Gluon TMDs at Small x

Bo-Wen Xiao

Institute of Particle Physics, Central China Normal University

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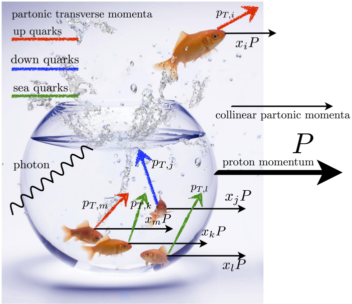
Emerging Spin and Transverse Momentum Effects in p+p and p+A Collisions

Outline

Transverse Momentum Dependent parton distributions



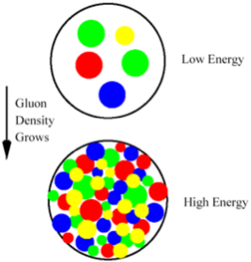
gluon pol.

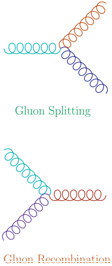
nucleon pol.	U	L	linear
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

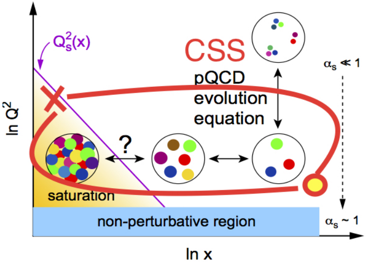
- As compared to Feynman PDFs, TMDs contain extra degrees of freedom ($k_\perp$).
- Unintegrated Gluon Distributions (UGDs) at small-x also depend on $k_\perp$.
- However, TMDs were mainly used in large-x region and often in spin physics.
- TMDs (Sudakov type logarithms) and UGDs (Small-x logarithms) evolve differently.

High density QCD

Saturation Phenomenon (Color Glass Condensate)







- Resummation of the $\alpha_s \ln \frac{1}{x} \Rightarrow$ BFKL equation. (In DIS, $x_{bj} = \frac{Q^2}{s}$)
- When too many gluons squeezed in a confined hadron, gluons start to overlap and recombine $\Rightarrow$ Non-linear dynamics $\Rightarrow$ BK equation
- Introduce the saturation momentum $Q_s(x)$ to separate the saturated dense regime $x < 10^{-2}$ from the dilute regime.

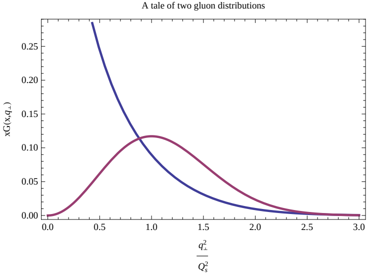
A Tale of Two Gluon Distributions[1]

In small-x physics, two gluon distributions are widely used:[Kharzeev, Kovchegov, Tuchin; 03]
I. Weizsäcker Williams gluon distribution([Kovchegov, Mueller, 98] and MV model):

$$xG_{\mathrm{WW}}(x,k_\perp) = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c}\int\frac{d^2r_\perp}{(2\pi)^2}\frac{e^{-ik_\perp\cdot r_\perp}}{r_\perp^2}\left[1-e^{-\frac{r_\perp^2 Q_{sg}^2}{4}}\right]$$

II. Color Dipole gluon distributions:

$$xG_{\mathrm{DP}}(x,k_\perp) = \frac{S_\perp N_c}{2\pi^2\alpha_s}k_\perp^2\int\frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}e^{-\frac{r_\perp^2 Q_{sq}^2}{4}} \Leftarrow \frac{1}{N_c}\mathrm{Tr}\left[U(r_\perp)U^\dagger(0_\perp)\right]$$



[1] As far as I know, the title is due to Y. Kovchegov and C. Dickens.

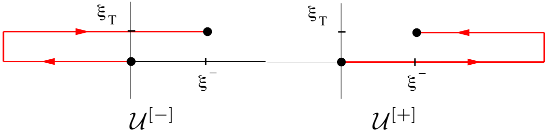
A Tale of Two Gluon Distributions

In terms of operators (known from TMD physics [Bomhof, Mulders and Pijlman, 06]), two gauge invariant gluon definitions: [Dominguez, Marquet, Xiao and Yuan, 11]
I. Weizsäcker Williams gluon distribution:

$$xG_{\mathrm{WW}}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$

II. Color Dipole gluon distributions:

$$xG_{\mathrm{DP}}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle.$$



Remarks:

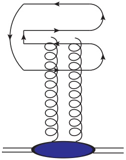
- The WW gluon distribution is the conventional gluon distributions.
- The dipole gluon distribution has no such interpretation.
- Two topologically different gauge invariant definitions.

A Tale of Two Gluon Distributions

[F. Dominguez, C. Marquet, Xiao and F. Yuan, 11]

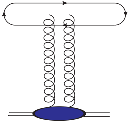
I. Weizsäcker Williams gluon distribution

$$xG_{\rm WW}(x,k_\perp) = \frac{2N_c}{\alpha_S}\int\frac{d^2R_\perp}{(2\pi)^2}\frac{d^2R'_\perp}{(2\pi)^2}e^{iq_\perp\cdot(R_\perp-R'_\perp)} \times\frac{1}{N_c}\left\langle \mathrm{Tr}\left[i\partial_i U(R_\perp)\right]U^\dagger(R'_\perp)\left[i\partial_i U(R'_\perp)\right]U^\dagger(R_\perp)\right\rangle,$$



II. Color Dipole gluon distribution:

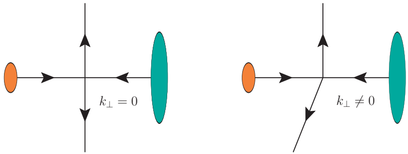
$$xG_{\rm DP}(x,k_\perp) = \frac{2N_c}{\alpha_s}\int\frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4}e^{iq_\perp\cdot(R_\perp-R'_\perp)} \left(\nabla_{R_\perp}\cdot\nabla_{R'_\perp}\right)\frac{1}{N_c}\left\langle\mathrm{Tr}\left[U(R_\perp)U^\dagger(R'_\perp)\right]\right\rangle_x ,$$



- Quadrupole $\Rightarrow$ Weizsäcker Williams gluon distribution;
- Dipole $\Rightarrow$ Color Dipole gluon distribution;
- Generalized universality in the large N_c limit in *ep* and *pA* collisions $\Rightarrow$ Effective dilute dense factorization.

A Tale of Two Gluon Distributions

In terms of operators, we find these two gluon distributions can be defined as follows:
I. Weizsäcker Williams gluon distribution: II. Color Dipole gluon distributions:



Questions:

- Modified Universality for Gluon Distributions:

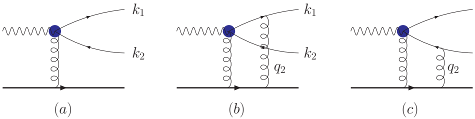
	Inclusive	Single Inc	DIS dijet	γ +jet	g+jet
xG_{WW}	$\times$	$\times$	$\surd$	$\times$	$\surd$
xG_{DP}	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

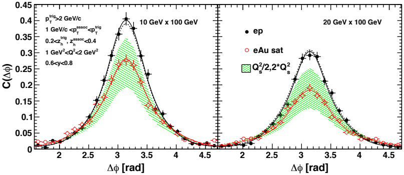
  $\times \Rightarrow$ Do Not Appear. $\surd \Rightarrow$ Apppear.

- Two fundamental gluon distributions which are related to the quadrupole and dipole amplitudes, respectively.

Dijet production in DIS

[L. Zheng, E. Aschenauer, J. H. Lee and BX, 14]



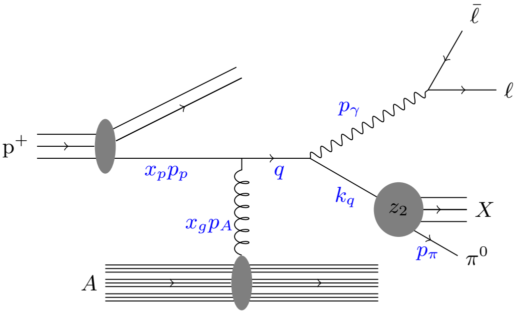


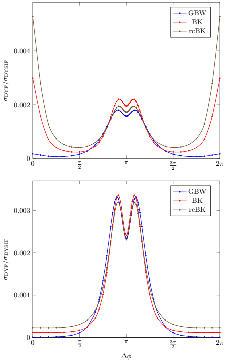
Remarks:

- For back-to-back correlation $|k_{1\perp}| \simeq |k_{2\perp}| \gg q_\perp = k_{1\perp} + k_{2\perp}$.
- Unique golden measurement for the Weizsäcker Williams gluon distributions.
- EIC will provide us perfect machines to study gluon fields inside protons/nuclei.

DY correlations in pA collisions

[Stasto, BX, Zaslavsky, 12]



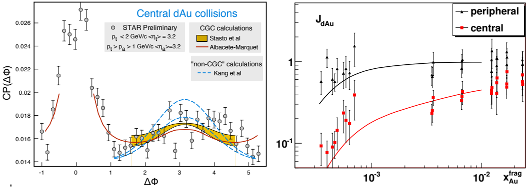


$M = 0.5, 4$GeV, $Y = 2.5$ at RHIC dAu.

- Partonic cross section vanishes at $\pi \Rightarrow$ Dip at π.
- Sudakov factorization may change the double peak structure.
- Prompt photon calculation [J. Jalilian-Marian, A. Rezaeian, 12].

Dihadron correlations in dAu collisions

$$C(\Delta\phi) = \frac{\int_{|p_{1\perp}|,|p_{2\perp}|} \frac{d\sigma^{pA\to h_1 h_2}}{dy_1 dy_2 d^2p_{1\perp} d^2p_{2\perp}}}{\int_{|p_{1\perp}|} \frac{d\sigma^{pA\to h_1}}{dy_1 d^2p_{1\perp}}} \qquad J_{dA} = \frac{1}{\langle N_{\rm coll}\rangle} \frac{\sigma^{\rm pair}_{dA}/\sigma_{dA}}{\sigma^{\rm pair}_{pp}/\sigma_{pp}}$$



Comparing to STAR and PHENIX data

- Physics predicted by [C. Marquet, 09].
- Further calculated in [Marquet, Albacete, 10; Stasto, BX, Yuan, 11]
- Physical picture: de-correlation of dijets due to dense gluonic matter.

Evolutions: TMDs vs UGDs

Evolutions are effectively resumming large logarithms:

- TMDs evolve with the CSS equation which resums Sudakov logarithms

$$\left[\frac{\alpha_s C_R}{2\pi}\ln^2\frac{Q^2}{k_\perp^2}\right]^n + \cdots, \quad \text{with} \quad Q^2 \gg k_\perp^2$$

- UGDs follow the small-x evolution equations, such as BK or JIMWLK which resums

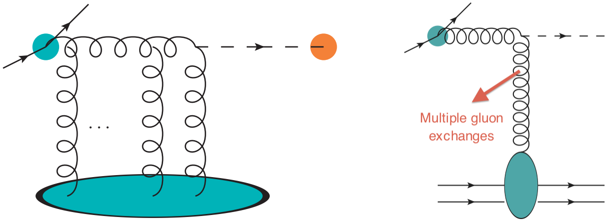
$$\left[\frac{\alpha_s N_c}{2\pi}\ln\frac{1}{x}\right]^n, \quad \text{with} \quad x = \frac{Q^2}{s} \ll 1$$

- Question: What happens when $s \gg Q^2 \gg k_\perp^2$?

Gedanken experiment: Higgs production in pA collisions

[A. Mueller, BX and F. Yuan, 12, 13] The effective Lagrangian:

$$\mathcal{L}_{eff} = -\frac{1}{4} g_\phi \Phi F^a_{\mu\nu} F^{a\mu\nu}$$

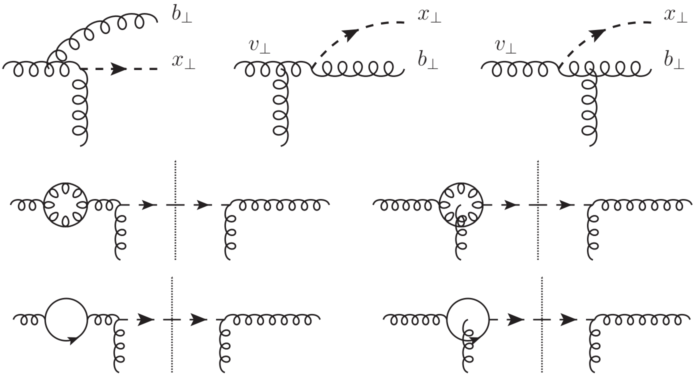


$$\frac{d\sigma^{(\mathrm{LO})}}{dy d^2 k_\perp} = \sigma_0 \int \frac{d^2 x_\perp d^2 x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot (x_\perp - x'_\perp)} x g_p(x) S_Y^{WW}(x_\perp, x'_\perp) \ ,$$

- $S_Y^{WW}(x_\perp, y_\perp) = -\left\langle \mathrm{Tr}\left[\partial_\perp^\beta U(x_\perp) U^\dagger(y_\perp) \partial_\perp^\beta U(y_\perp) U^\dagger(x_\perp)\right]\right\rangle_Y$
- Only initial state interaction is present. $\Rightarrow$ WW gluon distribution.

Some Technical Details

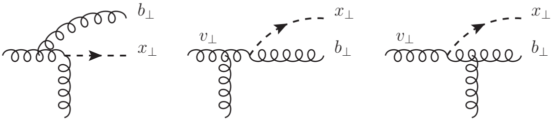
[A. Mueller, BX and F. Yuan, 12, 13] Typical diagrams:



- High energy limit $s \to \infty$ and $M^2 \gg k_\perp^2$. Use dimensional regularization.
- Power counting analysis: take the leading power contribution in terms of $\frac{k_\perp^2}{M^2}$.

Some Technical Details

[A. Mueller, BX and F. Yuan, 12, 13]



The phase space $(l^+, l^-, l_\perp)$ of the radiated gluon can be divided into three regions:

- (a) gluon is collinear to the incoming proton. $\Rightarrow$ DGLAP evolution.
  Subtraction of the collinear divergence and choose $\mu^2 = \frac{c_0^2}{R_\perp^2}$:

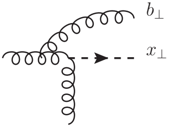
$$-\frac{1}{\epsilon} S^{WW}(x_\perp, y_\perp) \mathcal{P}_{gg}(\xi) \otimes xg\left(x, \frac{c_0^2}{R_\perp^2}\right) \quad \text{with} \quad \xi = \frac{l^+}{p^+}$$

- (b) gluon is collinear to the incoming nucleus. $\Rightarrow$ Small-x evolution.
  Subtraction of the rapidity divergence: $\Rightarrow$ non-linear small-x evolution equation

$$xg_p(x) \int_0^1 \frac{d\xi}{\xi} \int \mathbf{K}_{\mathrm{DMMX}} \otimes S^{WW}(x_\perp, y_\perp)$$

- (c) gluon is soft. $\Rightarrow$ Sudakov logarithms.

Separation of the small-x logarithm and Sudakov logarithms



(a)

- Consider the kinematic constraint for real emission before taking $s \to \infty$, and note that $x_g x_p s = M^2$

$$\int_{\frac{l_\perp^2}{x_p s}}^{1} \frac{\mathrm{d}\xi}{\xi} = \ln\left(\frac{x_p s}{l_\perp^2}\right) = \ln\frac{1}{x_g} + \ln\frac{M^2}{l_\perp^2}.$$

- Now we can take $s \to \infty$ and $x_g \to 0$, but keep $x_g x_p s = M^2$.
- The Sudakov contribution gives

$$\mu^{2\epsilon} \int \frac{\mathrm{d}^{2-2\epsilon} l_\perp}{(2\pi)^{2-2\epsilon}} e^{-il_\perp \cdot R_\perp} \frac{1}{l_\perp^2} \ln\frac{M^2}{l_\perp^2}$$
$$= \frac{1}{4\pi}\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln\frac{M^2}{\mu^2} + \frac{1}{2}\left(\ln\frac{M^2}{\mu^2}\right)^2 - \frac{1}{2}\left(\ln\frac{M^2 R_\perp^2}{c_0^2}\right)^2 - \frac{\pi^2}{12}\right]$$

Sudakov factor

Final results:

- At one-loop order: $R_\perp \equiv x_\perp - x'_\perp$

$$\frac{d\sigma^{(\text{1-Loop})}}{\sigma_0 dy d^2k_\perp} = \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot R_\perp} xg_p(x, \mu^2 = c_0^2/R_\perp^2) S^{WW}_{Y=\ln 1/x_g}(x_\perp, x'_\perp)$$
$$\times \left[1 + \frac{\alpha_s}{\pi} N_c \left(-\frac{1}{2}\ln^2 \frac{M^2R_\perp^2}{c_0^2} + \beta_0 \ln \frac{M^2R_\perp^2}{c_0^2} + \frac{\pi^2}{2}\right)\right].$$

- Collins-Soper-Sterman evolution:

$$\frac{d\sigma^{(\text{resum})}}{dyd^2k_\perp}|_{k_\perp \ll M} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot R_\perp} e^{-\mathcal{S}_{\text{sud}}(M^2, R_\perp^2)} S^{WW}_{Y=\ln 1/x_g}(x_\perp, x'_\perp)$$
$$\times x_p g_p(x_p, \mu^2 = \frac{c_0^2}{R_\perp^2}) \left[1 + \frac{\alpha_s}{\pi}\frac{\pi^2}{2} N_c\right],$$

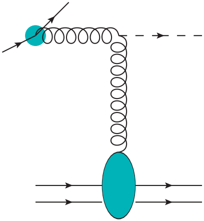
  where the Sudakov form factor contains all order resummation

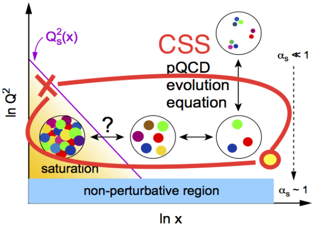
$$\mathcal{S}_{\text{sud}}(M^2, R_\perp^2) = \int_{c_0^2/R_\perp^2}^{M^2} \frac{d\mu^2}{\mu^2}\left[A \ln \frac{M^2}{\mu^2} + B\right].$$

- $A = \sum_{i=1}^{\infty} A^{(i)} \left(\frac{\alpha_s}{\pi}\right)^i$, we find $A^{(1)} = N_c$ and $B^{(1)} = -\beta_0 N_c$.

Sudakov resummation in saturation formalism

One-loop Calculation for Higgs, Heavy-Quarkonium and Dijet processes $\Rightarrow$ Sudakov factor in saturation physics. [A. Mueller, BX and F. Yuan, 13; P. Sun, J. Qiu, BX, F. Yuan, 13]

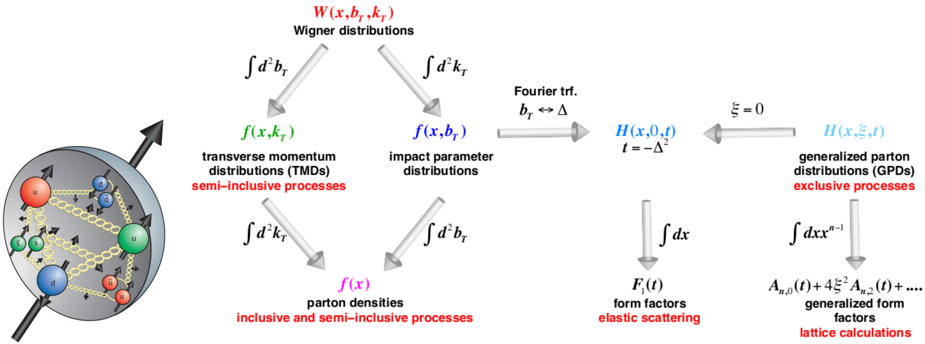




- Multiple scales problem. $k_\perp^2 \ll Q^2 \sim M^2 \ll s$.
- Joint Small-x $\left[\frac{\alpha_s N_c}{2\pi} \ln \frac{1}{x}\right]^n$ resummation and Sudakov factor $\left[\frac{\alpha_s C_R}{2\pi} \ln^2 \frac{Q^2}{k_\perp^2}\right]^n$ resummation.
- [Balitsky, Tarasov, 14] Starting from the same operator definition, xG_{WW}: TMD (moderate $x \sim \frac{Q^2}{s}$) and W.W. (small-x, high energy with fixed Q^2). Unified description of the TMD and small-x UGD.
- [Marzani, 15] Q_T resummation and small-x resummation.
- Evolution depends on the necessity.

3D Tomography of Proton

The bigger picture:



- In small-x physics (color glass condensate), we use different objects: dipole, quadrupole.
- Dipole, quadrupole $\Rightarrow$ Unintegrated Gluon Distributions (UGDs) at small-x.
- Impact parameter $b_\perp$ dependent UGDs $\Leftrightarrow$ gluon Wigner distributions? [Ji, 03]
- Can we measure the gluon Wigner distribution at small-x? Yes, we can!

The exact connection between dipole amplitude and Wigner distribution

[Hatta, Xiao, Yuan, 16] Definition of gluon Wigner distribution:

$$\begin{aligned} xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp) &= \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ixP^+\xi^- - iq_\perp\cdot\xi_\perp} \\ &\times \left\langle P+\frac{\Delta_\perp}{2} \middle| F^{+i}\left(\vec{b}_\perp+\frac{\xi}{2}\right) F^{+i}\left(\vec{b}_\perp-\frac{\xi}{2}\right) \middle| P-\frac{\Delta_\perp}{2} \right\rangle , \end{aligned}$$

Let us choose proper gauge link and define GTMD [Meissner, A. Metz and M. Schlegel, 09]

$$xG(x,q_\perp,\Delta_\perp) \equiv \int d^2b_\perp e^{-i\Delta\cdot b_\perp} xW_g^T(x,\vec{q}_\perp;\vec{b}_\perp).$$

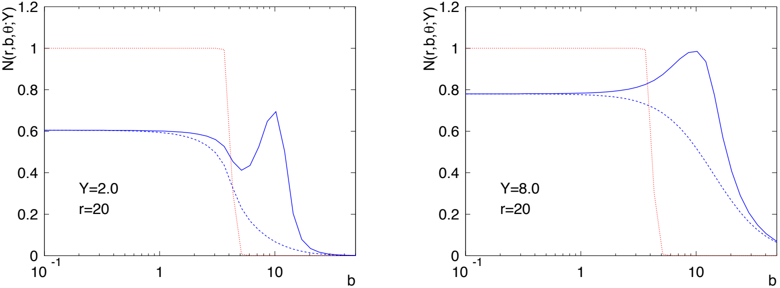
- With one choice of gauge link (dipole like) and $b_\perp = \frac{1}{2}(R_\perp + R'_\perp)$, we demonstrate

$$\begin{aligned} xG_{\mathrm{DP}}(x,q_\perp,\Delta_\perp) &= \frac{2N_c}{\alpha_s} \int \frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4} e^{iq_\perp\cdot(R_\perp-R'_\perp)+i\frac{\Delta_\perp}{2}\cdot(R_\perp+R'_\perp)} \\ &\times \left(\nabla_{R_\perp}\cdot\nabla_{R'_\perp}\right) \frac{1}{N_c} \left\langle \mathrm{Tr}\left[U(R_\perp)\,U^\dagger(R'_\perp)\right]\right\rangle_x . \end{aligned}$$

- $\int d^2\Delta_\perp xG_{\mathrm{DP}}(x,q_\perp,\Delta_\perp) \Rightarrow TMD$; $\int d^2q_\perp xG_{\mathrm{DP}}(x,q_\perp,\Delta_\perp) \Rightarrow GPD$ at small-x.

Angular correlations

$$N = 1 - \frac{1}{N_c} \left\langle \mathrm{Tr} \left[U\left(R_\perp\right) U^\dagger\left(R'_\perp\right) \right] \right\rangle_Y$$

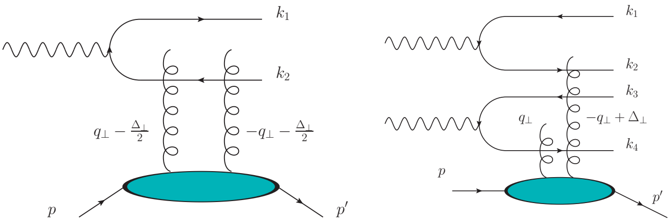


[Golec-Biernat, Stasto, 03] $b = \frac{1}{2}\left(R'_\perp + R_\perp\right)$ and $r = R'_\perp - R_\perp$

- Solving BK equation with impact parameter. (Need to improve our saturation models).
- Non-trivial angular correlation between b and r.
- In Wigner distribution, Non-trivial angular correlation between $\Delta_\perp$ and $q_\perp$.

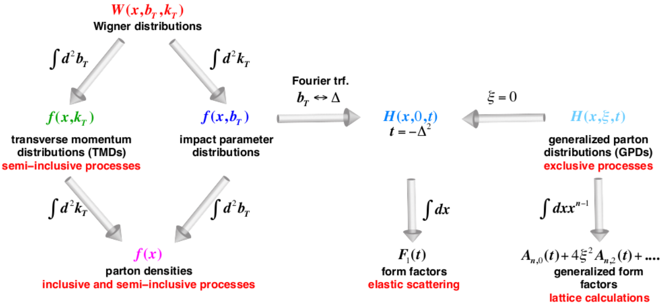
Probing 3D Tomography of Proton at small-x

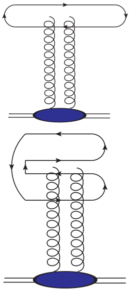
Diffractive back-to-back dijet productions:



- Measure final state proton recoil $\Delta_\perp$ as well as dijet momentum $k_{1\perp}$ and $k_{2\perp}$.
- We can obtain $xG_{\text{DP}}(x, q_\perp, \Delta_\perp)$ directly since $q_\perp \simeq P_\perp \equiv \frac{1}{2}(k_{2\perp} - k_{1\perp}) \gg \Delta_\perp$.
- By measuring $\langle \cos 2\left(\phi_{P_\perp} - \phi_{\Delta_\perp}\right)\rangle$, we can learn more about the low-x dynamics.
- WW Wigner (WWW) distribution can be also defined and measured.
- Linearly polarized Wigner distribution, etc. This is only the beginning.

Summary





- Towards unification of TMD physics and small-x physics (Definition and Evolution).
- Wigner distributions at small-x can be computed and measured.
- Gluon saturation could be the next interesting discovery in the near future.