Relativistic hydrodynamics for a lattice QCD Equation of State

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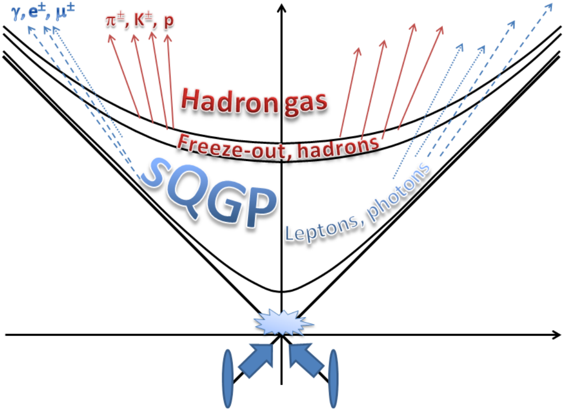
Outline

Time evolution of the sQGP

- sQGP: hot, expanding, strongly interacting, perfect quark fluid
- Hadrons created at the freeze-out
- Leptons, photons “shine through”



Hydrodynamics

- Hydrodynamics: applicable from 1 fm to the size of the Universe
- Relevant in high energy A+A and p+p as well
- Equations of hydro: highly non-linear, not straightforward to solve
- *Exact, analytic solutions*: important to determine initial and final state
- Non-relativistic hydro: many solutions, applicable, but inconsistent!
- Relativistic ideal hydro: formulated by Landau
  - Famous 1+1D solutions: Landau, Izv. Acad. Nauk SSSR **17**, 51 (1953)
    Hwa, Phys. Rev. D **10**, 2260 (1974)
    Bjorken, Phys. Rev. D **27**, 40 (1983)
  - Revival of interest, many new solutions, mostly 1+1D, few 1+3D (mostly by people in the audience here)
  - Physically relevant exact solutions still needed!
  - All (up to now) using constant Equation of State (speed of sound)!
- *Time evolution of sQGP? Temperature dependent EoS?*

Equations of perfect hydrodynamics

- Energy-momentum conservation:

$$\partial_\nu T^{\mu\nu} = 0 \text{ with } T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} \tag{1}$$

- In case of a *conserved charge n* (chem. pot. $\neq 0$):

$$\partial_\mu(nu^\mu) = 0 \tag{2}$$

- If no n: *entropy conservation* (via $\epsilon = p + T\sigma$ and $d\varepsilon = Td\sigma$):

$$\partial_\nu(\sigma u^\nu) = 0 \tag{3}$$

- Equation of State: $\epsilon = \kappa p$ ($\kappa =$ const. $\Rightarrow c_s^2 = \partial p/\partial\epsilon = 1/\kappa$)
- This is a full set of equations for $\{u^\mu, n, p\}$ or $\{u^\mu, \sigma, p\}$
- Temperature $p = nT$ (if $\exists$ n), or $p = T\sigma/(\kappa + 1)$ (if no n)
- The same (!) resulting temperature equation, *if* $\kappa =$ *const.*:

$$T\partial_\mu u^\mu + \kappa u^\mu \partial_\mu T = 0 \tag{4}$$

- All $\kappa =$ const. solutions valid for *both* $\{u^\mu, n, T\}$ *and* $\{u^\mu, \sigma, T\}$.

Outline

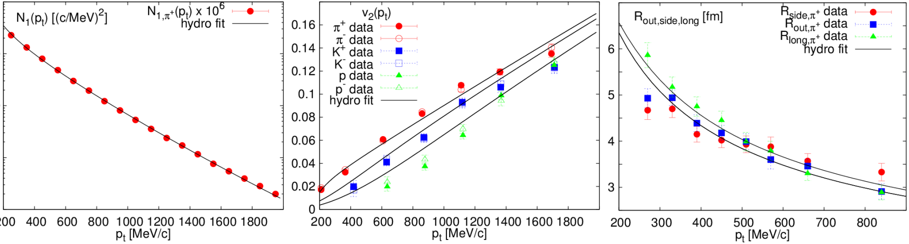
Hadronic observables compared to data

- Take first exact, analytic and truly 3D relativistic solution

  Csörgő, Csernai, Hama *et al.*, Heavy Ion Phys. **A21**, 73 (2004), nucl-th/0306004
- Calculate observables for identified particles
  - Transverse momentum distribution $N_1(p_t)$
  - Azimuthal anisotropy $v_2(p_t)$
  - Bose-Einstein correlation (HBT) radii $R_{out,side.long}(p_t)$
- Compared to data successfully (RHIC shown, LHC done as well)

  Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

  Data: PHENIX Coll., PRC**69**034909(2004), PRL**91**182301(2003), PRL**93**152302(2004)

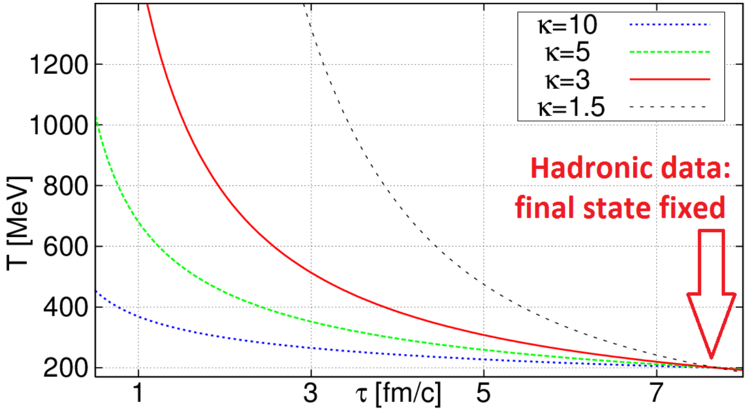


Time evolution of the sQGP matter?

- Hadronic observables fix *only the final state*!
- Different EoS lead to different initial states!

Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

Csanád, Nagy, Csörgő, Eur. Phys. J. **ST**, 19 (2008), arXiv:0710.0327

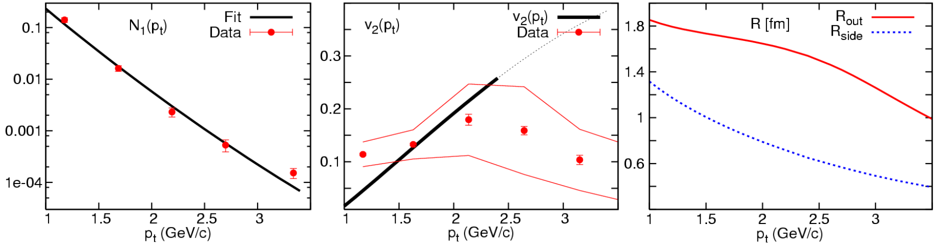


Direct photon observables compared to data

- Photons are created throughout the evolution
- *Their distribution reveals information about the EoS!*
- Compared to PHENIX data (spectra and flow) successfully
- Predicted photon HBT radii: $R_{\rm out} > R_{\rm side}$

Csanád, Májer, Central Eur. J. Phys. **10** (2012), arXiv:1101.1279

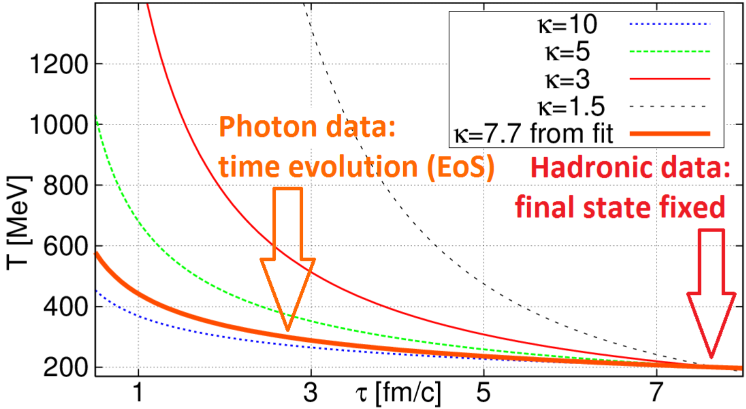
Data: PHENIX Collaboration, arXiv:0804.4168 and arxiv:1105.4126



- Result for the average EoS: $c_s = 0.36 \pm 0.02_{stat} \pm 0.04_{syst}$
- Time interval: $\tau_{\rm ini} \approx 0.7$ fm$/c$, $\tau_{\rm final} = 7.7$ fm$/c$ (from hadron fits)

Time evolution result with κ =const.

- The $c_s = 0.36$ result means $\kappa = 7.7$
- Average EoS, compare Lacey et al., nucl-ex/0610029
- Corresponds to to T_{ini} *of* $507 \pm 12_{stat} \pm 90_{syst}$ *MeV at* 0.7 *fm/c*



Csanád, Vargyas, Eur. Phys. J. A **44**, 473 (2010), arXiv:0909.4842

Outline

Solutions with arbitrary EoS?

- Constant EoS *may not be realistic* (temperature may change rapidly)
- If $\kappa(T)$, *new solutions* have to be found
- With a conserved charge (and $\epsilon = \kappa n T$), the temperature equation is:

$$T\partial_\mu u^\mu + \left[\kappa + T\frac{d\kappa}{dT}\right] u^\mu \partial_\mu T = 0. \tag{5}$$

- Works only if $d(\kappa T)/dT \neq 0$!
- In case of no conserved charges (and $\epsilon = \kappa T\sigma/(\kappa+1)$):

$$T\partial_\mu u^\mu + \left[\kappa + \frac{T}{\kappa+1}\frac{d\kappa}{dT}\right] u^\mu \partial_\mu T = 0, \tag{6}$$

- Remarkable: these are not the same, but coincide if $\kappa = \text{const.}$!
- If u^μ known, these *can be solved for an arbitrary $\kappa(T)$*!

A new $\kappa(T)$ type of solution without conserved charge

- A new solution for arbitrary $\kappa(T)$:
  Csanád, Nagy, Lökös, Eur. Phys. J. A, arXiv:1205.5965

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}, \tag{7}$$

$$u^\mu = \frac{x^\mu}{\tau}, \tag{8}$$

$$\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^{T} \left(\frac{\kappa(\beta)}{\beta} + \frac{1}{\kappa(\beta)+1} \frac{d\kappa(\beta)}{d\beta} \right) d\beta \tag{9}$$

β is the integration variable here, i.e. T

- Arbitrary $\kappa(T)$ functions may be used, the QCD one as well

A new $\kappa(T)$ type of solution with conserved charge

- A new solution with conserved charge n, for arbitrary $\kappa(T)$:
  Csanád, Nagy, Lökös, Eur. Phys. J. A, arXiv:1205.5965

$$n = n_0 \frac{\tau_0^3}{\tau^3}, \tag{10}$$

$$u^\mu = \frac{x^\mu}{\tau}, \tag{11}$$

$$\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^{T} \left(\frac{1}{\beta} \frac{d\kappa(\beta)\beta}{d\beta} \right) d\beta \tag{12}$$

- Arbitrary $\kappa(T)$ functions may be used
- For some choices of the $\kappa(T)$ function this solution becomes ill-defined
- If $d(\kappa T)/dT \leq 0$, the last equation cannot be inverted
- Same problem as mentioned earlier

A new $\kappa(p)$ type of solution without conserved charge

- A partly implicit new solution:
  Csanád, Nagy, Lökös, Eur. Phys. J. A, arXiv:1205.5965

$$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}, \tag{13}$$

$$u^\mu = \frac{x^\mu}{\tau}, \tag{14}$$

$$\frac{\tau_0^3}{\tau^3} = \int_{p_0}^{p} \left(\frac{\kappa(\beta)}{\beta} + \frac{d\kappa(\beta)}{d\beta} \right) \frac{d\beta}{\kappa(\beta)+1} \tag{15}$$

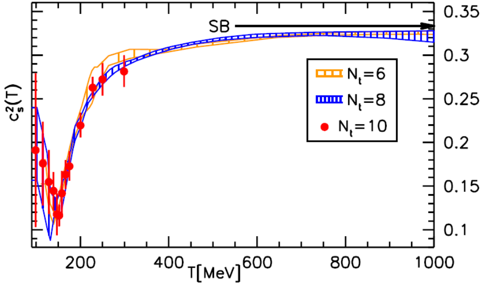
β is the integration variable here, i.e. p

- This solution may be used if κ is given as a function of pressure p.

Outline

Lattice QCD EoS

- lQCD EoS calculated for physical quark masses, continuum limit
  Borsányi, Fodor, Katz *et al.* JHEP **1011**, 077 (2010), arXiv:1007.2580

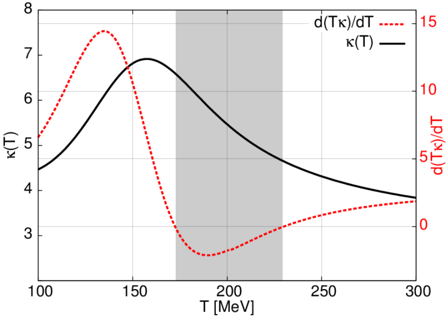


- Trace anomaly $I(T)/T^4$ is parametrized analytically in the cont. limit
- Pressure is given by $\frac{p(T)}{T^4} = \int \frac{dT}{T} \frac{I(T)}{T^4}$
- From $I = \epsilon - 3p \;\Rightarrow\; \kappa = I/p + 3$

Lattice QCD EoS

- Take EoS from lQCD: trace anomaly $I(T)/T^4$ parametrized
- *EoS in form of $\kappa(T)$ analytically given*

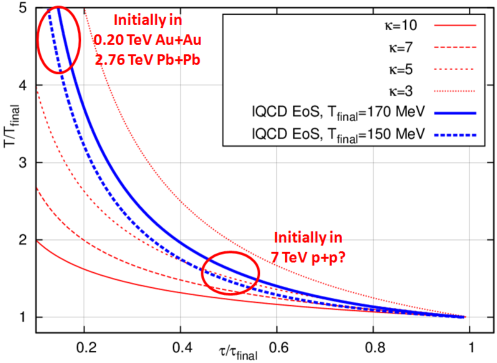
  Csanád, Nagy, Lökös, Eur. Phys. J. A, arXiv:1205.5965



- Problem: $d(\kappa T)/dT \leq 0$ at $T = 173 - 225$ MeV
- Recall slide no. 12: *$p = nT$ not compatible with this EoS*!

Results for the lattice QCD EoS

- $T(\tau)$ from QCD EoS + hydro (see arXiv:1205.5965)



- RHIC 200 GeV Au+Au: $\tau_{\mathrm{ini}}/\tau_{\mathrm{final}} \approx 0.1 - 0.2$
- T_{ini} higher then predicted from constant EoS!
- LHC 2.76 TeV Pb+Pb: thermalize fast, longer evolution: higher T_{ini}
- LHC 7 TeV p+p: $\tau_{\mathrm{ini}}/\tau_{\mathrm{final}} \approx 0.5 \Rightarrow$ compatible with $T_{\mathrm{ini}} \approx 250$ MeV?

Outline

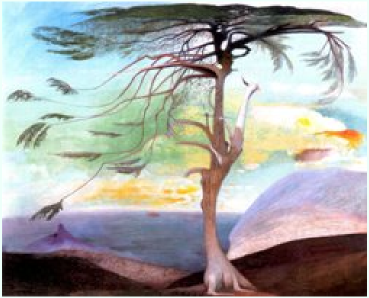
Summary

- Constant EoS solutions work on RHIC & LHC data
- Large temperature change: variable κ more realistic
- *Found first exact solutions* with arbitrary $\kappa(T)$ or $\kappa(p)$
- *Lattice QCD EoS applicable* when no conserved charges
- *Calculated* $T(\tau)$
- Only assuming a $\tau_{\text{final}}/\tau_{\text{ini}}$, one gets $T_{\text{final}}/T_{\text{ini}}$
- High temperatures not inconsistent with LHC p+p?

Thank you for your attention!

And let me invite you to the 12th Zimanyi School

ZIMÁNYI SCHOOL'12



T. Kosztka Csontváry: The Lonely Cedar

12. Zimányi

WINTER SCHOOL ON HEAVY ION PHYSICS

Dec. 3. - Dec. 7., Budapest, Hungary

József Zimányi (1931 - 2006)

http://zimanyischool.kfki.hu/12/

The sQGP is created at RHIC & LHC

- *Jet suppression*: new phenomenon of missing high energy jets
  PHENIX Coll., Phys.Rev.Lett. 88.022301 (2002)
- No jet suppression in d+Au: *new form of matter* in central Au+Au
  PHENIX Coll., Phys. Rev. Lett. 91, 072303 (2003)
- Collective dynamics: it is a *liquid*!
  PHENIX Coll., Nucl. Phys. A 757, 184-283 (2005)
- Scaling properties: appearance of *quark degrees of freedom*
  PHENIX Coll., Phys. Rev. Lett. 98, 162301 (2007)
- Energy loss of heavy quarks: nearly *perfect liquid*
  PHENIX Coll., Phys. Rev. Lett. 98, 172301 (2007)
- *Thermal photons*: very high initial temperature
  PHENIX Coll., Phys. Rev. Lett. 104, 132301 (2010)
- *Anisotropic flow*: extremely small kinematic viscosity
  ALICE Coll., Phys.Rev.Lett. 107, 032301 (2011),
  CMS Coll., Eur. Phys. J. C 72, 2012 (2012),
  ATLAS Coll., arXiv:1203.3087 (PRC)

Equations of hydrodynamics with a conserved charge

- There may be a *conserved charge or number n* (chem. pot. $\neq 0$)
- Basic eqs: continuity and energy-mom. conservation

$$\partial_\mu(nu^\mu) = 0 \text{ and } \partial_\nu T^{\mu\nu} = 0 \tag{16}$$

- Energy-momentum tensor in perfect fluid: $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$
- Equation of State (if $\kappa =$ const., $c_s^2 = \partial p/\partial\epsilon = 1/\kappa$): $\epsilon = \kappa p$
- This is *a full set of equations for u^μ, n and p*
- Introduce temperature with $p = nT$, temperature eq. from here:

$$T\partial_\mu u^\mu + \kappa u^\mu \partial_\mu T = 0 \tag{17}$$

- Write up *solutions for u^μ, n and T* instead!

Equations of hydrodynamics without conserved charges

- *Energy-momentum conservation is the same*
- Let us introduce *entropy density* σ
- Fundamental thermodynamical relations without a conserved charge

$$\varepsilon + p = T\sigma \;\Rightarrow\; d\varepsilon = Td\sigma \text{ and } dp = \sigma dT \tag{18}$$

- The *same continuity equation for* σ follows from here:

$$\partial_\nu(\sigma u^\nu) = 0, \tag{19}$$

- EoS can be used the same manner here, but *different* $p - T$ *relation*!

$$\epsilon = \kappa p \text{ and } p = T\sigma/(\kappa + 1) \tag{20}$$

- If $\kappa =$ const., we get *the same equation on the temperature as with* n:

$$T\partial_\mu u^\mu + \kappa u^\mu \partial_\mu T = 0. \tag{21}$$

- All $\kappa =$ const. solutions valid for *both* $\{u^\mu, n, T\}$ *and* $\{u^\mu, \sigma, T\}$.

A known ellipsoidal solution with constant κ

- First exact, analytic and truly 3D relativistic solution
  Csörgő, Csernai, Hama *et al.*, Heavy Ion Phys. **A21**, 73 (2004), nucl-th/0306004

$$u^\mu = \frac{x^\mu}{\tau}, \quad \text{with } \tau = \sqrt{x_\mu x^\mu}, \tag{22}$$

$$n = n_0 \frac{V_0}{V} \nu(s), \quad T = T_0 \left(\frac{V_0}{V}\right)^{1/\kappa} \nu(s)^{-1}, \tag{23}$$

- $\nu(s)$ is an arbitrary function and

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}, \quad V = \tau^3, \tag{24}$$

- X, Y, Z: principal axes of expanding ellipsoid, $X(t) = \dot{X}_0 t$ etc.
- This solution is non-accelerating, ie. obeys $u^\nu \partial_\nu u^\mu = 0$.
- Source function is:

$$S(x,p)d^4x = \mathcal{N} \frac{p_\mu \, d^3\Sigma^\mu(x) H(\tau) d\tau}{n(x) \exp\left(p_\mu u^\mu(x) / T(x)\right) - 1} \tag{25}$$

Direct photon observables compared to data

- Photons are created throughout the evolution
- *Their distribution reveals information about the EoS!*
- The source function of photon creation is assumed as:

$$S(x,p)d^4x = \mathcal{N}\frac{p_\mu u^\mu}{\exp\left(p_\mu u^\mu(x)/T(x)\right)-1}\,d^4x \qquad (26)$$

- Integrated over energy (i.e. four-momentum): emission $\propto T^4$
  - Analyzed systematic change with T power, based on

$$\text{rate}(A+B\to X) = n_A n_B \left\langle v\sigma_{A+B\to X}\right\rangle \propto T^6 \qquad (27)$$

- *Transverse mom. distribution, elliptic flow and HBT radii calculable*

Csanád, Májer, Central Eur. J. Phys. **10** (2012), arXiv:1101.1279

The two cases

Case	With n	Without n
Temperature	$\epsilon = \kappa n T$	$\epsilon = \kappa T \sigma / (\kappa + 1)$
Conservation eq.	$\partial_\mu (n u^\mu) = 0$	$\partial_\mu (\sigma u^\mu) = 0$
Temperature eq.	$T \partial_\mu u^\mu + \left[\kappa + T \frac{d\kappa}{dT}\right] u^\mu \partial_\mu T = 0$	$T \partial_\mu u^\mu + \left[\kappa + \frac{T}{\kappa+1} \frac{d\kappa}{dT}\right] u^\mu \partial_\mu T = 0$
Solution	$n = n_0 \frac{\tau_0^3}{\tau^3}$ $u^\mu = \frac{x^\mu}{\tau}$ $\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^{T} \left(\frac{1}{\beta} \frac{d\kappa(\beta)\beta}{d\beta} \right) d\beta$	$\sigma = \sigma_0 \frac{\tau_0^3}{\tau^3}$ $u^\mu = \frac{x^\mu}{\tau}$ $\frac{\tau_0^3}{\tau^3} = \exp \int_{T_0}^{T} \left(\frac{\kappa(\beta)}{\beta} + \frac{1}{\kappa(\beta)+1} \frac{d\kappa(\beta)}{d\beta} \right) d\beta$
Valid for	$\frac{d(T\kappa(T))}{dT} > 0$	$\frac{\kappa(T)}{T} + \frac{1}{\kappa(T)+1} \frac{d\kappa(T)}{dT} > 0$
Problem	Yes	No