Ivan Vitev

Initial-State in-Medium Bremsstrahlung Contribution to Soft and Hard Processes in Reactions with Nuclei

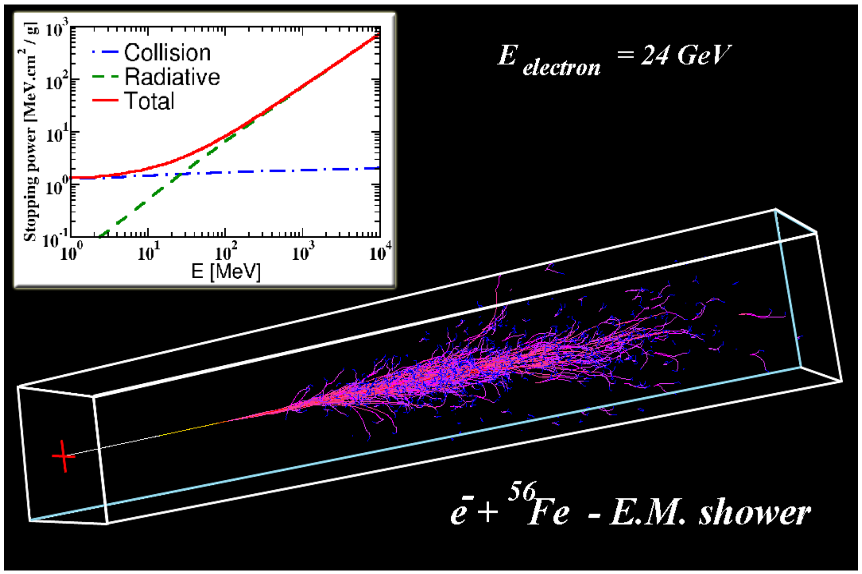
Emerging Spin and Transverse Momentum Effects in p+p and p+A Collisions. Brookhaven National Laboratory, February 8-10, 2016

Outline of the talk

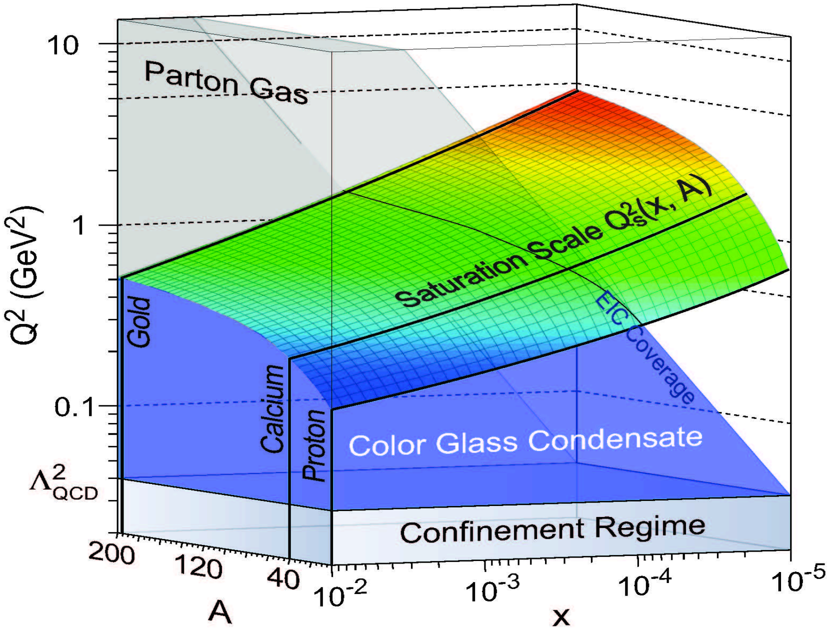
- Motivation to study p(d)+A at RHIC and LHC (and also at low fixed target energies)
- Global observables, soft particle multiplicities, and azimuthal asymmetries in p(d)+A reactions
- The effect of coherent medium induced bremsstrahlung on hadron and jet production
- Toward better understanding of medium-induced radiative corrections to hard processes
- Conclusions

Motivation – fundamental many-body QCD

- p(d)+A reactions provide a laboratory to study the perturbative QCD dynamics and the transport properties of cold nuclear matter – the shortest radiation length in nature, dynamical mass generation in strong magnetic fields, and a transverse saturation scale



NIST simulation (2007)



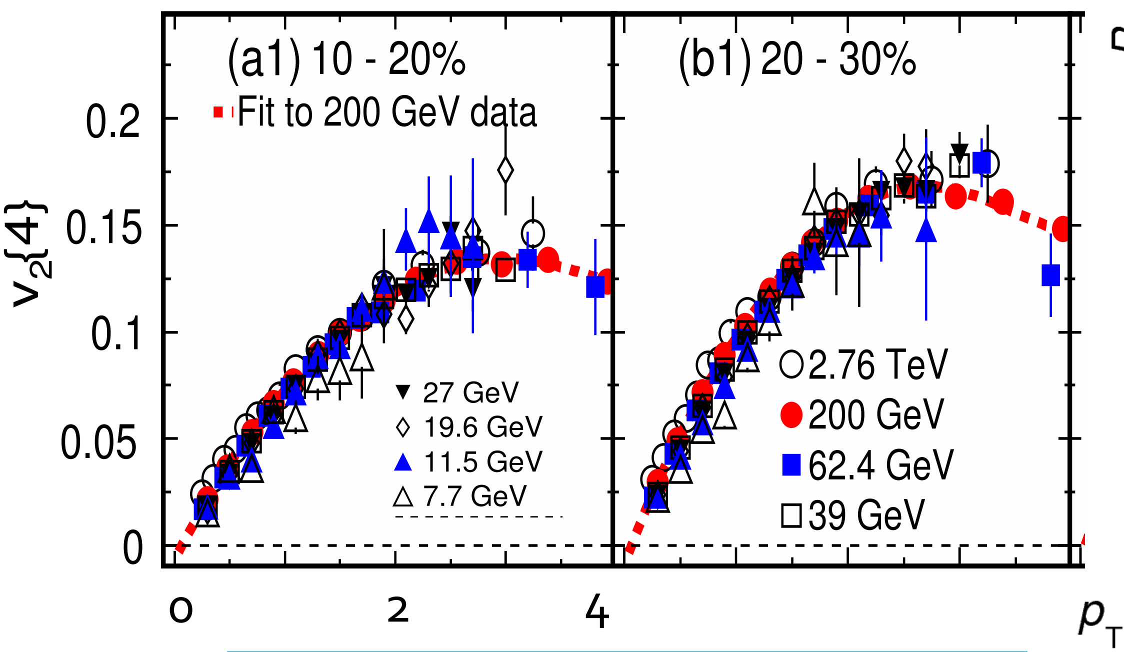
D. Boer et al. (2011)

Motivation – a new paradigm for heavy ion collisions (HIC)

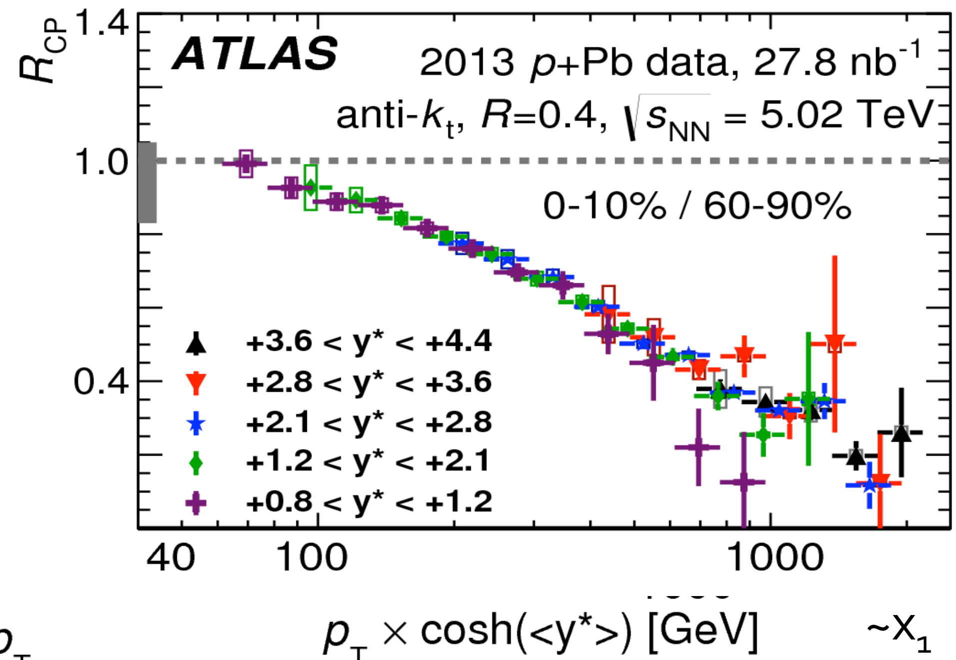
- A lot of exciting and surprising (certainly in magnitude) results have come form the p(d)+A programs at RHIC and LHC – asymmetries and suppression qualitatively similar to the ones observed in A+A

$$\frac{dN^h}{dyp_T dp_T d\phi} = \frac{1}{2\pi}\frac{dN^h}{dyp_T dp_T}\left(1+2\sum_n v_n(\cos n\phi)\right)$$

$$R_{AB}(p_T) = \frac{1}{\langle N_{bin}\rangle}\frac{d\sigma^{AB}}{dydp_T} / \frac{d\sigma^{pp}}{dydp_T}$$



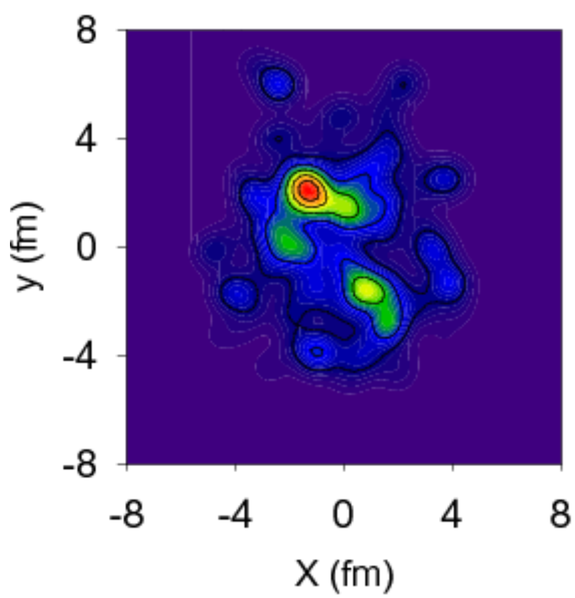
Y. Li [STAR collab.] NPA (2014)



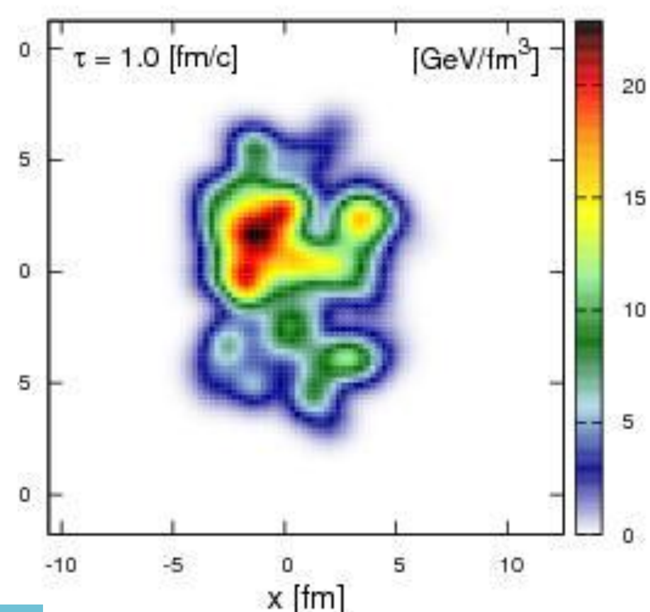
G. Aad et al. PLB (2015)

Fluctuating initial conditions and azimuthal asymmetries

- Important advance in understanding soft particle production was the inclusion of fluctuations of the initial nuclear geometry – hot spots, asymmetries (odd v_n coefficients)



A. Chaudhuri PRC (2013)



W. Quian et al. JPG (2014)

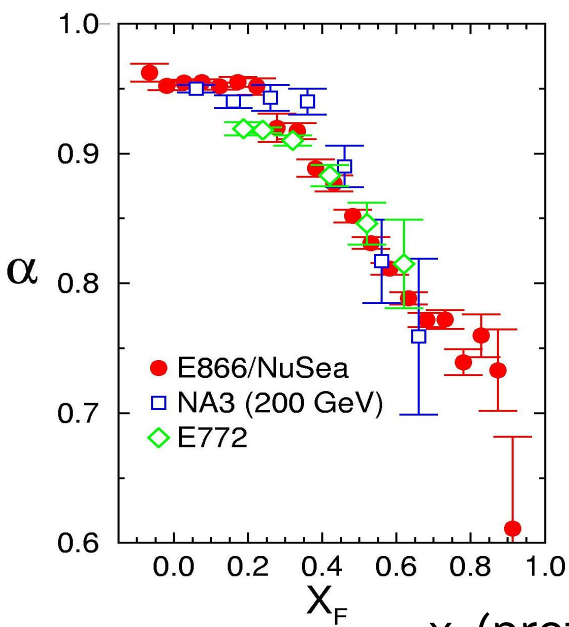
- First models to translate the asymmetries into final-state observables – hydrodynamic and CGC models.
- Correlations form 2 gluon production

Some indications about physics form lower energies

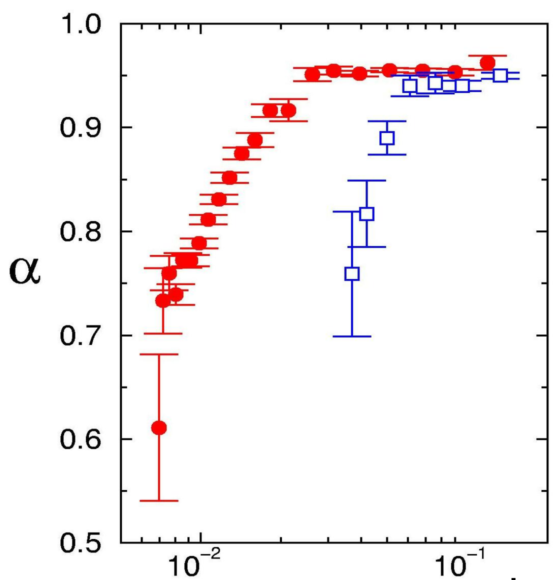
- Scaling with Feynman x [Tree level for inclusive processes is $x_F = x_1 - x_2 \sim x_1$] (not with x_2)

E772 data

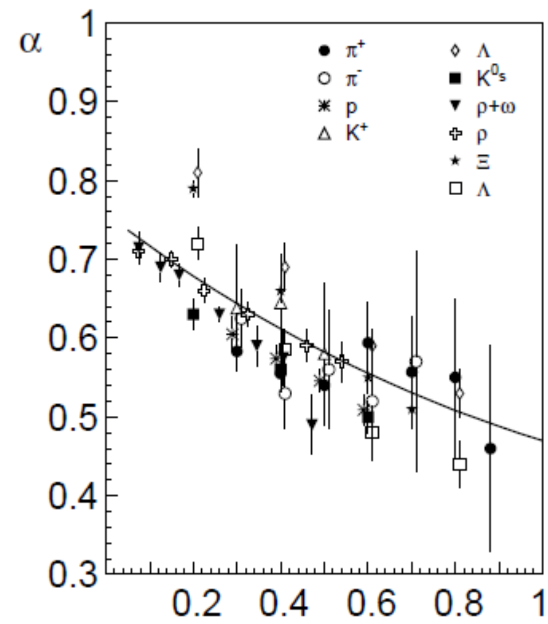
p+A at $s^{1/2}$ = 38.8 GeV



~x_1 (proton)



X_2 ~ (nucleus)



x_F ~x_1 (proton)

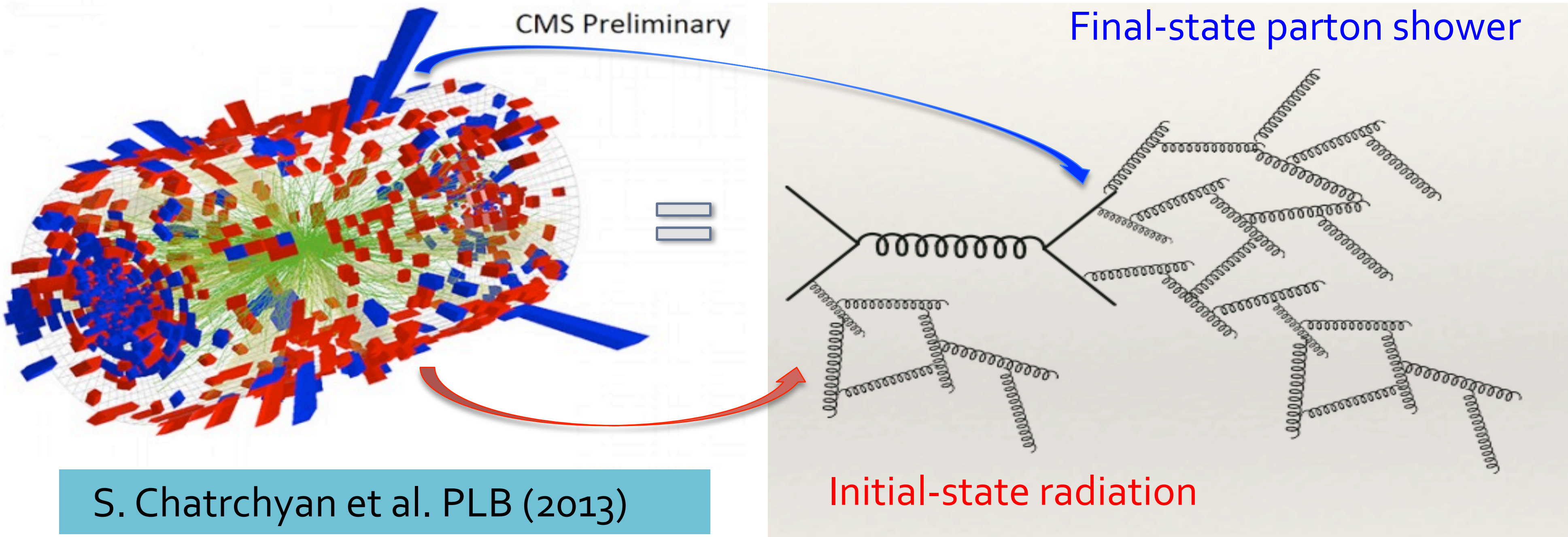
$$\sigma(pA) = \sigma^{pp} A^{\alpha}$$

$$R(pA) = A^{1-\alpha}$$

K. Eskola et al. (2009),
Many others

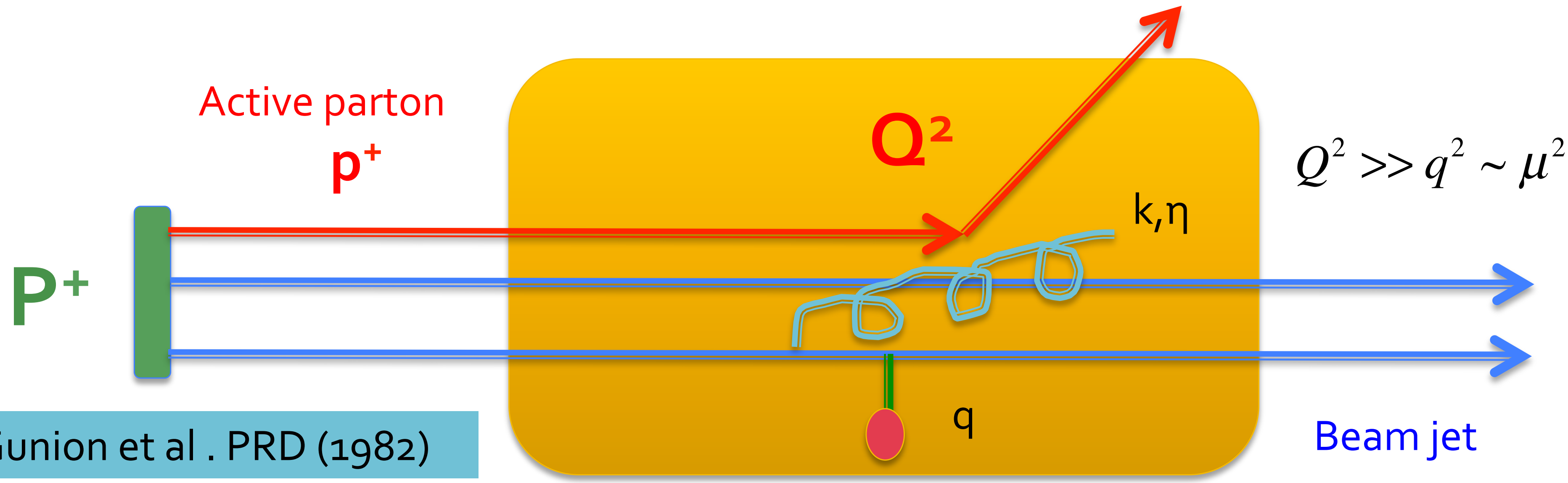
B. Kopeliovich et al. (2005)

Micro-droplets of QGP versus initial-state bremsstrahlung



- In the description of high energy processed significant effort has been devoted to understand the parton shower. We explore how this parton shower technology can be applied to p(d)+A
- In this case look at initial-state medium induced radiation

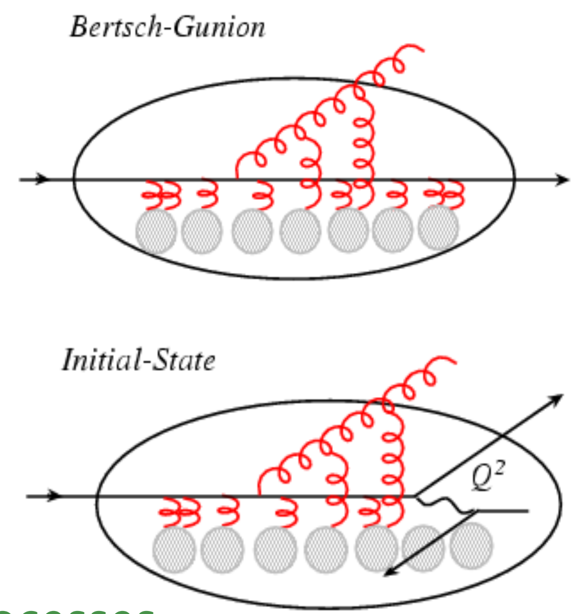
Initial-state bremsstrahlung in more detail



J. Gunion et al . PRD (1982)

$$\frac{dN_g^1}{d\eta d^2\mathbf{k} d^2\mathbf{q}} = \frac{C_R \alpha_s}{\pi^2} \frac{\mu^2}{\pi(q^2+\mu^2)^2} \frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

- Uniform in rapidity soft particle production
- Bremsstrahlung preferentially in the direction of the momentum transfer



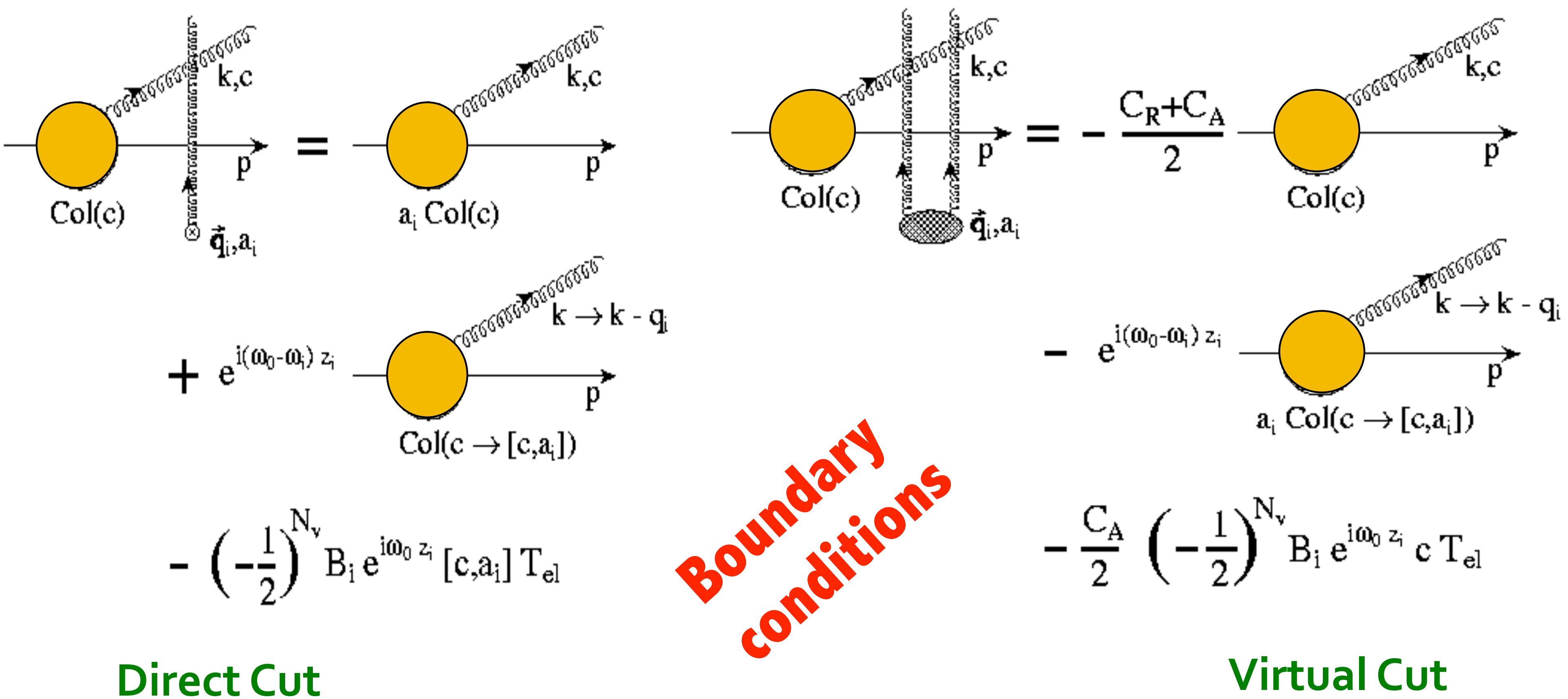
$P^+ \rightarrow P^+(1 - \varepsilon)$

Energy loss or backward rapidity shift for hard processes

A method to evaluate the soft bremsstrahlung spectrum

- An exact solution exists in the soft gluon emission limit only. Strong cancelation between direct and virtual cuts

M. Gyulassy et al . NPB (2001)



Bertsch-Gunion bremsstrahlung from correlated multiple scattering

- Bertsch-Gunion spectrum to any number of correlated multiple soft interactions in a QCD medium.

Integral along the path of propagation

Distribution of transverse Momentum transfers

$$k^+ \frac{dN^g(BG)}{dk^+ d^2\mathbf{k}} = \frac{C_R \alpha_s}{\pi^2} \left[\prod_{i=1}^{n} \int \frac{d\Delta z_i}{\lambda_g(z_i)} \right] \left[\prod_{j=1}^{n} \int d^2\mathbf{q}_j \left(\frac{1}{\sigma_{el}(z_j)} \frac{d\sigma_{el}(z_j)}{d^2\mathbf{q}_j} - \delta^2(\mathbf{q}_j) \right) \right]$$

Color current propagators

$$\times \mathbf{B}_{(2\cdots n)(1\cdots n)} \cdot \left[\mathbf{B}_{(2\cdots n)(1\cdots n)} + 2 \sum_{i=2}^{n} \mathbf{B}_{(i+1\cdots n)(i\cdots n)} \cos\left(\sum_{j=2}^{i} \omega_{(j\cdots n)} \Delta z_j \right) \right]$$

$$\mathbf{C}_{(i_1\cdots i_m)} = \frac{\mathbf{k} - \mathbf{q}_{i_1} - \cdots - \mathbf{q}_{i_m}}{(\mathbf{k} - \mathbf{q}_{i_1} - \cdots - \mathbf{q}_{i_m})^2} ,$$

$$\mathbf{B}_{(i_1\cdots i_m)(j_1 j_2\cdots i_n)} = \mathbf{C}_{(i_1\cdots j_m)} - \mathbf{C}_{(j_1 j_2 \cdots j_n)}$$

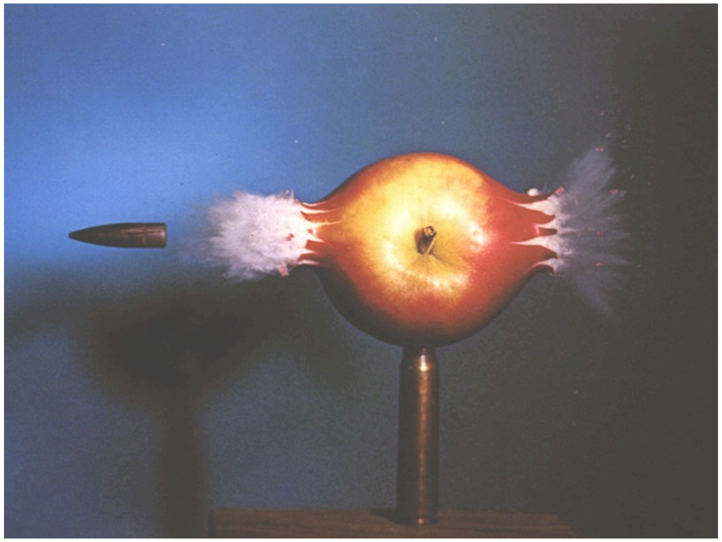
Interference phases

$$\tau^{-1}_{(i_1\cdots i_m)} = \omega_{(i_1\cdots i_m)} = \frac{(\mathbf{k} - \mathbf{q}_{i_1} - \cdots - \mathbf{q}_{i_m})^2}{k^+}$$

- In order to obtain solutions like that there are essential approximations. a) The jet does not deflect from the original direction. b) only possible in the soft gluon emission limit

Very high energy fully coherent limit

- Asymptotic no-hard-scattering initial conditions correspond to the BG problem



Beam jets, very high energy in the rest frame of the nucleus

$$\cos\left(\frac{(\mathbf{k}-\sum_i \mathbf{q_i})}{2\omega}\Delta z\right) \sim 1$$

- Bremsstrahlung is in the direction of the cumulative momentum transfer

M. Gyulassy et al . PRD (2014)

$$dN^{VGB}_{coh}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d^2\mathbf{Q}\; P^{el}_n(\mathbf{Q})\; dN^{GB}(\mathbf{k},\mathbf{Q})$$

Opacity $\chi = \dfrac{L}{\lambda}$

$$P^{el}_n(\mathbf{Q}) = \exp[-\chi]\frac{\chi^n}{n!}\int \left\{\prod_{j=1}^{n} \frac{d^2\mathbf{q}_j}{\sigma_{el}}\frac{d\sigma_{el}}{d^2\mathbf{q}_j}\right\}\delta^2(\mathbf{Q}-(\mathbf{q}_1+\cdots+\mathbf{q}_n))$$

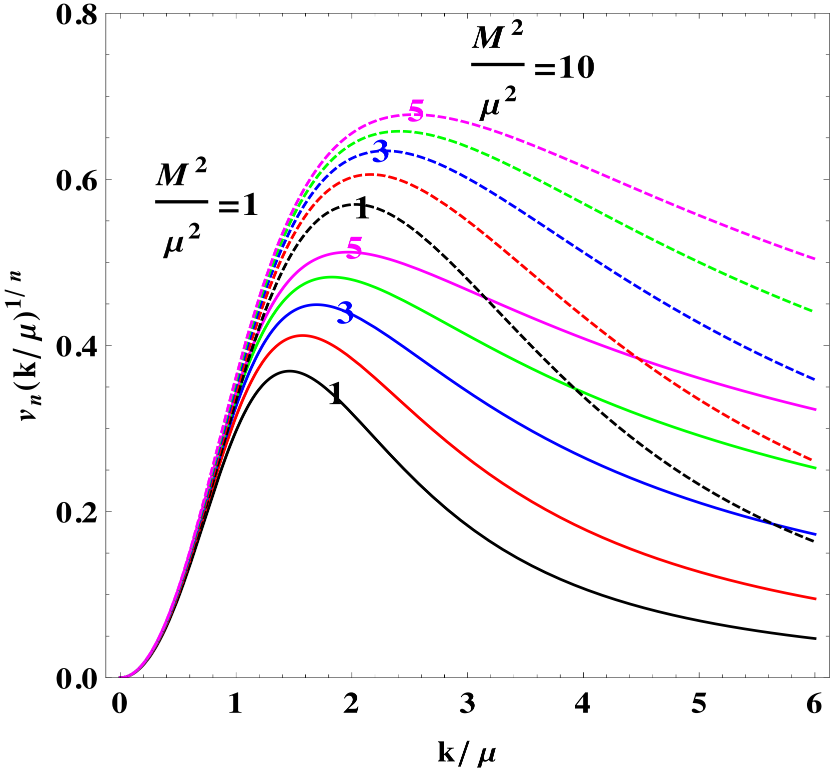
Azimuthal moments form non-Abelian bremsstrahlung

- Given its directional nature, the non-abelian bremsstrahlung and related soft particle multiplicities exhibit non-vanishing Fourier harmonics

$$v_1^{GB}(k,q,\psi) = \cos[\psi](A_{kq} - \sqrt{A_{kq}^2 - 1}) \quad ,$$
$$\lim_{\mu\to 0} v_1^{GB}(k,q,0) = (k/q)\ \theta(q-k) \quad ,$$
$$v_n^{GB}(k,q,\psi) = \cos[n\psi]\ (v_1^{GB}(k,q,0))^n \quad ,$$
$$\lim_{\mu\to 0} v_n^{GB}(k,q,0) = (k/q)^n\ \theta(q-k) \quad .$$

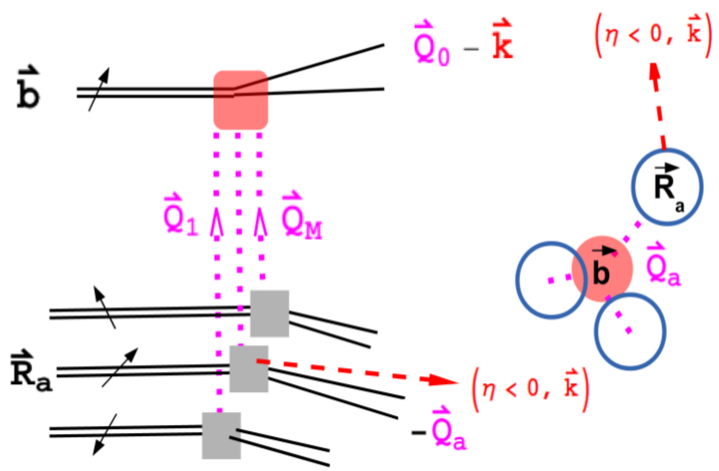
- Scaling relation for the azimuthal harmonics

$$[v_n^{GB}(k,q,0)]^{1/n} = [v_m^{GB}(k,q,0)]^{1/m}$$



M. Gyulassy et al . PRD (2014)

Target and beam contributions



- Including both target and beam jets (N target dipoles in M clusters)

$$dN^{M,N} = dN_P^N(\eta, \mathbf{k}_1; \mathbf{Q}_P) + dN_T^{M,N}(\eta, \mathbf{k}_1; \{\mathbf{Q}_a\})$$

$$= \sum_{a=0}^{M} \frac{B_{1a}}{(\mathbf{k}_1 + \mathbf{Q}_a)^2 + \mu_a^2}$$

- The generalization to multiple gluon emission (which accounts for the soft particle multiplicity) proceeds as follows

$$dN_{2\ell}^{M}(\eta_1, \mathbf{k}_1, \cdots, \eta_{2\ell}\mathbf{k}_{2\ell}) = \prod_{i=1}^{2\ell} \left(\sum_{a_i=0}^{M} \frac{B_{k_i a_i}}{A_{k_i a_i} - \cos(\phi_i + \psi_{a_i})} \right)$$

N target dipoles in M clusters
+ the beam color dipole

M. Gyulassy et al . PRD (2014)

Azimuthal asymmetries through multi-particle cumiulants

- Particle cumulants – proposed to help remove ``non-flow" sources of correlations such as momentum conservation, back to back dijet, and Bose statistics effects

$$(v_n\{2\})^2 \equiv \left\langle e^{in(\phi_1-\phi_2)} \right\rangle \equiv \langle |v_2|^2 \rangle$$

A. Bazdak et al . NPA (2015)

$$(v_n\{4\})^4 \equiv \left\langle -e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle + 2\left\langle e^{in(\phi_1-\phi_3)} \right\rangle \left\langle e^{in(\phi_2-\phi_4)} \right\rangle = 2\langle |v_2|^2 \rangle^2 - \langle |v_n|^4 \rangle$$

$$(v_n\{6\})^6 \equiv (\left\langle e^{in(\phi_1+\phi_2+\phi_3-\phi_4-\phi_5-\phi_6)} \right\rangle - 9\left\langle |v_2|^2 \right\rangle \left\langle |v_n|^4 \right\rangle + 12\left\langle |v_2|^2 \right\rangle^3)/4$$

- The scaling relation for v_n using 2ℓ multi-particle cumulants from color scintillation antenas (CSA)

$$\bar{v}_n\{2\ell\} \equiv (v_m^{M=1}\{2\ell\}(k,\cdots,k;\bar{Q}))^{n/m} \Rightarrow$$

$$\langle |v_n|^4 \rangle = \left\langle |v_2|^2 \right\rangle^2$$

$$\langle |v_6|^6 \rangle = \langle |v_2|^2 \rangle \langle |v_n|^4 \rangle = \left\langle |v_2|^2 \right\rangle^3$$

The cumulant relations naturally arise form coherent bremsstrahlung

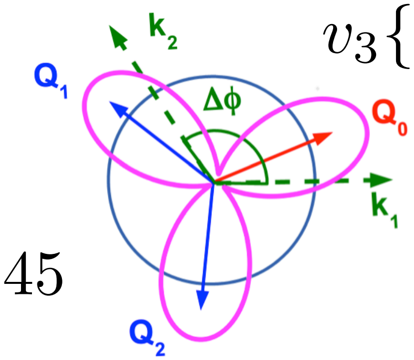
Examples of non-Abelian bremsstrahlung event asymmetries

- Equally distributed in azimuth momentum transfers

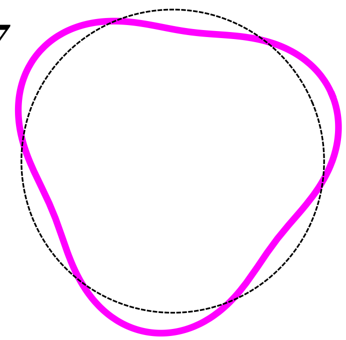
Momentum transfers $Q_a^2 = N/M\mu^2$ **Angles** $\{\psi_a\} = 2\pi a/n$

$$v_n\{2\}(k_1, k_2) = \delta_{n,3}$$
$$v_3^{GB}(k_1, Q_0) v_3^{GB}(k_2, Q_0)$$

$$v_3\{2\} = 0.45$$

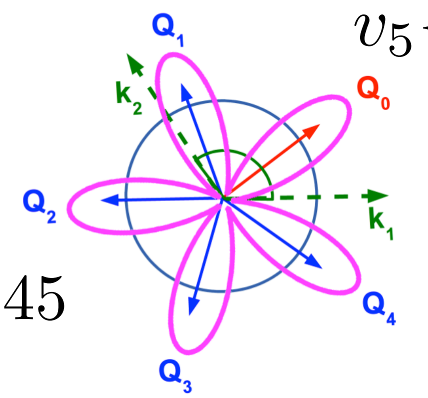


$$v_3\{2\} = 0.07$$

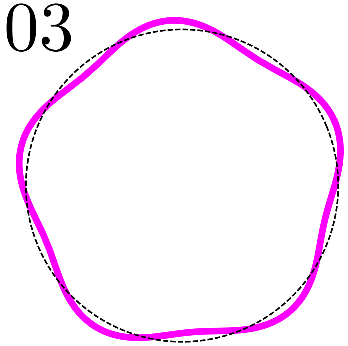


$$v_n\{2\}(k_1, k_2) = \delta_{n,5}$$
$$v_5^{GB}(k_1, Q_0) v_5^{GB}(k_2, Q_0)$$

$$v_5\{2\} = 0.45$$



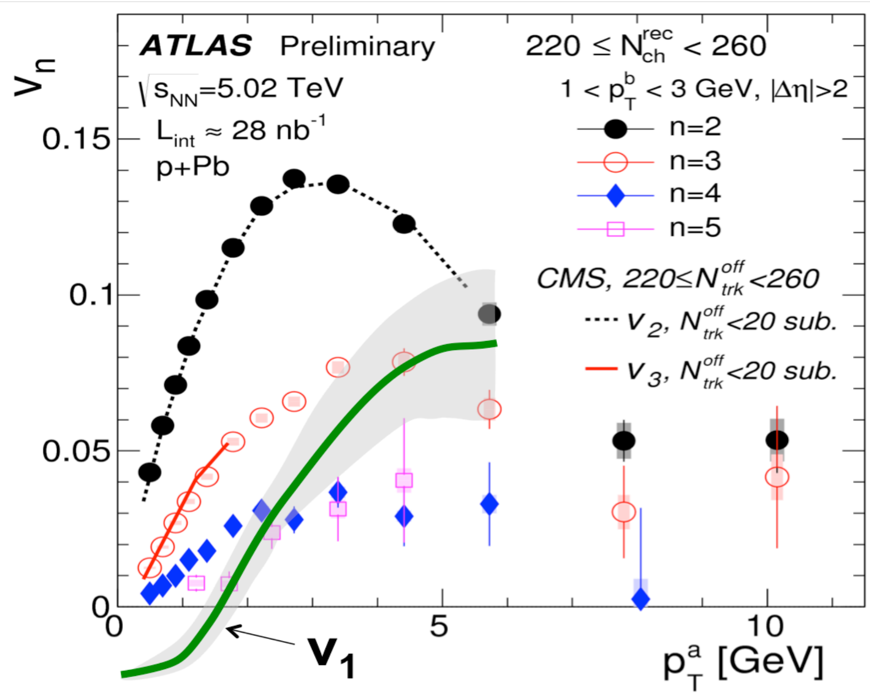
$$v_5\{2\} = 0.03$$



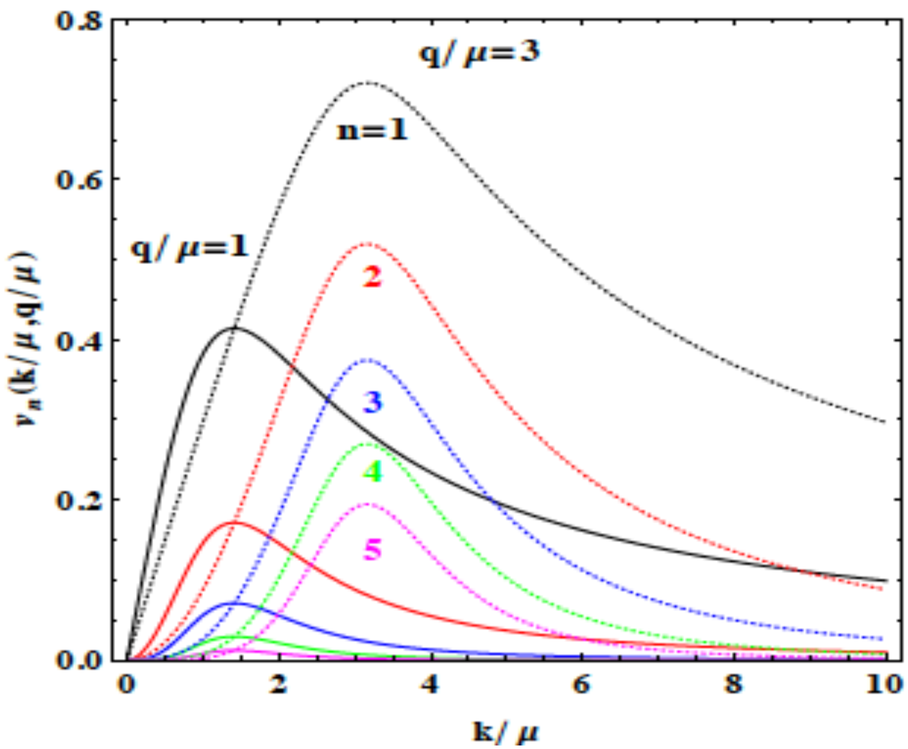
- My co-authors intended to incorporate this physics in HIJING

Qualitative comparison

- Definitely gets the ordering of "flow coefficient" starting with n=2 (n=1 appears to have a different behavior)



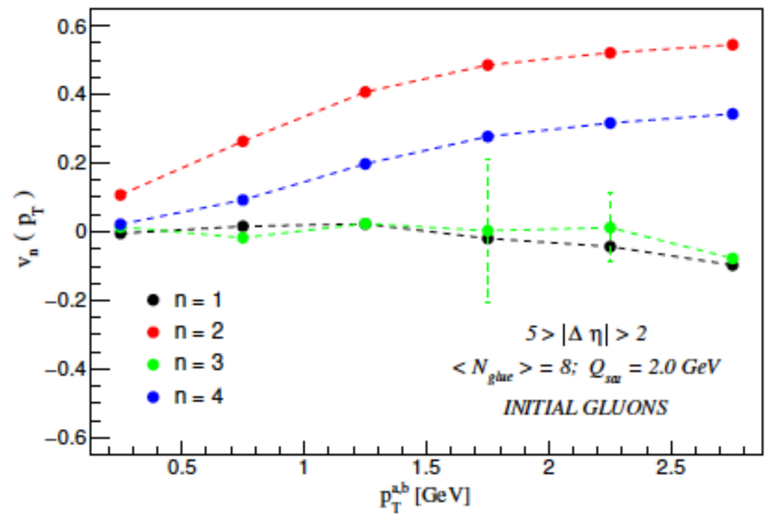
ATLAS collab. NPA (2014)

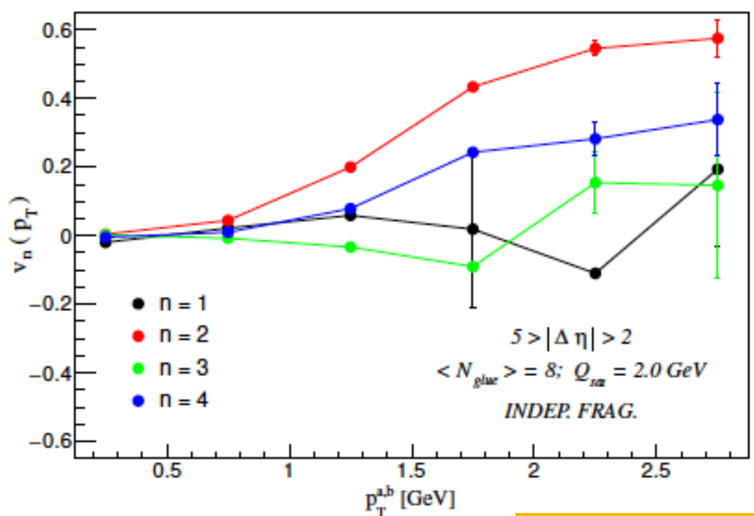


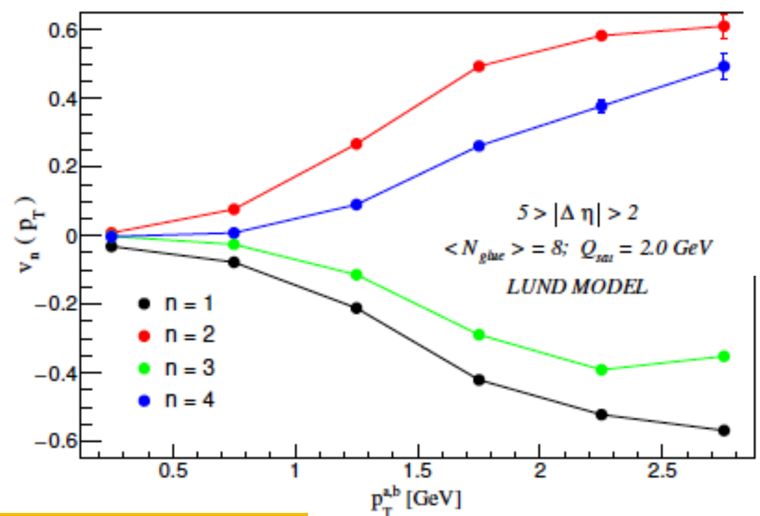
A. Espozito et al. PLB (2015)

Hadronization corrections

- Hadronization corrections are important. Below 2 GeV v_n is always changed. Above 2 GeV the hadronization scheme matters







A. Espozito et al. PLB (2015)

- Can't ignore

Initial-state energy loss phenomenology

- Implications for hard processes

$$\Delta E^{\mathrm{rad.}}_{\mathrm{initial-state}} \sim \kappa_{LPM}\, C_R \alpha_s E \frac{L}{\lambda_g}$$

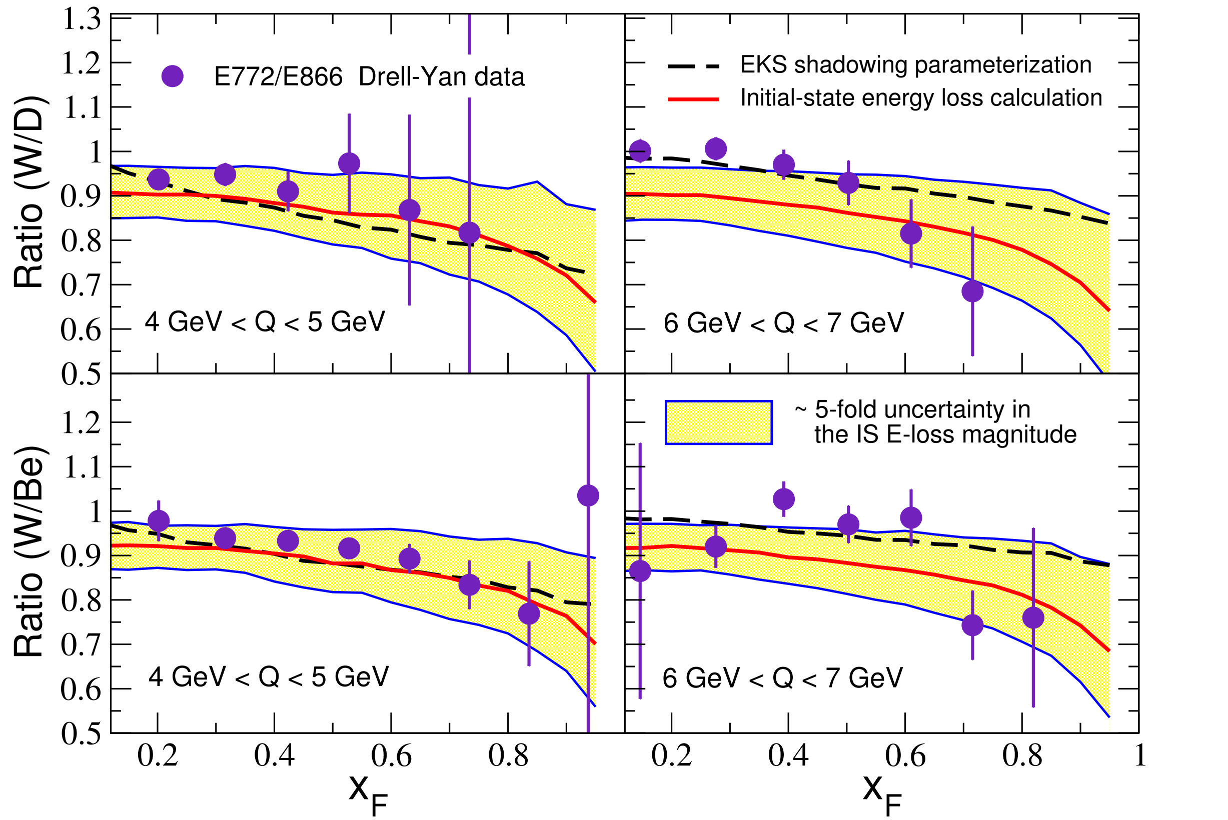
B. Neufeld et al . PLB (2011)

- The functional form allows to define radiation length

$$-dE/dx = E/X_0$$

Shortest radiation length in nature Xo ~ 100 fm (few %)

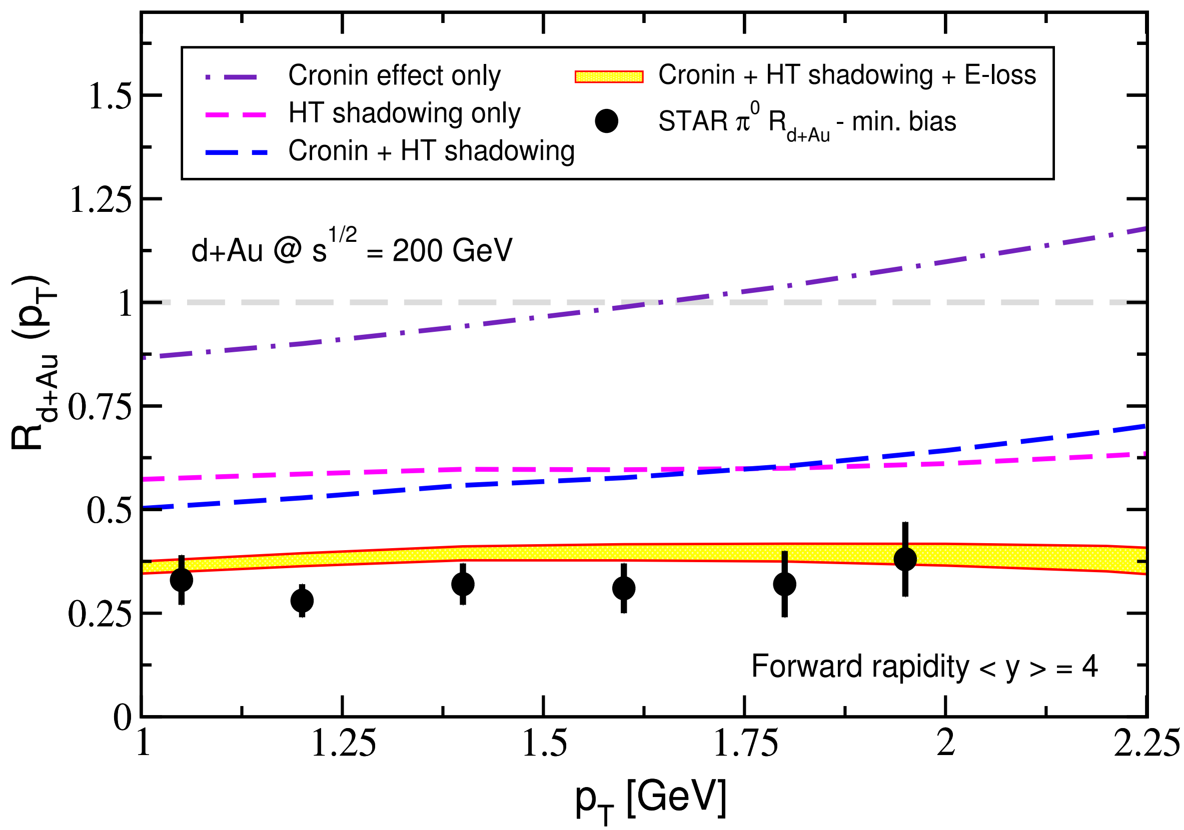
- Suppression scaling with x_F



Hadron production at forward rapidities

- Multiple nuclear effects play a role in the modification of the transverse momentum distributions

B. Neufeld et al . PLB (2010)



- CNM energy loss

$$x_1 \to x_1/(1-\epsilon_{\rm eff})$$

- Coherent power corrections

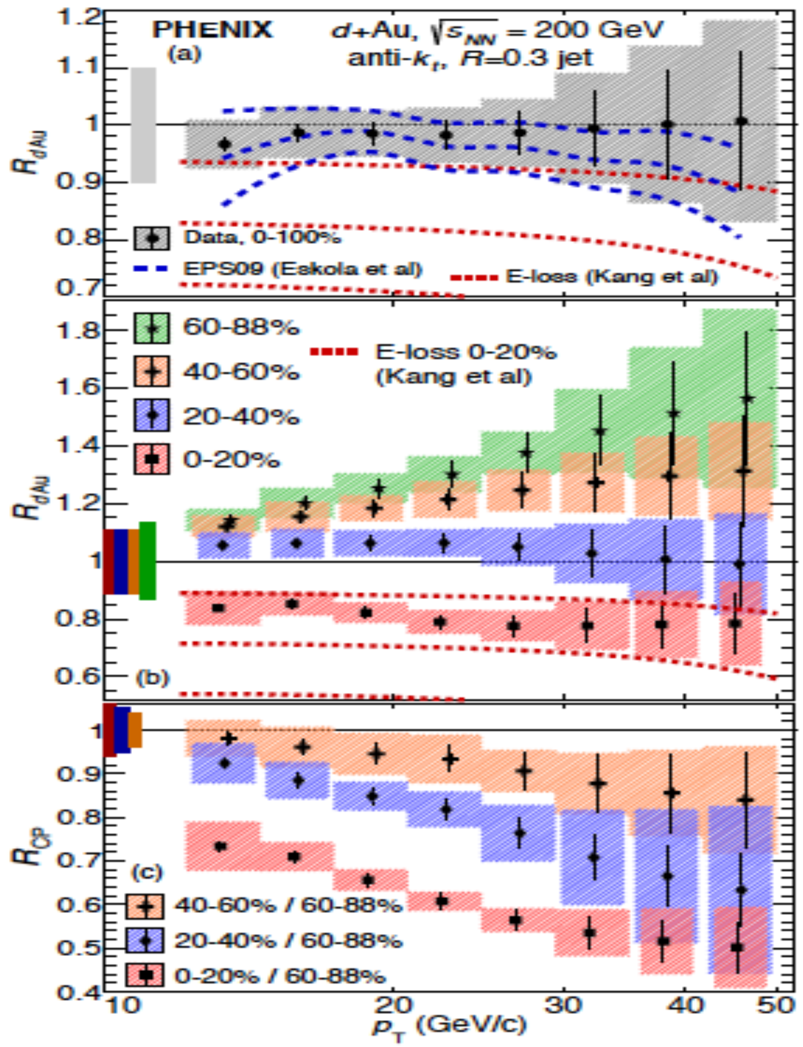
$$m_{dyn}^2 = \mu^2 A^{1/3}$$

- Cronin effect

$$\langle k_T^2\rangle_{pA} = \langle k_T^2\rangle_{pp} + 2\frac{\mu^2 L}{\lambda}\xi$$

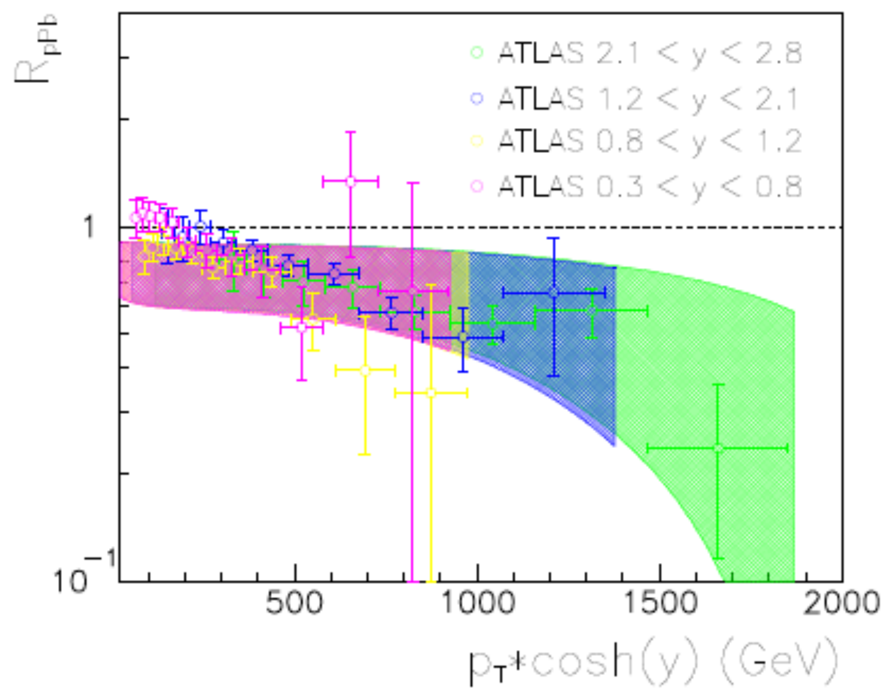
Jet production in p+A

$$R_{AA}(p_T) = \frac{1}{\langle N_{bin} \rangle} \frac{d\sigma^{pA}}{dydp_T} / \frac{d\sigma^{pp}}{dydp_T}$$



- CNM energy loss

$$x_1 \rightarrow x_1/(1-\epsilon_{\rm eff})$$

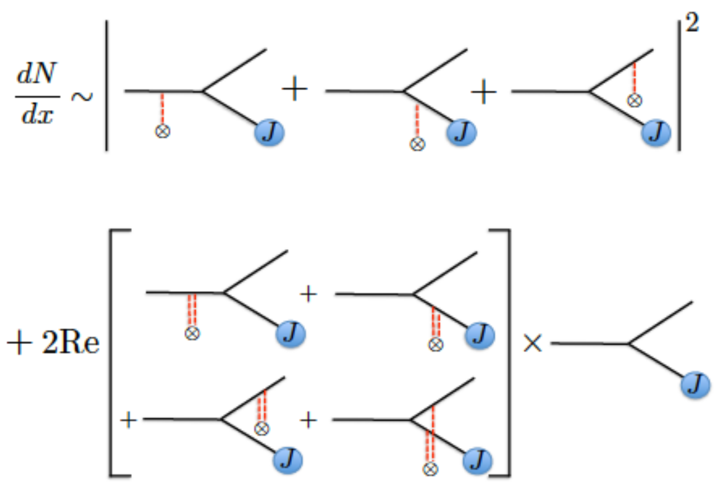


$$p_T \times \cosh(y) \propto x_a \sqrt{s}$$

Z. Kang et al . PRC (2014)

Toward a better understanding of initial-state active parton

- Attempt to go beyond the energy loss approximation. Better separation between the active parton (large Q^2) and beam jets



G. Ovanesyan et al. (2015)

For experts: x <-> 1-x
+ functions, A δ(x), coefficients can be extracted form flavor and momentum sum rules

- Checked that reproduce the DGLAP splitting kernels in the absence of medium

$$\left(\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp}\right)_{q\to qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1+(1-x)^2}{x} \frac{1}{\boldsymbol{k}_\perp^2},$$

$$\left(\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp}\right)_{g\to gg} = \frac{\alpha_s}{2\pi^2} 2C_A \Big(\frac{1-x}{x} + \frac{x}{1-x} + x(1-x)\Big) \frac{1}{\boldsymbol{k}_\perp^2},$$

$$\left(\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp}\right)_{g\to q\bar{q}} = \frac{\alpha_s}{2\pi^2} T_R \ \left(x^2+(1-x)^2\right) \frac{1}{\boldsymbol{k}_\perp^2},$$

$$\left(\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp}\right)_{q\to gq} = \left(\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp}\right)_{q\to qg} (x\to 1-x).$$

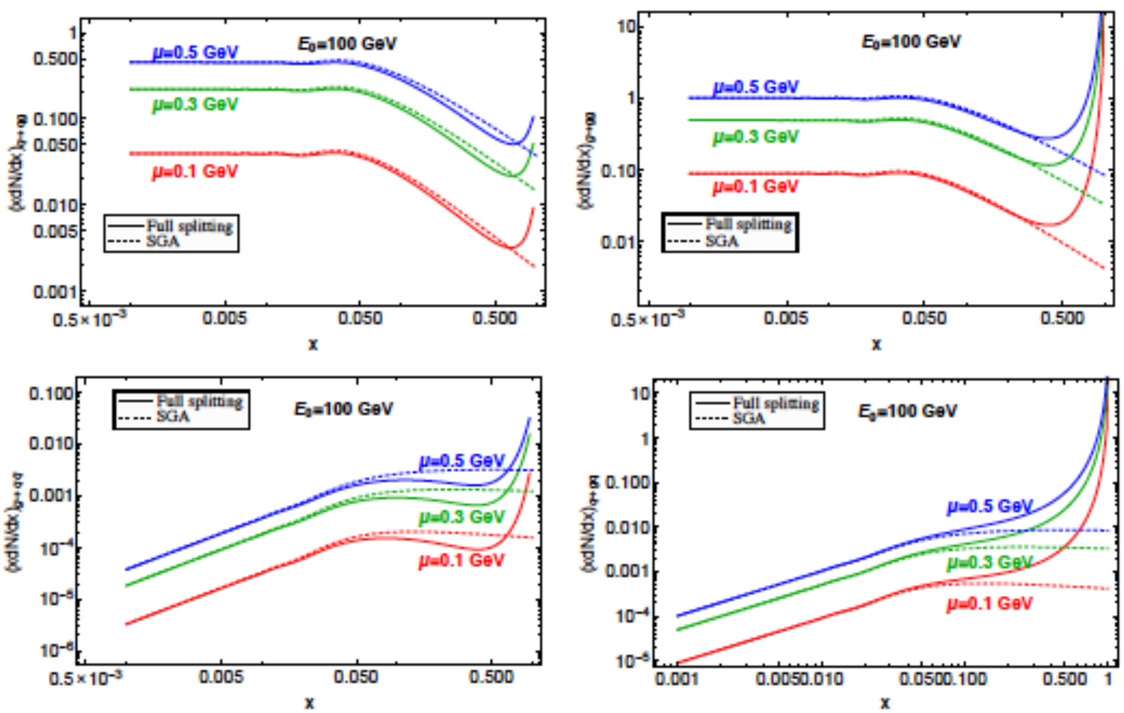
Initial-state in-medium radiative corrections of active partons

- Results (example)

$$\left(\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp}\right)_{q\to qg}$$

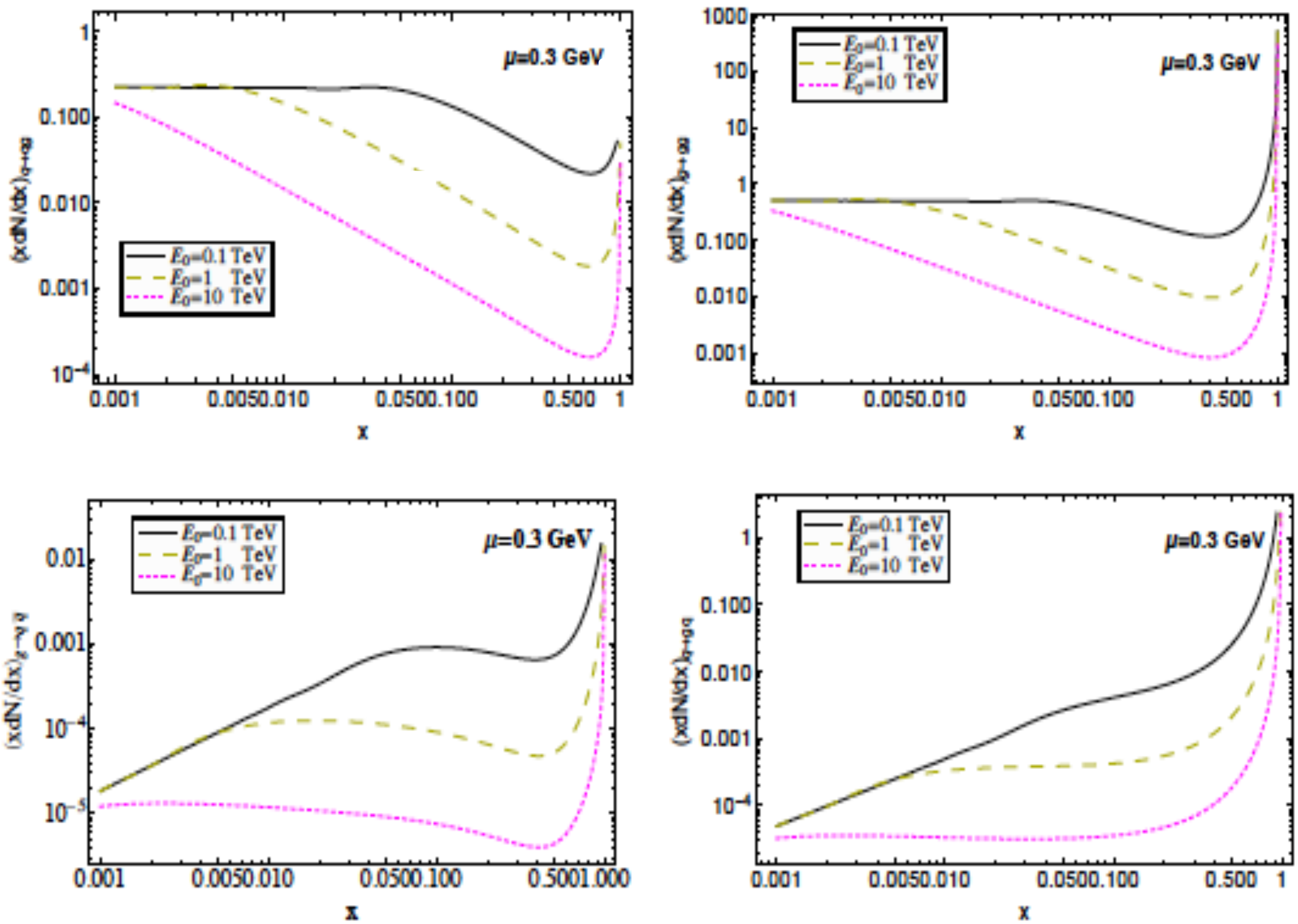
$$= C_F \frac{\alpha_s}{2\pi^2}\frac{1+(1-x)^2}{x}\int\frac{\mathrm{d}z}{\lambda_g(z)}\mathrm{d}^2\boldsymbol{q}_\perp\frac{1}{\sigma_{\mathrm{el}}}\frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}^2\boldsymbol{q}_\perp}\left[\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)(1-\cos(\Omega_1-\Omega_2)\Delta z)\right.$$

$$\left.+\frac{1}{\boldsymbol{C}_\perp^2}-\frac{1}{\boldsymbol{A}_\perp^2}+\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)(1-\cos\Omega_3\Delta z)-\frac{1}{N_c^2}\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\right)(1-\cos(\Omega_1-\Omega_2)\Delta z)\right],$$



- Large x behavior arises form helicity flips
- Large x divergent behavior does not come into play

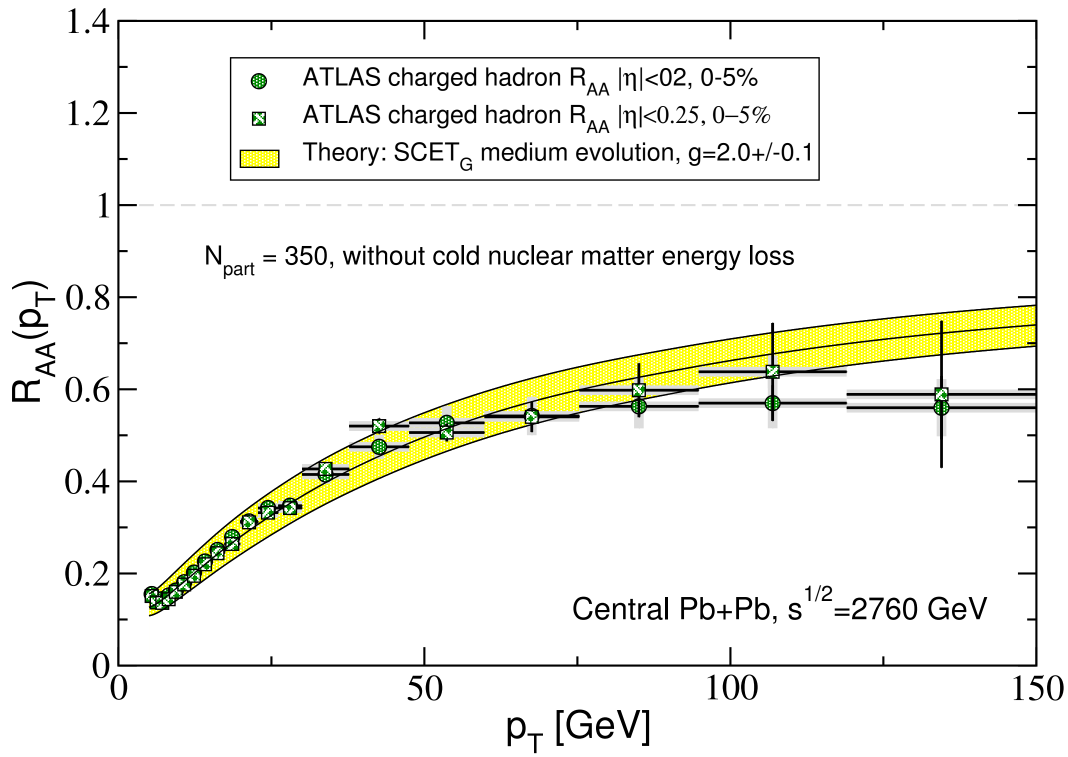
Energy dependence of initial-state medium-induced splittings



- Variation in energy does not eliminate the significance of the x> 0.5 region
- We can see at high energies suppression with ~ 1/E.

G. Ovanesyan et al. (2015)

Potential applications



$$P_i^{\rm full}(x, \boldsymbol{k}_\perp; \beta) = P_i^{\rm vac}(x) + P_i^{\rm med}(x, \boldsymbol{k}_\perp; \beta)$$

- Beam functions and modification to the parton distribution functions from corrections to the DGLAP evolution equations

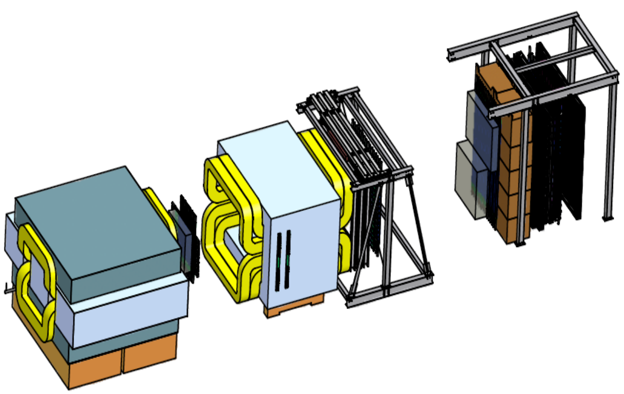
$$\frac{dD_{h/q}(z,Q)}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_z^1 \frac{dz'}{z'}\left[P_{q\to qg}(z')D_{h/q}\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z')D_{h/g}\left(\frac{z}{z'},Q\right)\right]$$

$$\frac{dD_{h/g}(z,Q)}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_z^1 \frac{dz'}{z'}\left[P_{g\to gg}(z')D_{h/g}\left(\frac{z}{z'},Q\right) + P_{g\to q\bar{q}}(z')\sum_q D_{h/q}\left(\frac{z}{z'},Q\right)\right]$$

Z. Kang et al. PRL (2014), … (2015)

Opportunities in p+A

- Better constrain initial state bremsstrahlung contribution/ effect on hard processes (but also azimuthal asymmetries)
- Given the large uncertainties in hadronization, need of Monte Carlos, fluctuatons, asymmetries appear more challenging
- With better undersanding of the complementary coherent Bertsch-Gunion regime (high energy) and radiative corrections to hard processes, effects on particle and jet production can be disentangled.



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Conclusions

- There is renewed strong interest in cold nuclear matter effects (CNM) in p+A. While these effects have been studied and incorporated in A+A calculations, only recently has the larger community been intrigued by them – due to their magnitude and qualitative similarity to A+A
- Measurements in p+A have stimulated interesting theoretical developments, such as hydrodynamics in tiny systems, some CGC phenomenology. More limited to specific kinematics/CM energy
- Initial state parton scattering and non-Abelian bremsstrahlung does not suffer from such limitations and, is common to both global event-shape observables and hard probes, including initial-state CNM effects. (well studied in final-state jet quenching in the QGP.
- With momentum fluctuations included, the non-abelian bremsstrahlung form beam and target jets describes all qualitative features of the azimuthal asymmetries observed in p+A. Expect dilution in A+A. Sensitivity to hadronization.
- For hard probes phenomenology in p+A, CNM energy loss plays a significant role, which becomes dominant near kinematic thresholds and leads to sizeable suppression of jets, hadrons and quarkonia that scales with $x_F \sim x_1 \sim p_T \exp(y)/s^{1/2} \sim p_T \cosh(y)/s^{1/2}$.
- New theoretical work underway to understand the coherent BG radiation and the radiative corrections to hard scattering.

Soft Collinear Effective Theory with Glauber gluons

- Jet physics presents a multiscale problem, EFT treatment

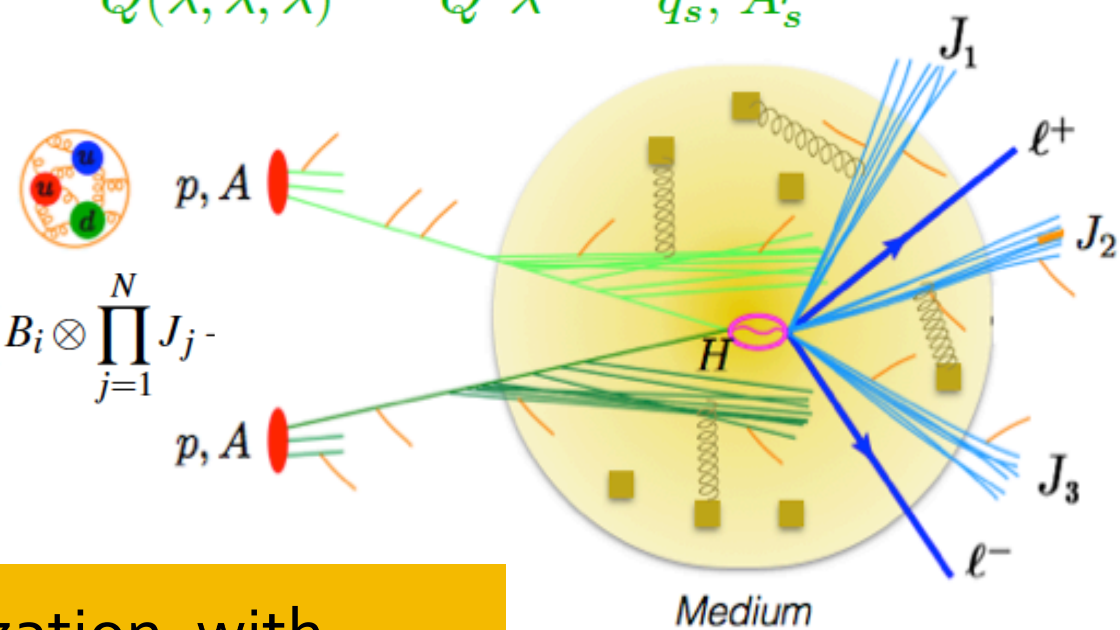
C. Bauer et al. PRD (2001)

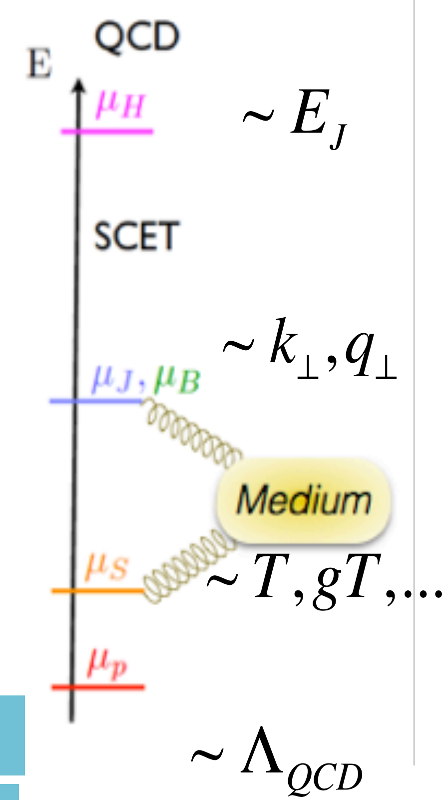
D. Pirol et al. PRD (2004)

SCET (Soft Colliner Effective Theory)

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

$$\sigma = \mathrm{Tr}(HS) \otimes \prod_{i=1}^{n_B} B_i \otimes \prod_{j=1}^{N} J_j$$



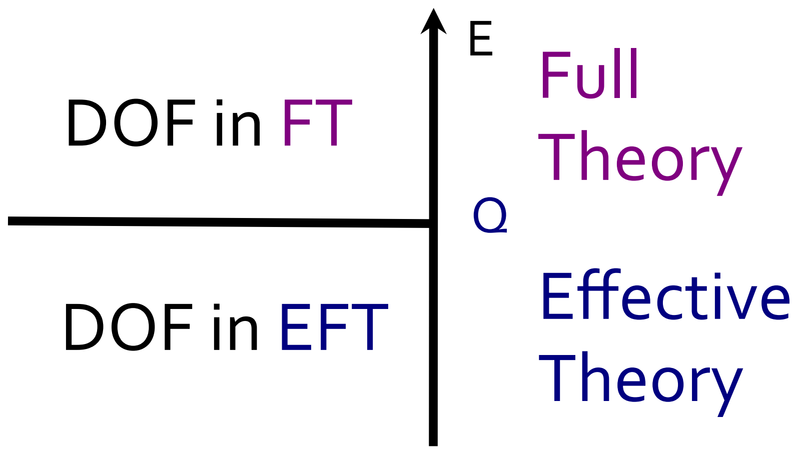


- Factorization, with modified J, B, S

Idilbi et al. PRD (2008)

Ovanesyan et al. JHEP (2011)

Examples of effective field theories [EFTs]



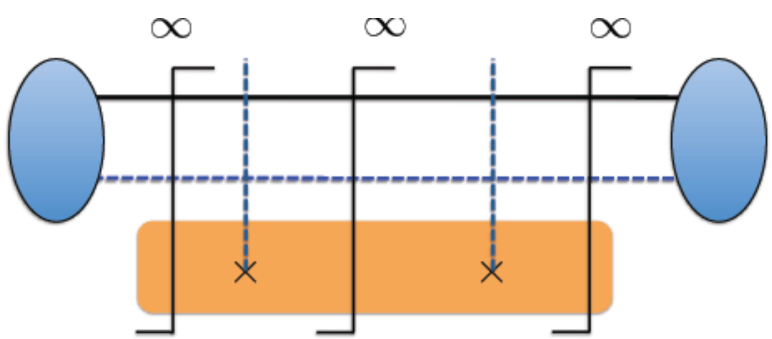
- Simple but powerful idea to concentrate on the significant degrees of freedom [DOF]. Manifest power counting

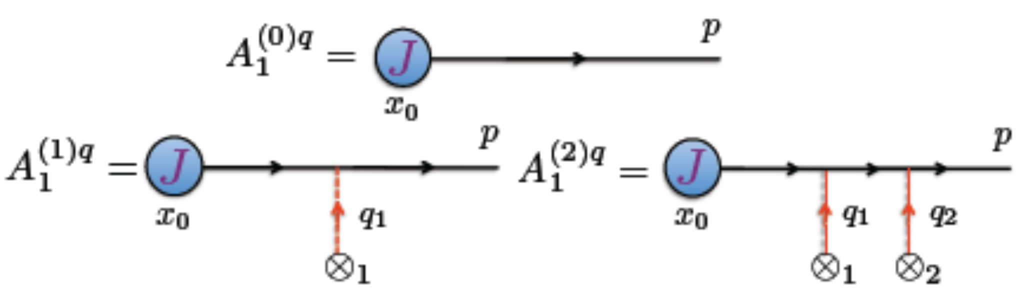
	Q	power counting	DOF in FT	DOF in EFT
Chiral Perturbation Theory (ChPT)	Λ_{QCD}	p/Λ_{QCD}	q, g	K,π
Heavy Quark Effective Theory (HQET)	m_b	Λ_{QCD}/m_b	ψ,A	h_v,A_s
Soft Collinear Effective Theory (SCET)	Q	$p_\perp/Q$	ψ,A	ξ_n, A_n, A_s

III. Main results: jet broadening

- Jet broadening and its gauge invariance

M. Gyulassy et al. (2001)





Classes of diagrams (single Born, double Born). Reaction Operator

- General result. Will evaluate the broadening (or lack off) of jets

$$\frac{dN^{(n)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \prod_{i=1}^{n}\int_{z_{i-1}}^{L}\frac{dz_i}{\lambda}\int d^2\mathbf{q}_{\perp i}\left[\frac{1}{\sigma_{el}(z_i)}\frac{d\sigma_{el}(z_i)}{d^2\mathbf{q}_{\perp i}}\left(e^{-\mathbf{q}_{\perp i}\cdot\vec{\nabla}_{\mathbf{p}_\perp}}\right) - \delta^2(\mathbf{q}_\perp)\right]\frac{dN^{(0)}(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp}$$

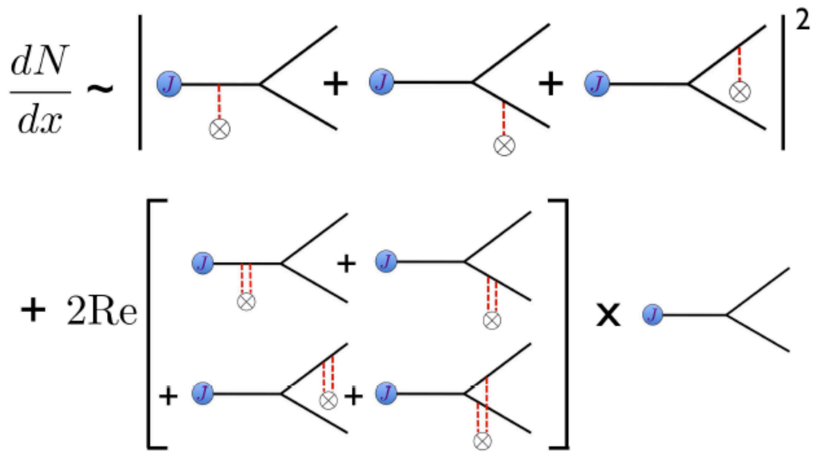
- In special cases such as constant density and the Gaussian approximation

Starting with a collinear beam of quarks/gluons

we recover M. Gyulassy et al. (2002)

$$\frac{dN(\mathbf{p}_\perp)}{d^2\mathbf{p}_\perp} = \frac{1}{2\pi}\frac{e^{-\frac{p^2}{2\chi\mu^2\xi}}}{\chi\,\mu^2\,\xi} \qquad \chi = \frac{L}{\lambda}$$

III. Main results: in-medium splitting / parton energy loss



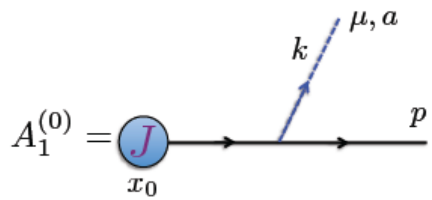
Gluon splitting functions factorize form the hard scattering cross section only for spin averaged processes

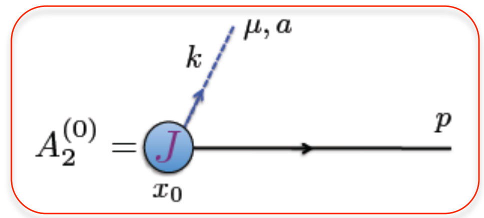
Altarelli-Parisi splitting

G. Altarelli et al. (1978)

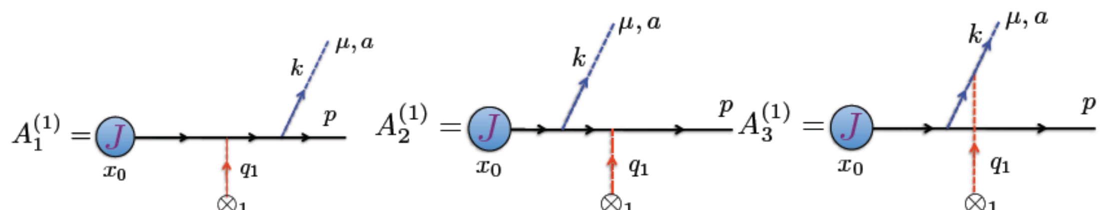
- Note that a collinear Wilson line appears in the R_ξ gauge

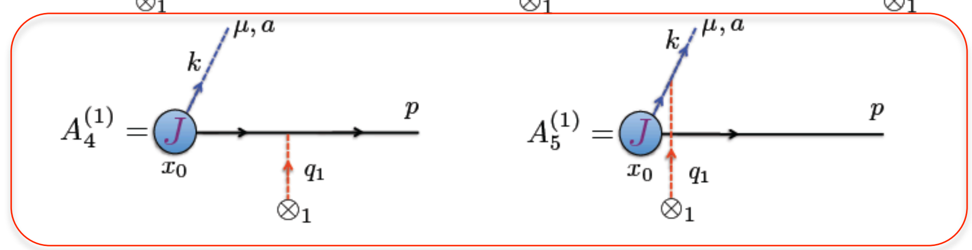
Single Born diagrams





$$\Gamma_W^{\alpha,a}(k) = gT_r^a \frac{\bar{n}^\alpha}{k^+ + i\epsilon}$$





Toward a better understanding of initial-state radiative corrections

- Gluon splittings

$$\left(\frac{\mathrm{d}N}{\mathrm{d}x\,\mathrm{d}^2\boldsymbol{k}_\perp}\right)_{\begin{Bmatrix} g\to gg \\ g\to q\bar{q}\end{Bmatrix}} = \begin{Bmatrix} \frac{\alpha_s}{2\pi^2}\,2C_A\left(\frac{x}{1-x}+\frac{1-x}{x}+x(1-x)\right) \\ \frac{\alpha_s}{2\pi^2}T_R\left(x^2+(1-x)^2\right)\end{Bmatrix} \int \mathrm{d}\Delta z \begin{Bmatrix} \frac{1}{\lambda_g(z)} \\ \frac{1}{\lambda_q(z)} \end{Bmatrix} \int \mathrm{d}^2\boldsymbol{q}_\perp \frac{1}{\sigma_{el}} \frac{\mathrm{d}\sigma_{el}^{\text{medium}}}{\mathrm{d}^2\boldsymbol{q}_\perp}$$

$$\times\left[\left(2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)^2+2\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}+\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)\cdot\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\right)-2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)\right.$$

$$\times\cos[(\Omega_1-\Omega_2)\Delta z]+\begin{Bmatrix} -\frac{1}{2} \\ \frac{1}{N_c^2-1}\end{Bmatrix}\left(2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}\cdot\left(2\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])\right.$$

$$\left.\left.-2\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2}\right)(1-\cos[\Omega_3\Delta z])\right)\right].$$