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TMD Fragmentation and PDF at NNLO in QCD

In collaboration with
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arXiv:1509.06392
and arXiv:1511.05590
and work in progress

Status and Outline

DY, SIDIS, ee to 2j, TMD factorization

- Transverse momentum distributions involve non-perturbative QCD effects which go beyond the usual PDF formalism. New factorization theorem is required. (Collins '11; Echevarría, Idilbi, S. '12 (EIS))

$q^2 = Q^2 \gg q_T^2$ Q=M=di-lepton invariant mass

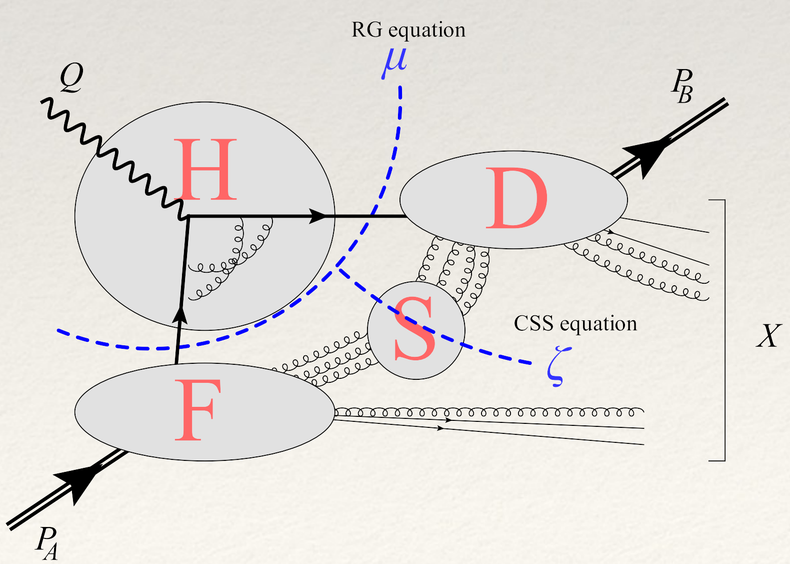
$$q_T^2 \sim \Lambda_{QCD}^2 \longrightarrow \tilde{M} = H(Q^2/\mu^2)\, \tilde{F}_n(x_n, b; Q^2, \mu^2)\, \tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$$

TMDs offer a description of very different experiments which share a common QCD bulk

Example: DY type experiments → TMDPDFs

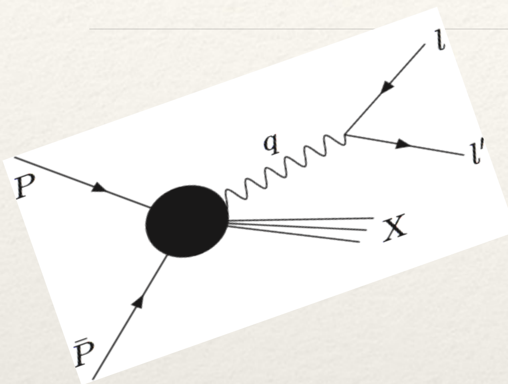
SIDIS → TMDPDF and TMDFF

e+e- to 2j → TMDFFs



TMDs are defined in b-space: in momentum space, they are correlation functions of parton momenta/spin

TMD's factorization: principles and formalism



Factorized hadronic tensor

$$q^2 = Q^2 \gg q_T^2$$

Q=M=dilepton invariant mass

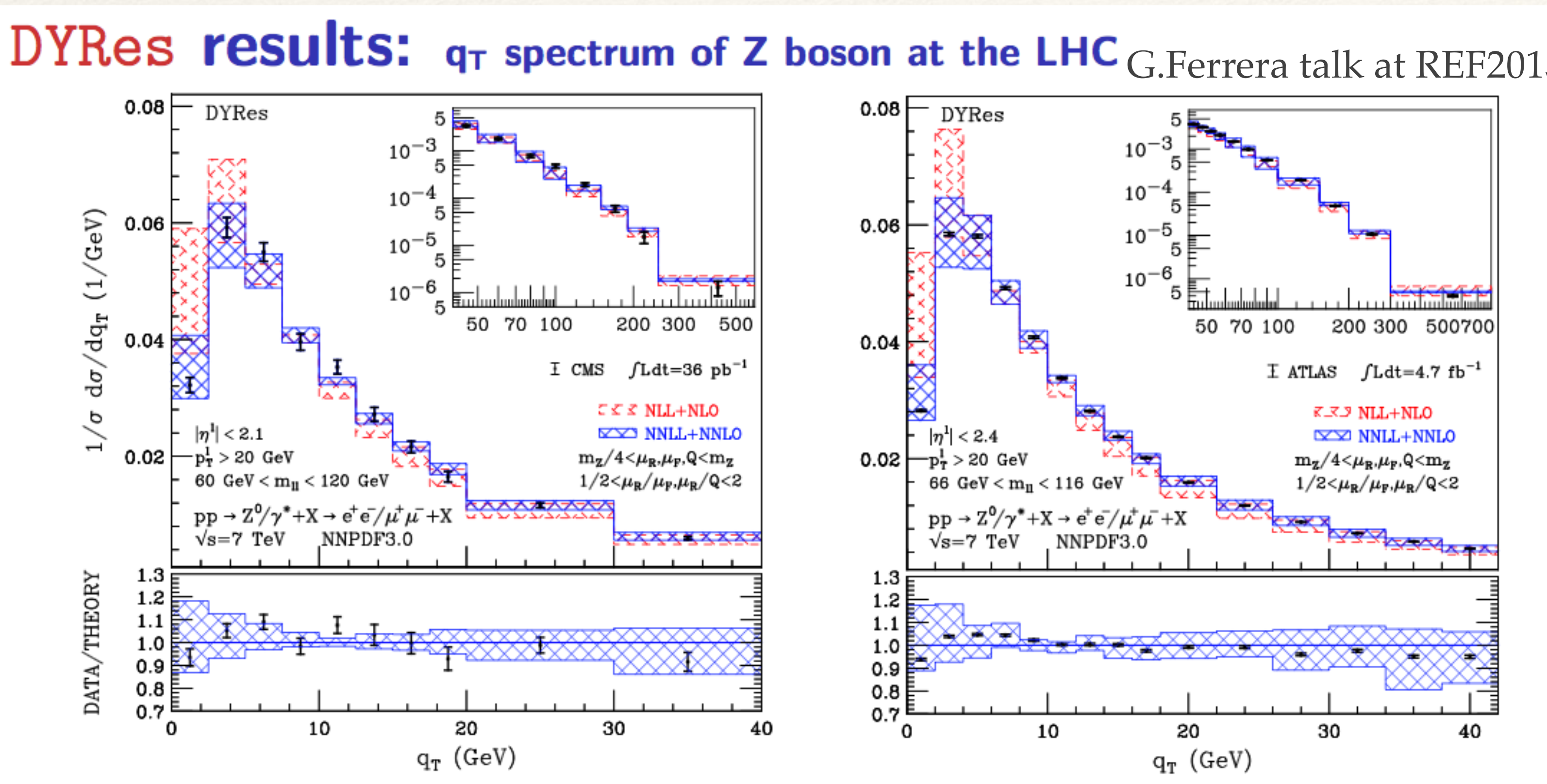
$q_T^2 \sim \Lambda^2_{QCD}$ $\Rightarrow$ $\tilde{M} = H(Q^2/\mu^2)\,\tilde{F}_n(x_n, b; Q^2, \mu^2)\,\tilde{F}_{\bar{n}}(x_{\bar{n}}, b; Q^2, \mu^2)$

$q_T^2 \gg \Lambda^2_{QCD}$ $\Rightarrow$ $\tilde{M} = H(Q^2/\mu^2)\,\tilde{C}_n(b^2\mu^2, Q^2/\mu^2)\,\tilde{C}_{\bar{n}}(b^2\mu^2, Q^2/\mu^2)\,f_n(x_n;\mu^2)\,f_{\bar{n}}(x_{\bar{n}};\mu^2)$

The factorization theorem predicts that each coefficient
can be extracted on its own:
this checked only at 1 loop up to now

All coefficients are extracted matching effective field theories. During the matching the IR parts have to be regulated consistently above and below the matching scales

An example:Z-production at LHC

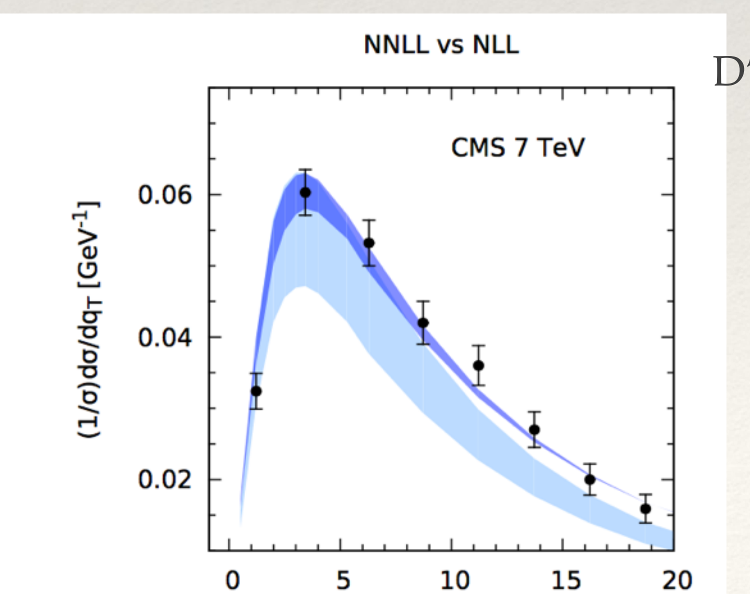


G.Ferrera talk at REF2015

Main differences

DYRes includes full 2-loop matching onto PDFs, but no low energy DY infos. Minimal Subtraction.

D`Alesio et al 2014, partially includes the 2-loop matching and full low energy DY infos (2 parameters)



D’ Alesio et al. 2014

A TMD approach has the potential to drastically reduce errors! (including information from other experiments)

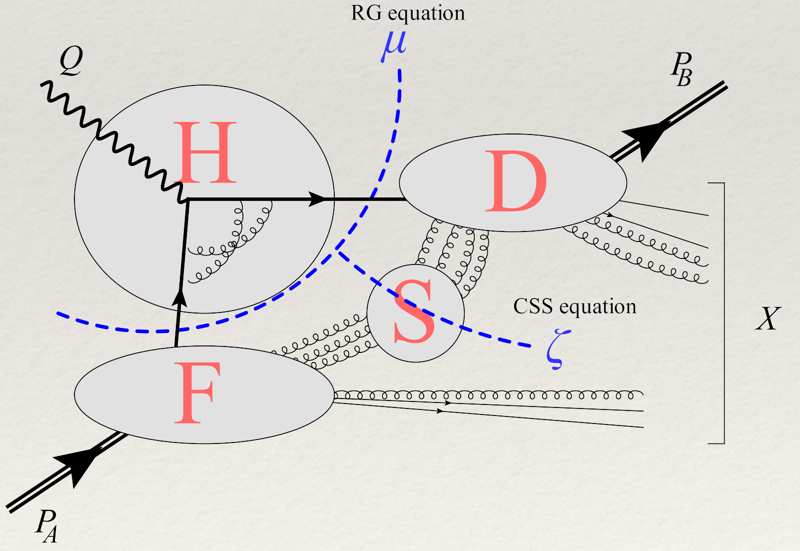
Caveat: THE COMPLETE NNLO PERTURBATIVE QCD INFORMATION MUST BE INCLUDED

DY, SIDIS, ee to 2j, TMDs status

A complete analysis of the TMDs requires:

- A complete knowledge of the perturbative structure of TMDs at NNLO: the perturbative knowledge should be maximally implemented in programs (Required to match the LHC and future colliders program).
- Consequently, a correct estimate of perturbative QCD errors and understanding of model dependence (see for instance: D' Alesio, Echevarría, Melis, S., JHEP 1411 (2014) 098 and arXiv:1510.0288)

The status of perturbative knowledge at NNLO is not homogeneous (even for the unpolarized case!)



Construction of (un)polarized TMDPDFs

In the asymptotic limit (High Q, qT) of **each** TMDPDF

PDF

$$\tilde{F}_{q/N}(x,b_T;\zeta,\mu) = \left(\frac{\zeta}{C_\zeta \mu_b^2}\right)^{-D^R(b_T;\mu)} \sum_{j=q,\bar{q},g} \tilde{C}_{q/j}(x_A,b_T;C_\zeta\mu_b^2,\mu) \otimes f_{j/N}(x_N;\mu) M(x_N,b_T,\zeta)$$

OPE to PDF, valid ONLY for qT>> Λ_{QCD}

Process independent
Non-perturbative correction

This construction formally recovers the perturbative limit.

2-loop matching of PDFs deduced from the calculation of the cross section [Firenze (Catani et al.2008), Zurich (Gehrmann. et al. 2012-2014)].
M.G. Echevarría, I.S., A. Vladimirov: coming soon!
Direct application of the TMD formalism and agreement with previous works

$$C^{Q}_{g\leftarrow g} = \mathcal{O}(\alpha^0) \quad \tilde{C}^{Q}_{q\leftarrow q} = \mathcal{O}(\alpha_s^0)$$
$$C^{Q}_{g\leftarrow q} = \mathcal{O}(\alpha^1) \quad \tilde{C}^{Q}_{q\leftarrow g} = \mathcal{O}(\alpha_s^1)$$
$$\tilde{C}^{Q}_{q\leftarrow \bar{q}} = \mathcal{O}(\alpha_s^2)$$
$$\tilde{C}^{Q}_{q\leftarrow q'} = \mathcal{O}(\alpha_s^2)$$

Starting order

Status: This formula predicts that one TMDPDF matches onto a sum of PDFs (FLAVOUR MIXING).
Currently all analysis of low energy data have fully exploited this up to first order

Construction of (un)polarized TMDFFs

In the asymptotic limit (High Q, qT) of **each** TMDFF

FF

$$\tilde{D}_{q/N}(z,b_T;\zeta,\mu)=\left(\frac{\zeta}{\mu_b}\right)^{-D(b,\mu)}\sum_j\int_z^1\frac{d\tau}{\tau^{3-2\varepsilon}}C^{\not{Q}}_{q\to j}\left(\frac{z}{\tau},b_T;\mu_b,\mu\right)d_{j/N}(\tau,\mu)M_j(\tau,b_T,\zeta)$$

OPE to FF, valid ONLY for qT>> Λ_{QCD}

Process independent
Non-perturbative correction

This construction formally recovers the perturbative limit.

M.G. Echevarría, I.S., A.Vladimirov
arXiv:1509.06392 and work in progress

$$C^{\not{Q}}_{g\to g}=\mathcal{O}(\alpha^0)\qquad C^{\not{Q}}_{q\to q}=\mathcal{O}(\alpha_s^0)$$

$$C^{\not{Q}}_{g\to q}=\mathcal{O}(\alpha^1)\qquad C^{\not{Q}}_{q\to g}=\mathcal{O}(\alpha_s^1)$$

$$C^{\not{Q}}_{q\to\bar{q}}=\mathcal{O}(\alpha_s^2)$$

$$C^{\not{Q}}_{q\to q'}=\mathcal{O}(\alpha_s^2)$$

Starting order

$\mathcal{O}(\alpha_s^2)$?

(Recent phenomenological work
Bacchetta et al. , 2015, in ee)

Status: This formula predicts that one TMDFF matches onto a sum of FFs. (FLAVOR MIXING)
Currently all analysis of low energy data have fully exploited this up to first order

TMD structures in SIDIS (EIS formulation)

$$W = H(Q/\mu) \int \frac{db}{(2\pi)^2} D_{A/f}(z_A, b; \mu, \zeta_A) F_{f'/B}(z_B, b; \mu, \zeta_B)$$

- Unpolarized TMDFF case

Only one Soft Function for all spin (in)dependent TMDs!!

$$\Delta^{(0)}_{i\to h}(z, \hat{\boldsymbol{P}}_{h\perp}) = \frac{1}{2z} \int \frac{dy^+ d^2\boldsymbol{b}_\perp}{(2\pi)^3} e^{i(y^+ k^-_{\bar{n}} - \boldsymbol{b}_\perp \cdot \boldsymbol{k}_{\bar{n}\perp})} \mathrm{Tr} \sum_X \langle 0| \left[\tilde{W}^{T\dagger}_{\bar{n}} \psi\right] (y^+, 0^-, \boldsymbol{b}_\perp) |X; P_h\rangle \not{n} \langle X; P_h| \left[\bar{\psi} \tilde{W}^T_{\bar{n}}\right] (0) |0\rangle \Big|_{\mathrm{zb}}$$

$$S(\boldsymbol{k}_{s\perp}) = \int \frac{d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_\perp \cdot \boldsymbol{k}_{s\perp}} \frac{1}{N_c} \langle 0| \mathrm{Tr} \left[\tilde{S}^{T\dagger}_n \tilde{S}^T_{\bar{n}}\right] (0^+, 0^-, \boldsymbol{b}_\perp) \left[\tilde{S}^{T\dagger}_{\bar{n}} \tilde{S}^T_n\right] (0) |0\rangle$$

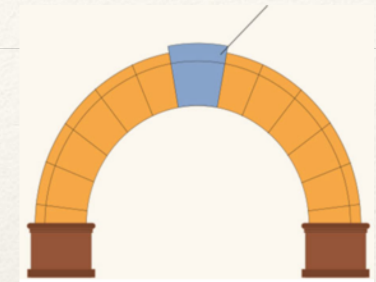
$$\boxed{\tilde{D}_{i\to h}(z, \mathbf{L}_\mu, \mathbf{l}_{\zeta_D}) = \tilde{\Delta}^{(0)}_{i\to h}(z, \mathbf{L}_\mu, \mathbf{l}_{\zeta_D}, \mathbf{L}_{\delta^+})\, \tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta^+})}$$

- Rapidity divergences canceled within one TMDFF

$$\mathbf{L}_X \equiv \ln(X^2 b_T^2 e^{2\gamma_E}/4).$$
$$\mathbf{l}_\zeta = \ln(\mu^2/\zeta)$$

Rapidity divergences and the Soft Function key stone

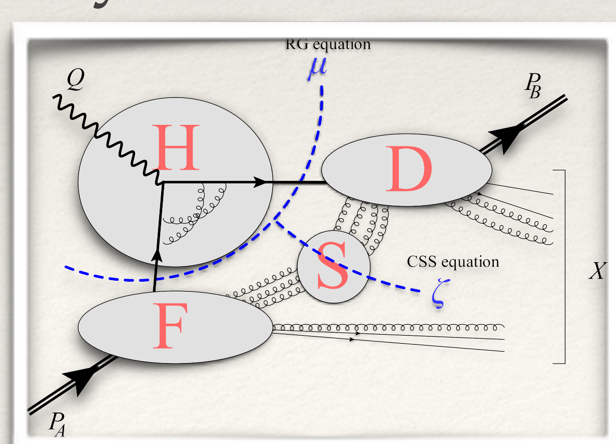
$$S(b_T) = \frac{1}{N_c}\langle 0|[-\infty_n, b_T, \infty_{\bar{n}}][\infty_{\bar{n}}, 0, -\infty_n]|0\rangle, \quad [\gamma] \sim P\exp\left(-ig\int_\gamma A_\gamma\right)$$



- The rapidity divergences arise in the limit $k^+ \to \infty,\ k^- \to 0,\ k^+k^-$ fixed
- The Soft Function is undefined without a regulator for rapidity divergences
- The log of the Soft Function is linear in the rapidity regulator at all orders

$$\tilde{S}(\mathbf{L}_\mu, \mathbf{L}_{\sqrt{\delta^+\delta^-}}) = \tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta^+})\,\tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta^-})$$

- By definition the Soft Function is just 1 in the limit $b_T \to 0$



$$\mathbf{L}_X \equiv \ln(X^2 b_T^2 e^{2\gamma_E}/4)$$
$$\mathbf{l}_\zeta = \ln(\mu^2/\zeta)$$

One loop Soft Function

Origin of divergences: A.Idilbi, M.G. Echevarria, I.S. arXiv:1310.8541, Int.J.Mod.Phys.Conf.Ser. 25 (2014) 1460005

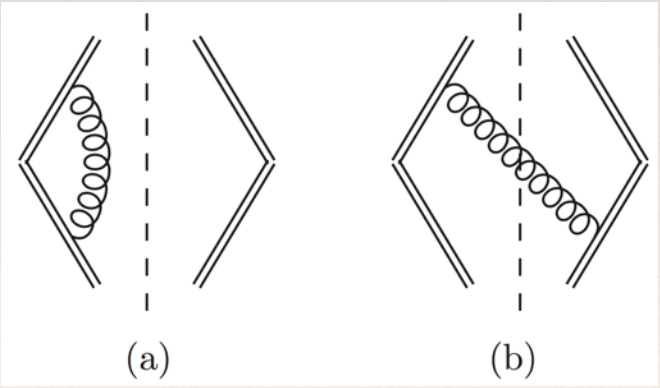
$$\boxed{S_1^v = -2ig_s^2 C_F \delta^{(2)}(\vec{k}_{s\perp})\mu^{2\varepsilon} \int \frac{d^dk}{(2\pi)^d} \frac{1}{[k^+ - i\delta^+][k^- + i\delta^-][k^2 - \lambda^2 + i0]} + h.c.}$$

$$\boxed{S_1^r = 4\pi g_s^2 C_F \mu^{2\varepsilon} \int d^d\boldsymbol{k}_s\, e^{i\boldsymbol{k}_s\boldsymbol{b}} \int \frac{d^dk}{(2\pi)^d} \frac{\delta^{(2)}(\boldsymbol{k}+\boldsymbol{k}_s)\delta(k^2-\lambda^2)\theta(k^+)}{(k^+ + i\delta^+)(k^- - i\delta^-)} + h.c.}$$

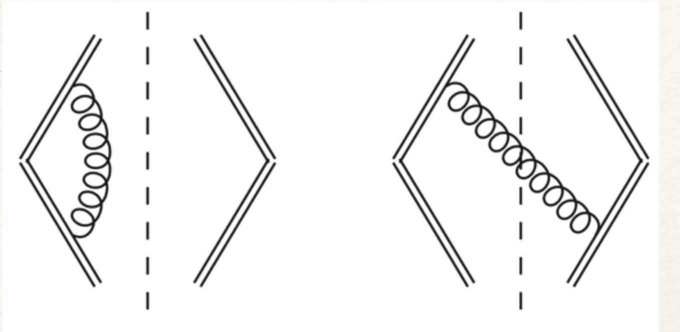
d: UV-Dim.Reg.

λ : IR div.

$\delta^{\pm}$: rapidity div.



One loop Soft Function



Origin of divergences: A.Idilbi, M.G. Echevarria, I.S. arXiv:1310.8541, Int.J.Mod.Phys.Conf.Ser. 25 (2014) 1460005

$$\tilde{S}_1^v = \frac{\alpha_s C_F}{2\pi}\left[\frac{-2}{\varepsilon_{\rm UV}^2} + \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\delta^+\delta^-}{\mu^2} + \ln^2\frac{\lambda^2}{\mu^2} - 2\ln\frac{\lambda^2}{\mu^2}\ln\frac{\delta^+\delta^-}{\mu^2} + \frac{\pi^2}{6}\right]$$

$$\tilde{S}_1^r = \frac{\alpha_s C_F}{2\pi}\left[L_\perp^2 + 2L_\perp\ln\frac{\delta^+\delta^-}{\mu^2} + 2\ln\frac{\lambda^2}{\mu^2}\ln\frac{\delta^+\delta^-}{\mu^2} - \ln^2\frac{\lambda^2}{\mu^2}\right]$$

ε_{UV} : Dim. Reg.

λ : IR div.

$\delta^\pm$: rapidity div.

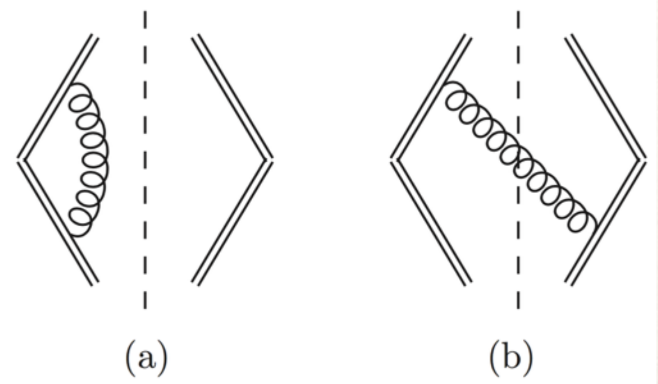
$$L_\perp = \ln\frac{\mu^2 b_T^2 e^{2\gamma_E}}{4}$$

Summing up only UV and rapidity divergences survive:

$$\tilde{S}_1 = \frac{\alpha_s C_F}{2\pi}\left[\frac{-2}{\varepsilon_{\rm UV}^2} + \frac{2}{\varepsilon_{\rm UV}}\ln\frac{\delta^+\delta^-}{\mu^2} + L_\perp^2 + 2L_\perp\ln\frac{\delta^+\delta^-}{\mu^2} + \frac{\pi^2}{6}\right]$$

To compute a NNLO calculation we need this result at all orders in ε

One loop Soft Function



Modified delta-regularization:

$$P\exp\left[-ig\int_0^\infty d\sigma A_\pm(\sigma n)\right] \to P\exp\left[-ig\int_0^\infty d\sigma A_\pm(\sigma n)e^{-\delta^\pm|\sigma|}\right]$$

$$\mathrm{S}_v^{[1]} = -\frac{2g^2}{(4\pi)^{\frac{d}{2}}}\boldsymbol{\delta}^{-\epsilon}\Gamma^2(\epsilon)\Gamma(1-\epsilon) + \ h.c.$$

Mixed divergences cancel at all orders

$$\mathrm{S}_r^{[1]} = \frac{2g^2}{(4\pi)^{\frac{d}{2}}}\left[\boldsymbol{\delta}^{-\epsilon}\Gamma^2(\epsilon)\Gamma(1-\epsilon) - \boldsymbol{B}^\epsilon\Gamma(-\epsilon)\left(L_+ - \psi(-\epsilon) - \gamma_E\right)\right] + \ h.c.$$

Summing up only UV and rapidity divergences survive:

$$S^{[1]} = -4\boldsymbol{B}^\epsilon\Gamma(-\epsilon)\left(L_0 - \psi(-\epsilon) - \gamma_E\right)$$

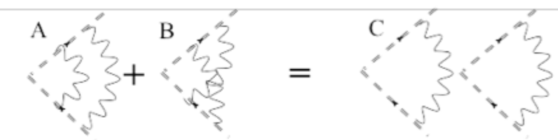
The trivial limit ($b_T = 0$) easily recovered

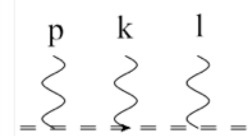
$$\boldsymbol{\delta} = \pm\delta^+\delta^-, \quad \boldsymbol{B} = \frac{b_T^2}{4}$$

$$L_0 = \ln\left(\frac{\boldsymbol{B}|\boldsymbol{\delta}|}{e^{-2\gamma_E}}\right)$$

Two loops Soft Function

Modified delta-regularization
preserves abelian exponentiation





$$= \frac{1}{(p^+ + i\delta)(p^+ + k^+ + 2i\delta)(p^+ + k^+ + l^+ + 3i\delta)}$$

δ-regularization preserving exponentiation

The regularization should be implemented on the level of operator

$$P\exp\left[-ig\int_0^\infty d\sigma A_\pm(\sigma n)\right] \longrightarrow P\exp\left[-ig\int_0^\infty d\sigma A_\pm(\sigma n)e^{-\delta^\pm|\sigma|}\right]$$

Then exponentiation is exact

$$\mathrm{Diag}_A + \mathrm{Diag}_B = \frac{\mathrm{Diag}_C^2}{2}$$

The structure of each diagram recalls the one-loop result

$$\text{Diagram} = \mu^{2\epsilon}\left(A_1\boldsymbol{\delta}^{-2\epsilon} + A_2\boldsymbol{\delta}^{-\epsilon}\boldsymbol{B}^{\epsilon} + A_3\boldsymbol{B}^{2\epsilon}\right)$$

$A_1,\ A_2 = 0$ in the sum of all diagrams

A_3 in the sum of all diagrams is linear in rapidity logs

The Soft Function is linear in rapidity logs at all orders in ϵ

Two loops Soft Function

The D-function which governs the TMD evolution kernel is the finite part of

$$\boxed{D = \frac{1}{2}\frac{d\ln\tilde{S}}{d\mathbf{l}_\delta}\Big|_{finite}}$$

First direct calculation of the D- function at two loops

$$D = \sum_n \sum_{k=0}^{n} a_s^n \boldsymbol{L}_\mu^k d^{(n,k)}$$

$$d^{(2,2)} = \frac{\Gamma^{(0)}\beta_0}{4} = C_F\left(\frac{11}{3}C_A - \frac{4}{3}T_R N_f\right)$$

$$d^{(2,1)} = \frac{\Gamma^{(1)}}{2} = 2C_F\left(\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_R N_f\right)$$

$$d^{(2,0)} = C_F\left(\left(\frac{404}{27} - 14\zeta_3\right)C_A - \frac{112}{27}T_R N_f\right).$$

Two loops Soft Function

The D-function which governs the TMD evolution kernel is the finite part of

$$D = \left[\frac{\ln^{-1} \tilde{S}}{2} \frac{d}{d \ln |\boldsymbol{\delta}|} \ln \tilde{S} \right] |_{finite}$$

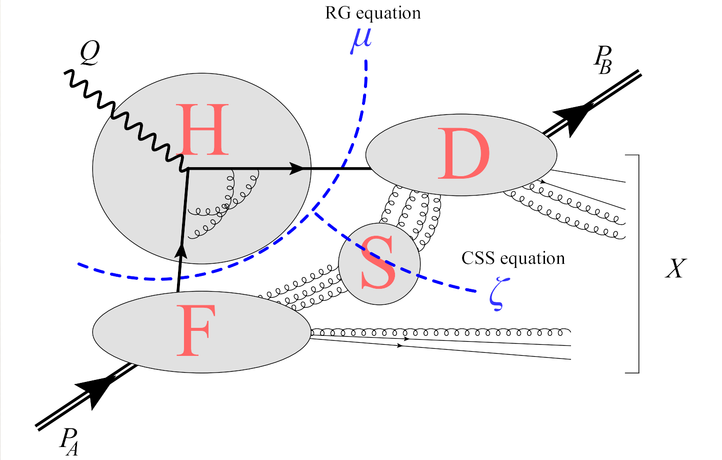
Comparing quark and gluon soft functions, the difference is just the Casimir scaling

The Casimir scaling of D's is valid at all order in perturbation theory

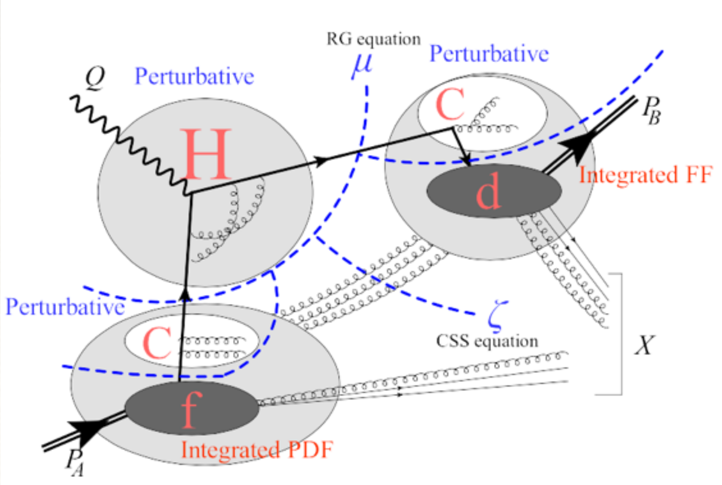
$$\frac{D}{C_F} = \frac{D_g}{C_A}$$

TMDFF at NNLO

- The Universal Soft function (Spin independent)
- ***The unsubstracted TMDs***



$$\tilde{D}_{i\to h}(z, \mathbf{L}_\mu, \mathbf{l}_{\zeta_D}) = \tilde{\Delta}^{(0)}_{i\to h}(z, \mathbf{L}_\mu, \mathbf{l}_{\zeta_D}, \mathbf{L}_{\delta+})\, \tilde{S}^{\frac{1}{2}}(\mathbf{L}_\mu, \mathbf{L}_{\delta+})$$



$$\tilde{D}_{i\to h}(z, \mathbf{L}_\mu, \mathbf{l}_\zeta) = \int_z^1 \frac{d\tau}{\tau^{3-2\epsilon}} C_{i\to j}\Big(\frac{z}{\tau}, \mathbf{L}_\mu, \mathbf{l}_\zeta\Big) d_{j\to h}(\tau, \mu)$$

TMDFF expansion

Perturbative expansion:

$$\tilde{C}^{[0]}_{j\to f} = \delta(1-z)$$

$$\tilde{C}^{[1]}_{i\to j} = \tilde{D}^{[1]}_{i\to j} - \frac{d^{[1]}_{i\to j}}{z^{2-2\epsilon}}$$

$$\tilde{C}^{[2]}_{i\to f} = \boxed{\tilde{D}^{[2]}_{i\to f}} - \tilde{C}^{[1]}_{i\to k} \otimes \frac{d^{[1]}_{k\to f}}{z^{2-2\epsilon}} - \frac{d^{[2]}_{i\to f}}{z^{2-2\epsilon}}$$

Missing piece

To recover the SF from the soft limit of FF (zero-bin correspondence) :

$$\frac{1}{(k_1^+ - i0)(k_2^+ - i0)\ldots(k_n^+ - i0)} \to \frac{1}{(k_1^+ - i\delta^+/z)(k_2^+ - 2i\delta^+/z)\ldots(k_n^+ - ni\delta^+/z)}$$

TMDFF expansion

1-loop $$D^{[1]}_{q/q} = \tilde{\Delta}^{[1]}_{q/q} - \tilde{S}^{[1]}_{+} - Z^{[1]}_2 + Z^{[1]}_D$$

Rapidity divergences cancel here!

$\sim \delta(1-z)$

2-loops $$\tilde{D}^{[2]}_{i/f} = \underbrace{\overbrace{\tilde{\Delta}^{[2]}_{i/f} - \tilde{S}^{[1]}_{+}\tilde{\Delta}^{[1]}_{i/f}}^{\text{cros.rap.div.free}} - \tilde{S}^{[2]}_{+}\delta_{if} + \frac{3\tilde{S}^{[1]}_{+}\tilde{S}^{[1]}_{+}}{2}\delta_{if}}_{\text{rap.div.free}}$$

$$+ \left(Z^{[1]}_D - Z^{[1]}_2\right)\left(\tilde{\Delta}^{[1]}_{i/f} - \frac{\tilde{S}^{[1]}_{+}\delta_{if}}{2}\right)$$

$$+ \left(Z^{[2]}_D - Z^{[2]}_2 - Z^{[1]}_2 Z^{[1]}_D + Z^{[1]}_2 Z^{[1]}_2\right)\delta_{if} \, .$$

Pure UV

RGE for TMD's

$$\mu^2 \frac{d}{d\mu^2} \tilde{D}_{i\to h} = \frac{1}{2} \gamma_D^i \, \tilde{D}_{i\to h}$$

$$\gamma_D = \Gamma^i_{cusp} \mathbf{l}_\zeta - \gamma_V^i$$

$$\zeta \frac{d}{d\zeta} \tilde{D}_{i\to h} = -\mathcal{D}^i \, \tilde{D}_{i\to h} \, , \quad 2\mu^2 \frac{d}{d\mu^2} \mathcal{D}^i = \Gamma^i_{cusp}$$

RGE for TMD fragmentation

$$\mathbf{l}_X \equiv \ln(\mu^2/X)$$

RGE for coefficients

$$\zeta \frac{d}{d\zeta} \tilde{C}_{i\to j} = -\mathcal{D}^i \, \tilde{C}_{i\to j}$$

$$\mu^2 \frac{d}{d\mu^2} \tilde{C}_{i\to j} = \tilde{C}_{i\to k} \otimes \mathcal{K}^i_{k\to j}$$

$$\mathcal{K}^i_{k\to j}(z) = \frac{\delta_{kj}}{2} (\Gamma^i_{cusp} \mathbf{l}_\zeta - \gamma_V^i) \delta(\bar{z}) - \frac{\mathcal{P}_{k\to j}(z)}{z^2} .$$

Re-writing of the coefficient

$$\tilde{C}_{i\to j} = \exp\left[-\mathcal{D}^i \mathbf{L}_{\sqrt{\zeta}} \right] \tilde{C}_{i\to j}$$

Structure of the result

Log structure of the result

$$\tilde{\mathcal{C}}_{ij}^{[n]} = \sum_{k=0}^{2n} \tilde{\mathcal{C}}_{ij}^{(n;k)} \mathbf{L}_\mu^k$$

Recursion relation

$$(k+1)\tilde{\mathcal{C}}_{i\to j}^{(n;k+1)} = \sum_{r=1}^{n} \frac{\Gamma_{cusp}^{[r]}}{2} \tilde{\mathcal{C}}_{i\to j}^{(n-r;k-1)} - \frac{\gamma_V^{i[r]} - 2(n-r)\beta^{[r]}}{2} \tilde{\mathcal{C}}_{i\to j}^{(n-r;k)} - \tilde{\mathcal{C}}_{i\to k}^{(n-r;k)} \otimes \frac{\mathcal{P}_{k\to j}^{[r]}}{z^2}$$

The original piece of the 2-loop coefficient is

$$\tilde{\mathcal{C}}_{q\to q}^{(2;0)}(z) = C_F^2 Q_F(z) + C_F C_A Q_A(z) + T_R N_f Q_N(z)$$

$$\tilde{\mathcal{C}}_{q\to \bar{q}}^{(2;0)}(z) = C_F \left(C_F - \frac{C_A}{2} \right) Q_{q\bar{q}}(z)$$

Sample of the result

Log structure of the result

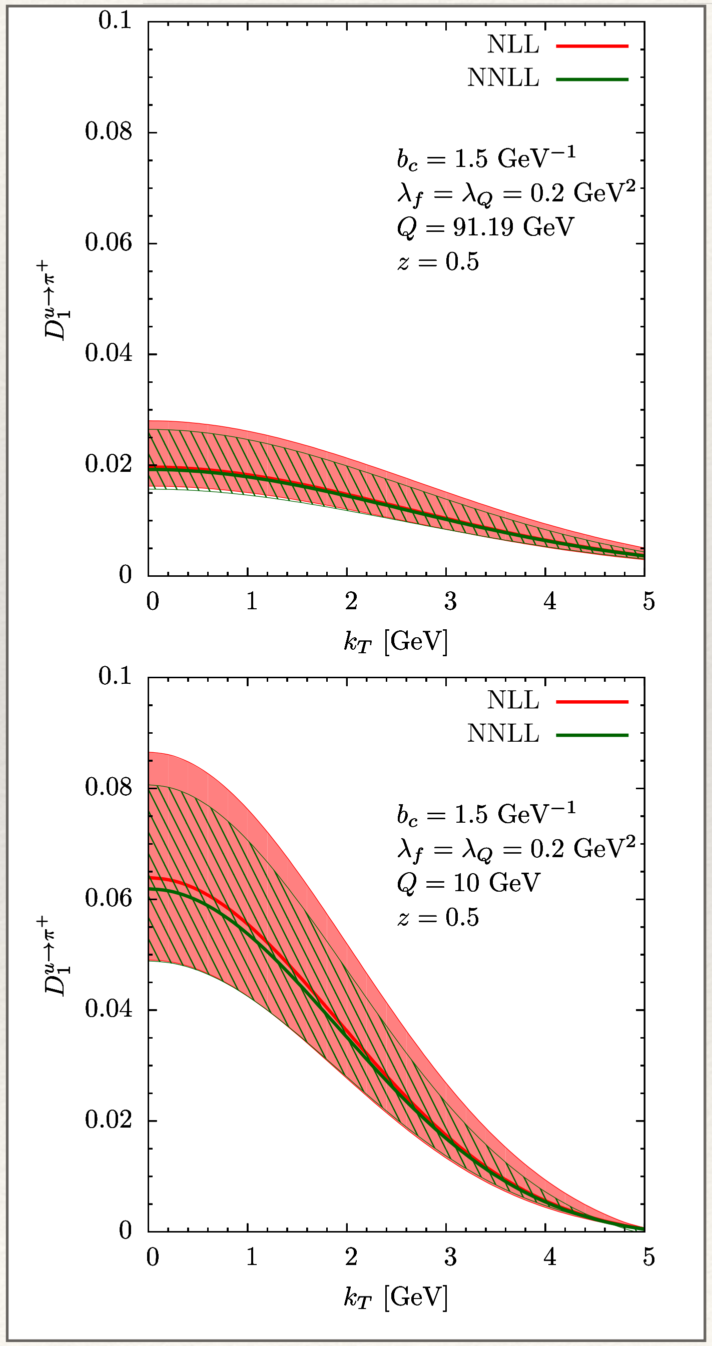
$$\boxed{\tilde{\mathcal{C}}_{ij}^{[n]} = \sum_{k=0}^{2n} \tilde{\mathcal{C}}_{ij}^{(n;k)} \mathbf{L}_\mu^k}$$

$$\tilde{\mathcal{C}}_{q\to q}^{(2;0)}(z) = C_F^2 Q_F(z) + C_F C_A Q_A(z) + T_R N_f Q_N(z),$$
$$\tilde{\mathcal{C}}_{q\to \bar{q}}^{(2;0)}(z) = C_F\left(C_F - \frac{C_A}{2}\right) Q_{q\bar{q}}(z).$$

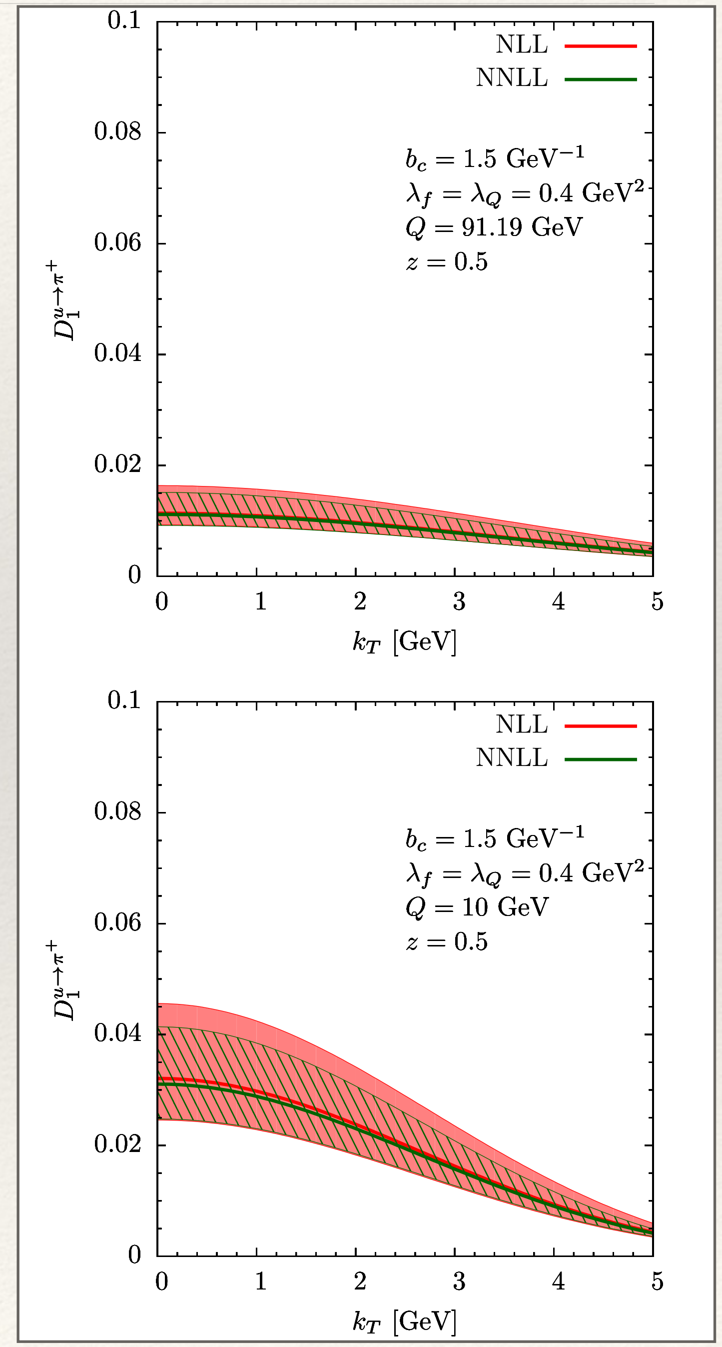
$$\boxed{\begin{aligned} Q_N(z) = &\frac{1}{z^2}\left[\left(\frac{2}{3}\ln^2 z - \frac{20}{3}\ln z + \frac{112}{27}\right)p(z) - \frac{16}{3}\bar{z}\ln z - \frac{4}{3}\bar{z}\right]_+ \\ &+ \delta(\bar{z})\left(-\frac{2717}{162} + \frac{25\pi^2}{9} + \frac{52}{9}\zeta_3\right)\end{aligned}}$$

Full result in arXiv:1509.06392 and upcoming publication

FF phenomenology



Starting up



Conclusions

- The comprehension of transverse momentum spectra for all energy ranges requires the correct inclusion of non-perturbative effects: TMDs.
- The check of the universality of TMD evolution requires the same degree of precision for TMDPDF and TMDFF: NNLO.
- The perturbative part of TMDs should be used at highest available order to control the perturbative series (NNLL only achieved in a limited set of TMDs): Here a direct calculation of D-function at NNLO! Casimir scaling of the D function established at all orders in perturbation theory.)
- The control of perturbative error is fundamental to understand the nature of non-perturbative effects.
- We have completed the calculation of the universal Soft Function and concluding the matching of the ALL unpolarized TMDFF onto FF at NNLO using the EIS formulation. (TMDPDF onto PDF also checked.)
- The Soft Function can be used for the evaluation of the matching of all (polarized) TMDs.
- The so called “Phase I” of TMD history is (almost) over

Thanks!!!

TMD’s FAQ

The question is:
What is a TMD?

The question is:
Who cares?