Collins effect in SIDIS, e^+e^-, and pp collisions

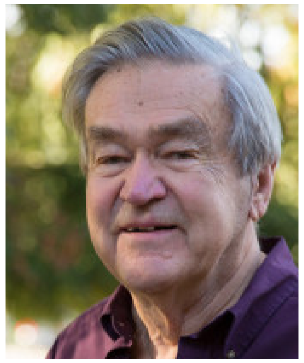
Alexei Prokudin

Spin and QCD

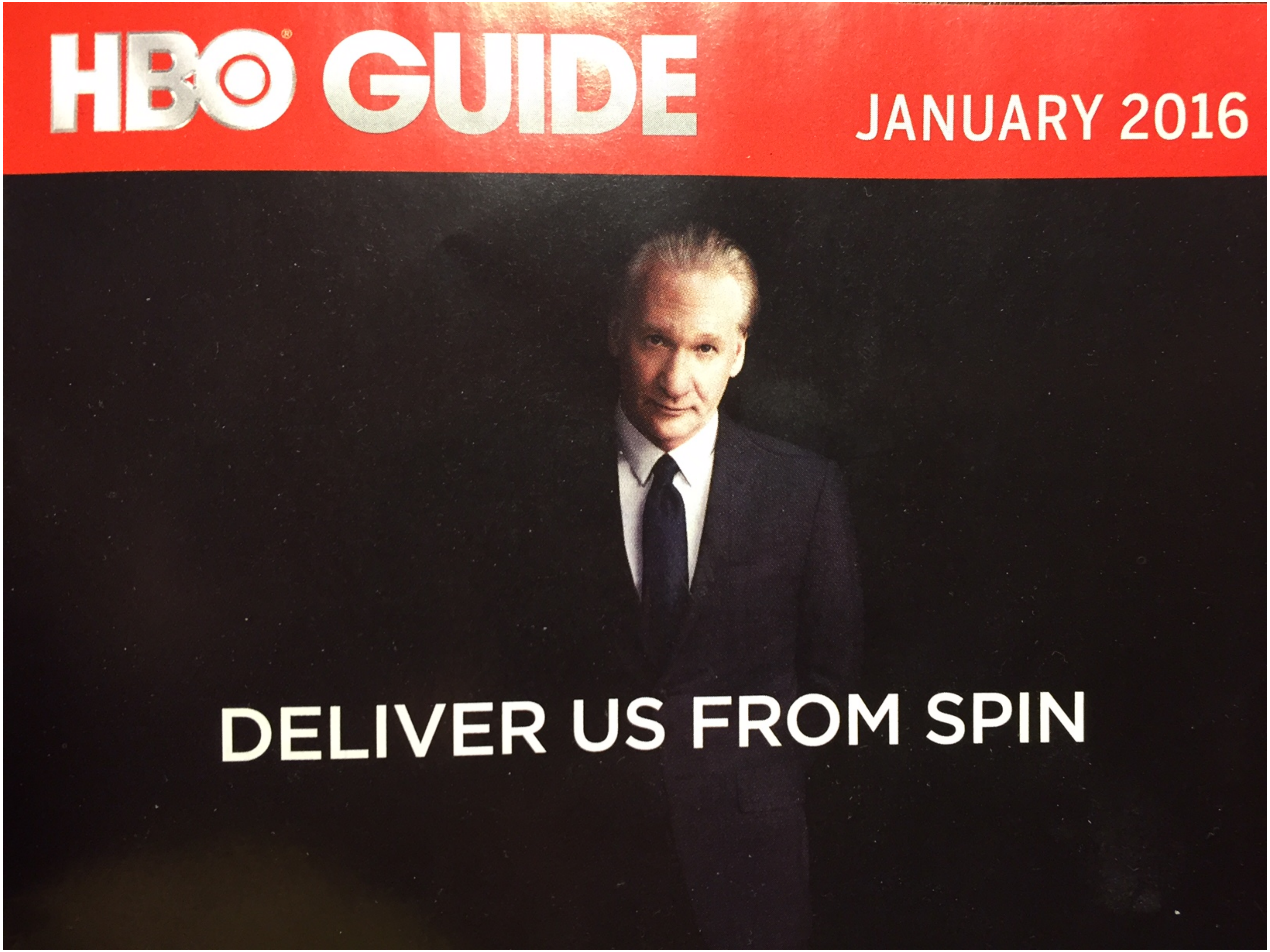
"Experiments with spin have killed more theories than any other single physical parameter"

Elliot Leader, Spin in Particle Physics, Cambridge U. Press (2001)

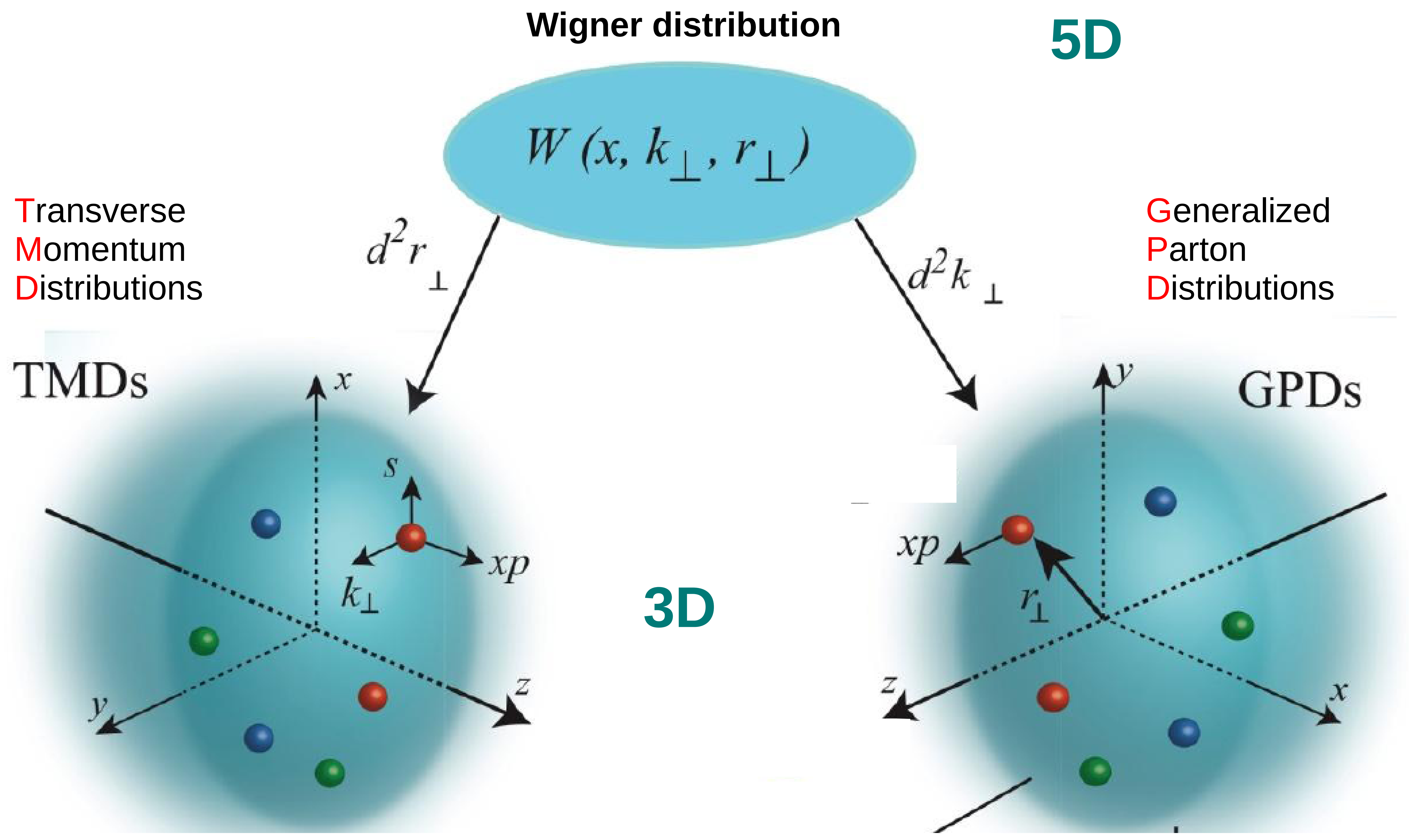
"Polarization data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection."



J. D. Bjorken, Proc. Adv. Research Workshop on QCD Hadronic Processes,
St. Croix, Virgin Islands (1987).


HBO GUIDE
JANUARY 2016
DELIVER US FROM SPIN

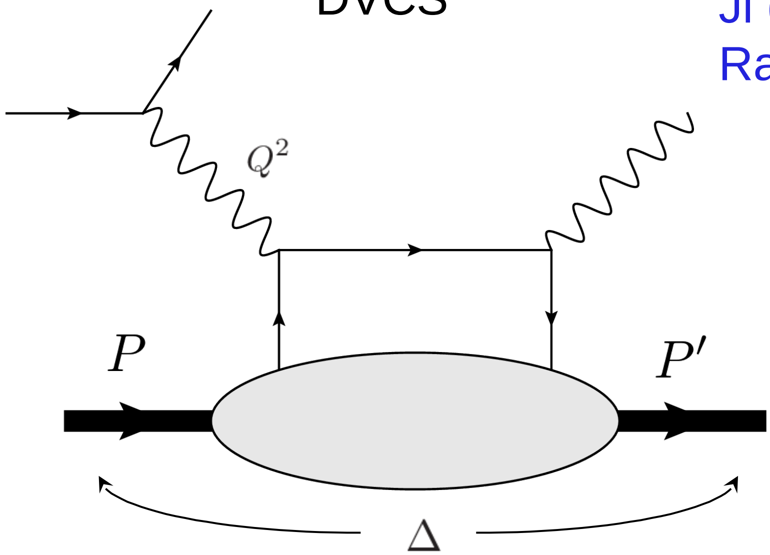
Unified View of Nucleon Structure



GPDs

DVCS

Ji (1997)
Radyushkin (1997)



Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

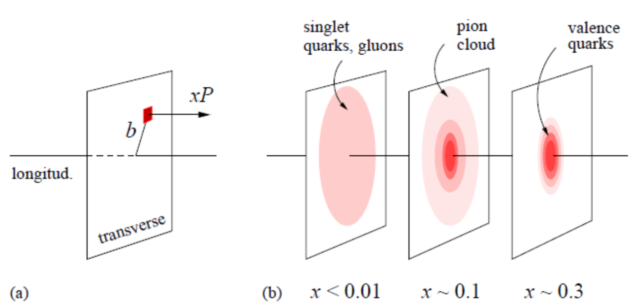
Connection to 3D structure

Burkardt (2000)
Burkardt (2003)

$$\rho(x, \vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

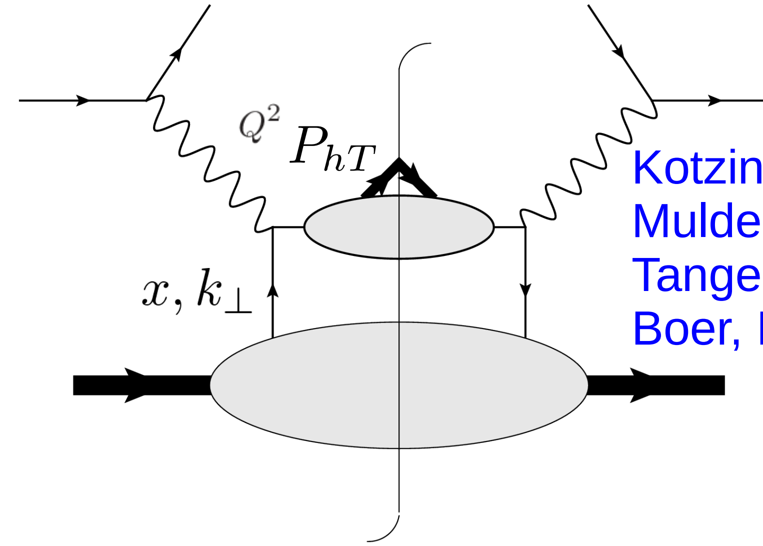
Drell-Yan frame $\Delta^+ = 0$

Weiss (2009)



TMDs

SIDIS



Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Q^2 ensures hard scale, pointlike interaction

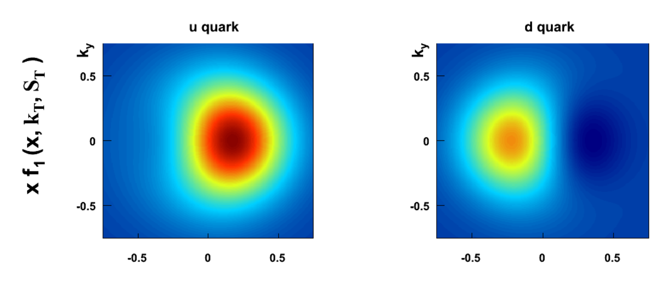
P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure

Ji, Ma, Yuan (2004)
Collins (2011)

$$\tilde{f}(x, \vec{b}_T) = \int d^2k_\perp e^{i\vec{b}_T \cdot \vec{k}_\perp} f(x, \vec{k}_\perp)$$

$\vec{b}_T$ is the transverse separation of parton fields in configuration space



AP (2012)

TMD distributions

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD distributions

N \ q	U	L	T
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Transversity TMD

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD Fragmentation Functions

N \ q	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$H_{1T}^{\perp}$	G_{1T}	H_1 $H_{1T}^{\perp}$

8 functions describing fragmentation of a quark into spin ½ hadron

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

TMD Fragmentation Functions

N \ q	U	L	T
U	D_1		$H_1^{\perp}$
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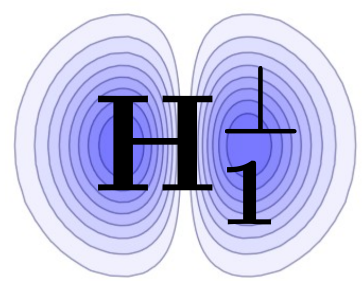
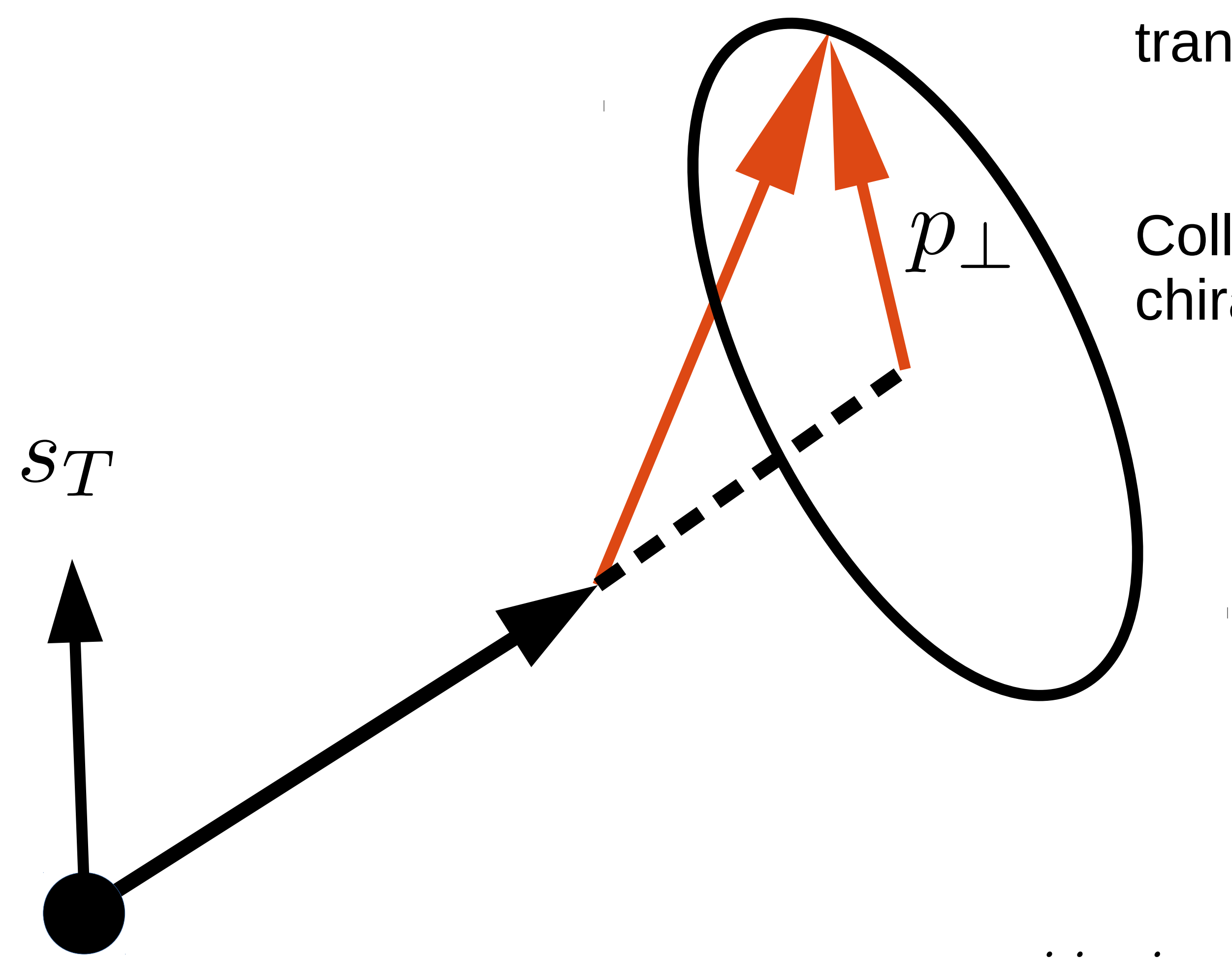
8 functions describing fragmentation of a quark into spin ½ hadron

Collins FF

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)


$\mathbf{H}_1^\perp$

Collins effect



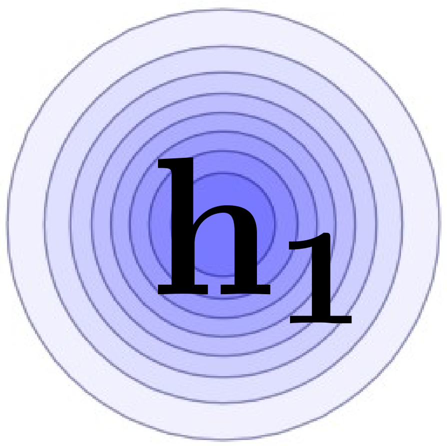
Collins function is generated by correlation of transverse momentum of the quark and transverse spin of the quark

Collins function is chiral odd and needs another chiral odd function to be measured

Collins FF

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \ldots - \frac{\varepsilon_T^{ij} p_\perp^j}{z M_h} H_1^{\perp h/q}(z, p_\perp^2)$$

Collins (1992)


h_1

Twist-2 collinear PDFs

Quark-quark correlator can be decomposed by means of
3 Parton Distributions Functions (PDF) in collinear (kt integrated) case

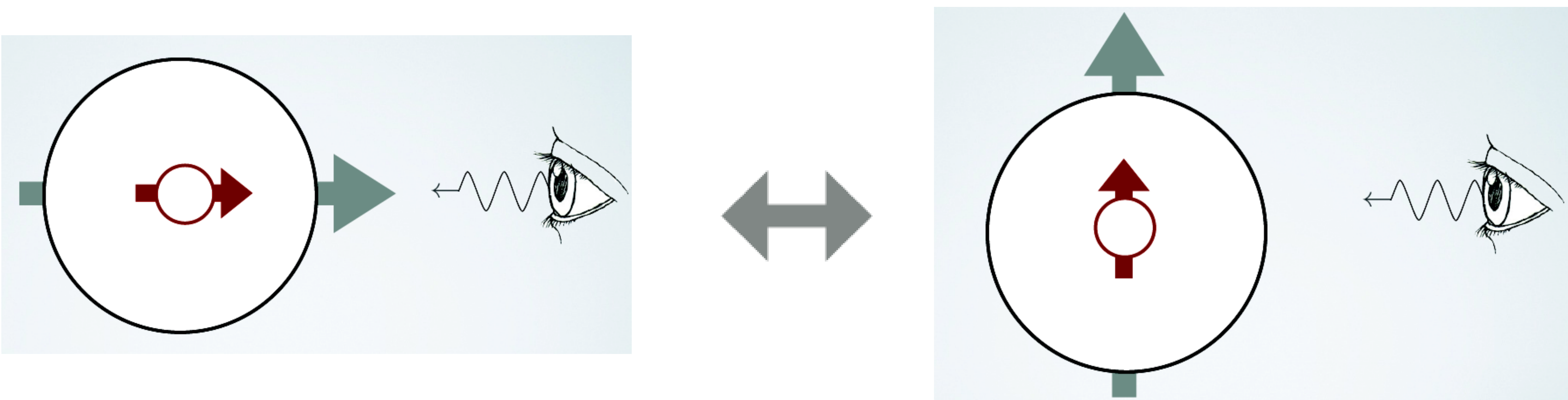
$$\Phi(x;P,S) = \frac{1}{2}\left\{ f_1(x)\ \not{P} + S_L g_1(x)\gamma_5\ \not{P} + \frac{1}{2} h_1(x)\gamma_5 [\not{S}_T, \not{P}] \right\}$$

Quark-quark correlator can be decomposed by means of
3 Parton Distributions Functions (PDF) in collinear (kt integrated) case

$$\Phi(x;P,S) = \frac{1}{2}\left\{ \underbrace{f_1(x)}_{\text{Unpolarised PDF}}\, \not{P} + S_L \underbrace{g_1(x)}_{\text{Helicity distribution}}\gamma_5\, \not{P} + \frac{1}{2}\underbrace{h_1(x)}_{\text{Transversity distribution}}\gamma_5 [\not{S}_T, \not{P}] \right\}$$

h_1

Helicity distribution | Transversity distribution



Boost and rotation do not commute → helicity and transversity are different!

Transversity is the only source of information
on tensor charge

$$\delta q = \int_0^1 dx (h_1^q(x) - h_1^{\bar{q}}(x))$$

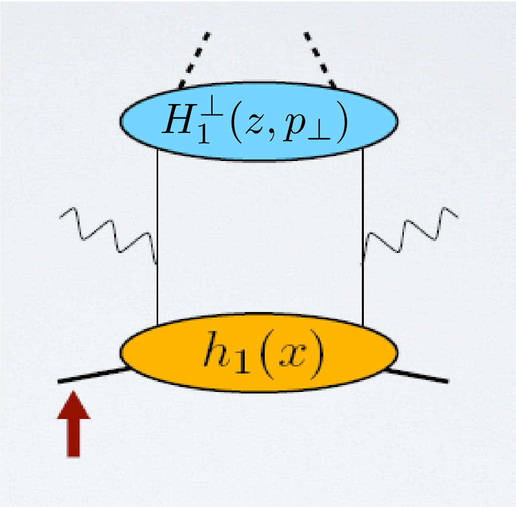
Fundamental quantity

Caveat: no sum rules

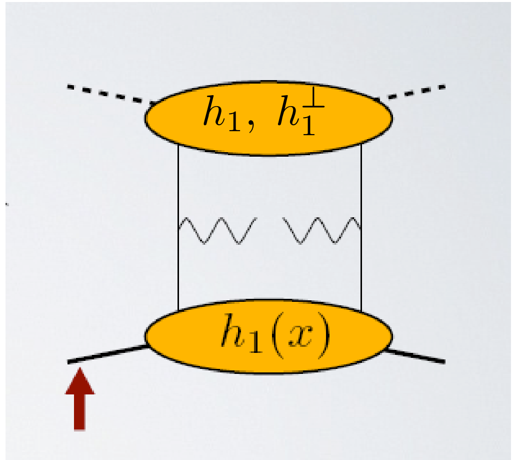
Collins effect and TMD factorization

Transversity and Collins FF how measure?

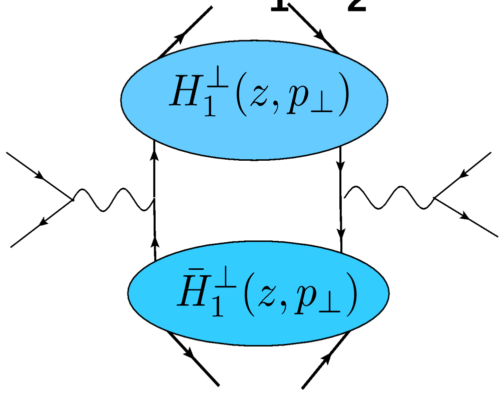
Semi Inclusive DIS (SIDIS)



Drell-Yan



$e^+e^- \rightarrow h_1\ h_2\ X$



✔ TMD Factorization proven

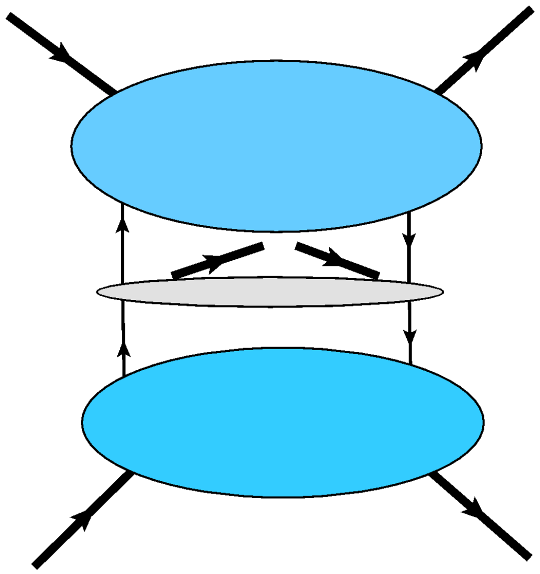
✔ Separation of different contributions is possible

✘ Implementation of TMD factorization is more difficult compared to collinear factorization

See talks by John Collins, Bowen Wang, Leonard Gamberg

Transversity and Collins FF how measure?

pp → π X scattering, A_N



✘ TMD Factorization may not be valid

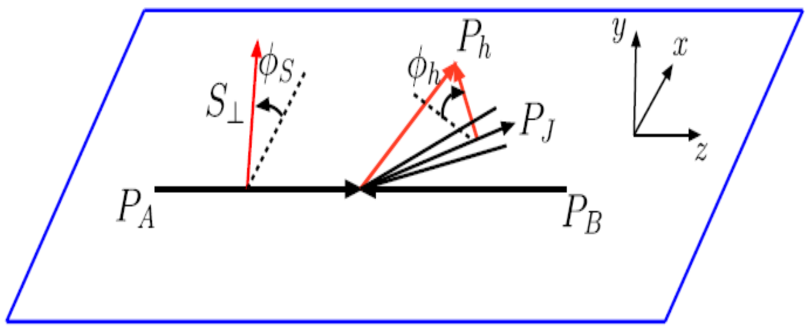
✘ Contribution from many channels, Sivers effect etc

✔ Possibility to study factorization breaking effects

See talk by Cristian Pisano

Transversity and Collins FF how measure?

pp scattering, pion in jet A_N



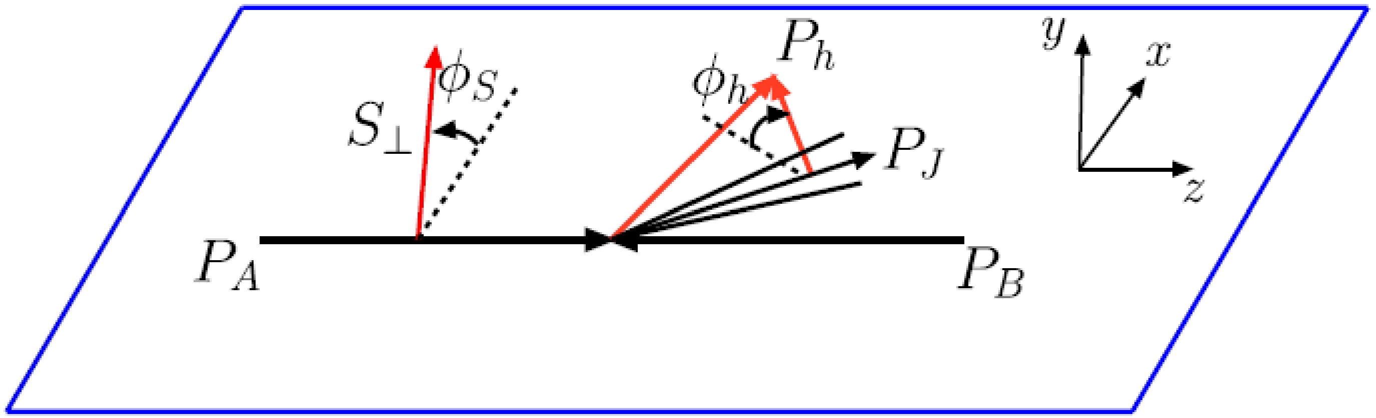
✔✘ TMD Factorization may hold for this channel

✔ Contribution from Collins effect

PP and measurement of Collins FF

Collins asymmetry can be studied in pp: $p(P_A, S_\perp) + p(P_B) \to \text{jet}(P_J) + X \to H(P_h) + X$

Feng Yuan (2008)



$$\frac{d\sigma}{d\text{PS}} = \frac{d\sigma_{UU}}{d\text{PS}} + |S_\perp| \frac{|P_{hT}|}{M_h} \sin(\phi_h - \phi_s) \frac{d\sigma_{TU}}{d\text{PS}}$$

$$d\sigma_{TU} \propto h_1(x_A) \otimes f_1(x_B) \otimes H_1^\perp(z)$$

PennState Berks

Collinear methods:
Collins effect and dihadron method and twist-3

Transversity and Collins FF how measure?

Dihadron method

✔✘ Factorization proven at one loop

✔ Collinear quantities

See talks by Alessandro Bacchetta, Marco Radici

Twist-3

✔✘ More contributions than in case of TMD. Methods to separate contributions are needed

✔ Twist-3 functions are related to TMD functions

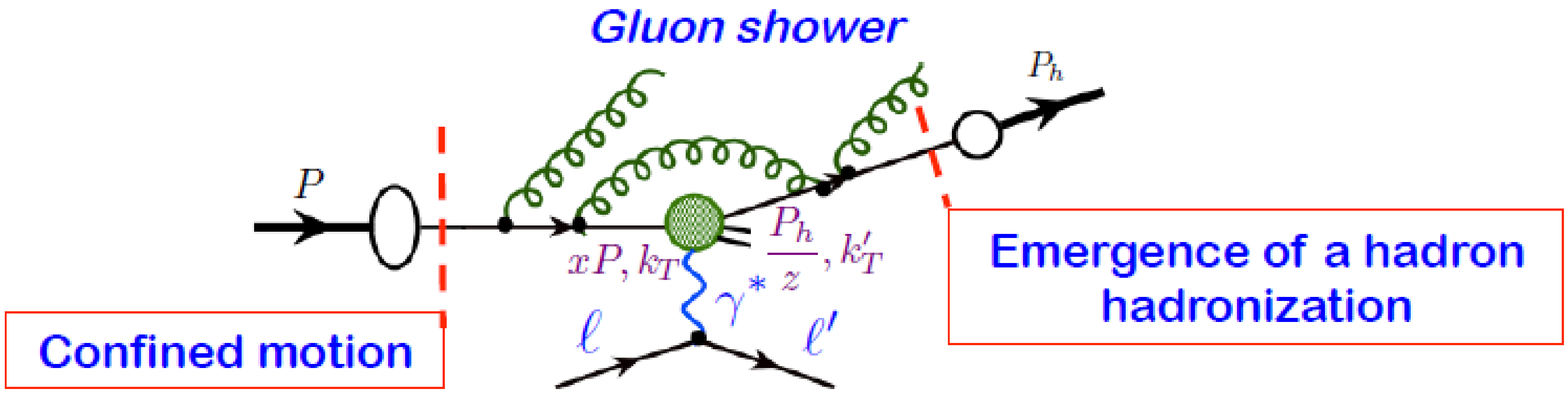
✔✘ Evolution generally involve two arguments making it more difficult to implement

See talk by Andreas Metz

Why evolution is interesting?

Why QCD evolution is interesting?

Study of evolution gives us insight on different aspects and origin of confined motion of partons, gluon radiation, parton fragmentation



Evolution allows to connect measurements at very different scales

→Note that DGLAP evolution gives definite predictions that depend only on initial conditions TMD evolution is more complex and include non perturbative functions, thus results of application of TMD evolution will strongly depend on the choice of non perturbative input

TMD evolution

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

John Collins, 2011

$$\frac{d\tilde{K}(b_T, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

See talks by John Collins,
Ignazio Scimemi

$$\frac{d \ln \tilde{F}(x, b_T, \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu), \zeta)$$

TMD evolution

TMD functions are measured at scale Q $\quad f(x,k_\perp;Q)$

Evolution is performed in Fourier space

$$\tilde{f}(x,b_T;Q) = \int d^2k_\perp e^{-ik_\perp b_T} f(x,k_\perp;Q)$$

Standard CSS formalism, evolution starts from

$$\mu_b = c/b_T,\ c = 2e^{-\gamma_E}$$

in order to allow for OPE of TMDs and relation to collinear functions

$$\tilde{f}(x,b_T;Q) = \tilde{f}(x,b;\mu_b)e^{-S_{pert}(b_T)}$$

$$S_{pert}(b) = \int_{\mu_b}^{Q} \frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2} + B\right)$$

Perturbative Sudakov factor, serves to evolve TMD from scale μ_b to Q

$$A = \sum_{n=1}\left(\frac{\alpha_s}{\pi}\right)^n A^{(n)} \qquad B = \sum_{n=1}\left(\frac{\alpha_s}{\pi}\right)^n B^{(n)}$$

TMD evolution

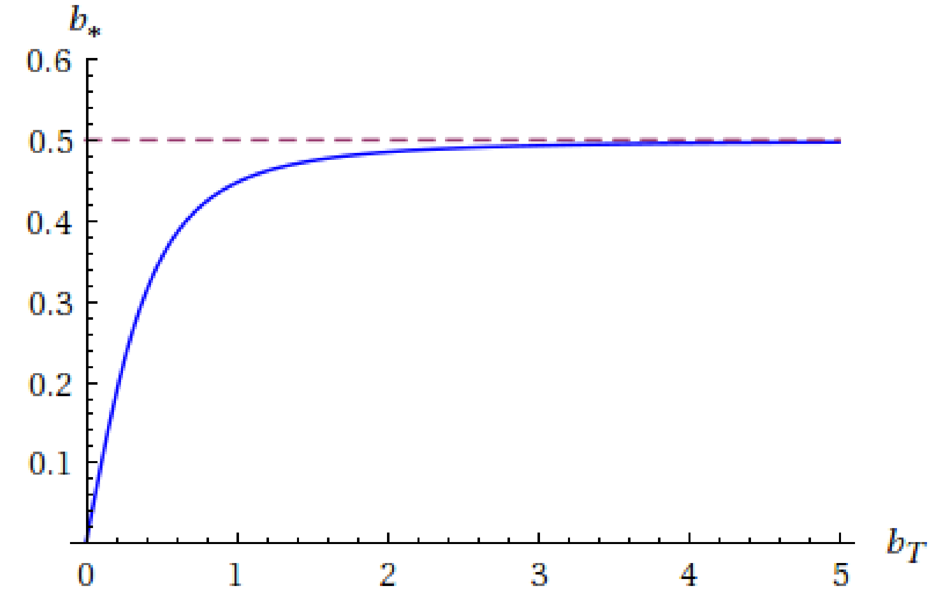
Calculation is perturbative, valid only in region $b \ll 1/\Lambda_{QCD}$

Fourier transform in momentum space involves non-perturbative region

$$f(x,k_\perp;Q) = \int_0^\infty \frac{bdb}{2\pi} J_0(k_\perp b)\tilde{f}(x,b;Q)$$

Non perturbative region needs to be treated.
Common method b_* prescription

$$b_* = \frac{b}{\sqrt{1+b^2/b_{max}^2}}$$



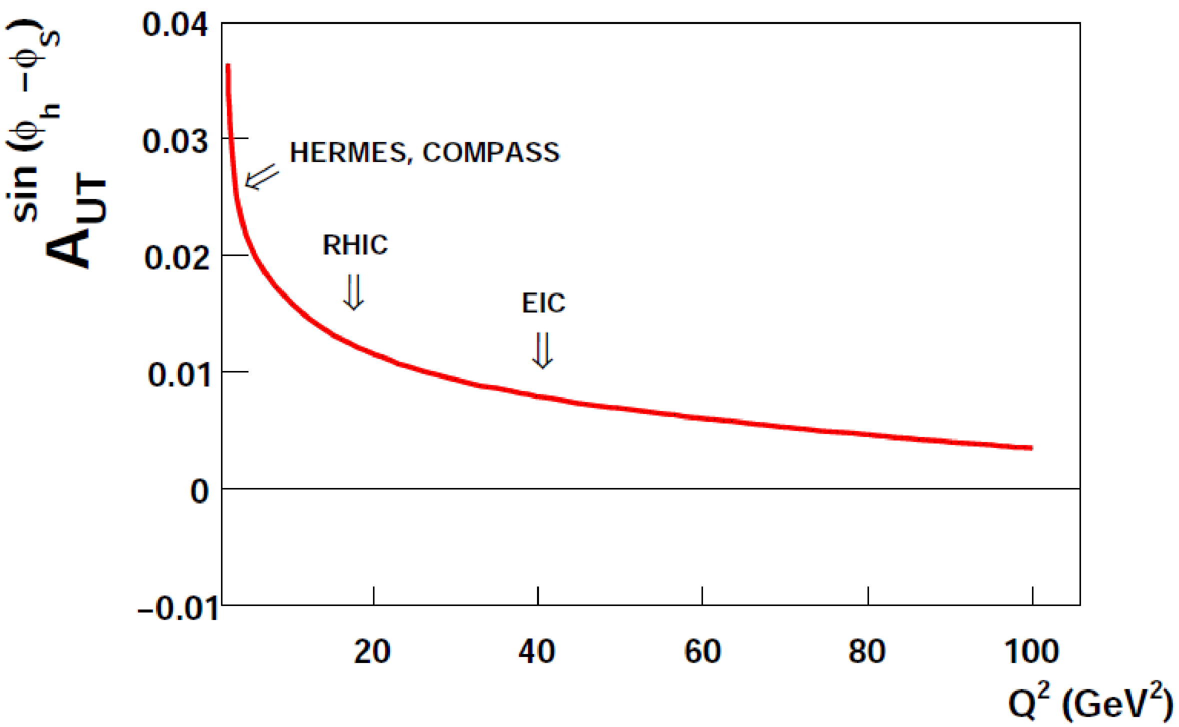
$$\tilde{f}(x,b;Q) = \tilde{f}(x,b_*;c/b_*)e^{-S_{pert}(b_*)}e^{-S_{NP}(b)}$$

Non perturbative Sudakov factor

Is evolution “fast” or “slow”?

Evolution depends of non perturbative Sudakov form factor

Aybat, Prokudin, Rogers (2011)



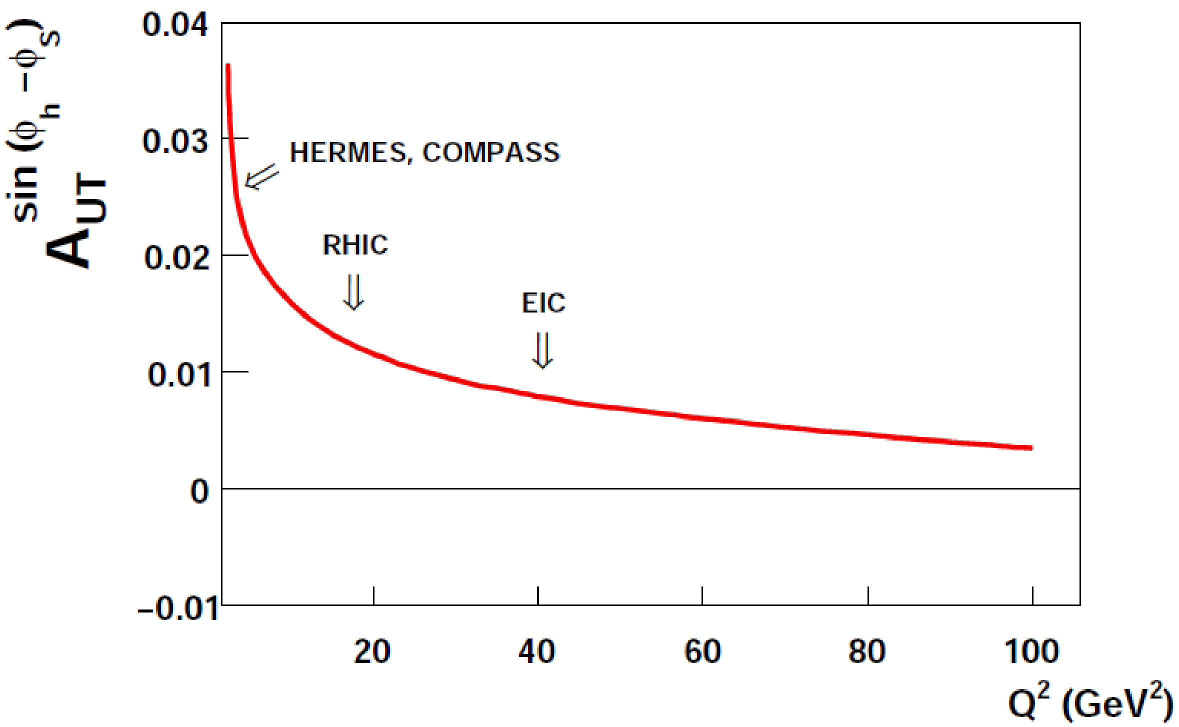
$$S_{NP} \sim g_2 b^2 \ln(Q^2/Q_0^2)$$

g_2 evaluated from high-energy data

Is evolution "fast" or "slow"?

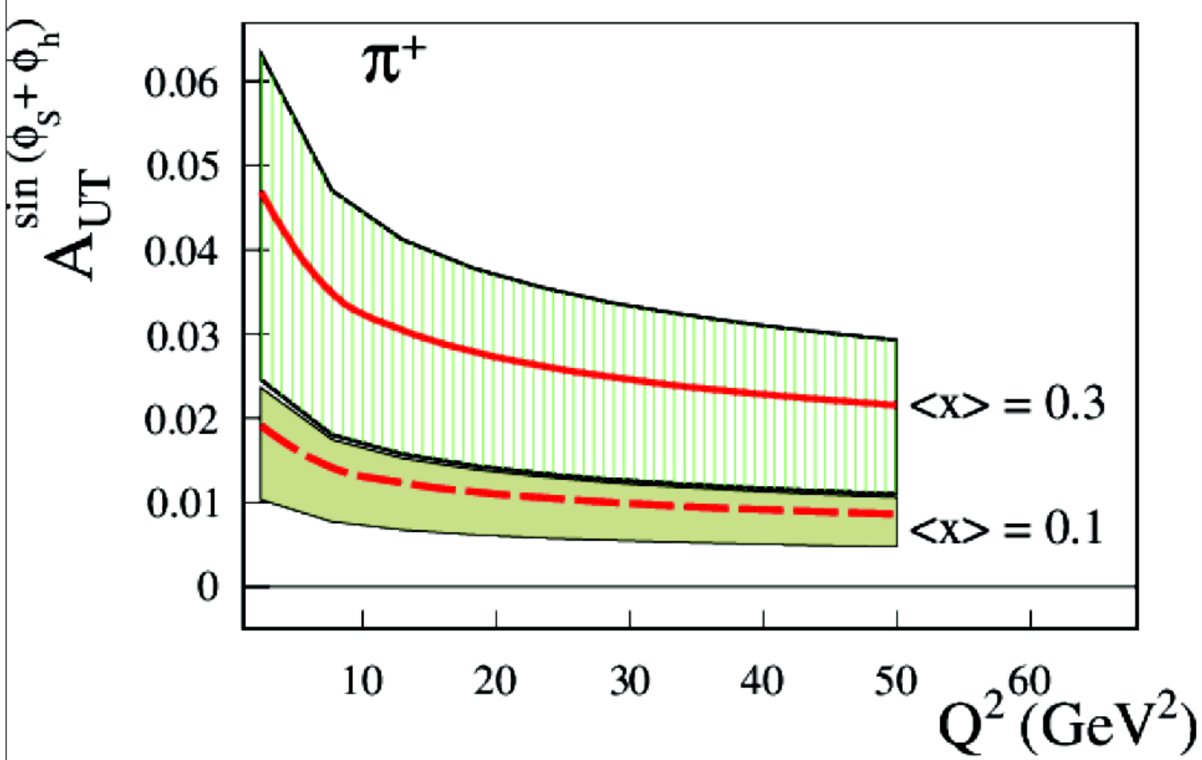
Evolution depends of non perturbative Sudakov form factor

Aybat, Prokudin, Rogers (2011)



$$S_{NP} \sim g_2 b^2 \ln(Q^2/Q_0^2)$$

Kang-Prokudin-Sun-Yuan 2014

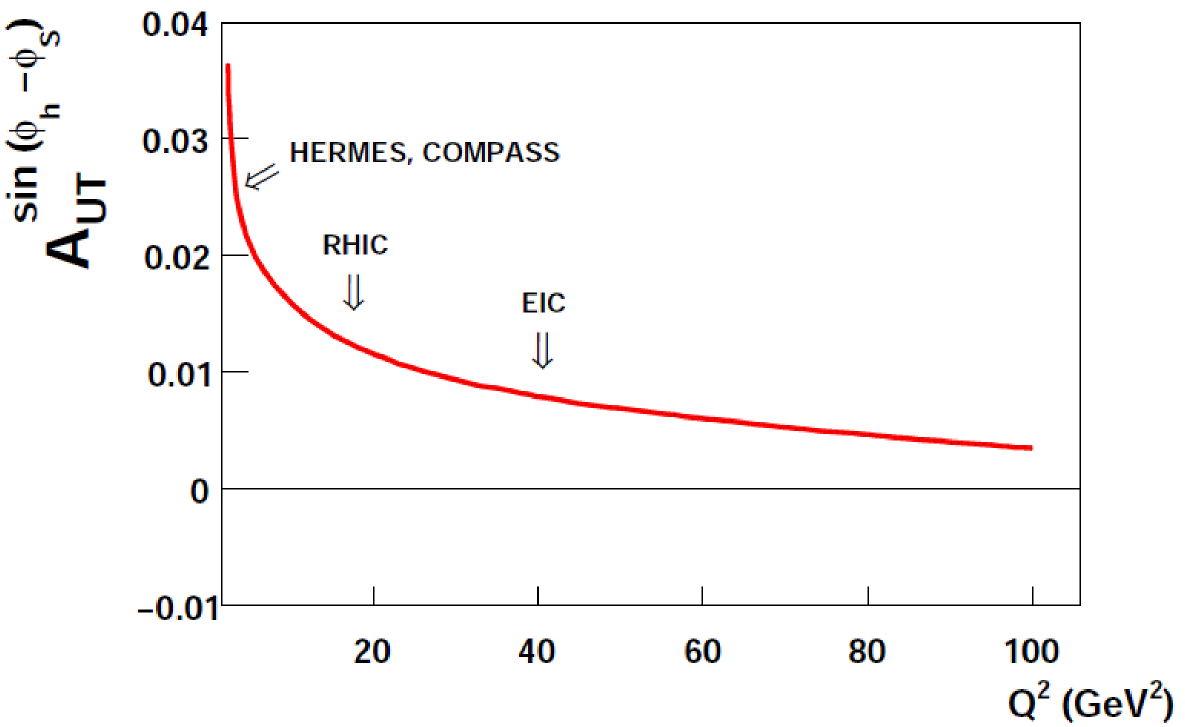


$$S_{NP} \sim g_2 \ln(1 + b^2/b_{max}^2) \ln(Q^2/Q_0^2)$$

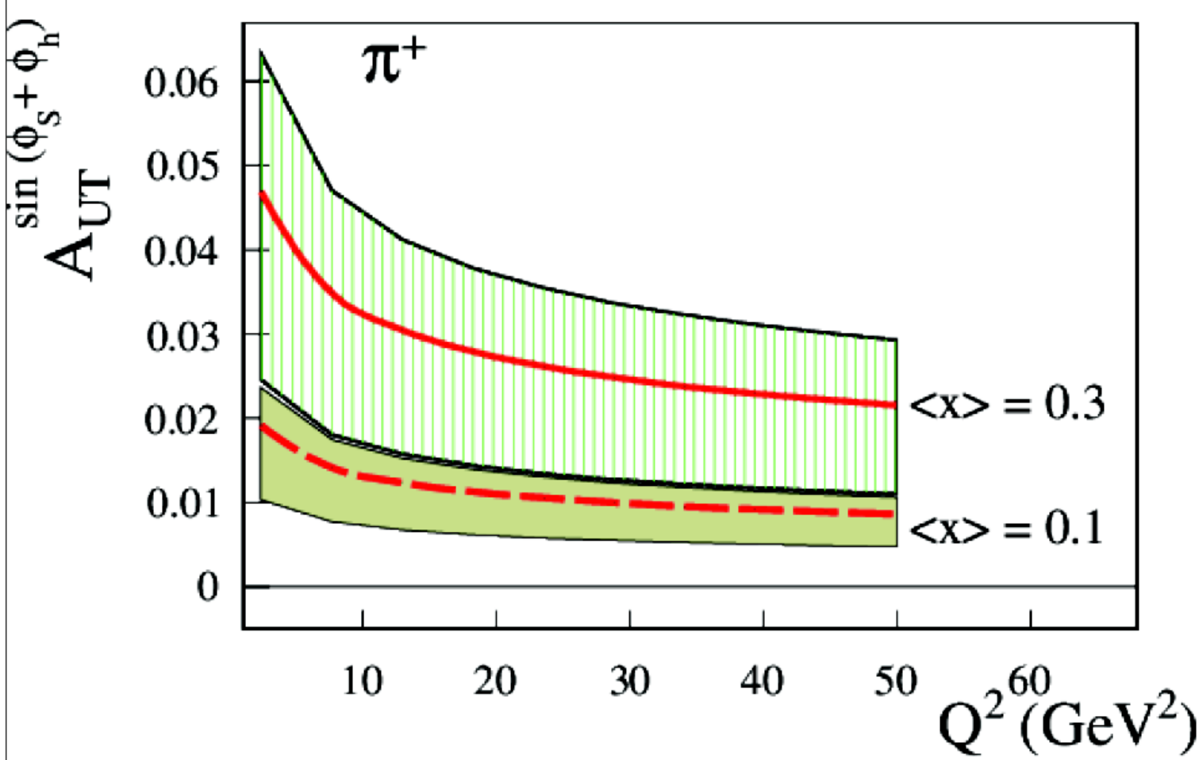
g_2 evaluated from low and high-energy data

Is evolution “fast” or “slow”?

Aybat, Prokudin, Rogers (2011)



Kang-Prokudin-Sun-Yuan 2014



$S_{NP} \sim g_2 b^2 \ln(Q^2/Q^2_0)$ ... $\ln(Q^2/Q_0^2)$

...rgy data

The same evolution equations may result in different numerical results.
Global data analysis is needed.

TMD evolution

Relation to collinear functions at small values of b:

$$\tilde{f}^{j}(x, b_*; c/b_*) = \sum_{j'=q,g} \int_x^1 \frac{d\hat{x}}{\hat{x}} C_{j/j'}\left(\frac{x}{\hat{x}}, b_*; c/b_*\right) f^{j'}(x; c/b_*)$$

$$C = \sum_{n=1} \left(\frac{\alpha_s}{\pi}\right)^n C^{(n)}$$ Wilson coefficient

Collinear PDF

For transversity and helicity TMDs: Bacchetta-Prokudin 2013

For Collins function (relation to collinear **twist-3** function): Yuan-Zhou 2009, Kang 2011

In future also gluon functions will be important

For gluon twist-3 function: Dai-Kang-Prokudin-Vitev 2014

Taking into account Wilson coefficients is very important!
Large K factors of collinear computations between LO and NLO!

TMD evolution in a nut shell

Precision of extraction depends on precision of calculations

Leading Log (LL):	$A^{(1)}$		
Next-to Leading Log (NLL):	$A^{(1,2)}$	$B^{(1)}$	$C^{(1)}$
Next-to-Next-to Leading Log (NNLL):	$A^{(1,2,3)}$	$B^{(1,2)}$	$C^{(1)}$

Precision is important!

$C^{(1)}$ means that one should use NLO collinear distributions

Transversity at NLL

Kang-Prokudin-Sun-Yuan 2014

First NLL' extraction from the data

$$A^{(1,2)} \quad B^{(1)} \qquad {\color{red}C^{(1)}}$$

Collins function is related to twist-3 function

$$\tilde{H}_1^{\perp,\alpha}(z_h, b; \mu_b) \sim \left(\frac{-ib^\alpha}{2z_h}\right) H^{(3)}(z_h; \mu_b)$$

We solve also DGLAP equations for transversity and (diagonal) Collins FF

Diagonal part for twist-3 Collins function is:

Yuan-Zhou 2009, Kang 2011

$$\frac{d}{d\ln\mu^2} H_q^{(3)}(z_h, \mu) = \frac{\alpha_s}{2\pi} P_{i\to q}^H \otimes H_i^{(3)}$$

$$P_{i\to q}^{H_1}(\hat{x}) = \delta_{iq} C_F \left(\frac{2\hat{z}}{(1-\hat{z})_+} + \frac{3}{2}\delta(1-\hat{z})\right)$$

SIDIS data used: HERMES, COMPASS, JLAB – 140 points

e+e- data used: BELLE, BABAR including PT dependence – 122 points

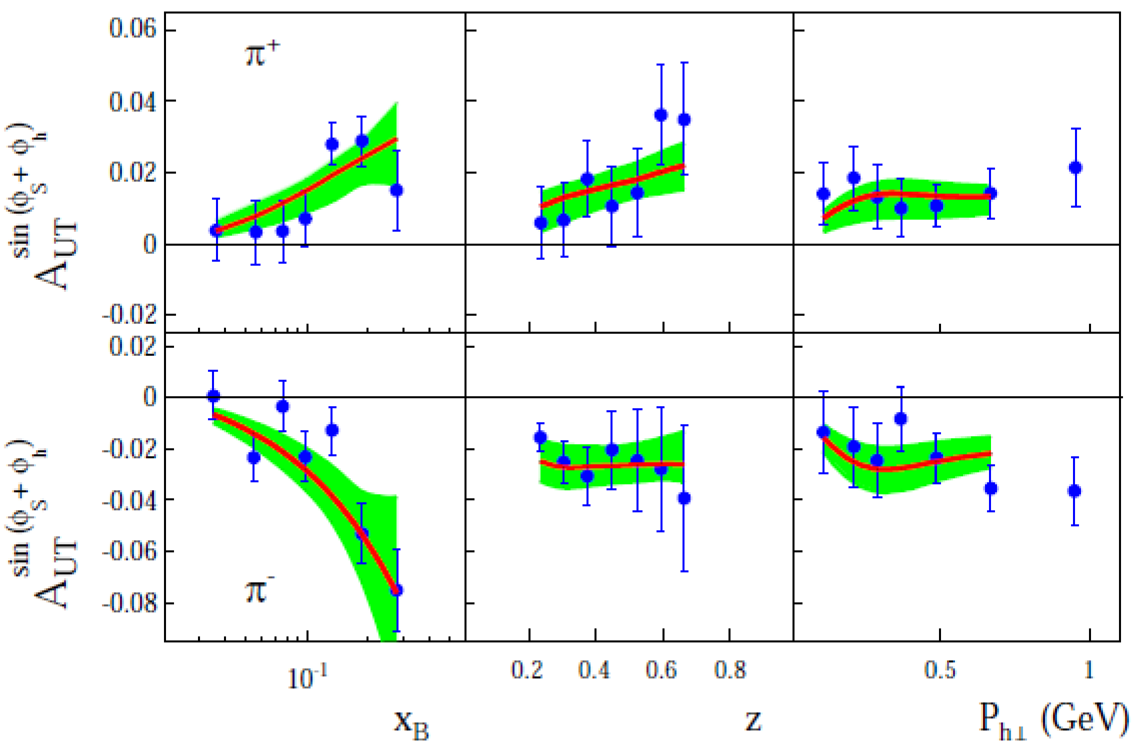
$$\chi^2/\mathrm{d.o.f.} \simeq 0.88$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

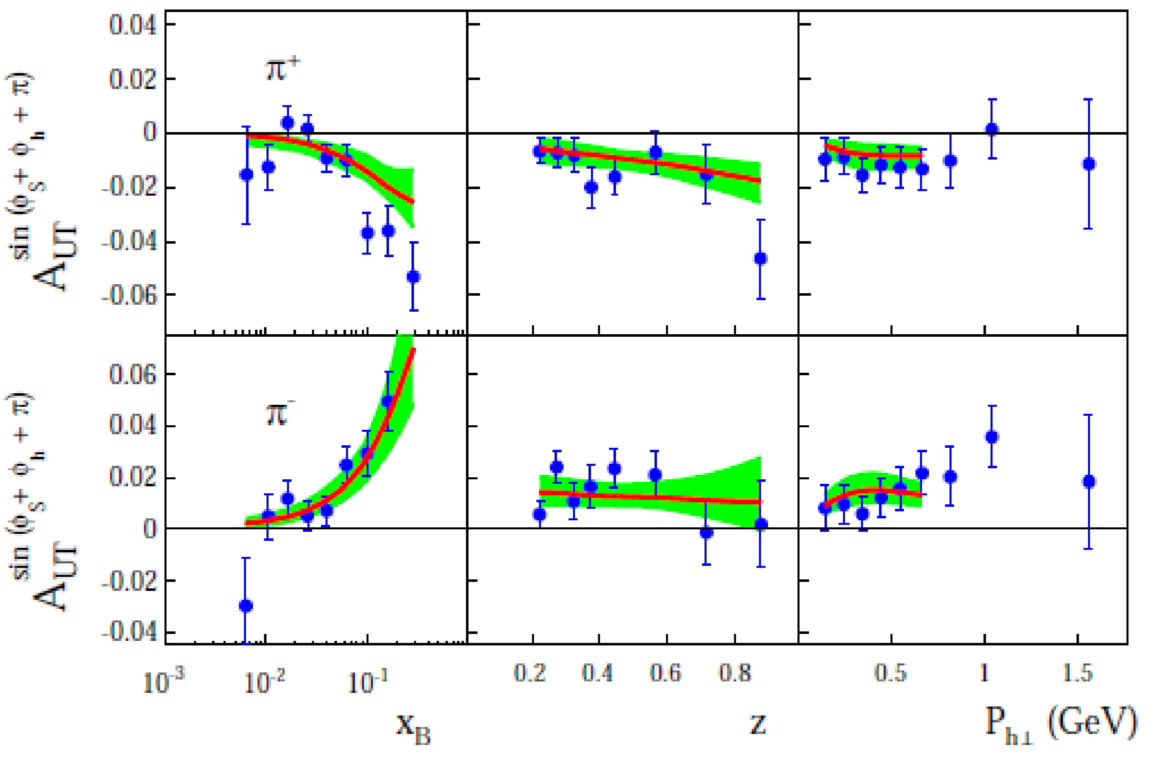
$$\ell P \to \pi^{\pm} X$$

HERMES



$$1 \lesssim \langle Q^2 \rangle \lesssim \ 6 \ \mathrm{GeV}^2$$

COMPASS



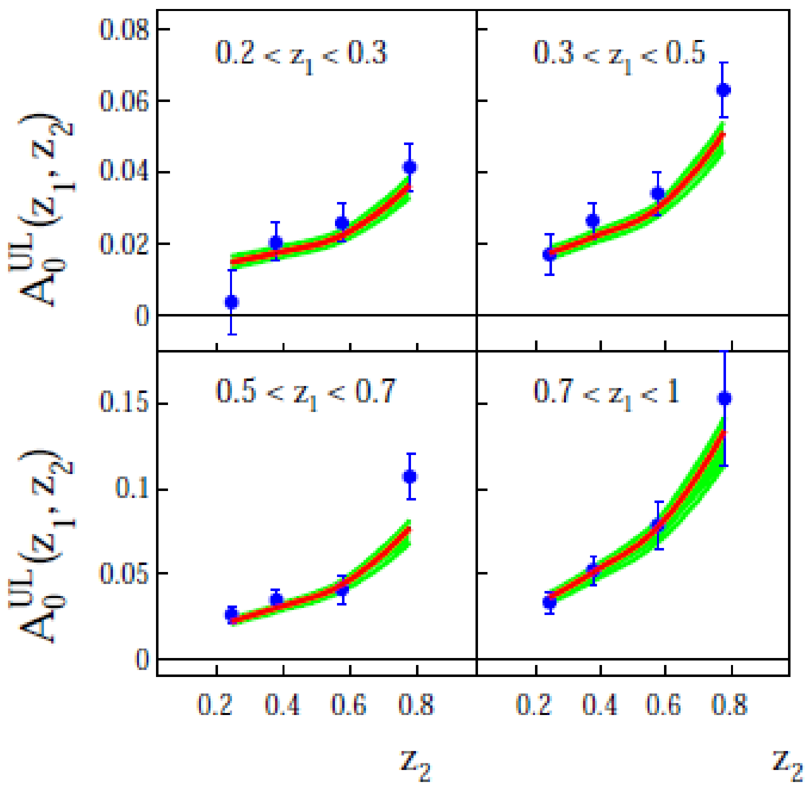
$$1 \lesssim \langle Q^2 \rangle \lesssim \ 21 \ \mathrm{GeV}^2$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

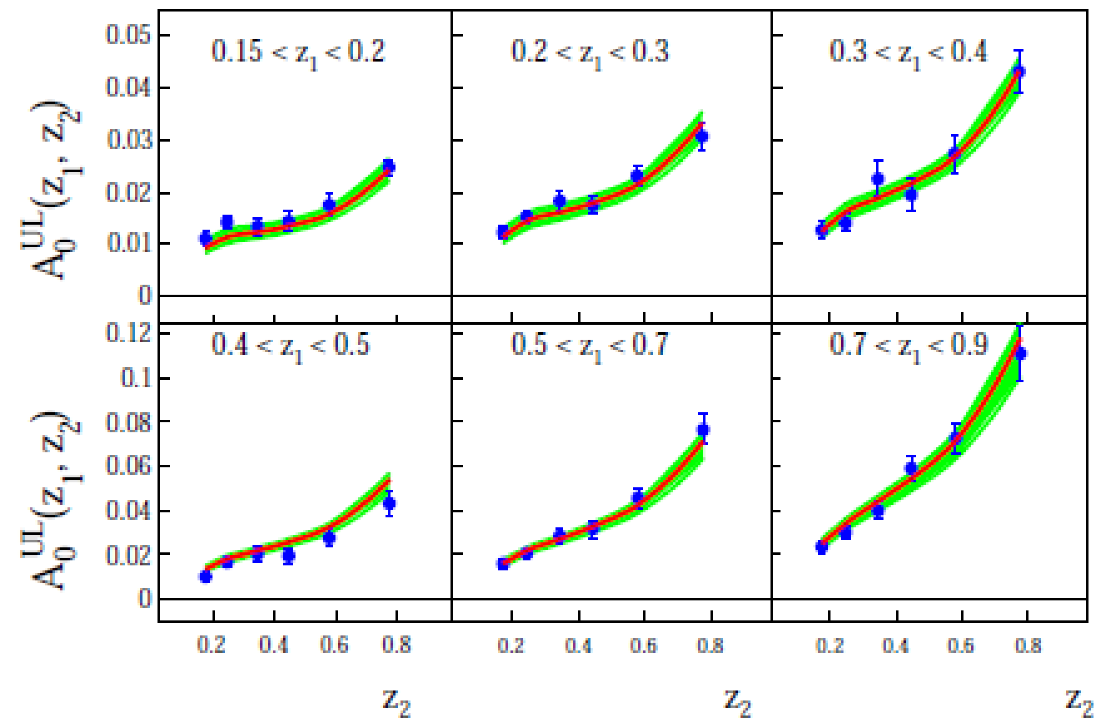
$$e^+e^- \to \pi\pi X$$

BELLE



$$Q^2 = 110\ \mathrm{GeV}^2$$

BABAR

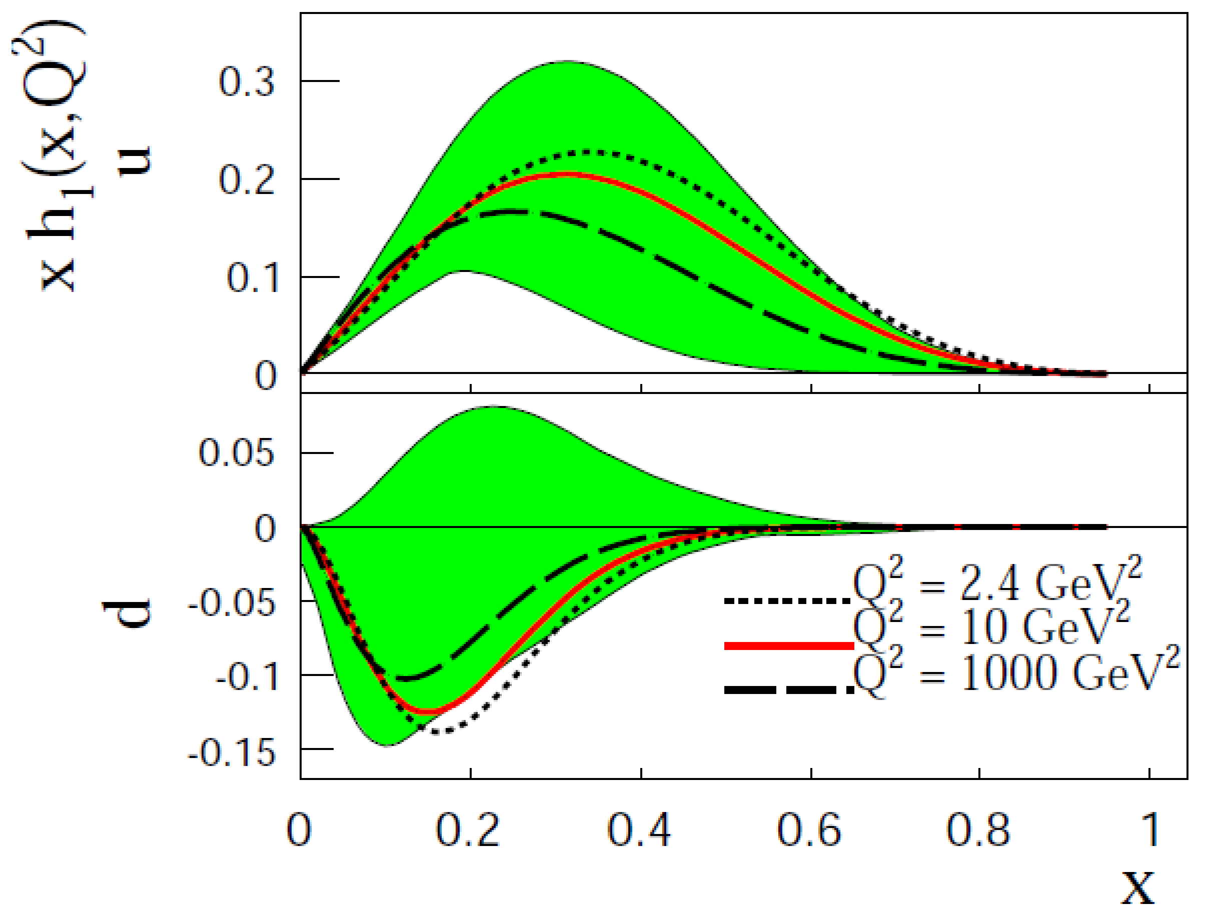


$$Q^2 = 110\ \mathrm{GeV}^2$$

Transversity and Collins FF

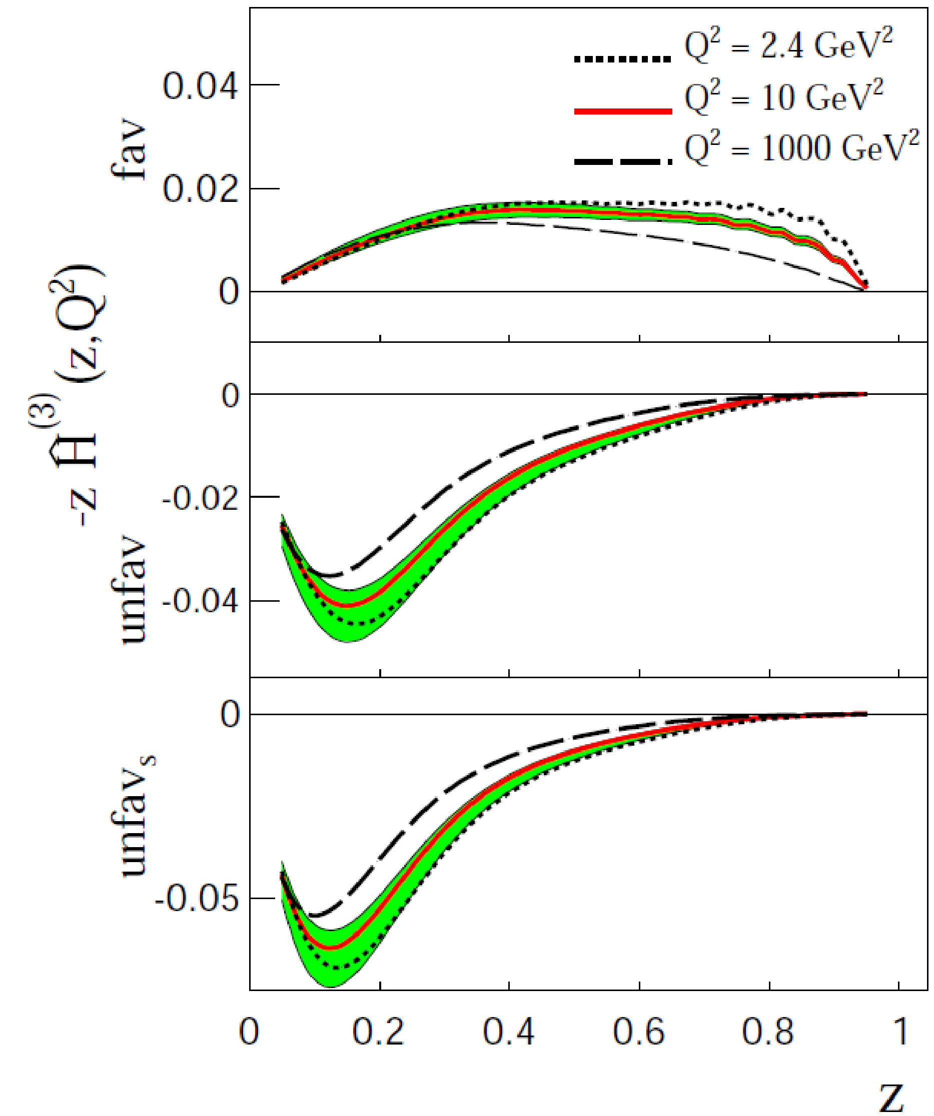
Kang-Prokudin-Sun-Yuan 2014

Transversity



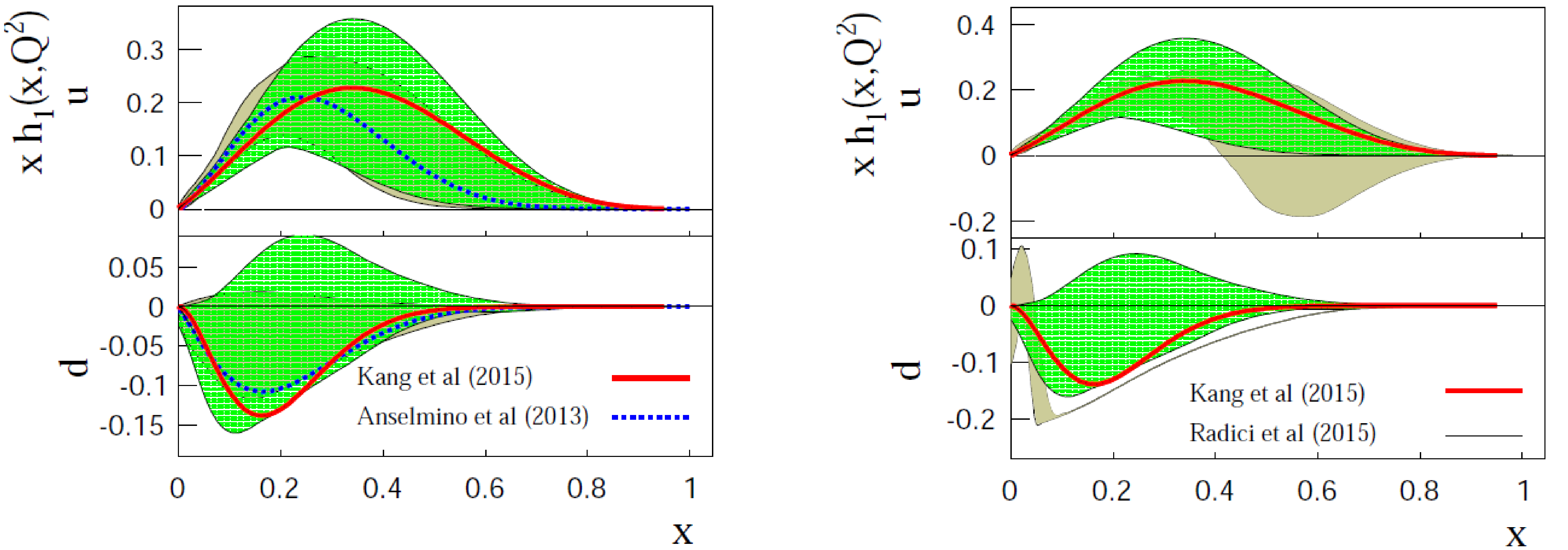
Positive u and negative d transversity

Collins



Positive favoured and negative unfavoured Collins FF

Transversity

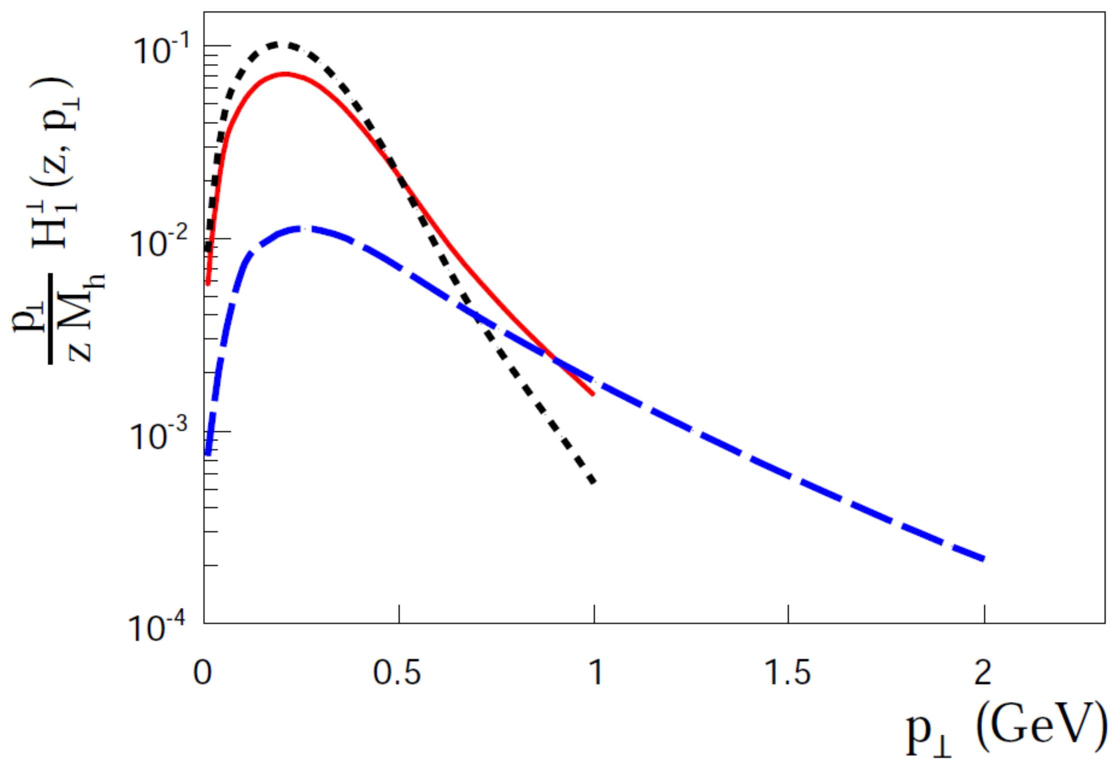


Compatible with LO extraction Anselmino et al 2013, Radici et al (2015)

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2014

It does not mean LO fit is impossible

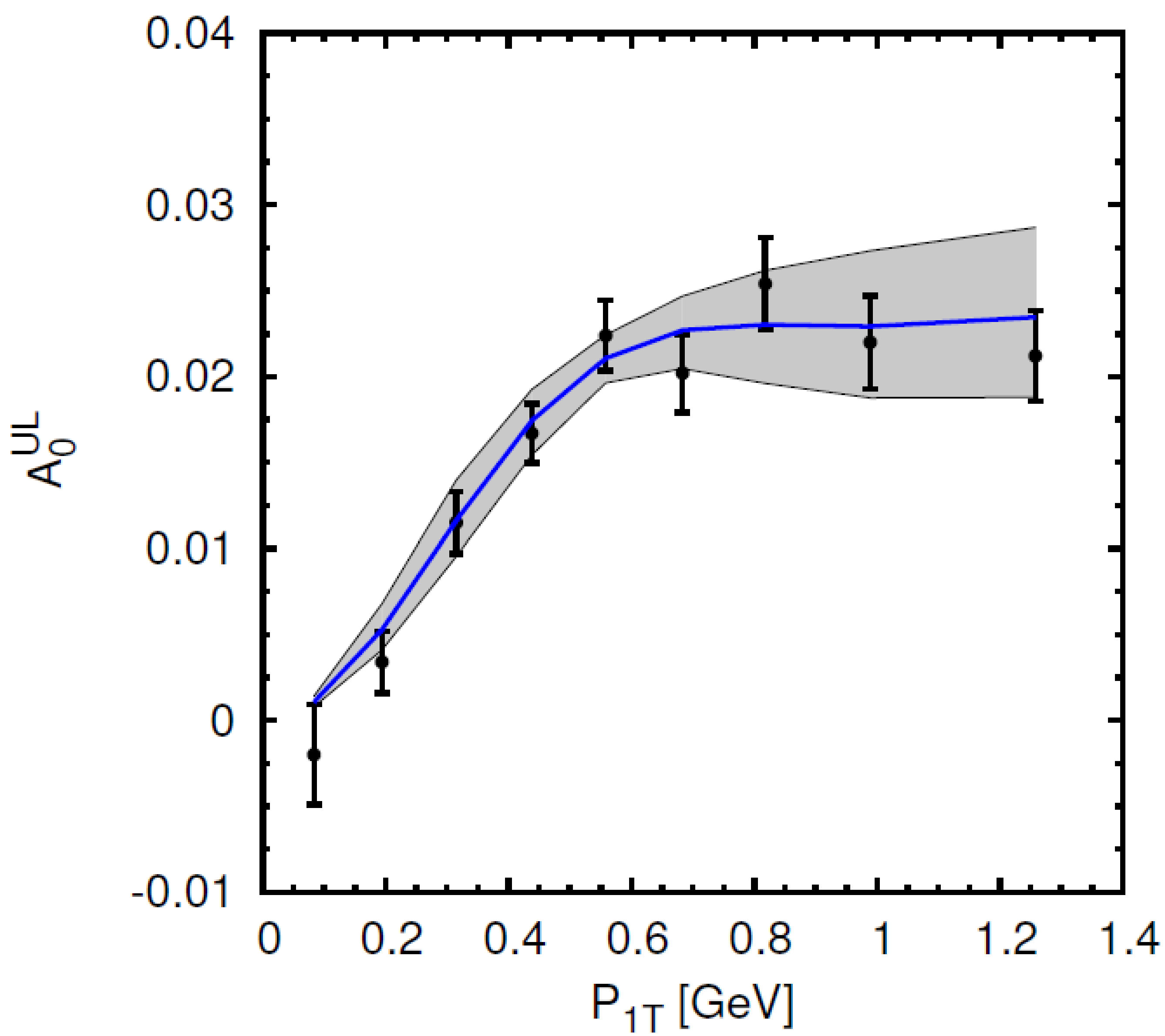


TMD evolution:

$Q^2 = 2.4\ \mathrm{GeV}^2$

$Q^2 = 10\ \mathrm{GeV}^2$

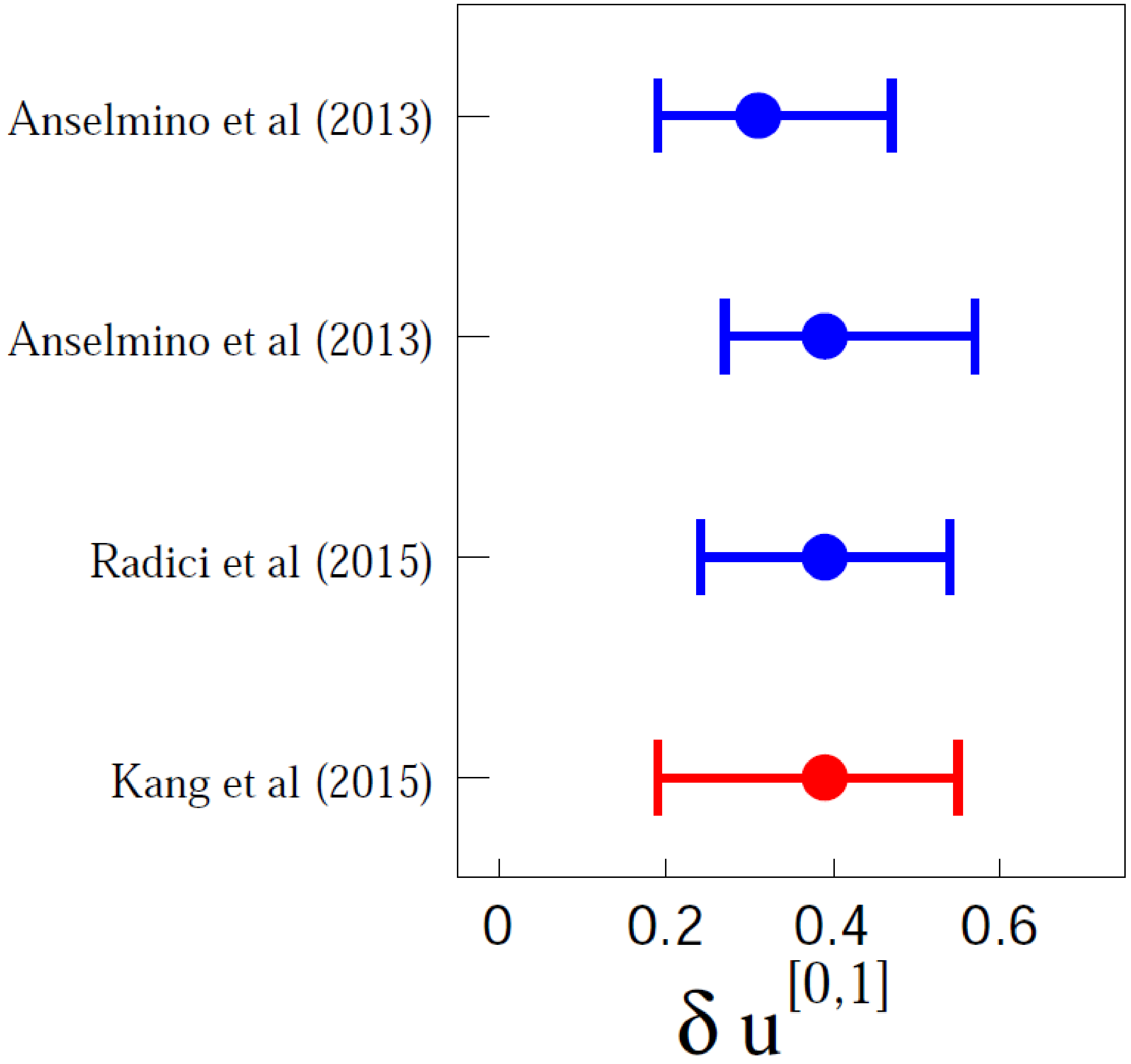
$Q^2 = 1000\ \mathrm{GeV}^2$

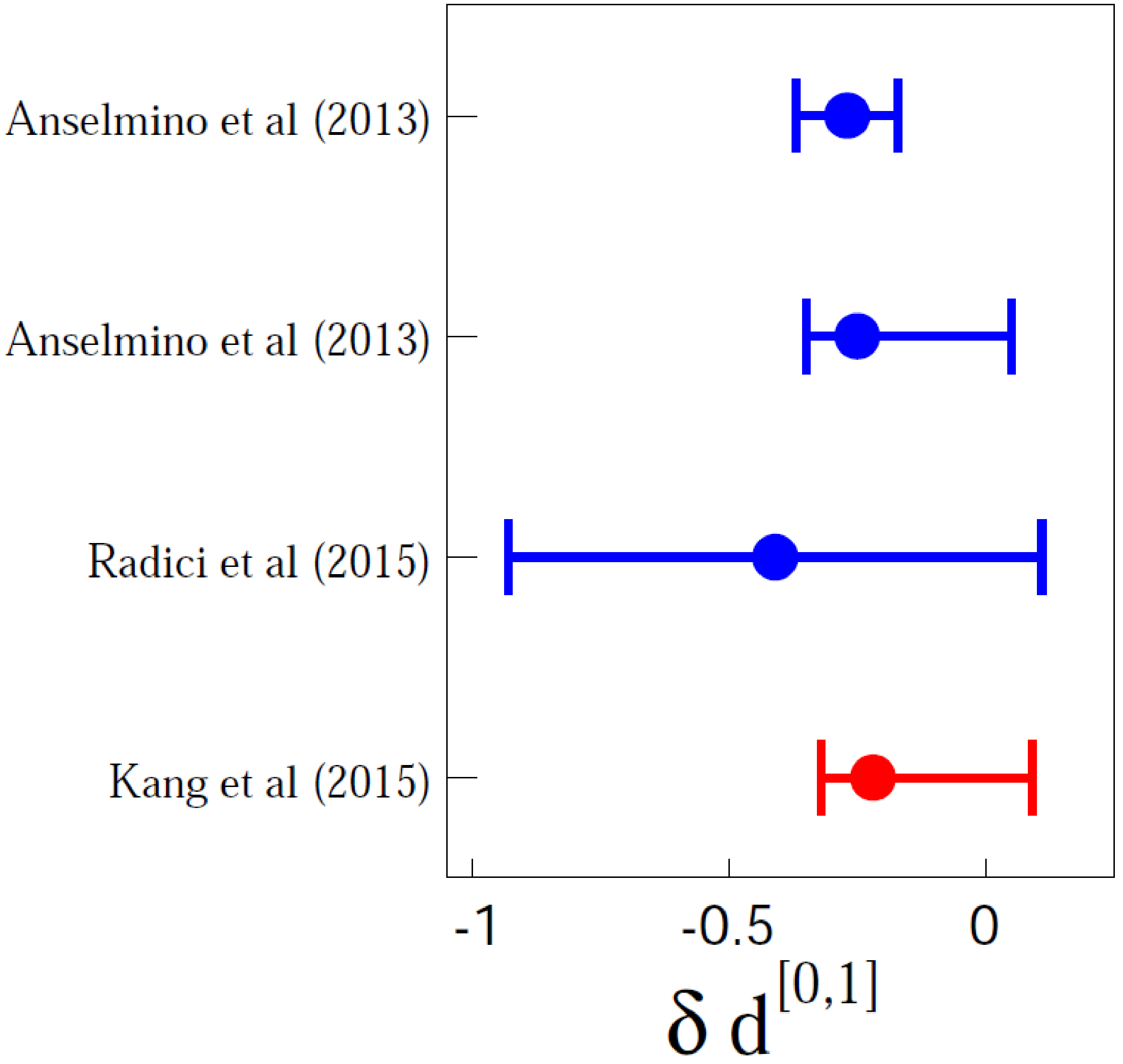


Anselmino et al (2015)

No TMD evolution, LO fit

Tensor charge





Collins effect in pp

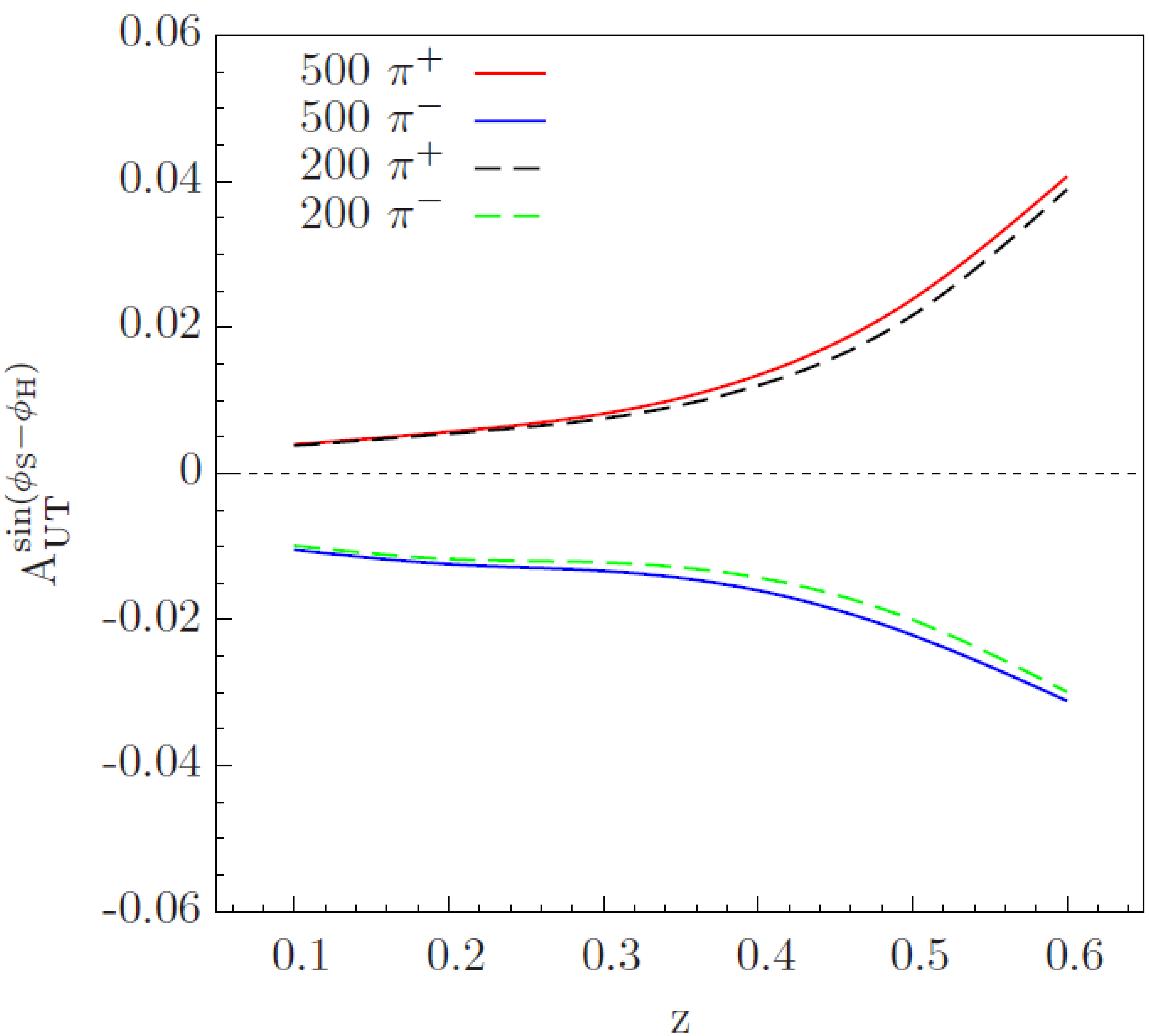
Hadron in jet asymmetry

Courtesy of Zhongbo Kang

Cross section of STAR measurement:

$$\frac{d\sigma}{dy d^2 p_\perp^{\rm jet} dz d^2 j_T} = F_{UU} + \sin(\phi_S - \phi_H) F_{UT}^{\sin(\phi_S - \phi_H)}$$

$$A_{UT}^{\sin(\phi_S - \phi_H)}(p_\perp^{\rm jet}, z, j_T) = \frac{F_{UT}^{\sin(\phi_S - \phi_H)}}{F_{UU}}$$



No TMD evolution

$$\langle j_T^2 \rangle = 0.12 \ (GeV^2)$$

Anselmino et al (2015)

Rapidity $0 < y < 1$

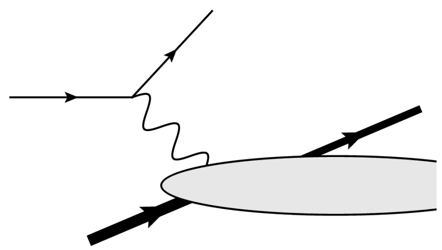
Jet transverse momenta

$$\langle p_\perp^{jet} \rangle = 12.9 \ GeV @ \sqrt{s} = 200 \ GeV$$

$$\langle p_\perp^{jet} \rangle = 31 \ GeV @ \sqrt{s} = 500 \ GeV$$

Complementarity of SIDIS, e+e- and Drell-Yan, and hadron-hadron

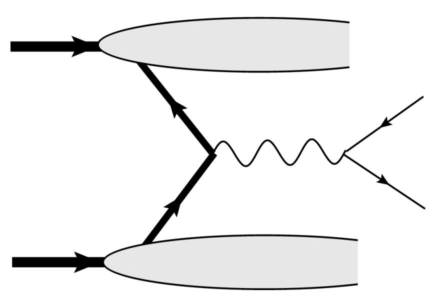
Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them



$$f(x) \otimes D(z)$$

Semi Inclusive DIS –
convolution of distribution functions and fragmentation functions

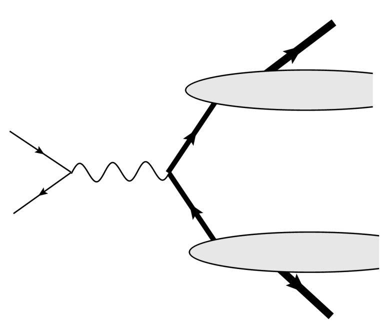
$$\ell + P \to \ell' + h + X$$



$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

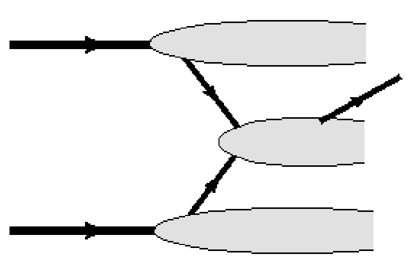
$$P_1 + P_2 \to \bar{\ell}\ell + X$$



$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \to h_1 + h_2 + X$$



$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and fragmentation functions

$$h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

Combining measurements from all above is important