Small-x improved TMD factorization for forward particle production and possible pA measurements

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P. Kotko, K. Kutak, CM, E. Petreska, S. Sapeta and A. van Hameren
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Motivations

- cross sections in the Bjorken limit of QCD

$$s \to \infty \ , \quad Q^2 \to \infty$$
$$Q^2/s = x \ \ \text{fixed}$$

are expressed as a 1/Q^2 "twist" expansion $$d\sigma = \sum_p f_p \otimes d\hat{\sigma} + O(1/Q^2)$$

collinear factorization: parton content of proton described by k_T-integrated distributions

sufficient approximation for most high-p_T processes

TMD factorization: involves transverse-momentum-dependent (TMD) distributions

needed in particular cases, TMD-pdfs are process dependent

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TMD factorization: involves transverse-momentum-dependent (TMD) distributions

needed in particular cases, TMD-pdfs are process dependent

- cross sections in the Regge limit of QCD

$$s \to \infty\ ,\quad x \to 0$$
$$xs = Q^2 \ \text{ fixed}$$

are expressed as a $1/s$ "eikonal" expansion $$d\sigma = \sum_p f_p \otimes d\hat{\sigma} + O(1/s)$$

k_T factorization: parton content described by unintegrated parton distributions (u-pdfs)

we would like to understand: - the connection between TMD & k_T factorizations
- how TMD-pdfs and u-pdfs are related

Conclusions from talk 6 years ago

- considering the SIDIS process, we have shown that

  TMD factorization (valid at large Q^2)
  and k_T factorization (valid at small x)

  CM, Xiao and Yuan (2009)

  are consistent with each other in the overlapping domain of validity

- the SIDIS measurement provides direct access to the transverse momentum distribution of partons

  the saturation regime, characterized by $Q_s^2 \simeq \Lambda_{QCD}^2\,(A/x)^{1/3}$, can be easily investigated

  even if Q^2 is much bigger than Q_s^2,
  the saturation regime will be important when $P_\perp^2 \sim Q_s^2$

- this is an encouraging start, but now we would like to understand the relations between TMD and k_T factorization breaking

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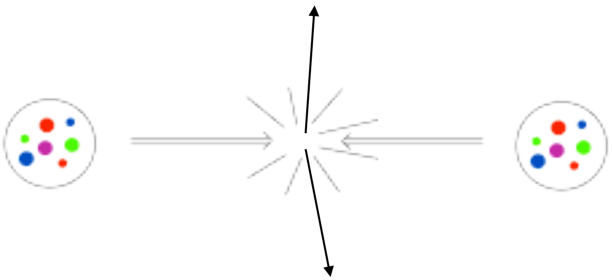
$\longrightarrow$ study process where factorization breaks: di-jets
(and forward production to have small x)

Di-jet final-state kinematics

final state : $k_1, y_1 \qquad k_2, y_2$

$$x_p = \frac{k_1\, e^{y_1} + k_2\, e^{y_2}}{\sqrt{s}} \qquad x_A = \frac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}}$$

scanning the wave functions:



$$x_p \sim x_A < 1$$

central rapidities probe moderate x

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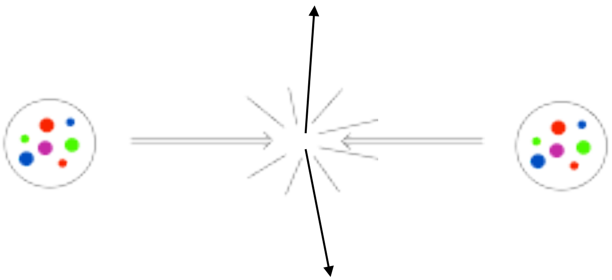

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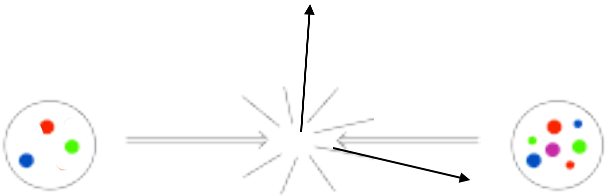
x_p increases $\downarrow$ x_A ~ unchanged



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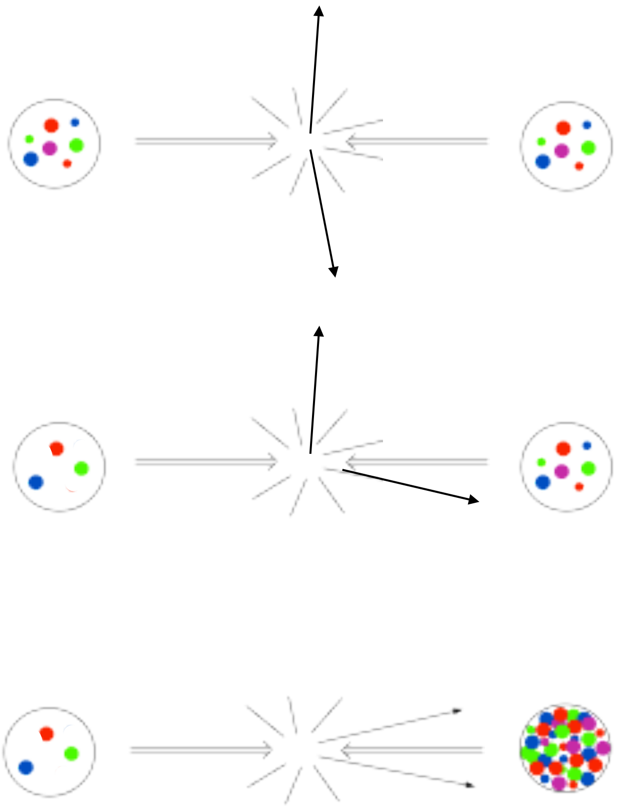


$$x_p \sim x_A < 1$$

central rapidities probe moderate x

x_p increases | x_A ~ unchanged

$$x_p \sim 1,\, x_A < 1$$

forward/central doesn’t probe much smaller x

x_p ~ unchanged | x_A decreases

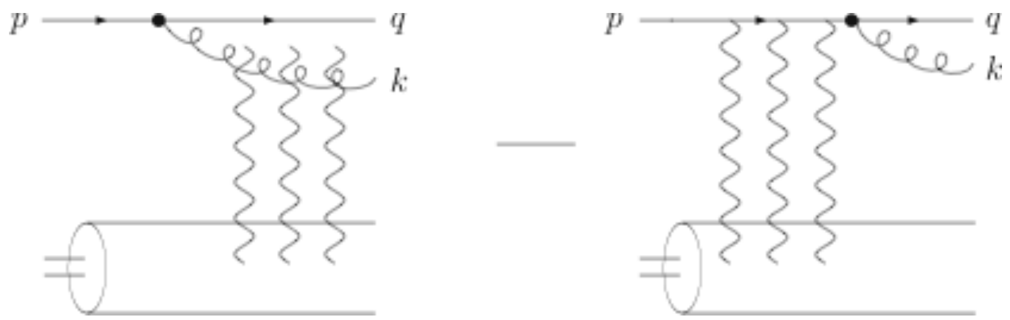
$$x_p \sim 1,\, x_A << 1$$

forward rapidities probe small x

Color Glass Condensate (CGC) calculation of forward di-jets

Saturation calculation

CM (2007)



b: quark in the amplitude
x: gluon in the amplitude
b': quark in the conj. amplitude
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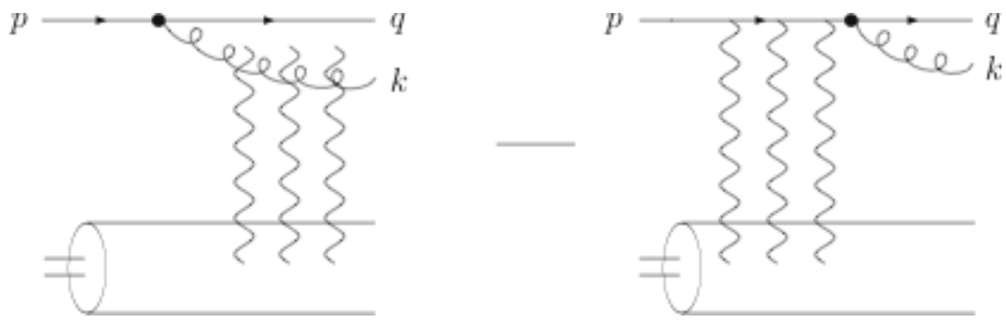

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collinear factorization of quark density in deuteron

Fourier transform $k_\perp$ and $q_\perp$ into transverse coordinates

$$\frac{d\sigma^{dAu\to qgX}}{d^2k_\perp dy_k d^2q_\perp dy_q} = \alpha_S C_F N_c\, x_d q(x_d,\mu^2)\int \frac{d^2x}{(2\pi)^2}\frac{d^2x'}{(2\pi)^2}\frac{d^2b}{(2\pi)^2}\frac{d^2b'}{(2\pi)^2}\, \overbrace{e^{ik_\perp.(\mathbf{x}'-\mathbf{x})}e^{iq_\perp.(\mathbf{b}'-\mathbf{b})}}$$

$$\left|\Phi^{q\to qg}(z,\mathbf{x}-\mathbf{b},\mathbf{x}'-\mathbf{b}')\right|^2 \Big\{S^{(4)}_{qg\bar{q}g}[\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A] - S^{(3)}_{qg\bar{q}}[\mathbf{b},\mathbf{x},\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A]$$

$$-S^{(3)}_{\bar{q}gq}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{x}',\mathbf{b}';x_A] + S^{(2)}_{q\bar{q}}[\mathbf{b}+z(\mathbf{x}-\mathbf{b}),\mathbf{b}'+z(\mathbf{x}'-\mathbf{b}');x_A]\Big\}$$

pQCD $q \to qg$ wavefunction

$$z = \frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k}+|q_\perp|e^{y_q}}$$

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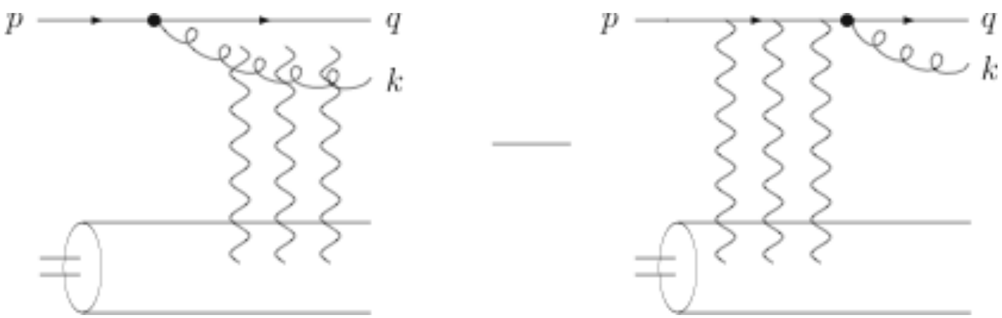

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pQCD $q \to qg$ wavefunction

interaction with target nucleus

$$z = \frac{|k_\perp|e^{y_k}}{|k_\perp|e^{y_k} + |q_\perp|e^{y_q}}$$

n-point functions that resums the powers of $g_S A$ and the powers of $\alpha_S \ln(1/x_A)$

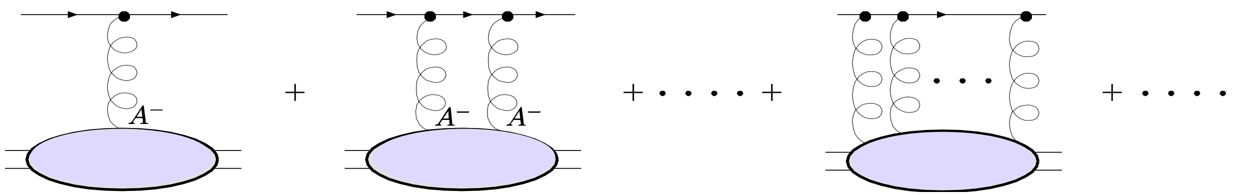
Scattering on the dense target

- this is described by Wilson lines

scattering of a quark:

$$W_F[\alpha](\mathbf{x}) = \mathcal{P}\exp\left\{ig_s \int dx^+ T^c \alpha_c(x^+, \mathbf{x})\right\}$$

α dependence kept implicit in the following



in the CGC framework, any cross-section is determined by colorless combinations of Wilson lines $S[\alpha]$, averaged over the CGC wave function $\langle S \rangle_x = \int D\alpha \, |\Phi_x[\alpha]|^2 S[\alpha]$

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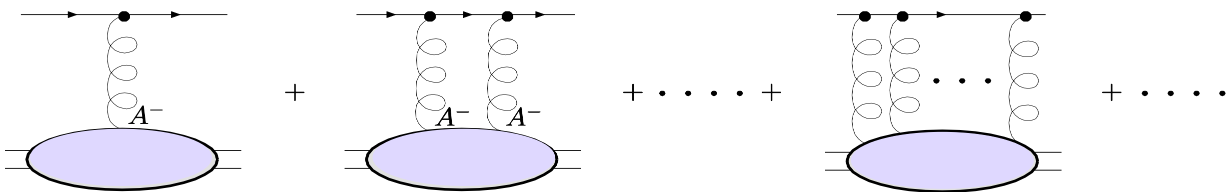

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- the 2-point function or dipole amplitude

the $q\bar{q}$ dipole scattering amplitude:

$\langle T_{q\bar{q}}(\mathbf{x},\mathbf{y})\rangle_x$ or $\langle T_{q\bar{q}}(\mathbf{r},\mathbf{b})\rangle_x$

$$T_{q\bar{q}}(\mathbf{x},\mathbf{y}) = 1 - \frac{1}{N_c}Tr(W_F^\dagger(\mathbf{y})W_F(\mathbf{x}))$$

$\mathbf{x}$: quark transverse coordinate

$\mathbf{y}$: antiquark transverse coordinate

this is the most common Wilson-line average

2- 4- and 6-point functions

- coming back to the double-inclusive cross-section

the scattering off the CGC is expressed through the following correlators of Wilson lines:

if the gluon is emitted before the interaction, four partons scatter off the CGC

$$S^{(4)}_{qg\bar{q}g}(\mathbf{b},\mathbf{x},\mathbf{b}',\mathbf{x}';x_A)=\frac{1}{C_F N_c}\left\langle \mathrm{Tr}\left(W_F(\mathbf{b})W_F^\dagger(\mathbf{b}')T^d T^c\right)[W_A(\mathbf{x})W_A^\dagger(\mathbf{x}')]^{cd}\right\rangle_{x_A}$$

if the gluon is emitted after the interaction, only the quarks interact with the CGC

$$S^{(2)}_{q\bar{q}}(\mathbf{b},\mathbf{b}';x_A)=\frac{1}{N_c}\left\langle \mathrm{Tr}\left(W_F(\mathbf{b})W_F^\dagger(\mathbf{b}')\right)\right\rangle_{x_A}$$

interference terms, the gluon interacts in the amplitude only (or c.c. amplitude only)

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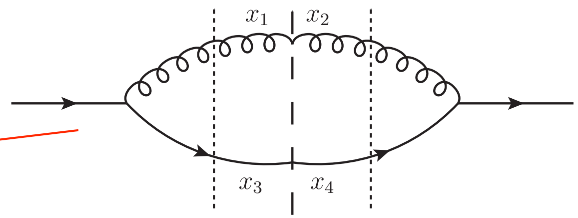
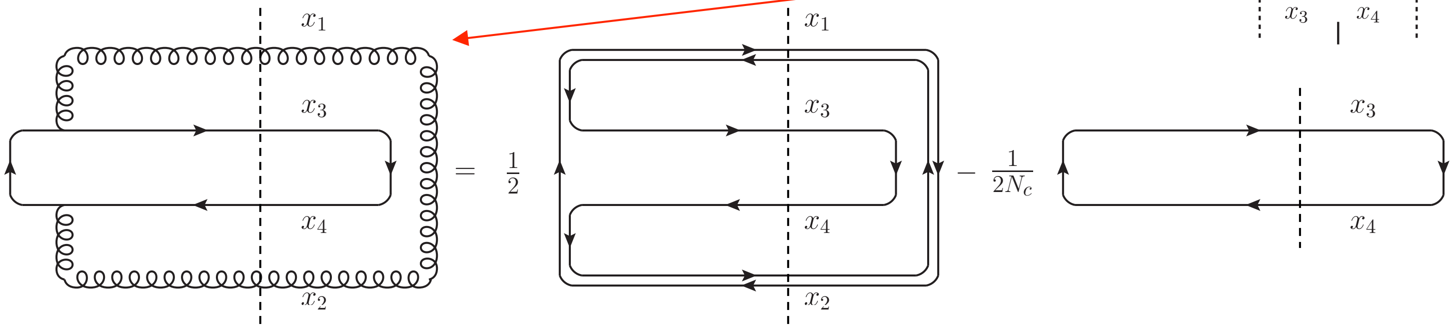
The large-Nc limit

in the large Nc limit, everything is expressed in terms of dipoles and quadrupoles





- in the large-Nc limit, the cross section is obtained from

$$S^{(4)} = \frac{1}{N_c} \left\langle \mathrm{Tr}(W_{\mathbf{x}} W_{\mathbf{y}}^\dagger W_{\mathbf{u}} W_{\mathbf{v}}^\dagger) \right\rangle_{x_A} \text{ and } S^{(2)} = \frac{1}{N_c} \left\langle \mathrm{Tr}(W_{\mathbf{x}} W_{\mathbf{y}}^\dagger) \right\rangle_{x_A}$$

(this is true for an arbitrary number of final-state particles measured)

Dominguez, CM, Stasto and Xiao (2013)

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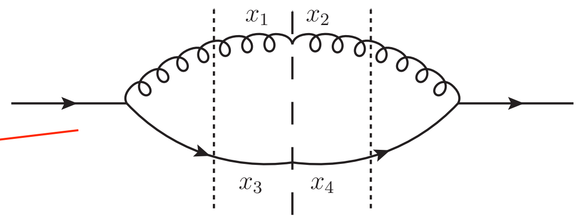

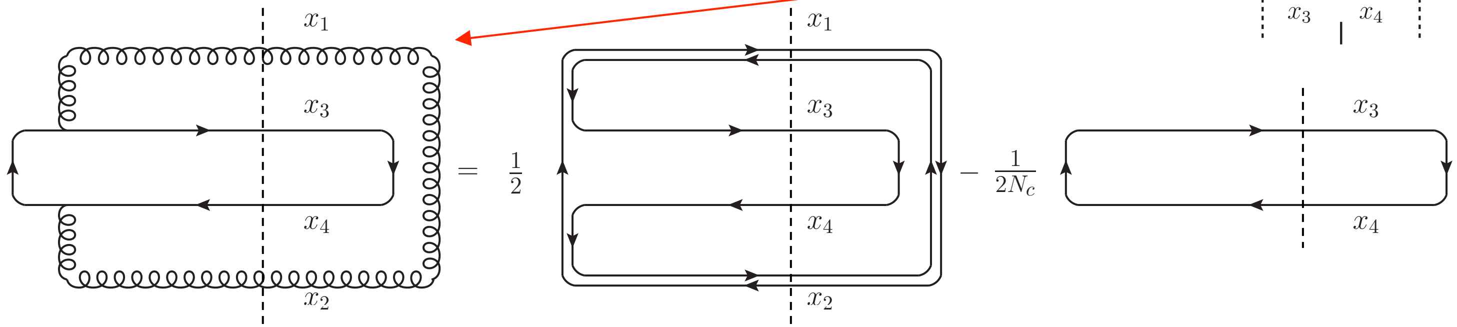

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- the 2-point function is fully constrained by e+A DIS and d+Au single hadron data

they are obtained from the dipole scattering amplitude

$$\mathcal{N}(x,r) \equiv 1 - S^{(2)}$$

r = dipole size

Connections with high-energy factorization and TMD factorization

The linear regime

$$|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$$

- taking all involved momenta >> Qs, the CGC formula reduces to

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \frac{\alpha_s^2}{\pi(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2)\ |\overline{\mathcal{M}_{ag^*\to cd}}|^2\ \mathcal{F}_{g/A}(x_2,k_t^2)\frac{1}{1+\delta_{cd}}\ .$$

this is the so-called high-energy factorization (HEF) formula

e.g. Kutak and Sapeta (2012)

$x_1 f_{a/p}(x_1,\mu^2)$	–	collinear PDF in p, suitable for $x_1 \sim 1$
$\lvert\overline{\mathcal{M}_{ag^*\to cd}}\rvert^2$	–	matrix element with off-shell incoming gluon
$\mathcal{F}_{g/A}(x_2,k_t)$	–	unintegrated gluon PDF in A, suitable for $x_2 \ll 1$

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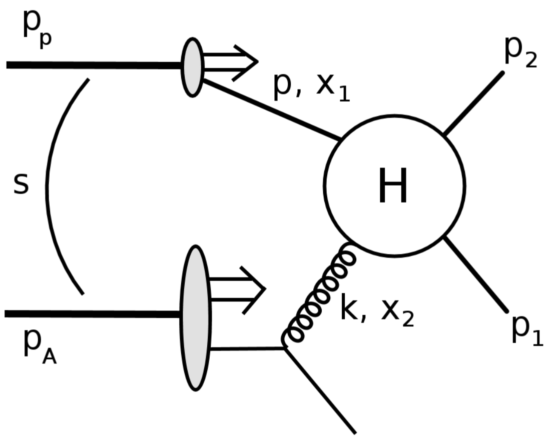
the unintegrated gluon density involved is also the also involved in deep inelastic scattering, it is related to the dipole scattering amplitude $\mathcal{N}(x,r)$

$$\mathcal{F}_{g/A}(x,k^2) = \frac{N_c}{\alpha_s (2\pi)^3} \int d^2b \int d^2r\ e^{-i\mathbf{k}\cdot\mathbf{r}} \nabla_r^2\ \mathcal{N}(x,r)$$

Recall dilute-dense kinematics

- large-x projectile (proton) on small-x target (proton or nucleus)



$$\hat{s} = (p+k)^2$$
$$\hat{t} = (p_2 - p)^2$$
$$\hat{u} = (p_1 - p)^2$$

Incoming partons' energy fractions:

$$\begin{aligned} x_1 &= \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \\ x_2 &= \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \end{aligned} \quad \xrightarrow{y_1, y_2 \gg 0} \quad \begin{aligned} x_1 &\sim 1 \\ x_2 &\ll 1 \end{aligned}$$

Gluon's transverse momentum (p_{1t}, p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos\Delta\phi$$

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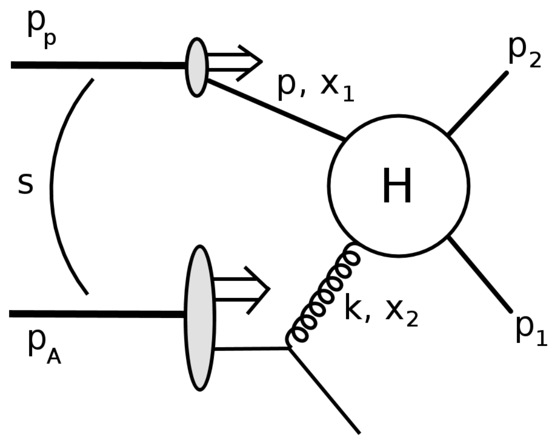

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- several momentum scales in the process

$|p_{1t}|, |p_{2t}| \gg Qs$ however, $|k_t|$ can be small or large

The back-to-back regime

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

- a factorization can be established in the small *x* limit, for nearly back-to-back di-jets

Dominguez, CM, Xiao and Yuan (2011)

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag\to cd} \Phi^{(i)}_{ag\to cd} \frac{1}{1+\delta_{cd}}$$

but it involves six unintegrated gluon distributions $\Phi^{(i)}_{ag\to cd}(x_2,k_t^2)$ (2 per channel)

and their associated hard matrix elements $K^{(i)}_{ag\to cd}$ are on-shell (i.e. $k_t = 0$)

this is the so-called Transverse Momentum Dependent (TMD) factorization formula

e.g. Bomhof, Mulders and Pijlman (2006)

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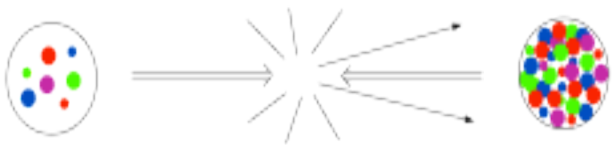
- only valid in asymmetric situations

Collins and Qiu (2007), Xiao and Yuan (2010)



does not apply with unintegrated parton densities for both colliding projectiles

TMD gluon distributions

- the naive operator definition is not gauge-invariant

$$\mathcal{F}_{g/A}(x_2,k_t) \overset{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\right|\mathrm{Tr}\left[F^{i-}\left(\xi^+,\boldsymbol{\xi}_t\right)F^{i-}(0)\right]\left|A\right\rangle$$

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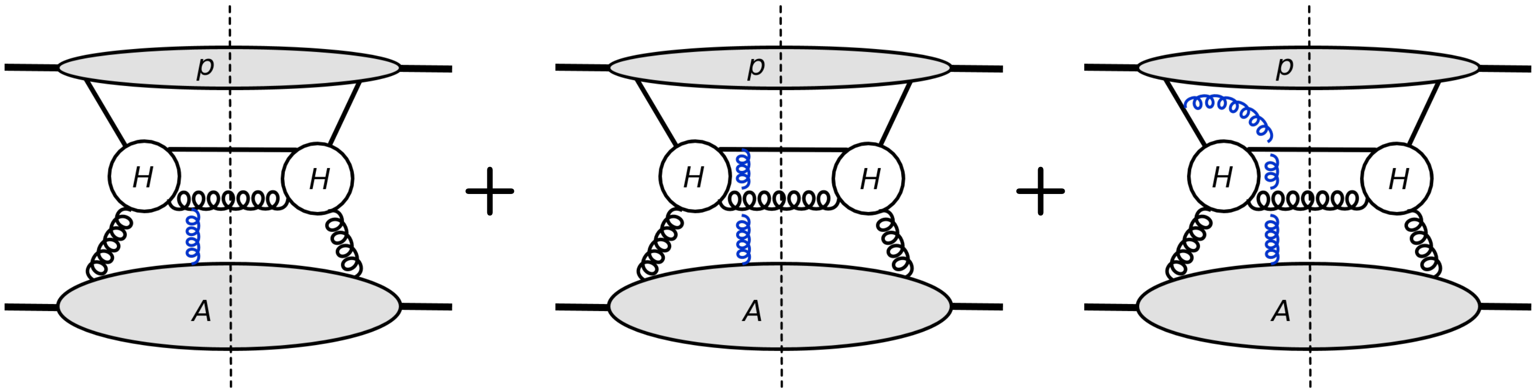
- a theoretically consistent definition requires to include more diagrams



$+$ similar diagrams with $2, 3, \ldots$ gluon exchanges

They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

Process-dependent TMDs

- the proper operator definition(s)

some gauge link $\mathcal{P}\exp\left[-ig\int_{\alpha}^{\beta} d\eta^{\mu} A^{a}(\eta) T^{a}\right]$

$$\mathcal{F}_{g/A}(x_2, k_t) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t\cdot\boldsymbol{\xi}_t} \left\langle A\right| \mathrm{Tr}\left[F^{i-}\left(\xi^+, \boldsymbol{\xi}_t\right) U_{[\xi,0]} F^{i-}(0)\right]\left|A\right\rangle$$

  - $U_{[\alpha,\beta]}$ renders gluon distribution gauge invariant

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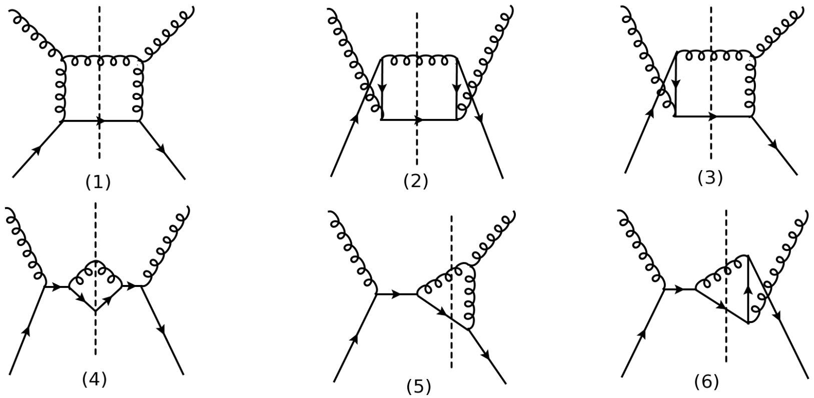
example for the $qg \to qg$ channel

each diagram generates a different gluon distribution



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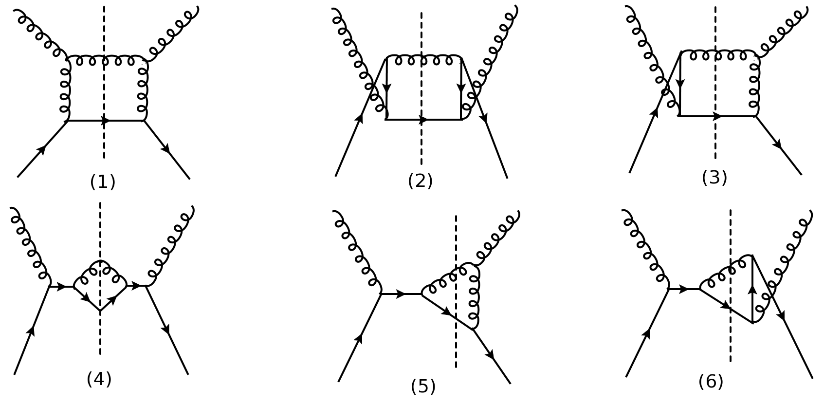

in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution) up to $\mathcal{O}\left(Q_s^2/k_t^2\right)$ corrections, and a single gluon distribution is sufficient

The six TMD gluon distributions

- correspond to a different gauge-link structure

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several paths are possible for the gauge links

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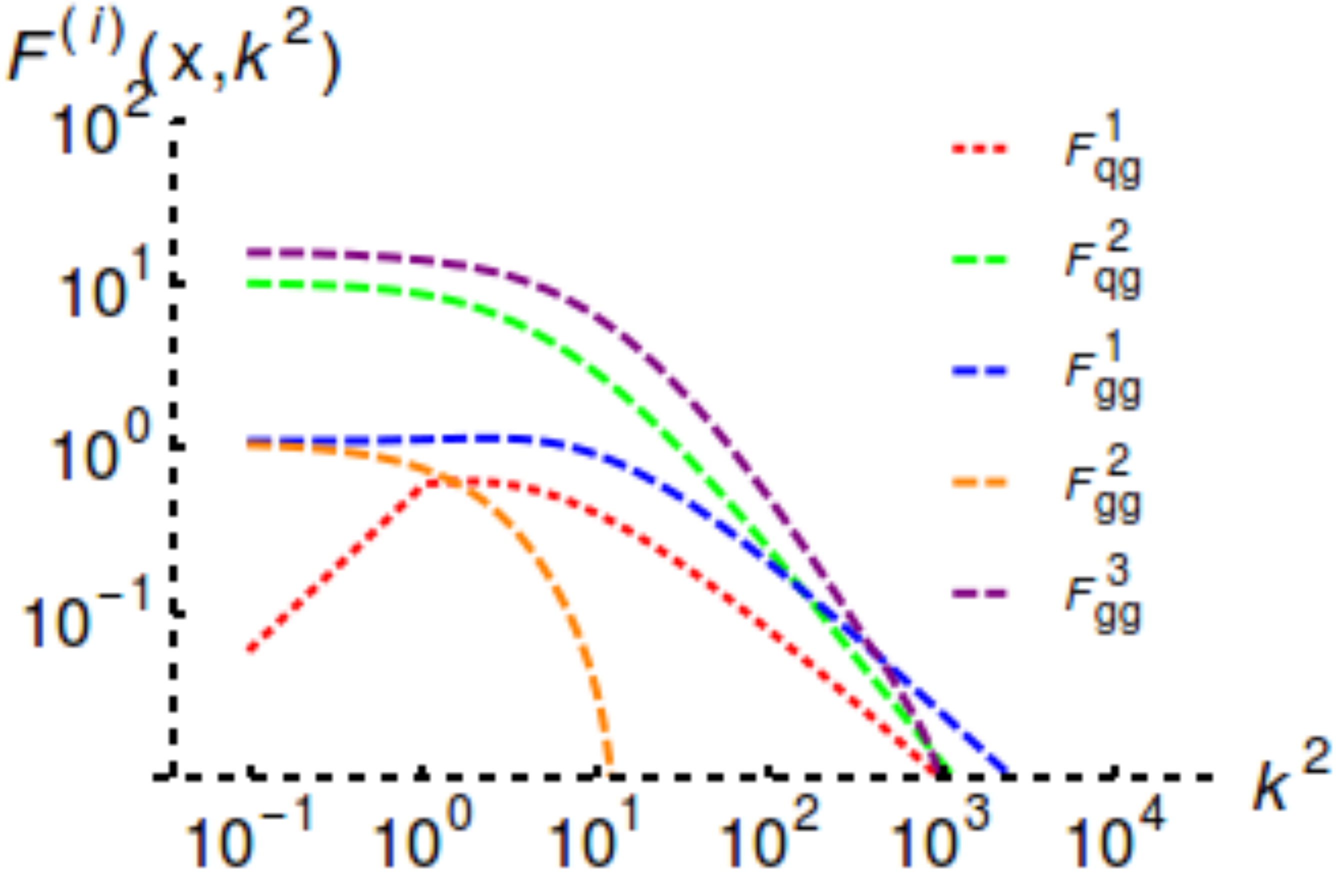
- they are independent and in general they all should be extracted from data

only one of them has the probabilistic interpretion
of the number density of gluons at small x_2

- in the Color Glass Condensate, (using some approximations), one can obtain relations between them

Some numerical results

the five gluon TMDs which survive in the large Nc limit



Combining both limits into a common factorization formula

Improved TMD factorization formula

$$|p_{1t}|, |p_{2t}| \gg Q_s$$

$$\frac{d\sigma^{pA\to \mathrm{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1x_2s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1,\mu^2) \sum_{i=1}^{2} K^{(i)}_{ag^*\to cd} \Phi^{(i)}_{ag\to cd} \frac{1}{1+\delta_{cd}}$$

practical for for an arbitrary $|k_t|$ value

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- in the back-to-back limit $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

$K^{(i)}_{ag^*\to cd} \to K^{(i)}_{ag\to cd}$ and the TMD formula is recovered

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- in the dilute limit $|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$ then $\Phi^{(i)}_{ag\to cd} \to \mathcal{F}_{a/g}/\pi$

  and since $\sum_{i=1}^{2} K^{(i)}_{ag^*\to cd} = |\overline{M_{ag^*\to cd}}|^2$ the HEF formula is recovered

The six 2-to-2 off-shell hard factors

- they can be computed in two independent ways:

  using Feynman diagrams, and using color-ordered amplitudes

i	1	2
$K^{(i)}_{gg^*\to gg}$	$\frac{N_c}{C_F}\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{N_c}{2C_F}\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K^{(i)}_{gg^*\to q\bar{q}}$	$\frac{1}{2N_c}\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}}$	$\frac{1}{4N_c^2C_F}\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}}$
$K^{(i)}_{qg^*\to qg}$	$-\frac{\bar{u}\left(\bar{s}^2+\bar{u}^2\right)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\,\bar{u}\hat{u}}\right)$	$-\frac{C_F}{N_c}\frac{\bar{s}\left(\bar{s}^2+\bar{u}^2\right)}{\bar{t}\hat{t}\hat{u}}$

$\hat{s},\hat{t},\hat{u}$ are the Mandelstam variables and $\bar{s},\bar{t},\bar{u}=\hat{s},\hat{t},\hat{u}(k_t=0)$

Conclusions I

- at leading order, for inclusive enough processes (like SIDIS) where factorization is “simple”:

  TMD factorization (valid at large Q^2) and k_T factorization (valid at small *x*)

  are consistent with each other in the overlapping domain of validity

- at leading order, for processes where factorization is more involved (like forward di-jets):

  TMD factorization (with several sub-process-dependent TMDs) and saturation calculations (which no more consist of k_T-factorized expressions)

  are consistent with each other in the overlapping domain of validity

- the breaking of k_T factorization at small-x is expected, understood, and is not a problem in saturation calculations:

  a more involved factorization is used, with more a appropriate description of the parton content of the proton (in terms of classical fields)

Conclusions II

- some features of saturation calculations can be imported into the TMD framework in order to improve it

- for instance, in the case of forward di-jet production:

  several gluon TMDs (as opposed to a single one) are crucial in the TMD factorization regime $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ which corresponds to nearly back-to-back jets, but the off-shellness of the small-x gluon is missing

  this off-shellness is crucial to recover the HEF regime $|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$ and can be restored

- also, the different TMDs can be related to each other at small-x

  one can use information extracted from one process to predict another

- the next step now is to connect the x evolution of u-pdfs and the scale evolution of TMD-pdfs $\longrightarrow$ Bowen Xiao's talk now