1 Statistical tests.

1.1 Mann-Whitney Test.

Let us suppose that our data can be separated into two samples. The only assumptions of the Mann-Whitney test are

- The two samples are randomly and independently drawn.
- The dependent variable is intrinsically continuous.
- Within the two samples it has to be meaningful to speak of "greater than," "less than," and "equal to."

We can achieve this kind of data by considering the differences $x_i = \frac{A_i}{D_i} - < \frac{A}{D} >$ in time, and by ordering them in increasing values. The positive values of x_i will belong to group A and the negative ones will belong to group B. We will then get a sequence of A's and B's of the kind AABABBBBAAABABABAAA, with N_A and N_B being the total number of A's and B's, and $N = N_A + N_B$. The next step is to assign *ranks*: a number is assigned to each measure from 1 to N, so that they are ordered from 1 to N. In the case in which the values of x_i, $...x_{i+j}$ are tied, their assigned rank is $i + j/2$. Let us now introduce the sums of the ranks of the elements in group A and B: S_A and S_B. Let S_{AB} be the total sum of the rankings, and μ_{AB} its average. Of course

$$S_{AB} = \sum_{i=1}^{N} i = \frac{N(N+1)}{2} \qquad \mu_{AB} = \frac{S_{AB}}{N} = \frac{N+1}{2} \tag{1.1}$$

The *null hypotesis* is that the quantities A and B are randomly distributed within the total group. If this were true, then the rankings would be evenly distributed within samples A and B, and their average would approximate the total average, i.e. the sums would approximate the values:

$$\mu_A = \frac{N_A(N+1)}{2} \qquad \mathcal{S}_B = \frac{N_B(N+1)}{2} \tag{1.2}$$

In situations where N ranks are distributed within two samples of sizes N_A and N_B the sampling distributions of S_A and S_B tend to approximate a normal distribution, providing that N_A and N_B are greater than 5. It can be shown that the distributions S_A and S_B have the same variance and standard deviation. The standard deviation is equal to:

$$\sigma_S = \sqrt{\frac{N_A N_B (N+1)}{12}} \tag{1.3}$$

Given the observed values of S_A and S_B one can refer them to the unit normal distribution (the normal distribution with $\sigma = 1$), and to the corresponding z-value, i.e. the number of standard deviations in the unit normal distribution. The expression for z is

$$z_{A/B} = \frac{(S_{A/B} - \mu_{A/B}) + \Delta}{\sigma_S} \tag{1.4}$$

where $S_{A/B}$ is the observed value of either S_A or S_B. The quantity Δ is a *correction for continuity* and it is equal to:

$$\begin{aligned} \Delta &= -0.5 \qquad & S_{A/B} > \mathcal{S}_{A/B} \\ \Delta &= +0.5 \qquad & S_{A/B} < \mathcal{S}_{A/B} \end{aligned} \tag{1.5}$$

P(direct.)	P(non direct.)	z critical
0.05	-	1.645
0.025	0.05	1.960
0.01	0.02	2.326
0.005	0.01	2.576
0.0005	0.001	3.291

Table 1: Critical values of z for directional and non-directional tests.

It is straightforward to prove that z_A and z_B have the same absolute value, but opposite sign, so that in the following we can just consider one of them. Table 1.1 shows the critical values of z for a directional (in which the sign of z is important) and non-directional test. Fig.1 shows the result of the Mann-Whitney test to the time dependence of A/D, for each bin. The z-values seem quite well contained within the values ± 1.654, with only a few ecceptions in the Bottom detector. This indicates that the values are randomly distributed with good approximation.

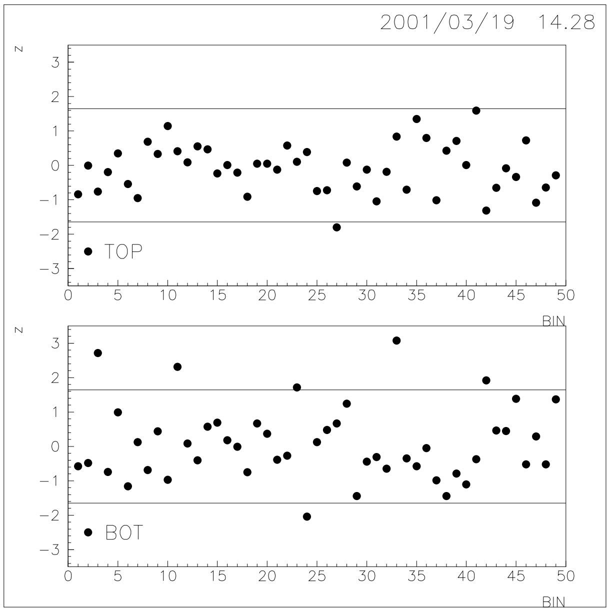

Fig. 1: The Mann-Whitney test applied to the time dependence of A/D. The area between the two orizontal lines at ± 1.645 indicates the region outside which the fluctuations cannot be considered randomly distributed.

1.2 Wilcoxon Test.

The Wilcoxon test, similarly to the Mann-Whitney test, relies on the assumption that the data is divided in two groups. Again, we can achieve this by considering the differences $x_i = \frac{A_i}{D_i} - < \frac{A}{D} >$. If $x_i < 0$, then the corresponding data point is labelled with A, otherwise it's a B. Data points with $x_i = 0$ are discarded. The values are ordered in the order they were collected. In this test one looks if heigher or lower values (with respect to the average) were randomly distributed in time. Table 1.2 is useful to explain how the test works.

Rank	1	2	3	-4	-5	-6	7	8	9	10	-11	-12	13	14	15	-16	-17	-18	-19
Data	A	A	A	B	B	B	A	A	A	A	B	B	A	A	A	B	B	B	B
Run	1	1	1	2	2	2	3	3	3	3	4	4	5	5	5	6	6	6	6

Table 2: Example of Ranks and Runs in the Wilcoxon Test.

To each data point is associated to a *rank* number, that is just the number that identifies the data point in the actual ordering. The rank for the A data points has opposite sign than the rank for the B data points. The *run* number instead identifies the cluster of A's and B's. If N is the total number of data points, then the sum of *unsigned* ranks is

$$S = \frac{N(N+1)}{2} \tag{1.6}$$

This is also the maximum possible (absolute) value of the sum of A or B ranks. The null hypotesis (in which the A's and B's are randomly distributed) is that the sum of A ranks and the sum of B ranks are roughly equal, so that the signed rank sum W should approximate zero within the limits of random variability. Indeed W belongs to a normal distribution with mean μ_W equal to zero, and standard deviation equal to:

$$\sigma_W = \sqrt{\frac{N(N+1)(2N+1)}{6}} \tag{1.7}$$

As in the Mann-Whitney test, we can create a z-value:

$$z = \frac{W - \mu_W \pm 0.5}{\sigma_W} = \frac{W \pm 0.5}{\sigma_W} \tag{1.8}$$

where ± 0.5 is the continuity correction already defined in the last paragraph. Fig.2 shows the results of the application of the Wilcoxon test to the time dependence of A/D. The small values of z indicate that no trend is present.

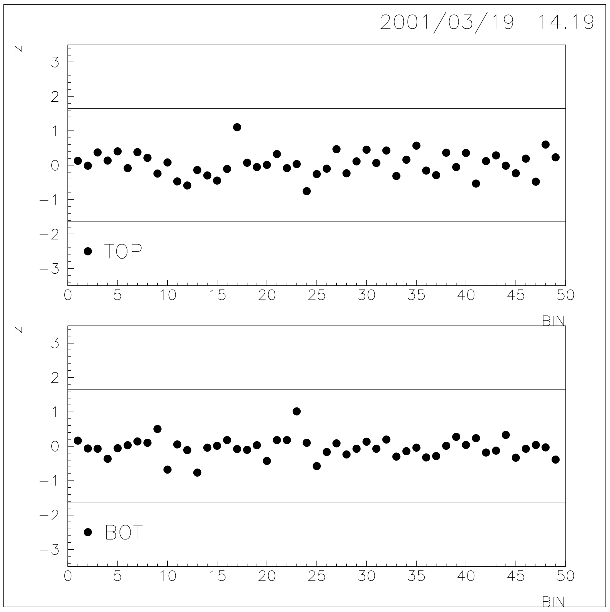

Fig. 2: The Wilcoxon test applied to the time dependence of A/D. The area between the two orizontal lines at ± 1.645 indicates the region outside which the fluctuations cannot be considered randomly distributed. The test shows no time dependence of the data.