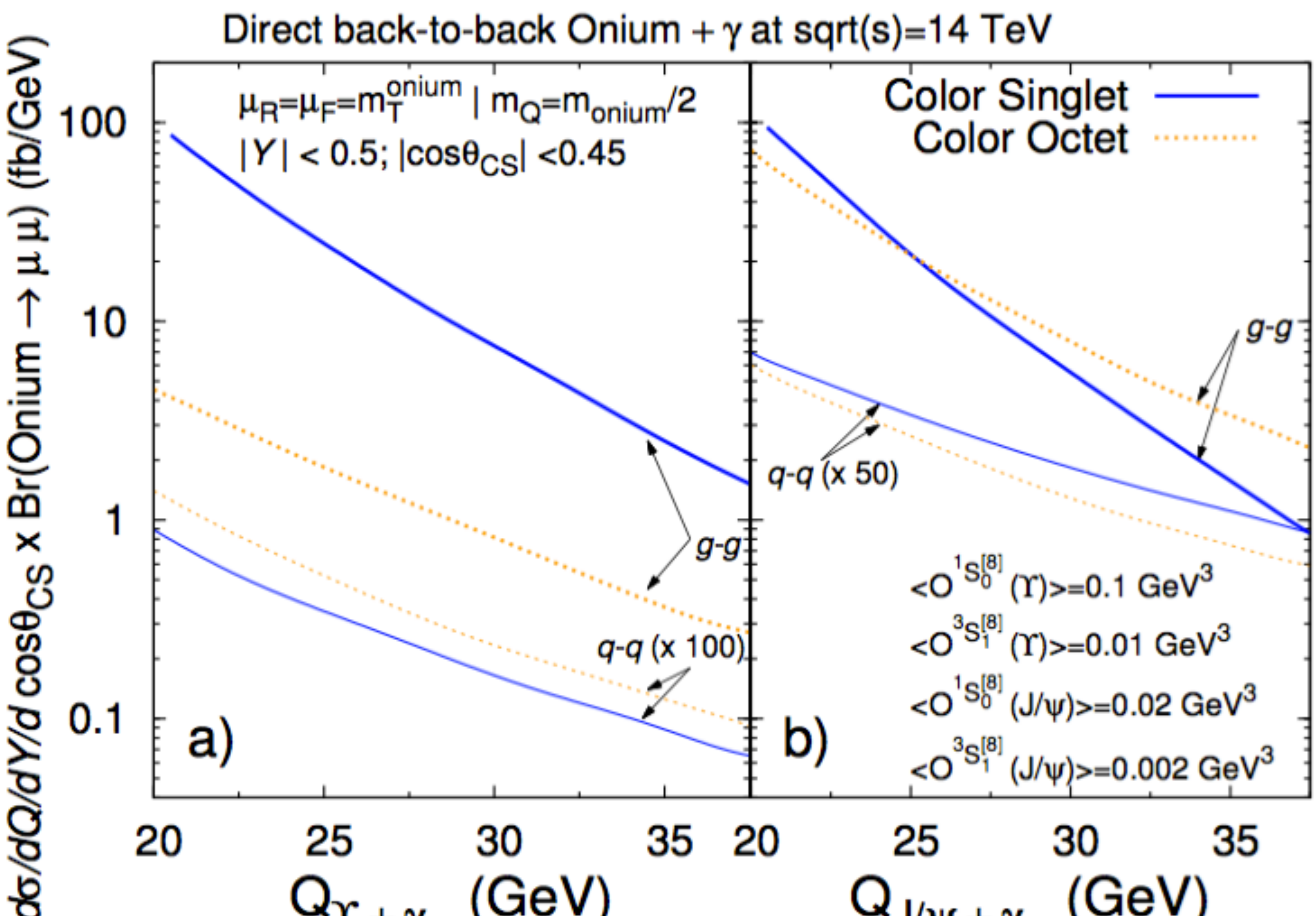
Single color singlet (CS) J/ψ or ϒ production from two gluons is not allowed by the Landau-Yang theorem, while color octet (CO) production involves a more complicated link structure. C-even (pseudo-)scalar quarkonium production is easier

D.B., Pisano, 2012

CS vs CO

In $\Upsilon+\gamma$ production the color singlet contribution dominates and in $J/\psi+\gamma$ production too for a specific range of invariant mass of the pair

Den Dunnen, Lansberg, Pisano, Schlegel, 2014



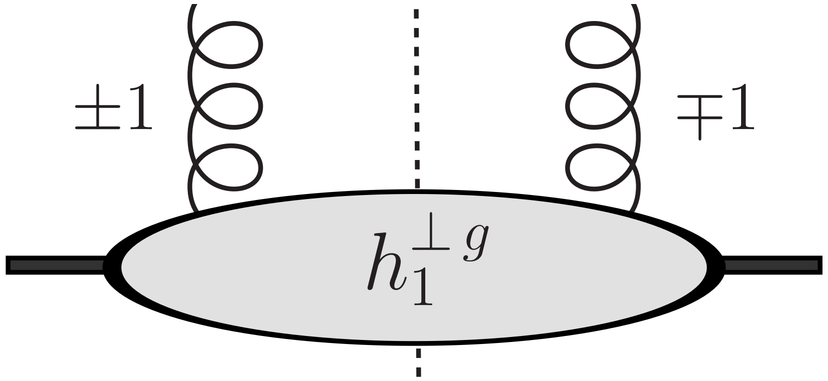
Linearly polarized gluons in unpolarized hadrons at small x

Gluon polarization inside unpolarized protons

Linearly polarized gluons can exist in **unpolarized** hadrons

[Mulders, Rodrigues, 2001]

It requires nonzero transverse momentum: TMD



an interference between ±1 helicity gluon states

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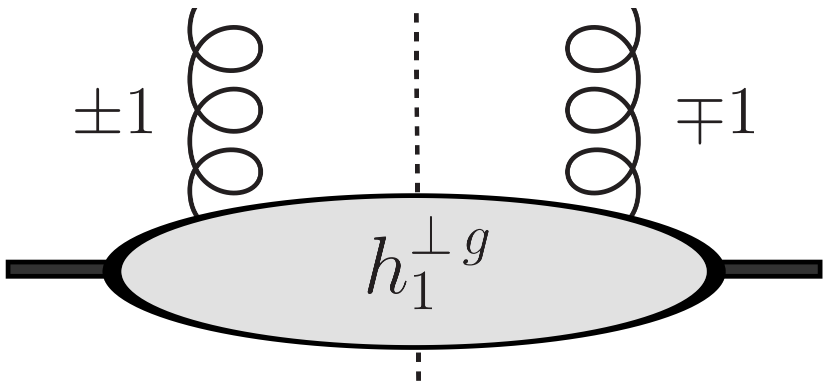
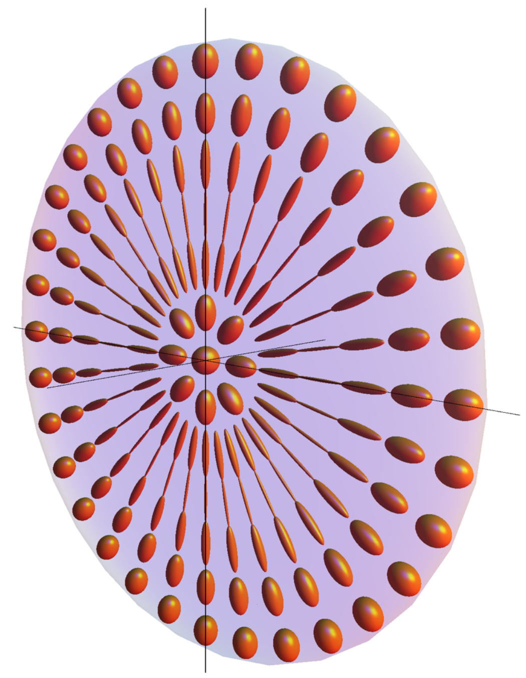


an interference between ± 1 helicity gluon states

It requires nonzero transverse momentum: TMD

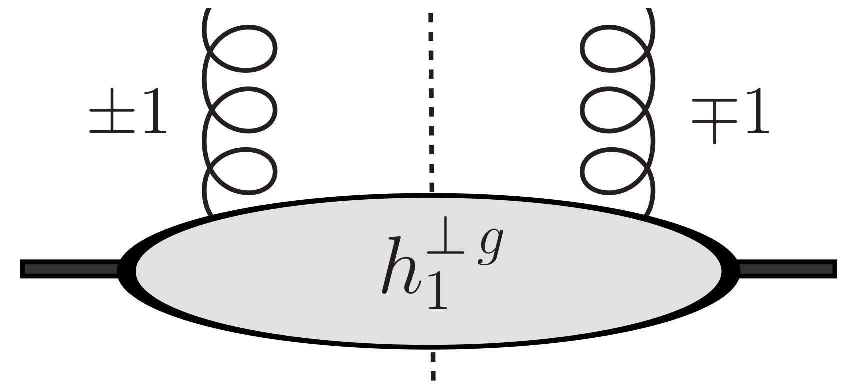
For $h_1^{\perp\, g} > 0$ gluons prefer to be polarized along k_T, with a $cos\ 2\phi$ distribution of linear polarization around it, where $\phi=\angle(k_T,\varepsilon_T)$



Gluon polarization inside unpolarized protons

Linearly polarized gluons can exist in **unpolarized** hadrons

[Mulders, Rodrigues, 2001]



an interference between ±1 helicity gluon states

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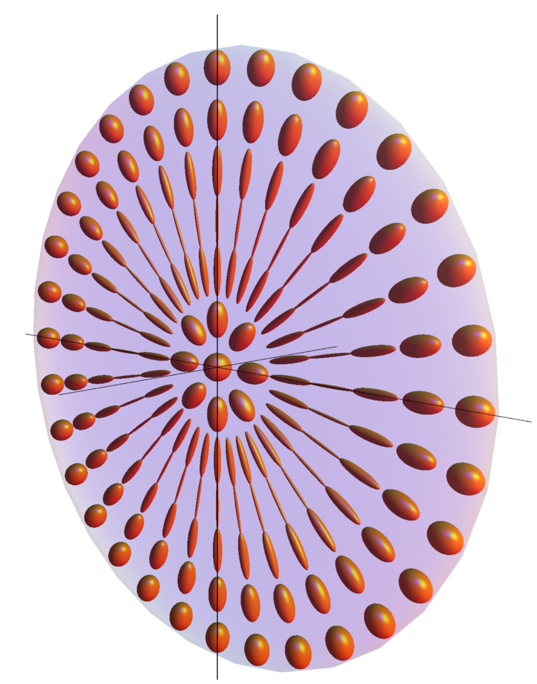

This TMD is k_T-even, chiral-even and T-even:

$$\Gamma_U^{\mu\nu}(x, \boldsymbol{p}_T) = \frac{x}{2} \left\{ -g_T^{\mu\nu}\, f_1^g(x, \boldsymbol{p}_T^2) + \left(\frac{p_T^\mu p_T^\nu}{M_p^2} + g_T^{\mu\nu} \frac{\boldsymbol{p}_T^2}{2M_p^2} \right) h_1^{\perp\, g}(x, \boldsymbol{p}_T^2) \right\}$$

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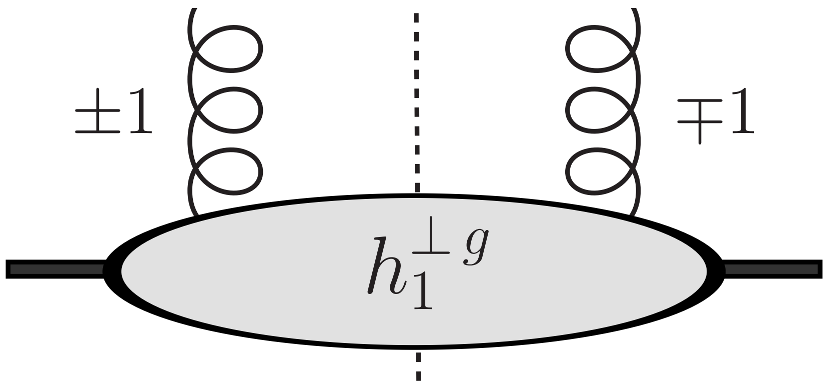

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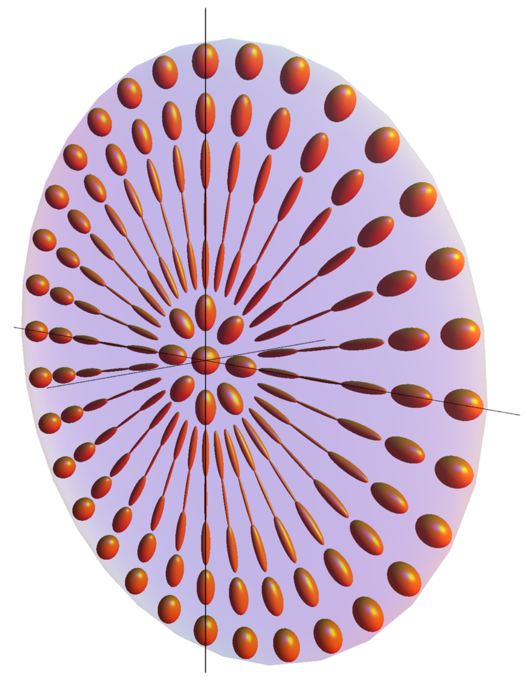

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For linearly polarized gluons also [+,+] = [−,−] and [+,−] = [−,+]

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	$pp \to \gamma\,\gamma\,X$	$pA \to \gamma^*\,\mathrm{jet}\,X$	$e\,p \to e'\,Q\,\overline{Q}\,X$ $e\,p \to e'\,j_1\,j_2\,X$	$pp \to \eta_{c,b}\,X$ $pp \to H\,X$	$pp \to J/\psi\,\gamma\,X$ $pp \to \Upsilon\,\gamma\,X$
$h_1^{\perp\,g\,[+,+]}$ (WW)	$\surd$	$\times$	$\surd$	$\surd$	$\surd$
$h_1^{\perp\,g\,[+,-]}$ (DP)	$\times$	$\surd$	$\times$	$\times$	$\times$

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Higgs and $0^{\pm+}$ quarkonium production allows to measure the linear gluon polarization using the angular independent p_T distribution

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EIC can probe the WW $h_1^{\perp g}$, while RHIC/LHC can probe both the WW and DP one

Qiu, Schlegel, Vogelsang, 2011; Jian Zhou , 2016; D.B., Brodsky, Pisano, Mulders, 2011; D.B., Pisano, 2012; Sun, Xiao, Yuan, 2011; D.B., den Dunnen, Pisano, Schlegel, Vogelsang, 2012; den Dunnen, Lansberg, Piano, Schlegel, 2014

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The small-x limit of the DP correlator in the TMD formalism:

$$\Gamma^{[+,-]\, ij}(x, \boldsymbol{k}_T) \xrightarrow{x \to 0} \frac{k_T^i k_T^j}{2\pi L} \Gamma_0^{[\Box]}(\boldsymbol{k}_T) \qquad U^{[\Box]} = U_{[0,y]}^{[+]} U_{[y,0]}^{[-]}$$

$$\Gamma_U^{ij}(x, \boldsymbol{k}_T) = \frac{x}{2} \left[-g_T^{ij}\, f_1(x, \boldsymbol{k}_T^2) + \frac{k_T^{ij}}{M^2}\, h_1^{\perp}(x, \boldsymbol{k}_T^2) \right] \xrightarrow{x \to 0} \frac{k_T^i k_T^j}{2M^2}\, e(\boldsymbol{k}_T^2)$$

$$\lim_{x \to 0} x f_1(x, \boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2} \lim_{x \to 0} x h_1^{\perp}(x, \boldsymbol{k}_T^2) = \frac{\boldsymbol{k}_T^2}{2M^2}\, e(\boldsymbol{k}_T^2)$$

In the TMD formalism the DP $h_1^{\perp g}$ becomes maximal when x → 0

D.B., Cotogno, van Daal, Mulders, Signori, Zhou, 2016

Polarization of the CGC

CGC framework calculations show the CGC gluons are in fact linearly polarized

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2 f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

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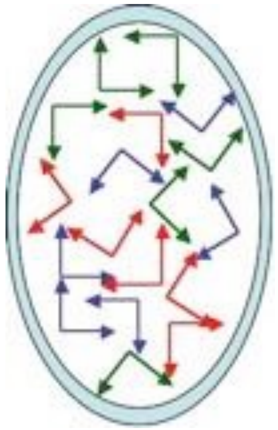

Metz, Zhou '11

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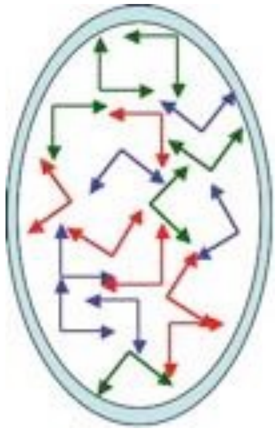

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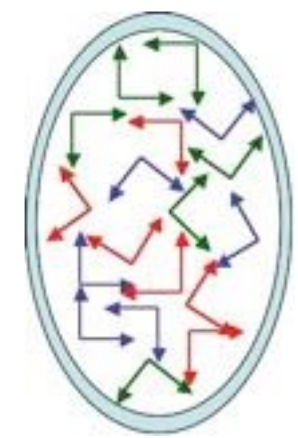

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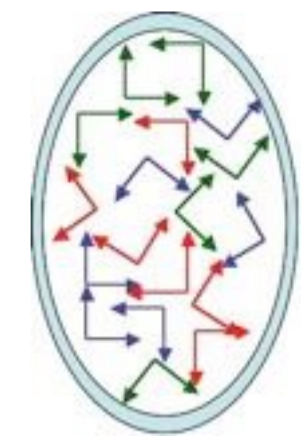

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The "k_T-factorization" approach (CCFM) yields maximum polarization too (but no process dependence):

$$\Gamma_g^{\mu\nu}(x, \boldsymbol{p}_T)_{\text{max pol}} = \frac{p_T^\mu p_T^\nu}{\boldsymbol{p}_T^2}\, x\, f_1^g$$

Catani, Ciafaloni, Hautmann, 1991

TMD evolution suppresses linear gluon polarization

Define the relative contribution of linearly polarized gluons in $pp \to HX$:

$$\mathcal{R}(Q_T) \equiv \frac{\mathcal{C}[w_H \, h_1^{\perp g} \, h_1^{\perp g}]}{\mathcal{C}[f_1^g \, f_1^g]}$$

$$w_H = \frac{(\boldsymbol{p}_T \cdot \boldsymbol{k}_T)^2 - \frac{1}{2}\boldsymbol{p}_T^2 \boldsymbol{k}_T^2}{2M^4}$$

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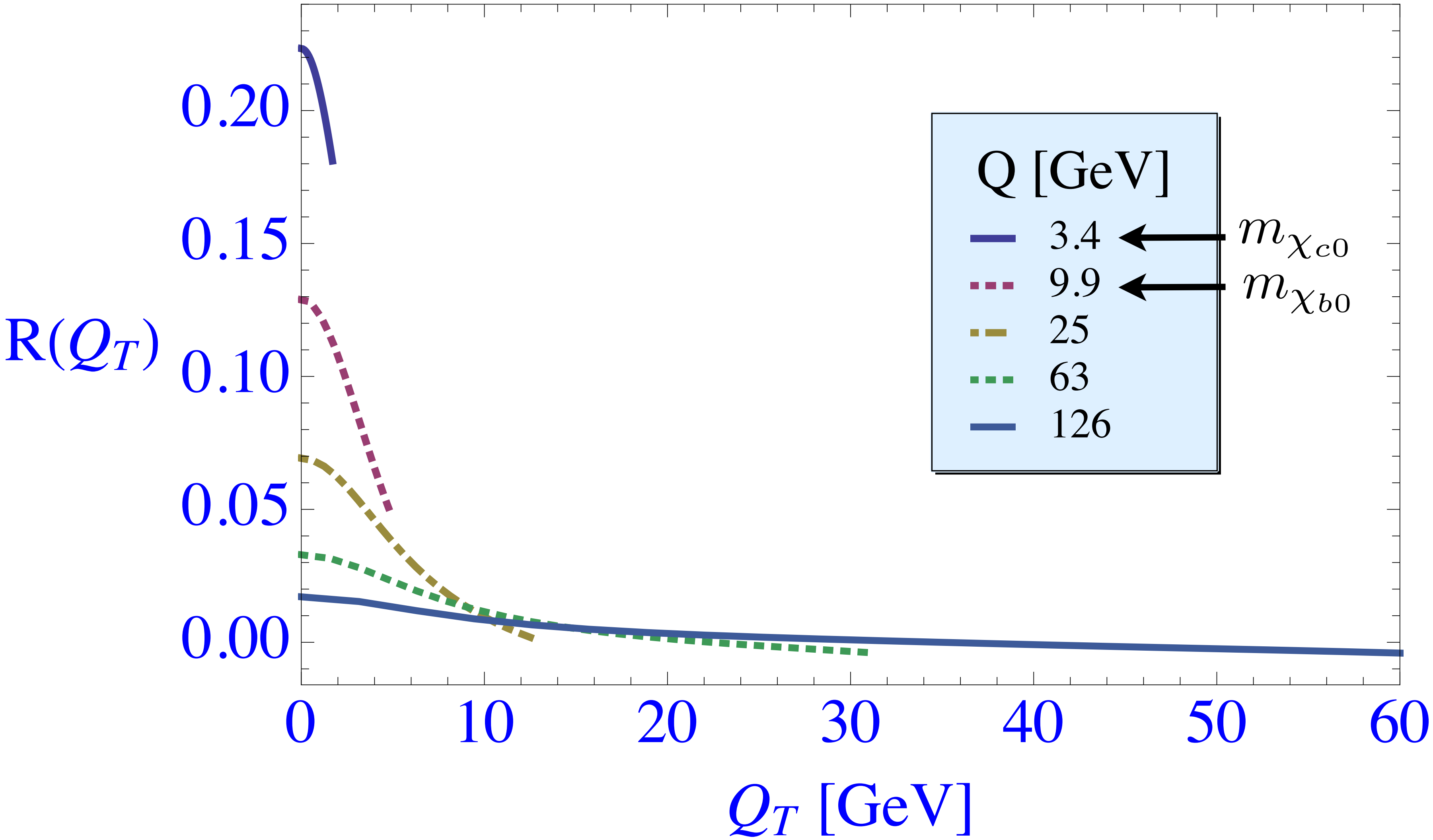
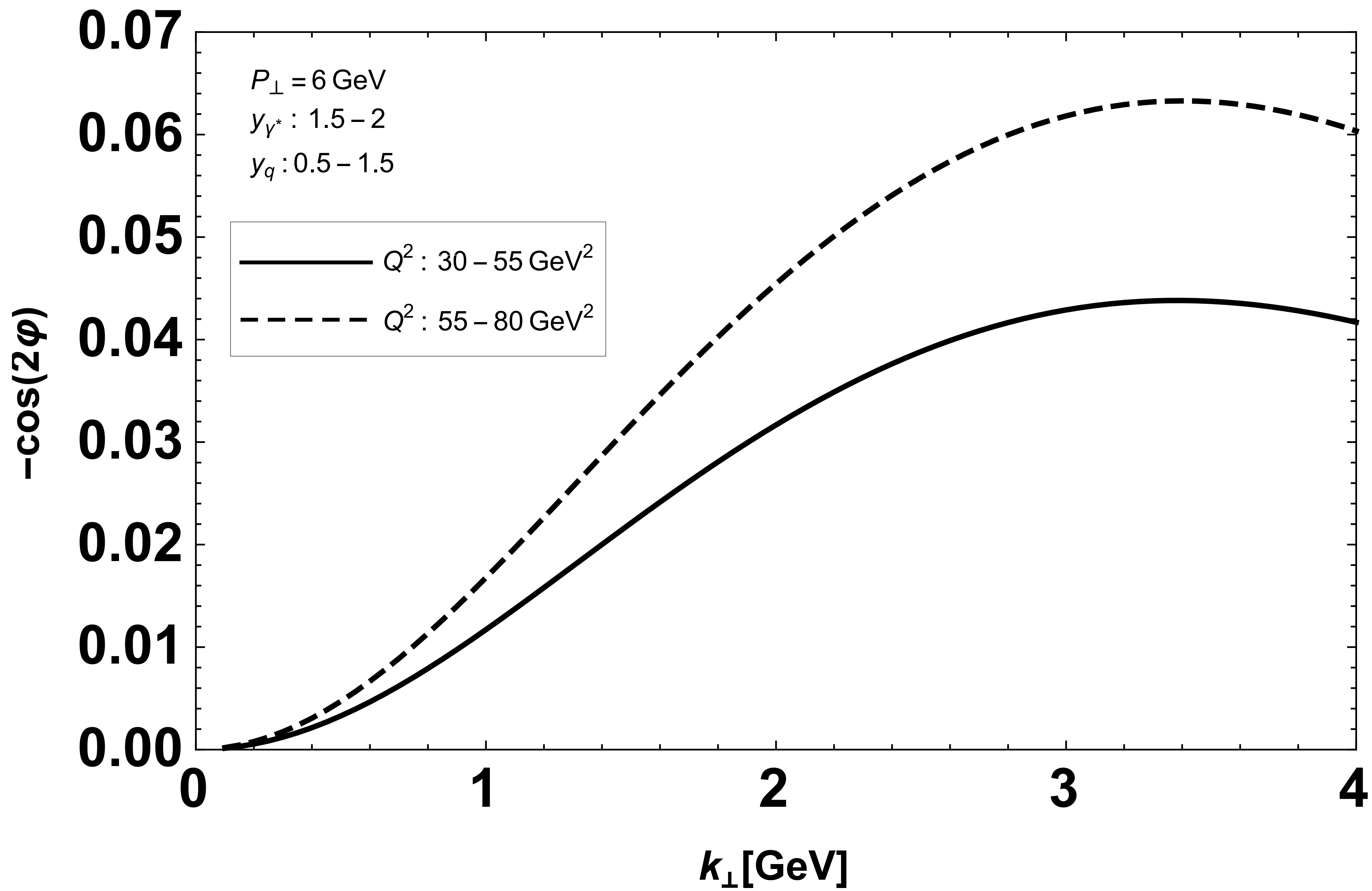
TMD evolution suppresses this ratio with increasing energy



D.B. & Den Dunnen 2014

Sudakov suppression of linear gluon polarization



Despite the maximal linear gluon polarization in pA→γ^* jet X at small x, there is Sudakov suppression of the cos(2φ) asymmetry: ~5% asymmetry at RHIC

D.B., Mulders, JianZhou, Ya-jin Zhou, 2017

Conclusions

“Must-do” experiments

Never done before yet:

- $p^{\uparrow}p$ or $p^{\uparrow}A \to \gamma^{*} X$ (quark Sivers, Fermilab E1039 experiment $\sqrt{s}$~15GeV in 2017)
- $p^{\uparrow}p$ or $p^{\uparrow}A \to \gamma\, \gamma\, X$ (sign of f-type (WW) gluon Sivers, relevant for EIC)
- $p^{\uparrow}p$ or $p^{\uparrow}A \to \gamma^{(*)}\, jet\, X$ (d-type (DP) gluon Sivers function & factorization test)
- $pA \to \gamma^{(*)}\, jet\, X$ (linear gluon polarization & Sudakov suppression test)
- $pp \to J/\Psi\, \gamma\, X$ (the unpolarized WW gluon TMD)

Improved precision needed:

- $p^{\uparrow}A \to h^{\pm} X$ (backward region, d-type (DP) gluon Sivers, spin-dependent odderon)
- $p^{\uparrow}p \to W^{\pm} X$ (sign change of quark Sivers)
- $p^{\uparrow}p \to jet\, jet\, X$ (1% level or better for color factor & factorization breaking test)

Processes have been considered before and most are part of RHIC Cold QCD plan, but several new scientific goals are added

Back-up slides

	Year	$\sqrt{s}$ (GeV)	Delivered Luminosity	Scientific Goals	Observable	Required Upgrade
Scheduled RHIC running	2017	$p^{\uparrow}p$ @ 510	400 pb^{-1} 12 weeks	Sensitive to Sivers effect non-universality through TMDs and Twist-3 $T_{q,F}(x,x)$ Sensitive to sea quark Sivers or ETQS function Evolution in TMD and Twist-3 formalism Transversity, Collins FF, linearly pol. Gluons, Gluon Sivers in Twist-3 First look at GPD Eg	A_N for γ, $W^{\pm}$, Z^0, DY $A_{UT}^{\sin(\phi_s-2\phi_h)}$ $A_{UT}^{\sin(\phi_s-\phi_h)}$ modulations of $h^{\pm}$ in jets, $A_{UT}^{\sin(\phi_s)}$ for jets A_{UT} for J/Ψ in UPC	A_N^{DY}: Postshower to FMS@STAR **None** **None**
	2023	$p^{\uparrow}p$ @ 200	300 pb^{-1} 8 weeks	subprocess driving the large A_N at high x_F and η evolution of ETQS fct. properties and nature of the diffractive exchange in p+p collisions.	A_N for charged hadrons and flavor enhanced jets A_N for γ A_N for diffractive events	Yes Forward instrum. **None** **None**
	2023	$p^{\uparrow}$Au @ 200	1.8 pb^{-1} 8 weeks	What is the nature of the initial state and hadronization in nuclear collisions Nuclear dependence of TMDs and nFF Clear signatures for Saturation	R_{pAu} direct photons and DY $A_{UT}^{\sin(\phi_s-\phi_h)}$ modulations of $h^{\pm}$ in jets, nuclear FF Dihadrons, γ-jet, h-jet, diffraction	R_{pAu}(DY):Yes Forward instrum. **None** Yes Forward instrum.
	2023	$p^{\uparrow}$Al @ 200	12.6 pb^{-1} 8 weeks	A-dependence of nPDF, A-dependence of TMDs and nFF A-dependence for Saturation	R_{pAl}: direct photons and DY $A_{UT}^{\sin(\phi_s-\phi_h)}$ modulations of $h^{\pm}$ in jets, nuclear FF Dihadrons, γ-jet, h-jet, diffraction	R_{pAl}(DY): Yes Forward instrum. **None** Yes Forward instrum.
Potential future running	202X	$p^{\uparrow}p$ @ 510	1.1 fb^{-1} 10 weeks	TMDs at low and high x quantitative comparisons of the validity and the limits of factorization and universality in lepton-proton and proton-proton collisions	A_{UT} for Collins observables, i.e. hadron in jet modulations at $\eta > 1$ and mid-rapidity observables as in 2017 run	Yes Forward instrum. **None**
	202X	$\vec{p}\vec{p}$ @ 510	1.1 fb^{-1} 10 weeks	$\Delta g(x)$ at small x	A_{LL} for jets, di-jets, h/γ-jets at $\eta > 1$	Yes Forward instrum.

Table 1-2: Summary of the Cold QCD physics program propsed in the years 2017 and 2023 and if an additional 500 GeV run would become possible.

Conclusions

- All TMDs are process dependent, with observable and testable effects

- At small x the unpolarized WW and DP gluon TMDs both matter and there are sufficient processes in ep and pp collisions to test the expectations

- Same applies to the linear polarization of gluons inside unpolarized hadrons:
  In pp collisions percent level effects, except in quarkonium production
  In ep collisions it could be much larger (10% or more) & its sign can be determined

- The CGC can be maximally polarized, although not all processes will be (fully) sensitive to it

- Two distinct gluon Sivers TMDs can be measured in $p^{\uparrow}p$ and $p^{\uparrow}A$ collisions (RHIC & AFTER@LHC), the WW-type allows for a sign-change test w.r.t. $ep^{\uparrow}$ (EIC)

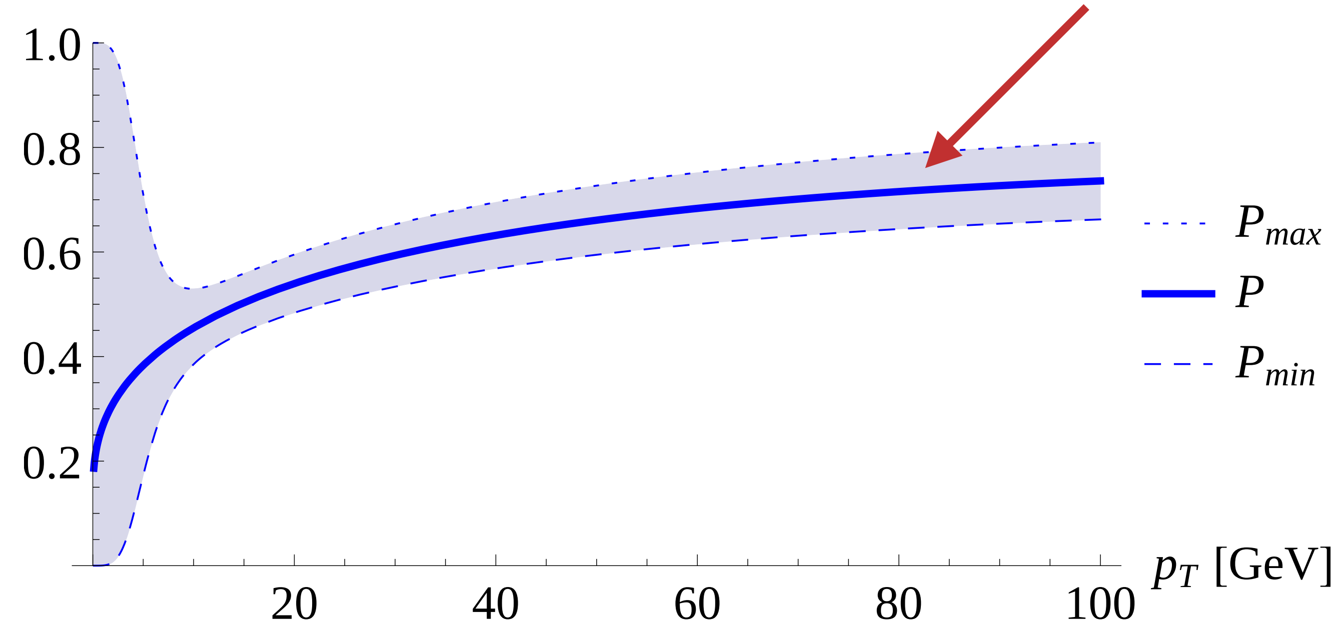
- As $x \to 0$ only the DP gluon Sivers TMD remains, which then corresponds to the spin-dependent odderon, a T-odd and C-odd single Wilson loop matrix element that determines A_N at negative x_F

Size of the effect

$$\frac{\alpha_s P' \otimes f_1}{\alpha_s P \otimes f_1}$$

Amount of linear gluon polarization:

D.B., Den Dunnen, Pisano, Schlegel '13



Ratio of large-k_T tails of $h_1^\perp$ and f_1 is large, does ***not*** mean large effects at large Q_T (observables involve ***integrals*** over all partonic k_T)

What matters is the small-b behavior of the Fourier transformed TMD:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

[Nadolsky, Balazs, Berger, C.-P. Yuan, 2007; Catani, Grazzini, 2010; P. Sun, B.-W. Xiao, F. Yuan, 2011]

The linear polarization starts at order α_s, leading to a **suppression** w.r.t. f_1

WW vs DP

At small x the unpolarized WW and DP gluon TMDs both matter and there are sufficient processes in ep and pp collisions to test the expectations

How different can the two unpolarized gluon distributions be?

The first transverse moment must coincide

$$\int d\boldsymbol{k}_T\, f_1^{g\,[+,+]}(x,\boldsymbol{k}_T^2) \;=\; \int d\boldsymbol{k}_T\, f_1^{g\,[+,-]}(x,\boldsymbol{k}_T^2)$$

Also the large k_T tail of the functions must coincide

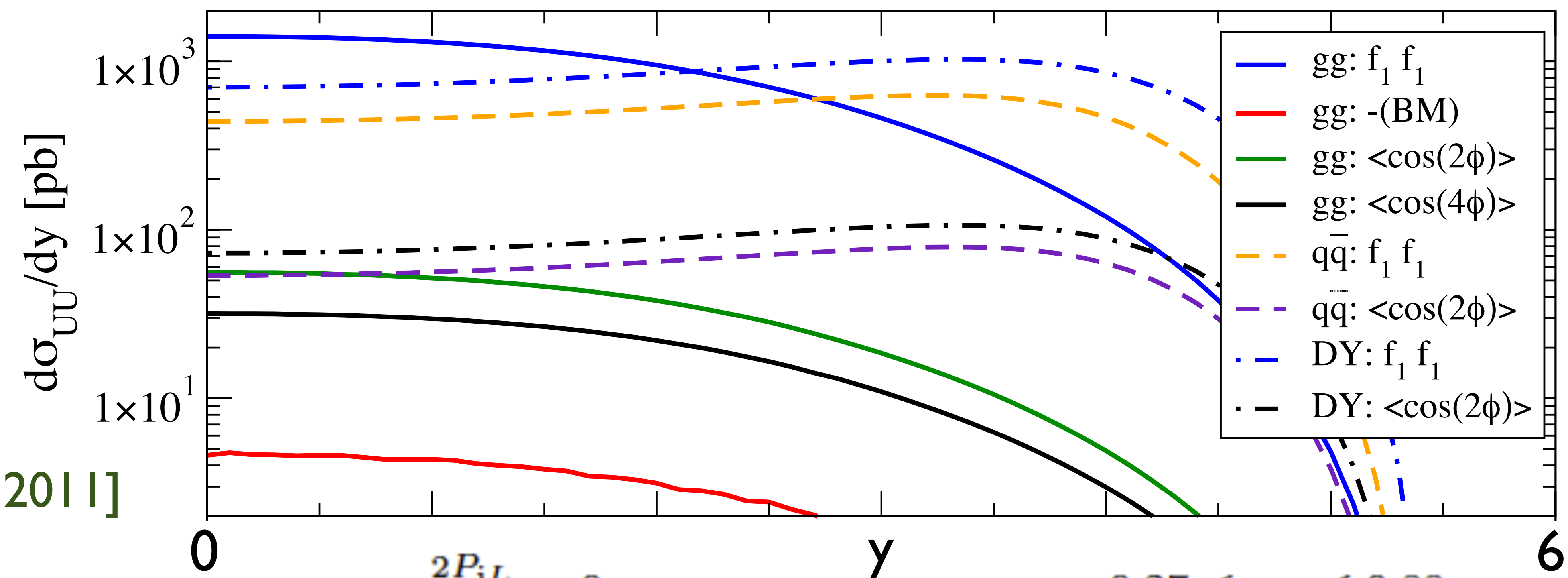
Therefore, the two functions can have rather different shapes and magnitudes

Angular distributions

Percent level effects
at RHIC energies in:

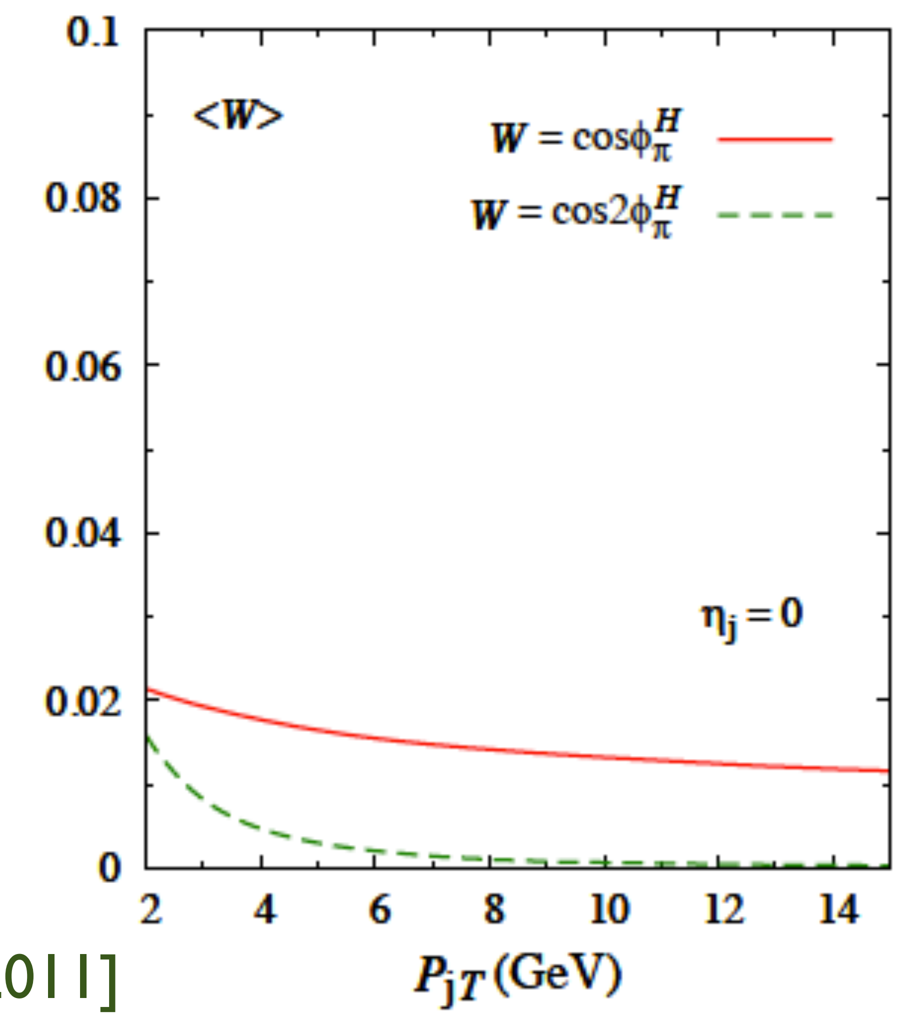
$pp \to \gamma\gamma X$

[Qiu, Schlegel, Vogelsang, 2011]

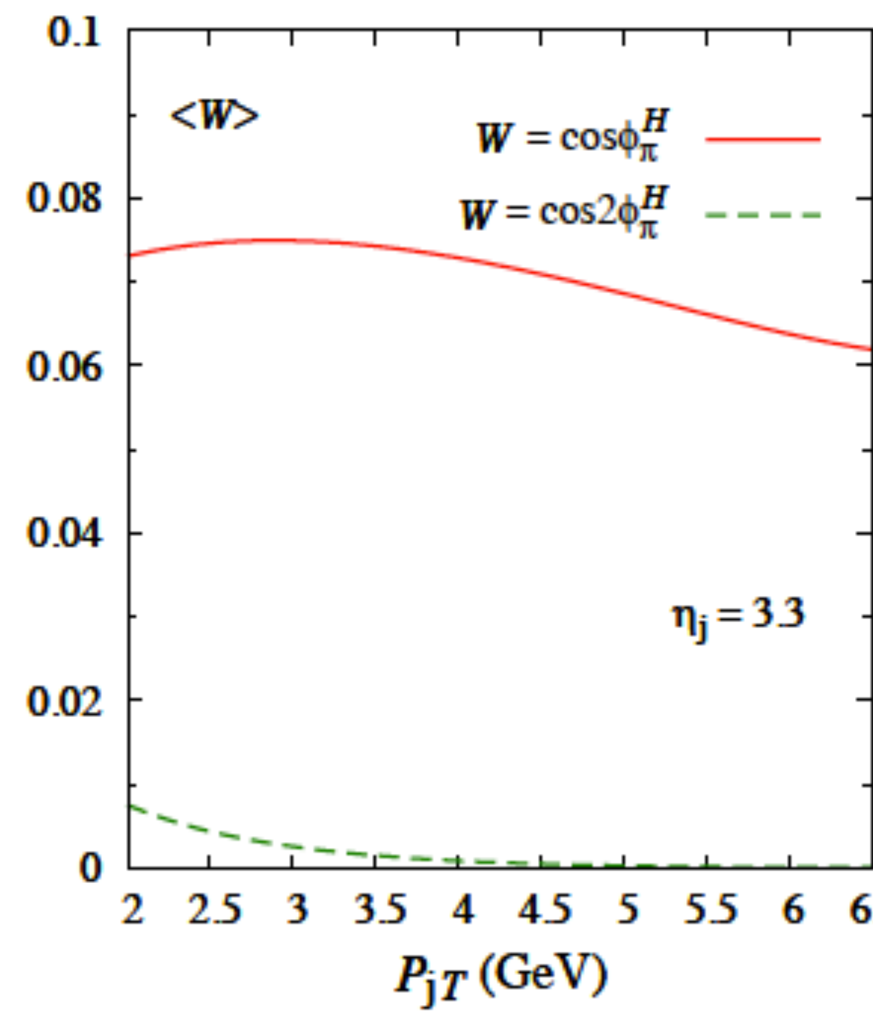


$pp \to$ (π *jet*) *X*

$x_F = \frac{2P_{jL}}{\sqrt{s}} = 0$



$0.27 \leq x_F \leq 0.88$



[D'Alesio, Murgia, Pisano, 2011]