xdvmp

Exclusive Diffractive Vector Meson Production

A implementation of the b-Sat/b-CGC Model

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What is xdvmp ?

xdvmp

Stand alone program in C++. Implements the KMW dipole based model: b-Sat & b-CGC

Features:

- b-Sat & b-CGC for e+p (following KMW)
- GaussLC and boosted Gaussian wave fct. implemented
- Calculates cross-sections for VM = J/ψ, φ, ρ
- DGLAP from Francois Gelis
- Output = ROOT histos
- Missing: Skewedness ($x \neq x'$) and Re(A) corrections (~20%)
- e+A (in principle only for b-Sat): following KT & KLV

In Practice : allows simulation (detector, acceptance, ...) by weighting with σ(<your favorite parameters x, Q^2, t, W, ...>) provided by xdvmp.

Implementation

xdvmp is a bunch of C and C++ source and header files plus a Makefile distributed via tarball (untar, make, run ☺)

Requires:

- ROOT libraries and header files
- GNU Scientific Library (gsl)
- Code from Francois (DGLAP) and Werner (α_s) [contained in package]

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Syntax:

xdvmp [-v] [-p rho|phi|jpsi] [-m bCGC|bSat] [-w GausLC|bGauss] [-A A] [-d]
      [-o] [-n] rootfile
-v       verbose mode (more print out)
-p part  VM to use: can be rho, phi, or jpsi (default = jpsi)
-m model dipole model to use: can be bSat or bCGC (default = bSat)
-w model VM wave function to use: can be GausLC or bGauss (default = bGauss)
-A A     mass number A of target (default = 1 (proton))
-d       generate dipole cross-sections histos
-o       generate overlap integrals histos
-n       do NOT calculate other cross-section histos
rootfile is the file to which all results (TF1 etc) are written
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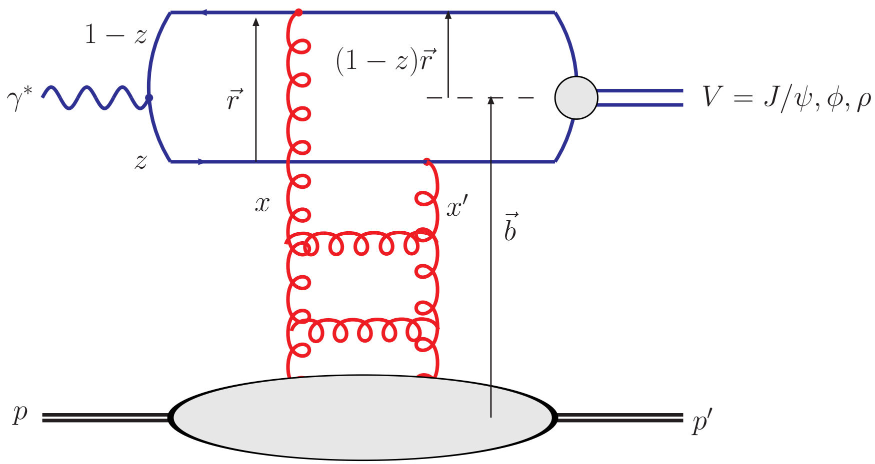
The Physics Behind xdvmp

Dipole Model & KWM

Based on:

Exclusive diffractive processes at HERA within the dipole picture, H. Kowalski, L. Motyka, G. Watt, PhysRev D74, 074016, arXiv:hep-ph/0606272v2



Dipole model:

1. γ^* fluctuates into $q\bar{q}$ pair
2. $q\bar{q}$ scatters elastically on p(A)
3. $q\bar{q}$ pair recombines into γ^*
4. γ^* decays into VM

Cross-section for production of final state VM:

$$\frac{d\sigma_{T,L}^{\gamma^* p \to Ep}}{dt} = \frac{1}{16\pi} \left| \mathcal{A}_{T,L}^{\gamma^* p \to Ep} \right|^2 = \frac{1}{16\pi} \left| \int d^2\boldsymbol{r} \int_0^1 \frac{dz}{4\pi} \int d^2\boldsymbol{b} \, (\Psi_E^* \Psi)_{T,L} \, e^{-i[\boldsymbol{b}-(1-z)\boldsymbol{r}]\cdot\boldsymbol{\Delta}} \frac{d\sigma_{q\bar{q}}}{d^2\boldsymbol{b}} \right|^2$$

Amplitude

Overlap between photon and VM wave function

Dipole Cross-Section

Overlap Function

$$(\Psi_V^*\Psi)_T = \hat{e}_f e \,\frac{N_c}{\pi z(1-z)}\, \left\{m_f^2 K_0(\epsilon r)\phi_T(r,z) - \left[z^2+(1-z)^2\right]\epsilon K_1(\epsilon r)\partial_r\phi_T(r,z)\right\},$$

$$(\Psi_V^*\Psi)_L = \hat{e}_f e\,\frac{N_c}{\pi} 2Q z(1-z)\,K_0(\epsilon r)\,\left[M_V\phi_L(r,z) + \delta\,\frac{m_f^2-\nabla_r^2}{M_V z(1-z)}\phi_L(r,z)\right],$$

Follows: Forshaw, Sandepen, Shaw description

where:

$$\hat{e}_f = 2/3,\ 1/3,\ \text{or}\ 1/\sqrt{2},\ \text{for}\ J/\psi,\ \phi,\ \text{or}\ \rho\ \text{mesons}$$

$$e = \sqrt{4\pi\alpha_{\rm em}}$$

$$\epsilon^2 \equiv z(1-z)Q^2 + m_f^2$$

z = fraction of photon's light cone momentum carried by quark

r = dipole size

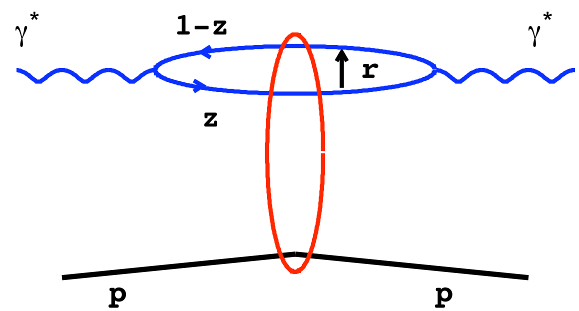
m_f = quark mass

M_V = vector meson mass

$N_c = 3$

$K_{0,1}$: Bessel functions

δ = 0 or 1 (model/author dependent - here always 1)



$\phi_{T,L}(r,z)$ = VM wave function

$$\nabla_r^2 \equiv (1/r)\partial_r + \partial_r^2$$

VM Wave Functions

Two models:

Gaus-LC

$$\phi_T(r,z) = N_T[z(1-z)]^2 \exp(-r^2/2R_T^2),$$
$$\phi_L(r,z) = N_L z(1-z)\exp(-r^2/2R_L^2).$$

Meson	M_V/GeV	f_V	m_f/GeV	N_T	R_T^2/GeV^{-2}	N_L	R_L^2/GeV^{-2}
J/ψ	3.097	0.274	1.4	1.23	6.5	0.83	3.0
ϕ	1.019	0.076	0.14	4.75	16.0	1.41	9.7
ρ	0.776	0.156	0.14	4.47	21.9	1.79	10.4

Parameters fixed by exp. measured decay width and N by normalization conditions

Boosted Gaussian

$$\phi_{T,L}(r,z) = \mathcal{N}_{T,L} z(1-z)\exp\left(-\frac{m_f^2\mathcal{R}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}^2} + \frac{m_f^2\mathcal{R}^2}{2}\right)$$

Meson	M_V/GeV	f_V	m_f/GeV	$\mathcal{N}_T$	$\mathcal{N}_L$	$\mathcal{R}^2$/GeV^{-2}	$f_{V,T}$
J/ψ	3.097	0.274	1.4	0.578	0.575	2.3	0.307
ϕ	1.019	0.076	0.14	0.919	0.825	11.2	0.075
ρ	0.776	0.156	0.14	0.911	0.853	12.9	0.182

Dipole Cross-Section: b-Sat Model

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_S(\mu^2) x g(x,\mu^2) T(b)\right)\right]$$

scale μ^2 is related to the dipole size r by $\mu^2 = 4/r^2 + \mu_0^2$.

xg is evolved from a scale μ_0^2 up to μ^2 using LO DGLAP

The initial gluon density at the scale μ_0^2 is taken in the form

$$xg(x,\mu_0^2) = A_g\, x^{-\lambda_g}\,(1-x)^{5.6}.$$

Gluon density for VM production is evaluated at scale:

$$x = x_B(1 + M_V^2/Q^2)$$

Proton shape: <u>Gaussian</u> or step function, here only former is used:

$$T_G(b) = \frac{1}{2\pi B_G} \mathrm{e}^{-\frac{b^2}{2B_G}},$$

$B_G = 4$ GeV^{-2} from fits to HERA data

Dipole Cross-Section: b-Sat Model

Parameters for: *xg()*

Model	$T(b)$	Q^2/GeV^2	$m_{u,d,s}/\text{GeV}$	m_c/GeV	μ_0^2/GeV^2	A_g	λ_g	$\chi^2/\text{d.o.f.}$
b-Sat	Gaussian	[0.25,650]	0.14	1.4	1.17	2.55	0.020	$193.0/160 = 1.21$
b-Sat	Gaussian	[0.25,650]	0.14	1.35	1.20	2.51	0.024	$190.2/160 = 1.19$
b-Sat	Gaussian	[0.25,650]	0.14	1.5	1.11	2.64	0.011	$198.1/160 = 1.24$
b-Sat	Gaussian	[0.25,650]	0.05	1.4	0.77	3.61	-0.118	$144.7/160 = 0.90$
b-Sat	Step	[0.25,650]	0.14	1.4	1.50	2.20	0.071	$199.6/160 = 1.25$

from fits to HERA F_2 data

How DGLAP is done:

http://ipht.cea.fr/pisp/gelis/Soft/DGLAP/index.html

The method used here for solving these integro-differential equations is based on an idea by **Laurent Schoeffel**, explained in the paper "***An elegant and fast method to solve QCD evolution equations***".

This method uses a decomposition of the parton distribution functions and of the splitting functions into a sum of Laguerre polynomials. The code for computing the coefficients of the Laguerre decomposition of the splitting functions is a mere translation into C of Laurent Schoeffel's original FORTRAN code.

To follow KMW only P_{gg} is used (easy to implement, just a flag)

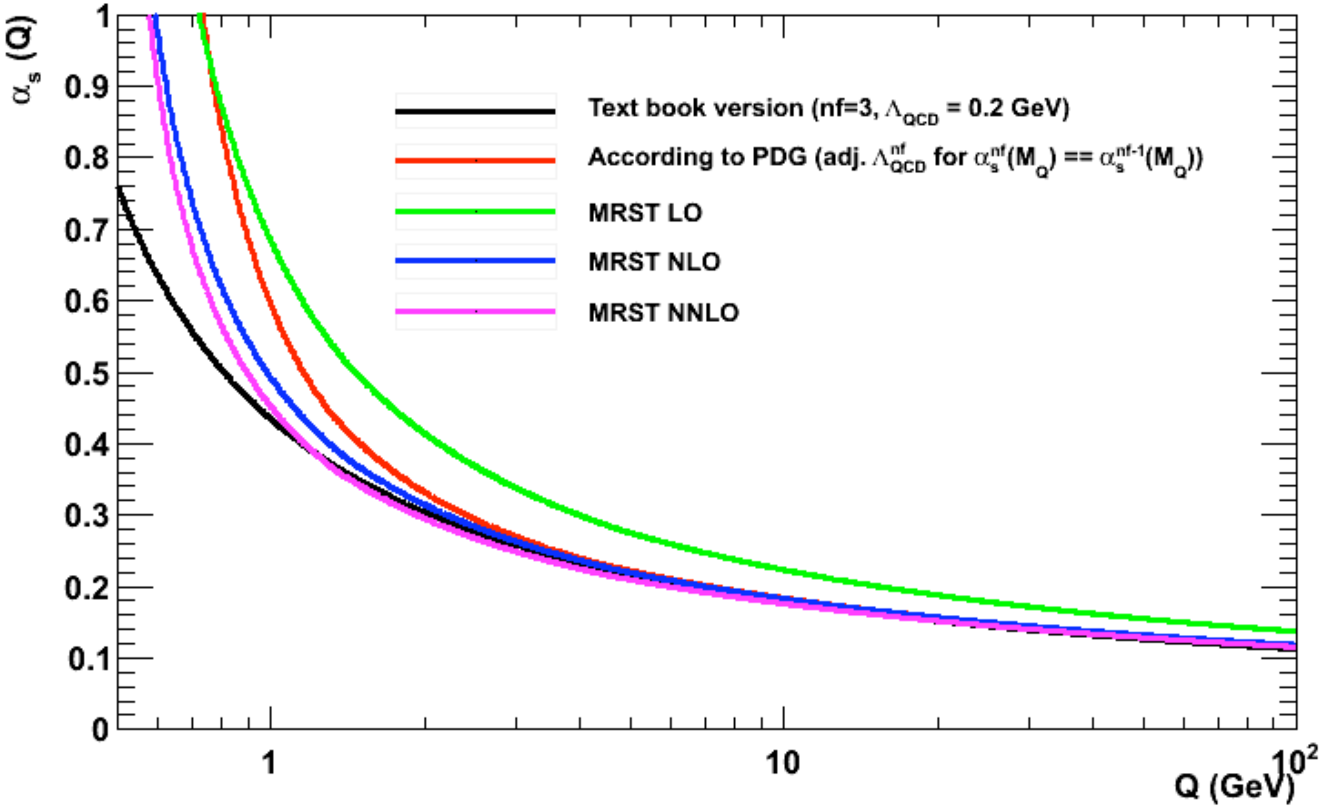
Dipole Cross-Section: b-Sat Model

A note on α_s:

KMW uses the text book version with n_f=3 and Λ_{QCD}=200 GeV.

I'm using MRST code I got from Werner (translated into C++).

Here's a comparison:



Note: since $d\sigma/db \sim \alpha_s xg$ knowledge of α_s matters

Dipole Cross-Section: b-CGC Model

KMW introduce b dependence to CGC model:

$$\frac{d\sigma_{q\bar{q}}}{d^2\boldsymbol{b}} \equiv 2\,\mathcal{N}(x,r,b) = 2 \times \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s + \frac{1}{\kappa\lambda Y}\ln\frac{2}{rQ_s}\right)} & : \quad rQ_s \leq 2 \\ 1 - e^{-A\ln^2(BrQ_s)} & : \quad rQ_s > 2 \end{cases},$$

where

depends only on x!

$$Q_s \equiv Q_s(x,b) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \left[\exp\left(-\frac{b^2}{2B_{\rm CGC}}\right)\right]^{\frac{1}{2\gamma_s}}$$

Model	Q^2/GeV^2	$m_{u,d,s}/\mathrm{GeV}$	m_c/GeV	$\mathcal{N}_0$	$x_0/10^{-4}$	λ	$\chi^2/\mathrm{d.o.f.}$
b-CGC	[0.25,45]	0.14	1.4	0.417	5.95	0.159	211.2/130 = 1.62

$$B_{\rm CGC} = 5.5 \ \mathrm{GeV}^{-2}$$

from fits to HERA F_2 data

Cross-Section

Having all the ingredients and after performing angular integrations:

$$\mathcal{A}_{T,L}^{\gamma^* p \to E p} = \mathrm{i} \int_0^\infty \mathrm{d}r \, (2\pi r) \int_0^1 \frac{\mathrm{d}z}{4\pi} \int_0^\infty \mathrm{d}b \, (2\pi b) \, (\Psi_E^* \Psi)_{T,L} \; J_0(b\Delta) \; J_0\left([1-z] r\Delta\right) \; \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2 \boldsymbol{b}},$$

$$t = -\Delta^2.$$

we are done for the e+p part

Integration is done via multidim. integration routines from ROOT which are essentially copies from CERNLIB, GSL, and others.

e+A

Based on
KT: Henri Kowalski , Derek Teaney, PRD68:114005, hep-ph/0304189
KLV: H. Kowalski, T. Lappi, R. Venugopalan, PRL100:022303, arXiv:
0705.3047 [hep-ph]

$$F(x,r^2) = \pi^2 \alpha_{\mathrm{s}} \left(\mu_0^2 + 4/r^2\right) xg\left(x, \mu_0^2 + 4/r^2\right) / (2N_{\mathrm{c}})$$

$$\frac{\mathrm{d}\sigma^A_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp} = 2\left[1 - e^{-r^2 F(x,r)\sum_{i=1}^{A} T_p(\mathbf{b}_\perp - \mathbf{b}_{\perp i})}\right],$$

for e+A

The average differential dipole cross-section can be approximated by:

$$\left\langle \frac{\mathrm{d}\sigma^A_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp} \right\rangle_{\mathrm{N}} \approx 2\left[1 - \left(1 - \frac{T_A(\mathbf{b}_\perp)}{2}\sigma^{\mathrm{p}}_{\mathrm{dip}}\right)^A\right]$$

where

$$\sigma_{qq}(x,r) = \int d^2b \, \frac{d\sigma_{qq}}{d^2b}.$$

which can be calculated from the b-Sat in e+p

e+A

$$\left\langle \frac{\mathrm{d}\sigma^A_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp} \right\rangle_{\mathrm{N}} \approx 2\left[1 - \left(1 - \frac{T_A(\mathbf{b}_\perp)}{2}\sigma^{\mathrm{p}}_{\mathrm{dip}}\right)^A\right]$$

R. Vogt, Nuclear Overlap Functions:

$$T_{\mathrm{A}} = \int dz \rho_{\mathrm{A}}(z, \vec{s})$$

Wood-Saxon:

$$\rho_{\mathrm{A}}(r) = \rho_0 \frac{1 + \omega(r/R_{\mathrm{A}})^2}{1 + \exp((r - R_{\mathrm{A}})/z)}$$

A	R_{A} (fm)	z (fm)	ω	ρ_0 (fm^{-3})
16	2.608	0.513	-0.051	0.1654
27	3.07	0.519	0.	0.1739
40	3.766	0.586	-0.161	0.1699
63	4.214	0.586	0.	0.1701
110	5.33	0.535	0.	0.1577
197	6.38	0.535	0.	0.1693
208	6.624	0.549	0.	0.1600

Note: Ramona's parameters give:
$\int T_A(b)\, db^2 = A$
while the above assumes
$\int T_A(b)\, db^2 = 1$

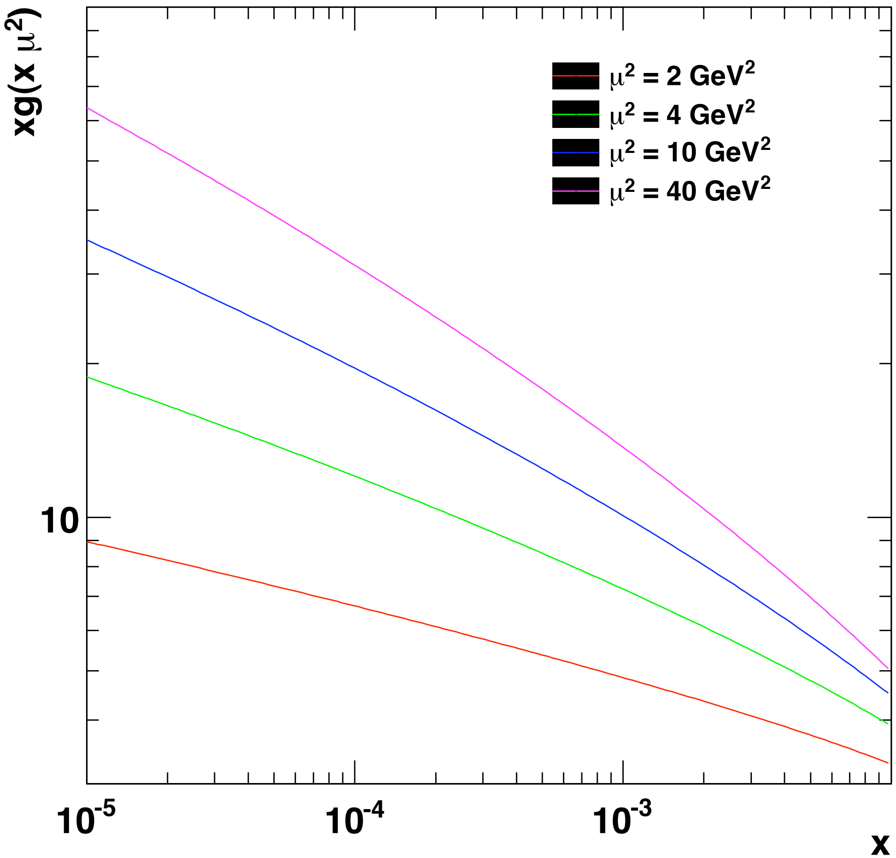
NB: Integrals of WS have no analytical solution. This and the $\sigma_{dip}{}^{p}$ integration make e+A a CPU hog.

Results

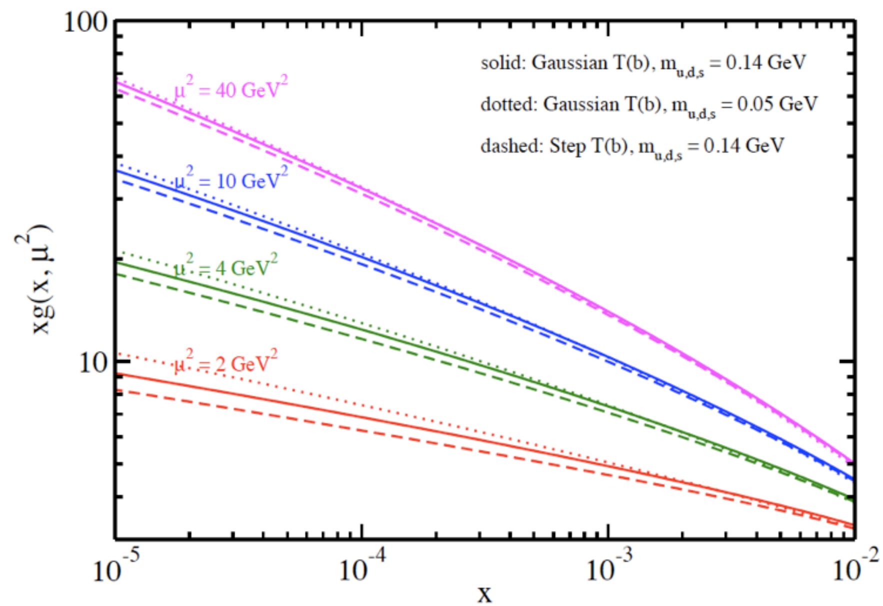
e+p: J/ψ production, b-Sat model

Gluon Distributions

xdvmp (DGLAP from F, Gelis)



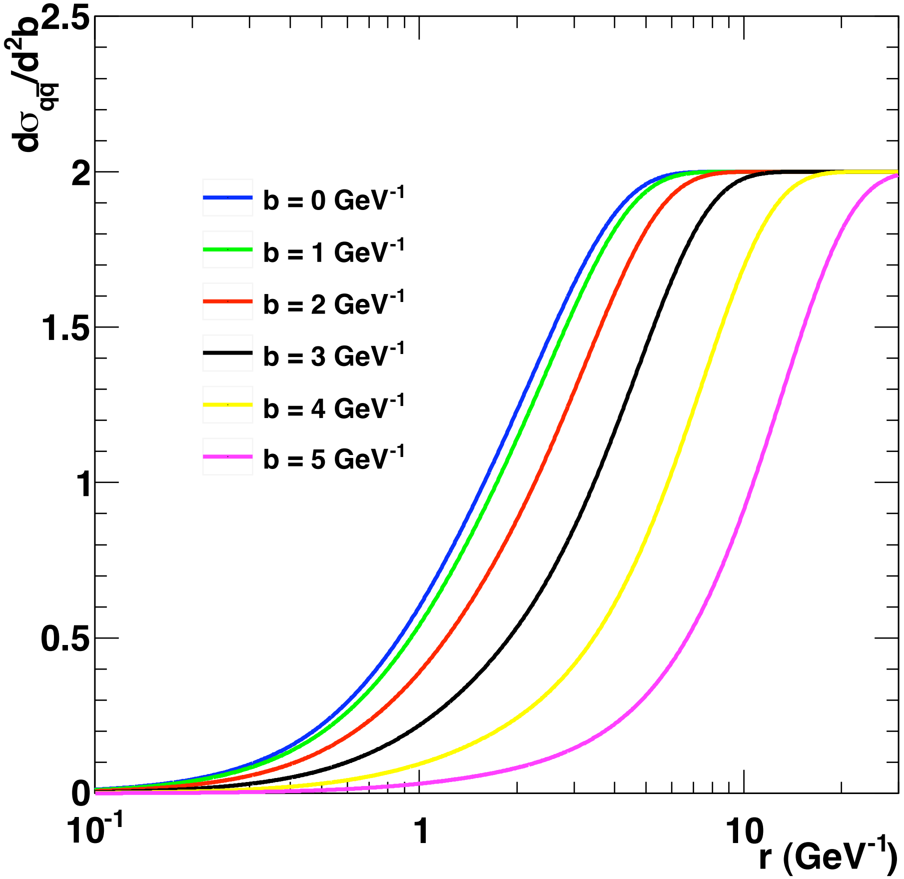
KMW



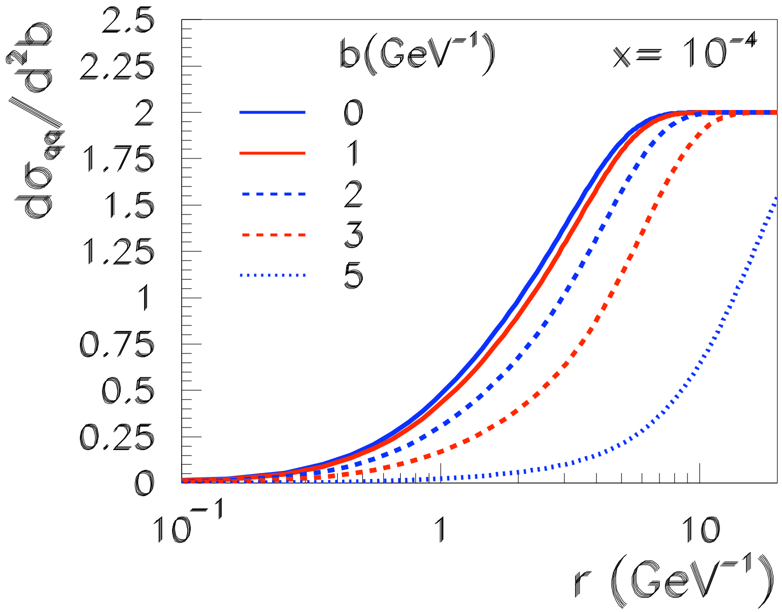
e+p: J/ψ production, b-Sat model

Dipole cross-section

xdvmp (boosted Gaussian)



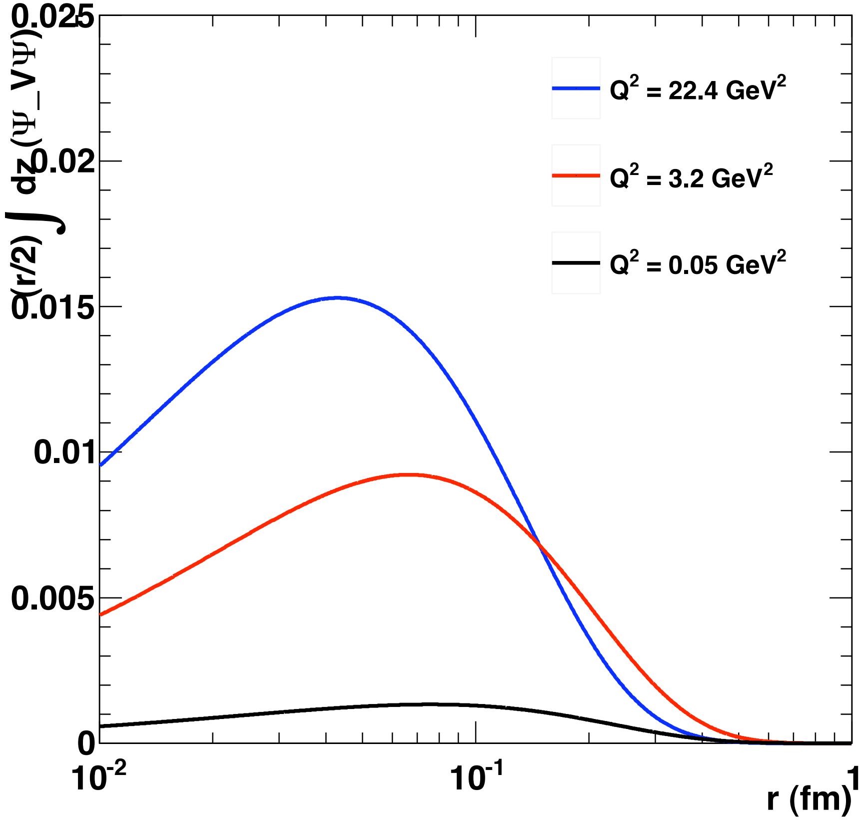
KMW



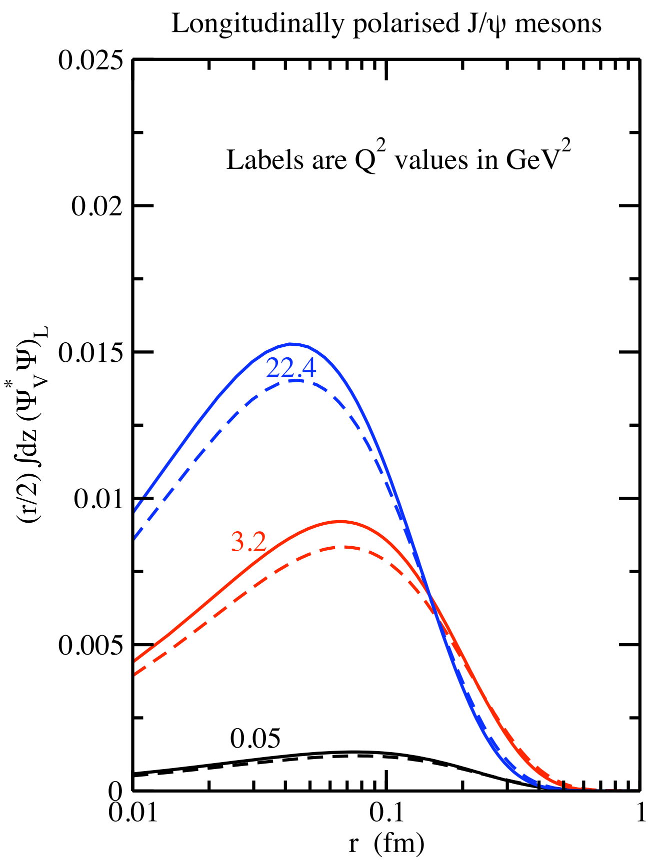
e+p: J/ψ production, b-Sat model

Overlap Functions (longitudinal polarized)

xdvmp



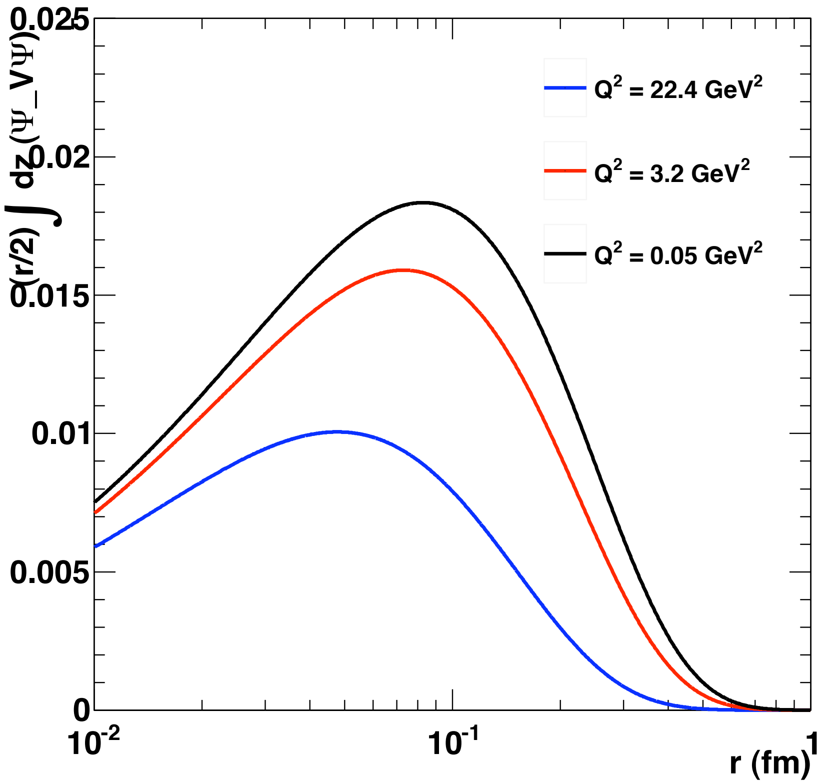
KMW



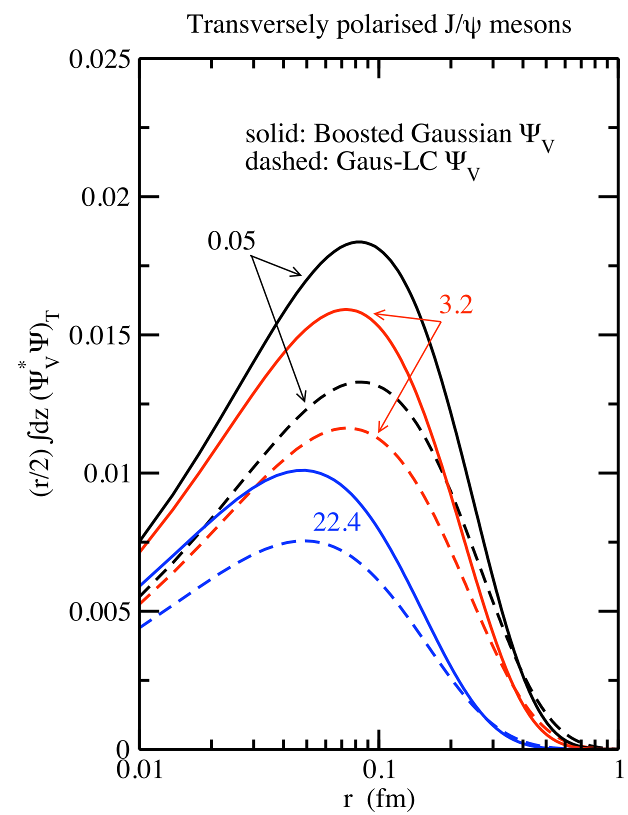
e+p: J/ψ production, b-Sat model

Overlap Functions (transversely polarized)

xdvmp



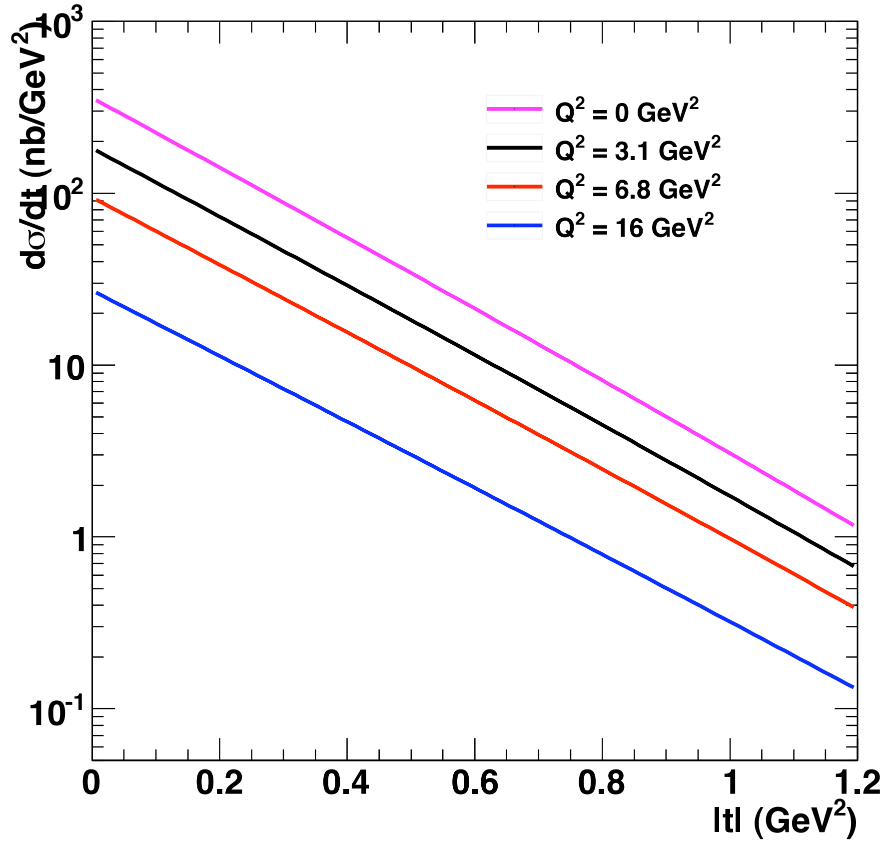
KMW



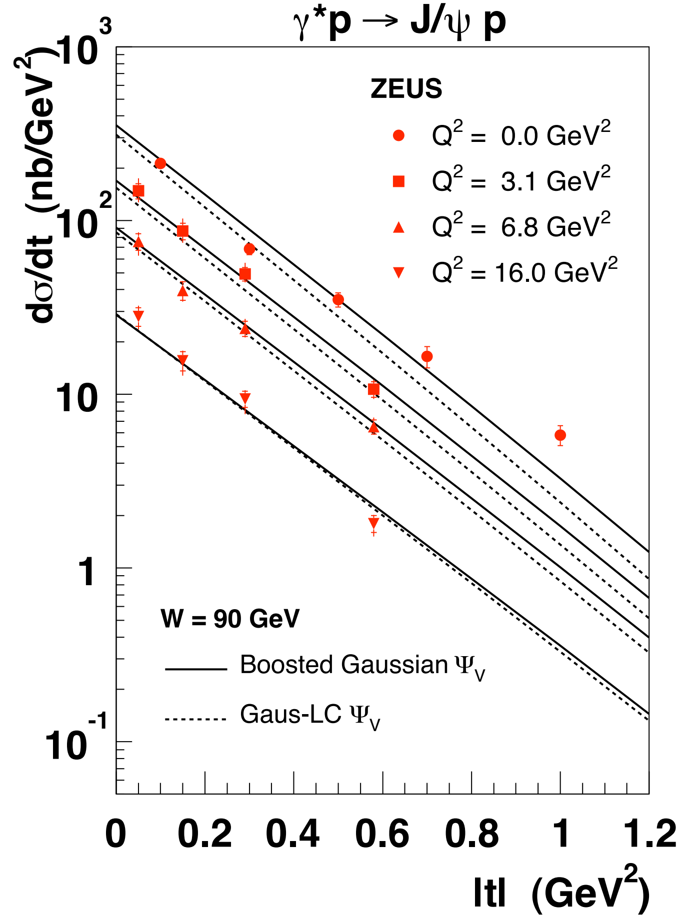
e+p: J/ψ production, b-Sat model

Cross-Sections

xdvmp



KMW

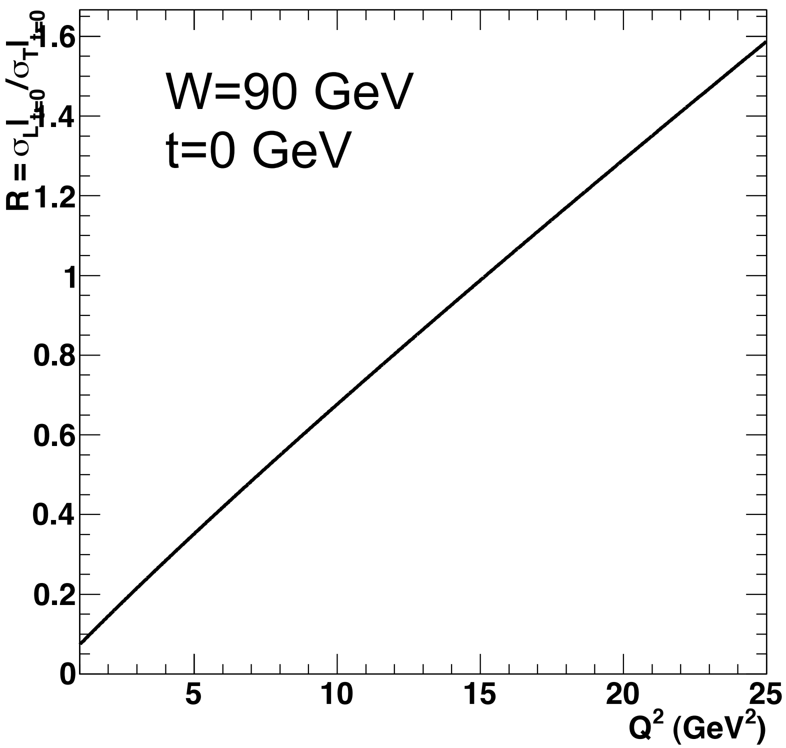


e+p: J/ψ production, b-Sat model

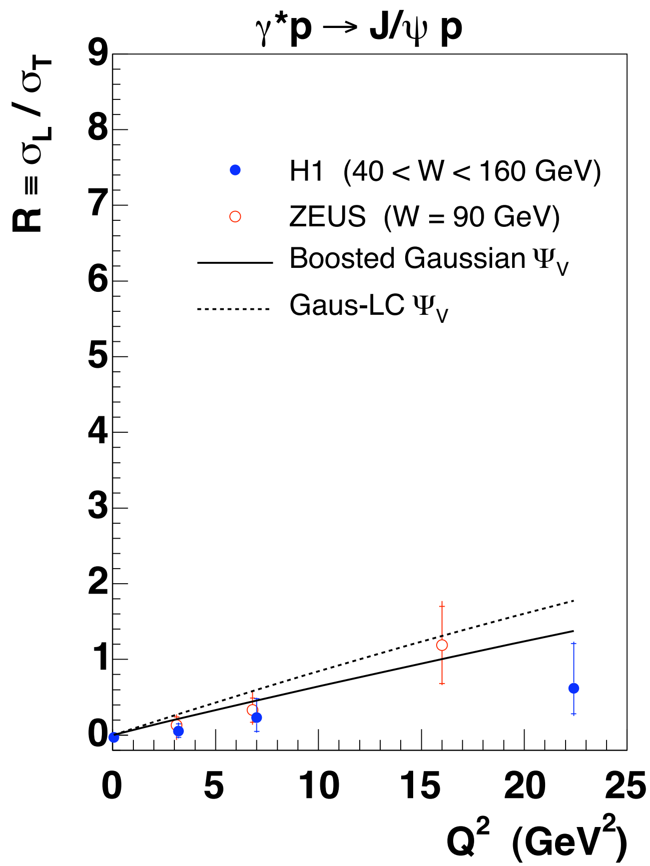
Cross-Sections

Note:
xdvmp at t=0
KMW integrated over t

xdvmp

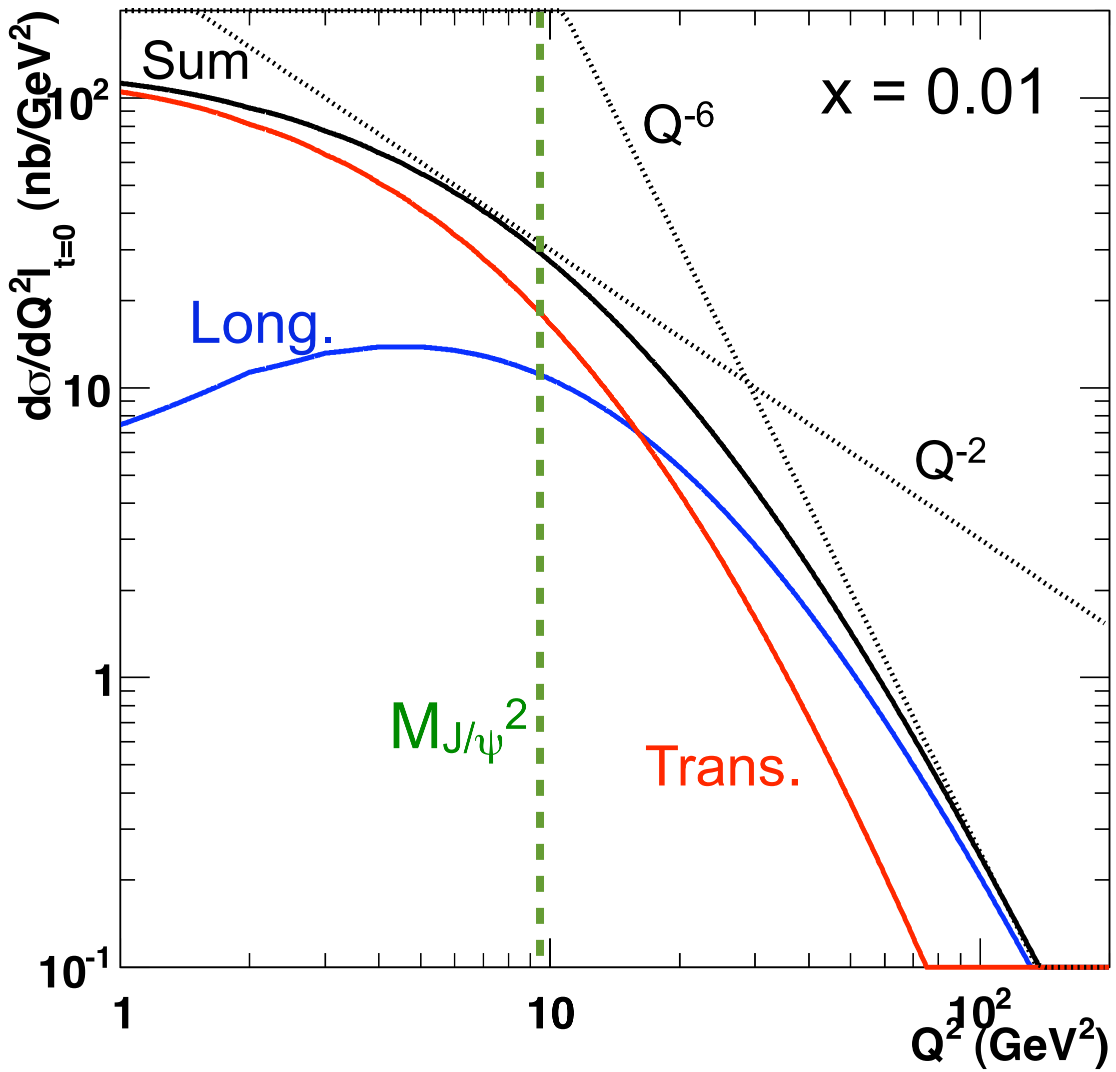


KMW



e+p: J/ψ production, b-Sat model

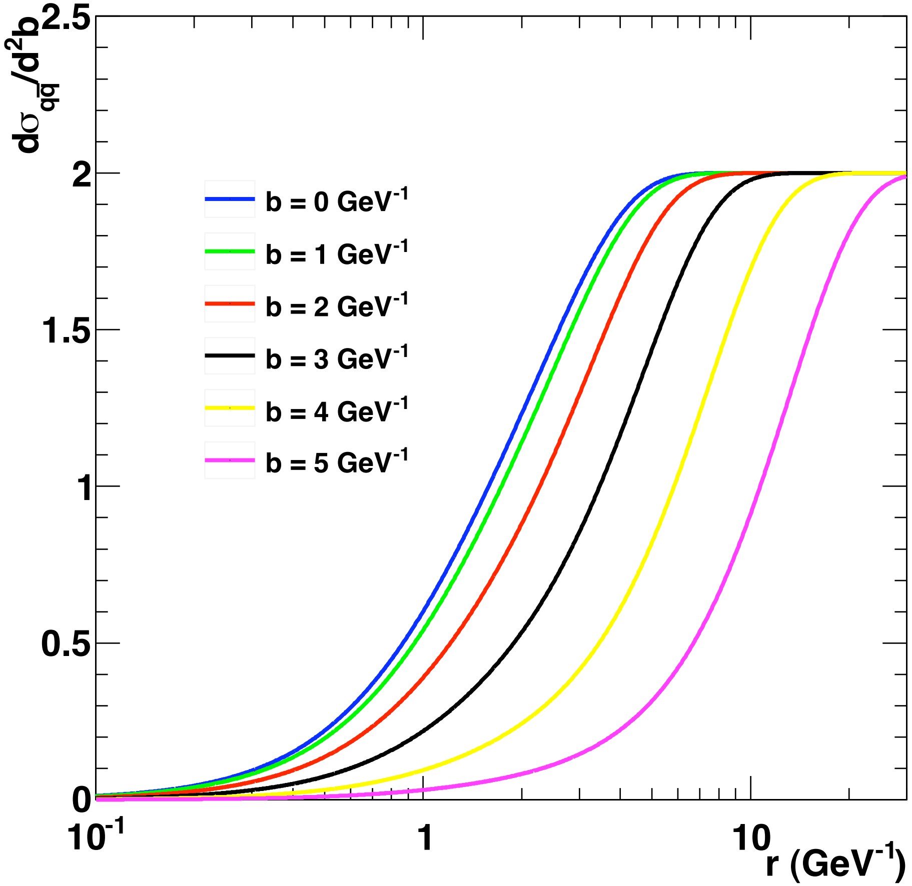
Cross-Sections: Q^2 Dependence



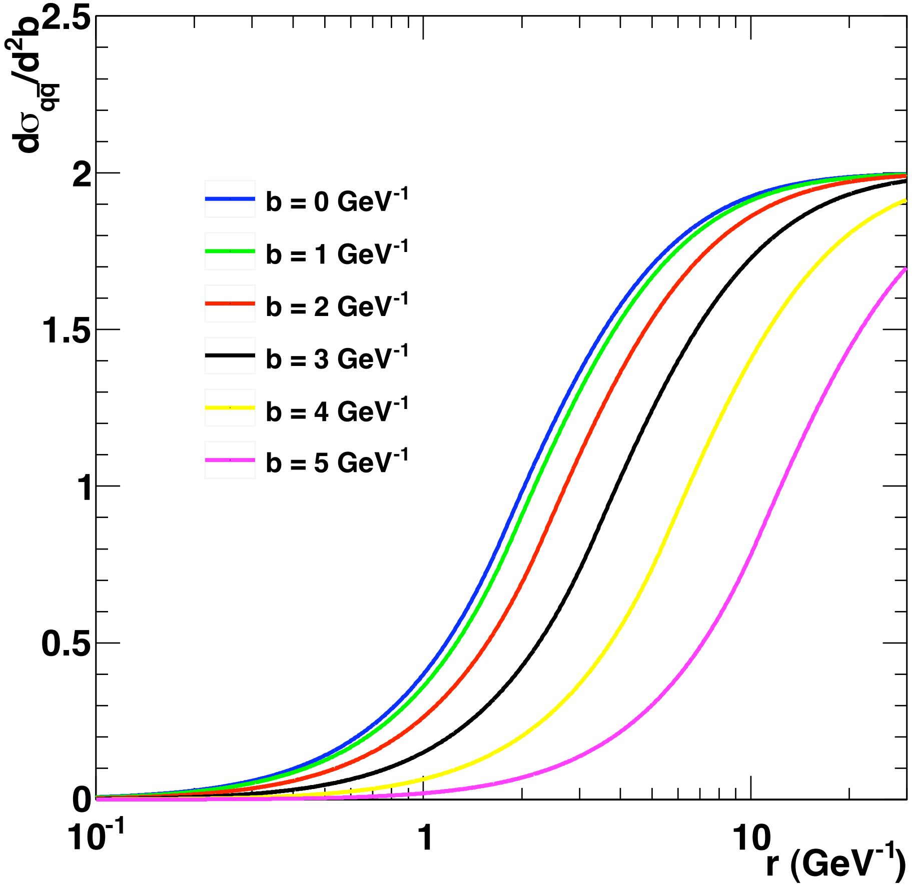
e+p: J/ψ production, b-CGC model

Dipole cross-section

xdvmp: b-Sat

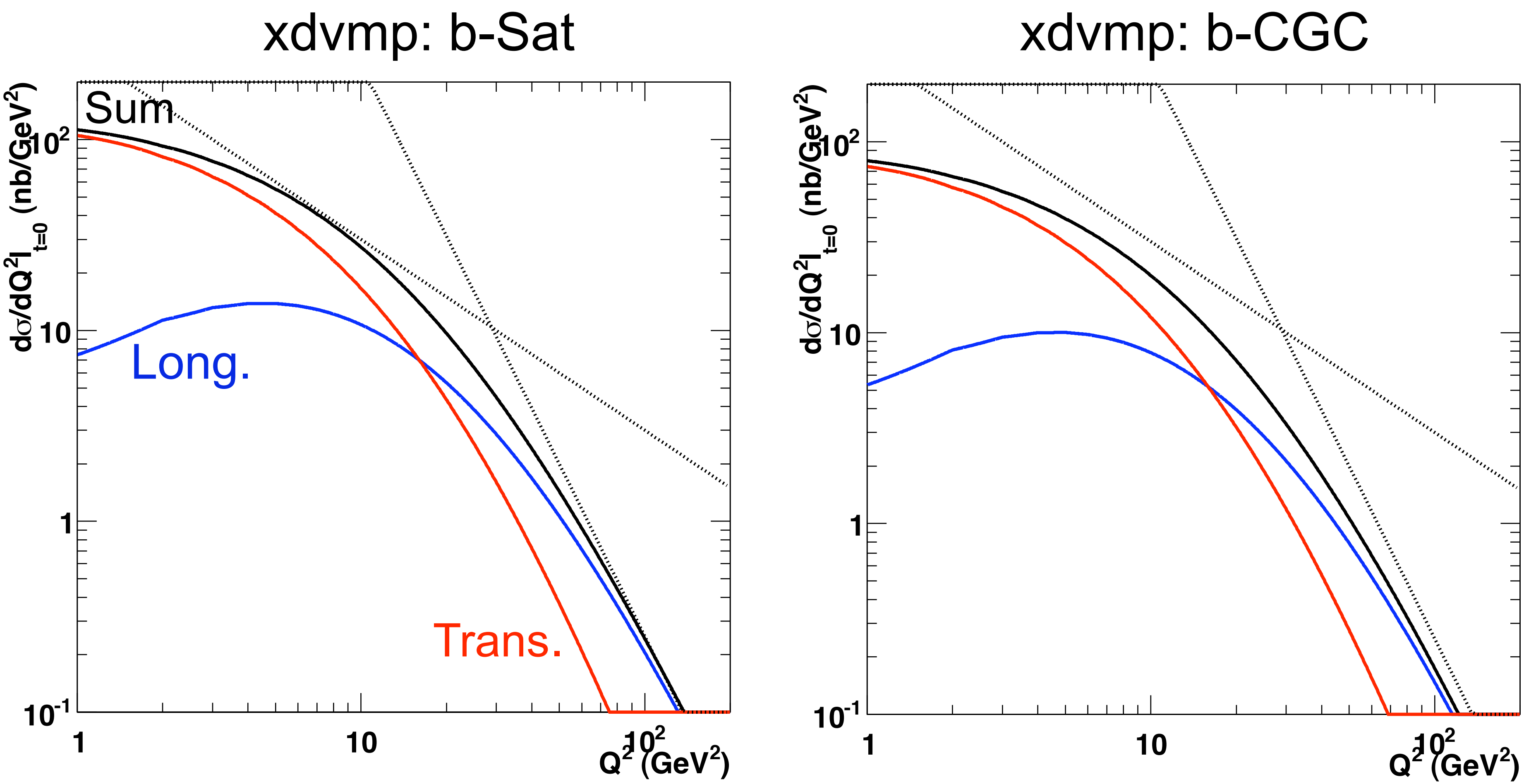


xdvmp: b-CGC



e+p: J/ψ production, b-CGC model

Cross-sections (Q^2 dependence)

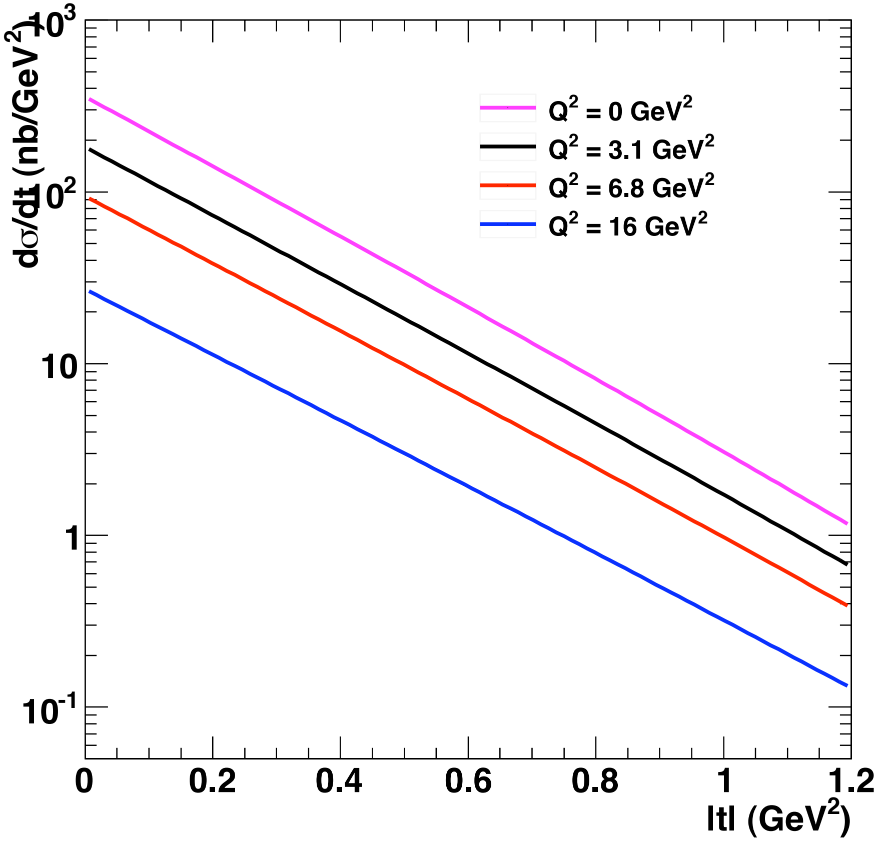


Note: increase of $d\sigma_L/dQ^2$ for $Q < M_{J/\psi}$ is a feature of the wave function not of the dipole

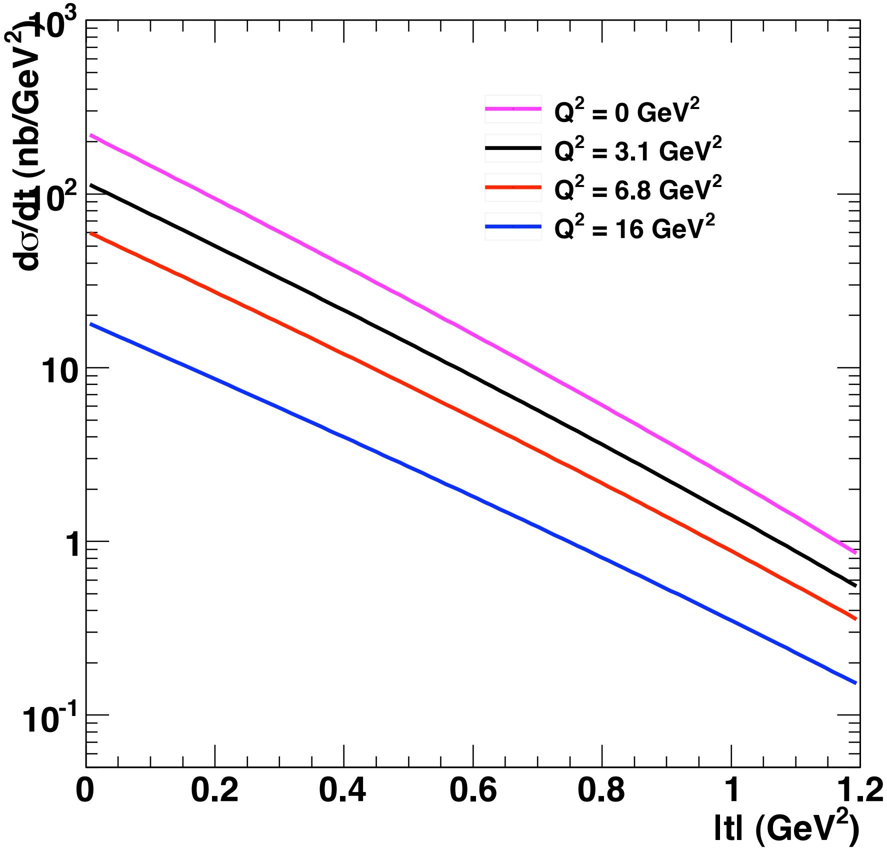
e+p: J/ψ production, b-CGC model

Cross-section

xdvmp: b-Sat

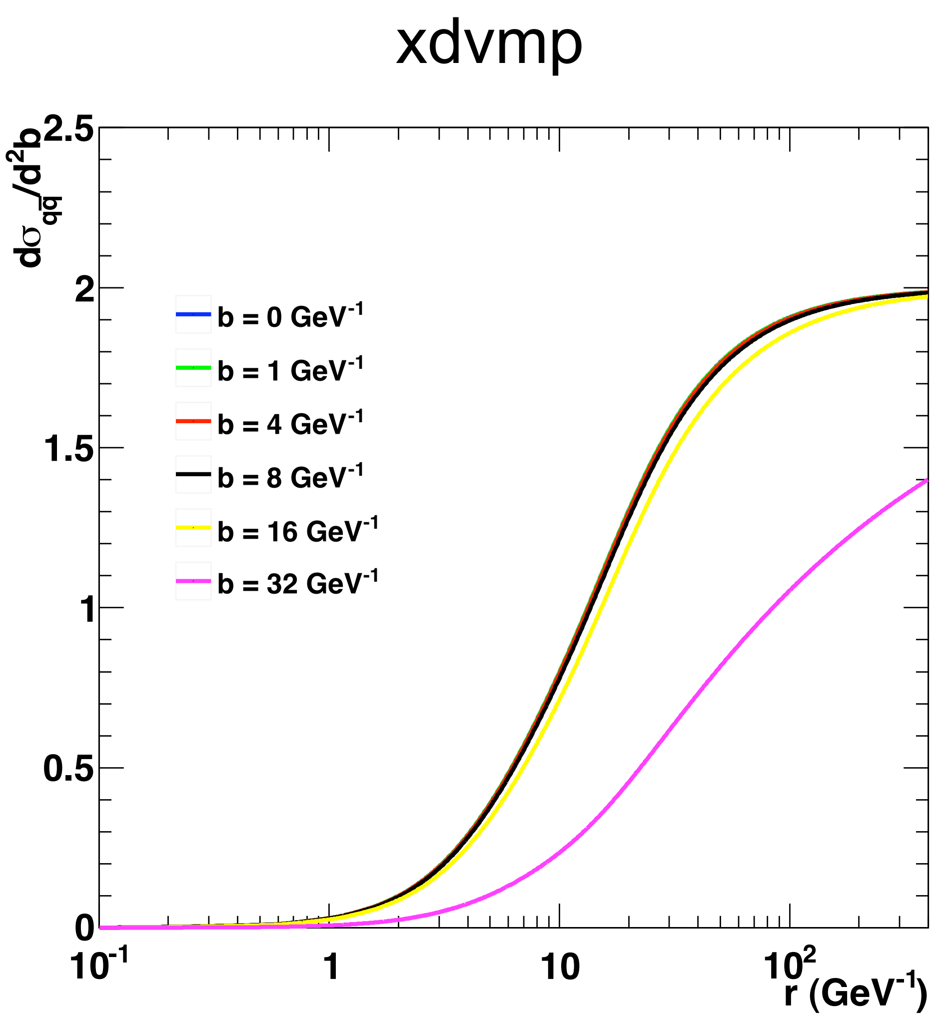


xdvmp: b-CGC



e+A: J/ψ production, b-Sat model

Dipole cross-section



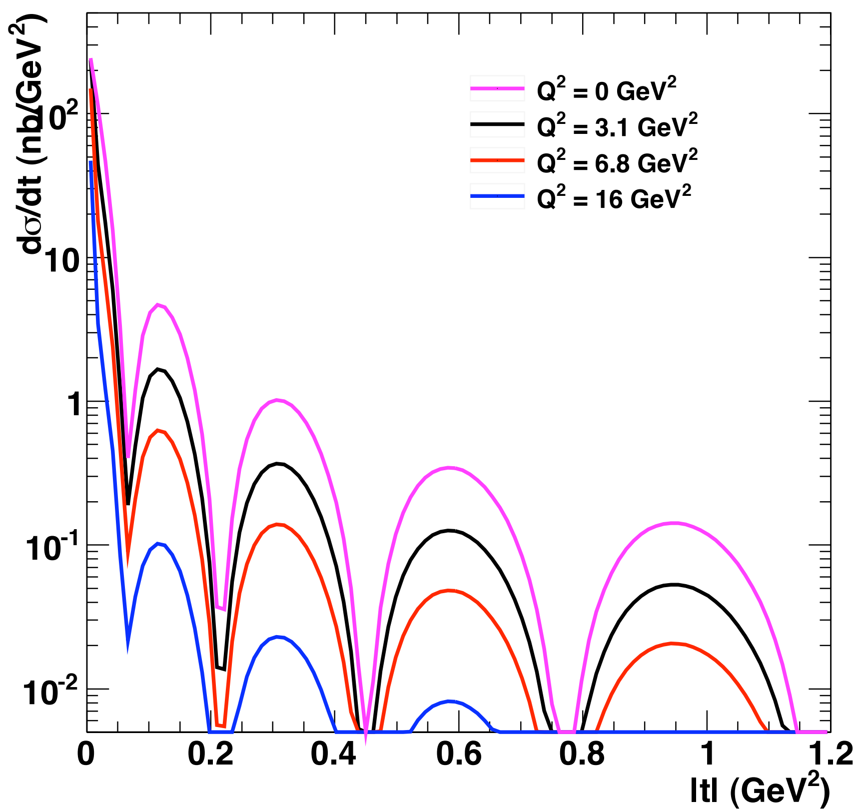
Here I'm using essentially the b-Sat model with a modified dipole cross-section as given in KLV paper:

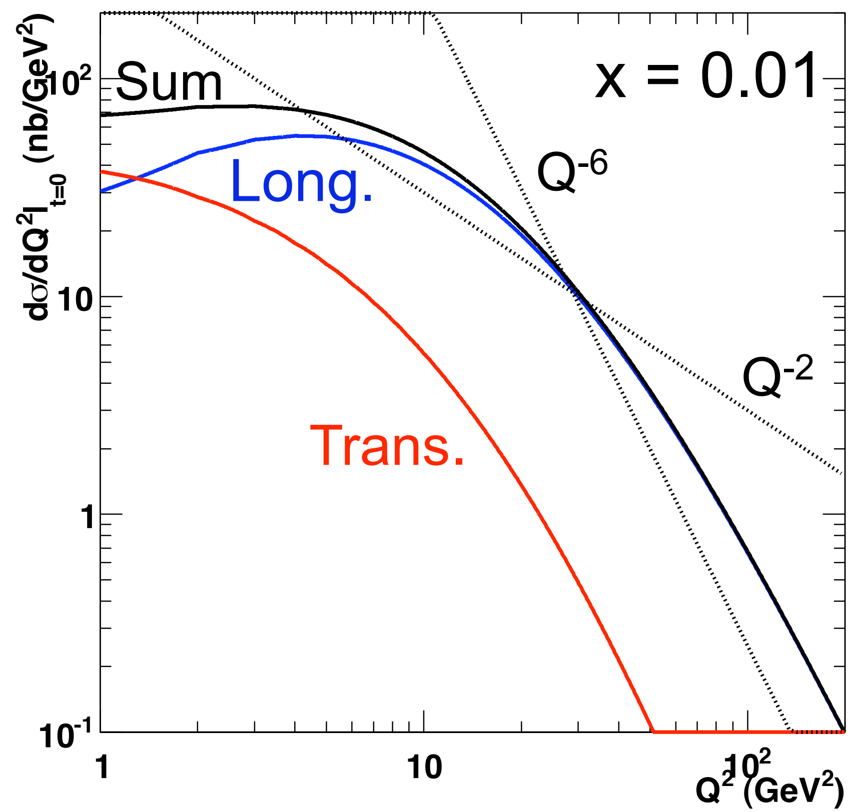
$$\langle \frac{d\sigma^A_{dip}}{d^2 b_\perp} \rangle \approx 2 \left[1 - \left(1 - \frac{T_A(b_\perp)}{2} \sigma^p_{dip} \right)^A \right]$$

T_A: Wood-Saxon using parameters from R. Vogt paper (only Pb, Au, Cd, Cu, Ca, Al, O implemented)

e+A: J/ψ production, b-Sat model

Cross-sections





Needs work and ideas ...

Summary

xdvmp is a C++ implementation of the b-Sat and b-CGC dipole models

- e+p is implemented, results agree with KWM results who successfully describe HERA data
- e+A exist but needs work
- Can be used for simulations through weighting of events with $\sigma(t,x,Q^2,...)$
- Needs more testing (and speed improvements)