TMD evolution in CSS

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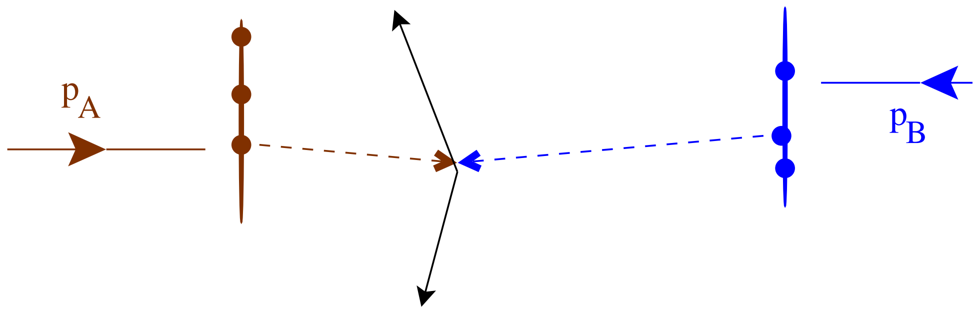
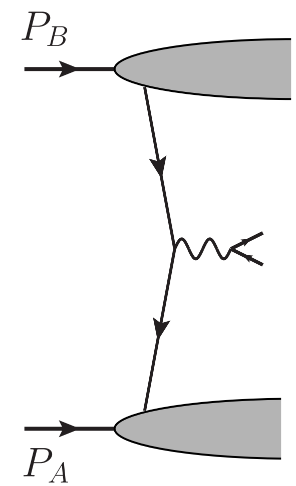
Basic parton model inspiration: Case of Drell-Yan at $q_\mathsf{T} \ll Q$





- $\boldsymbol{q}_\mathsf{T}$(leptons) $= \sum \boldsymbol{k}_\mathsf{T}$(quarks)

- Use parton distribution in x and $\boldsymbol{k}_\mathsf{T}$

- Same intuition applies to hadron distribution in jets, assisted by data, e.g.,
  - $e^+e^- \to$ (jet$_1$ + jet$_2$ $\to$) $h_1 + h_2 + X$,
  - $ep \to h + X$,
  - $pp \to$ (jet$_1$ + jet$_2$ + X $\to$) $h_1 + h_2 + X$ with high p_T jets and almost back-to-back hadrons. (Warning: factorization failure in QCD.)

- *But* parton model needs to be substantially modified in QCD

Summary

1. (TMD) factorization
2. Evolution equations à la CSS
3. Non-perturbative part
4. Consequences
5. Danger/opportunity areas

TMD factorization for DY in QCD at $q_{\mathsf{T}} \ll Q$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q \, \mathrm{d}\Omega} \simeq \frac{2}{s} \sum_j \frac{\mathrm{d}\hat{\sigma}_{j\bar{\jmath}}(Q, \mu \mapsto Q)}{\mathrm{d}\Omega} \int e^{i \boldsymbol{q}_{\mathsf{T}} \cdot \boldsymbol{b}_{\mathsf{T}}} \; \tilde{f}_{j/A}(x_A, \boldsymbol{b}_{\mathsf{T}}; Q^2, Q) \; \tilde{f}_{\bar{\jmath}/B}(x_B, \boldsymbol{b}_{\mathsf{T}}; Q^2, Q) \, \mathrm{d}^2 \boldsymbol{b}_{\mathsf{T}}$$

for unpolarized $p + p \to (\gamma^*(q) \to \mu^+ \mu^-) + X$, with $q = x_A P_A + x_B P_B + \boldsymbol{q}_{\mathsf{T}}$.

- Hard scattering $\mathrm{d}\hat{\sigma}$: perturbative
- TMD pdfs with 2 scale arguments . . .
- Extras:
  - Can add in polarization terms (Sivers, Boer-Mulders)
  - Need Y term (or . . .) to combine with collinear factorization at larger q_{T}

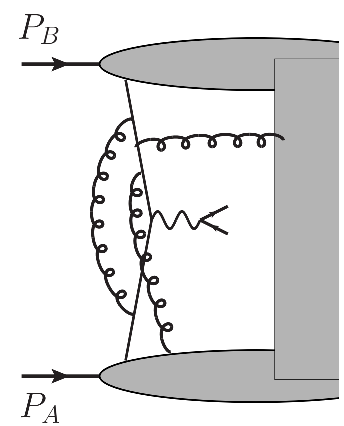
Old v. new CSS

- Originally (CSS, 1982, 1985):
  - Factorization into $H \times S \times \text{pdf} \times \text{pdf}$. ($H =$ "hard" factor; $S =$ "soft" factor.)
  - Reorganized for one process (mainly unpolarized DY) to combine S and H with pdfs, effectively.
  - Presented results in terms of parameterized "non-perturbative" functions, and parts involving perturbative quantities

- New (JCC, 2011)
  - Full proofs.
  - Better definitions of TMD functions.
  - With S incorporated into TMD functions
  - Keep H separate, and process dependent.
  - Emphasize presence of TMD functions (including all the spin-dependent ones).

[See JCC & Rogers (in preparation) for relationships.]

Need for evolution from QCD



When s and Q^2 are increased with x_A and x_B fixed:

- wider rapidity range for real and virtual emission
- wider k_T range for real emission

Hence q_T distribution broadens.

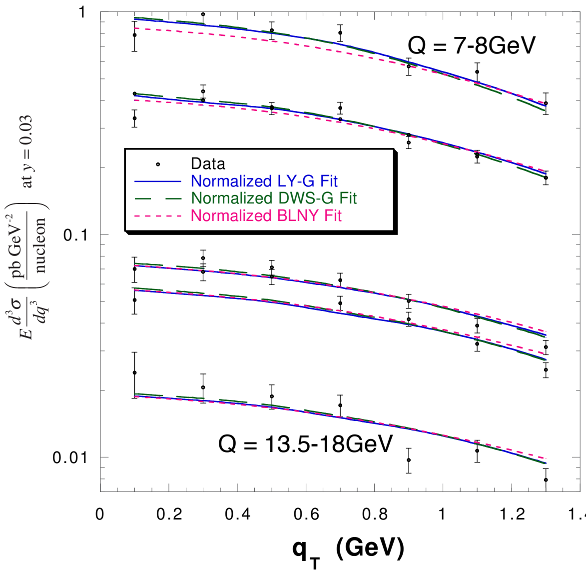
In TMD factorization, the effects are allowed for in properly defined TMD pdfs.

(Technical long story about Ward identities and Wilson lines!)

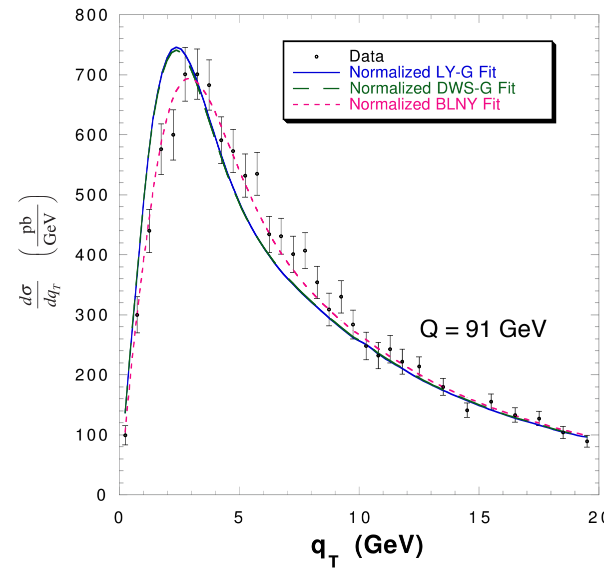
Geography of evolution

$q_{\rm T}$

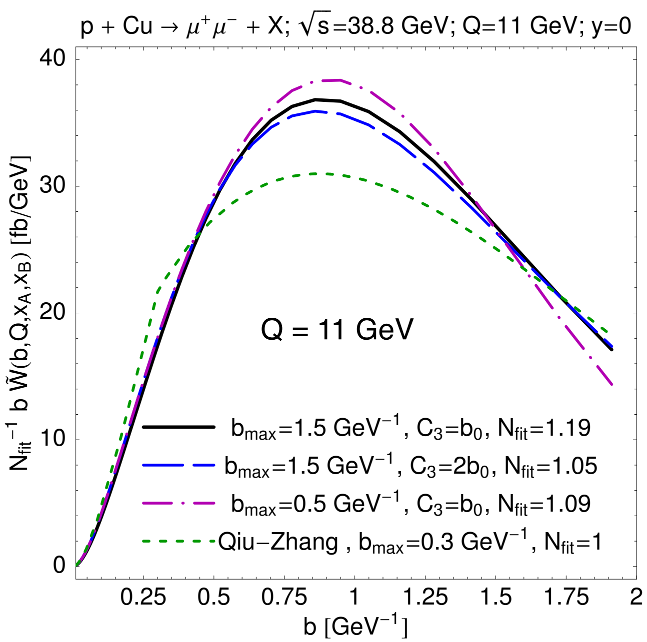
E605 Data

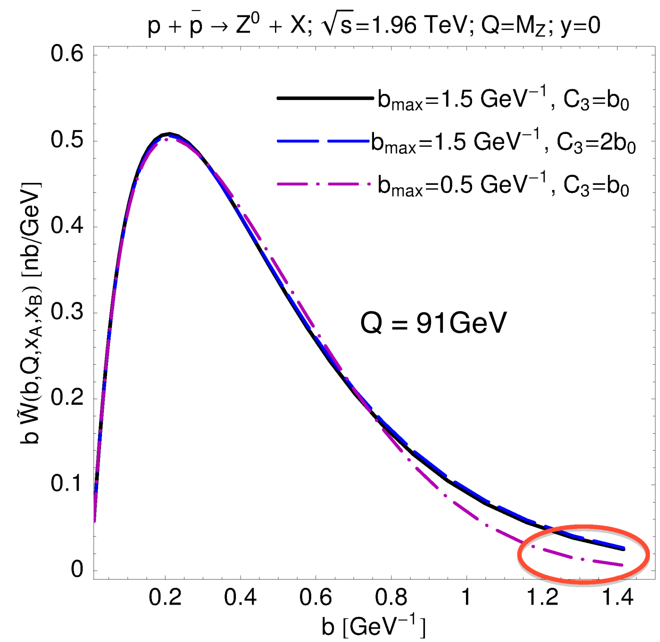


CDF Z Run 1



$b_{\rm T}$





Q: 7–18 GeV, $\sqrt{s} = 38.8\,\mathrm{GeV}$

$Q = m_Z$, $\sqrt{s} = 1800\,\mathrm{GeV}$

(Adapted from Landry et al., PRD 67,073016 (2003), Konychev & Nadolsky, PLB 633, 710 (2006))

Evolution (CSS and RG)

Use definitions of TMD pdfs with effective cut offs on

- rapidity of unobserved real emission; parameter $\zeta = M^2 x^2 e^{2(y_p - y_{\text{cut-off}})}$
- transverse momentum of virtual lines; parameter μ

Evolution on ζ:
$$\frac{\partial \ln \tilde{f}_{f/H}(x, b_\mathsf{T}; \zeta; \mu)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\mathsf{T}; \mu)$$

Combine with RG equations to get:

$$\begin{aligned}\frac{\mathrm{d} \ln \tilde{f}_{f/H}(x, b_\mathsf{T}; Q^2; Q)}{\mathrm{d} \ln Q} &= \gamma(\alpha_s(Q)) - \int_{Q_0}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_K(\alpha_s(\mu)) + \tilde{K}(b_\mathsf{T}; Q_0), \\ &= \gamma(\alpha_s(Q)) - \int_{\mu_b}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_K(\alpha_s(\mu)) + \tilde{K}(b_\mathsf{T}; \mu_b),\end{aligned}$$

Evolution kernel $\tilde{K}(b_\mathsf{T}, \mu)$ is strongly universal: independent of x, Q, flavor, type of TMD function.

Non-perturbative information is in

- Ordinary pdfs (via small-b_T OPE).
- Large b_T TMD pdfs: "intrinsic transverse momentum".
- $\tilde{K}(b_\mathsf{T}, \mu)$ at large b_T

Segregation of non-perturbative information à la CSS

For evolution

$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x, b_\mathsf{T}; Q^2; Q)}{\mathrm{d}\ln Q} = \gamma(\alpha_s(Q)) - \int_{\mu_b}^{Q} \frac{\mathrm{d}\mu}{\mu}\gamma_K(\alpha_s(\mu)) + \tilde{K}(b_\mathsf{T}; \mu_b)$$

$$= \gamma(\alpha_s(Q)) - \int_{\mu_{b_*}}^{Q} \frac{\mathrm{d}\mu}{\mu}\gamma_K(\alpha_s(\mu)) + \tilde{K}(b_*; \mu_{b_*}) - g_K(b_\mathsf{T}; b_{\max})$$

where *smooth* cutoff on perturbative part is $b_* = b_\mathsf{T}/\sqrt{1 + b_\mathsf{T}^2/b_{\max}^2}$

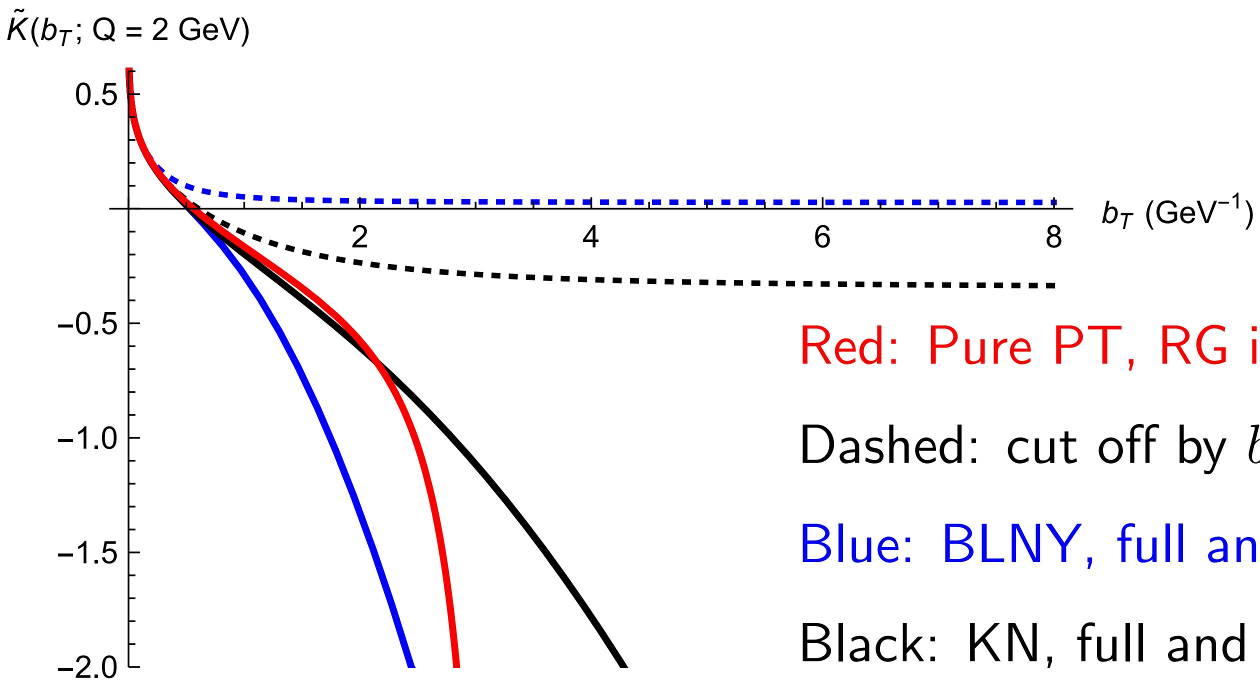
Similarly for TMD functions at large b_T, with $e^{-g_{j/A}(x, b_\mathsf{T})}$ factor.

Fits for $g_{j/A}$ and g_K can also allow for incomplete perturbative information.

Picturing segregation of non-perturbative information

For evolution

$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x, b_\mathrm{T}; Q^2; Q)}{\mathrm{d}\ln Q} = \gamma(\alpha_s(Q)) - \int_{\mu_{b_*}}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_K(\alpha_s(\mu)) + \tilde{K}(b_*; \mu_{b_*}) - g_K(b_\mathrm{T}; b_\mathrm{max})$$



Red: Pure PT, RG improved — No b_max

Dashed: cut off by b_max

Blue: BLNY, full and cutoff PT — $b_\mathrm{max} = 0.5\,\mathrm{GeV}^{-1}$

Black: KN, full and cutoff PT — $b_\mathrm{max} = 1.5\,\mathrm{GeV}^{-1}$

Typical b_T:	$0.5\,\mathrm{GeV}^{-1}$	$1.2\,\mathrm{GeV}^{-1}$	$3\,\mathrm{GeV}^{-1}$
Q:	$Q = m_Z$	$10\,\mathrm{GeV}$	$2\,\mathrm{GeV}$

- N.B. (RG improved) pert. calc. agrees with KN to $b_\mathrm{T} \simeq 2\,\mathrm{GeV}^{-1}$

Solutions for TMD pdfs

With maximal perturbative information

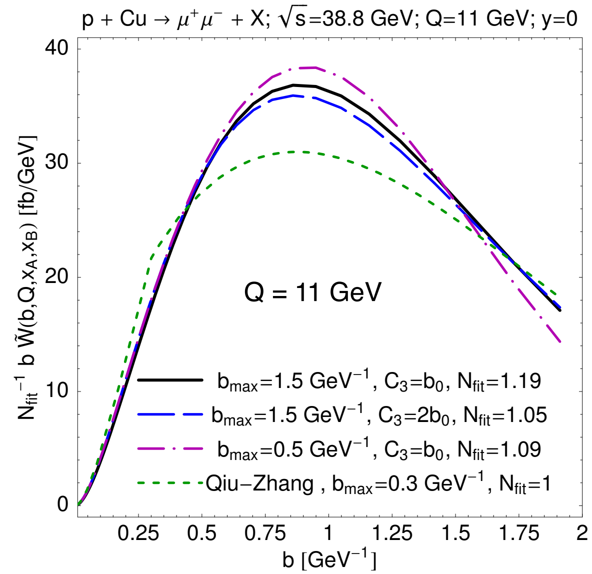
$$\tilde{f}_{j/H}(x, \boldsymbol{b}_\mathsf{T}; Q^2, Q) = \exp\left[-g_{j/A}(x_A, b_\mathsf{T}; b_{\max}) - g_K(b_\mathsf{T}; b_{\max}) \ln\frac{Q}{Q_0}\right]$$
$$\times \exp\left\{\tilde{K}(b_*; \mu_{b_*}) \ln\frac{Q}{\mu_{b_*}} + \int_{\mu_{b_*}}^{Q} \frac{\mathrm{d}\mu'}{\mu'}\left[\gamma_j(a_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(a_s(\mu'))\right]\right\}$$
$$\times \sum_{j_A}\int_{x_A}^{1}\frac{\mathrm{d}\xi}{\xi} f_{j_A/H}(\xi; \mu_{b_*})\ \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x}{\xi}, b_*; \mu_{b_*}^2, \mu_{b_*}, a_s(\mu_{b_*})\right).$$

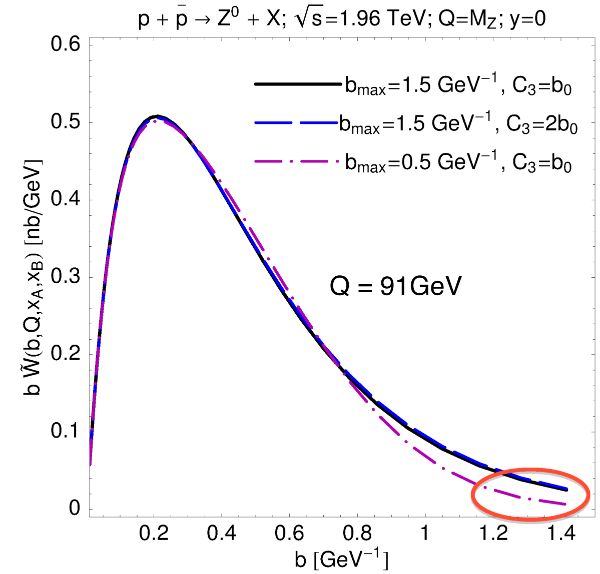
In terms of TMD function at Q_0:

$$\tilde{f}_{j/H}(x, \boldsymbol{b}_\mathsf{T}; Q^2, Q) = \tilde{f}_{j/H}(x, \boldsymbol{b}_\mathsf{T}; Q_0^2, Q_0) \times \exp\left\{\int_{Q_0}^{Q}\frac{\mathrm{d}\mu'}{\mu'}\left[\gamma_j(a_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(a_s(\mu'))\right]\right\}$$
$$\times \quad \exp\left\{\ln\frac{Q}{Q_0}\left[-g_K(b_\mathsf{T}; b_{\max}) + \tilde{K}(b_*; \mu_{b_*}) - \int_{\mu_{b_*}}^{Q_0}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(a_s(\mu'))\right]\right\}$$
$$= \mathsf{func.}(b_\mathsf{T}) \times \mathsf{func.}(Q) \times \left(\frac{Q}{Q_0}\right)^{\mathsf{func.}(b_\mathsf{T})}$$

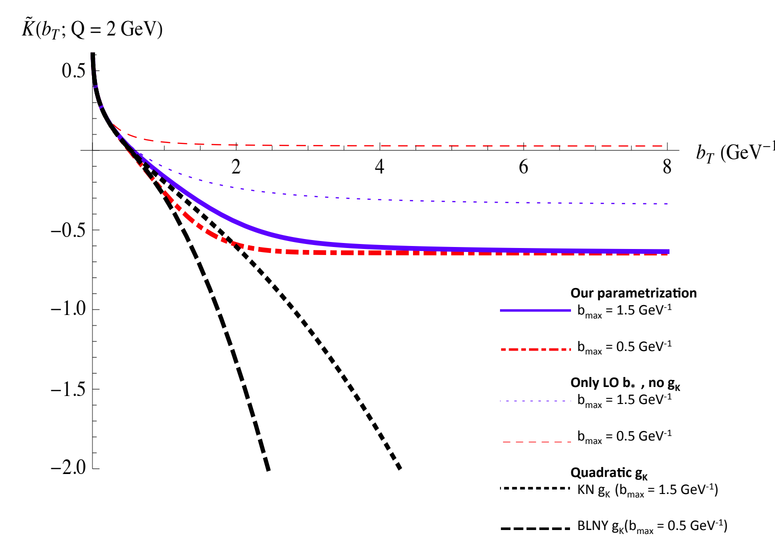
The meaning of $\tilde{K}(b_{\mathsf{T}})$

b_{T}-space DY:





$\tilde{K}(b_{\mathsf{T}}, Q = 2\,\mathrm{GeV}$:
Black: KN, BLNY
Blue, red: new parameterizations



$$\frac{\mathrm{d}\ln \tilde{f}_{f/H}(x, b_{\mathsf{T}}; Q^2; Q)}{\mathrm{d}\ln Q} = \gamma(\alpha_s(Q)) - \int_{Q_0}^{Q} \frac{\mathrm{d}\mu}{\mu}\gamma_K(\alpha_s(\mu)) \;+\; \tilde{K}(b_*; Q_0)$$

What happens at large b_{T}?

- With standard parameterizations, large b_{T} asymptote of "b_{T} cross section" and pdfs is

$$\text{coeff.} \times e^{-b_{\mathrm{T}}^2[\text{coeff}(x)+\text{const}\ln(Q^2/Q_0^2)]}$$

- At low Q, we get exponent that is too small to agree with data or may even be unphysical (negative).

- JCC & Rogers (PR D91, 074020 (2015)) proposed:
  - Modify parameterization to give constant $\tilde{K}$ at large b_{T},
  - while approximately preserving its form around $1.5\,\mathrm{GeV}^{-1} \sim 0.3\,\mathrm{fm}$, which dominates fits to DY data

  This gives slower evolution at the larger b_{T} values that are important at lower Q.

- Sun, Isaacson, Yuan & Yuan, arXiv:1406.3073 made fits with $\tilde{K} \propto \ln b_{\mathrm{T}}$ at large b_{T}. They obtained good agreement with data:
  - Fitted: Tevatron, fixed-target DY.
  - Predicted: SIDIS at HERMES.
  - But they neglected Y term!

Danger/opportunity areas

- Need better formulation of "Y term", with analysis of errors.
- TMD factorization failure in $pp \to (\text{jet}_1 + \text{jet}_2 + X \to)\ h_1 + h_2 + X$ etc: Understand the physics better.
- Forward SSA, etc
- Have we correctly analyzed role of non-perturbative physics, especially in hadronization?
- Effects of heavy quarks.
- Generally, in reporting fits, it's important to include
  - actual TMD pdfs (and fragmentation functions)
  - $\tilde{K}(b_{\mathsf{T}}, Q)$

  as well as the CSS "non-perturbative" functions.