

# $F_L$ and the statistics/rates needed to make the measurement

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The neutral current inclusive e-nucleon cross section can be written as a combination of two terms:

$$\sigma_r(x, Q^2, y) = \frac{d^2\sigma}{dx dQ^2} \cdot \frac{Q^4 x}{2\pi\alpha^2 Y_+} = F_2(x, Q^2) - \frac{y^2}{Y_+} \cdot F_L(x, Q^2),$$

“reduced cross section”

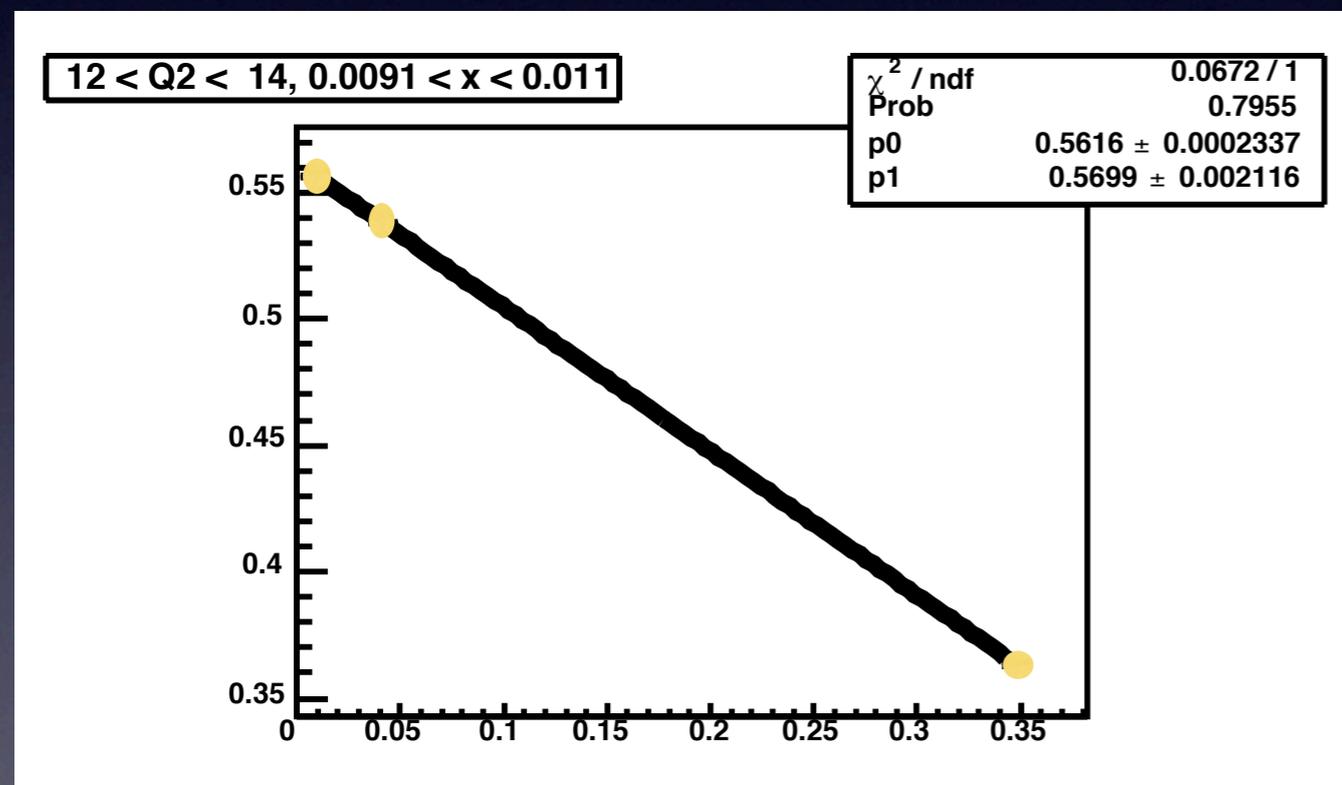
$F_2 \propto (\sigma_T + \sigma_L)$   $F_L \sim \sigma_L$  where  $\sigma_T$  and  $\sigma_L$  are the  $\gamma^*P$  cross sections for transverse and longitudinally polarized virtual photons. The value of  $F_2$  is determined by the sum of quark and anti-quark distributions, whereas  $F_L$  depends on quark + gluon distributions. Above some value of  $Q^2$ , quark PDFs are much smaller than gluon PDFs hence  $F_L$  is mainly “driven” by gluon PDFs

The measurement has experimental difficulties:

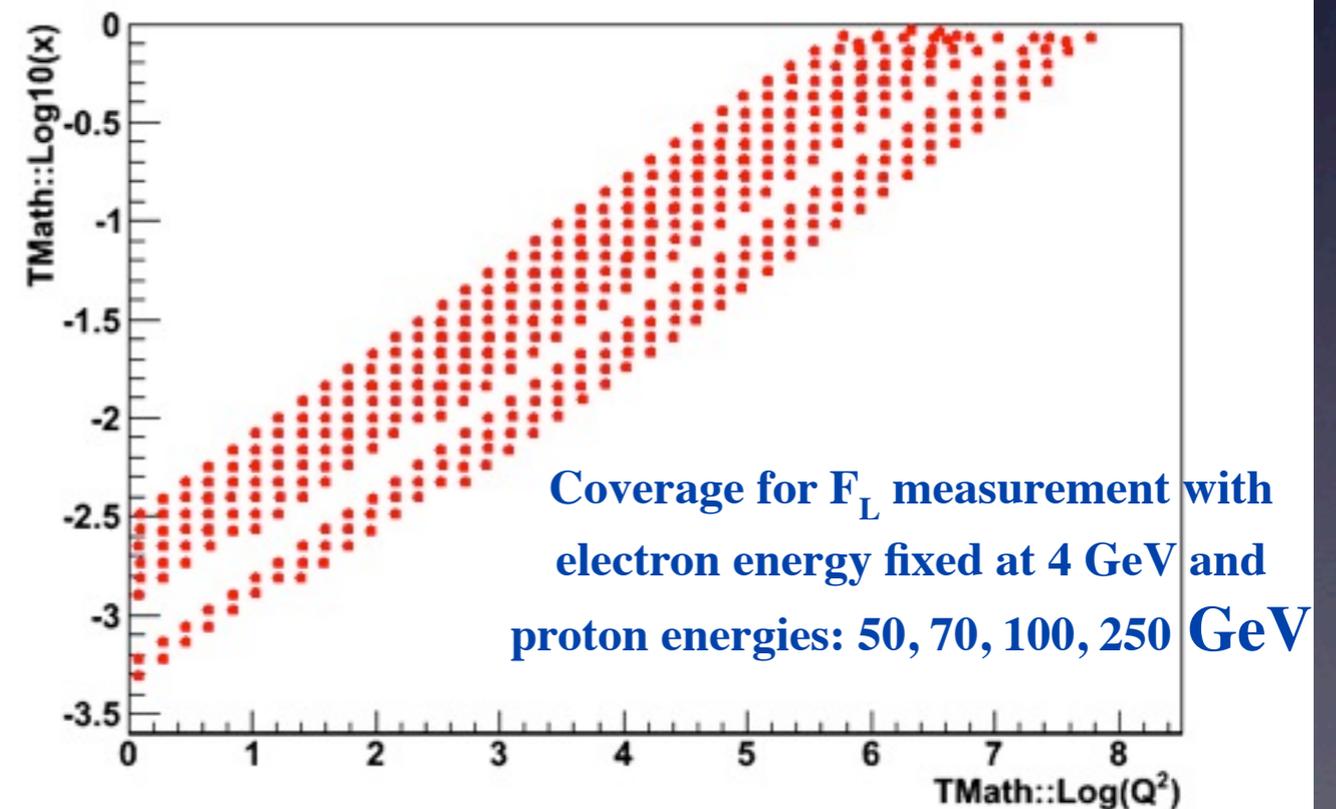
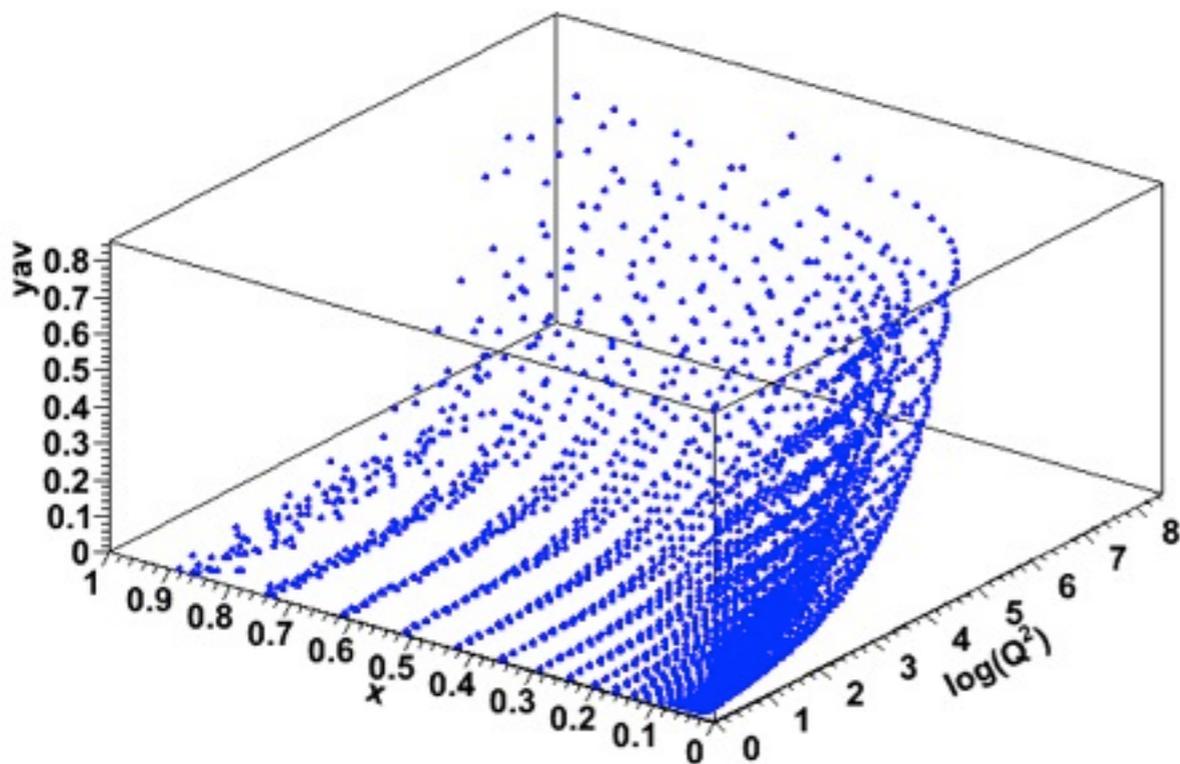
The highest sensitivity is obtained at high values of  $y = Q^2/sx$ , at low values of  $Q^2$  high  $y$  corresponds to low energy of the scattered e that can be obscured by hadronic background.

$F_L$  measurements are extracted from inclusive measurements of  $e+p$  or  $e+A$  at a set of collision energies. A most convenient set would have a fixed electron energy and changing  $p$  or  $A$  energy.

At a fixed  $Q^2$  and  $x$ , reduced cross sections are plotted as functions of  $y^2/Y^+$  and fitted to straight lines; the intercept is then  $F^2$  and the slope  $F_L$



This extraction technique is very much dependent on the lever arm on the  $y^2/Y^+$  variable (this set the first reduction in the reach in  $Q^2$  and  $x$  space).  
And very sensitive to statistical and systematic error.  
(Even with the best statistics it will be dominated by systematic errors)



# $F_L$ measurements with STAR TPC and electron beam fixed at 4 GeV



$$\theta_{\min} = 40^\circ$$

$$\theta_{\max} = 140^\circ$$

## Description of Jamie's work on $F_L$

I use a package Jamie put together to investigate the extraction of  $F_L$  from inclusive measurements.

I calculate cross-section in bins of  $Q^2$  and  $x$  for a set of  $e$  and  $p$  beam energies, using the MRST2002 parametrization.

Cross-sections are translated into counts for a set of given luminosities and each point is given error bars.

Additional errors (systematic) can then be added and the fits to straight lines in cross-section versus  $y^2/Y^+$  is done.

This first round of calculations have focused mainly in the reach in  $Q^2$  and  $x$  space. I have scaled the integrated luminosity used EIC work (...) by a factor of  $10^{-2}$  to bring it down to roughly the average luminosity pp 500 GeV (I picked this number as a boundary of stability for the STAR TPC)

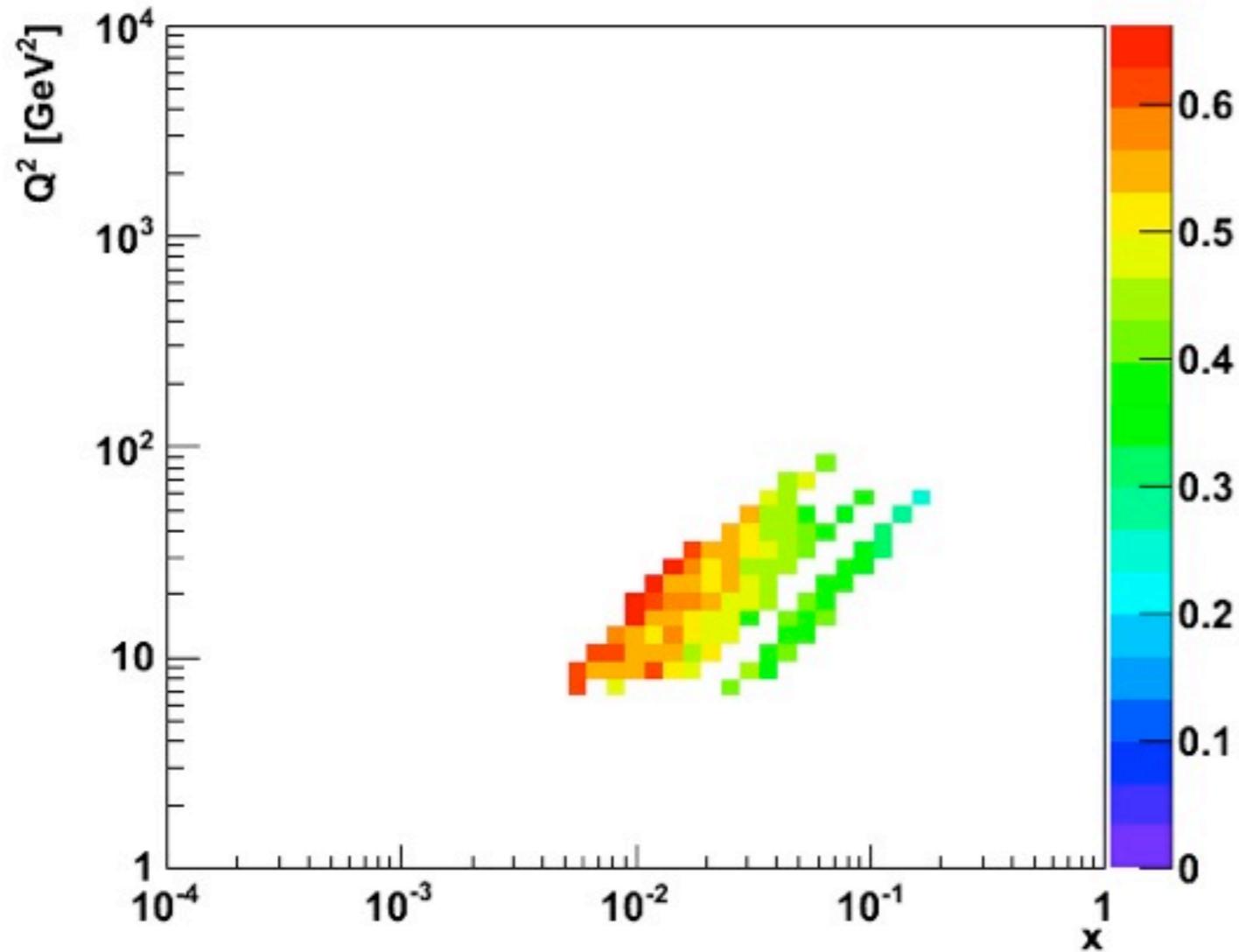
<u>p↑-p↑ operation</u>		(2009)	(2011/12)
Energy	GeV	100 / 250	100 / 250
No of bunches	...	109	109
Bunch intensity	$10^{11}$	1.3 / 1.1	1.3 / 1.5
<b>Average L</b>	<b><math>10^{30} \text{cm}^{-2} \text{s}^{-1}</math></b>	<b>28 / 55</b>	<b>30 / 150</b>
<b>Polarization P</b>	<b>%</b>	<b>56 / 34</b>	<b>70</b>

T. Roser RHIC retreat 2009

➤ eRHIC: 250 GeV p↑ × 10 – 20 GeV e↑ ;  $3 \times 10^{33} \text{cm}^{-2} \text{s}^{-1}$

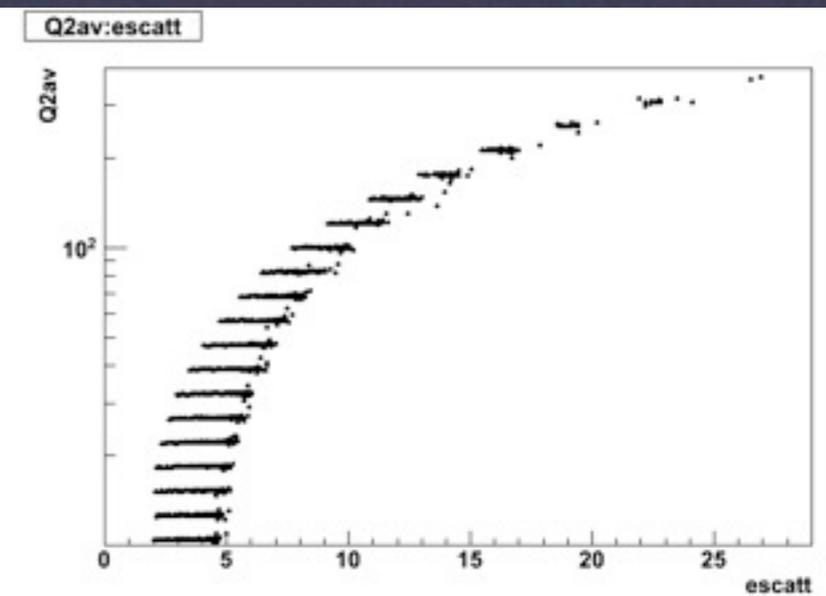
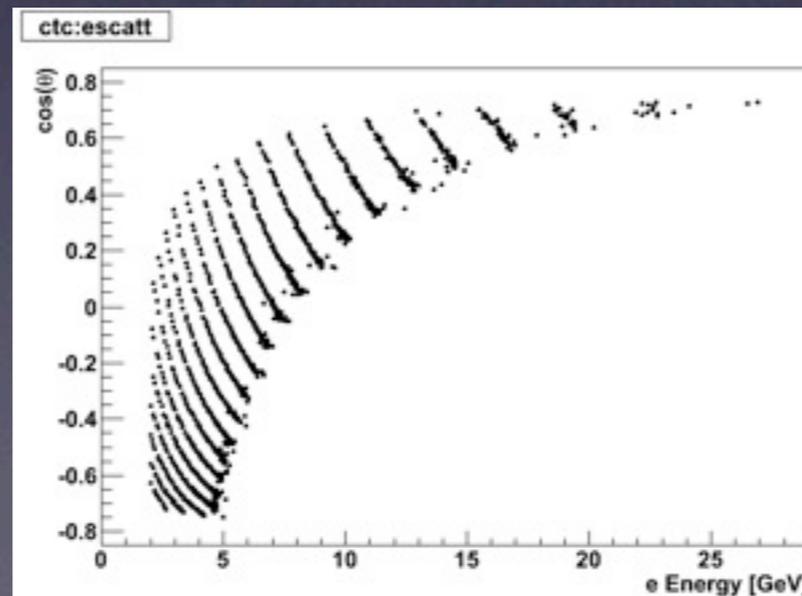
e: 4 GeV   p: 30 50 70 100 250 GeV

$F_L$  value, protons

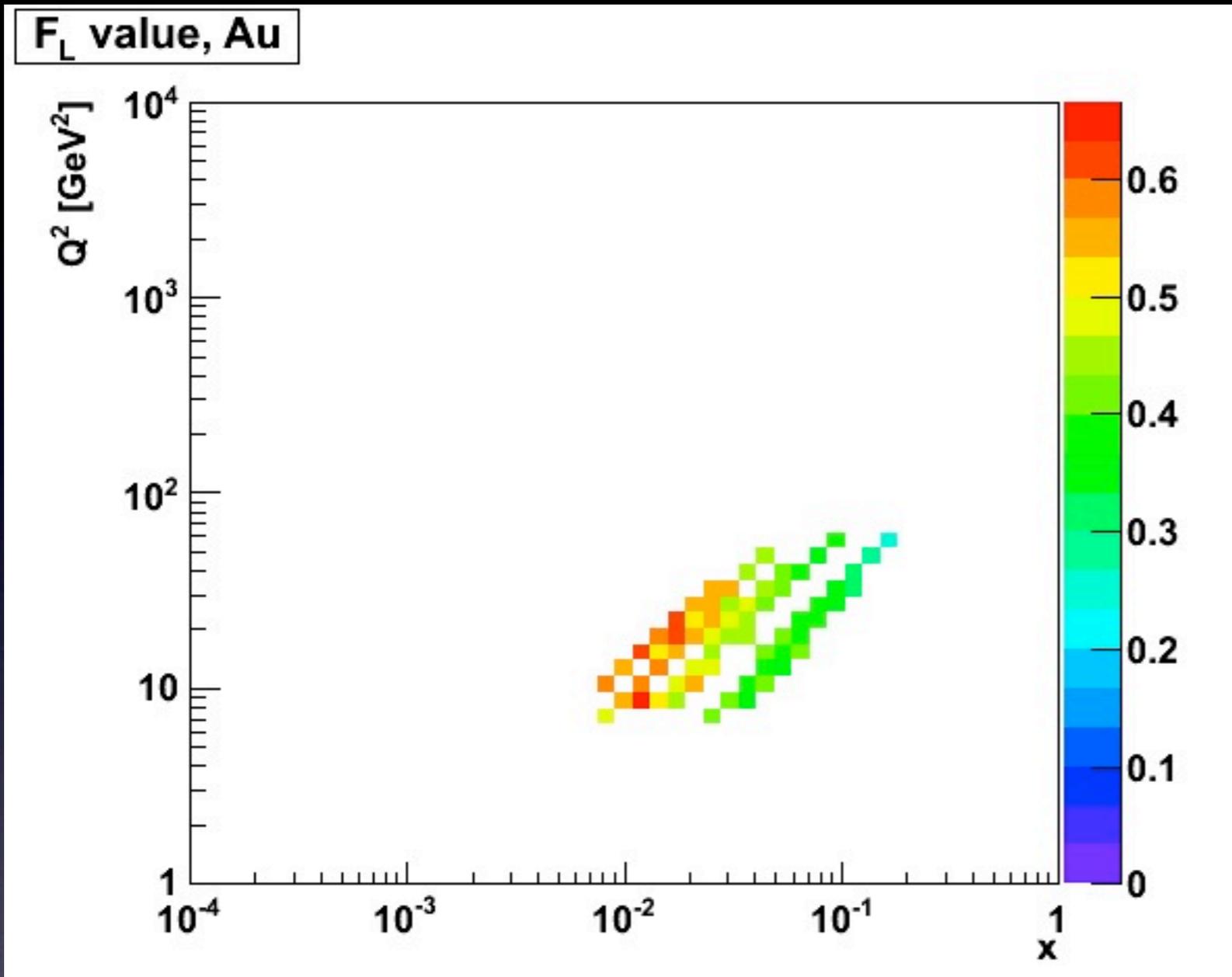


Events used in this plot have a lower limit on scattered electron energy at 2 GeV in order to be able to separate them from pions in the calorimeter.

This plot only displays  $F_L$  values with an overall uncertainty smaller than 10%



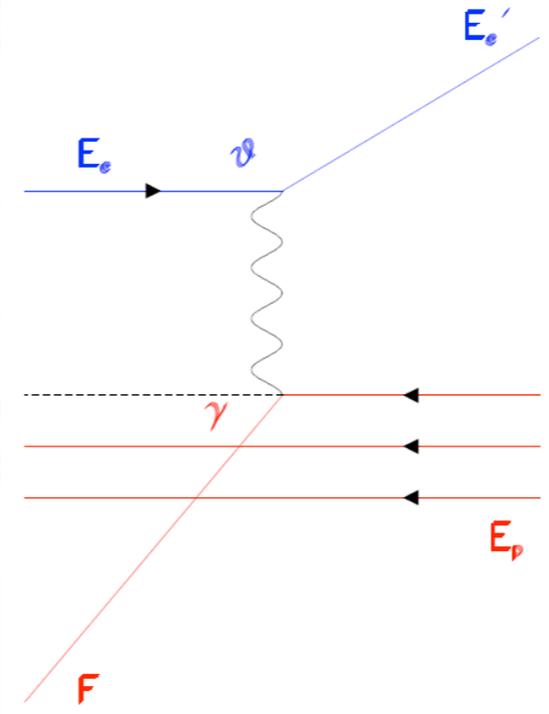
e: 4GeV p: 30 50 70 100



# DIS Kinematics

## □ Collider kinematics (1)

### ○ Electron method: scattered electron



$$F = \frac{p_{T,h}^2 + (E - p_z)_h^2}{2(E - p_z)_h}$$

$$\cot \gamma = \frac{p_{T,h}^2 - (E - p_z)_h^2}{p_{T,h}^2 + (E - p_z)_h^2}$$

$$x_e = \frac{Q_e^2}{sy_e} = \frac{E'_e \cos^2\left(\frac{\theta'_e}{2}\right)}{E_p \left(1 - \frac{E'_e}{E_e} \sin^2\left(\frac{\theta'_e}{2}\right)\right)}$$

### ○ Jacquet-Blondel method: hadronic final state

$$y_e = 1 - \frac{E'_e}{2E_e} (1 - \cos \theta'_e) = 1 - \frac{E'_e}{E_e} \sin^2\left(\frac{\theta'_e}{2}\right)$$

$$x_{JB} = \frac{Q_{JB}^2}{sy_{JB}}$$

$$p_{T,h}^2 = \left(\sum_h p_{x,h}\right)^2 + \left(\sum_h p_{y,h}\right)^2$$

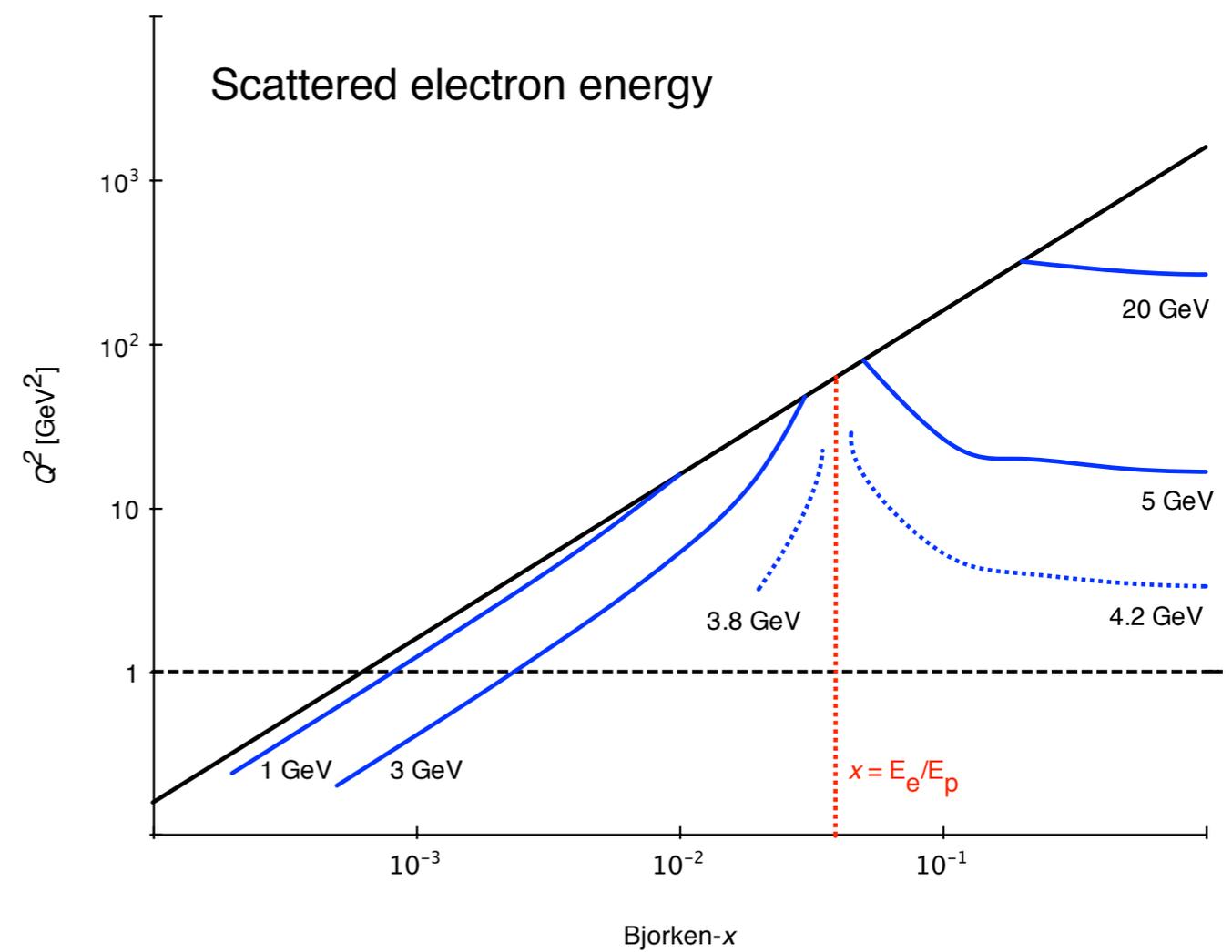
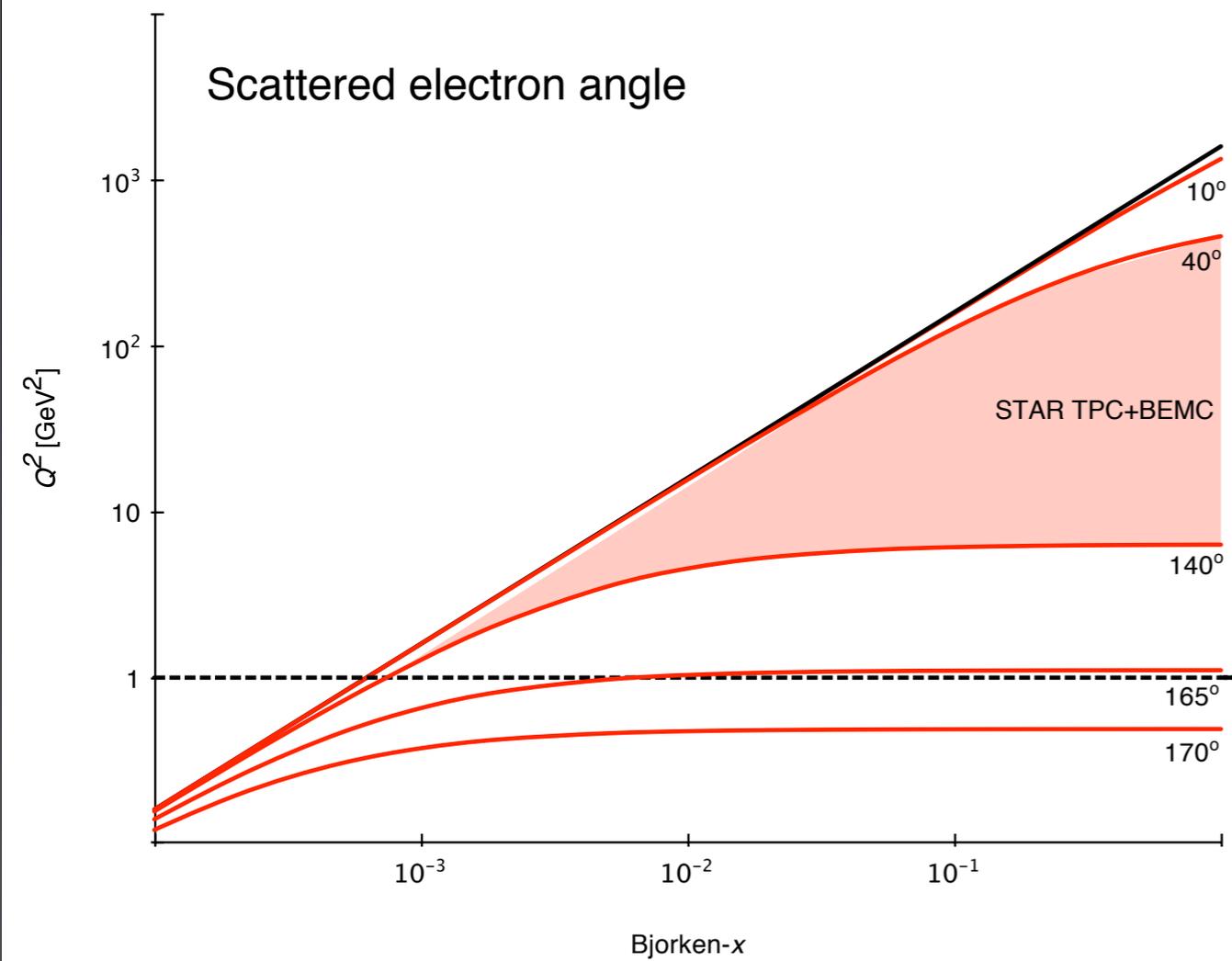
$$Q_e^2 = 2E_e E'_e (1 + \cos \theta'_e) = 4E_e E'_e \cos^2\left(\frac{\theta'_e}{2}\right) = \frac{p_{T,e}^2}{1 - y_e}$$

$$y_{JB} = \frac{(E - p_z)_h}{2E_e}$$

$$(E - p_z)_h = \sum_h (E_h - p_{z,h})$$

$$Q_{JB}^2 = \frac{p_{T,h}^2}{1 - y_{JB}}$$

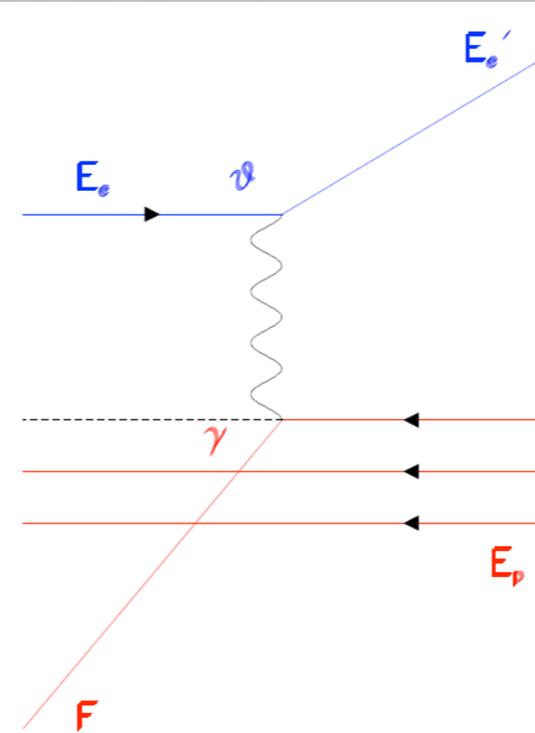
Electron issues: low- $x$  trigger,  
 electron energy resolution at intermediate to high  $x$ , and low- $Q^2$ ,  
 possibly high- $x$  and high- $Q^2$  acceptance.



# DIS Kinematics

## □ Collider kinematics (4)

### ○ Electron method: scattered electron



$$\left(\frac{\delta x_e}{x_e}\right) = \left(\frac{1}{y_e}\right) \frac{\delta E'_e}{E'_e} \otimes \left[\frac{x_e}{E_e/E_p} - 1\right] \tan\left(\frac{\theta'_e}{2}\right) \delta\theta'_e$$

$$\left(\frac{\delta y_e}{y_e}\right) = \left(1 - \frac{1}{y_e}\right) \frac{\delta E'_e}{E'_e} \otimes \left[\frac{1}{y_e} - 1\right] \cot\left(\frac{\theta'_e}{2}\right) \delta\theta'_e$$

$$\left(\frac{\delta Q_e^2}{Q_e^2}\right) = \frac{\delta E'_e}{E'_e} \otimes \tan\left(\frac{\theta'_e}{2}\right) \delta\theta'_e$$

$$F = \frac{p_{T,h}^2 + (E - p_z)_h^2}{2(E - p_z)_h}$$

$$\cot \gamma = \frac{p_{T,h}^2 - (E - p_z)_h^2}{p_{T,h}^2 + (E - p_z)_h^2}$$

### ○ Jacquet-Blondel method: hadronic final state

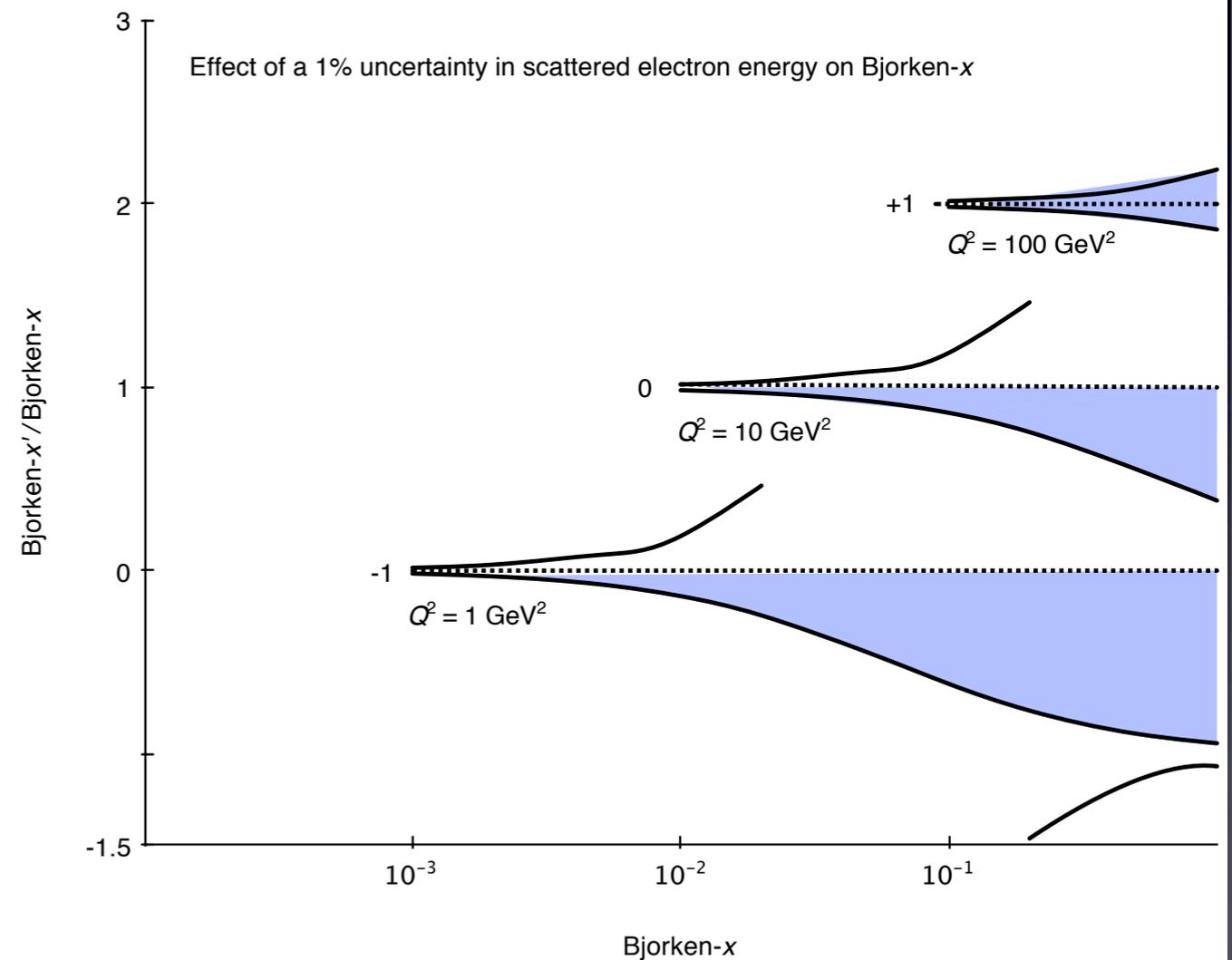
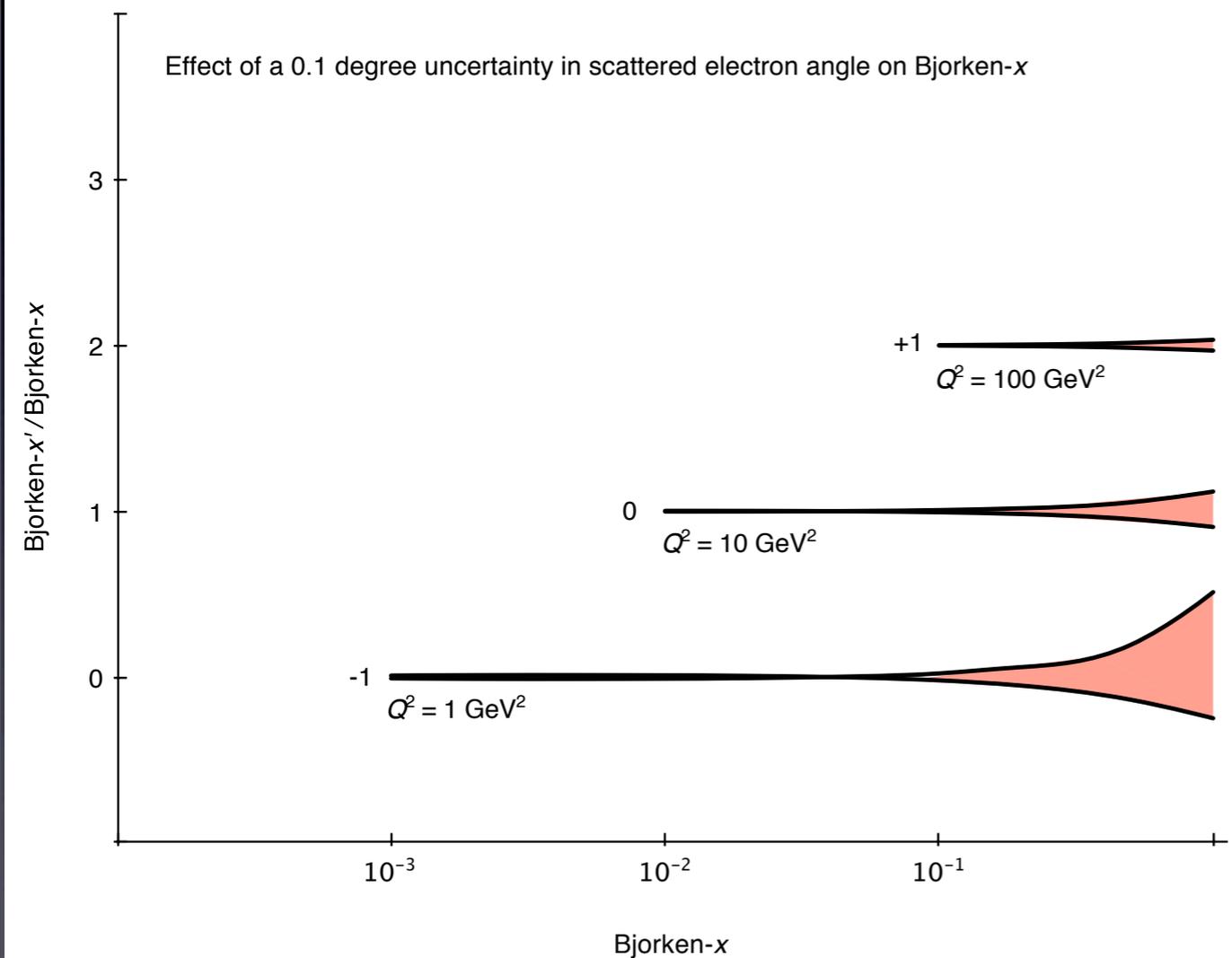
$$\left(\frac{\delta x_{JB}}{x_{JB}}\right) = \left(\frac{1}{1 - y_{JB}}\right) \frac{\delta F}{F} \otimes \left[2 \cot \gamma + \left(\frac{2y_{JB} - 1}{1 - y_{JB}}\right) \cot\left(\frac{\gamma}{2}\right)\right] \delta\gamma$$

$$\left(\frac{\delta y_{JB}}{y_{JB}}\right) = \frac{\delta F}{F} \otimes \cot\left(\frac{\gamma}{2}\right) \delta\gamma$$

$$\left(\frac{\delta Q_{JB}^2}{Q_{JB}^2}\right) = \left(\frac{2 - y_{JB}}{1 - y_{JB}}\right) \frac{\delta F}{F} \otimes \left[2 \cot \gamma + \left(\frac{y_{JB}}{1 - y_{JB}}\right) \cot\left(\frac{\gamma}{2}\right)\right] \delta\gamma$$

Electron issues: low- $x$  trigger,  
 electron energy resolution at intermediate to high  $x$ , and low- $Q^2$ ,  
 possibly high- $x$  and high- $Q^2$  acceptance.

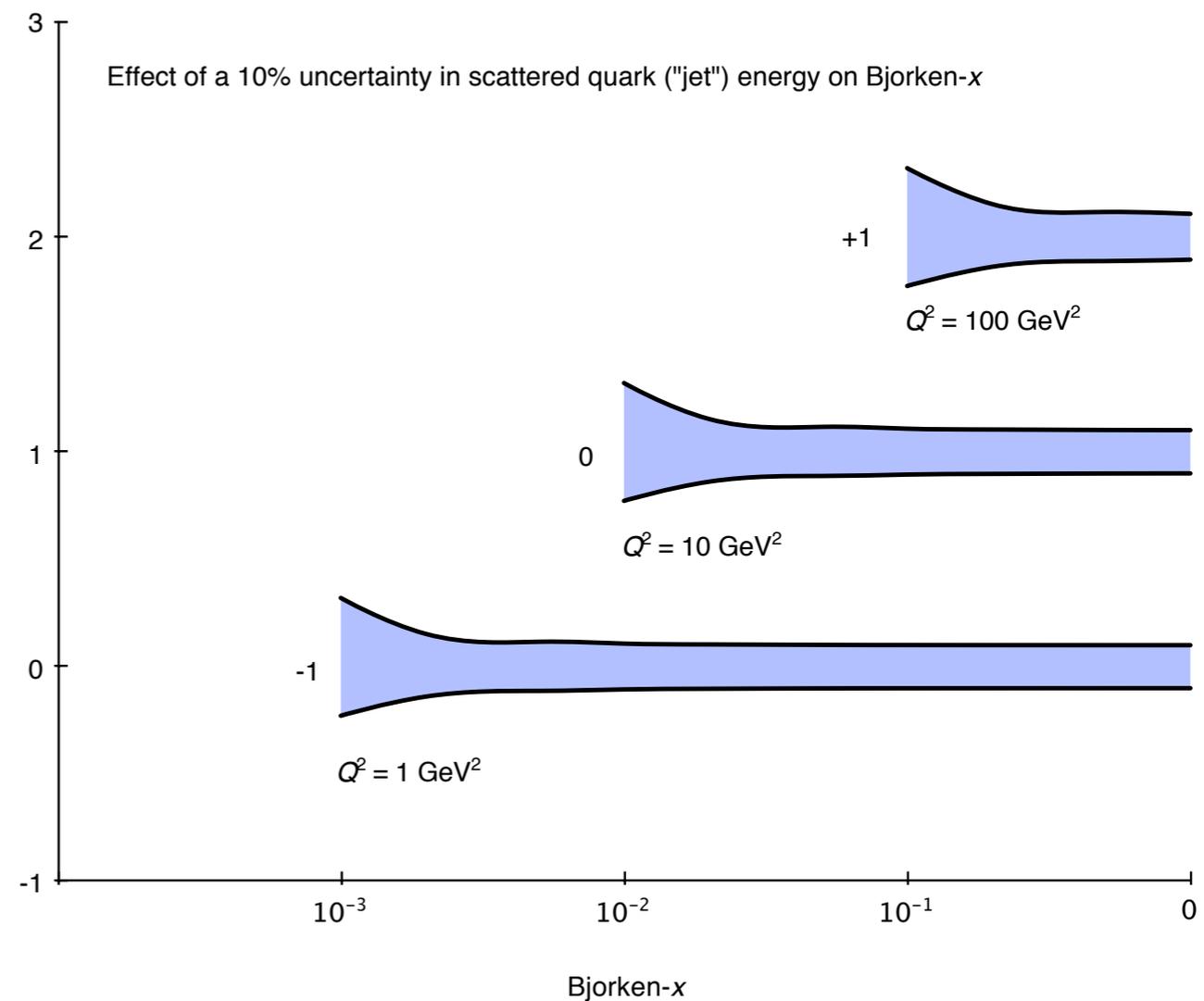
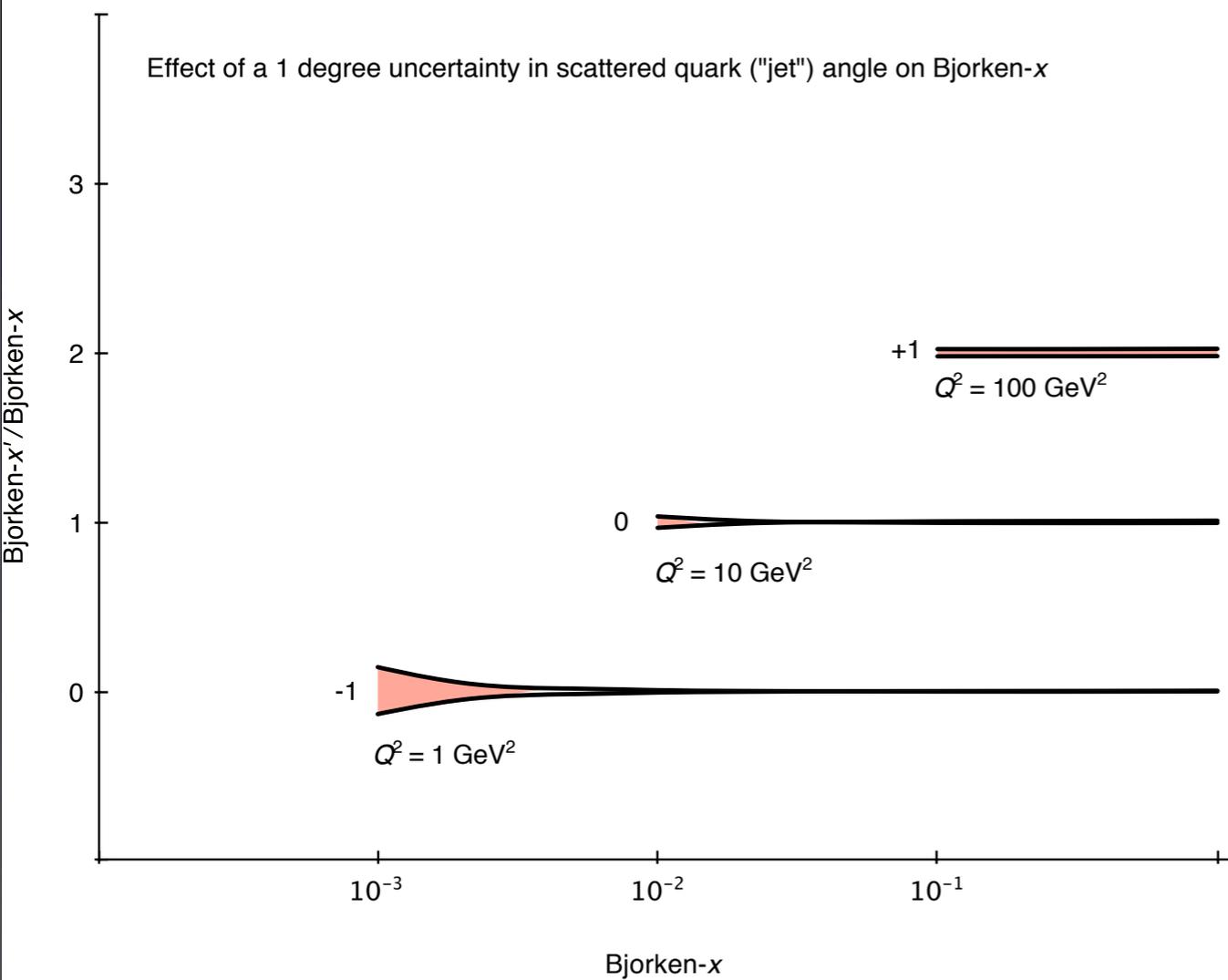
Resolution:



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The role of jets is principally at high- $x$ , and intermediate to high  $Q^2$

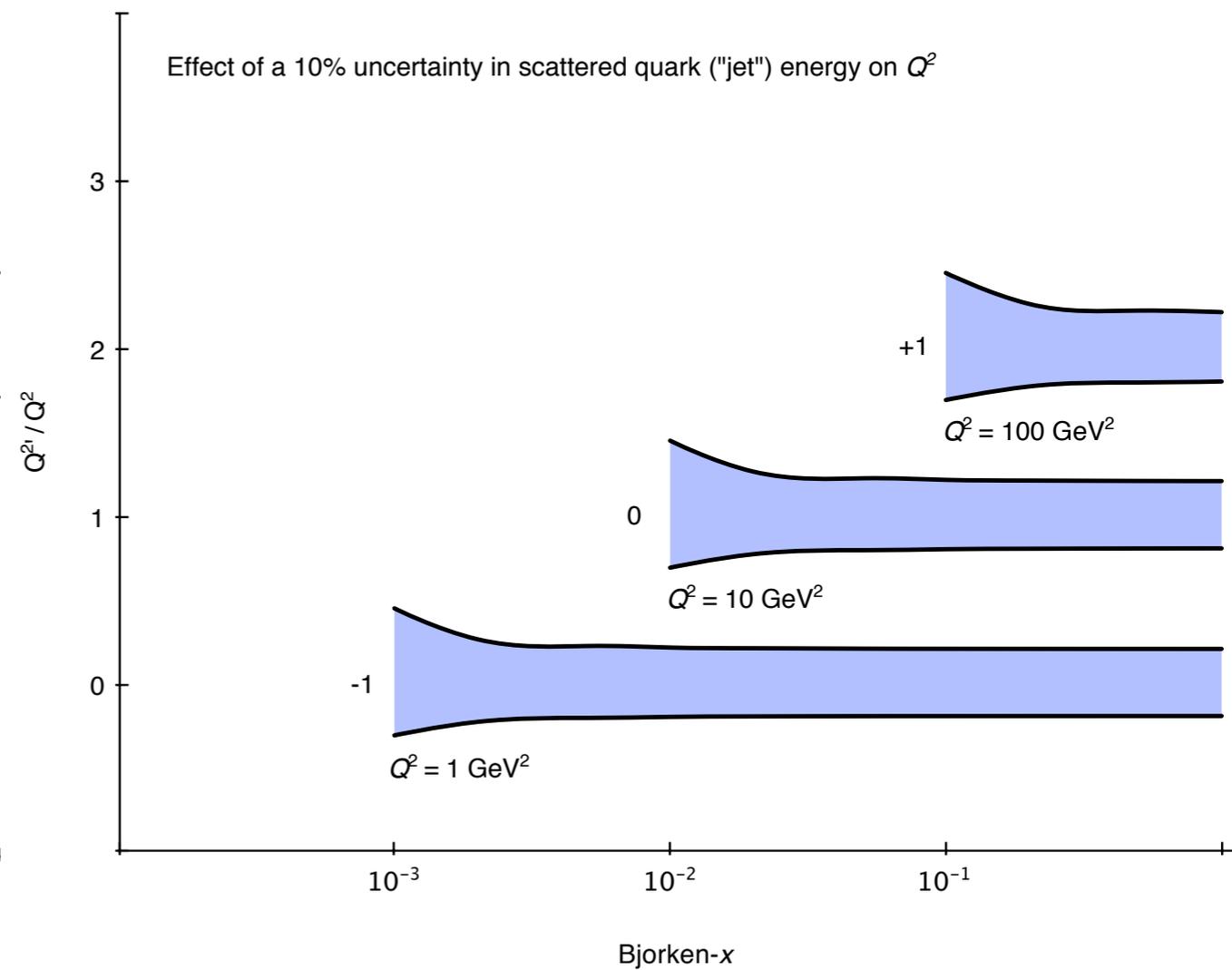
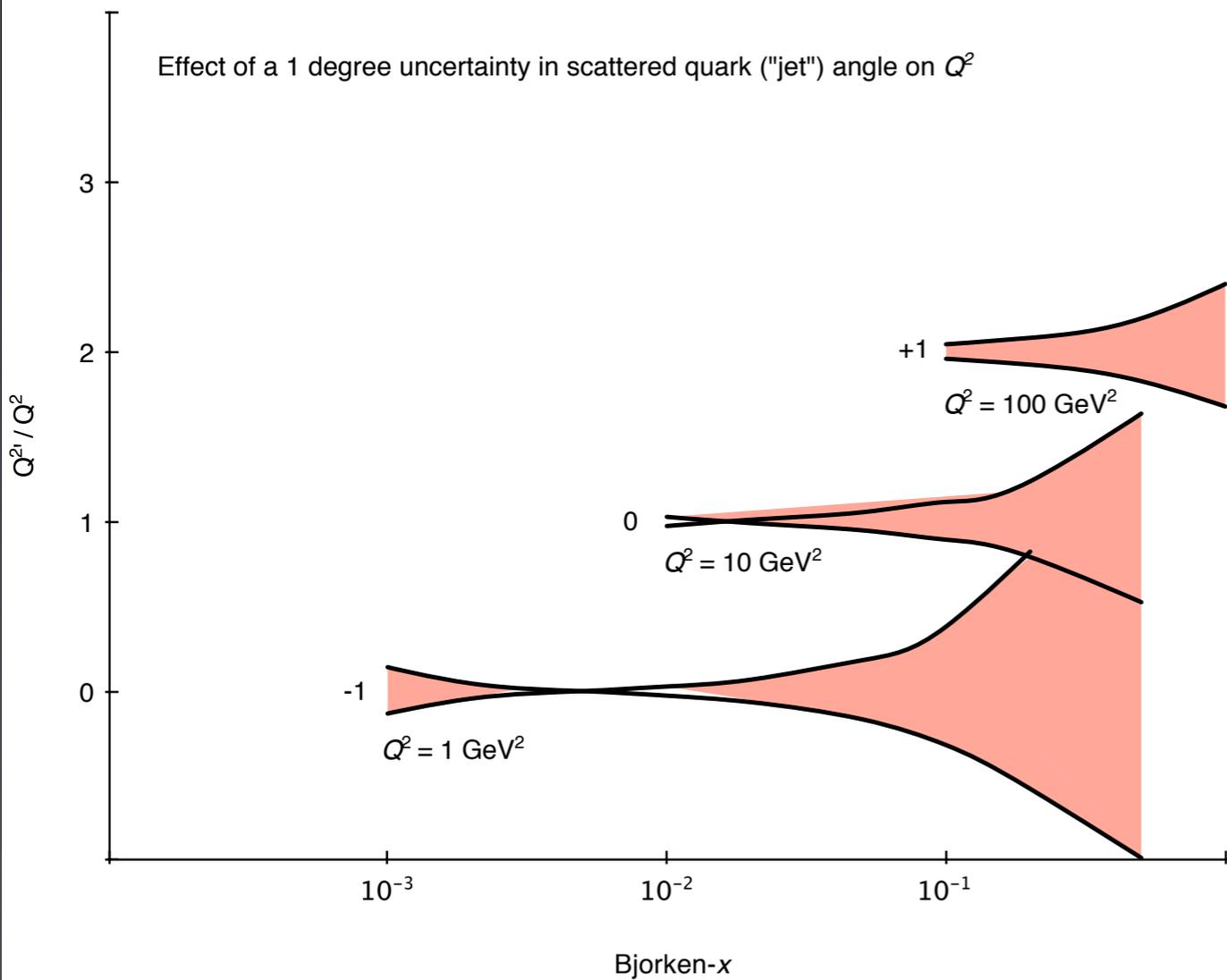
The effects of resolution on Bjorken- $x$ :



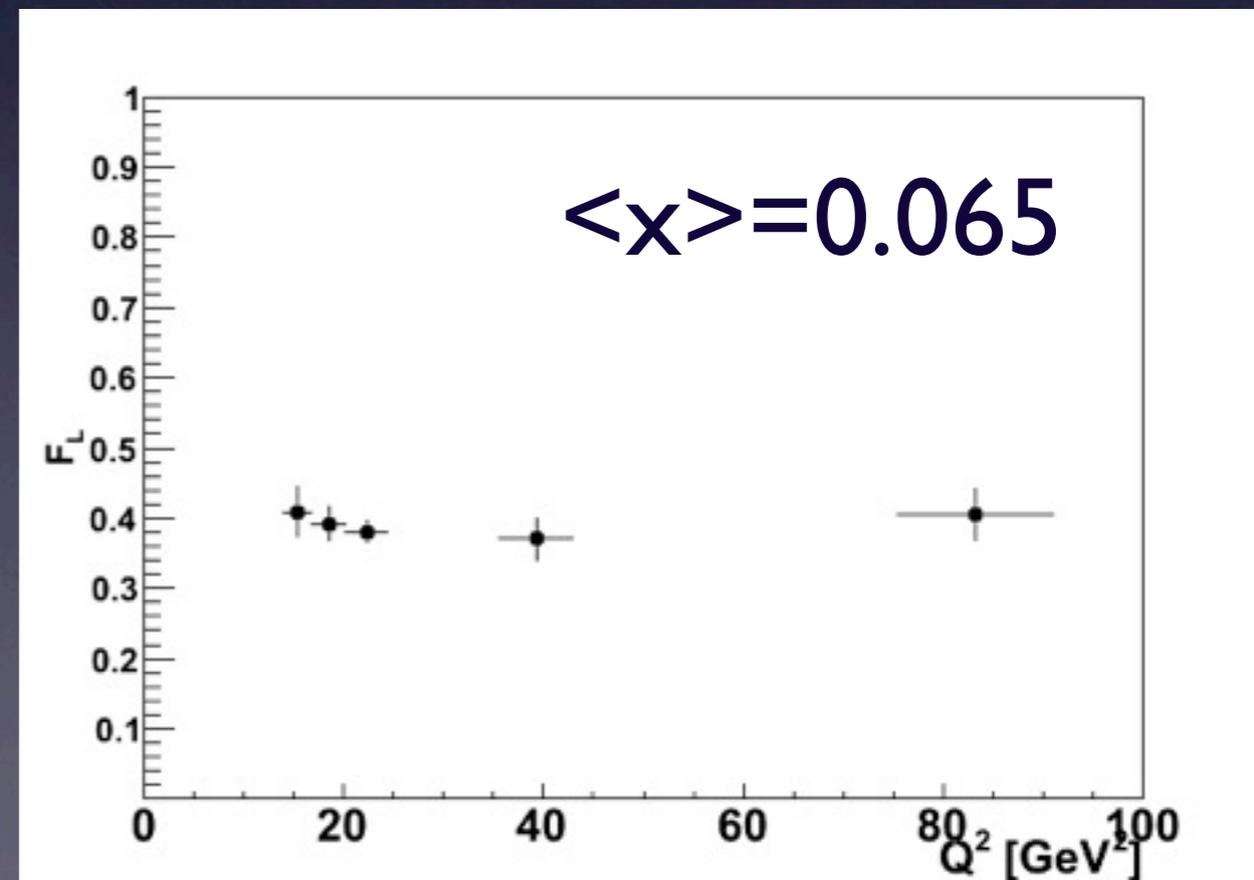
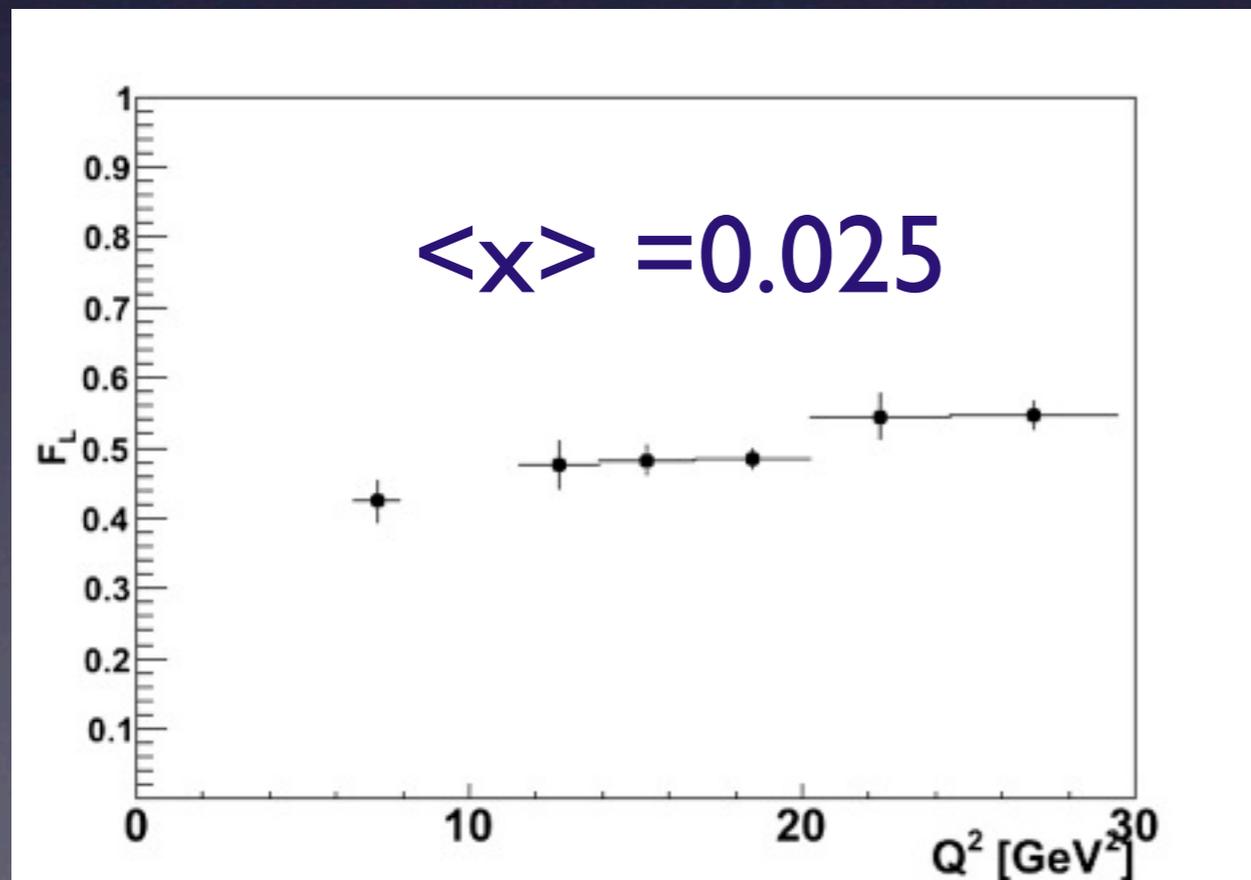
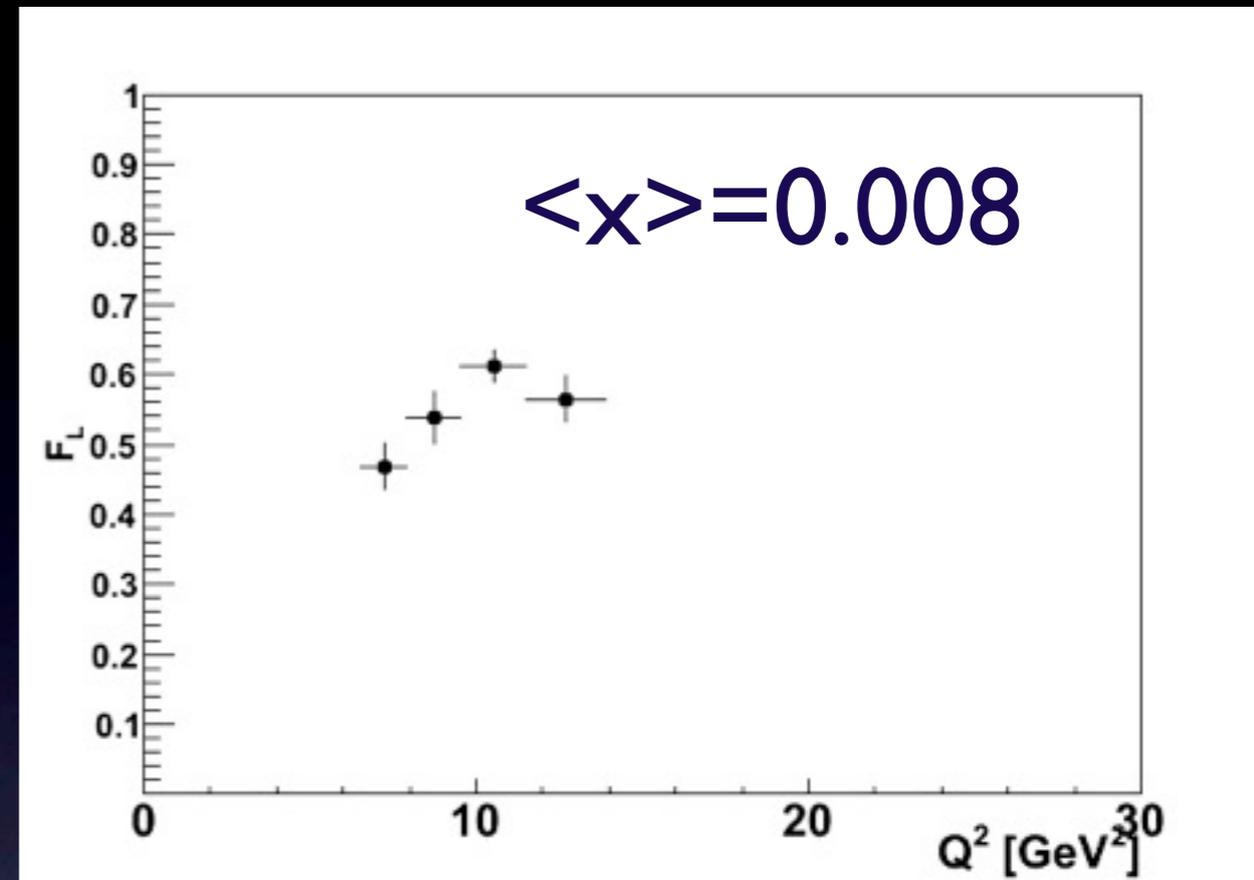
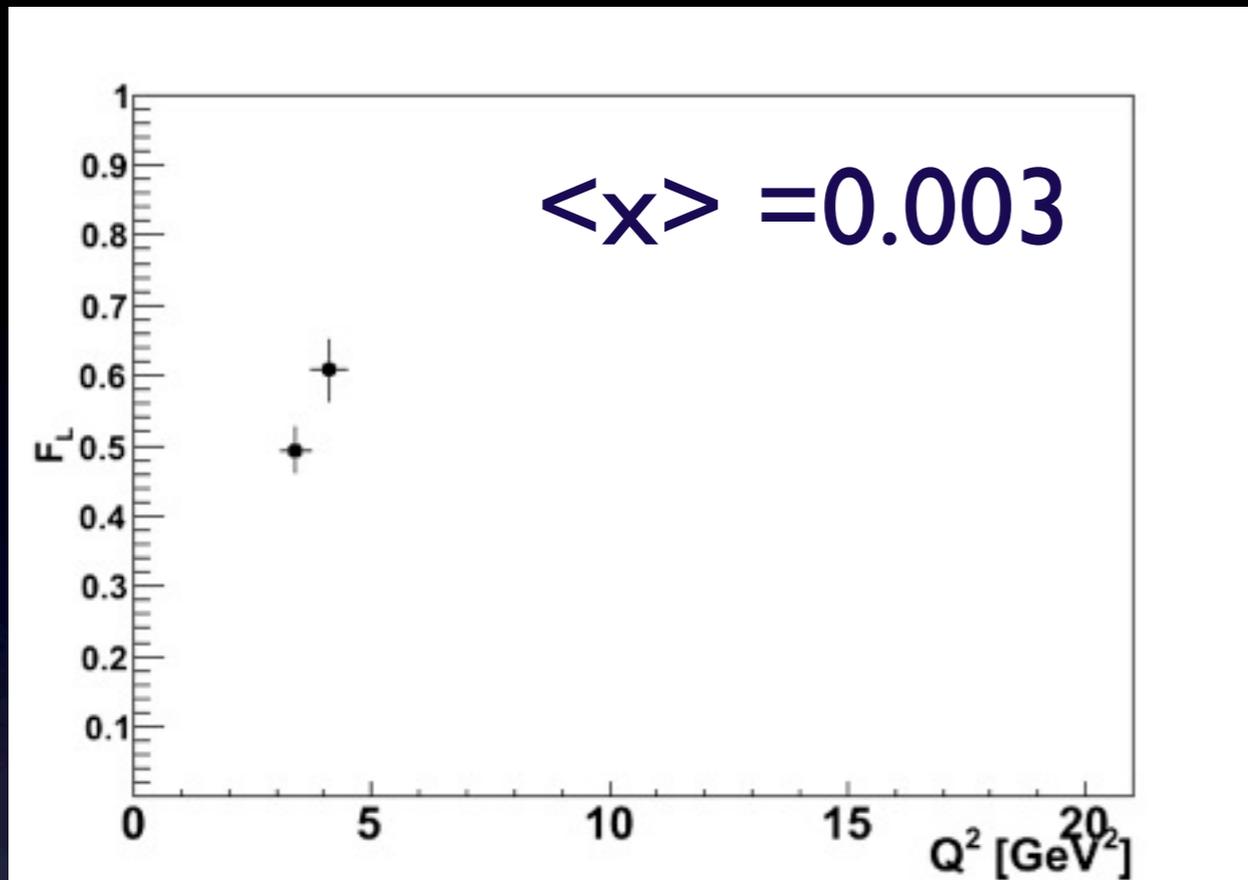
Ernst Sichtermann - LBNL

The role of jets is principally at high- $x$ , and intermediate to high  $Q^2$

The effects of resolution on  $Q^2$ :



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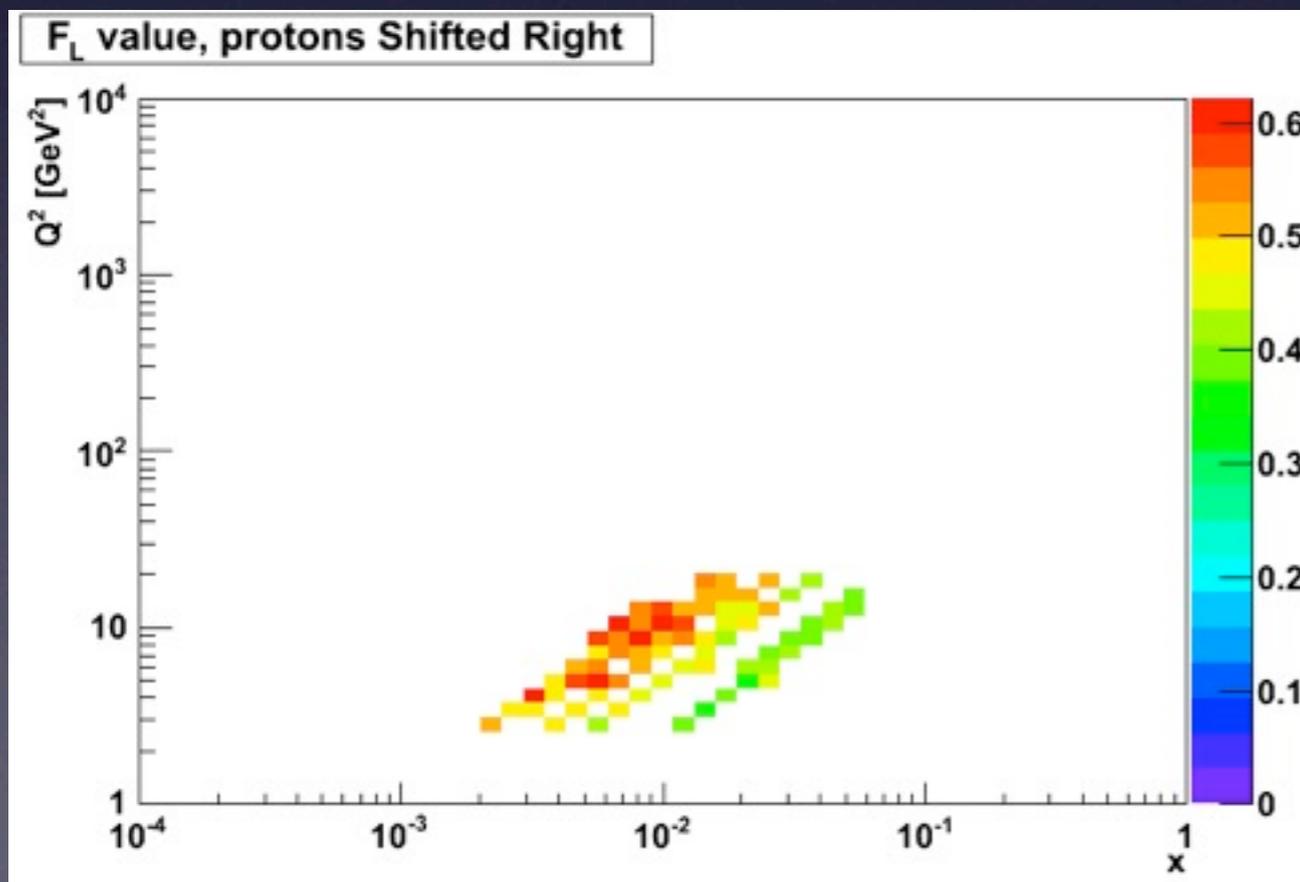
$$\theta_{\min} = 90^\circ$$

$$\theta_{\max} = 155^\circ$$

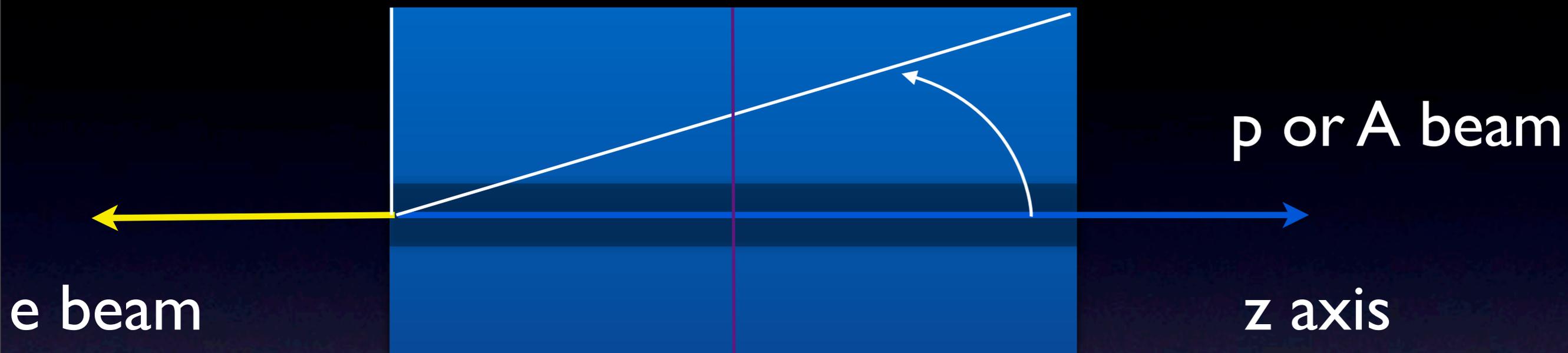
e beam



$\theta$  p or A beam  
z axis

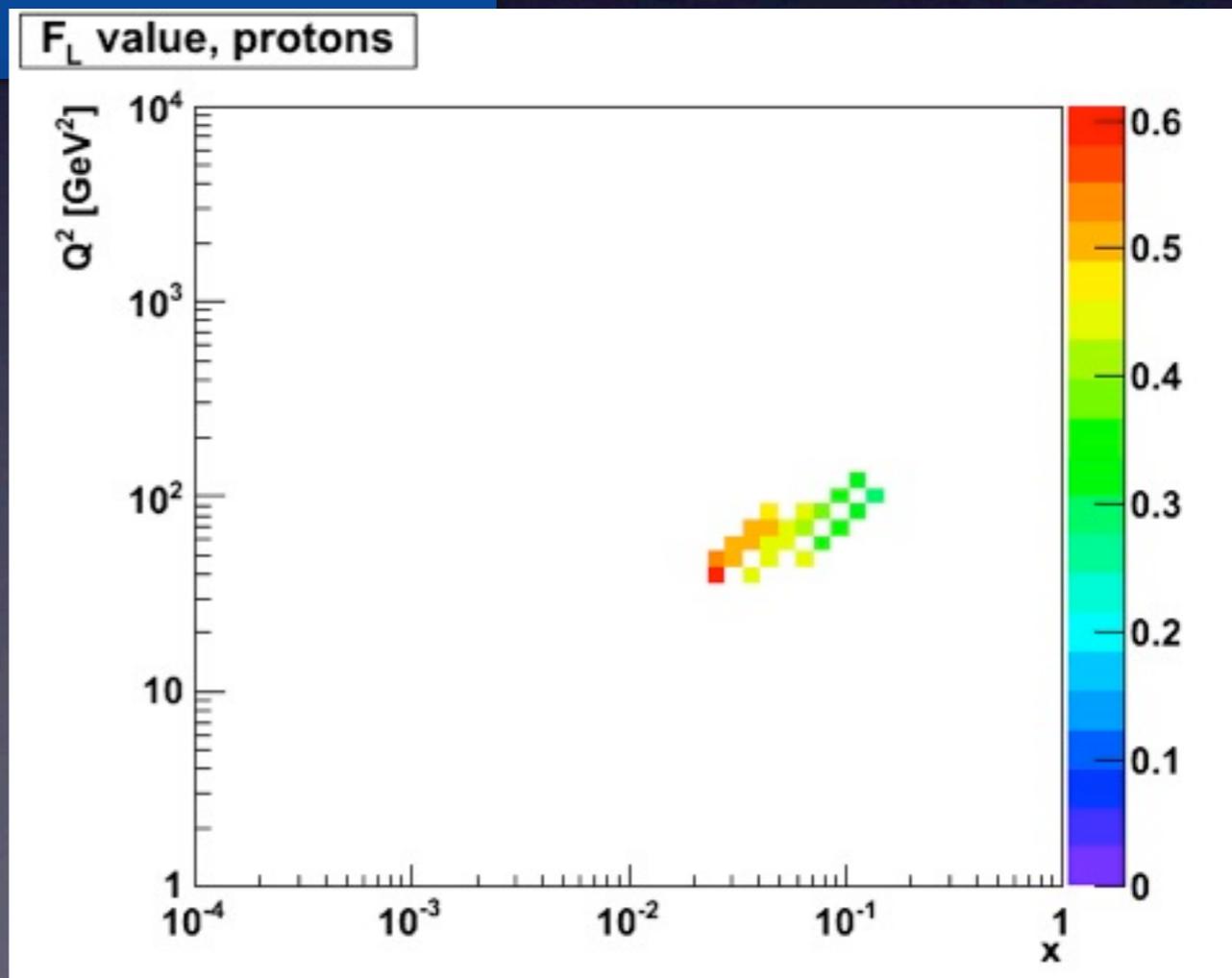


Displaced interactions  
give us access to lower  
values of  $Q^2$ .  
Tracking reconstruction  
becomes convoluted, but  
in principle still possible.

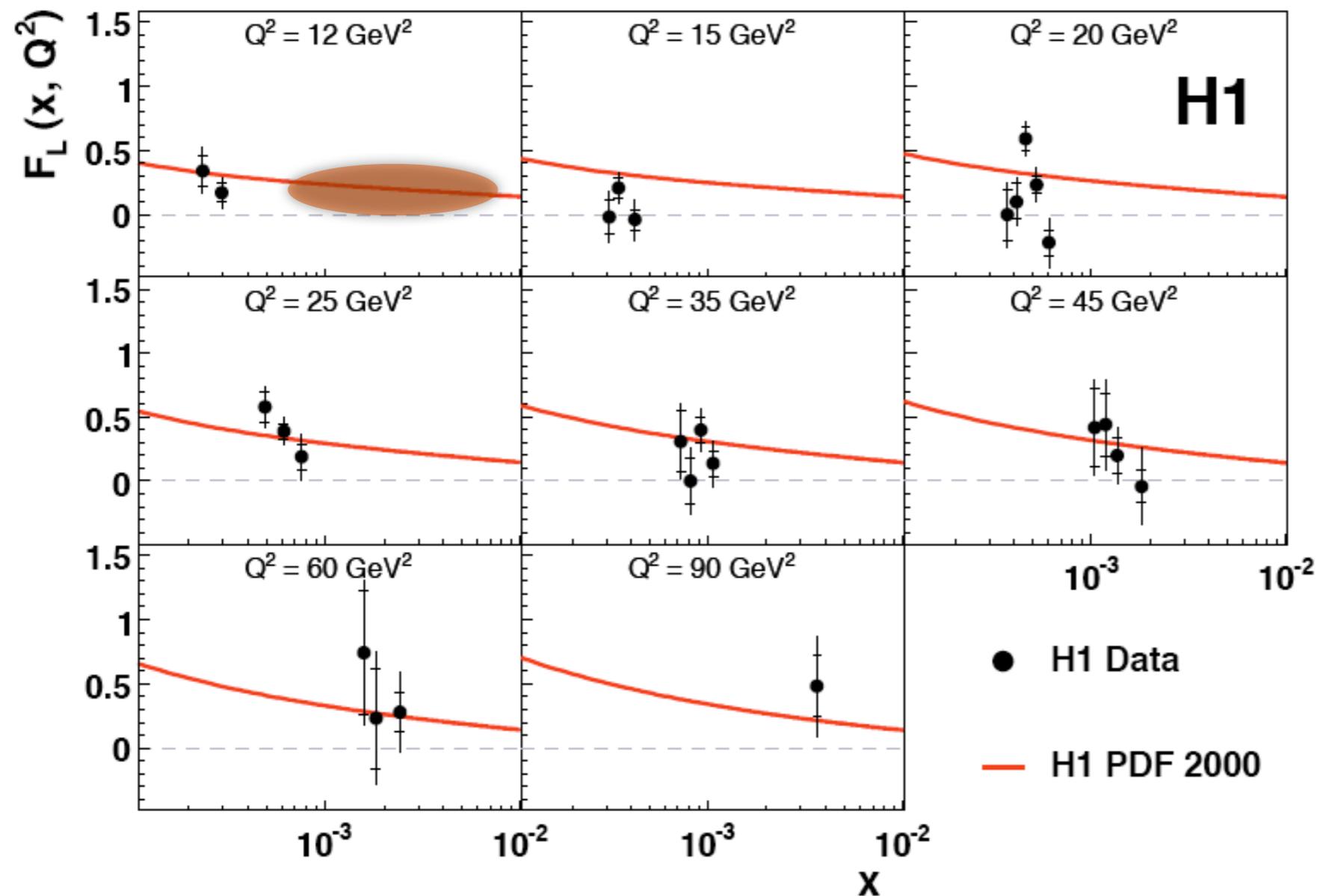


$$\theta_{\min} = 25^\circ$$

$$\theta_{\max} = 90^\circ$$



# FL measurement at HERA



STAR could reach values of  $Q^2$  as low as  $3 \text{ GeV}^2$  and provide coverage for:  $10^{-3} < x < 10^{-2}$ . It can add coverage between  $10^{-2}$  and  $10^{-1}$  in  $x$  for  $Q^2$  smaller than  $60 \text{ GeV}^2$ .

