Extraction of transversity in a collinear framework

Alessandro Bacchetta
in collaboration with M. Radici, A. Courtoy, A. Bianconi, M. Guagnelli

Funded by

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Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DI PAVIA

One slide on TMDs

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

One slide on TMDs

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

transversity

Twist-2 TMDs

Integrated over transv. momentum

quark pol.

nucleon pol.	U	L	T
U	f_1		
L		g_{1L}	
T			h_1

transversity

Twist-2 collinear PDFs

Integrated over transv. momentum

quark pol.

nucleon pol.

	U	L	T
U	f_1		
L		g_{1L}	
T			h_1

transversity

Twist-2 collinear PDFs

This is going to be a TMD-free talk (almost)

Why transversity is relevant?

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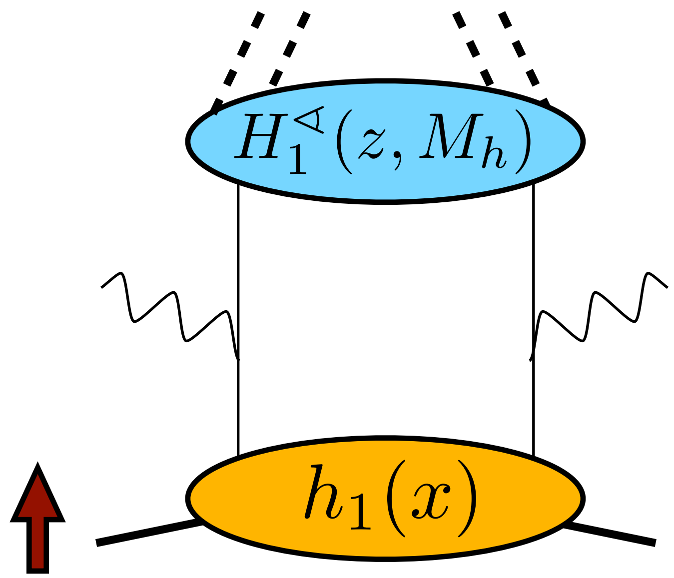
- Can be used to test details of perturbative QCD (factorization and evolution in a gluon-free sector)

Why transversity is relevant?

- Fundamental property of the nucleon
- Can test validity of approaches to nonperturbative QCD (e.g. models, lattice QCD calculations)
- Can be used to test details of perturbative QCD (factorization and evolution in a gluon-free sector)
- Can be used to put limits on couplings beyond Standard Model (tensor coupling)
  see, e.g., Courtoy et al. 1503.06814

Transversity observables (present)

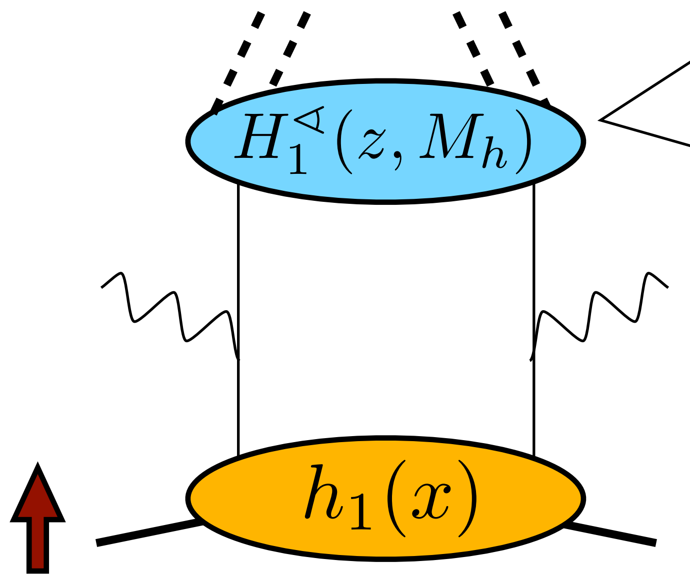
Collinear factorization



see A. Prokudin's talk

Transversity observables (present)

Collinear
factorization

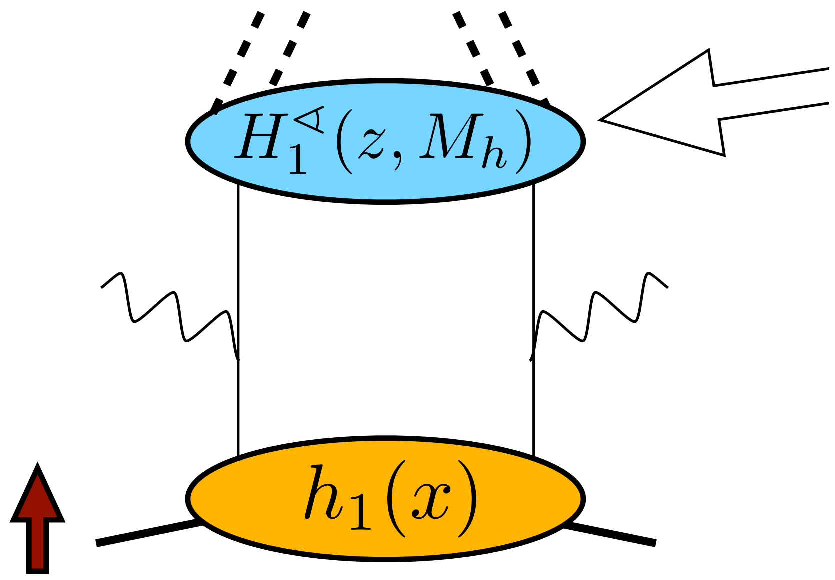


dihadron
interference FF

see A. Prokudin's talk

Transversity observables (present)

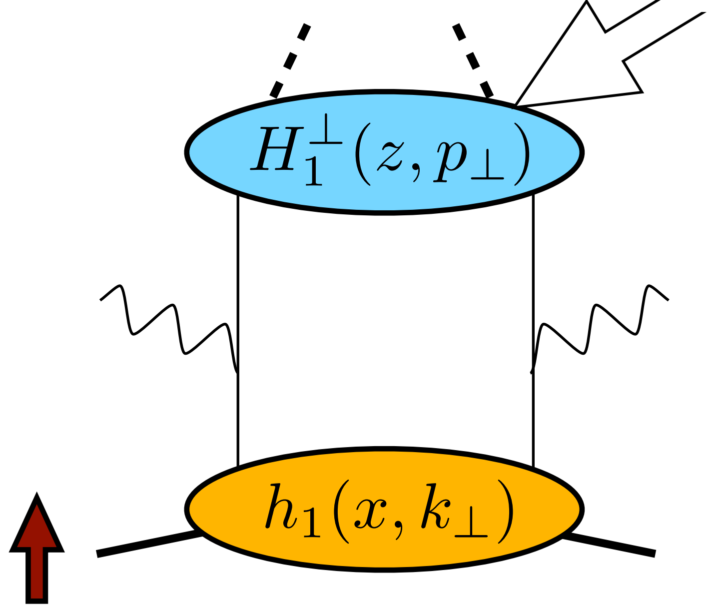
Collinear factorization



dihadron interference FF

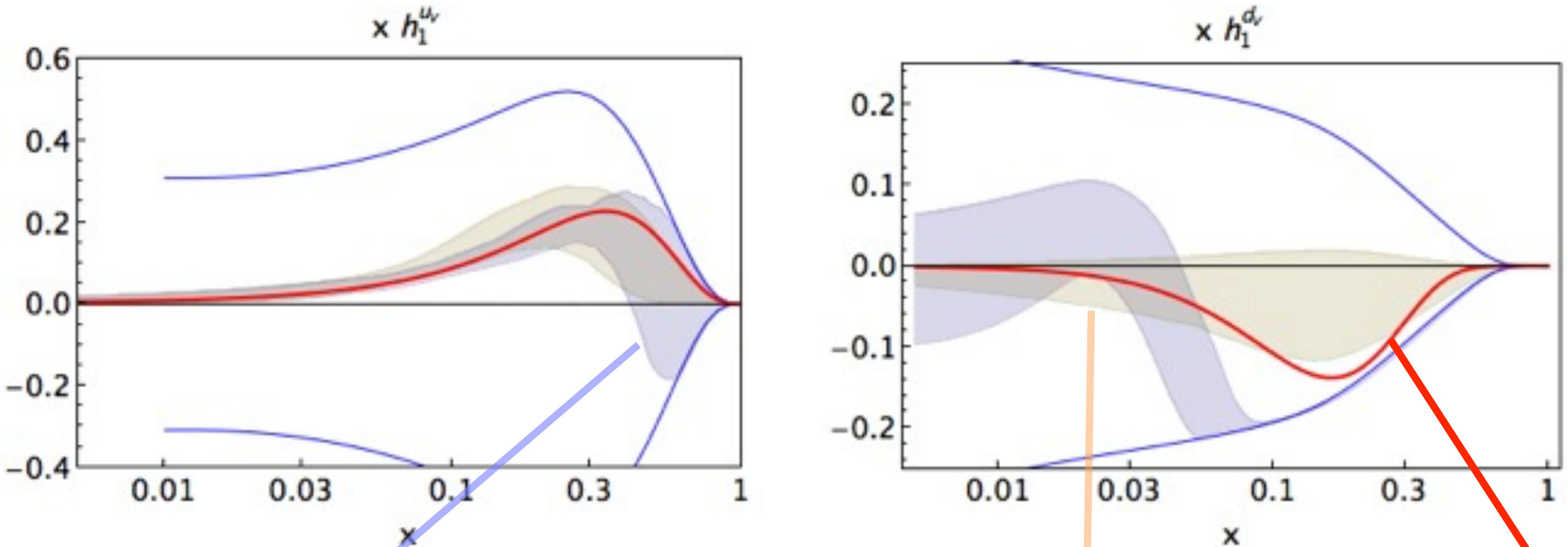
TMD factorization

Collins



see A. Prokudin's talk

The bottom line



Kang et al. 2015

arXiv:1505.05589

Pavia 2015

Radici et al., arXiv:1503.03495

Torino 2013

Anselmino et al.,
P.R.D87 (13) 094019

dihadron extraction

single-hadron extractions

Single hadron

see A. Prokudin's talk

SIDIS

$$A_{DIS}(x,z,P_{h\perp}^2) = -\langle C_y \rangle \frac{\sum_q e_q^2\, h_1^q(x,p_T^2) \otimes_C H_{1,q}^{\perp}(z,k_T^2)}{\sum_q e_q^2\, f_1^q(x,p_T^2) \otimes D_{1,q}(z,k_T^2)}$$

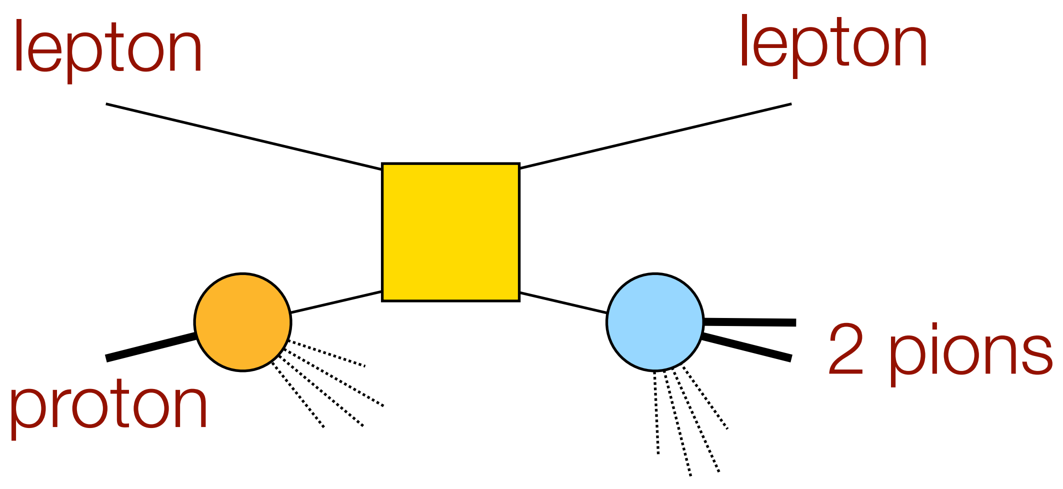
$$\ldots \otimes \ldots \rightarrow \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{p}_T) \ldots$$

Two hadrons

SIDIS

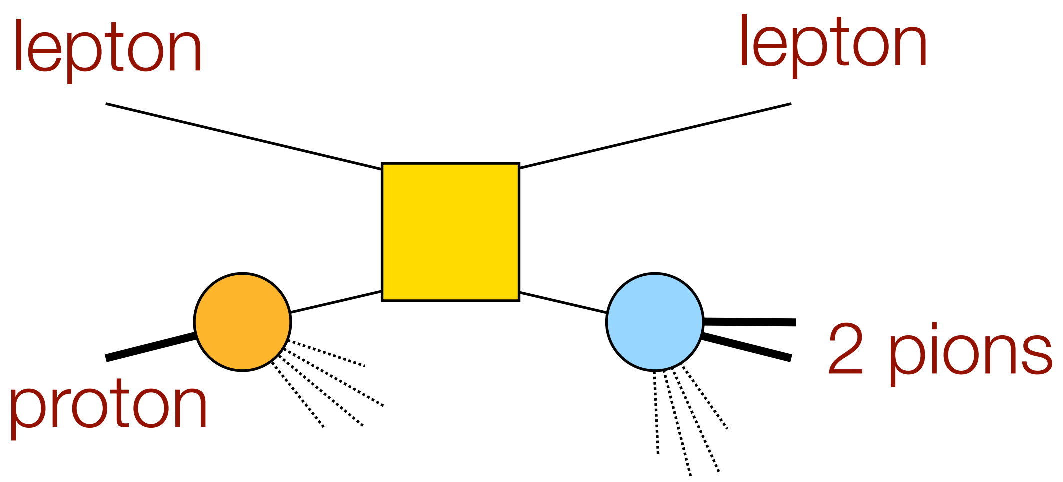
$$A_{DIS}(x, z, M_h^2) = -\langle C_y \rangle \, \frac{\sum_q e_q^2 \, h_1^q(x) \; \frac{|\boldsymbol{R}|}{M_h} H_{1,q}^{\sphericalangle}(z, M_h^2)}{\sum_q e_q^2 \, f_1^q(x) \; D_{1,q}(z, M_h^2)}$$

Relevant processes



SIDIS

Relevant processes



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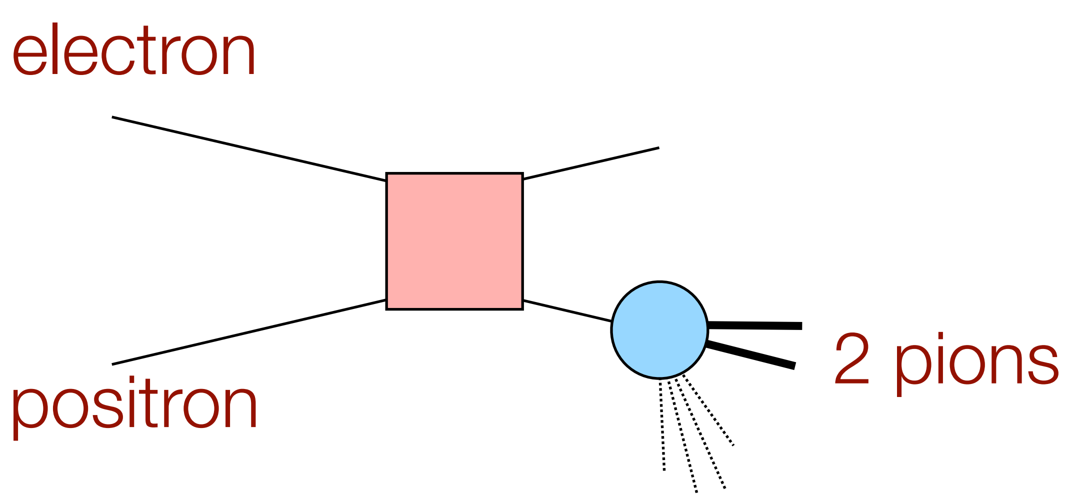
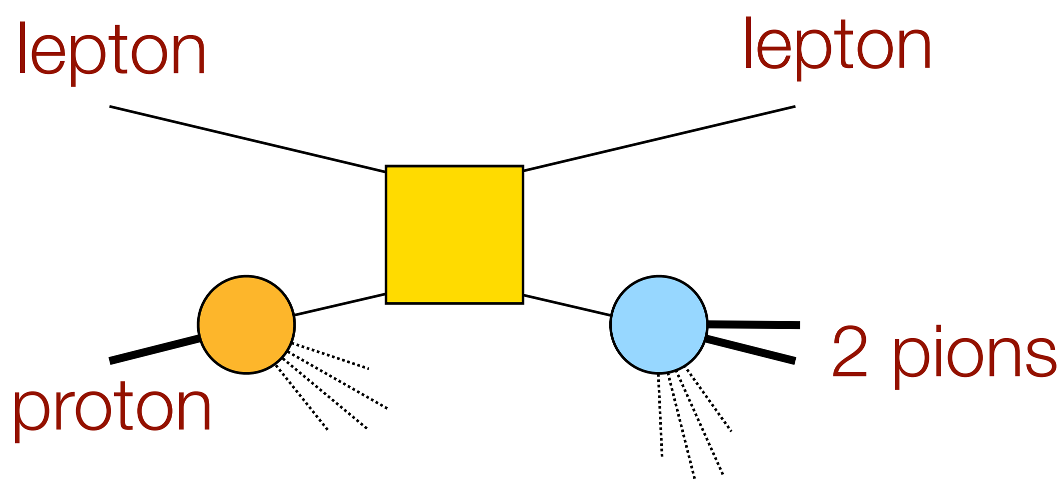
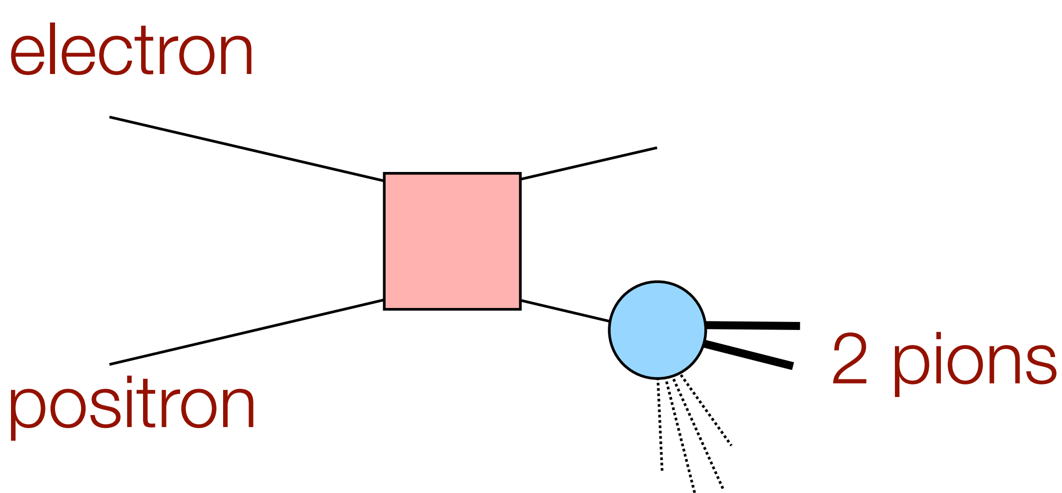


e^-e^+ to pions

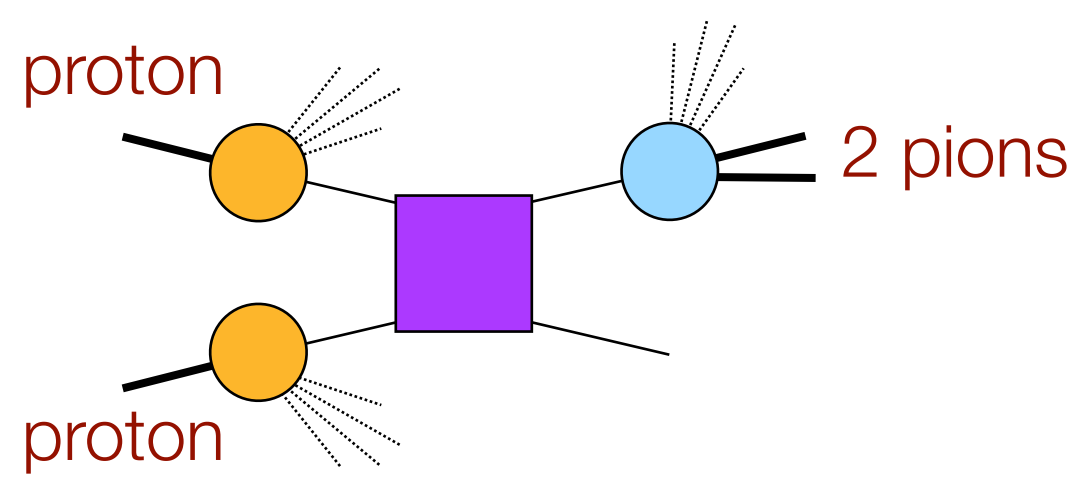
Relevant processes



SIDIS



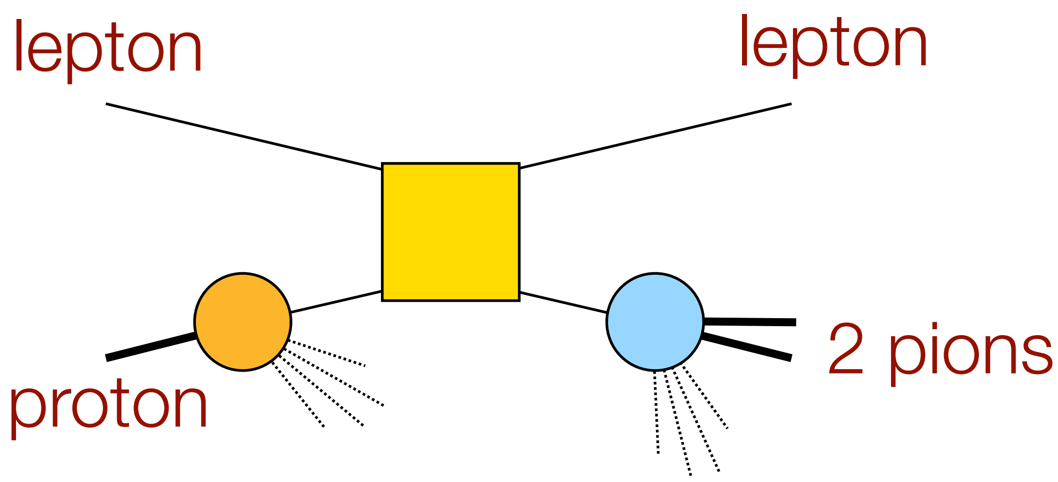
$e^{-}e^{+}$ to pions



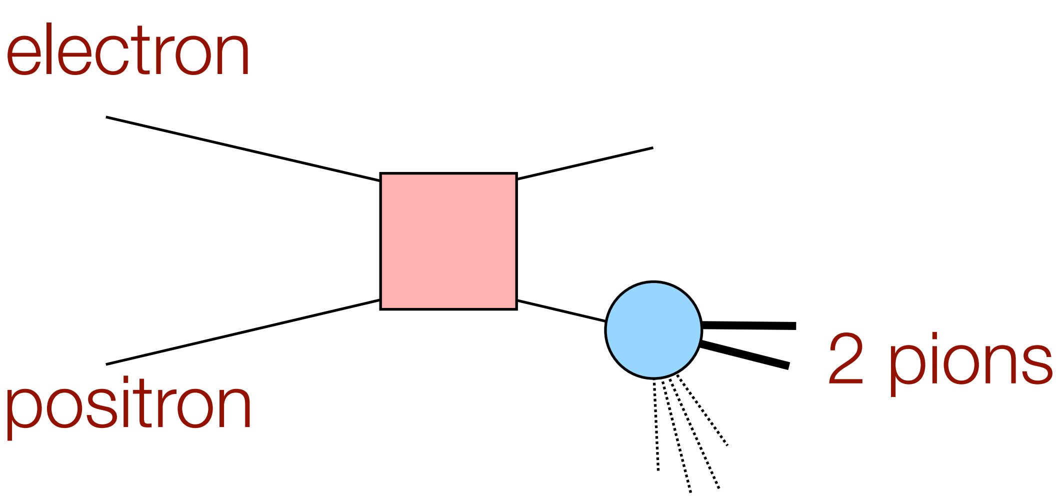
p-p to pions

Relevant processes

Collinear Factorization



SIDIS



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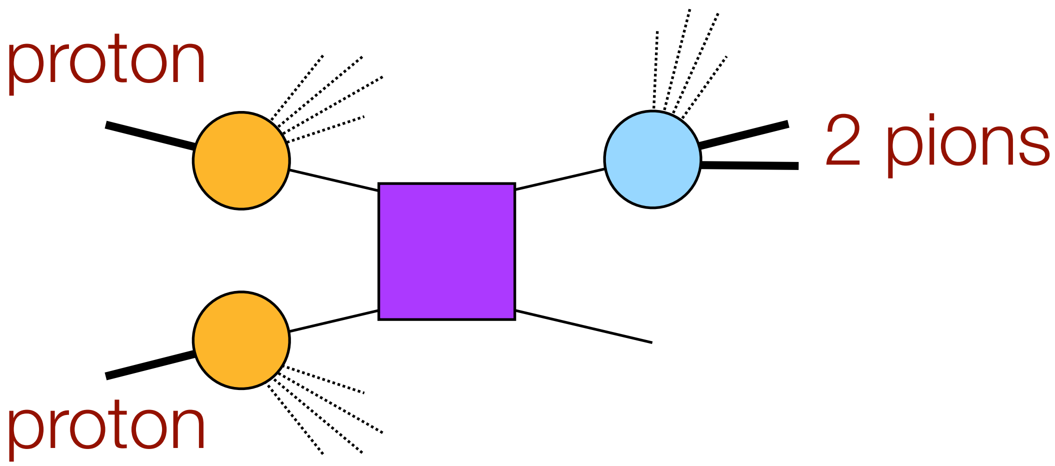
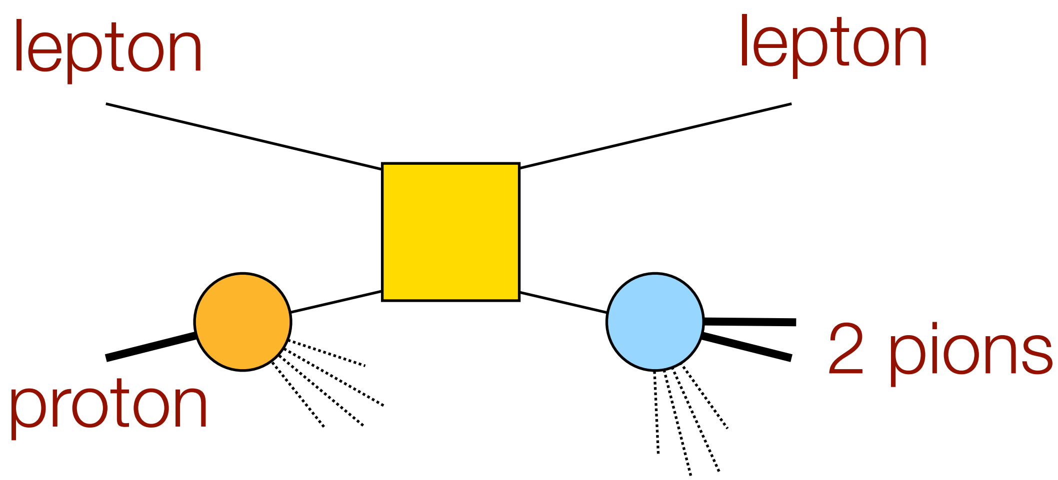


p-p to pions

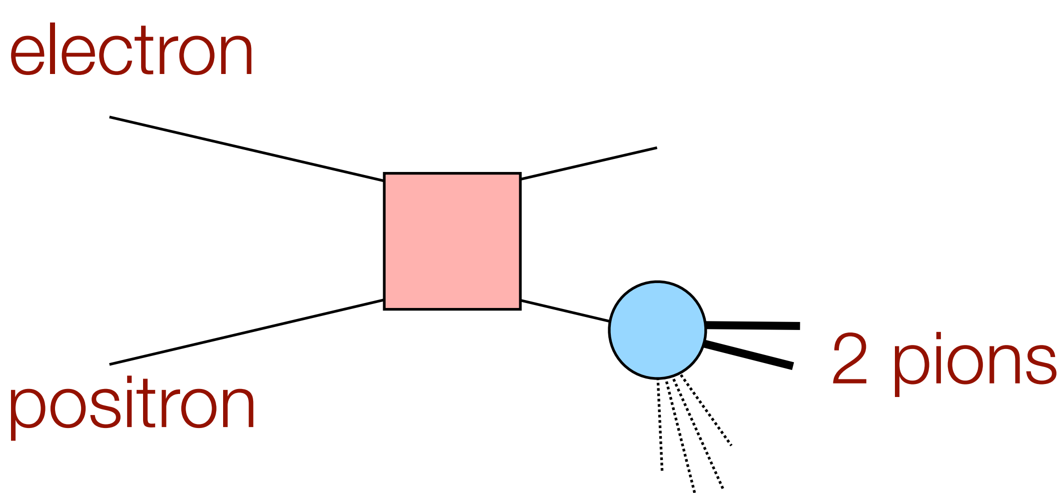
Relevant processes

Collinear Factorization

Universality



SIDIS



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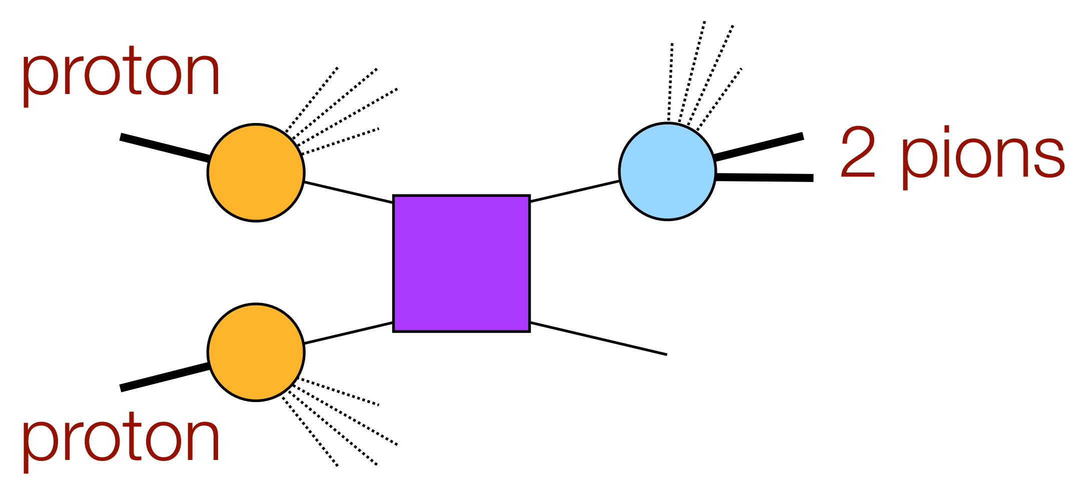
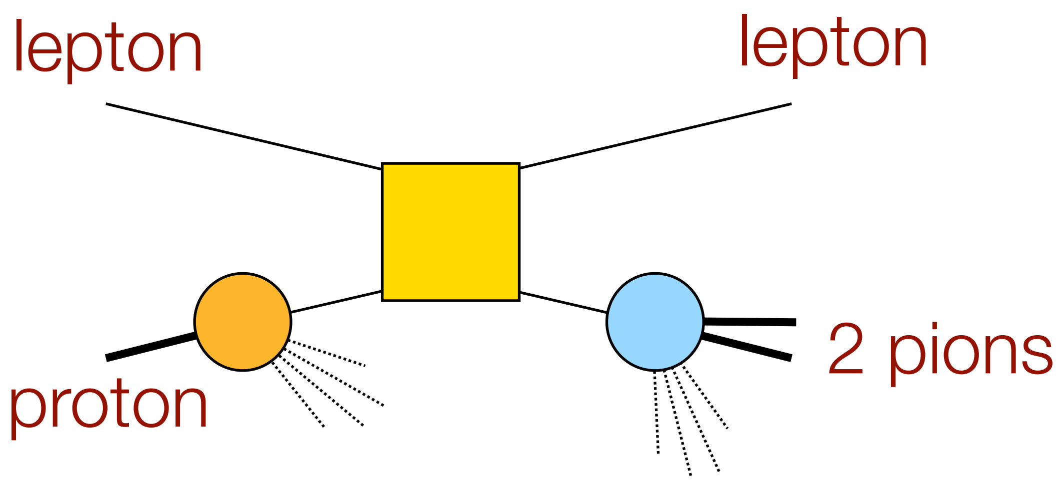


p-p to pions

Relevant processes



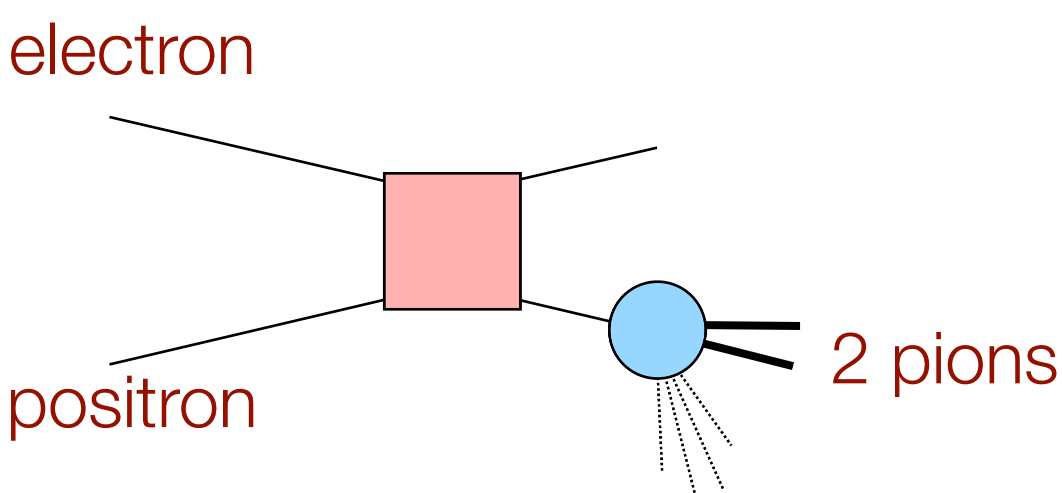
SIDIS

Collinear Factorization

Universality

DGLAP evolution

*Ceccopieri, Radici, Bacchetta, P.L.**B650** (07) 81*



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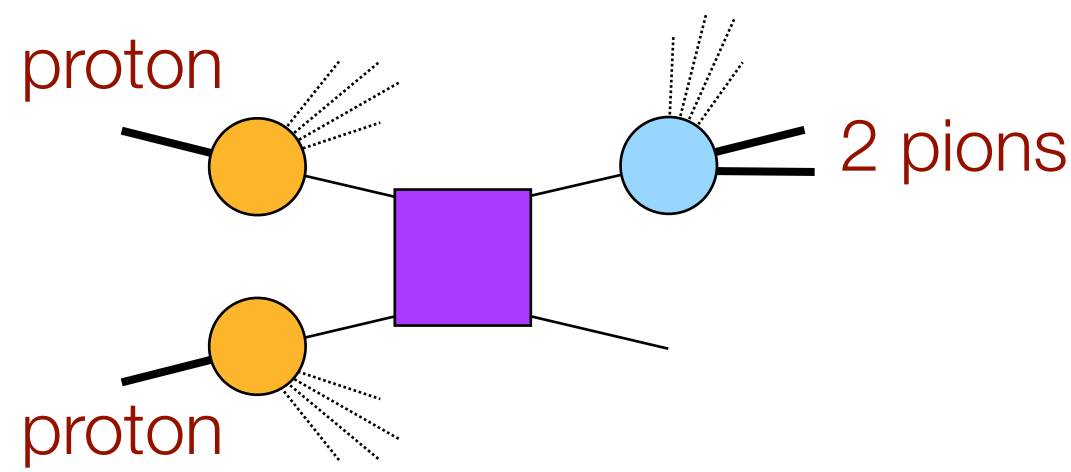

p-p to pions

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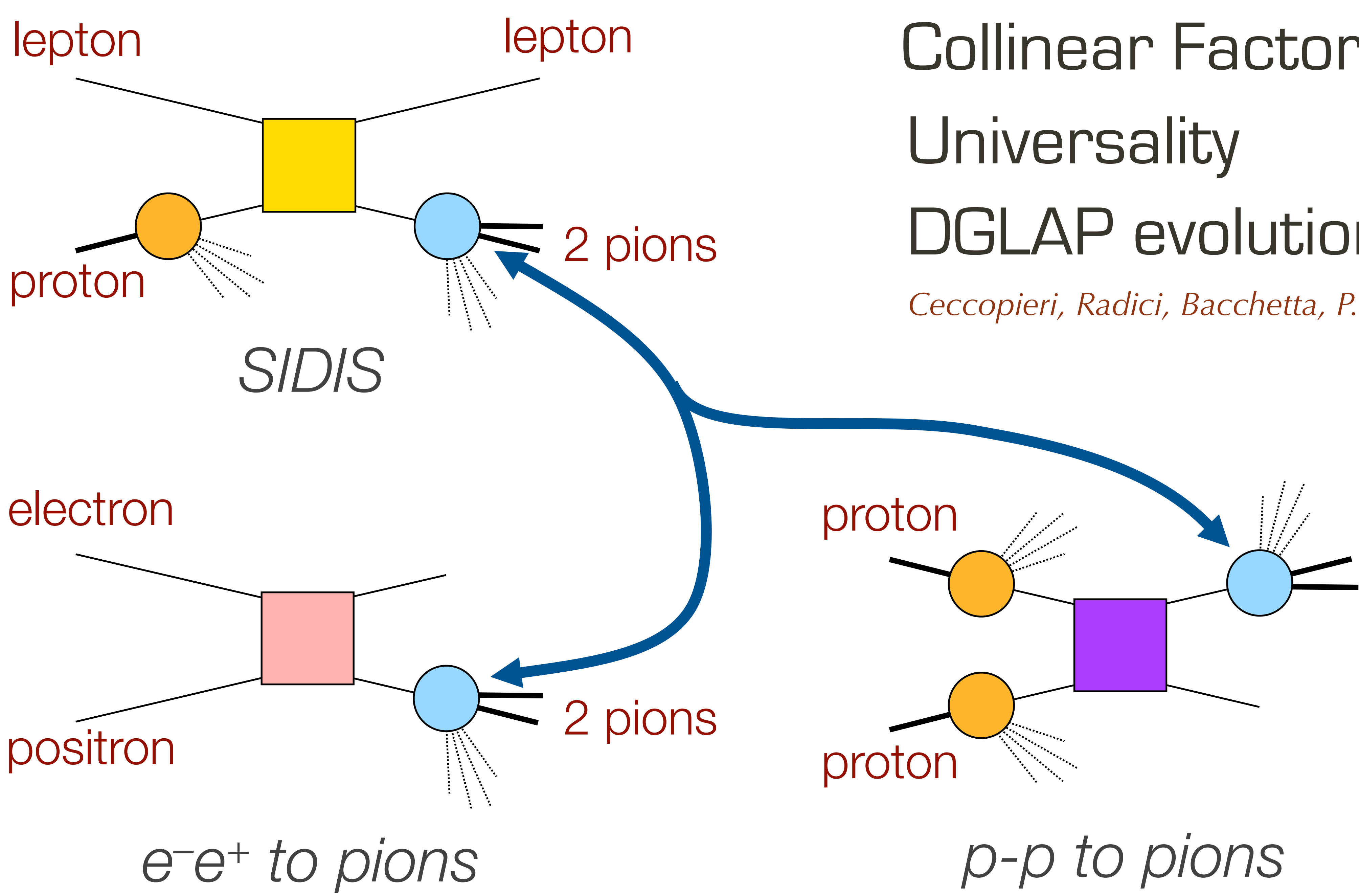

Collinear Factorization

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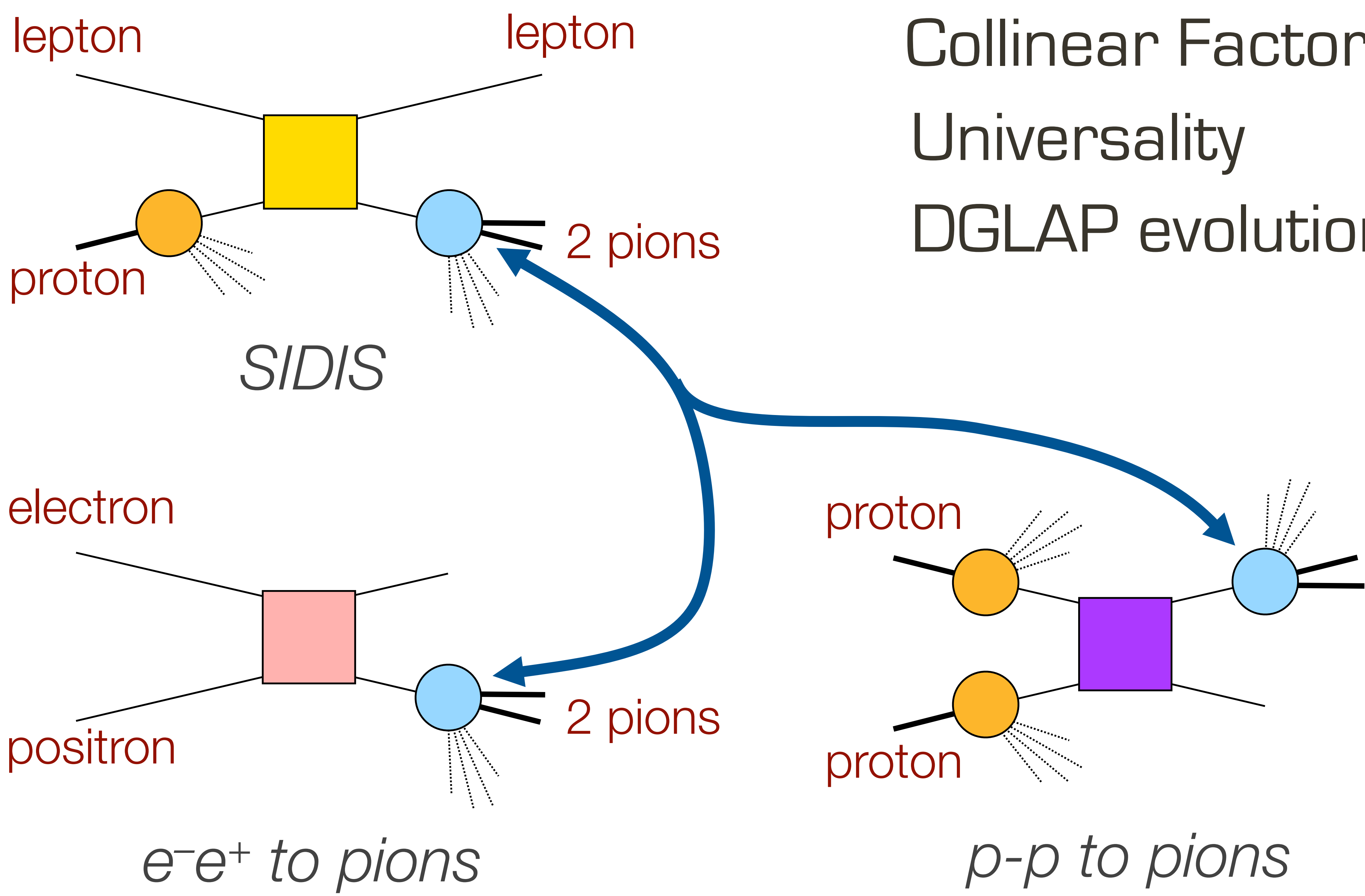
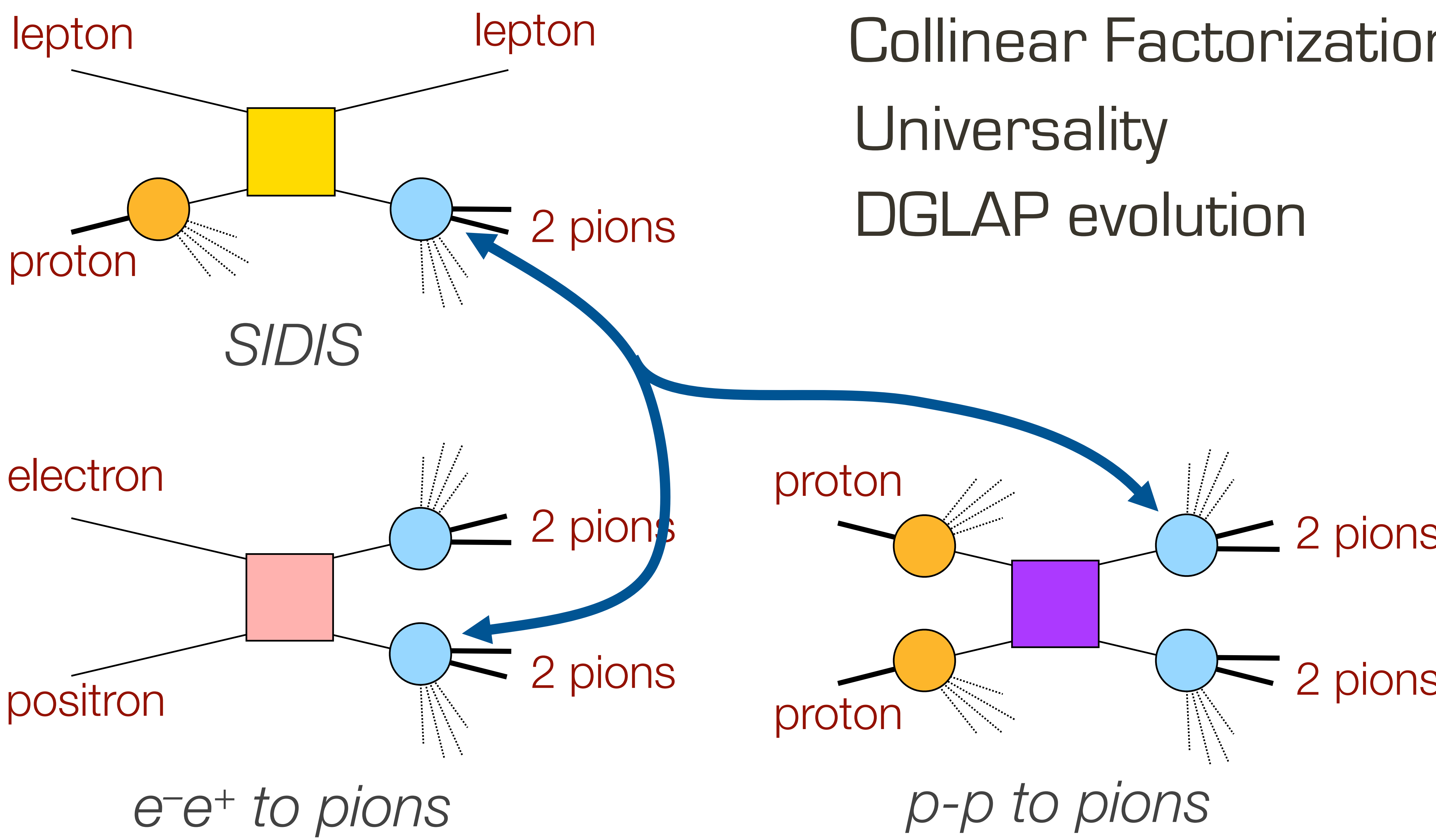


Collinear Factorization

Universality

DGLAP evolution

Relevant processes



Collinear Factorization
Universality
DGLAP evolution

Single hadron

see A. Prokudin's talk

SIDIS

$$A_{DIS}(x,z,P_{h\perp}^2) = -\langle C_y \rangle \frac{\sum_q e_q^2 \, h_1^q(x,p_T^2) \otimes_C H_{1,q}^{\perp}(z,k_T^2)}{\sum_q e_q^2 \, f_1^q(x,p_T^2) \otimes D_{1,q}(z,k_T^2)}$$

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e^+e^-

$$A_{e+e-}(z,\bar{z},Q_T^2) = -\frac{\langle \sin^2\theta_2\rangle}{\langle 1+\cos^2\theta_2\rangle} \frac{\sum_q e_q^2\; H_{1,q}^{\perp}(z,k_T^2) \otimes_C' H_{1,\bar{q}}^{\perp}(\bar{z},\bar{k}_T^2)}{\sum_q e_q^2\, D_{1,q}(z,k_T^2) \otimes' D_{1,\bar{q}}(\bar{z},\bar{k}_T^2)}$$

Two hadrons

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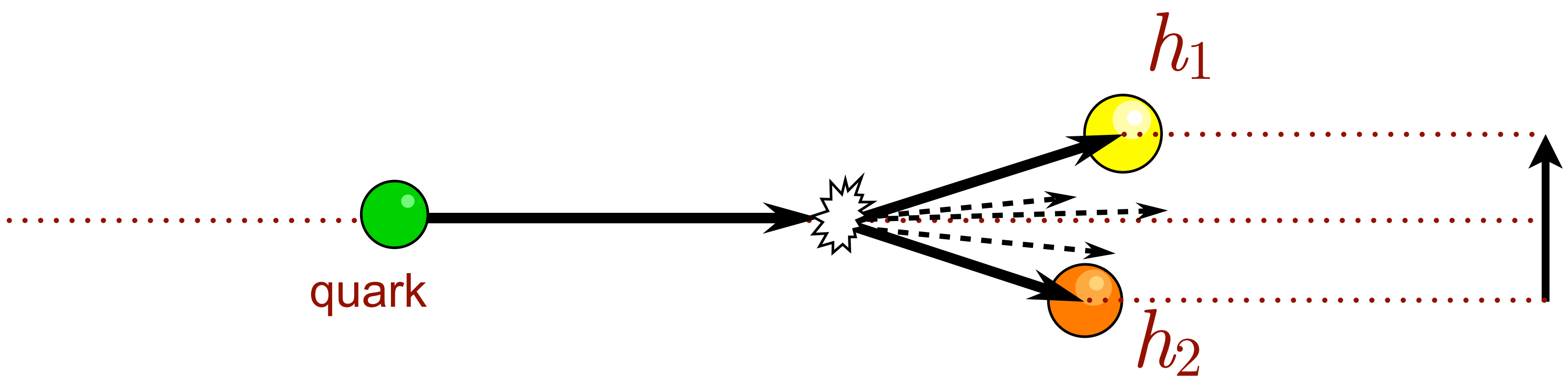
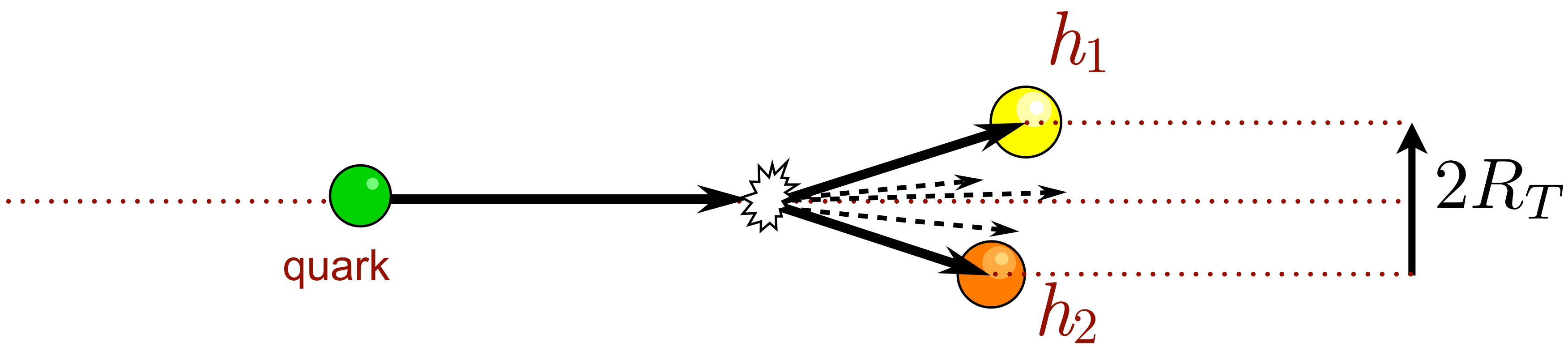
Dihadron fragmentation functions



$$R_T^\mu = g_T^{\mu\nu} R_\nu = R^\mu - \frac{\zeta_h}{2} P_h^\mu + x_B \frac{\zeta_h M_h^2 - (M_1^2 - M_2^2)}{Q^2 z_h} P^\mu$$

Dihadron fragmentation functions



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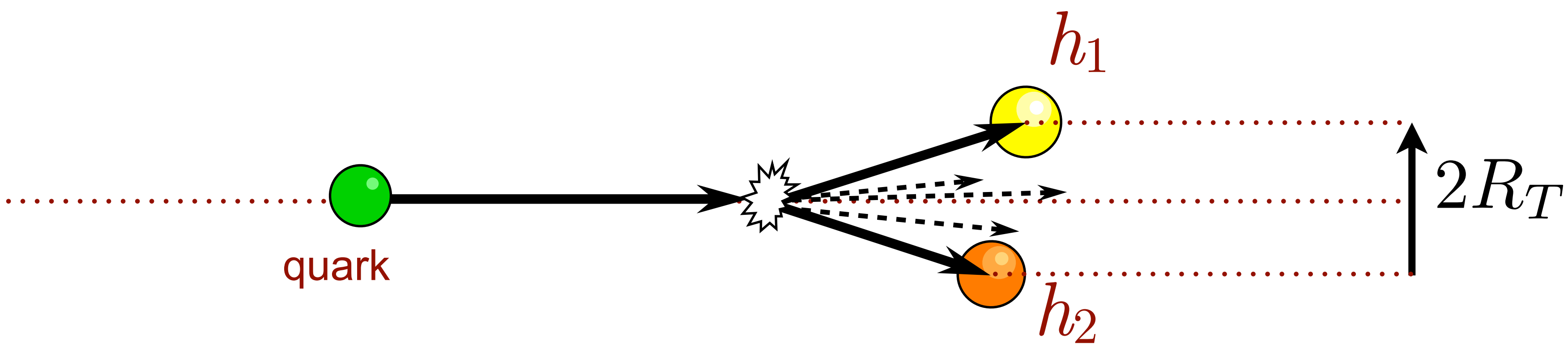
$$D_1^{q \to h_1 h_2}(z_1, z_2, R_T^2)$$

Dihadron fragmentation functions



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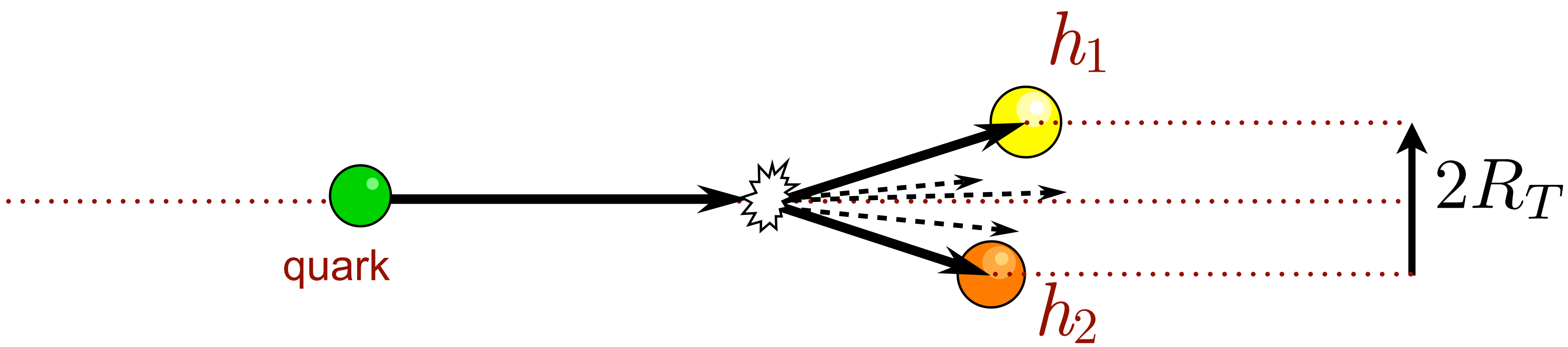
$$D_1^{q \to h_1 h_2}(z_1, z_2, R_T^2)$$

or

Dihadron fragmentation functions



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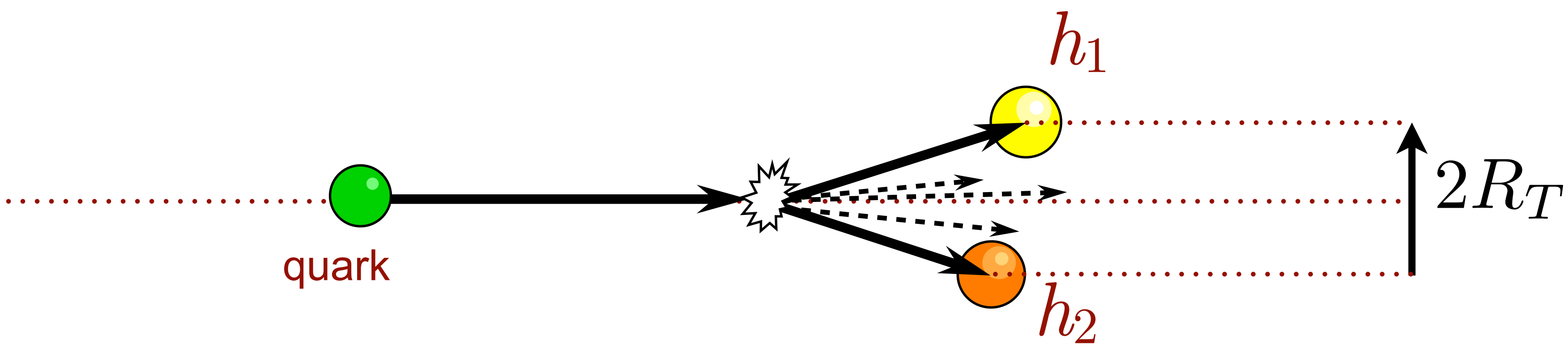
$D_1^{q \to h_1 h_2}(z_1, z_2, R_T^2)$ or $D_1^{q \to h_1 h_2}(z, \cos\theta, M_h)$

Dihadron fragmentation functions



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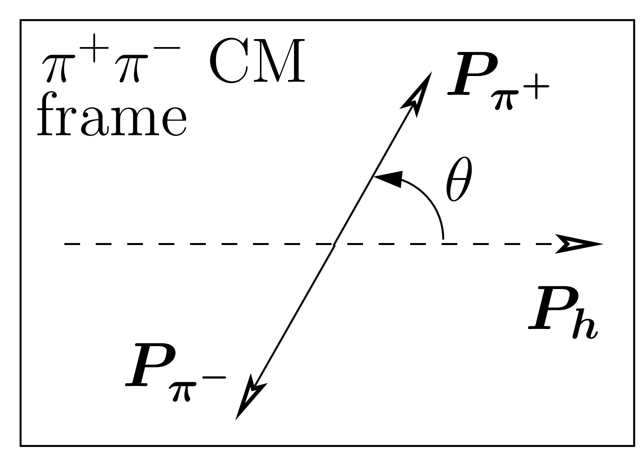
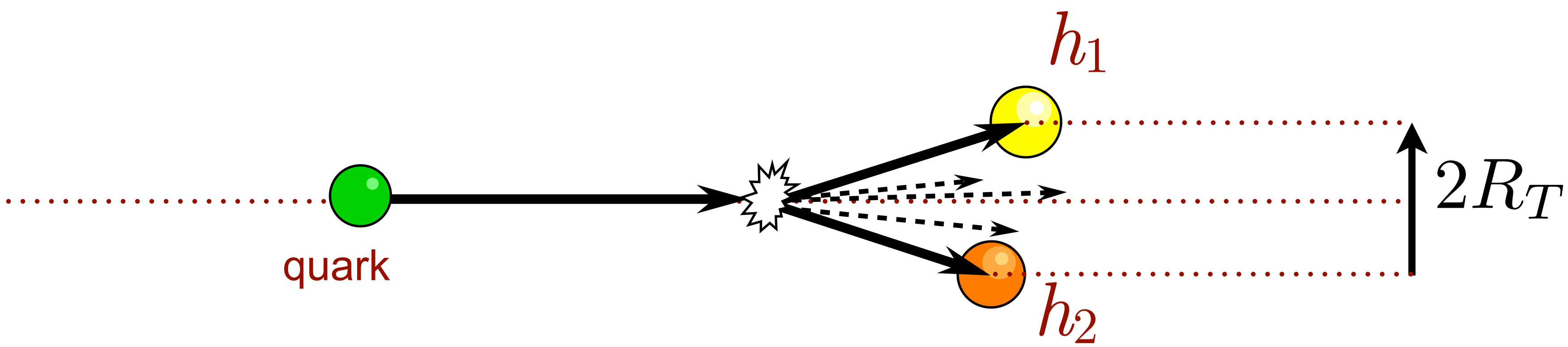


Dihadron fragmentation functions



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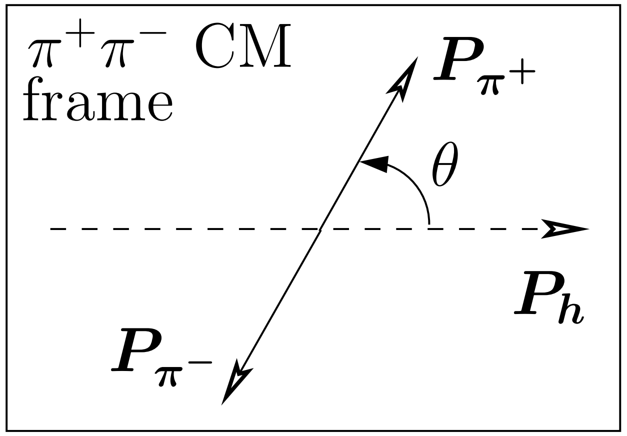
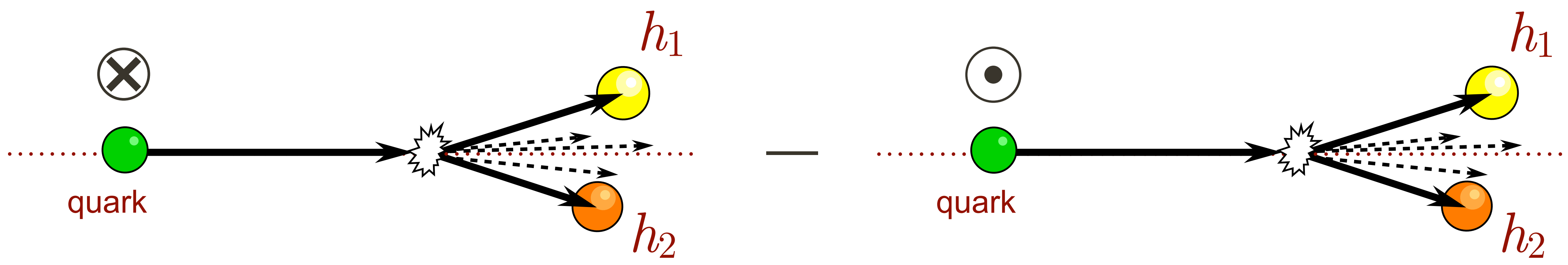
$D_1^{q\to h_1 h_2}(z_1, z_2, R_T^2)$ or $D_1^{q\to h_1 h_2}(z, \cos\theta, M_h)$

Unpolarized DiFF



Interference Fragmentation Function

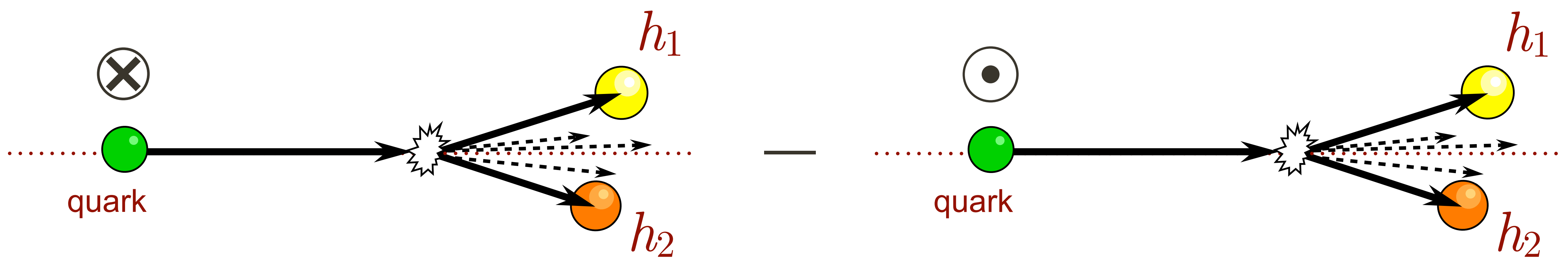
Collins, Heppelman, Ladinsky, NPB420 (94)



$$H^{\sphericalangle}_{1,q\to h_1 h_2}(z, \cos\theta, M_h)$$

Interference Fragmentation Function

Collins, Heppelman, Ladinsky, NPB420 (94)



$$H^{\sphericalangle}_{1,q\to h_1 h_2}(z, \cos\theta, M_h)$$

Does not vanish if integrated over transverse momentum

Interference Fragmentation Function

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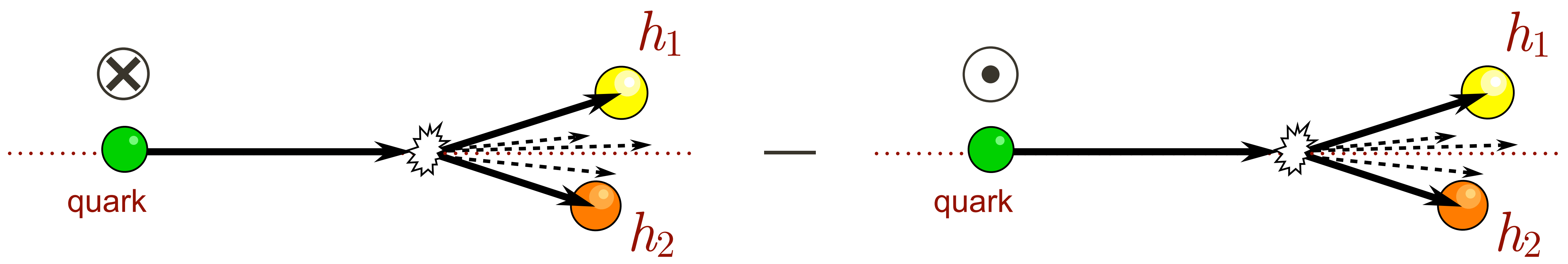

$$H^{\sphericalangle}_{1,q\to h_1 h_2}(z, \cos\theta, M_h)$$

Does not vanish if integrated over transverse momentum

(the two hadrons must be distinguishable)

Partial wave expansion

*Bacchetta & Radici, P.R. D**67** (03) 094002*

$$D_1(z,\cos\theta,M_h) \approx D_1(z,M_h) + D_{1,sp}(z,M_h)\ \cos\theta + \ldots$$

$$|\mathbf{R}_T|\ H_1^{\sphericalangle}(z,\cos\theta,M_h) \approx H_{1,sp}^{\sphericalangle}(z,M_h)\ \sin\theta + H_{1,pp}^{\sphericalangle}(z,M_h)\ \sin\theta\cos\theta + \ldots$$

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involved in recent
measured asymmetries

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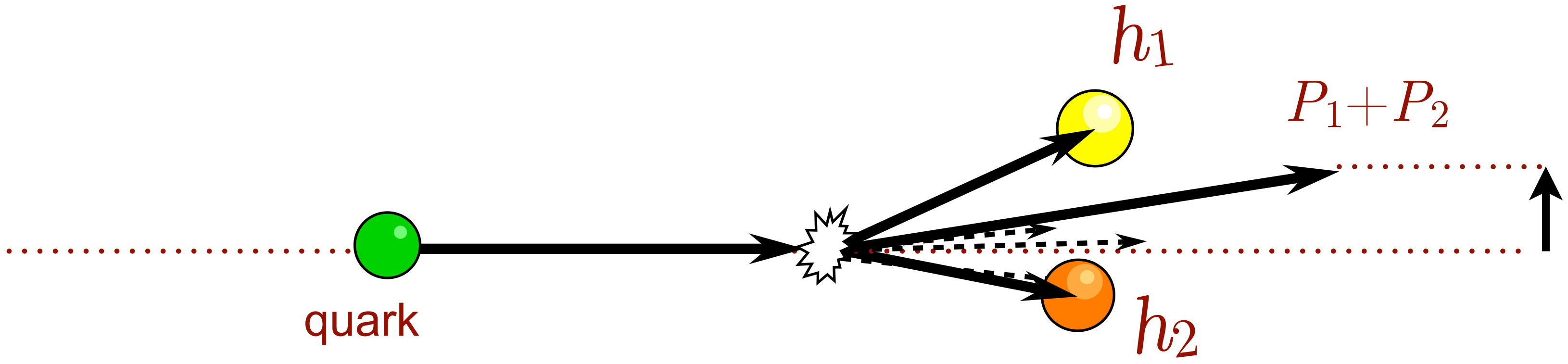
involved in recent
measured asymmetries

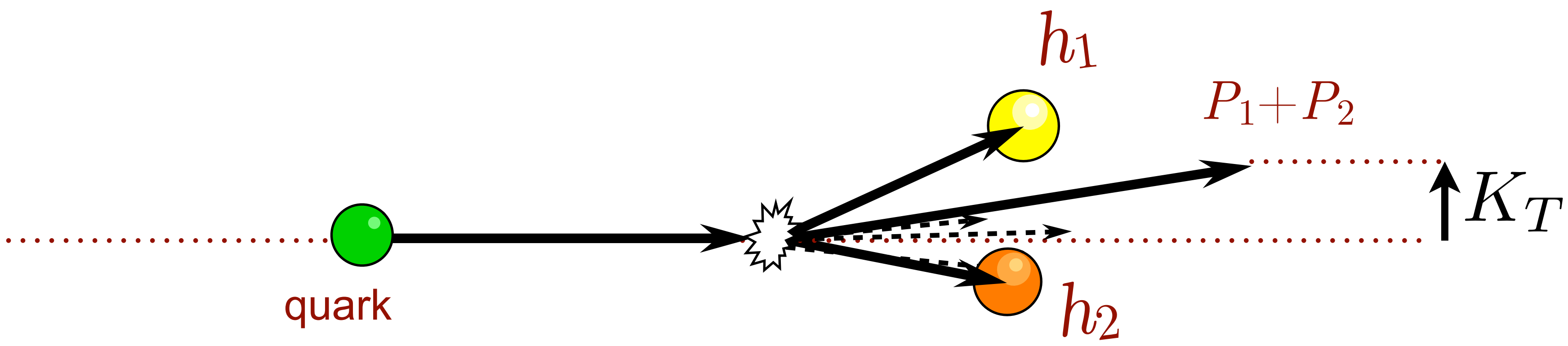
Caveat: dihadron fragmentation functions depend on three variables and effects of experimental acceptance are complicated

TMD dihadron FFs



Bianconi, Boffi, Jakob, Radici, PRD62 (00)
Boer, Jakob, Radici, P.R. D67 (03) 094003
Gliske, Bacchetta, Radici, Phys. Rev. D90 (14)

TMD dihadron FFs



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Boer, Jakob, Radici, P.R. D67 (03) 094003
Gliske, Bacchetta, Radici, Phys. Rev. D90 (14)

Determination of unpolarized DiFFs

Unpolarized cross section

$e^+e^- \rightarrow (\pi^+\pi^-) + X$

$$\frac{d\sigma^0}{dz\,dM_h} = \frac{4\pi\alpha^2}{Q^2} \sum_q e_q^2\, D_1^q(z, M_h)$$

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Data not yet available!

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Data not yet available!

Need multiplicities for

$e^+e^- \rightarrow (\pi^+\pi^-) + X$

or $e + p \rightarrow e' + (\pi^+\pi^-) + X$

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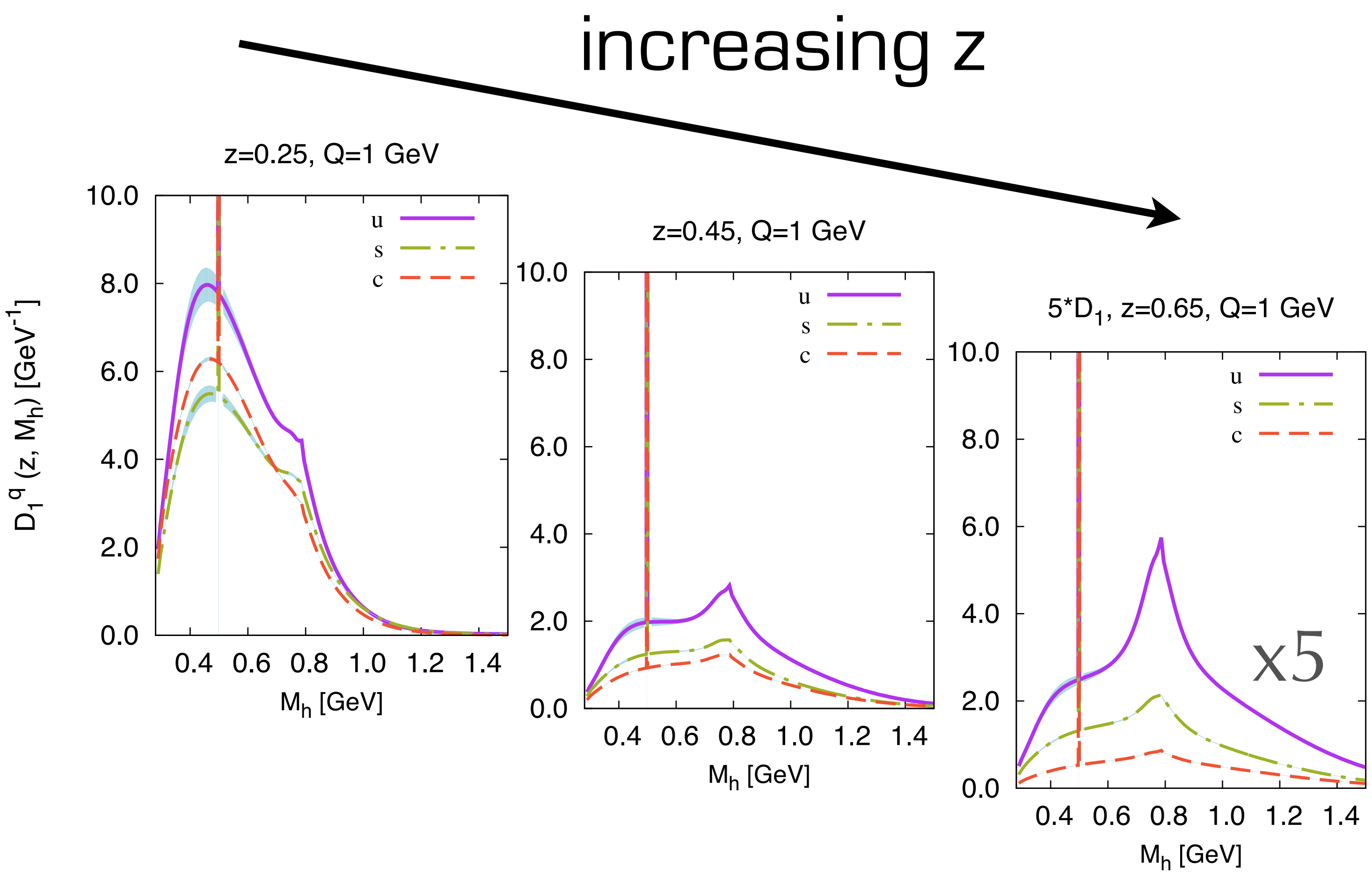
Data not yet available!

Temporary solution: use output of event generators (PYTHIA)

Results for unpolarized DiFF

Courtoy et al., P.R. D85 (12) 114023



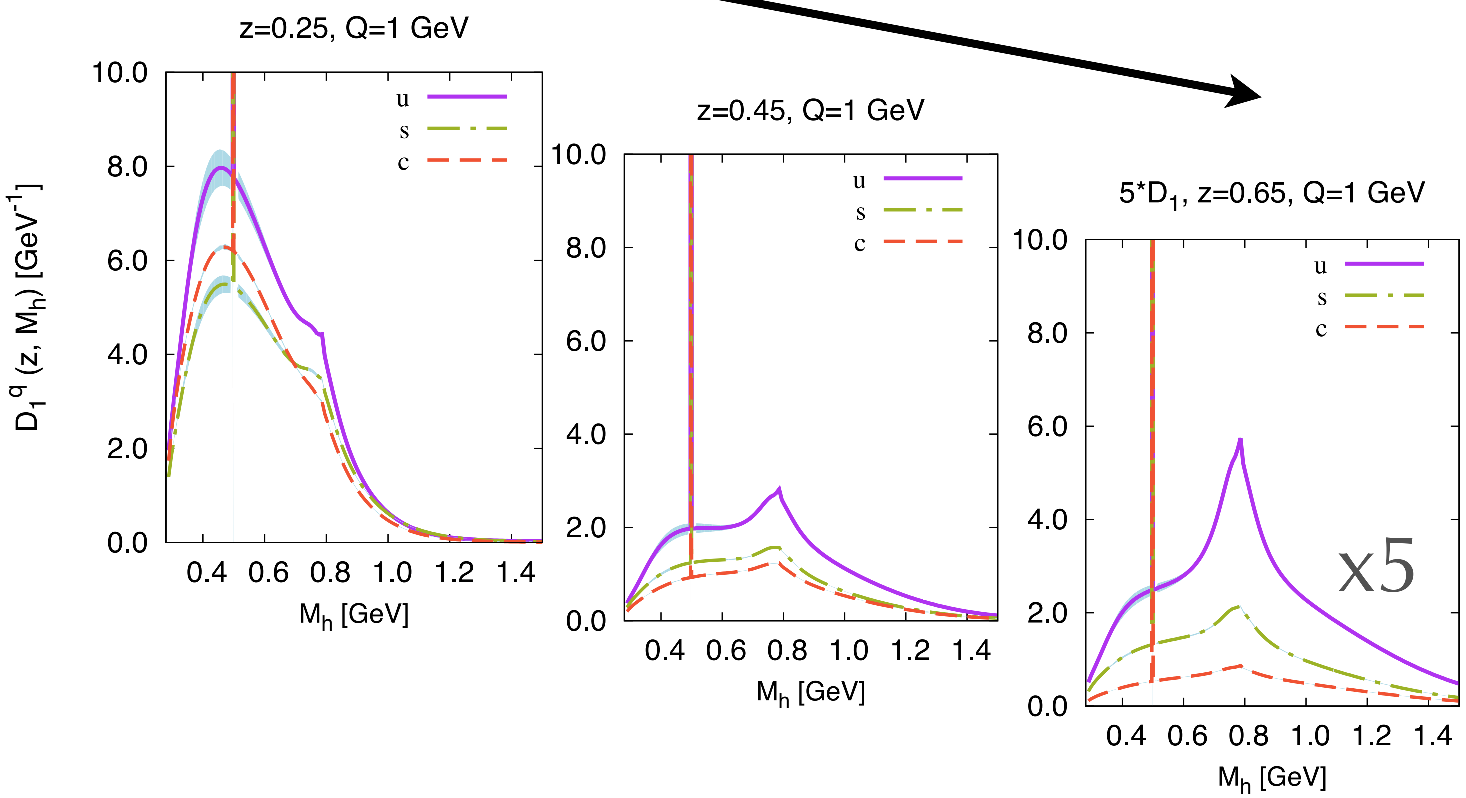
M_h behavior

$Q_0{}^2$ = 1 GeV^2

Results for unpolarized DiFF

Courtoy et al., P.R. D85 (12) 114023

increasing z



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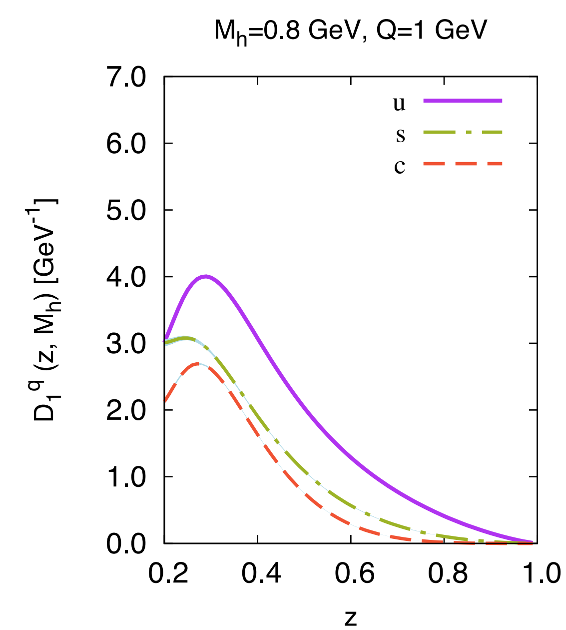

z behavior

Present limitations

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  Input $D_1^{q\to\pi+\pi-}(z,\, M_\mathrm{h})$ parametrized at initial scale Q_0^2= 1 GeV^2 then evolved at Q_0^2= 100 GeV^2

  Not so important for SIDIS, but can be very important for pp collisions

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  Not so important for SIDIS, but can be very important for pp collisions
- Need of model assumptions

  model-inspired fitting function (K^0, ω, ρ resonances + continuum)

  charge conjugation + isospin

  $u = \bar{u} \quad d = \bar{d} \quad s = \bar{s} \quad c = \bar{c}$

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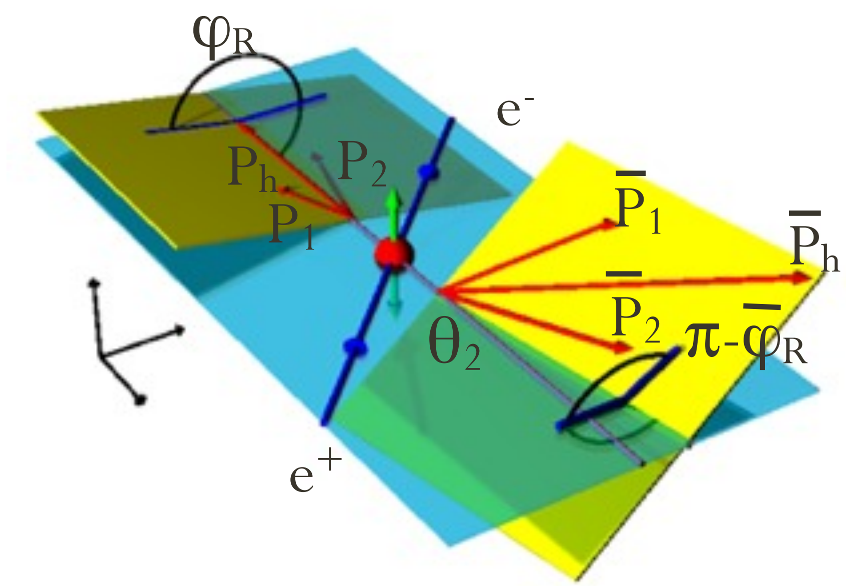
- Approach valid for $M_\mathrm{h} \gg Q$

*see Zhou and Metz, P.R.L. **106** (11) 172001*

Extraction of Interference FF

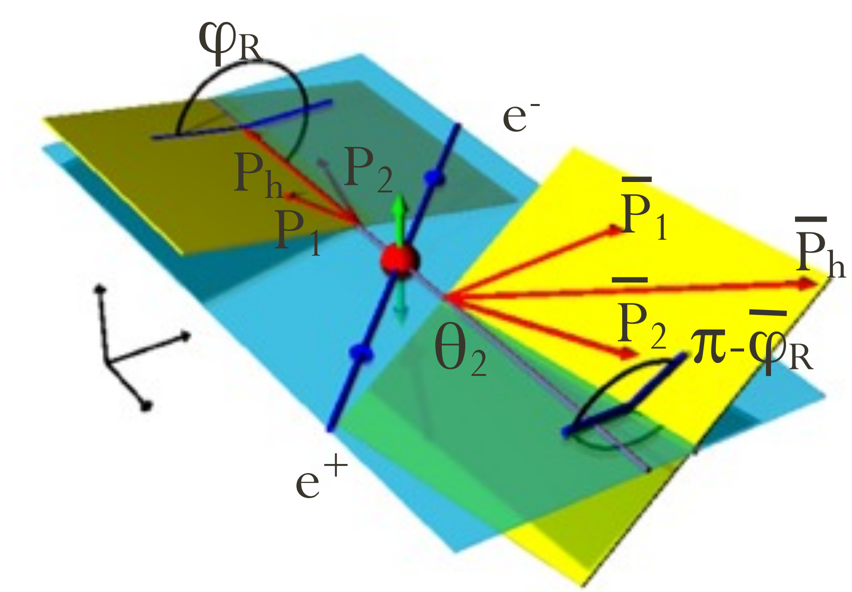


picture from BELLE

*Artru & Collins, Z.Ph. C**69** (96) 277*

*Boer, Jakob, Radici, P.R. D**67** (03) 094003*

Extraction of Interference FF



picture from BELLE

$$A_{e+e-}(z, M_h^2, \bar{z}, \bar{M}_h^2) = -\frac{\langle \sin^2\theta_2\rangle \langle \sin\theta\rangle\, \langle \sin\bar{\theta}\rangle}{\langle 1+\cos^2\theta_2\rangle}\, \frac{\sum_q e_q^2\, \frac{|\boldsymbol{R}|}{M_h}\, H_{1,q}^{\sphericalangle}(z, M_h^2)\, \frac{|\bar{\boldsymbol{R}}|}{\bar{M}_h}\, H_{1,\bar{q}}^{\sphericalangle}(\bar{z}, \bar{M}_h^2)}{\sum_q e_q^2\, D_{1,q}(z, M_h^2)\, D_{1,\bar{q}}(\bar{z}, \bar{M}_h^2)}$$

*Artru & Collins, Z.Ph. C**69** (96) 277*

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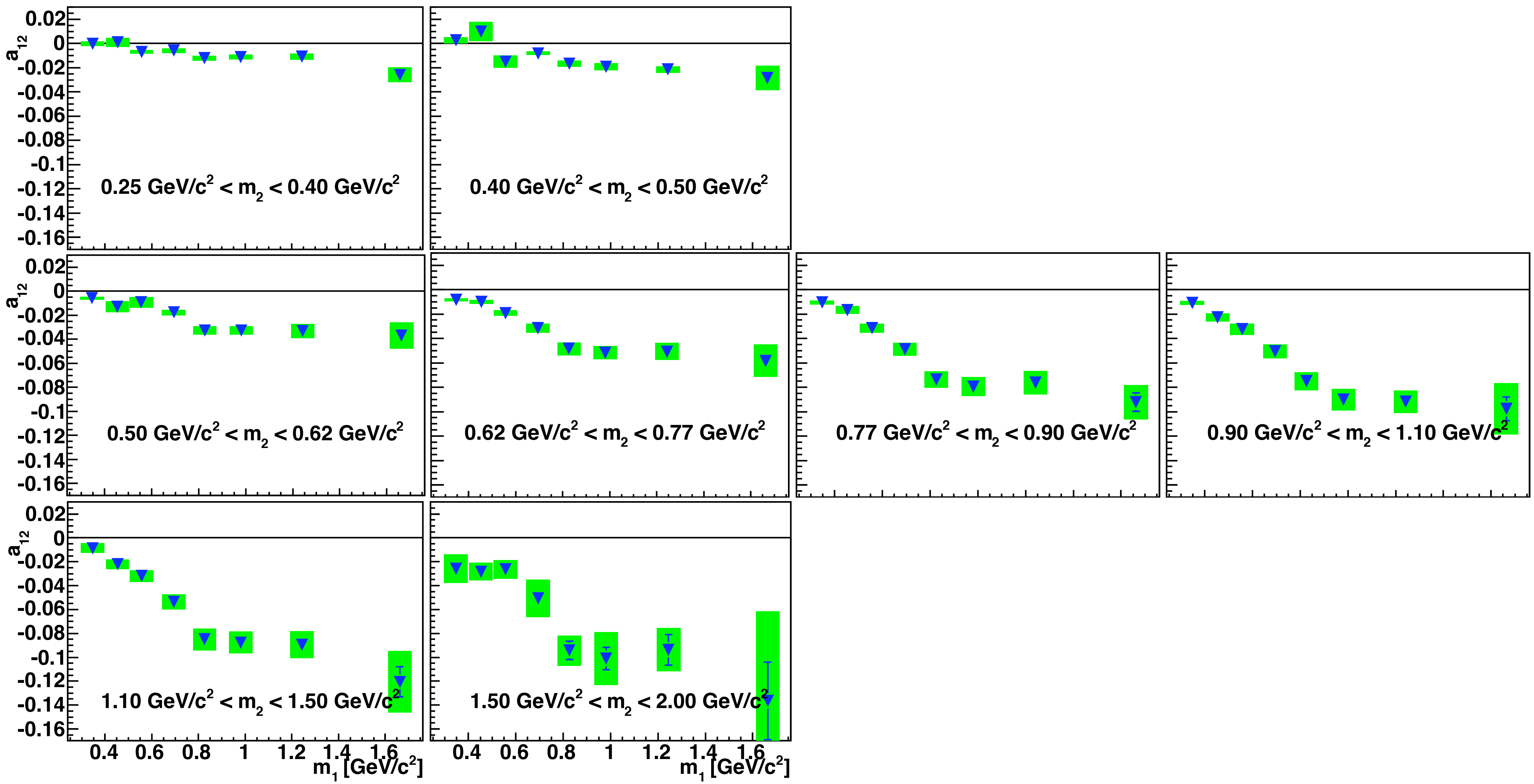
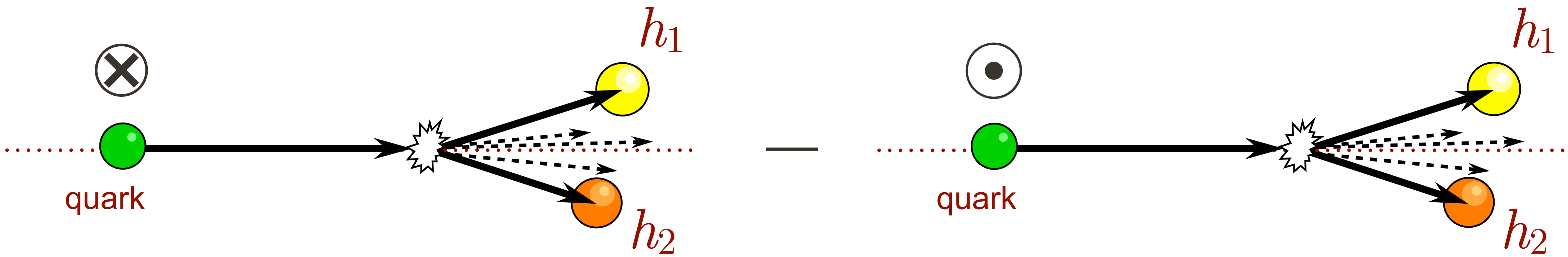
Extraction of Interference FF



Vossen, Seidl et al. (Belle), PRL 107 (2011)

Assumptions



For $\pi^+ \pi^-$

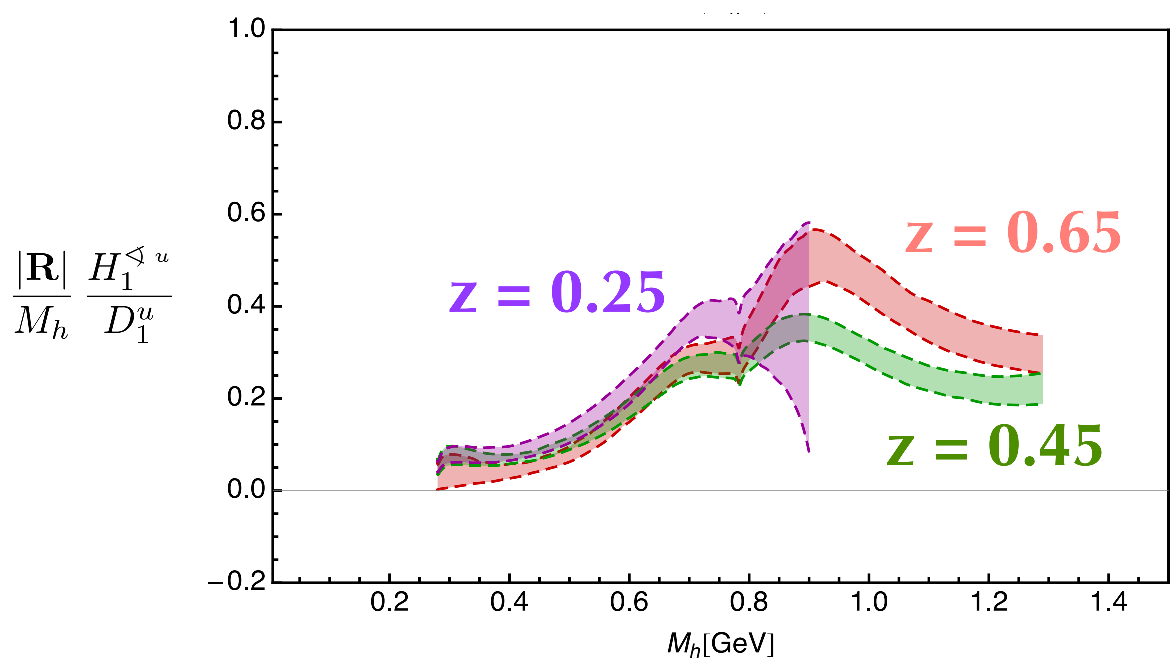
$$H_1^{\sphericalangle\, u} = -H_1^{\sphericalangle\, d} = -H_1^{\sphericalangle\, \bar{u}} = H_1^{\sphericalangle\, \bar{d}} , \quad H_1^{\sphericalangle\, s} = -H_1^{\sphericalangle\, \bar{s}} = 0$$

Most recent results

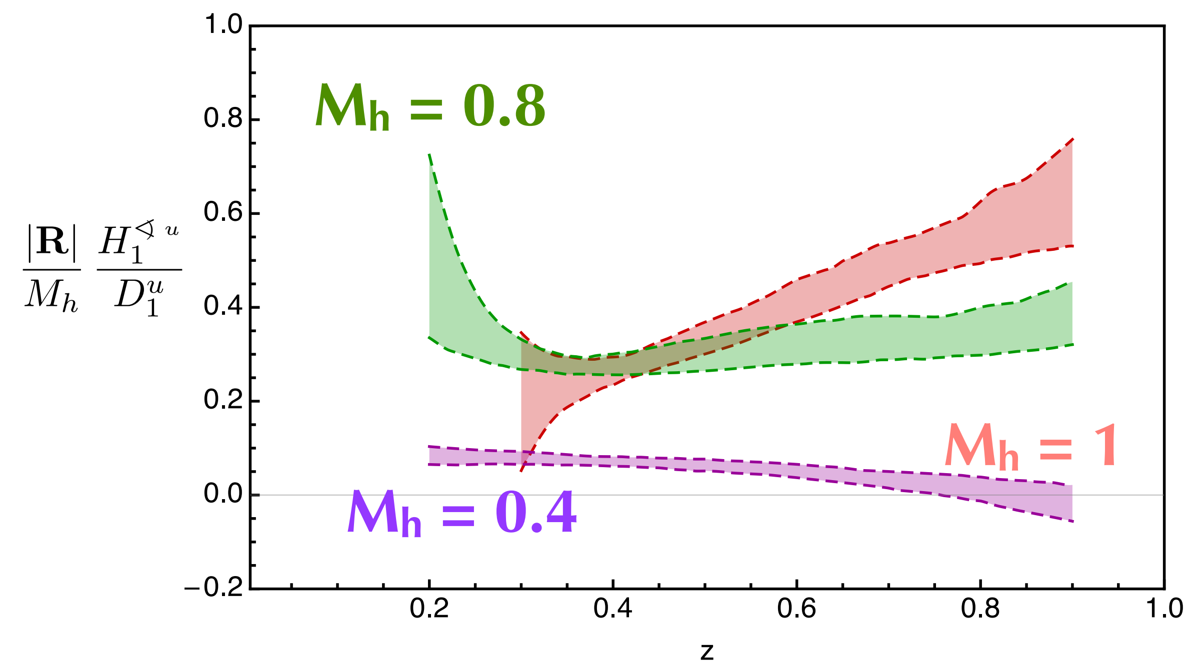
Radici et al., JHEP **1505** (15) 123

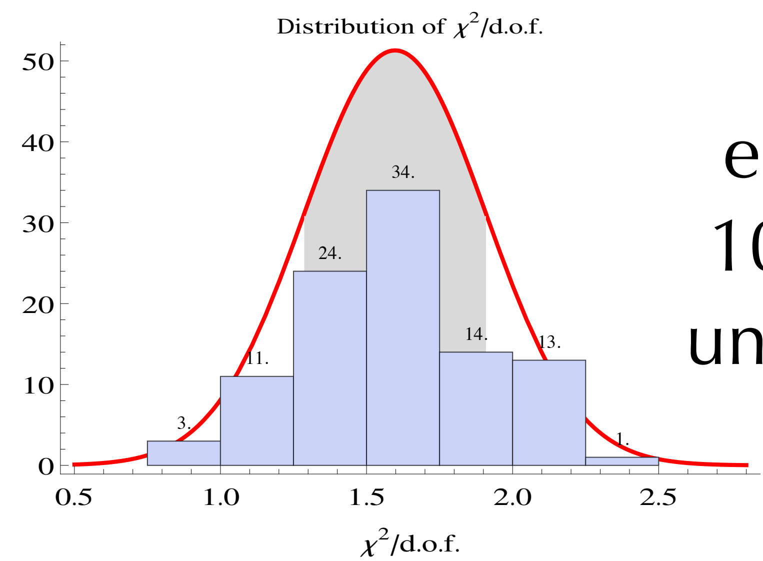
$Q_0^2 = 1\ \mathrm{GeV}^2$

M_h behavior



z behavior

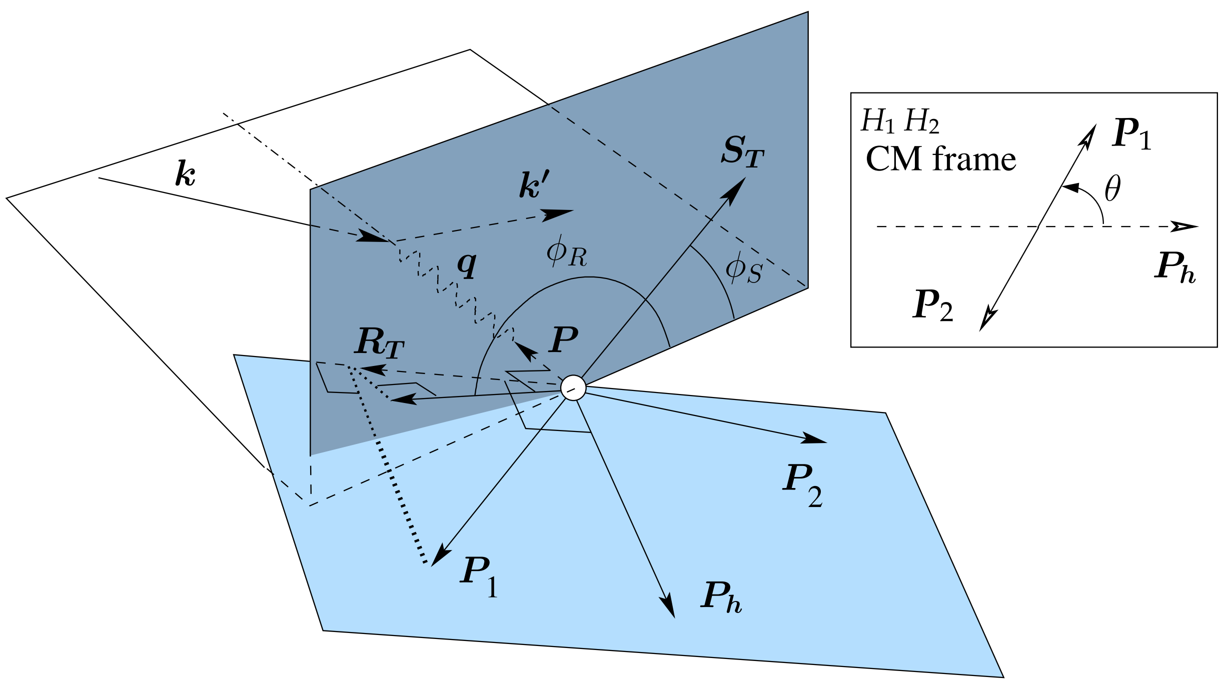




error analysis using
100 replicas of data
uncertainty driven by
exp. errors

Transversity extraction

*Radici, Jakob, Bianconi, P.R. D**65** (02) 074031*
*Bacchetta & Radici, P.R. D**67** (03) 094002*

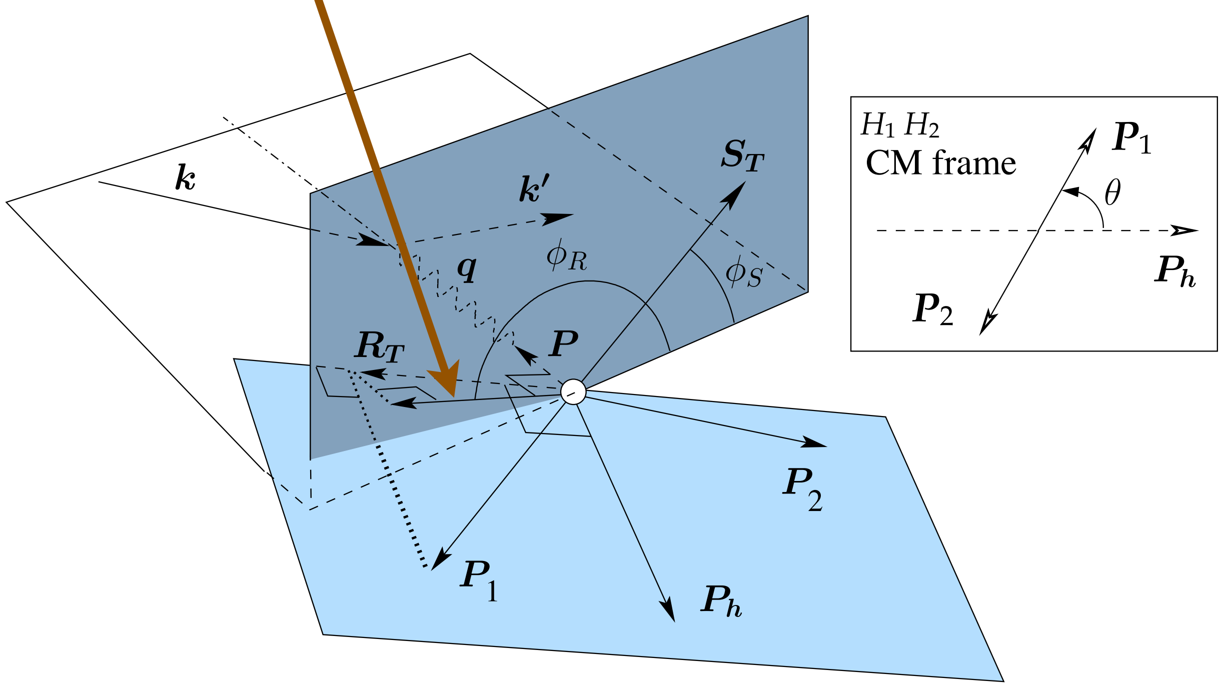


$$A_{\text{SIDIS}}(x, z, M_h; Q) = -\frac{B(y)}{A(y)} \frac{|\boldsymbol{R}|}{M_h} \frac{\sum_q e_q^2\, h_1^q(x; Q^2)\, H_1^{\sphericalangle\, q}(z, M_h; Q^2)}{\sum_q e_q^2\, f_1^q(x; Q^2)\, D_1^q(z, M_h; Q^2)}$$

Transversity extraction

*Radici, Jakob, Bianconi, P.R. D**65** (02) 074031*
*Bacchetta & Radici, P.R. D**67** (03) 094002*

$$\boldsymbol{R}_{T\perp}\big|_{\mathrm{TRF}} = \frac{z_2 \boldsymbol{P}_{1T} - z_1 \boldsymbol{P}_{2T}}{z} + \mathcal{O}\left(\frac{1}{Q^3}\right)$$



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x-dependent expressions

$$n_q^{\uparrow} = \int dz \int dM_h^2 \, \frac{|\mathbf{R}|}{M_h} \, H_{1,sp}^{\sphericalangle\, q}(z, M_h^2)$$

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$$n_q = n_{\bar{q}} \qquad \begin{aligned} n_q^{\uparrow} &= -n_{\bar{q}}^{\uparrow} \\ n_u^{\uparrow} &= -n_d^{\uparrow} \end{aligned}$$

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proton hermes COMPASS

$$\begin{aligned} x\, h_1^p(x;Q^2) &\equiv x\, h_1^{u_v}(x;Q^2) - \tfrac{1}{4}\, x h_1^{d_v}(x;Q^2) \\ &= -\frac{A_{\text{SIDIS}}^p(x;Q^2)}{n_u^{\uparrow}(Q^2)} \, \frac{A(y)}{B(y)} \, \frac{9}{4} \sum_{q=u,d,s} e_q^2 \, n_q(Q^2)\, x f_1^{q+\bar{q}}(x;Q^2) \end{aligned}$$

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proton

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deuteron

$$\begin{aligned} x\, h_1^D(x;Q^2) &\equiv x\, h_1^{u_v}(x;Q^2) + x h_1^{d_v}(x;Q^2) \\ &= -\frac{A_{\text{SIDIS}}^D(x;Q^2)}{n_u^{\uparrow}(Q^2)} \, 3 \sum_{q=u,d,s} \left[e_q^2 \, n_q(Q^2) + e_{\tilde{q}}^2 \, n_{\tilde{q}}(Q^2) \right] \, x f_1^{q+\bar{q}}(x;Q^2) \end{aligned}$$

Literature

data		extraction
hermes	proton target *Airapetian et al., JHEP **0806** (08) 017*	$$xh_1^{u_v}(x) - \frac{1}{4}\, xh_1^{d_v}(x)$$ *Bacchetta, Courtoy, Radici, P.R.L. **107** (11) 012001*
COMPASS	proton + deuteron *Adolph et al., P.L. **B713** (12)*	$$xh_1^{u_v}(x)\ ,\quad xh_1^{d_v}(x)$$ *Bacchetta, Courtoy, Radici, JHEP **1303** (13) 119*
COMPASS	new proton data *Braun et al., E.P.J. Web Conf. **85** (15) 02018*	**new fit** *Radici et al., JHEP **1505** (15) 123*

Choice of functional form

at starting scale $Q_0^2 = 1\ \mathrm{GeV}^2$

$$x h_1^{q_v}(x) = \tanh\left[\sqrt{x}\ \left(A_q + B_q x + C_q x^2 + D_q x^3\right)\right]\ \left[x\,\mathrm{SB}_q(x) + x\,\overline{\mathrm{SB}}_{\bar{q}}(x)\right]$$

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satisfies **S**offer **B**ound at any Q^2

$$2|h_1^q(x,Q^2)| \le 2\ \mathrm{SB}_q(x) = |f_1^q(x) + g_1^q(x)|$$

Choice of functional form

at starting scale $Q_0^2 = 1$ GeV^2

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rigid

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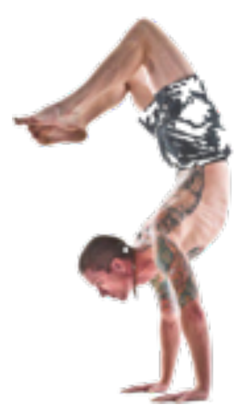
rigid

flexible



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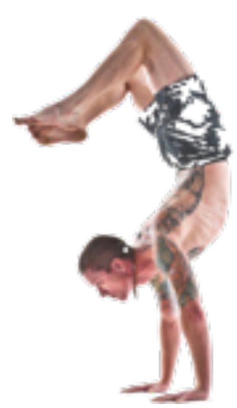
rigid

flexible

extra-flexible



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Replica method

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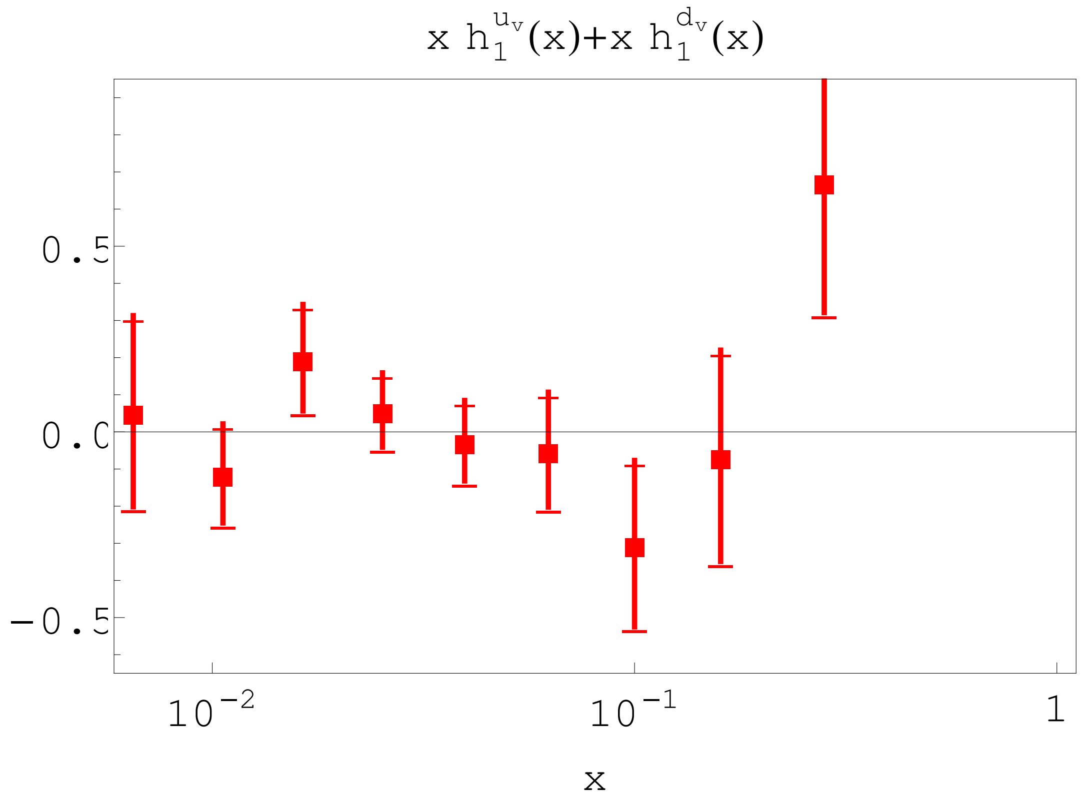
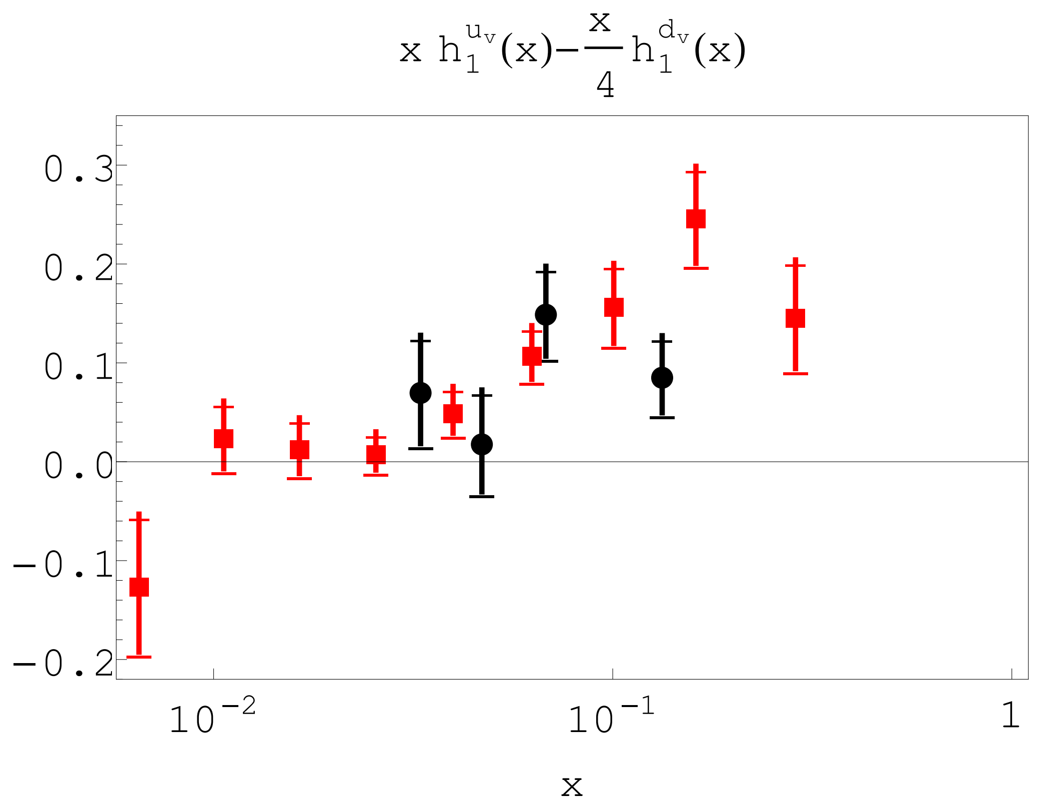
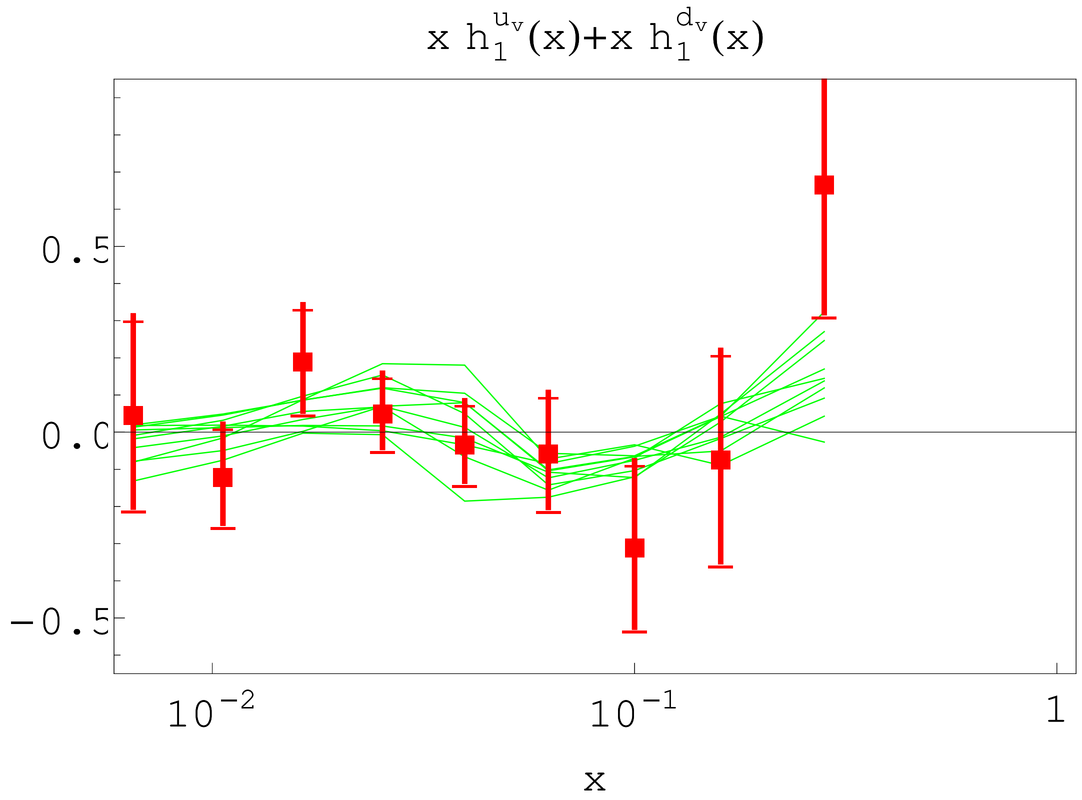
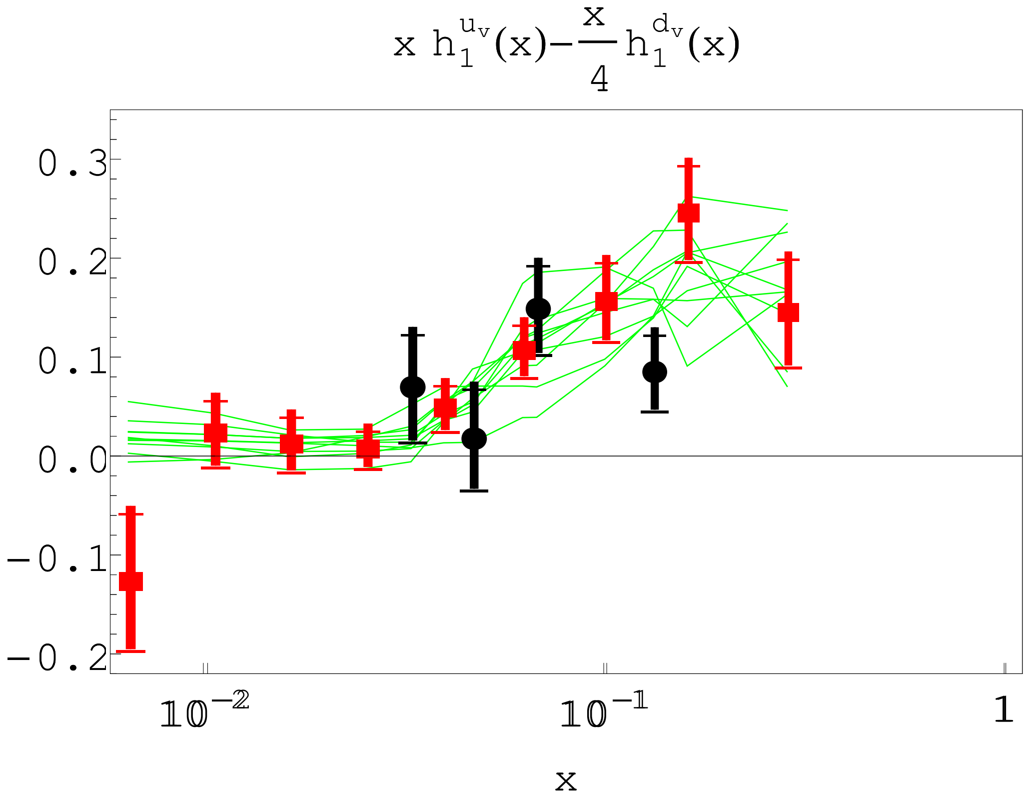


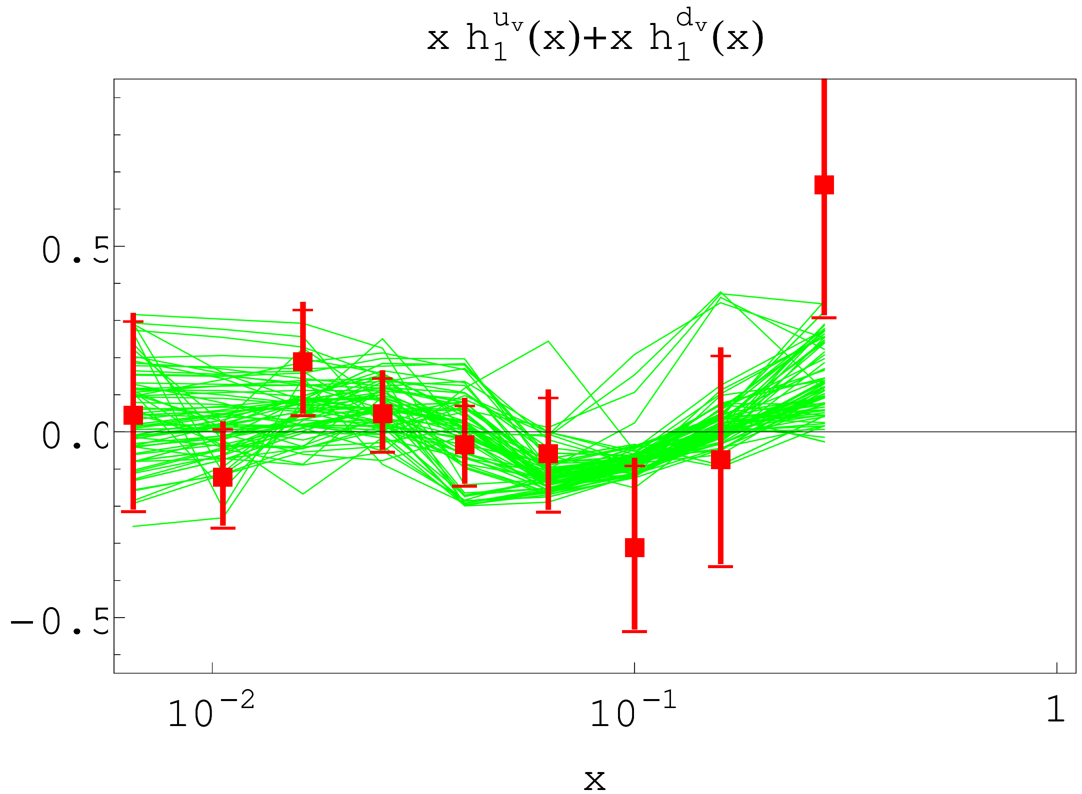


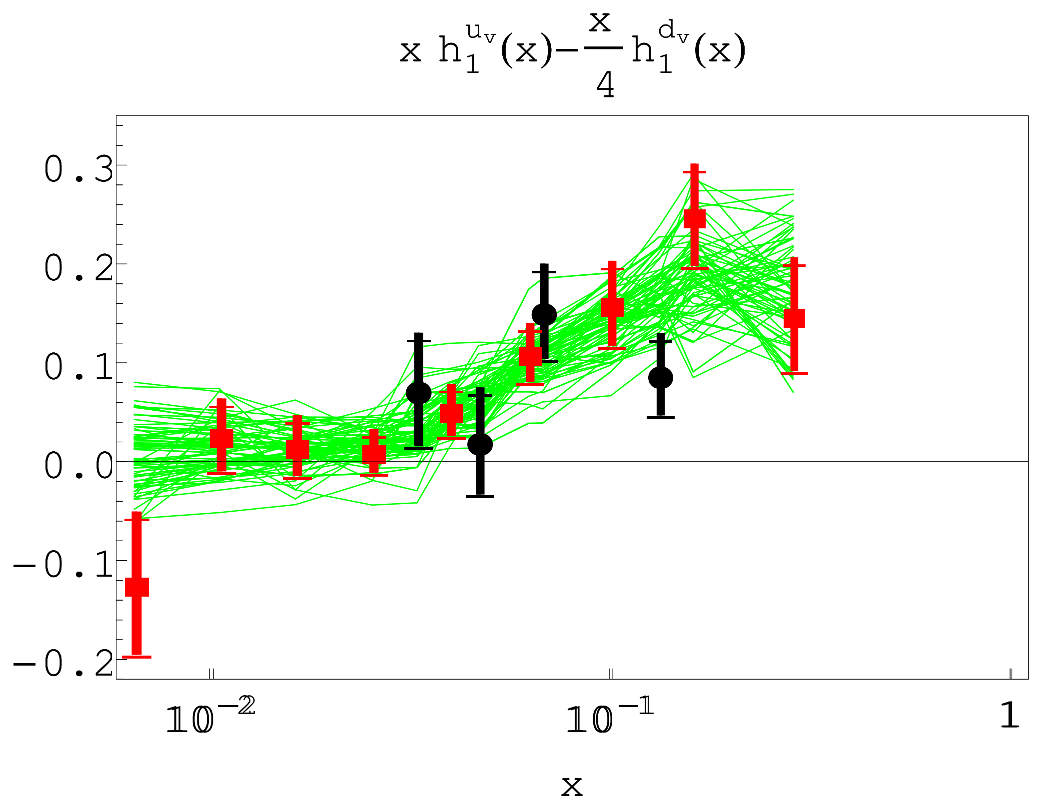
Replica method





Replica method





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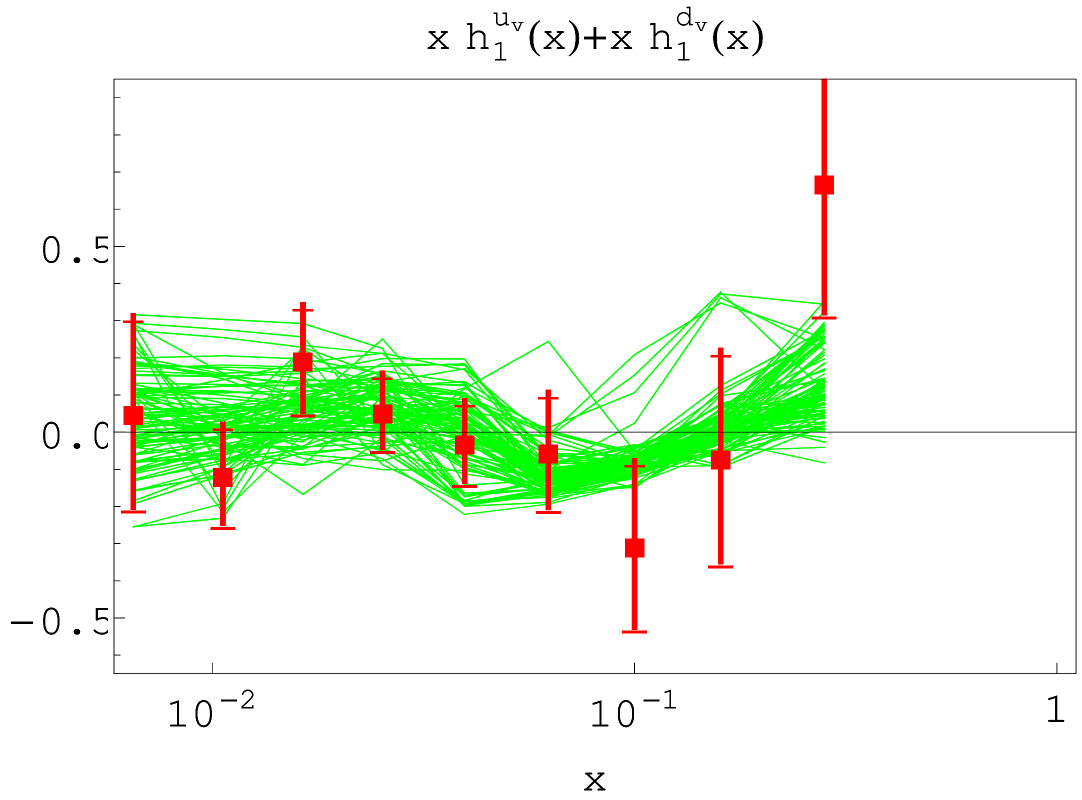

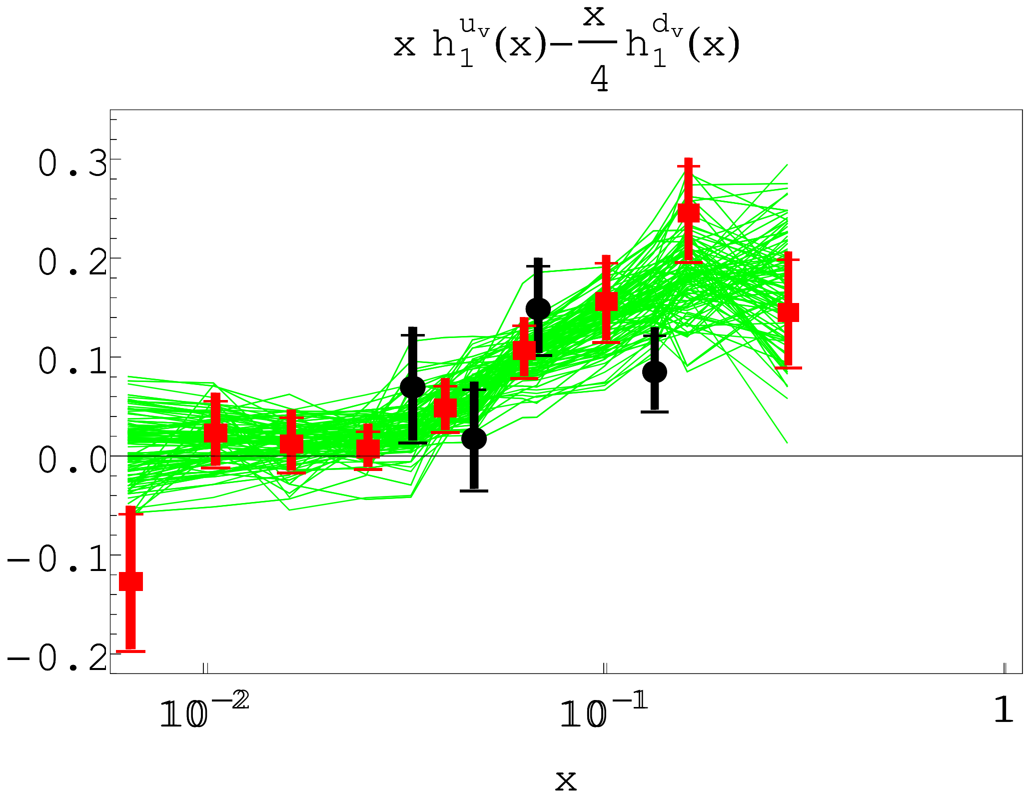

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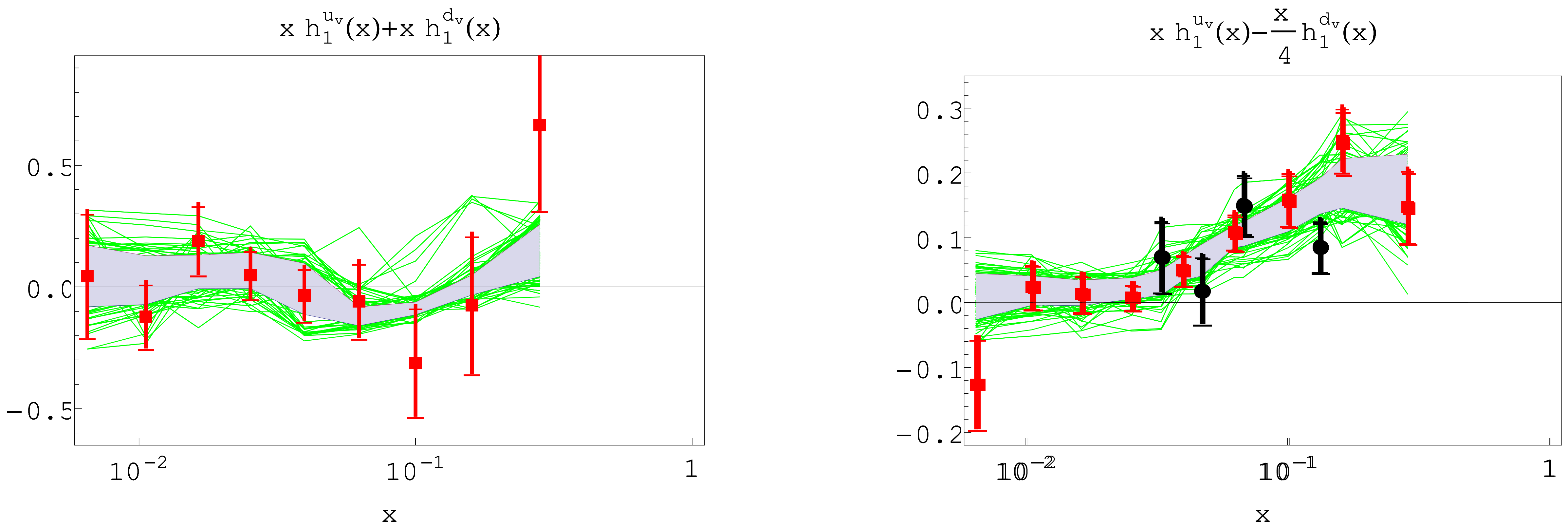

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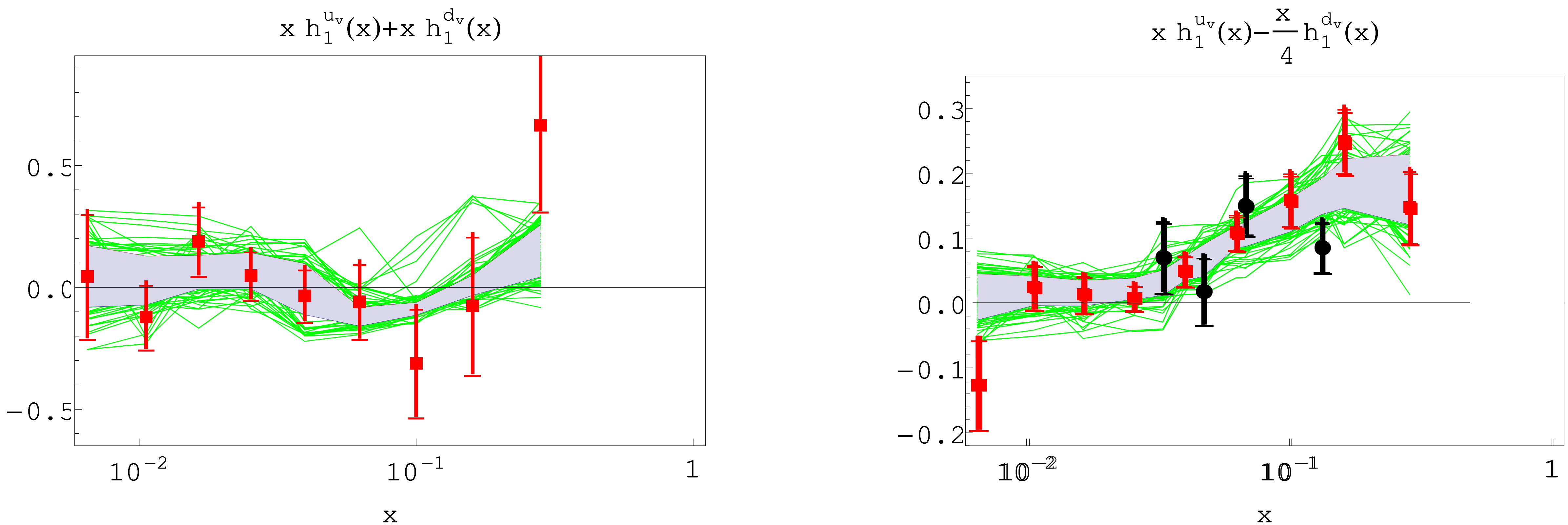
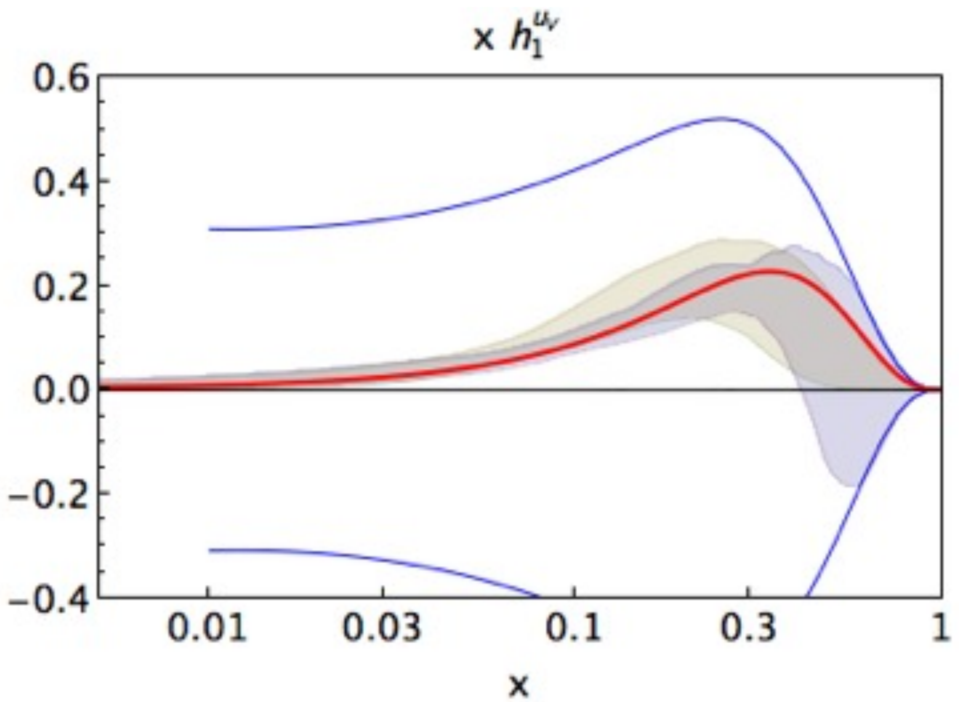
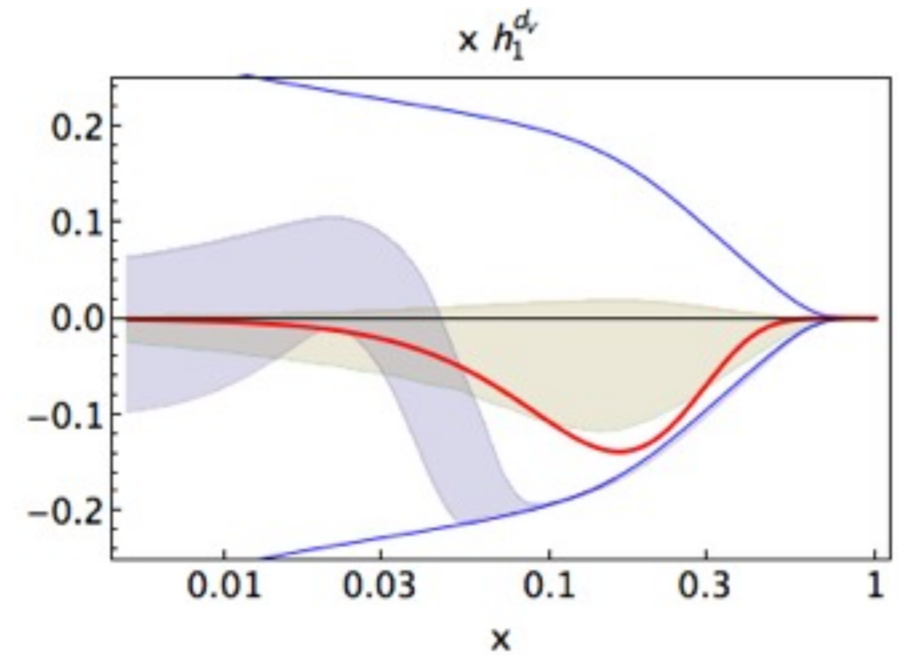


χ^2/d.o.f.	$\alpha_s(M_Z^2) = 0.125$	$\alpha_s(M_Z^2) = 0.139$
rigid	1.42	1.46
flexible	1.65	1.71
extraflexible	1.97	2.07

Results



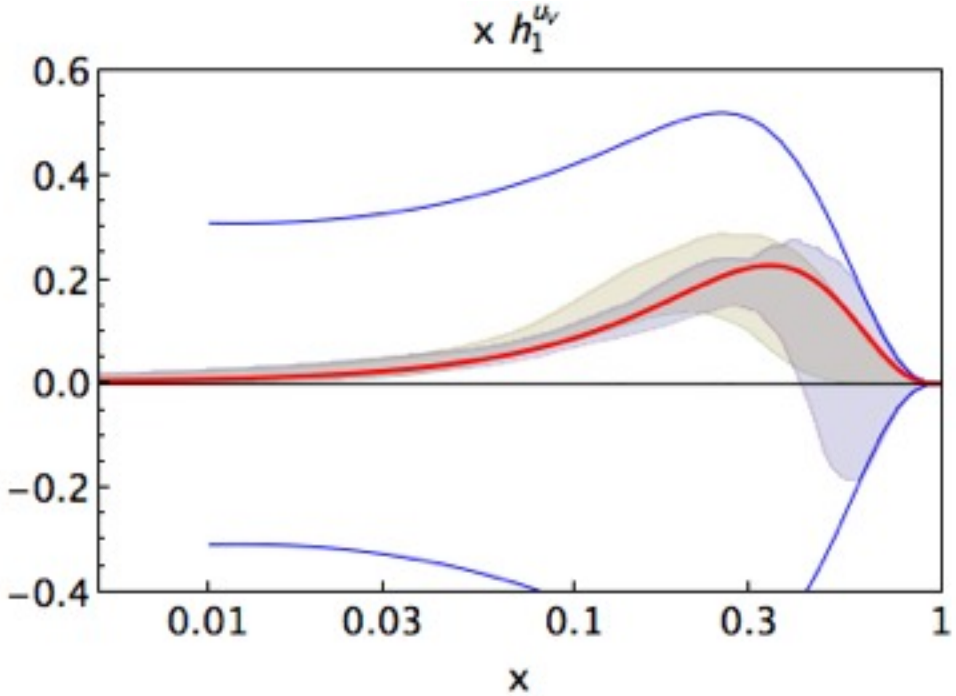
Flexible

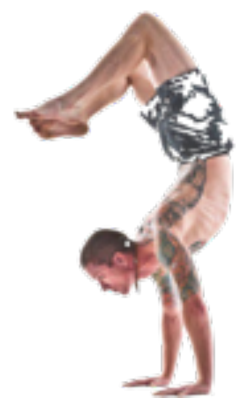


Extra flexible

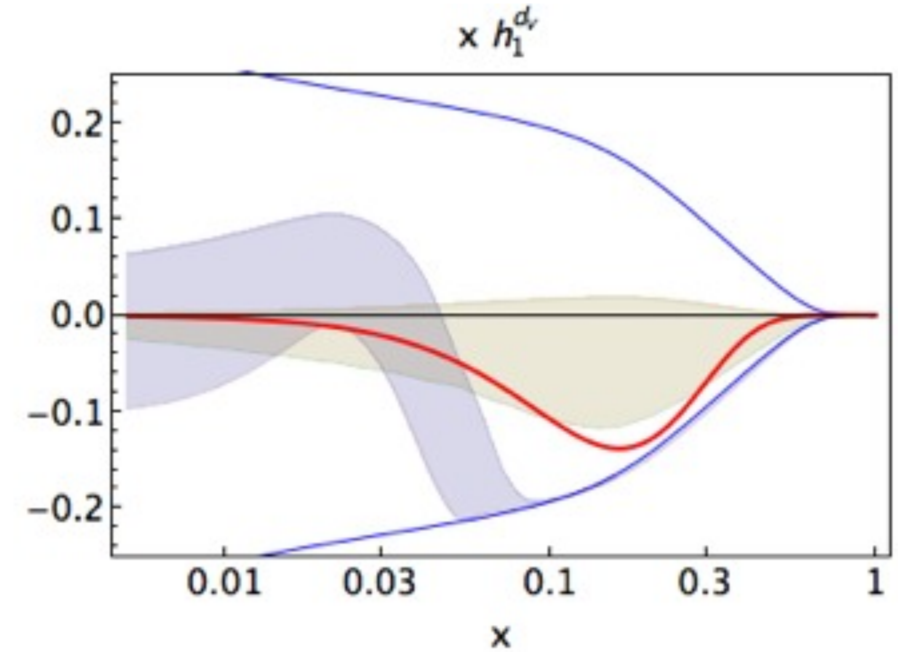
If not otherwise stated, we usually quote the results of the flexible scenario and α_S=0.125

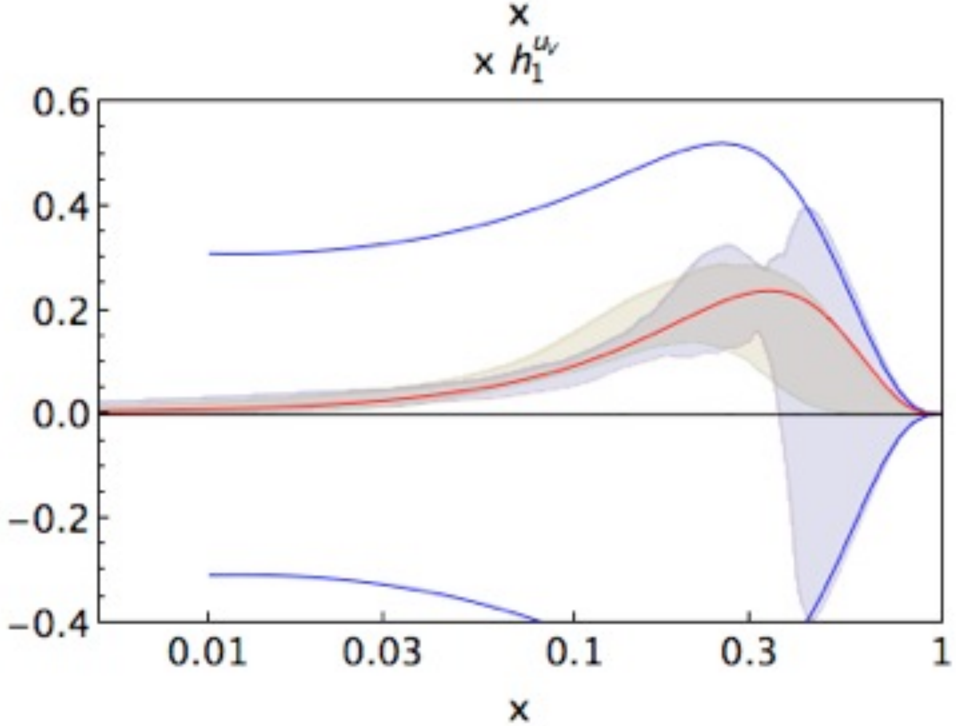
Results





Flexible





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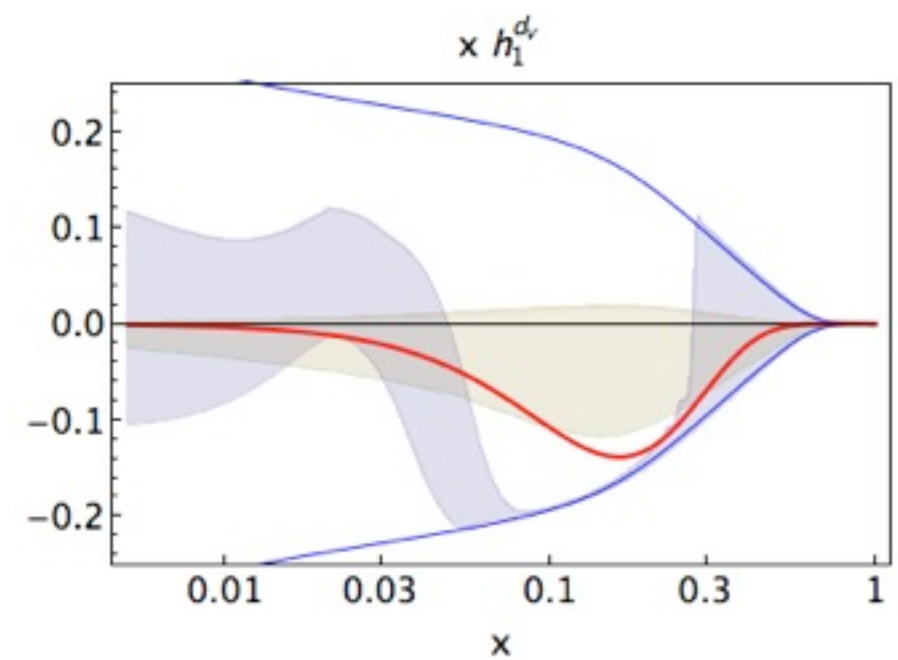
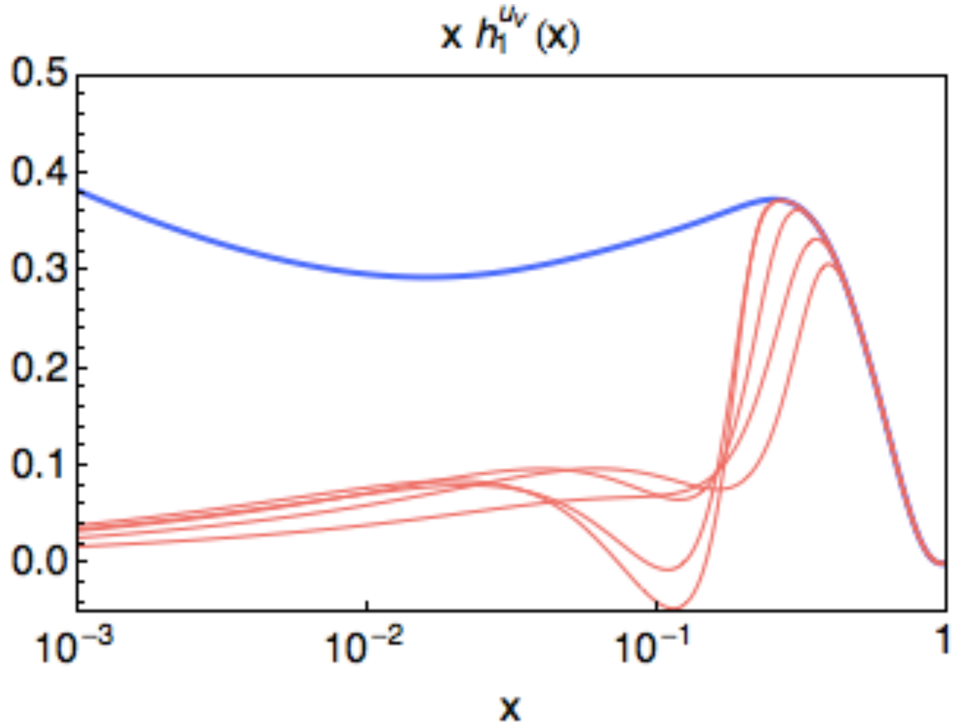
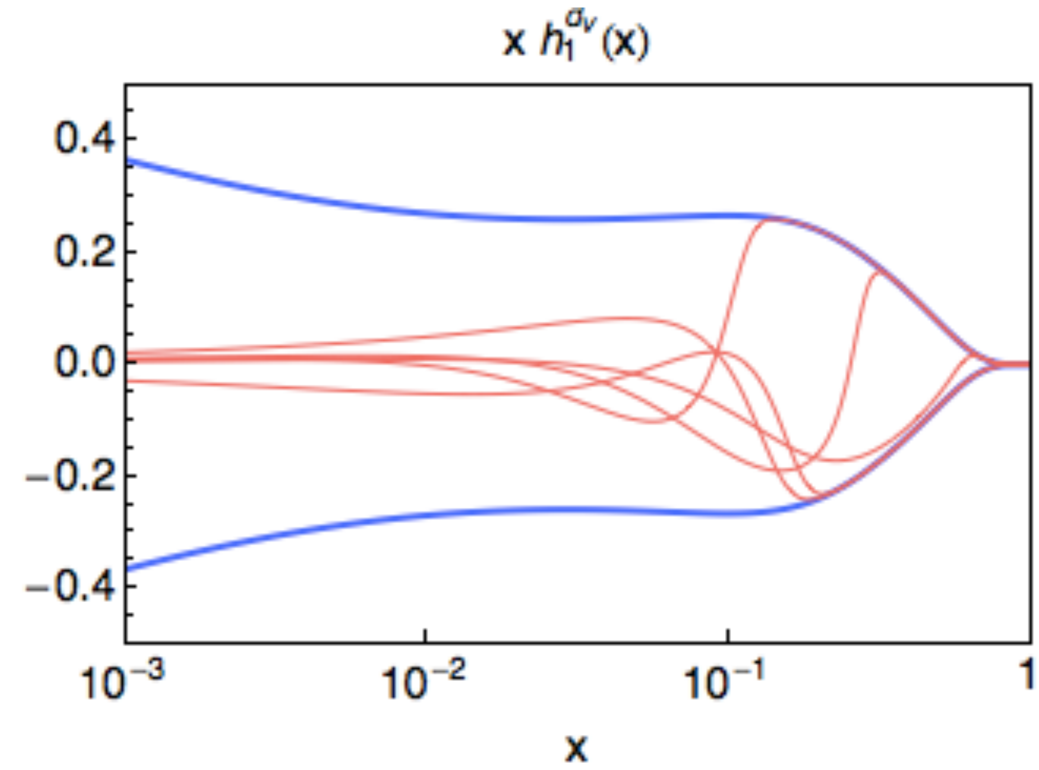


If not otherwise stated, we usually quote the results of the flexible scenario and α_S=0.125

Replicas outside 68% band

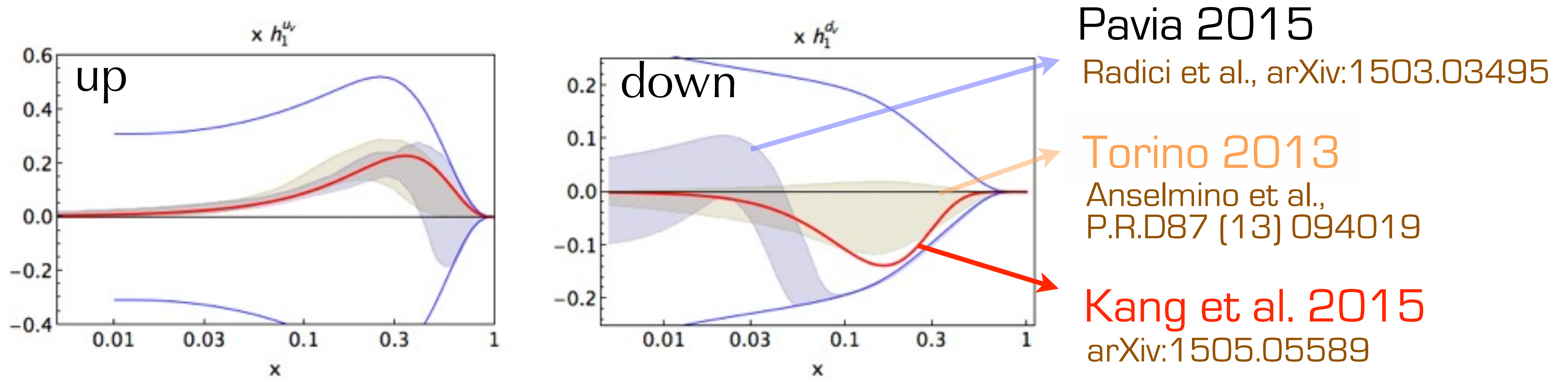




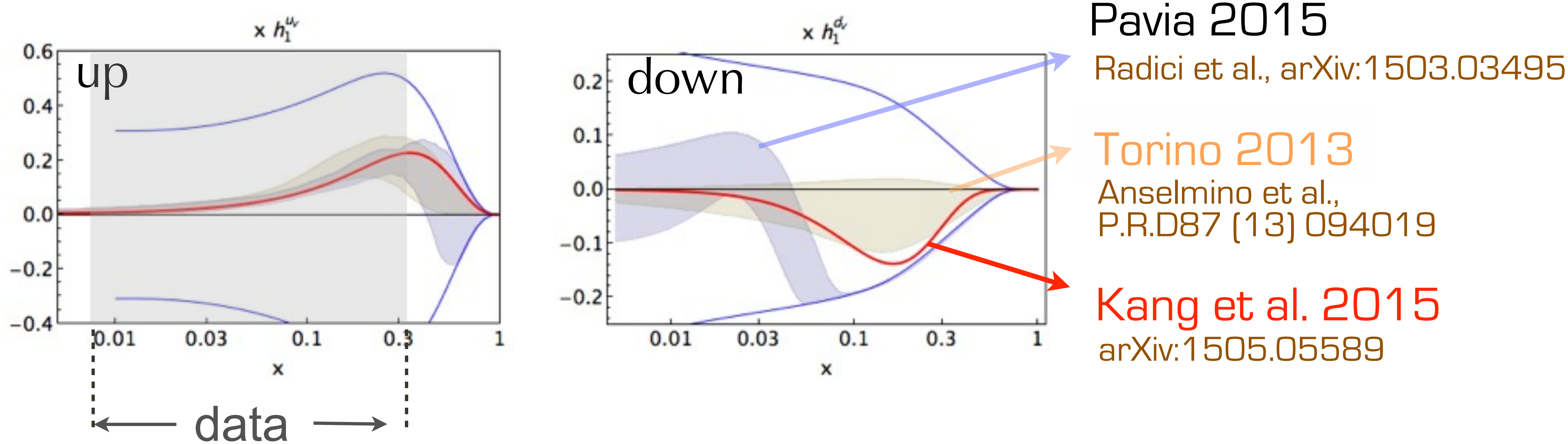
χ^2/dof

1.56557
1.42199
1.79911
2.07397
1.75523

Comparison with other extractions



Comparison with other extractions



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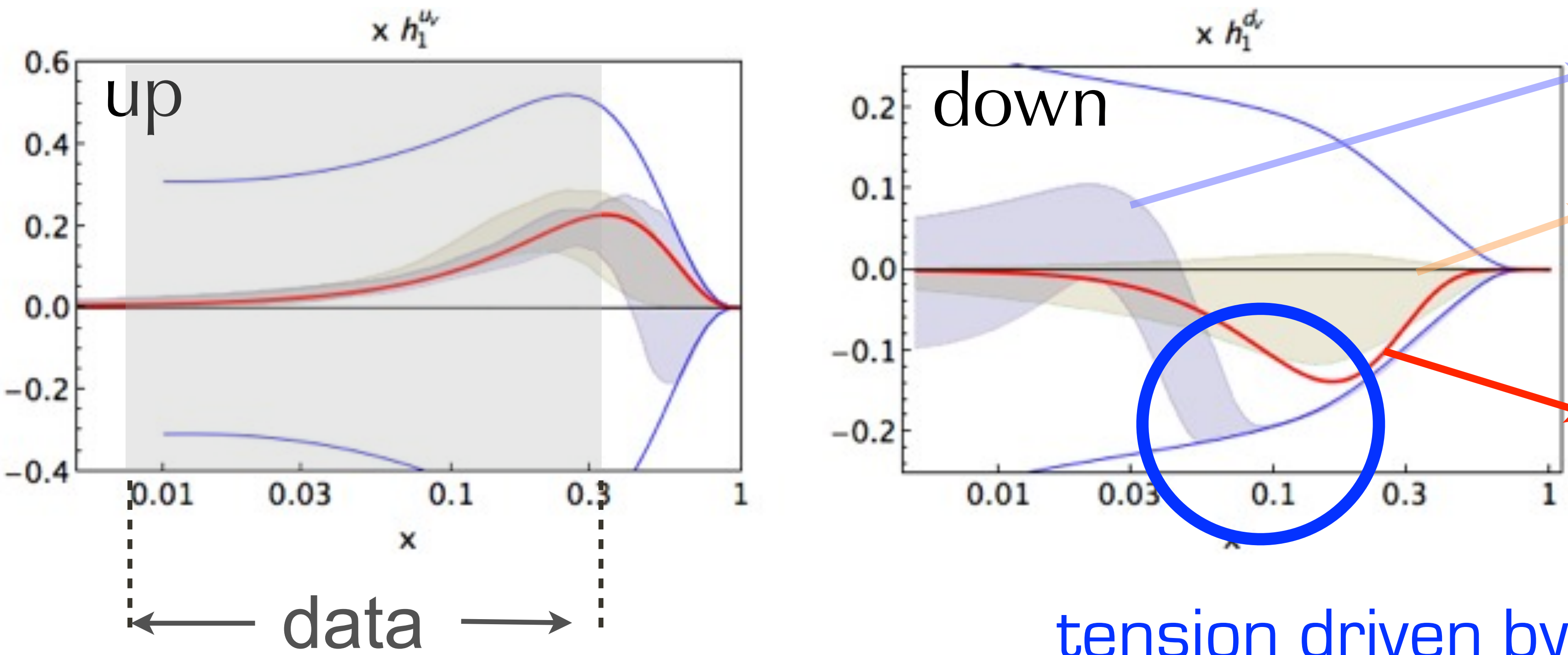


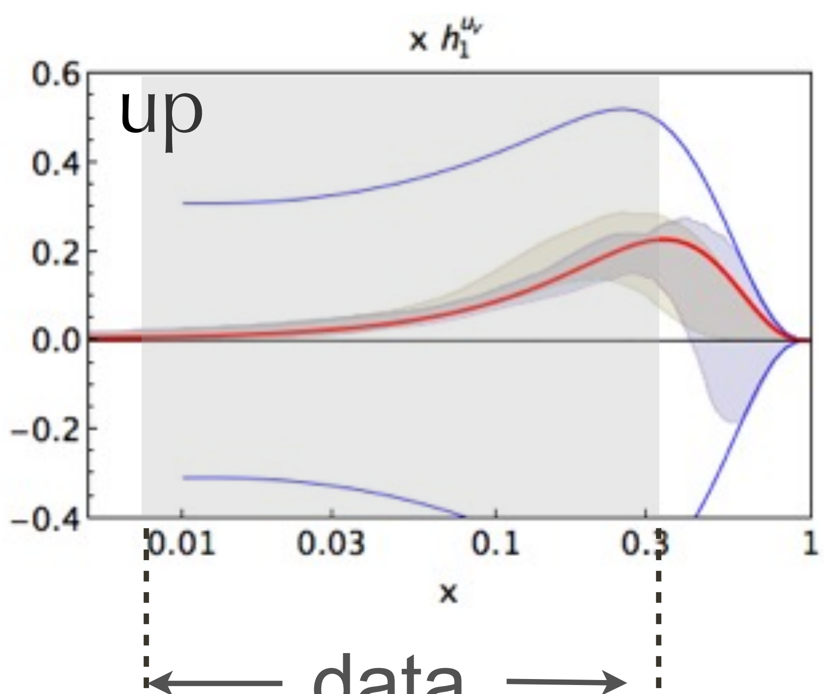
Pavia 2015
Radici et al., arXiv:1503.03495

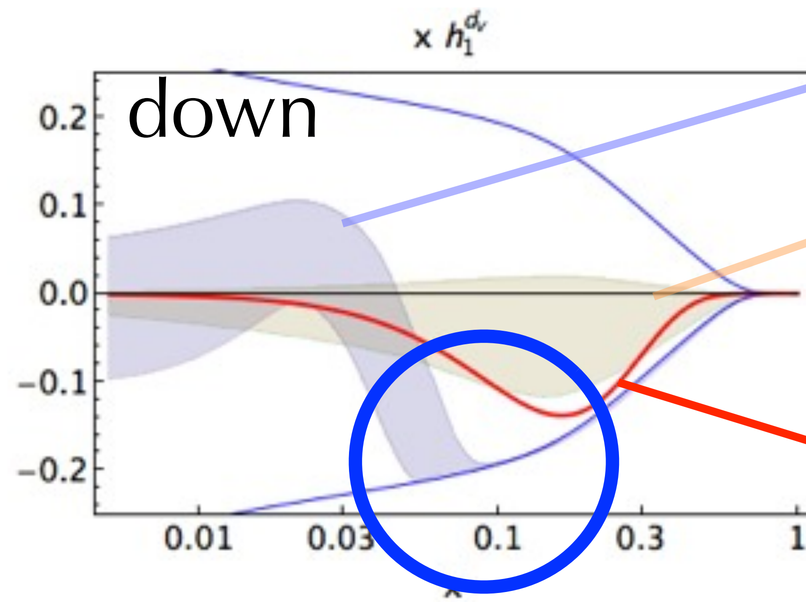
Torino 2013
Anselmino et al., P.R.D87 (13) 094019

Kang et al. 2015
arXiv:1505.05589

tension driven by COMPASS deuteron data

Comparison with other extractions





Pavia 2015

Radici et al., arXiv:1503.03495

Torino 2013

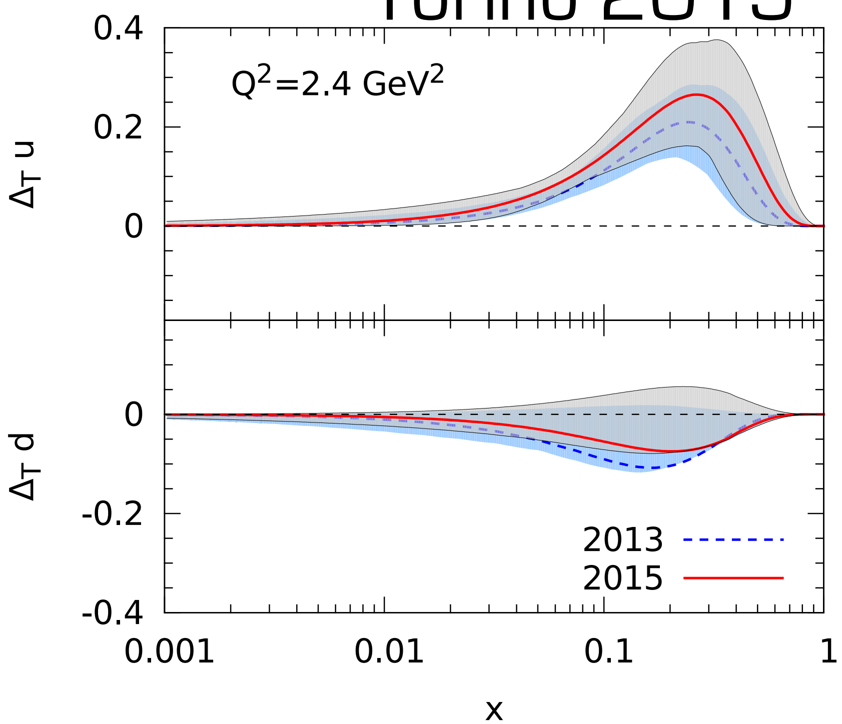
Anselmino et al., P.R.D87 (13) 094019

Kang et al. 2015

arXiv:1505.05589

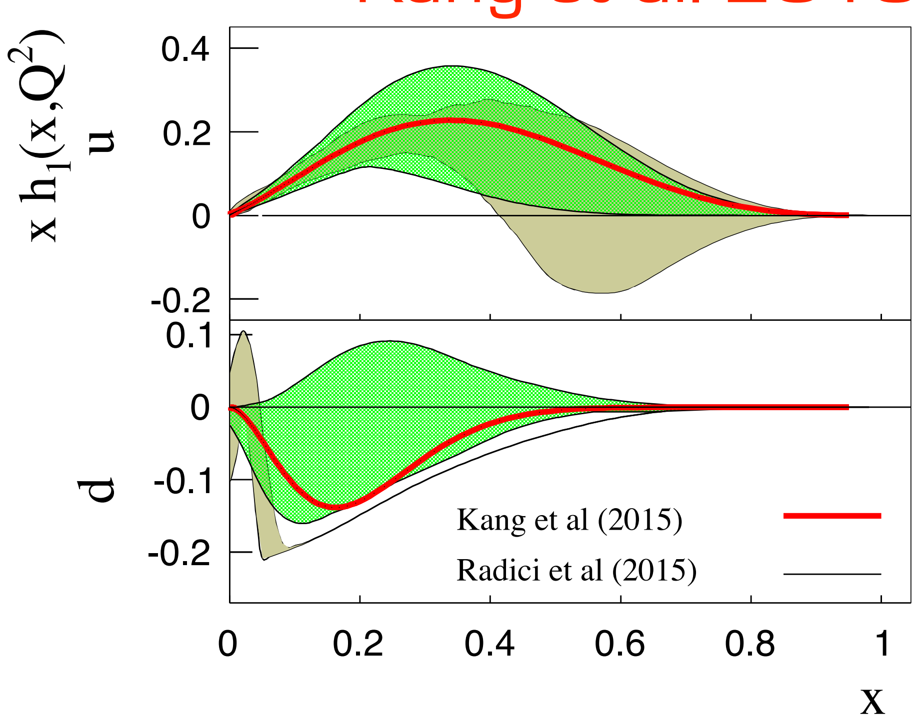
tension driven by COMPASS deuteron data

Torino 2015

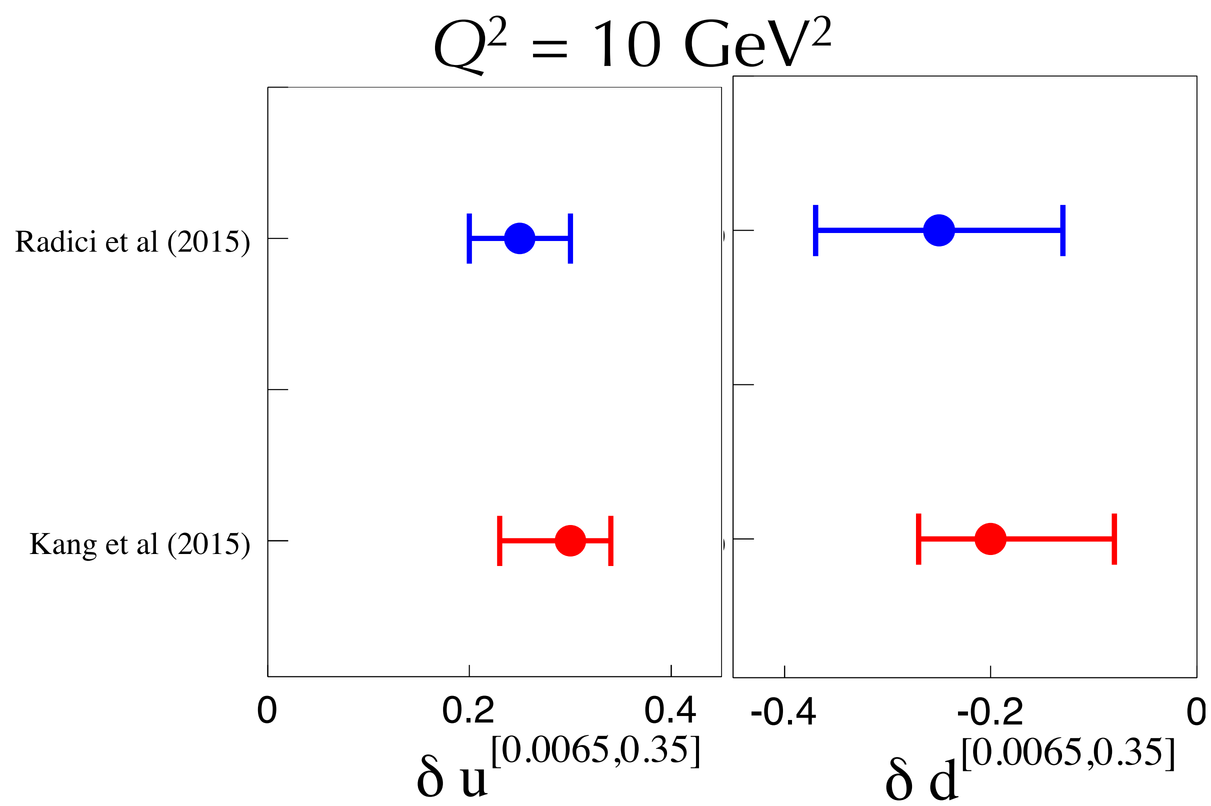


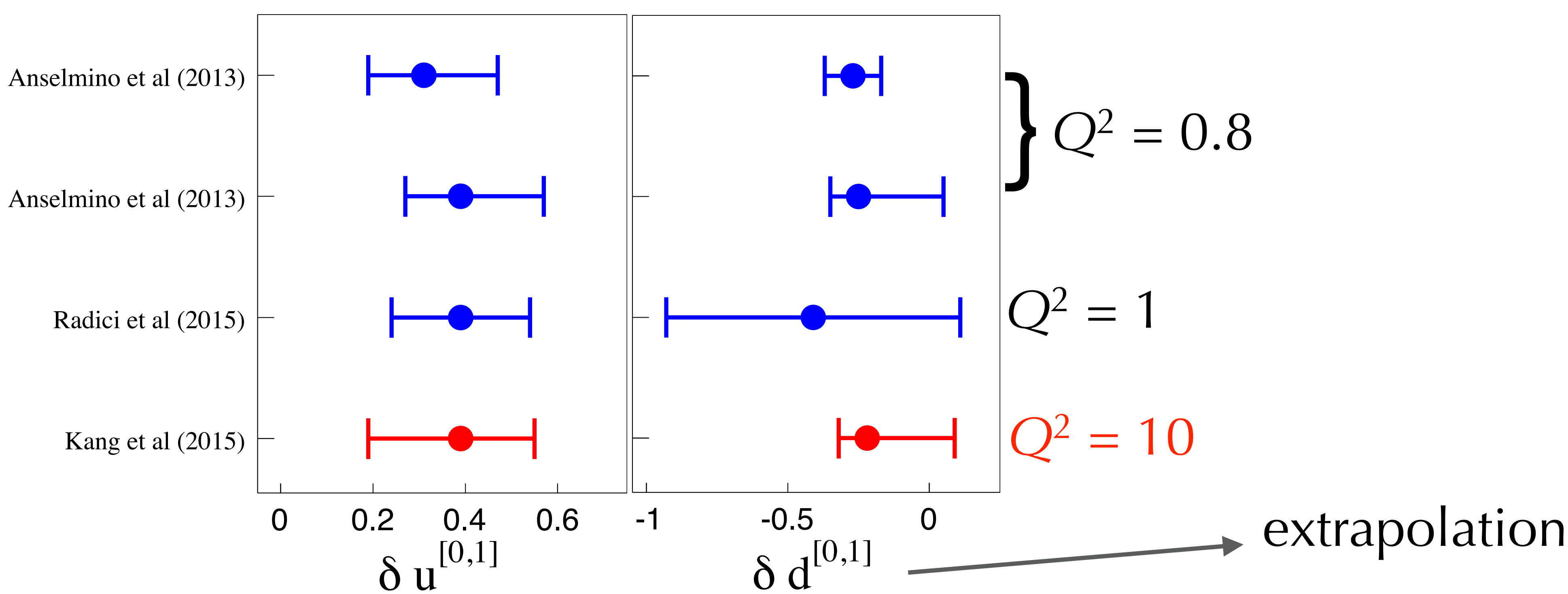
Anselmino et al., arXiv:1510.05389

Kang et al. 2015



Tensor charges





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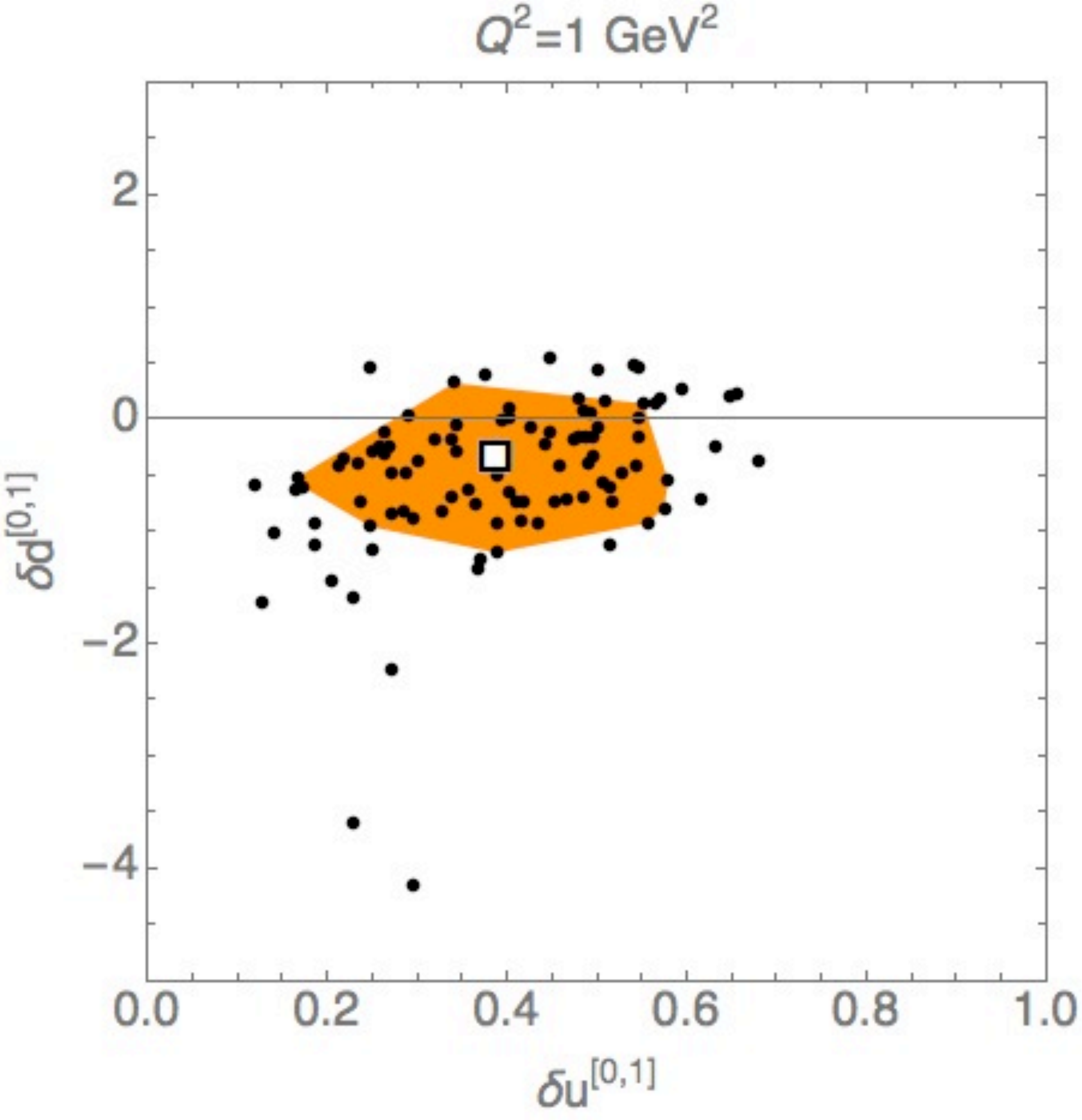

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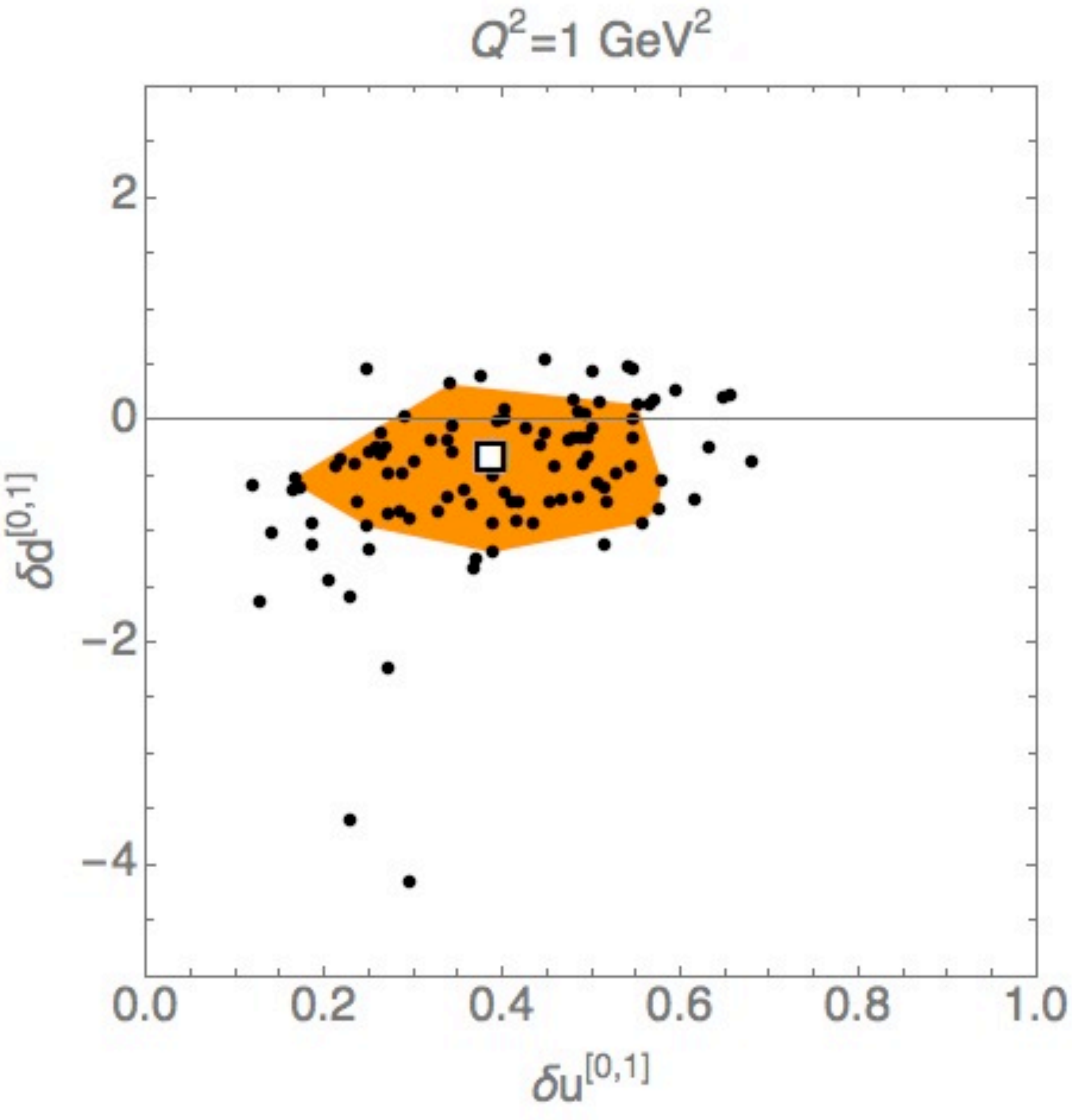
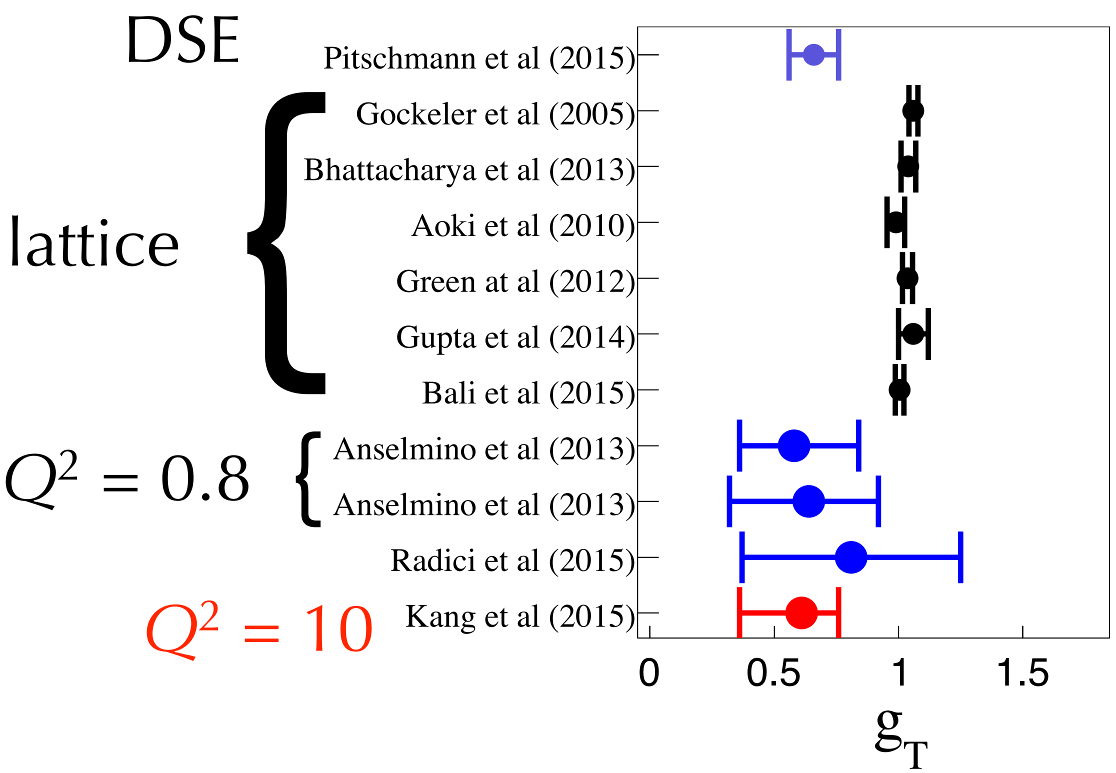


isovector tensor charge

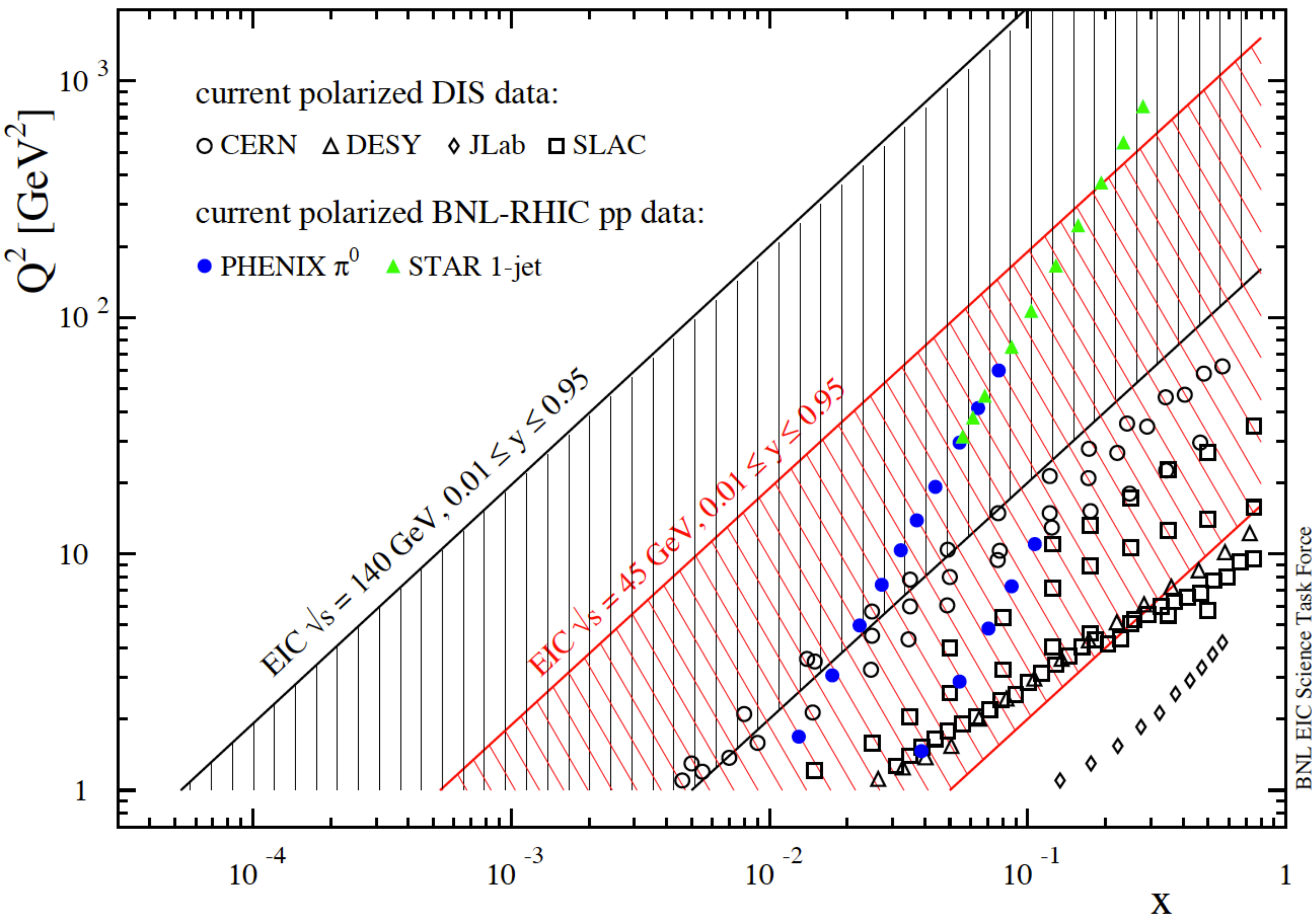
$g_T = \delta u - \delta d$

$Q^2 = 4$ GeV^2



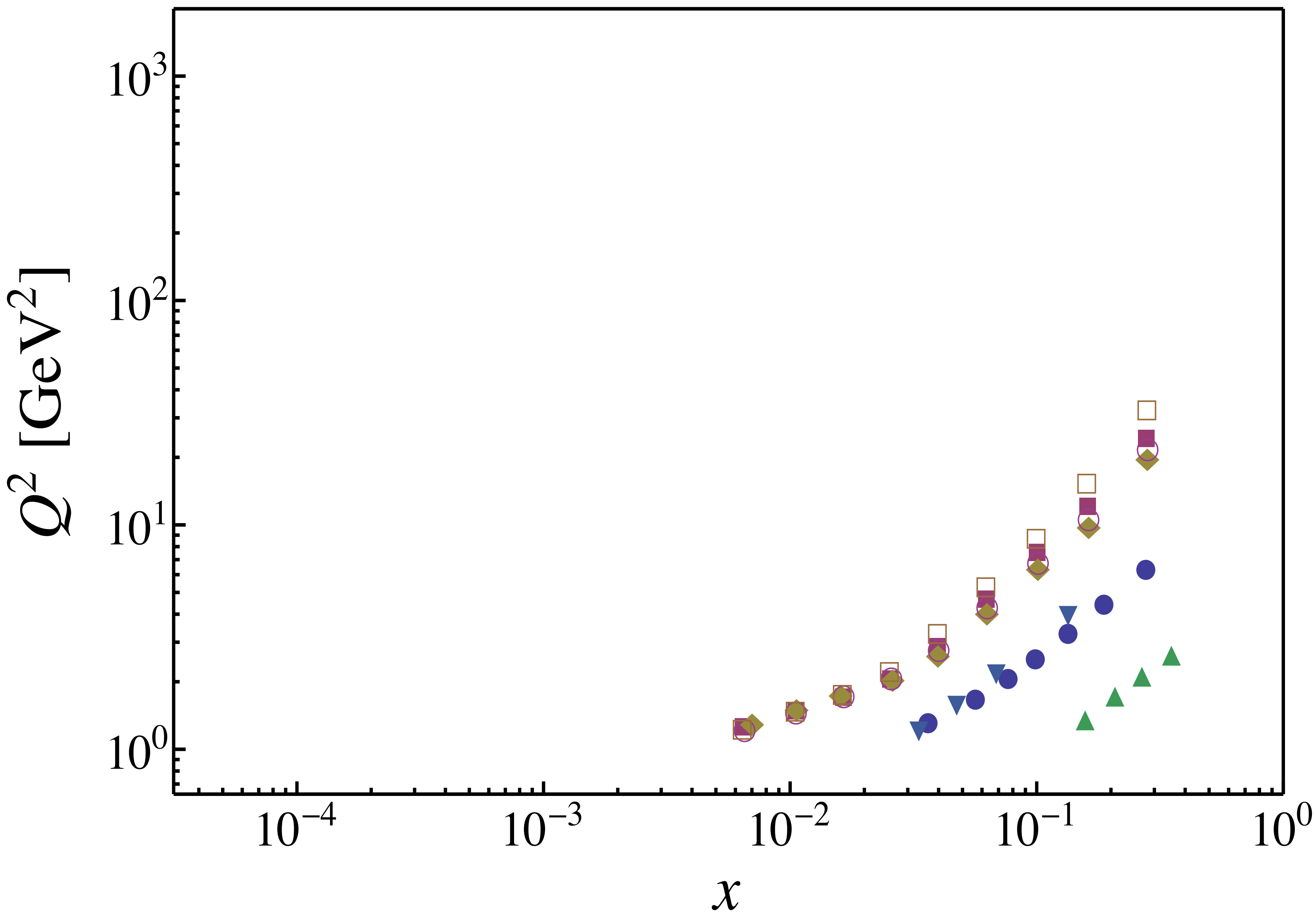
x-Q² coverage: helicity

M. Stratmann, talk at DIS2012



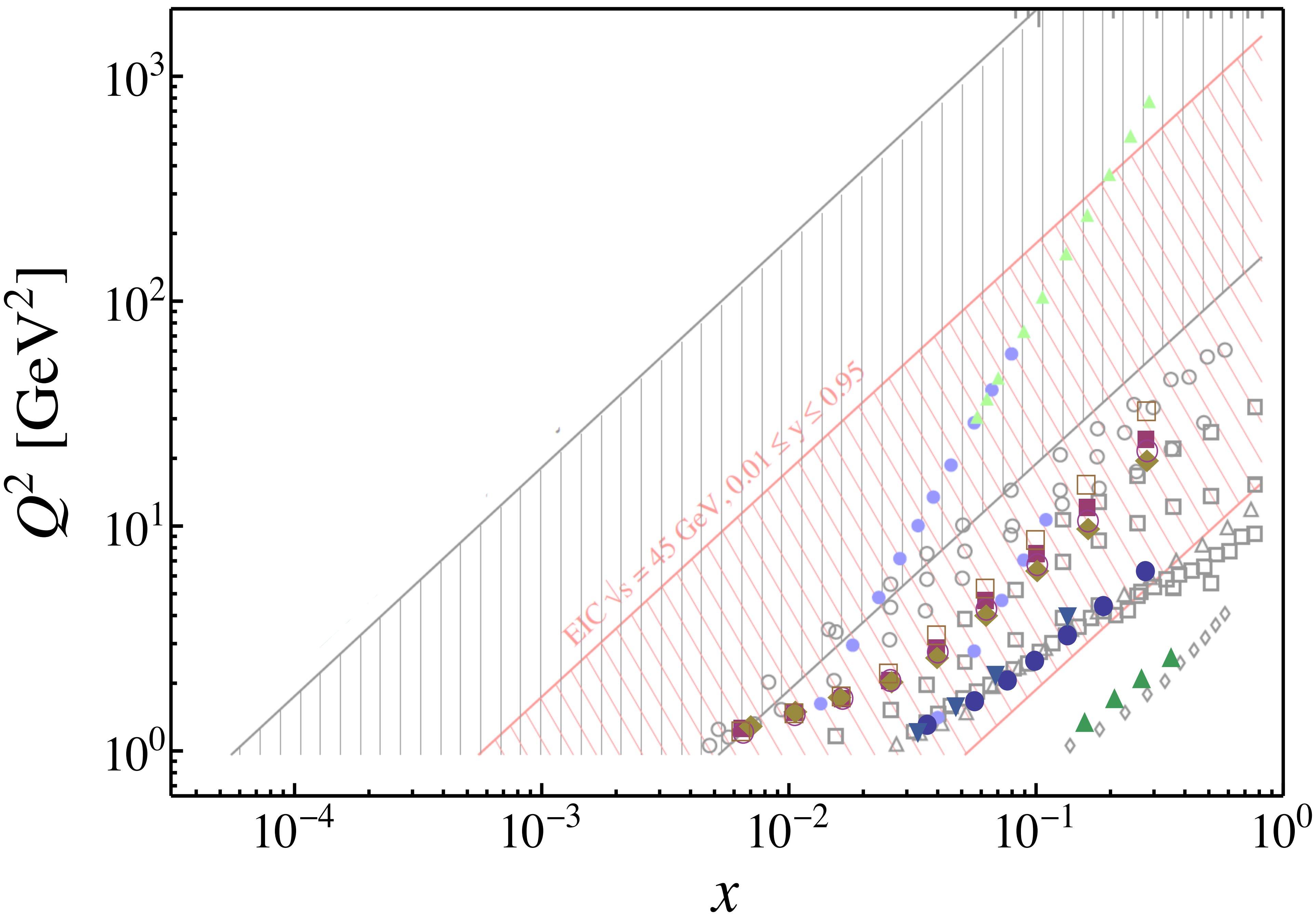
About 500 data points

x-Q² coverage: transversity



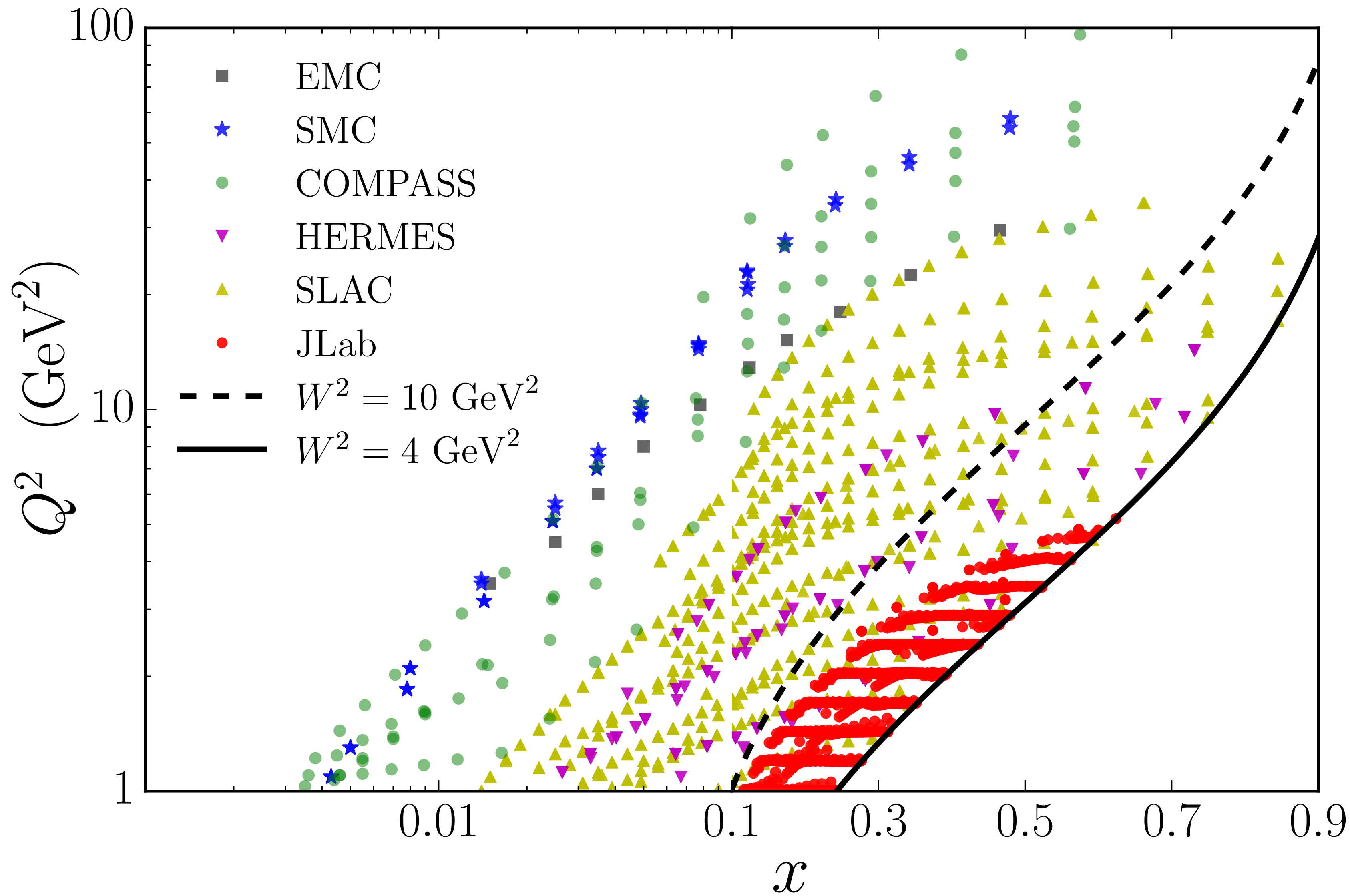
About 100 data points

x-Q² coverage: transversity



Data points: helicity

Sato, Melnitchouk, Kuhn, Ethier, Accardi, arXiv:1601.07782



2500 points

1800 points excluding g_2-related measurements

500 points excluding g_2-related and JLab

Conclusions & Needs…

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- Need of polarized pp collisions (STAR) → see next talk