Gluon TMD studies at RHIC

Daniël Boer

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university of
groningen

Outline

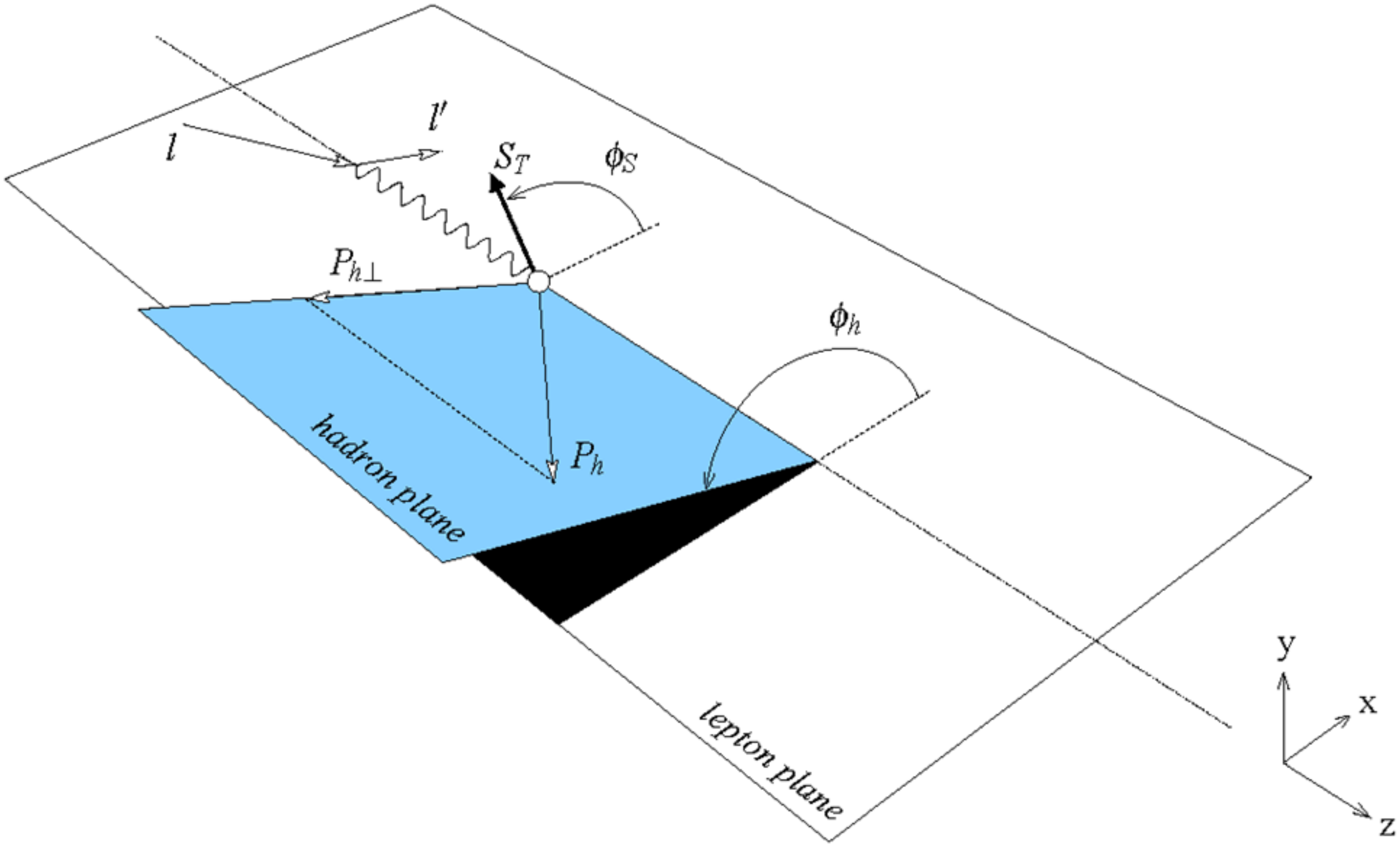
- Gluon TMDs
- Linearly polarized gluons in unpolarized protons
- Gluon Sivers effect
- Inherent process dependence
- Small x: “a tale of two gluon distribution functions”

Gluons TMDs

Typical TMD processes

Semi-inclusive DIS is a process sensitive to the transverse momentum of quarks

$$e\,p \to e'\,h\,X$$



D-meson pair production is sensitive to transverse momentum of gluons

$$e\,p \to e'\,D\,\bar{D}\,X$$

in the back-to-back correlation limit (φ around π)

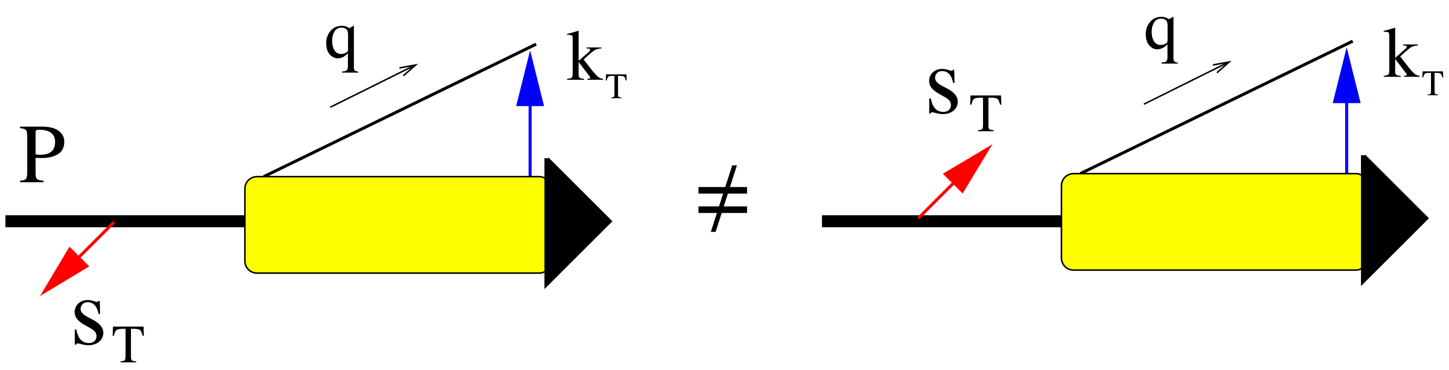
Transverse Momentum of Quarks

TMD = *transverse momentum dependent* parton distribution

Because of the additional k_T dependence there are more TMDs than collinear pdfs

The transverse momentum dependence can be correlated with the spin, e.g.

D. Sivers ('90):



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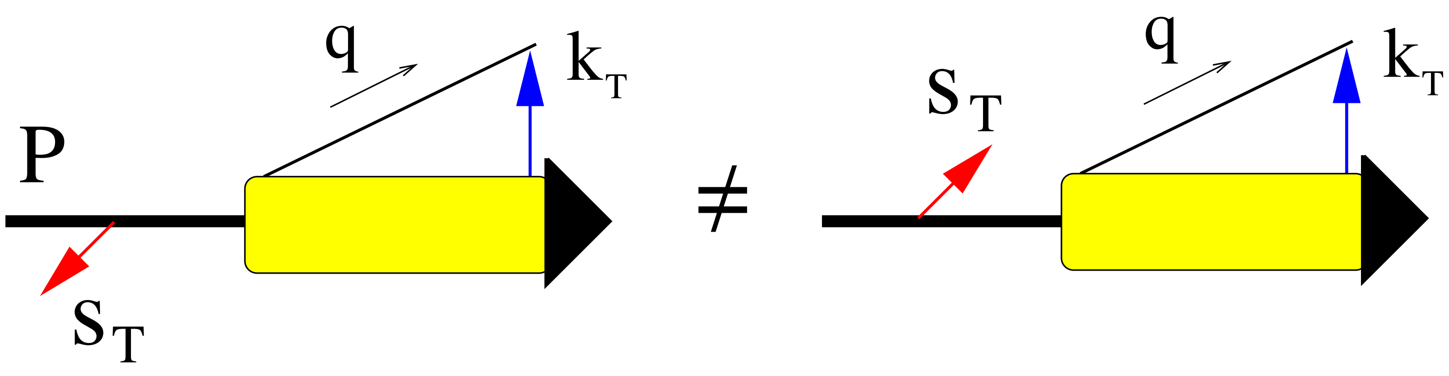

$k_T \times S_T$

Quark correlator:

Sivers function

$$\Phi(x, \boldsymbol{k}_T) = \frac{M}{2} \Big\{ f_1(x, \boldsymbol{k}_T^2) \frac{\not{P}}{M} + \boxed{f_{1T}^{\perp}(x, \boldsymbol{k}_T^2)} \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k_T^\rho S_T^\sigma}{M^2} + g_{1s}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{P}}{M}$$

$$+ h_{1T}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{S}_T \not{P}}{M} + h_{1s}^{\perp}(x, \boldsymbol{k}_T^2) \frac{\gamma_5 \not{k}_T \not{P}}{M^2} + h_1^{\perp}(x, \boldsymbol{k}_T^2) \frac{i \not{k}_T \not{P}}{M^2} \Big\}$$

[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98]

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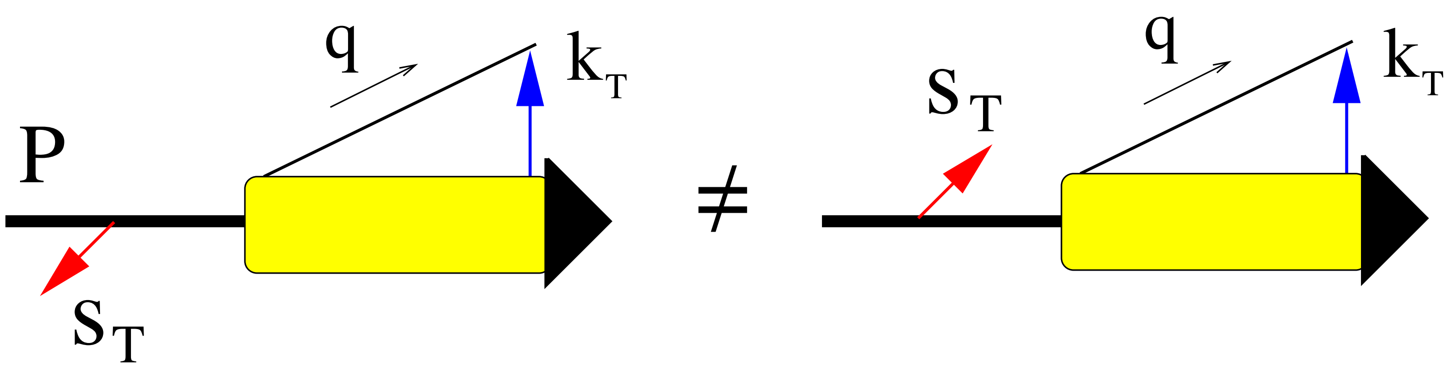

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k_T-odd

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[Ralston, Soper '79; Sivers '90; Collins '93; Kotzinian '95; Mulders, Tangerman '95; D.B., Mulders '98]

Transverse Momentum of Gluons

Idem for the gluon correlator:

$$\Gamma^{\mu\nu;\rho\sigma}(k;P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{i\,k\cdot\xi}\langle P,S|F^{\mu\nu}(0)\,\mathcal{U}(0,\xi)\,F^{\rho\sigma}(\xi)|P,S\rangle$$

$$\begin{aligned}
\Gamma_2^{\alpha\beta}(x,\boldsymbol{k}_T) &= \int dk^-\;\Gamma^{+\alpha;+\beta}(k;P,S) \\
&= \frac{x\,P^+}{2}\Bigg(-g_T^{\alpha\beta}\,G(x,\boldsymbol{k}_T) - g_T^{\alpha\beta}\,\frac{\epsilon_T^{ij}\,k_{Ti}S_{T\,j}}{M}\,G_T(x,\boldsymbol{k}_T) \\
&\qquad + \left(k_T^\alpha k_T^\beta + \frac{1}{2}\,g_T^{\alpha\beta}\,\boldsymbol{k}_T^2\right)\frac{H^\perp(x,\boldsymbol{k}_T)}{M^2} \\
&\qquad - i\,\epsilon_T^{\alpha\beta}\left[\lambda\,\Delta G_L(x,\boldsymbol{k}_T) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}\Delta G_T(x,\boldsymbol{k}_T)\right] \\
&\qquad - \frac{k_T^{\{\alpha}\epsilon_T^{\beta\}i}k_{Ti}}{2M^2}\left[\lambda\,\Delta H_L^\perp(x,\boldsymbol{k}_T) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{S}_T}{M}\Delta H_T^\perp(x,\boldsymbol{k}_T)\right] \\
&\qquad - \frac{k_T^{\{\alpha}\epsilon_T^{\beta\}i}S_{T\,i} + S_T^{\{\alpha}\epsilon_T^{\beta\}i}k_{Ti}}{4M}\left[\Delta H_T(x,\boldsymbol{k}_T) - \Delta H_T^{\perp(1)}(x,\boldsymbol{k}_T)\right]\Bigg)
\end{aligned}$$

[Mulders, Rodrigues '01]

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\end{aligned}$$

gluon Sivers function

unpolarized gluon distribution function

linearly polarized gluon distribution

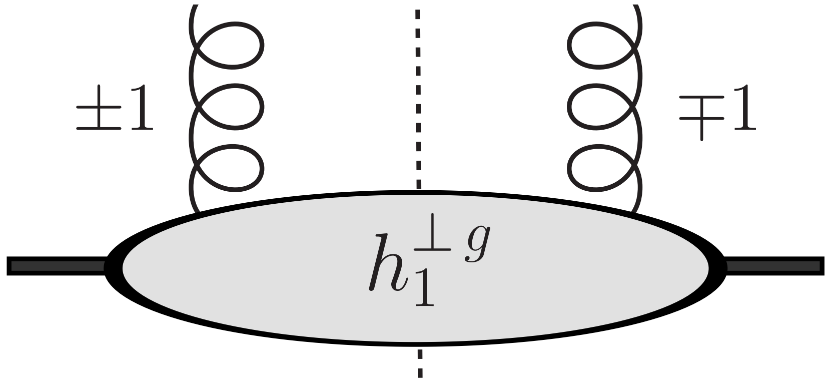
[Mulders, Rodrigues '01]

Linearly polarized gluons in unpolarized hadrons

Gluon polarization inside unpolarized protons

Linearly polarized gluons can exist in **unpolarized** hadrons

[Mulders, Rodrigues, 2001]



an interference between ±1 helicity gluon states

It requires nonzero transverse momentum: TMD

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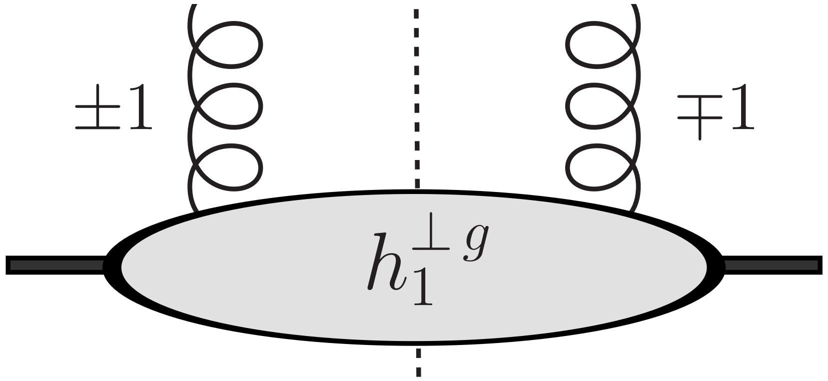

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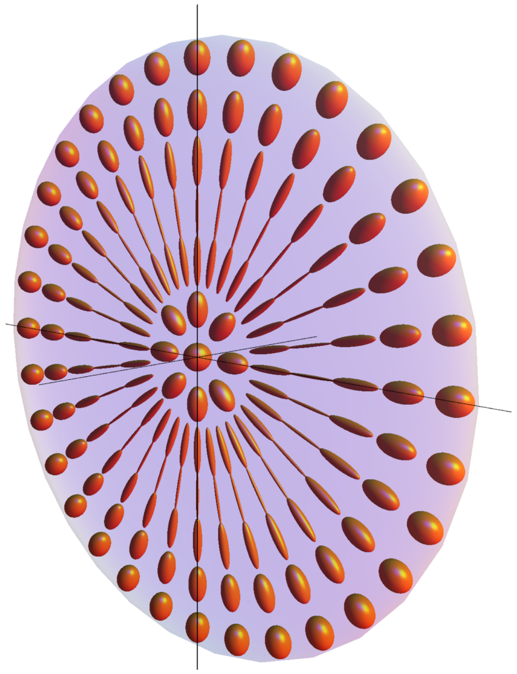
It requires nonzero transverse momentum: TMD

For $h_1^{\perp\, g} > 0$ gluons prefer to be polarized along k_T, with a $cos\ 2\phi$ distribution of linear polarization around it, where $\phi = \angle(k_T, \varepsilon_T)$



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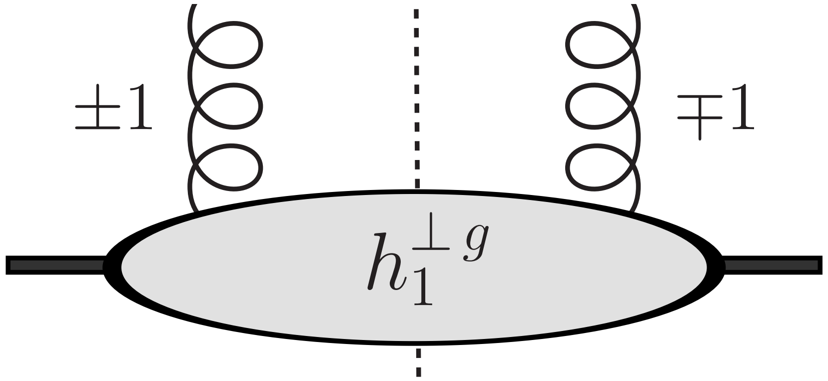

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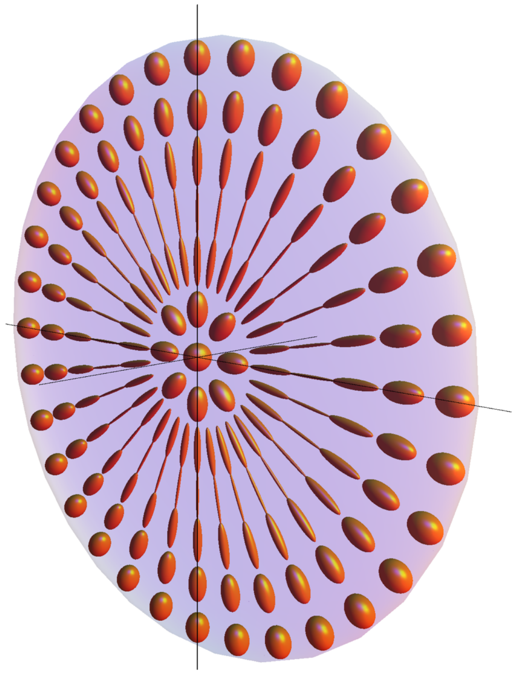


This TMD is k_T-even, chiral-even and T-even:

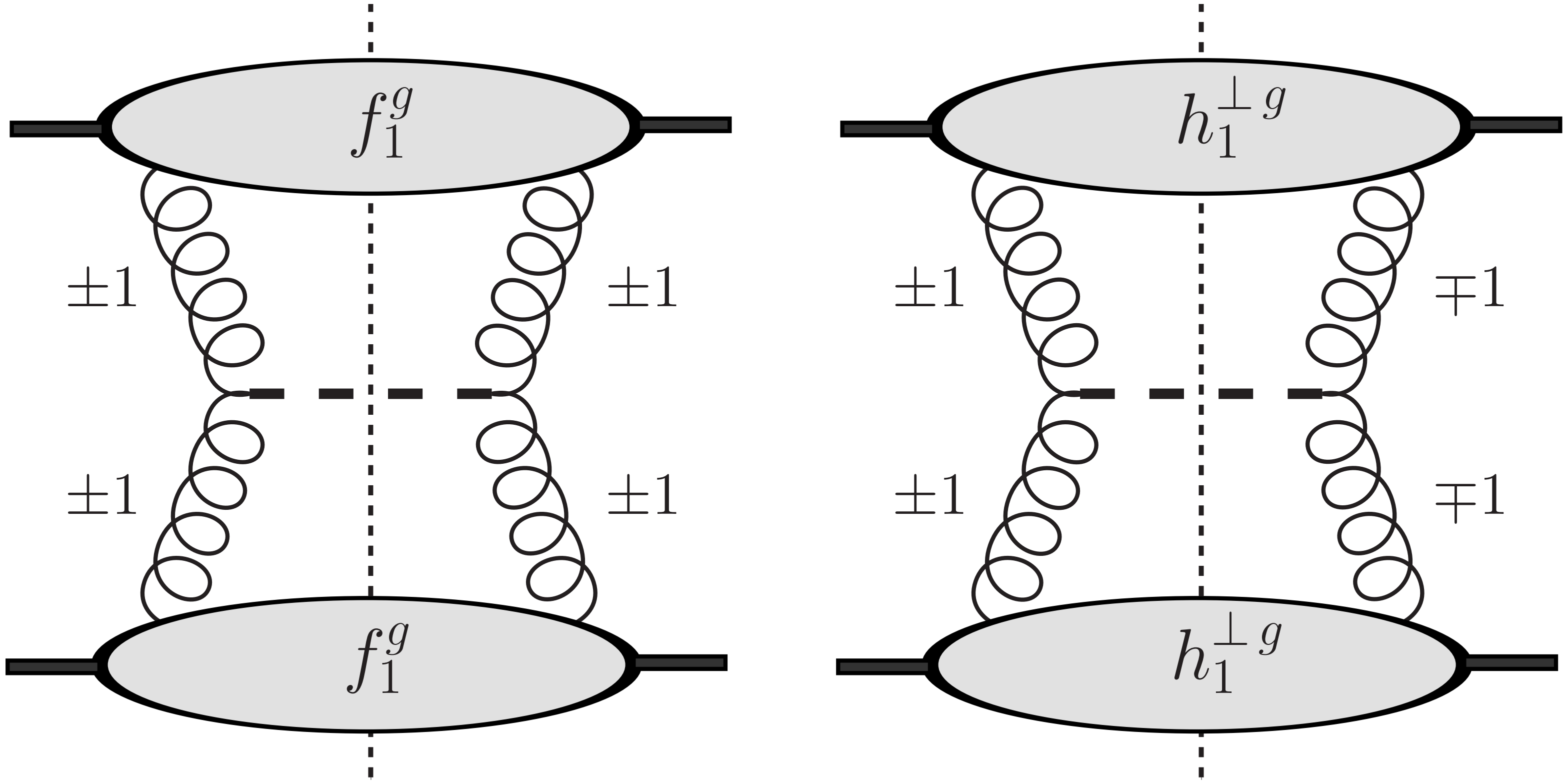
$$\Gamma_g^{\mu\nu}(x, \boldsymbol{k}_T) = \frac{n_\rho\, n_\sigma}{(p{\cdot}n)^2} \int \frac{d(\xi{\cdot}P)\, d^2\xi_T}{(2\pi)^3}\ e^{ik\cdot\xi}\, \langle P|\, \mathrm{Tr}\big[\, F^{\mu\rho}(0)\, F^{\nu\sigma}(\xi)\,\big]\, |P\rangle\, \big]_{\mathrm{LF}}$$

$$= -\frac{1}{2x}\left\{ g_T^{\mu\nu}\ f_1^g - \left(\frac{k_T^\mu k_T^\nu}{M^2} + g_T^{\mu\nu}\, \frac{\boldsymbol{k}_T^2}{2M^2} \right) h_1^{\perp g} \right\}$$

Sensitive processes

Linearly polarized gluons can be probed in:

- $pp \to \gamma\gamma X$ [Nadolsky, Balazs, Berger, C.-P. Yuan, 2007; Qiu, Schlegel, Vogelsang, 2011] RHIC
- $pp \to HX$ [Catani, Grazzini, 2010; Sun, Xiao, Yuan, 2011; D.B., Den Dunnen, Pisano, Schlegel, Vogelsang, 2012] LHC
- $pp \to [Q\bar{Q}]X$ with $J^{PC}=0^{\pm +}$ [D.B., Pisano, 2012] LHC
- $pp \to J/\psi\, \gamma\, X$ and $\Upsilon\, \gamma\, X$ [Den Dunnen, Lansberg, Pisano, Schlegel, 2014] LHC
- $pp \to (\pi\ \text{jet})\ X$ [D'Alesio, Murgia, Pisano, 2011] RHIC
- $pp \to H\ \text{jet}\ X$ [D.B., Pisano, 2015] LHC
- $ep \to e'\, Q\, \bar{Q}\, X$ and $ep \to e'\ \text{jet jet}\ X$ [D.B., Brodsky, Mulders, Pisano, 2010] EIC



Insensitive processes

Linearly polarized gluons **cannot** be probed in:

• $pp \to \gamma\ jet\ X$ [D.B, Mulders, Pisano, 2008]	Power suppressed
• $pp \to J/\psi\ X$ *or* $\Upsilon\ X$ [D.B., Pisano, 2012]	Landau-Yang theorem
• $pp \to Q\ \bar{Q}\ X$ [Akcakaya, Schäfer, Zhou, 2013]	No TMD factorization unless small x
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When color flow is in too many directions: *factorization breaking*

[Collins & J. Qiu '07; Collins '07; Rogers & Mulders '10]

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When color flow is in too many directions: *factorization breaking*

[Collins & J. Qiu '07; Collins '07; Rogers & Mulders '10]

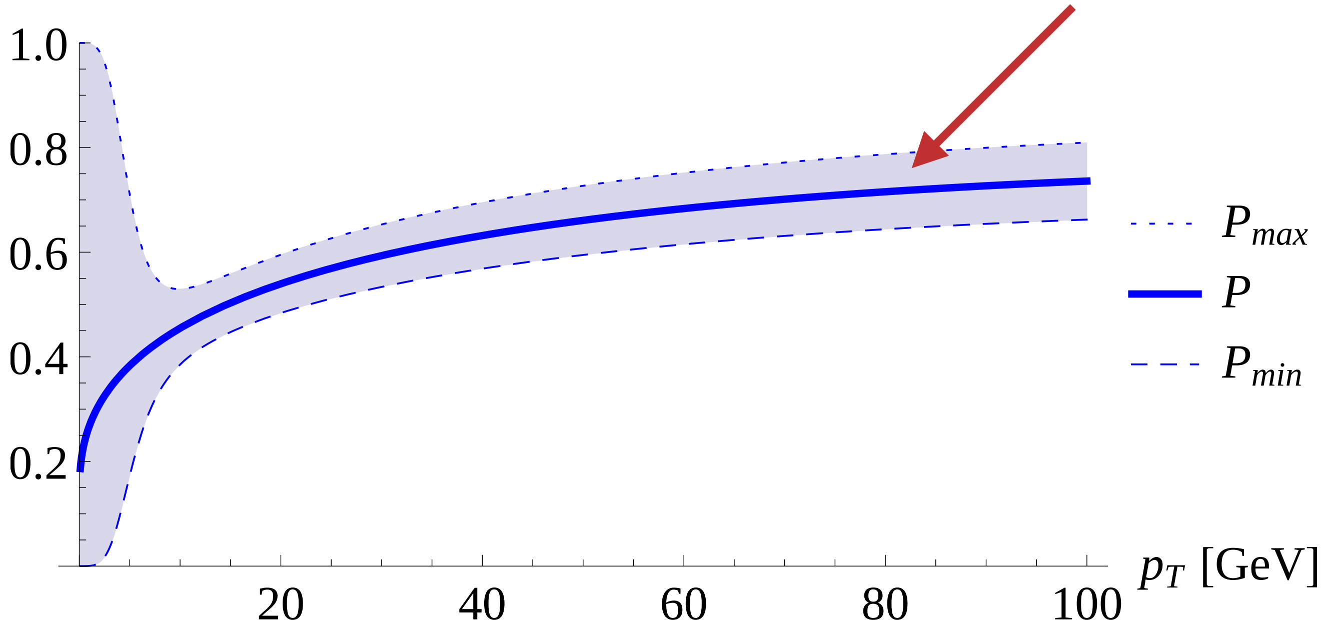
Such processes may become effectively TMD factorizing at small x
(hybrid factorization)

see e.g. Mueller, Xiao, Yuan, 2013

Size of the effect

Amount of linear gluon polarization:

D.B., Den Dunnen, Pisano, Schlegel ’13



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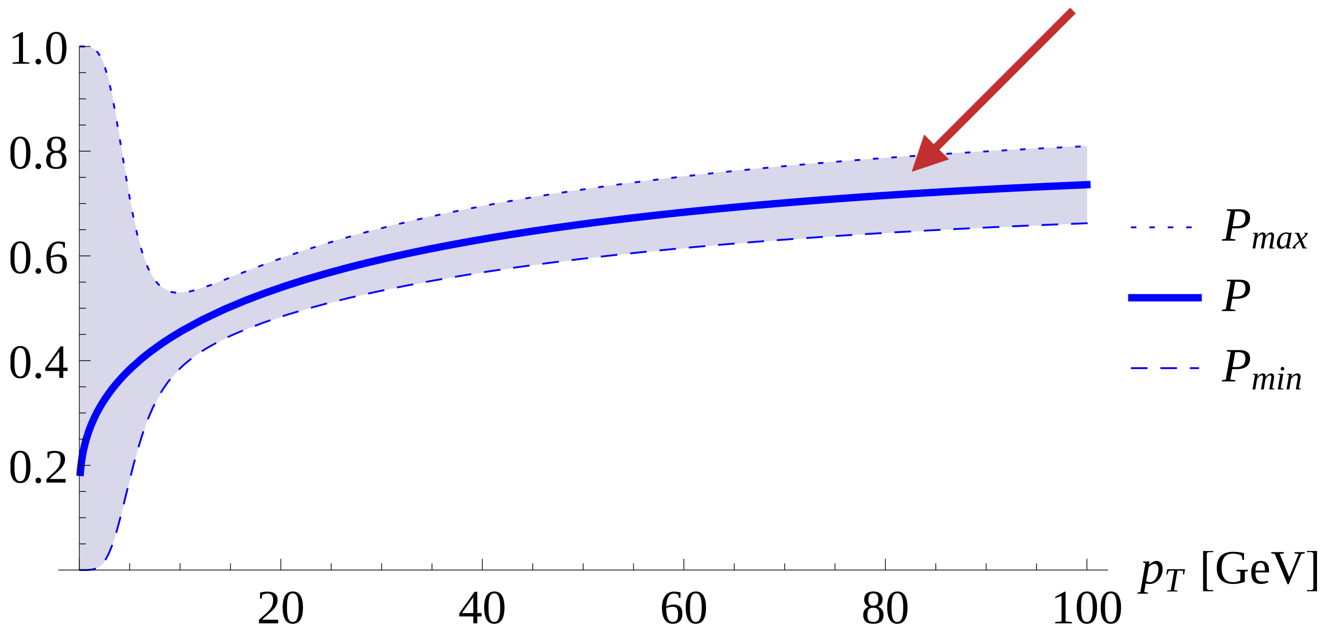

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What matters is the small-b behavior of the Fourier transformed TMD:

$$\tilde{f}_1^g(x, b^2; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s)$$

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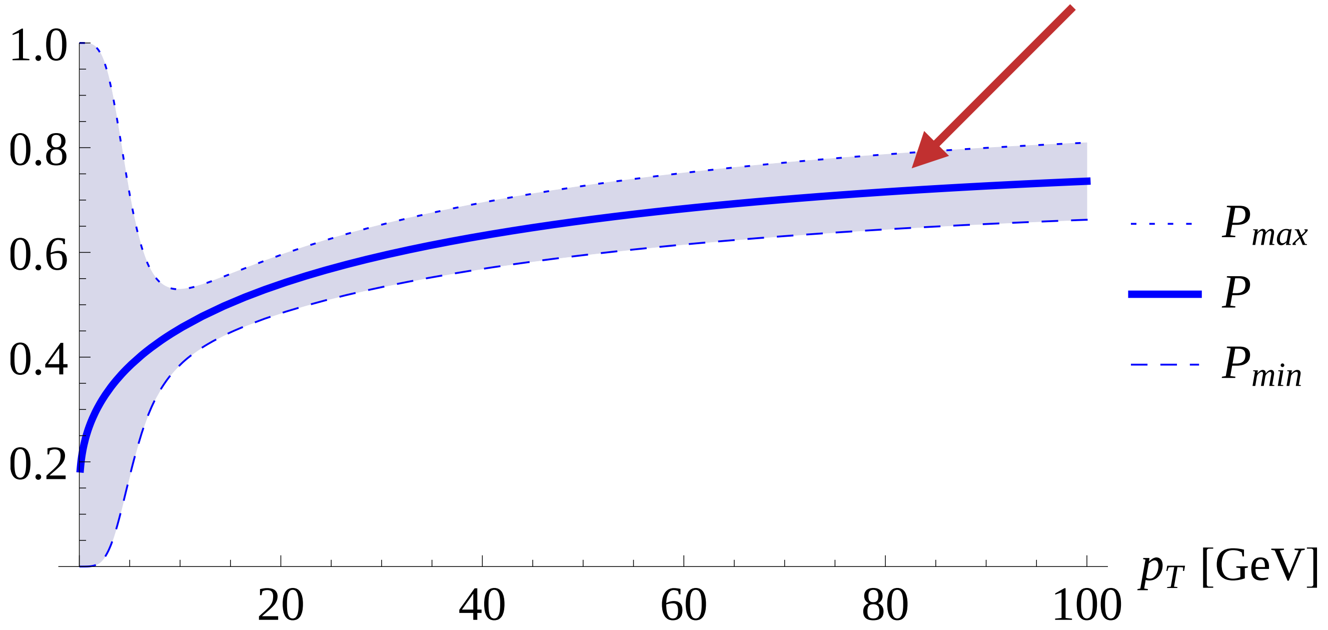

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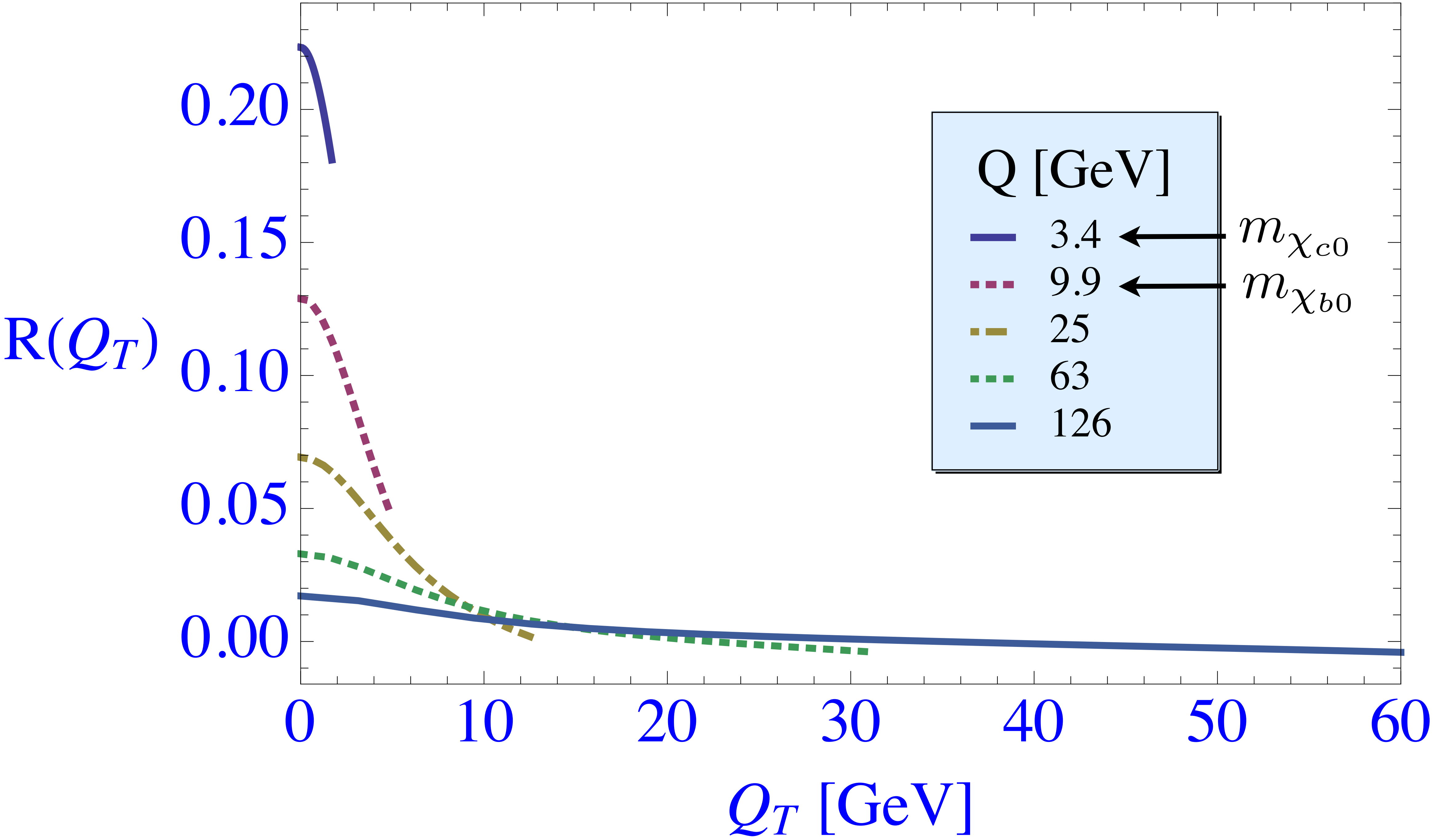
[Nadolsky, Balazs, Berger, C.-P. Yuan, 2007; Catani, Grazzini, 2010; P. Sun, B.-W. Xiao, F. Yuan, 2011]

The linear polarization starts at order α_s, leading to a **suppression** w.r.t. f_1

TMD evolution of CS scalar production



$x_A = x_B = Q/(8\text{TeV})$

MSTW08 LO gluon distribution

D.B. & den Dunnen, 2014

Conclusion: in Higgs production linear gluon polarization contributes at few % level

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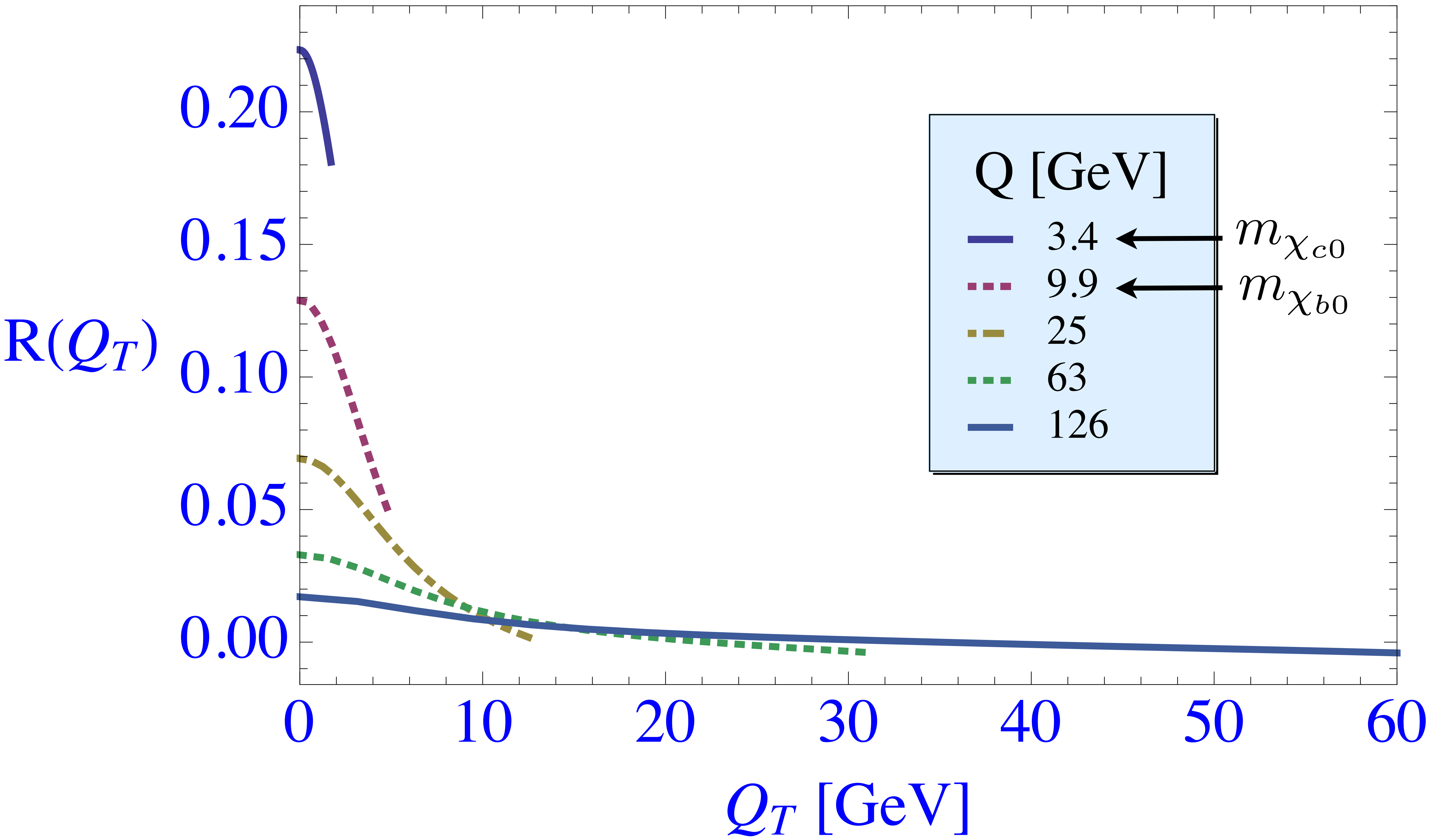

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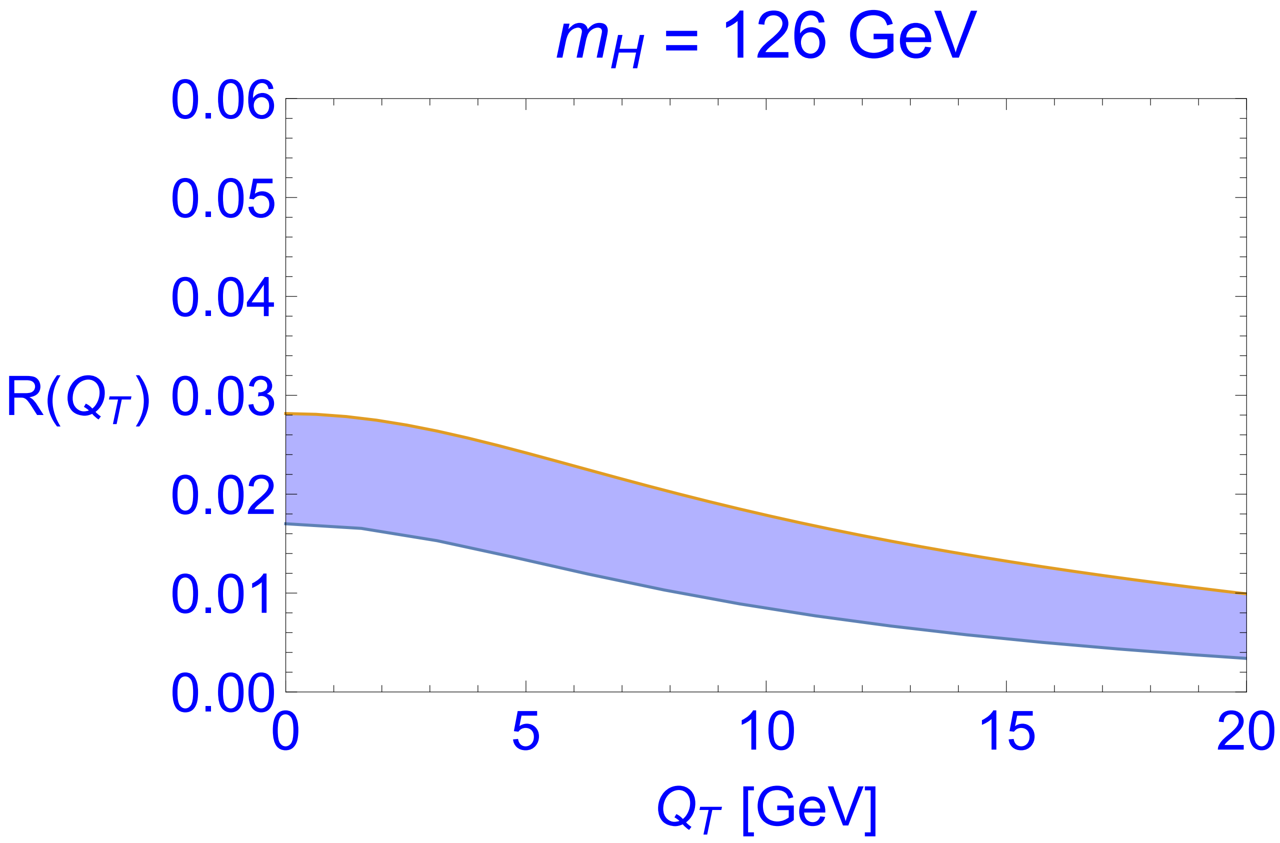
MSTW08 LO gluon distribution

Fall-off at Q_T=0: $Q^{-0.85}$

D.B. & den Dunnen, 2014

Conclusion: in Higgs production linear gluon polarization contributes at few % level

Range of predictions



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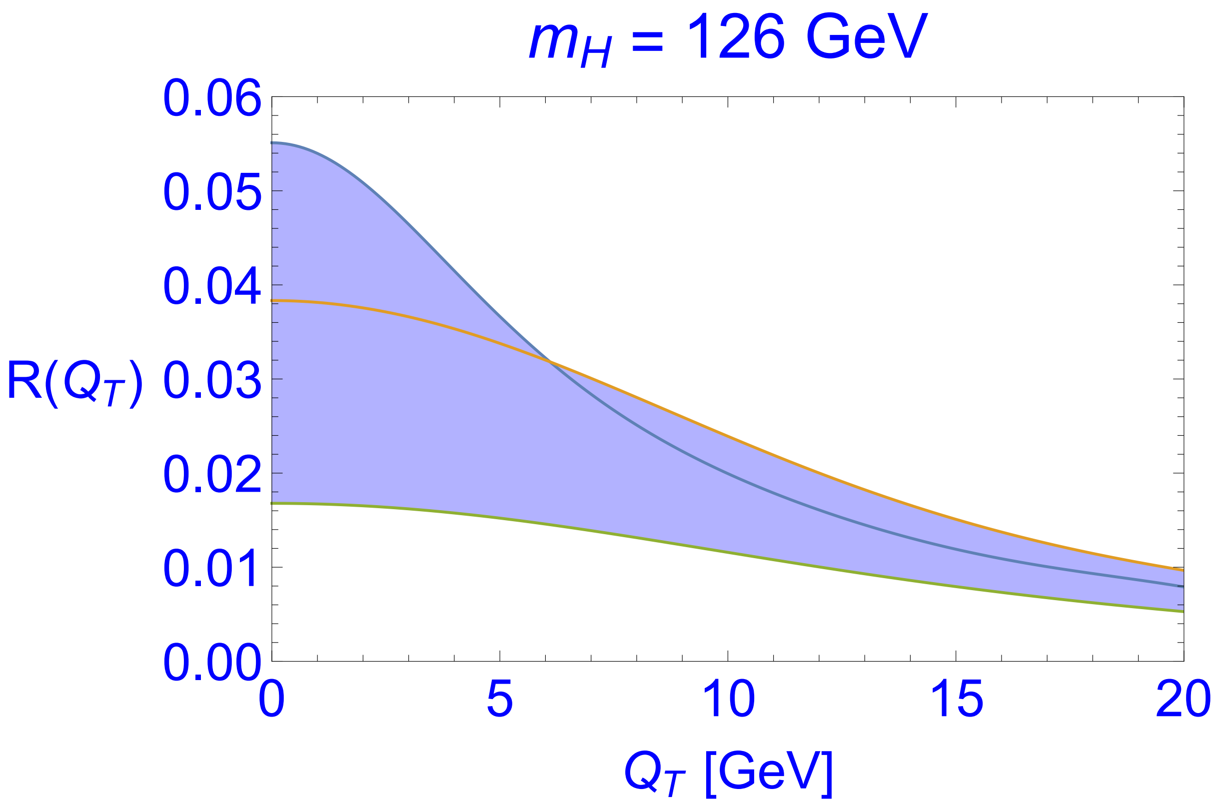

Echevarria, Kasemets, Mulders, Pisano, 2015

Left: variation of nonperturbative input for the TMDs and of the treatment of the very small b region (b<1/Q)

Right: variation of the nonperturbative Sudakov factor and the renormalization scale

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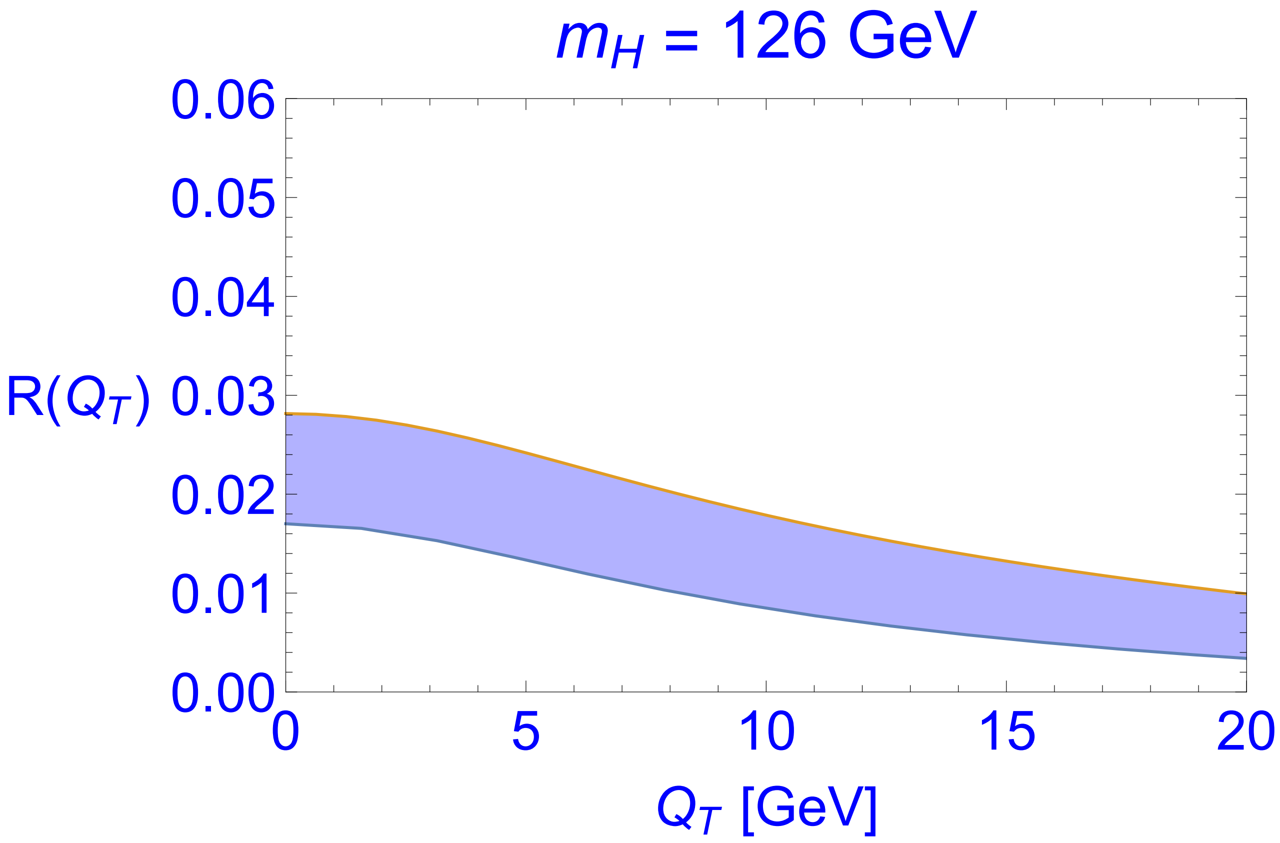

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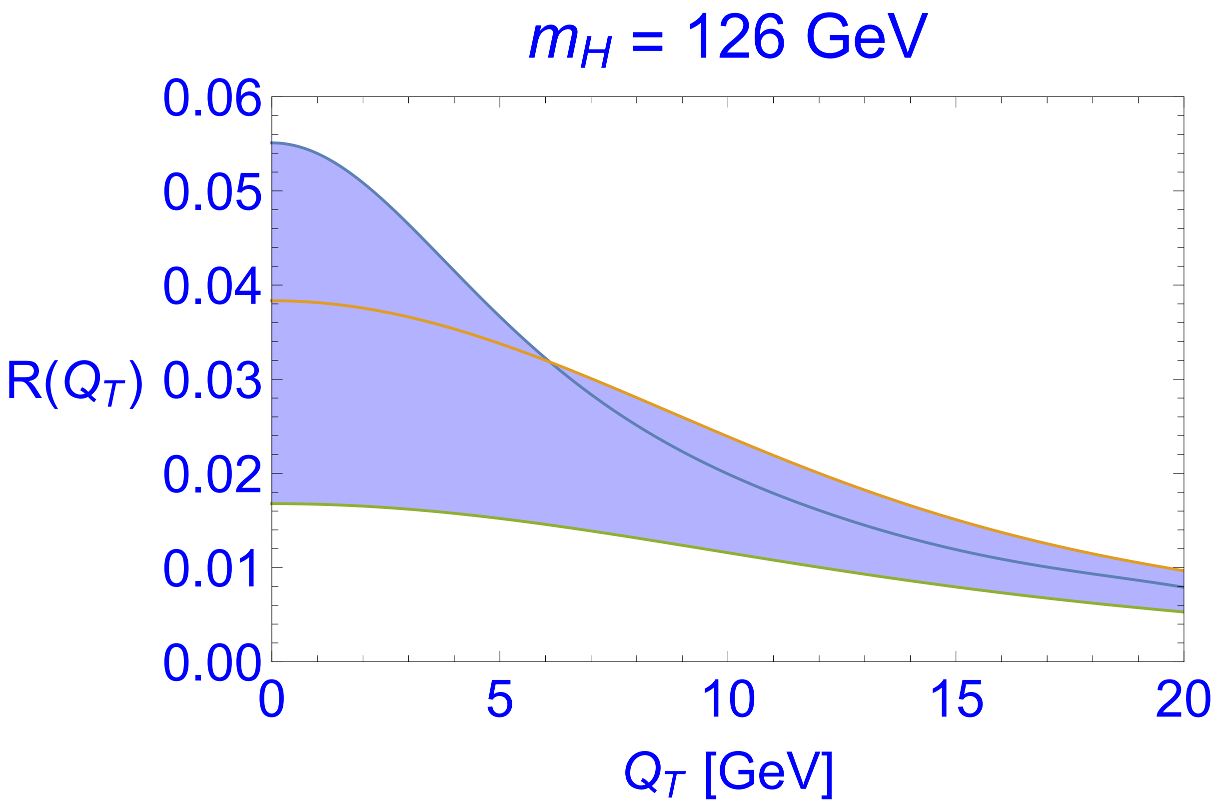

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Right: variation of the nonperturbative Sudakov factor and the renormalization scale

Conclusions:

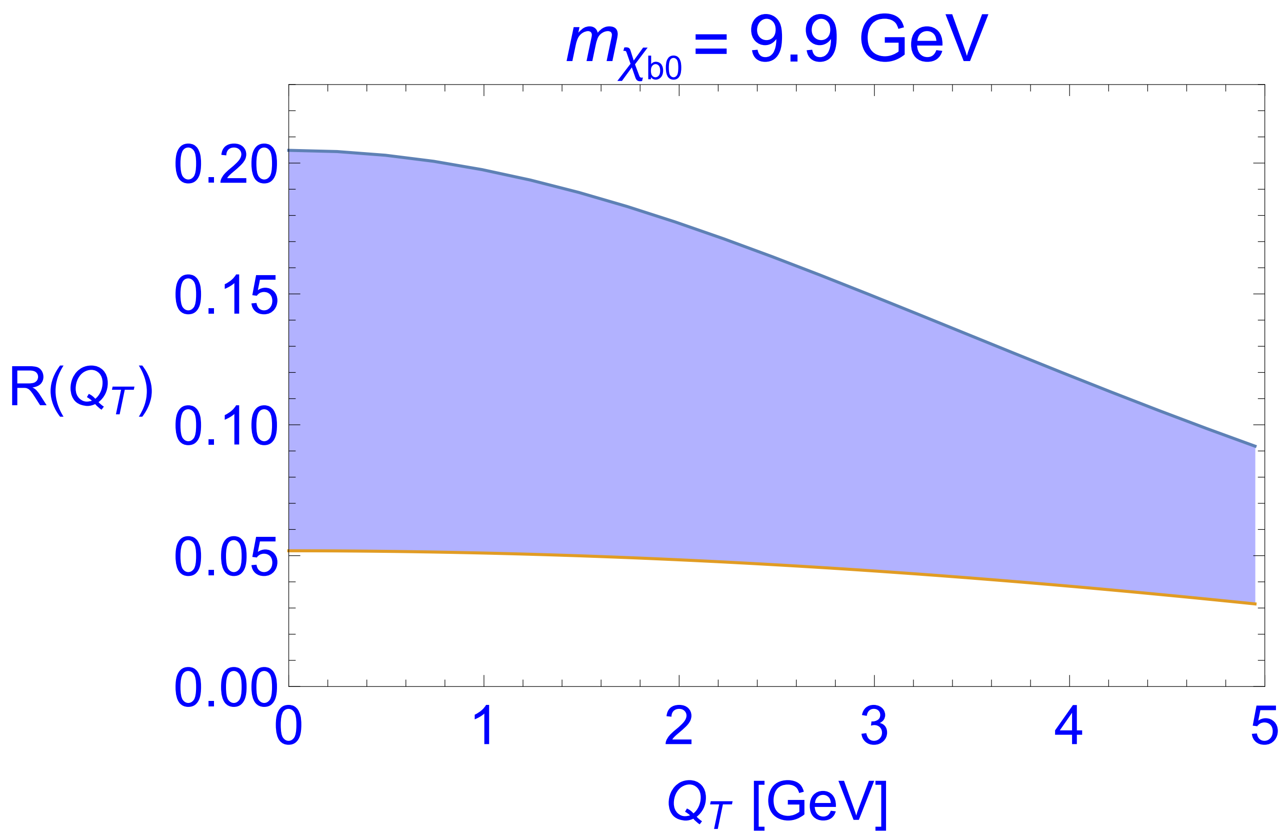
- effect of linear gluon polarization in Higgs production on the order of 2-5%
- extraction of $h_1^{\perp g}$ from Higgs production may be too challenging

Bottomonium production

More promising may be C-even (pseudo-)scalar quarkonium production

D.B., Pisano, 2012

The range of predictions for bottomonium production:



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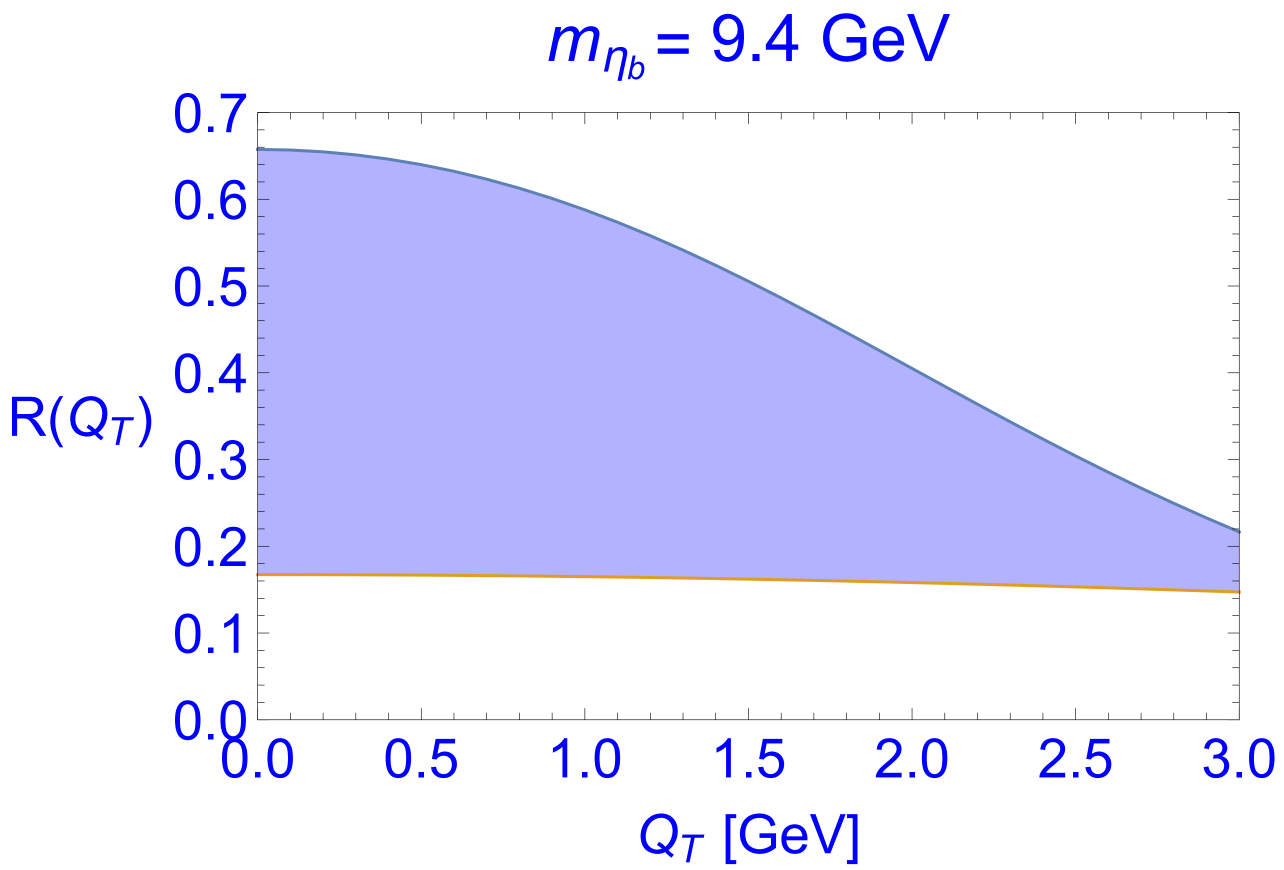


Echevarria, Kasemets, Mulders, Pisano, 2015

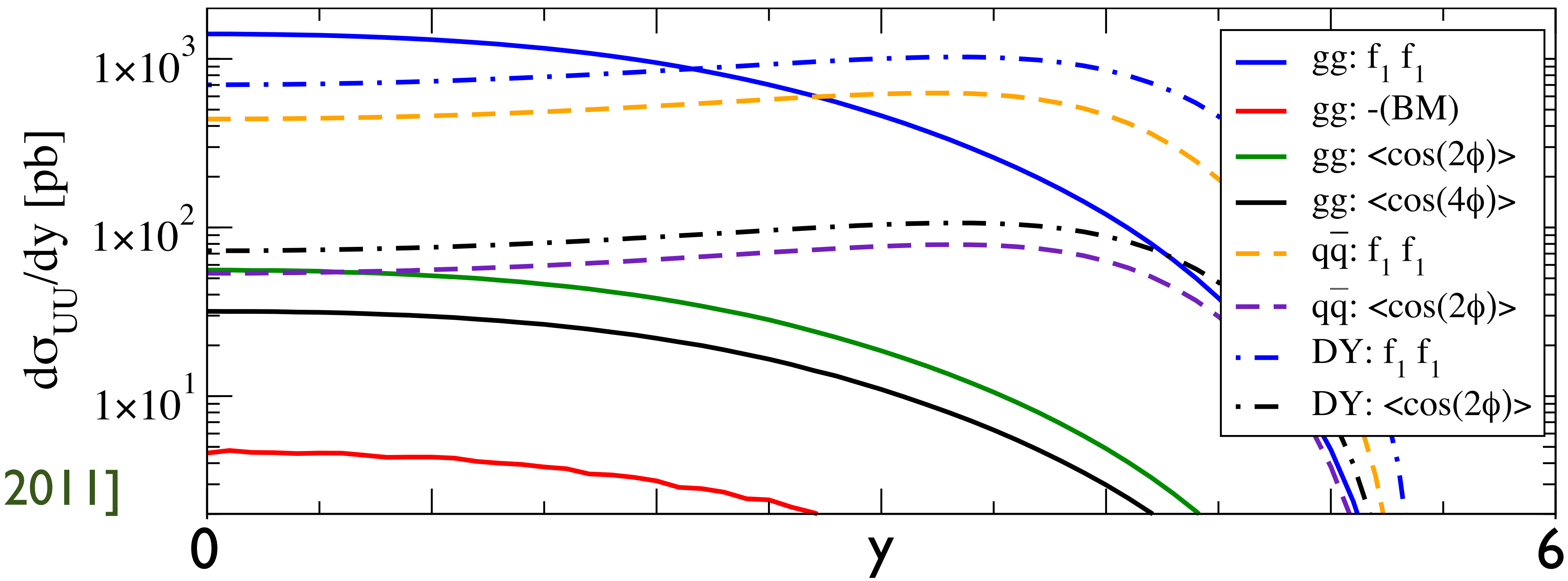
Conclusion: very large theoretical uncertainties in quarkonium production (more sensitive to unknown nonperturbative part than Higgs production), but larger effects

Angular distributions at RHIC

Percent level effects
at RHIC energies in:

$pp \rightarrow \gamma\gamma X$

[Qiu, Schlegel, Vogelsang, 2011]



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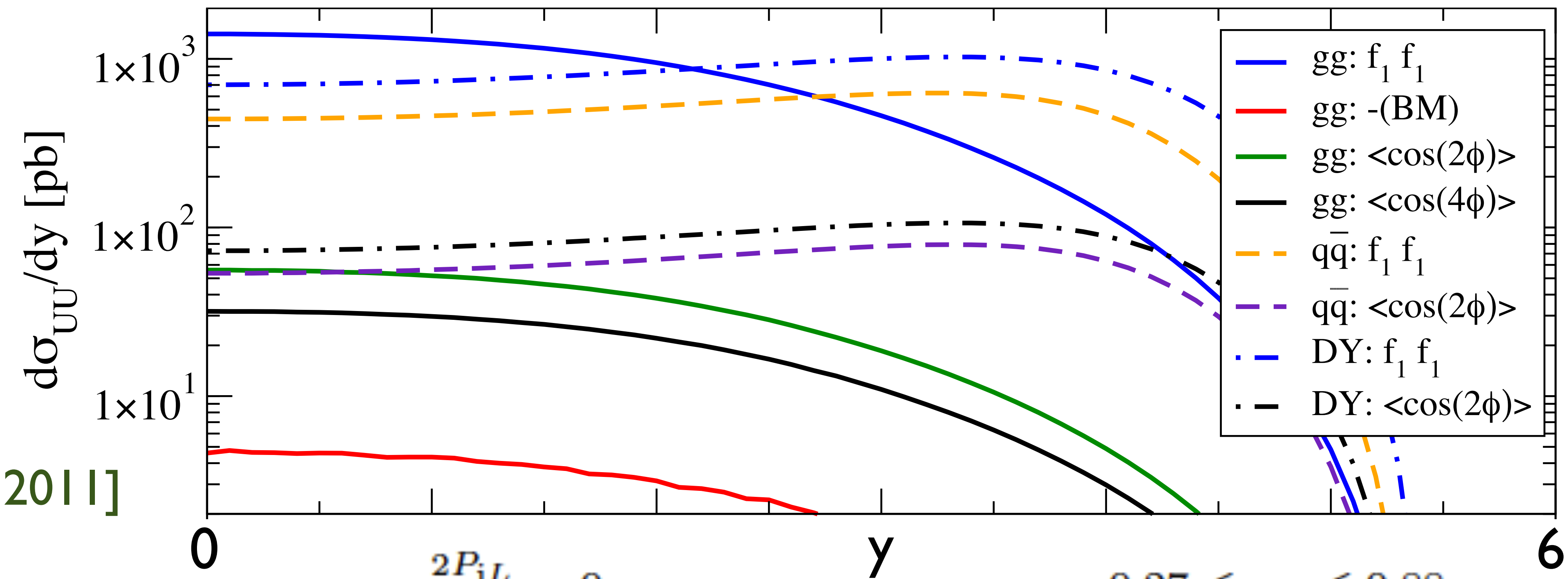
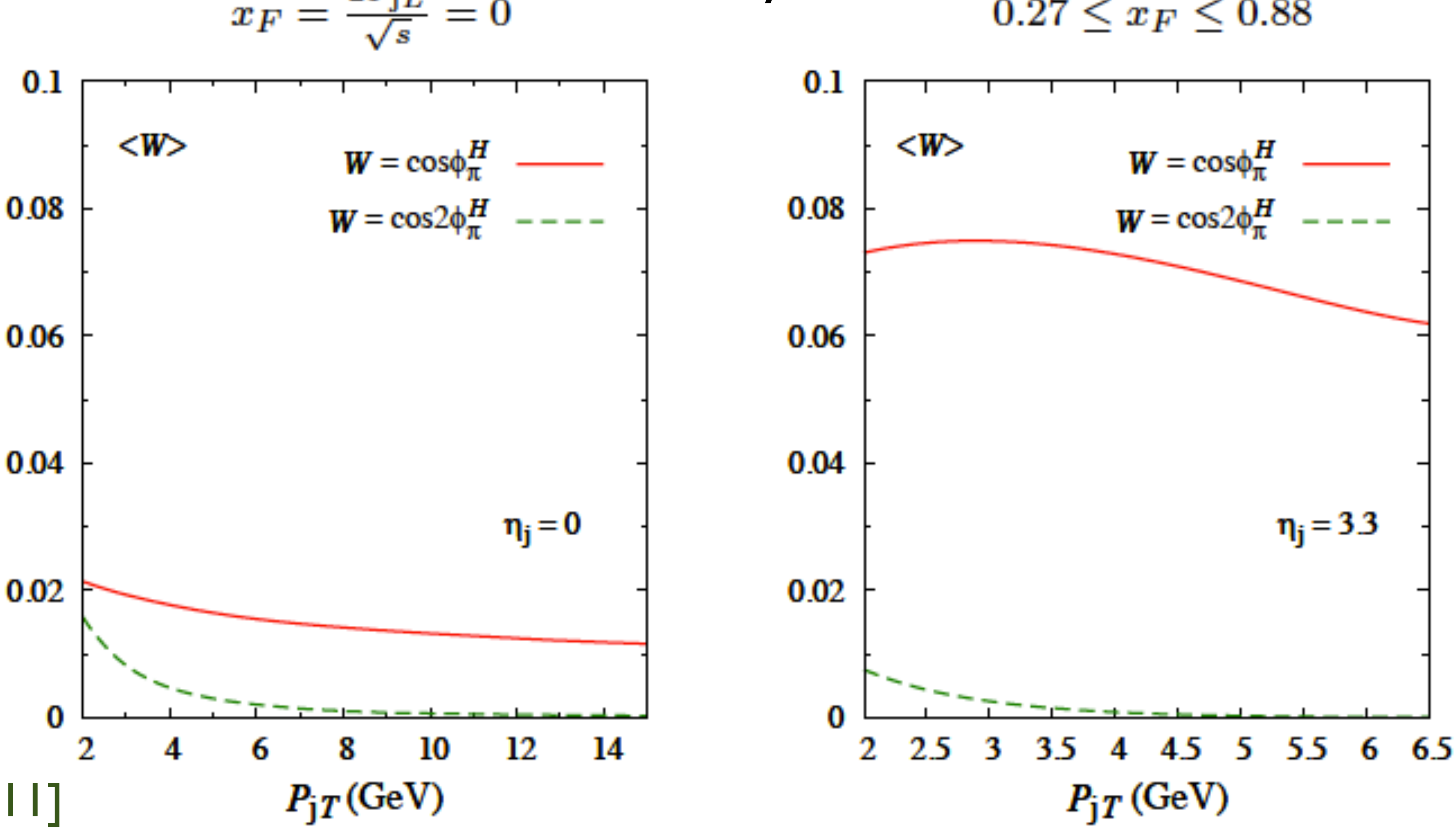


[Qiu, Schlegel, Vogelsang, 2011]

$pp \rightarrow$ (π *jet*) *X*

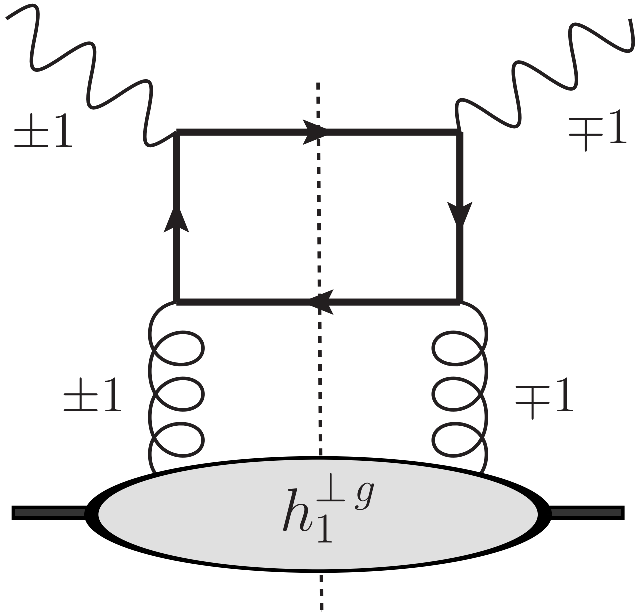


[D'Alesio, Murgia, Pisano, 2011]

Heavy quark electro-production

$h_1^{\perp g}$ can be probed in open charm and bottom quark electro-production
Here it appears by itself, so larger effects are expected and its sign can be probed

$$ep \to e' Q\bar{Q} X$$



Unlike Higgs production one needs to study angular distributions now, e.g. a $\cos 2(\phi_T - \phi_\perp)$ asymmetry, where $\phi_{T/\perp}$ are the angles of $K_\perp^Q \pm K_\perp^{\bar{Q}}$

[D.B., Brodsky, Mulders & Pisano, 2010]

Best measured at an Electron-Ion Collider

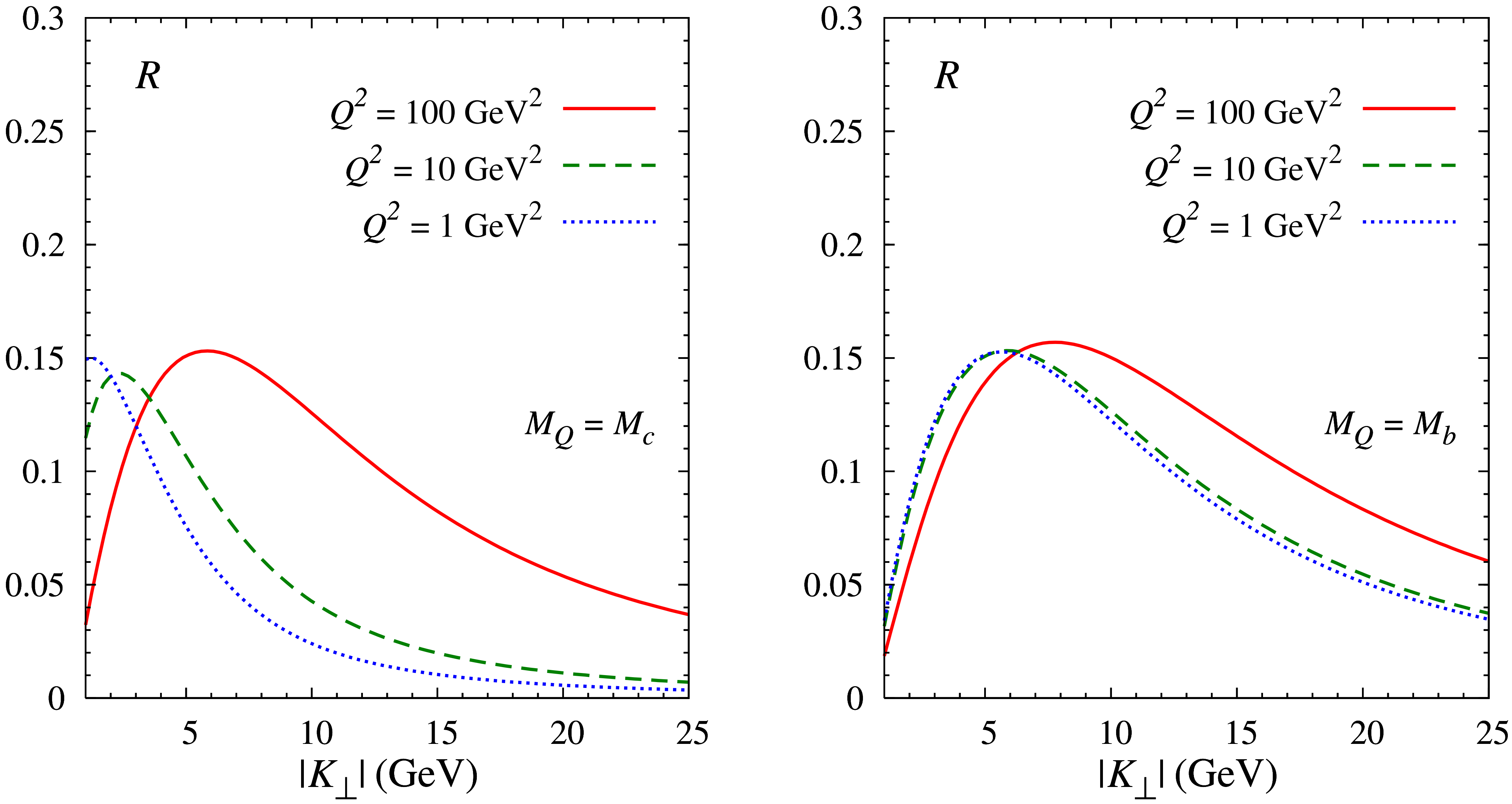
Because of problems with factorization in $\mathrm{pp} \to Q\bar{Q}X$

[Rogers & Mulders, 2010]

Maximum asymmetries in heavy quark production

$ep \to e'Q\bar{Q}X$

$R = \text{bound on } |\langle \cos 2(\phi_T - \phi_\perp) \rangle|$



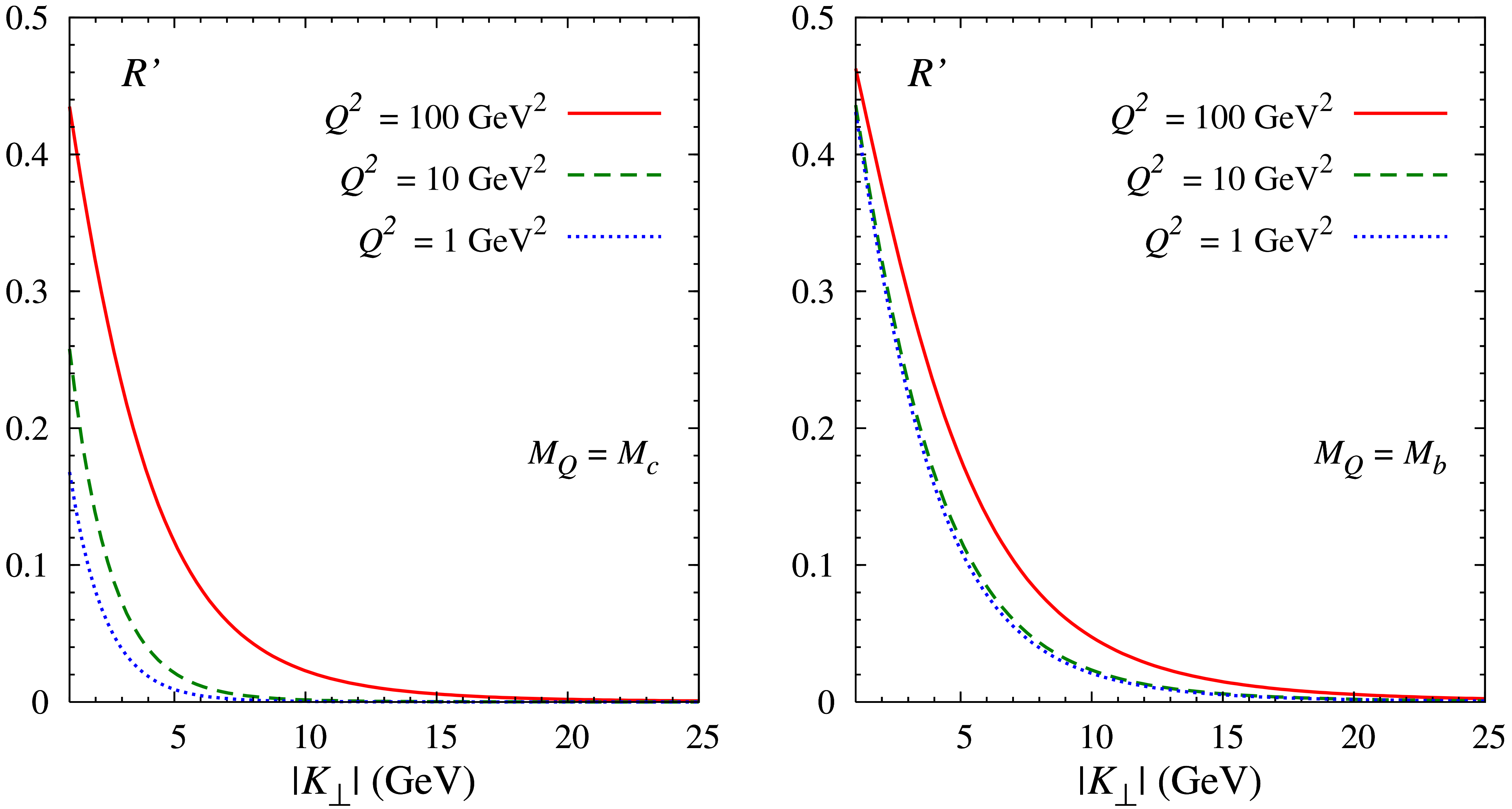
[Pisano, D.B., Brodsky, Buffing & Mulders, 2013]

Conclusion: asymmetries can be substantial
(Note that the maximum is to a large extent Q^2 and M_Q independent)

Maximum asymmetries in heavy quark production

There are also angular asymmetries w.r.t. the lepton scattering plane, which are mostly relevant at smaller |K⊥|

$ep \to e' Q\bar{Q}X$ $R' = \text{bound on } |\langle \cos 2(\phi_\ell - \phi_T)\rangle|$



[Pisano, D.B., Brodsky, Buffing & Mulders, 2013]

Gluon Sivers effect

Small gluon Sivers effect?

Arguments suggesting gluon Sivers is small:

- Burkardt sum rule already (approximately) satisfied by up and down quarks

$$\sum_{a=q,g} \int f_{1T}^{\perp(1)a}(x)\, dx = 0$$

- small Sivers asymmetry on deuteron target as found by COMPASS

[Brodsky & Gardner, 2006]

- 1/N$_c$ suppressed at not too small x (x~1/N$_c$), of order of the flavor singlet u+d

[Efremov, Goeke, Menzel, Metz, Schweitzer, 2005]

- small A$_N$ at midrapidity (small gluon Sivers function in the GPM)

[Anselmino, D'Alesio, Melis & Murgia, 2006; D’Alesio, Murgia, Pisano, 2015]

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[Anselmino, D'Alesio, Melis & Murgia, 2006; D'Alesio, Murgia, Pisano, 2015]

Note however that A_N in pion production is not a TMD factorizing process
COMPASS high-pT hadron pairs and other constraints are about fairly large x

Gluon Sivers function is constrained to be $\lesssim$ 30% of nonsinglet quark Sivers function
This is of natural size and will lead to smaller asymmetries, but not necessarily tiny

D.B., Lorcé, Pisano & Zhou, 2015

Gluon Sivers effect

Open charm and bottom quark electro-production is the 'golden channel' for the gluon Sivers function at EIC:

$$e\,p^{\uparrow} \to e'\,Q\bar{Q}\,X$$

For some model study, see D.B. Diehl, Milner *et al.*, arXiv:1108.1713

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One can also measure it in $p^{\uparrow}p$ and $p^{\uparrow}A$ collisions (RHIC, AFTER@LHC), in processes for which TMD factorization holds or may hold (CS dominance):

$$p^{\uparrow}\, p \to \gamma\, \mathrm{jet}\, X$$

$$p^{\uparrow}\, p \to \gamma\, \gamma\, X$$

Schmidt, Soffer, Yang, 2005
Bacchetta, Bomhof, D’Alesio, Mulders, Murgia, 2007
Qiu, Schlegel, Vogelsang, 2011

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Dunnen, Lansberg, Pisano, Schlegel, 2014
Lansberg *et al.*, 2014; Lansberg, Shao, 2015

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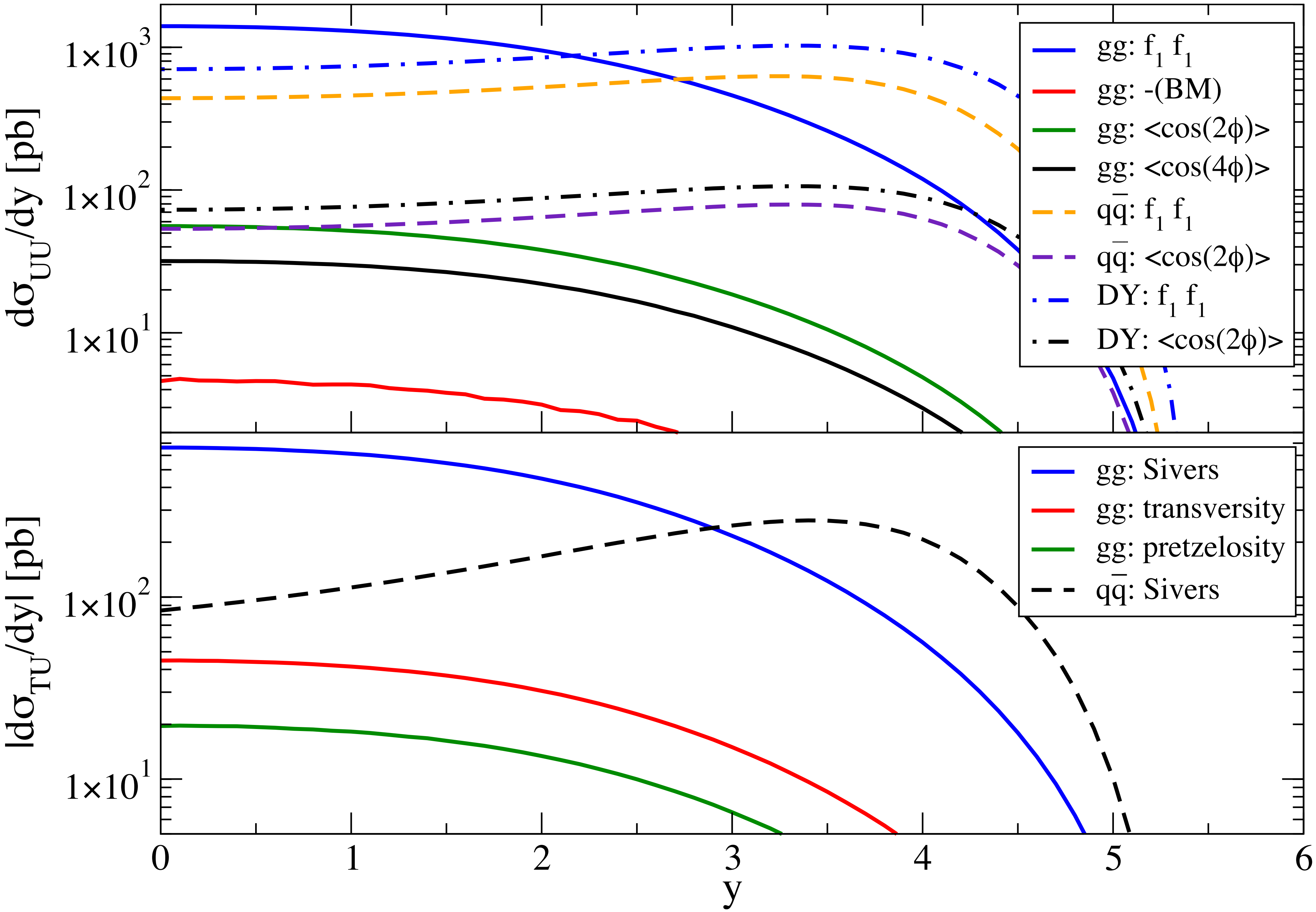
Such *pp* measurements are **complementary** to *ep*, as TMDs are process dependent

Photon pair production

$pp \to \gamma\gamma X$

$p^{\uparrow}p \to \gamma\gamma X$



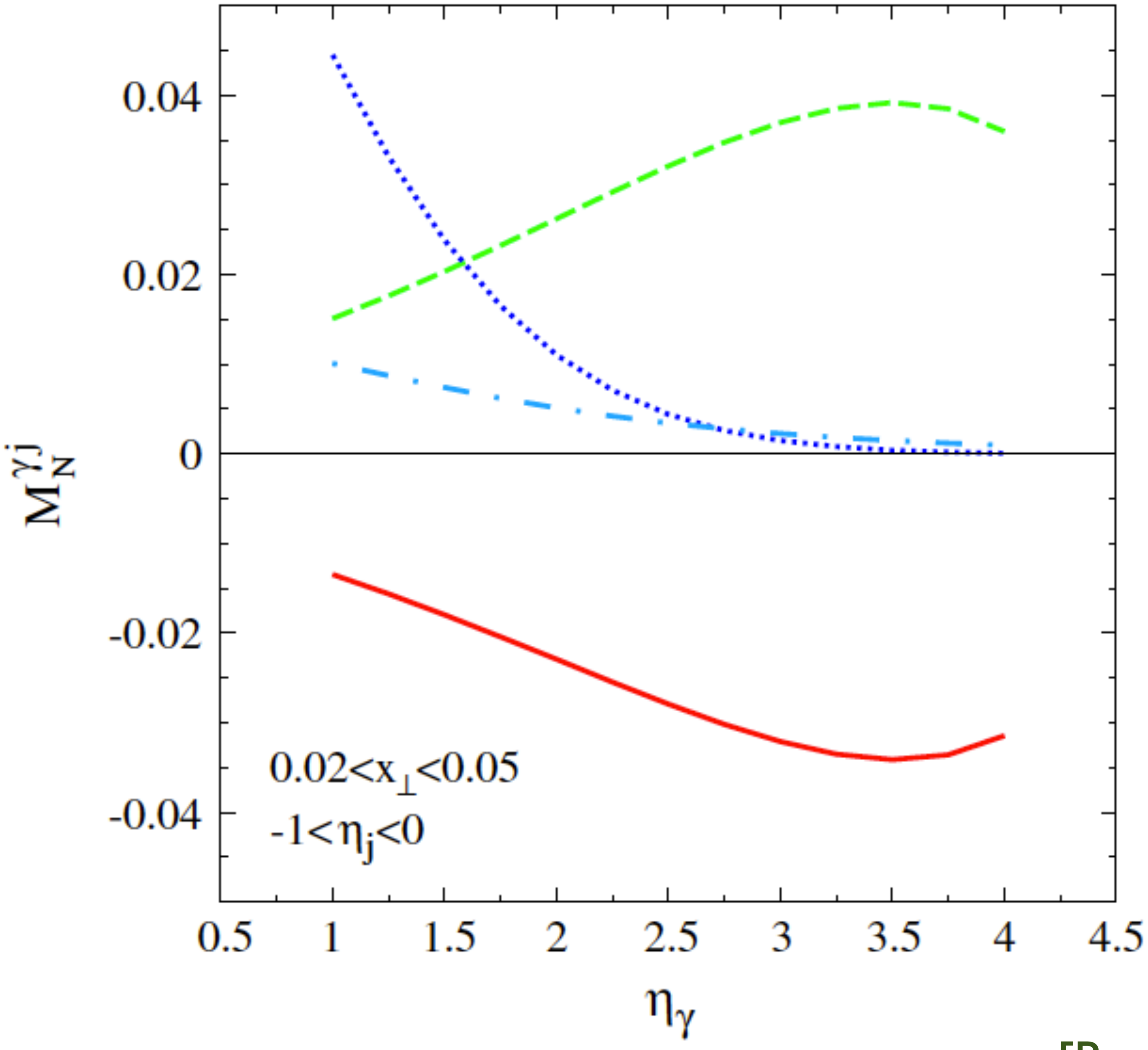
$\sqrt{s}$=500 GeV, $p_T^{\gamma} \geq 1$ GeV, integrated over $4 < Q^2 < 30$ GeV^2, $0 \leq q_T \leq 1$ GeV

At photon pair rapidity $y < 3$ gluon Sivers dominates and max($d\sigma_{TU}/d\sigma_{UU}$) ~ 30-50%

[Qiu, Schlegel, Vogelsang, 2011]

Photon-jet production

$$M_N^{\gamma j}(\eta_\gamma, \eta_j, x_\perp) = \frac{\int d\phi_j\, d\phi_\gamma \frac{2|\boldsymbol{K}_{\gamma\perp}|}{M} \sin(\delta\phi)\cos(\phi_\gamma)\, \frac{d\sigma}{d\phi_j\, d\phi_\gamma}}{\int d\phi_j\, d\phi_\gamma\, \frac{d\sigma}{d\phi_j\, d\phi_\gamma}}$$



Prediction for the azimuthal moment at $\sqrt{s}$=200 GeV, $p_T^\gamma \geq 1$ GeV, integrated over $-1 \leq \eta_j \leq 0$, $0.02 \leq x_\perp \leq 0.05$

Dashed line: GPM

Solid line: using gluonic-pole cross sections

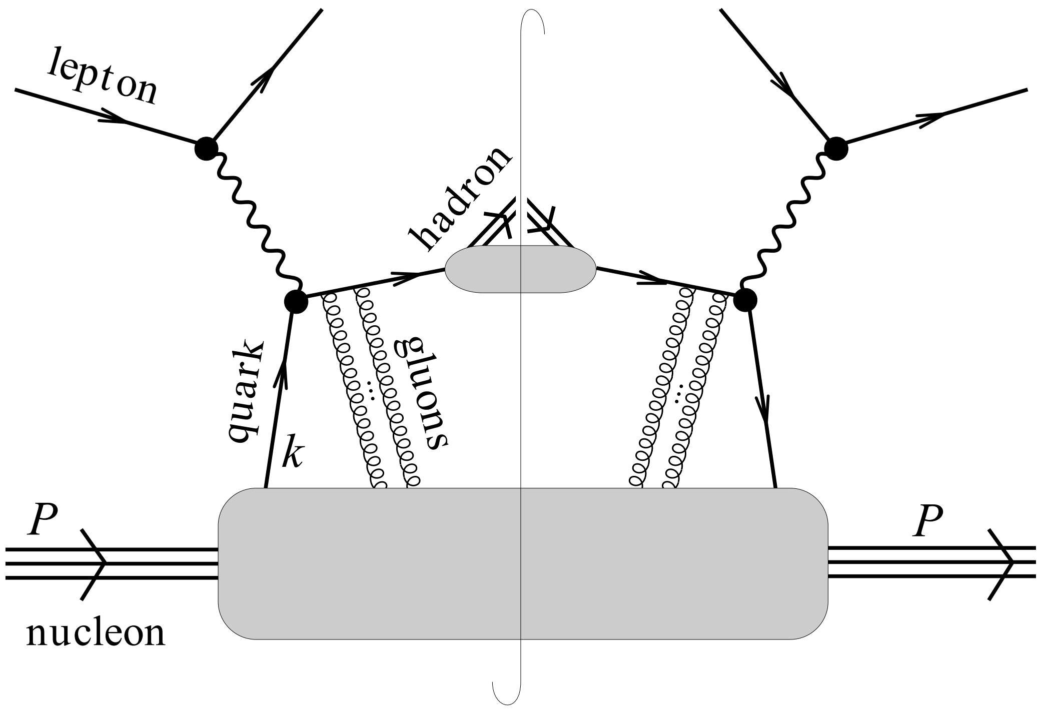
Dotted line: maximum contribution from the gluon Sivers function (absolute value)

Dot-dashed line: maximum contribution from the Boer-Mulders function (abs. value)

[Bacchetta, Bomhof, D'Alesio, Mulders, Murgia, 2007]

Process dependence

Initial and final state interactions



summation of all gluon rescatterings leads to path-ordered exponentials in the correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

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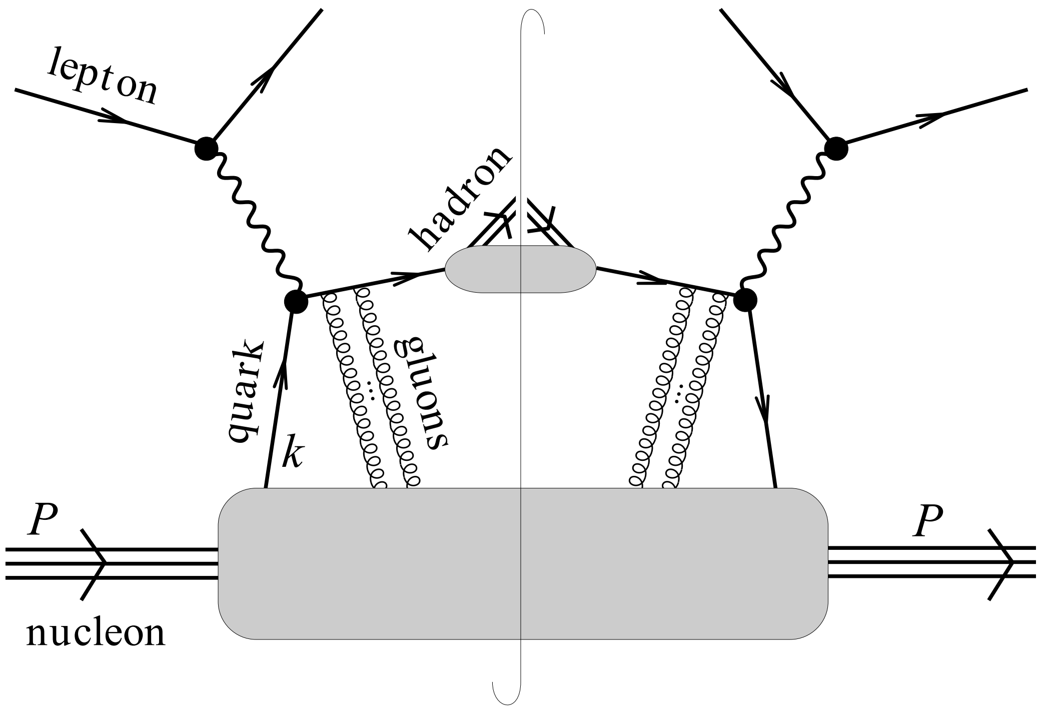

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Resulting Wilson lines depend on whether the color is incoming or outgoing

[Collins & Soper, 1983; DB & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, X. Ji & F. Yuan, 2003; DB, Mulders & Pijlman, 2003]

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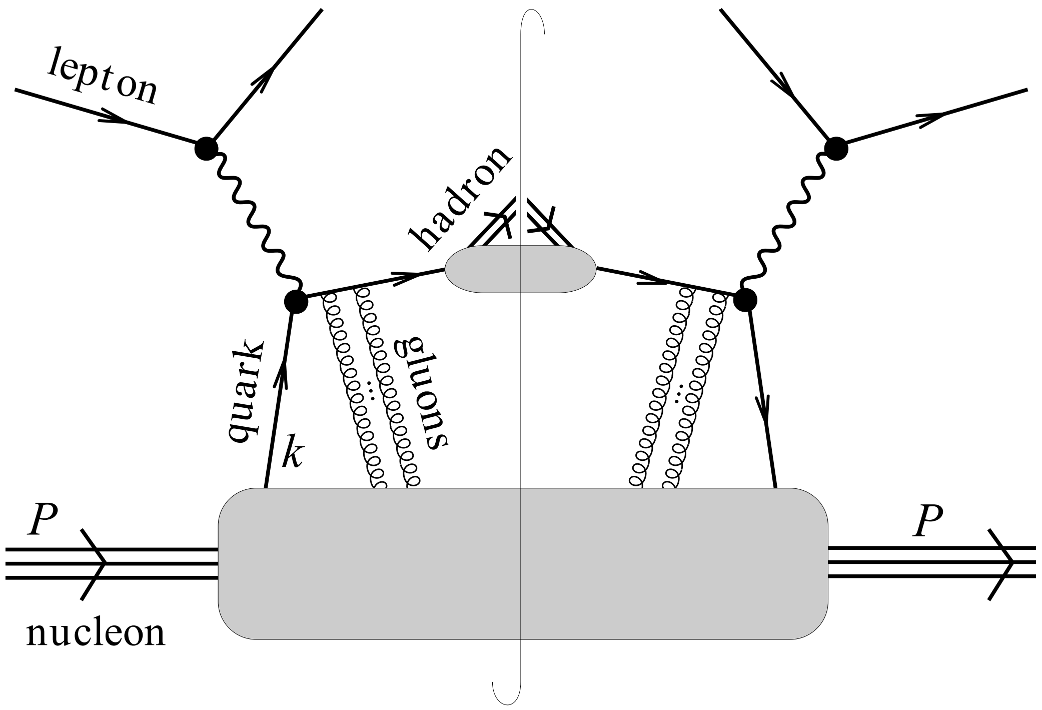

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Initial and final state interactions (ISI/FSI) affect some observables differently

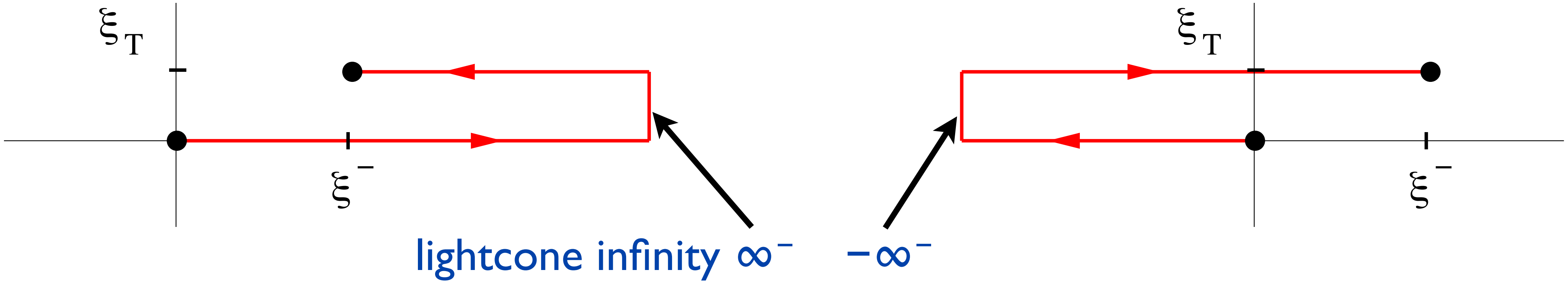
[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

Process dependence of quark Sivers TMD

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing

[Belitsky, X. Ji & F. Yuan '03]

$\gamma^* p \rightarrow h\, X$ (SIDIS) $\qquad\qquad$ $pp \rightarrow \gamma^* X$ (Drell-Yan)



One can use parity and time reversal invariance to relate these Sivers functions:

$$f_{1T}^{\perp[\mathrm{SIDIS}]} = -f_{1T}^{\perp[\mathrm{DY}]}$$

[Collins '02]

Although this process dependence can be calculated, not all Sivers functions from all processes can be related to each other!

pp measurements can be entirely **complementary** to those in *ep*

Process dependence of gluon Sivers TMD

$$\Gamma_g^{\mu\nu[\mathcal{U},\mathcal{U}']}(x,k_T) \equiv 2\int \frac{d(\xi\cdot P)\,d^2\xi_T}{(xP\cdot n)^2(2\pi)^3} e^{i(xP+k_T)\cdot\xi}\mathrm{Tr}_c\Big[\langle P|F^{n\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{n\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}|P\rangle\Big]_{\xi\cdot P'=0}$$

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$$e\,p^\uparrow \to e'\,Q\bar{Q}\,X$$

$$\gamma^*\,g \to Q\bar{Q}$$

This subprocess probes a gluon correlator with two + links (both future pointing)

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In the kinematic regime where gluons in the polarized proton dominate, one effectively selects the subprocess:

$$q\,g \to \gamma\,q$$

This subprocess probes a gluon correlator with a + and − link (future and past pointing), enclosing a whole area

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These processes probe 2 distinct, **independent** gluon Sivers functions

Related to antisymmetric (f^{abc}) and symmetric (d^{abc}) color structures

Bomhof, Mulders, 2007; Buffing, Mukherjee, Mulders, 2013

Process dependence of gluon Sivers TMD

$\gamma^* \, g \to Q\bar{Q}$ probes [+,+]

$q \, g \to \gamma \, q$ probes [+,−]

Conclusion: these two gluon Sivers TMD studies at EIC and at RHIC or AFTER@LHC are complementary

D.B., Lorcé, Pisano & Zhou, arXiv:1504.04332

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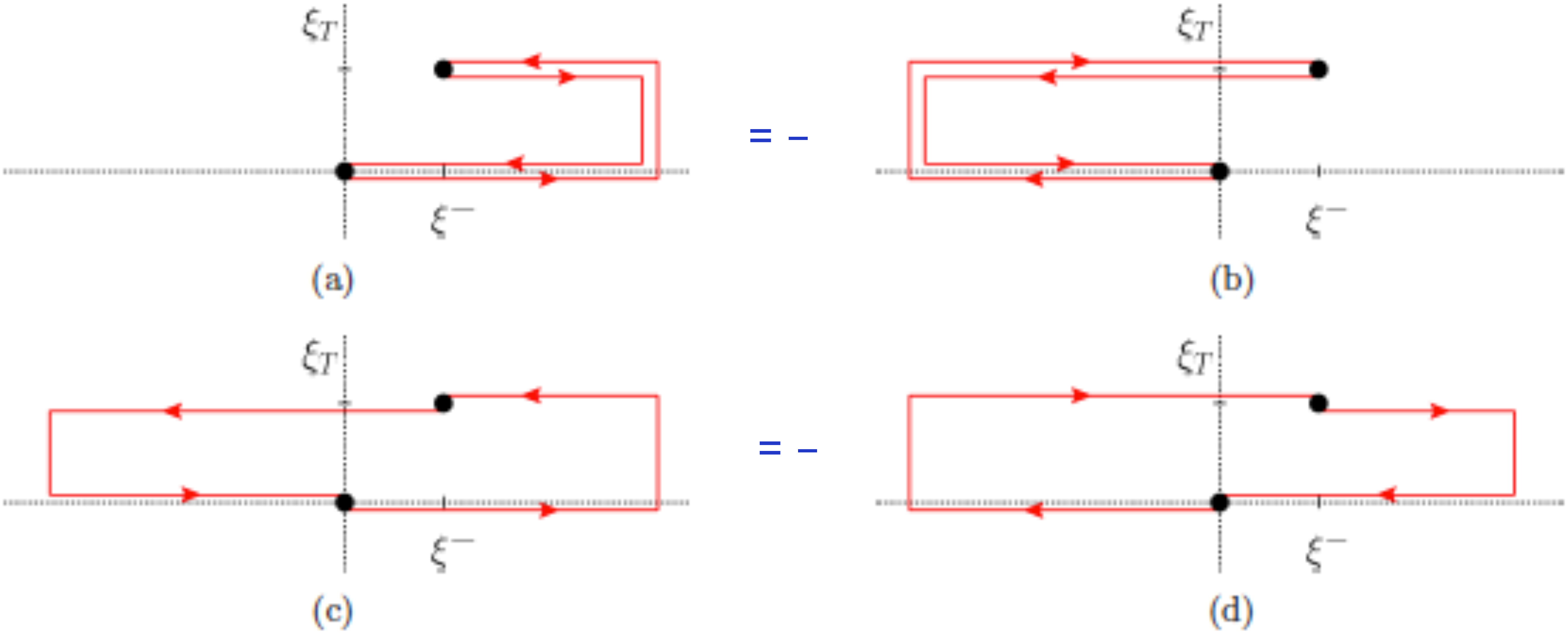
For f-Sivers function: [+,+] = − [−, −]   For d-Sivers function: [+,−] = − [−,+]



(a) (b)

(c) (d)

Process dependence of gluon TMDs

For f-Sivers function: [+,+] = − [−, −]

New prediction: the gluon Sivers function is of opposite sign in

$e\,p^{\uparrow} \to e'\,Q\bar{Q}\,X$ versus $p^{\uparrow}\,p \to \underbrace{\gamma\,\gamma}\,X$ [−, −]

[+,+]

Or any other color singlet state
in gg dominated kinematics

A sign change relation for gluon Sivers TMDs

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[+,+]

[−, −]

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A sign change relation for gluon Sivers TMDs

On the other hand, for $h_1^{\perp g}$ it holds that [+,+] = [−,−] and [+,−] = [−,+]

gg → H and gg → [$Q\bar{Q}$] probe [−,−], hence EIC and LHC can probe same $h_1^{\perp g}$

But e.g. gg → H+g probes a more complicated link structure

Process dependence of gluon TMDs

Is this TMD nonuniversality a polarization issue only? No!

This process dependence is also present for the unpolarized gluon TMD, as was first realized in a small-x context

Dominguez, Marquet, Xiao, Yuan, 2011

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Kharzeev, Kovchegov & Tuchin (2003): ``A tale of two gluon distributions"
They noted that there are two distinct but equally valid definitions for the small-x gluon distribution, the WW and the dipole (DP) distributions

The explanation turns out to be in the process dependence of the gluon distribution, in other words, its sensitivity to the ISI/FSI in a process

The difference between the WW and DP distributions would disappear without ISI/FSI

TMDs at small x

WW vs DP

At small x (and large N$_c$) there are two unpolarized gluon distributions that matter

Dominguez, Marquet, Xiao, Yuan, 2011

$$xG^{(1)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \qquad [+,+]$$

$$xG^{(2)}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \langle P|\mathrm{Tr}\left[F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}\right]|P\rangle \qquad [+,-]$$

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[+,−]

At small x they correspond to the Weizsäcker-Williams (WW) and dipole (DP) distributions, which are generally different:

$$xG^{(1)}(x,k_\perp) = -\frac{2}{\alpha_S}\int\frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2}\, e^{-ik_\perp\cdot(v-v')}\left\langle \mathrm{Tr}\left[\partial_i U(v)\right]U^\dagger(v')\left[\partial_i U(v')\right]U^\dagger(v)\right\rangle_{x_g}$$

WW

$$xG^{(2)}(x,q_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s}S_\perp\int\frac{d^2r_\perp}{(2\pi)^2}e^{-iq_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr}U(0)U^\dagger(r_\perp)\right\rangle_{x_g}$$

DP

WW vs DP

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Different processes probe one or the other or a mixture:

	DIS	DY	SIDIS	$p\,A \to h\,X$	$pA \to \gamma\,\text{jet}\,X$	Dijet in DIS	Dijet in pA
$f_1^{g\,[+,+]}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$
$f_1^{g\,[+,-]}$ (DP)	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\surd$

For dijet in pA the result requires large N_c , otherwise additional functions appear

Polarization of the CGC

	DIS	DY	SIDIS	$p\,A \to h\,X$	$pA \to \gamma^*\,\text{jet}\,X$	Dijet in DIS	Dijet in pA
$h_1^{\perp\,g\,[+,+]}$ (WW)	×	×	×	×	×	√	√
$h_1^{\perp\,g\,[+,-]}$ (DP)	×	×	×	×	√	×	√

γ+jet in pA in leading power *not* sensitive to $h_1^{\perp g}$ [D.B., Mulders, Pisano, 2008]

γ^*+jet in pA *is* sensitive to $h_1^{\perp g}$ [talk by Jian Zhou]

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$h_1^{\perp\,g\,[+,-]}$ (DP)	×	×	×	×	√	×	√

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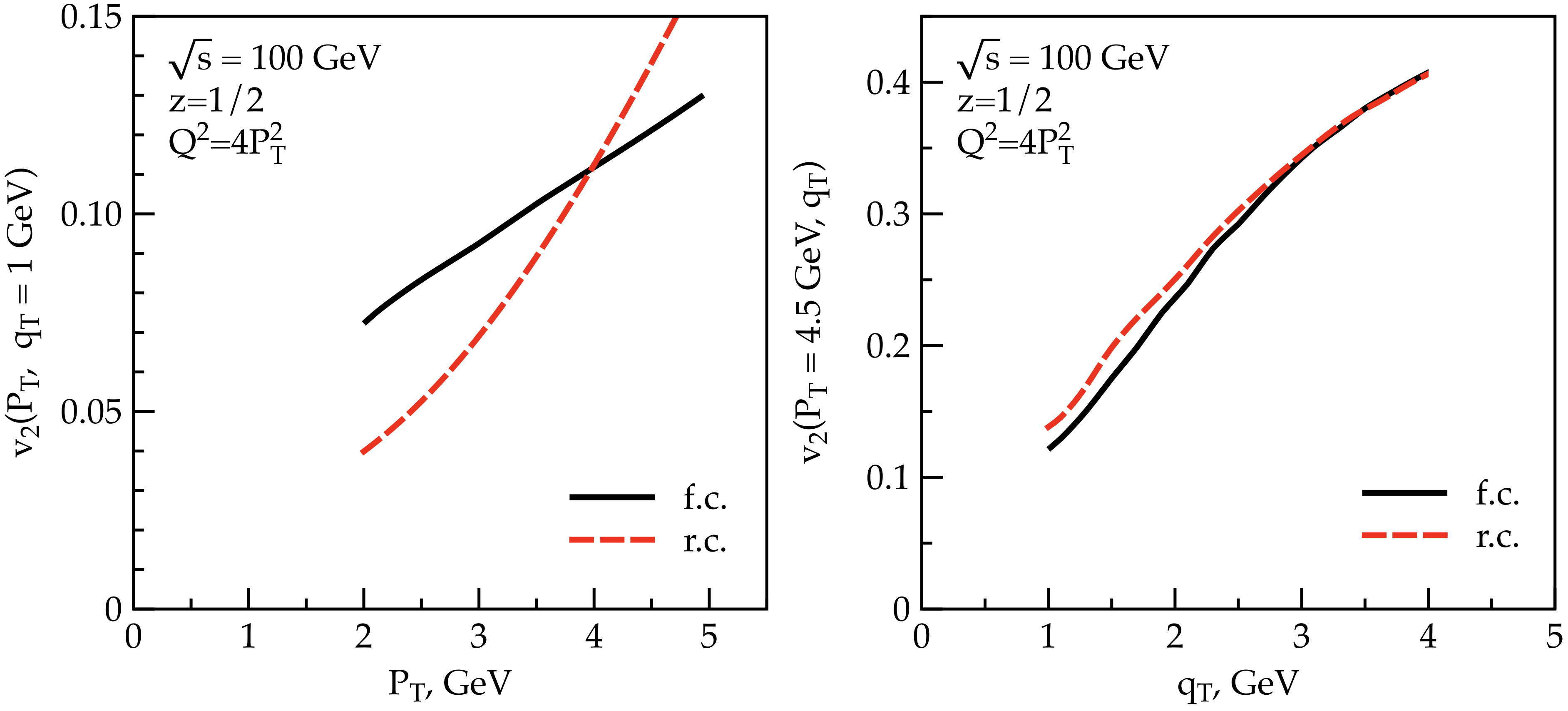
WW $h_1^{\perp g}$ accessible in dijet DIS at a high-energy EIC

[Metz, Zhou 2011; Pisano, D.B., Brodsky, Buffing & Mulders, 2013]



Large effects are found

Dumitru, Lappi, Skokov, 2015

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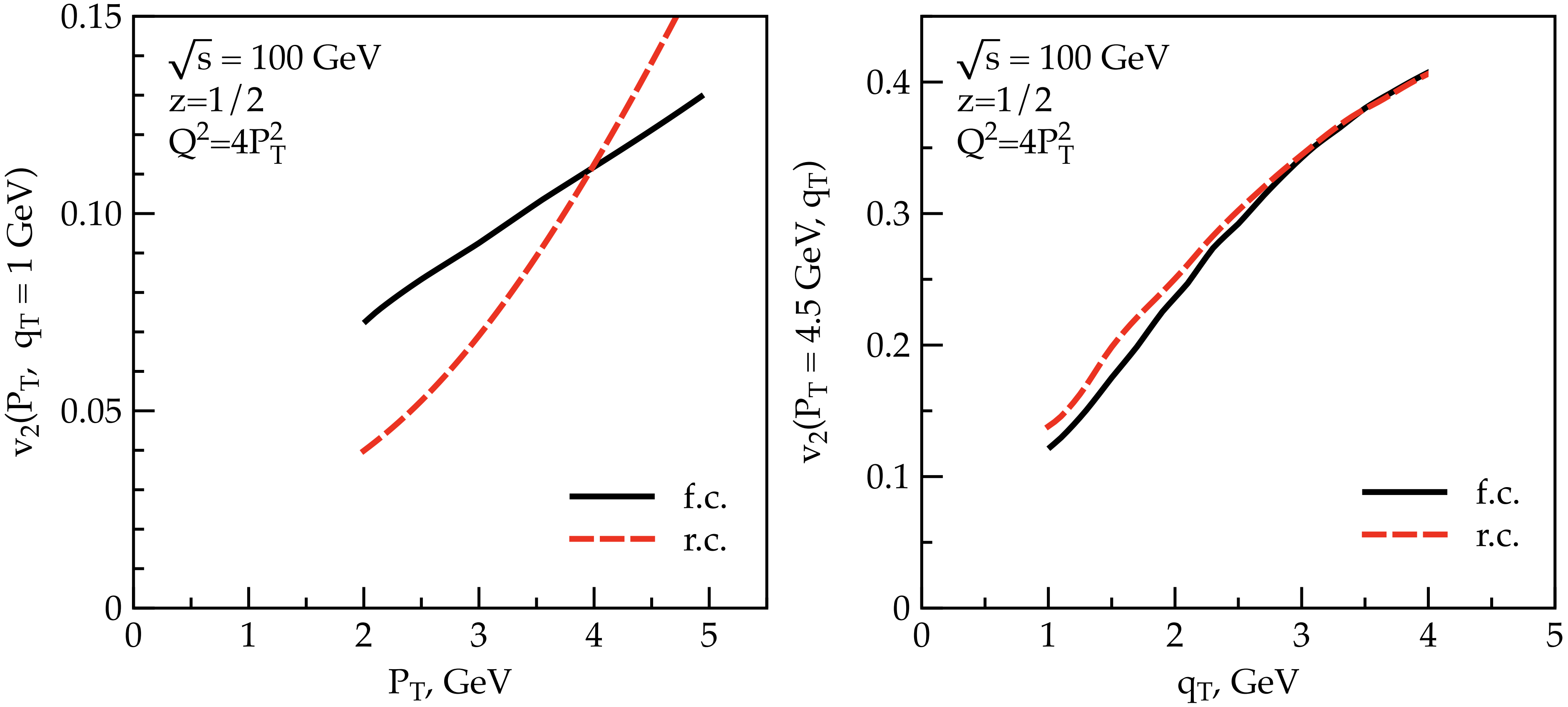

Large effects are found
Dumitru, Lappi, Skokov, 2015

Offers possibility to measure the polarization of the CGC

Gluon Sivers effect at small x

	DIS	DY	SIDIS	$p^\uparrow A \to h\, X$	$p^\uparrow A \to \gamma^{(*)}$ jet X	Dijet in DIS	Dijet in $p^\uparrow A$
$f_{1T}^{\perp\, g\, [+,+]}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$
$f_{1T}^{\perp\, g\, [+,-]}$ (DP)	$\times$	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\surd$

EIC

EIC

Christ & Lee, 1966
Qiu & Sterman, 1998

backward hadron production

At small x the WW or f-type Sivers function vanishes in leading logarithmic order

It has an additional suppression factor x compared to the unpolarized gluon TMD

Gluon Sivers effect at small x

	DIS	DY	SIDIS	$p^\uparrow A \to h\, X$	$p^\uparrow A \to \gamma^{(*)}\,\text{jet}\, X$	Dijet in DIS	Dijet in $p^\uparrow A$
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$f_{1T}^{\perp\, g\,[+,-]}$ (DP)	$\times$	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\surd$

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Christ & Lee, 1966
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The DP-type Sivers turns out to be the ***spin-dependent odderon***

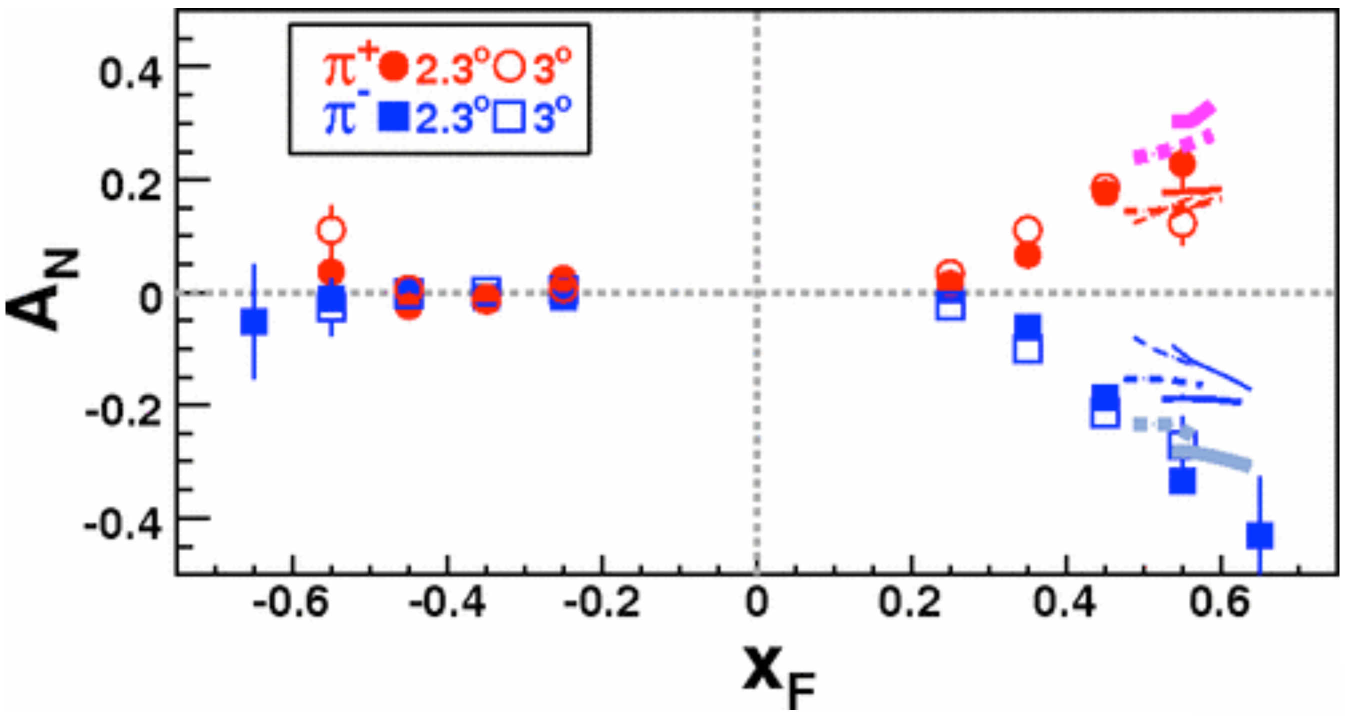
Zhou, 2013

$$\Gamma^{(T-\text{odd})}_{(d)} \equiv \left(\Gamma^{[+,-]} - \Gamma^{[-,+]}\right) \propto \text{F.T.}\, \langle P, S_T | \text{Tr}\left[U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T)\right] | P, S_T \rangle$$

D.B., Echevarria, Mulders, Zhou, 2015

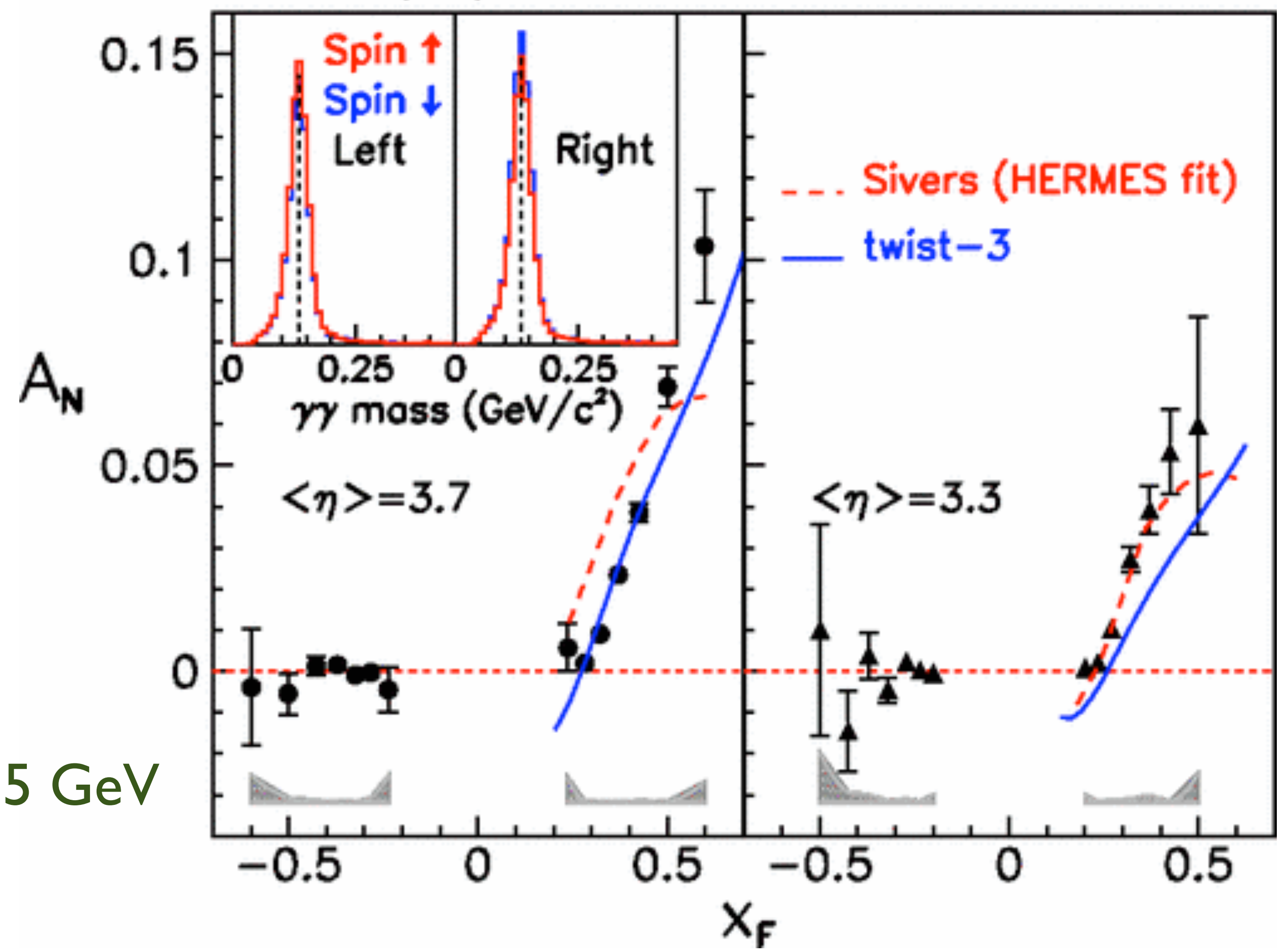
Can be probed at RHIC in DY, backward hadron and γ *jet* production

$p^{\uparrow}p \to h^{\pm}$ X at $x_F < 0$



BRAHMS, 2008 $\sqrt{s}$ = 62.4 GeV
low p_T, up to roughly 1.2 GeV
where gg channel dominates

p+p → π^0+X at $\sqrt{s}$=200 GeV



spin-dependent odderon is C-odd,
whereas gg in the CS state is C-even

expect smaller asymmetries
in neutral pion and jet production

STAR, 2008
$\sqrt{s}$ = 200 GeV
p_T between 1 and 3.5 GeV

Conclusions

Conclusions

- Linear polarization of gluons in unpolarized hadrons can affect many processes
  In pp collisions percent level effects, except in quarkonium production
  In ep collisions it could be much larger (10% or more) & its sign can be determined

- Open heavy quark pair or di-jet production in DIS may exhibit large $h_1^{\perp g}$ effects
  It probes the WW distribution, like Higgs or scalar quarkonium production at LHC
  At small x it would allow a study of the polarization of the CGC

- Gluon TMDs are inherently process dependent, which implies complementarity of certain studies of the gluon Sivers TMD at EIC and RHIC/AFTER@LHC
  A sign-change test for the gluon Sivers function is possible as well

- Promising channels for gluon TMD studies at RHIC: $\gamma\gamma$, $\gamma^{(*)}$+*jet*, *J*/ψ+γ production
  and processes that are effectively expressed in terms of TMD at small x such as backward $h^{\pm}$ production to study the DP Sivers a.k.a. spin-dependent odderon

Back-up slides

Quarkonium production

C-even (pseudo-)scalar quarkonium production promising for studying $h_1^{\perp g}$

Using the CSM model and LO NRQCD we obtain:

$$\frac{d\sigma(\eta_Q)}{dy\, d^2\boldsymbol{q}_T} = \frac{2}{9}\,\frac{\pi^3\alpha_s^2}{M^3\, s}\,\langle 0|\mathcal{O}_1^{\eta_Q}({}^1S_0)|0\rangle\,\mathcal{C}\left[f_1^g\, f_1^g\right]\,\left[1 - R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q0})}{dy\, d^2\boldsymbol{q}_T} = \frac{8}{3}\,\frac{\pi^3\alpha_s^2}{M^5\, s}\,\langle 0|\mathcal{O}_1^{\chi_{Q0}}({}^3P_0)|0\rangle\,\mathcal{C}\left[f_1^g\, f_1^g\right]\,\left[1 + R(\boldsymbol{q}_T^2)\right]$$

$$\frac{d\sigma(\chi_{Q2})}{dy\, d^2\boldsymbol{q}_T} = \frac{32}{9}\,\frac{\pi^3\alpha_s^2}{M^5\, s}\,\langle 0|\mathcal{O}_1^{\chi_{Q2}}({}^3P_2)|0\rangle\,\mathcal{C}\left[f_1^g\, f_1^g\right]$$

D.B., Pisano, PRD 86 (2012) 094007

These are color singlet model expressions, which at least may be justified for C=+ bottomonium states

Bodwin, Braaten, Lepage, 1995; Hägler, Kirschner, Schäfer, Teryaev, 2001; Maltoni, Polosa, 2004; Bodwin, Braaten, Lee, 2005; …

Bottomonium production

To extract R(Q$_T$) one can consider 3 bottomonia and ratios of ratios:

$$\frac{\sigma(\chi_{b0})}{\sigma(\eta_b)}\,\frac{d\sigma(\eta_b)/d^2\boldsymbol{q}_T}{d\sigma(\chi_{b0})/d^2\boldsymbol{q}_T} \approx \frac{1+R(\boldsymbol{q}_T^2)}{1-R(\boldsymbol{q}_T^2)}$$

$$\frac{\sigma(\chi_{b0})}{\sigma(\chi_{b2})}\,\frac{d\sigma(\chi_{b2})/d^2\boldsymbol{q}_T}{d\sigma(\chi_{b0})/d^2\boldsymbol{q}_T} \approx 1+R(\boldsymbol{q}_T^2)$$

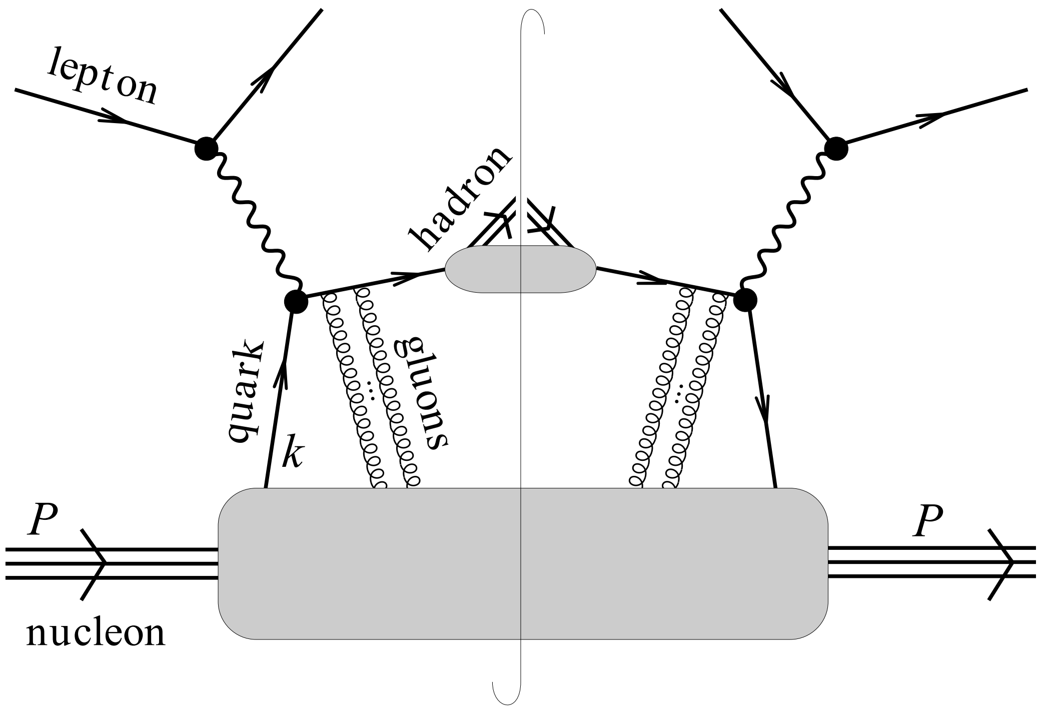
Uncertainties about the hadronic wave function (approximately) cancel

Very small scale differences: $m_{\eta_b} \approx m_{\chi_{b0}} \approx m_{\chi_{b2}}$

Therefore, hardly any TMD evolution effects

Of course, not easy experimentally, but much bigger effects are expected

Initial and final state interactions



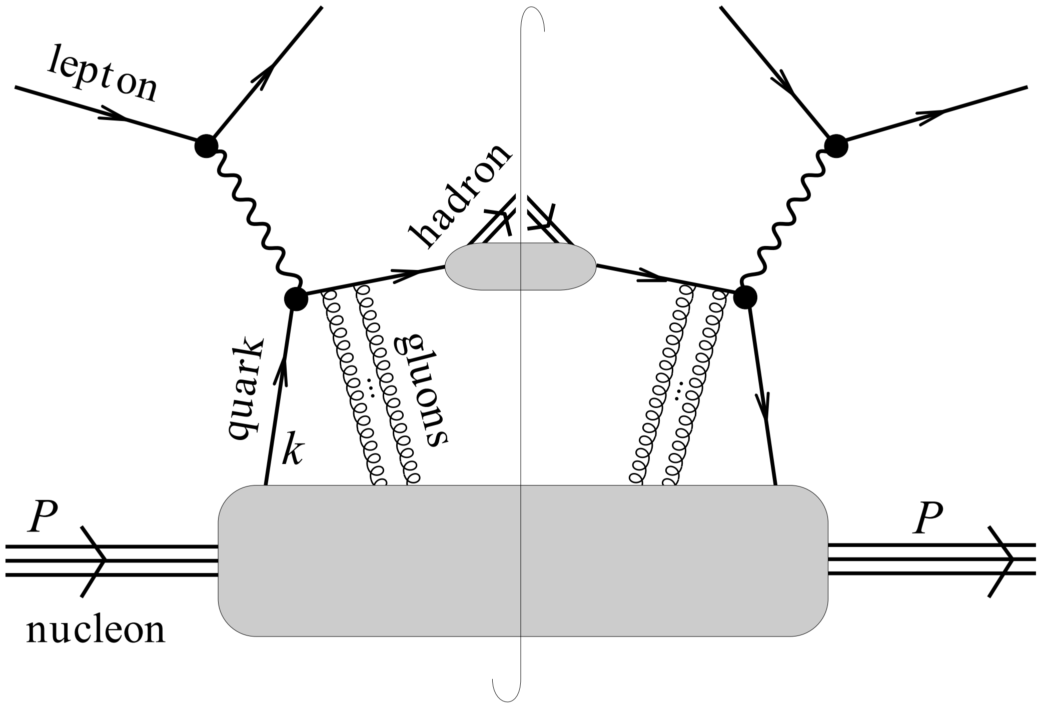
summation of all gluon rescatterings leads to path-ordered exponentials in the correlators

$$\mathcal{L}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

$$\Phi \propto \langle P|\overline{\psi}(0)\mathcal{L}_{\mathcal{C}}[0,\xi]\psi(\xi)|P\rangle$$

Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

Initial and final state interactions



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Efremov & Radyushkin, Theor. Math. Phys. 44 ('81) 774

Resulting Wilson lines depend on whether the color is incoming or outgoing

[Collins & Soper, 1983; DB & Mulders, 2000; Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, X. Ji & F. Yuan, 2003; DB, Mulders & Pijlman, 2003]

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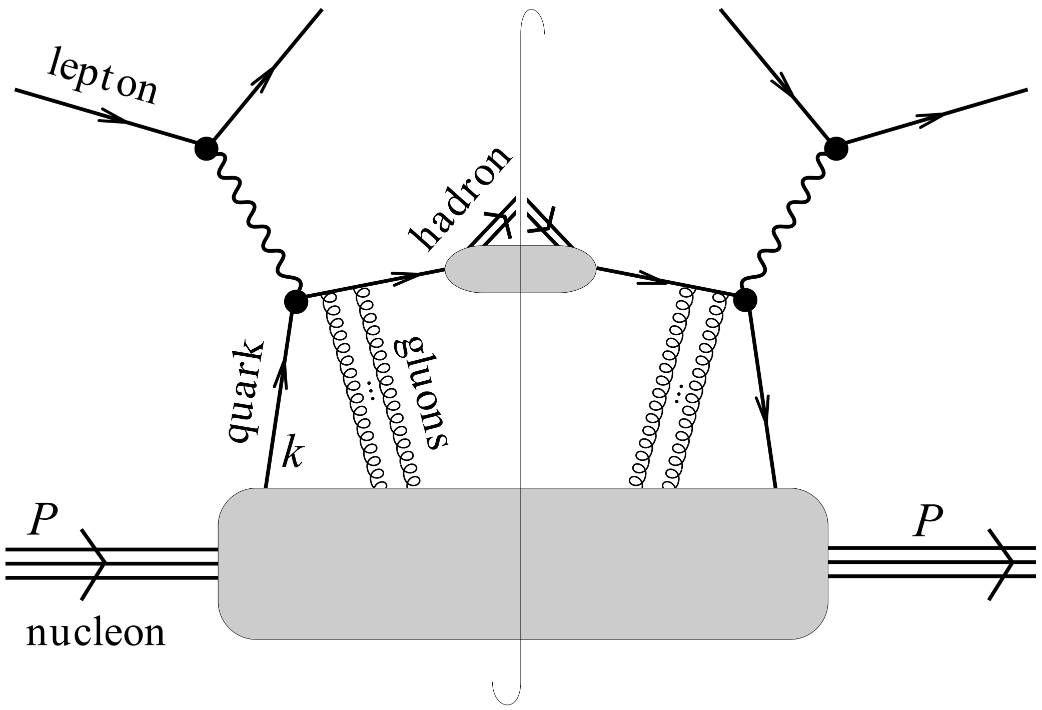

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This does not automatically imply that the ISI and/or FSI affect observables, but it turns out that they do in certain cases, for example, Sivers asymmetries

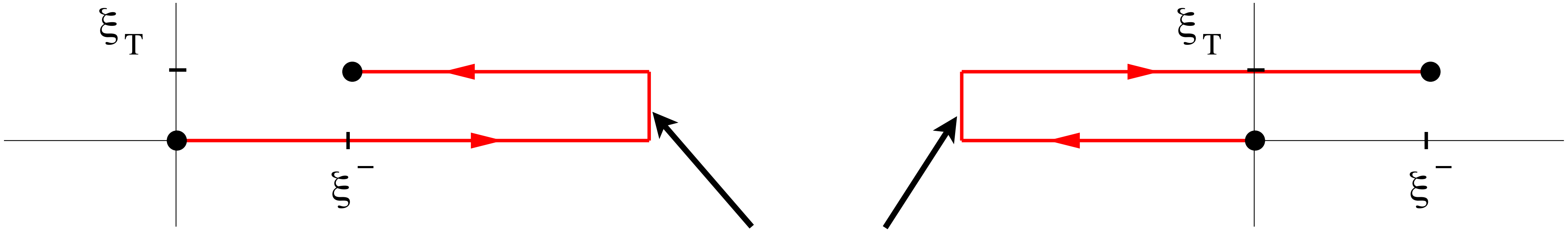
[Brodsky, Hwang & Schmidt, 2002; Collins, 2002; Belitsky, Ji & Yuan, 2003]

Process dependence of quark Sivers TMD

Gauge invariant definition of TMDs in semi-inclusive DIS contains a future pointing Wilson line, whereas in Drell-Yan (DY) it is past pointing

[Belitsky, X. Ji & F. Yuan '03]

$\gamma^* p \rightarrow h\ X$ (SIDIS) $pp \rightarrow \gamma^* X$ (Drell-Yan)



lightcone infinity ∞^- $-\infty^-$

One can use parity and time reversal invariance to relate these Sivers functions:

$$f_{1T}^{\perp[\text{SIDIS}]} = -f_{1T}^{\perp[\text{DY}]}$$

[Collins '02]

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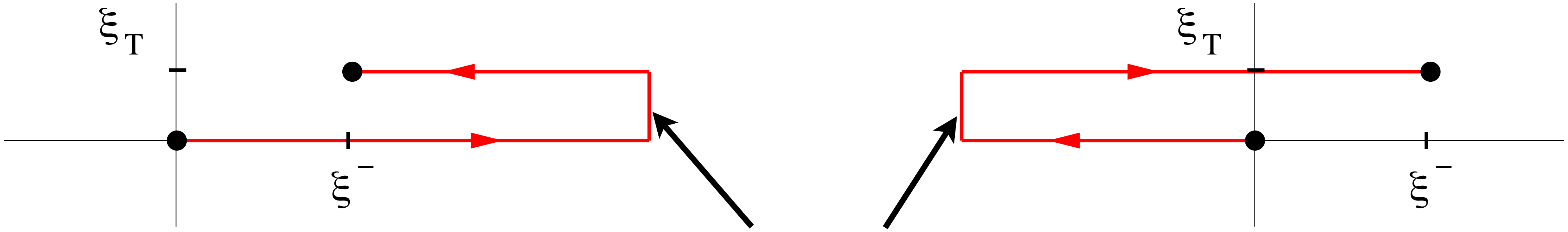

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[Bomhof, Mulders & Pijlman '04; Buffing, Mulders '14]

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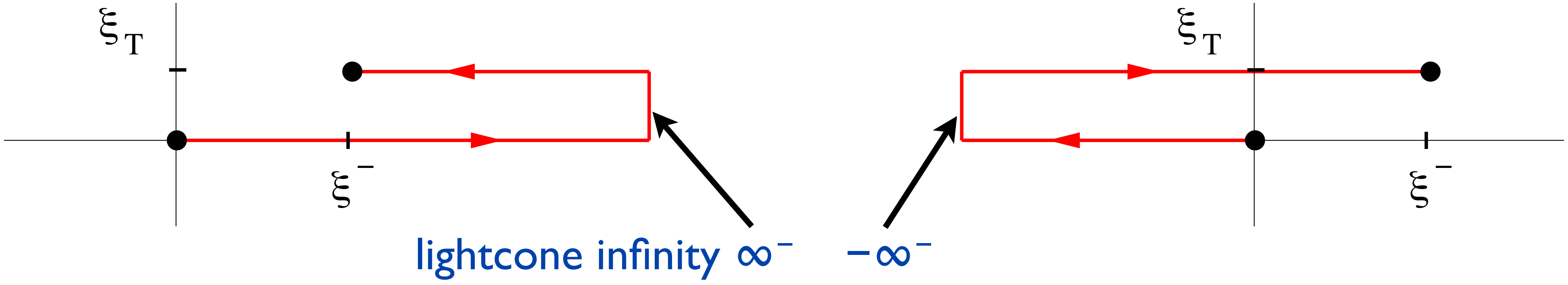

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[Bomhof, Mulders & Pijlman '04; Buffing, Mulders '14]

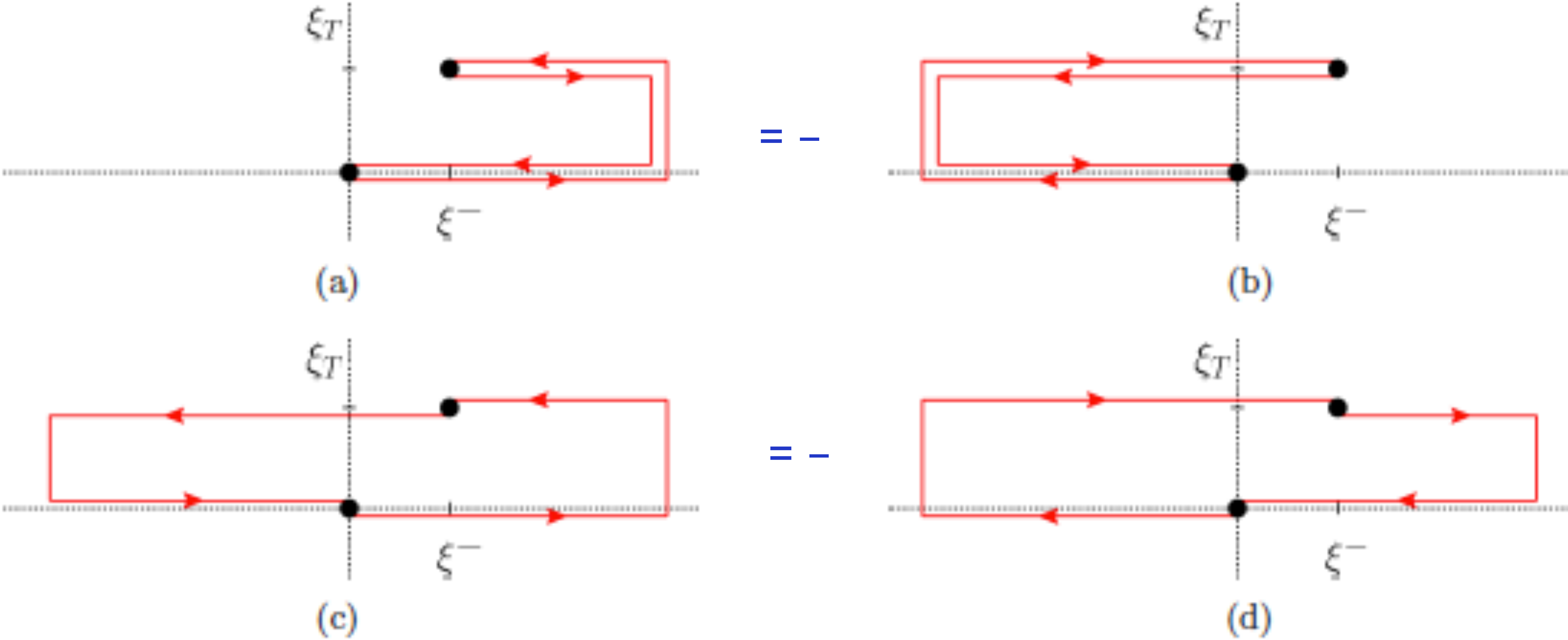
When color flow is in too many directions: *factorization breaking*

[Collins & J. Qiu '07; Collins '07; Rogers & Mulders '10]

Process dependence of gluon Sivers TMD

For the f-Sivers function the gluon correlator with two + links is equal to minus the one with two − links

For the d-Sivers function the gluon correlator with +,− links is equal to minus the one with −,+ links



Conclusion: the proposed gluon Sivers TMD studies at EIC and at RHIC or AFTER@LHC are complementary

D.B., Lorcé, Pisano & Zhou, arXiv:1504.04332

MV model

In the MV model one may not notice the origin for the difference between WW and DP, because the two TMDs become related:

$$xG_g^{(2)}(x,q_\perp) \overset{\text{MV}}{\propto} q_\perp^2 \nabla_{q_\perp}^2 xG_g^{(1)}(x,q_\perp)$$

Processes involving $G^{(1)}$ (WW) [+,+] in the MV model can be expressed in terms of $G^{(2)} \sim C(k_\perp)$

$$C(\boldsymbol{k}_\perp) = \int \mathrm{d}^2\boldsymbol{x}_\perp\, \mathrm{e}^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} \langle U(0)U^\dagger(\boldsymbol{x}_\perp)\rangle$$

$$\gamma\, A \to Q\bar{Q}\, X$$

Gelis, Peshier, 2002

$$\frac{\mathrm{d}\sigma_\mathrm{T}}{\mathrm{d}y\,\mathrm{d}k_\perp} = \pi R^2 \frac{2N_\mathrm{c}(Z\alpha)^2}{3\pi^3} \ln\left(\frac{\gamma}{2mR}\right) k_\perp C(k_\perp) \times \left\{1 + \frac{4(k_\perp^2 - m^2)}{k_\perp\sqrt{k_\perp^2 + 4m^2}} \operatorname{arcth} \frac{k_\perp}{\sqrt{k_\perp^2 + 4m^2}}\right\}$$

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Different processes probe one or the other or a mixture:

	DIS	DY	SIDIS	$p\,A \to h\,X$	$pA \to \gamma\,\text{jet}\,X$	Dijet in DIS	Dijet in pA
$f_1^{g\,[+,+]}$ (WW)	×	×	×	×	×	√	√
$f_1^{g\,[+,-]}$ (DP)	√	√	√	√	√	×	√

For dijet in pA the result requires large N_c , otherwise additional functions appear

Finite N_c: Akcakaya, Schäfer, Zhou, 2013; Kotko, Kutak, Marquet, Petreska, Sapeta, van Hameren, 2015

WW vs DP

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The p_T widths of TMDs are process dependent, and as a consequence, it gives an additional process dependence to p_T broadening (eA-ep versus pA-pp)

D.B., Buffing, Mulders, 2015

Linear gluon polarization at small x

The WW and DP $h_1^{\perp g}$ distributions will be different too:

$$h_{1,WW}^{\perp g} \ll f_{1,WW}^{\perp g} \quad \text{for } k_\perp \ll Q_s, \qquad h_{1,WW}^{\perp g} = 2 f_{1,WW}^{\perp g} \quad \text{for } k_\perp \gg Q_s$$

$$x h_{1,DP}^{\perp g}(x, k_\perp) = 2 x f_{1,DP}^{g}(x, k_\perp)$$

Metz, Zhou '11

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$$\Gamma_g^{\mu\nu}(x, \boldsymbol{p}_T)_{\text{max pol}} = \frac{1}{x} \frac{p_T^\mu p_T^\nu}{\boldsymbol{p}_T^2} f_1^g$$

Catani, Ciafaloni, Hautmann, 1991

Applied to Higgs production by A.V. Lipatov, Malyshev, Zotov, 2014

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The perturbative tail of $h_1^{\perp g}$ has a $1/x$ growth, which keeps up with f_1:

$$\tilde{h}_1^{\perp g}(x, b^2; \mu_b^2, \mu_b) = \frac{\alpha_s(\mu_b) C_A}{2\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left(\frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu_b) + \mathcal{O}(\alpha_s^2)$$

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There is no theoretical reason why it should be small, especially at small x