Physics 131, Lecture 3
Motion in two or three dimensions

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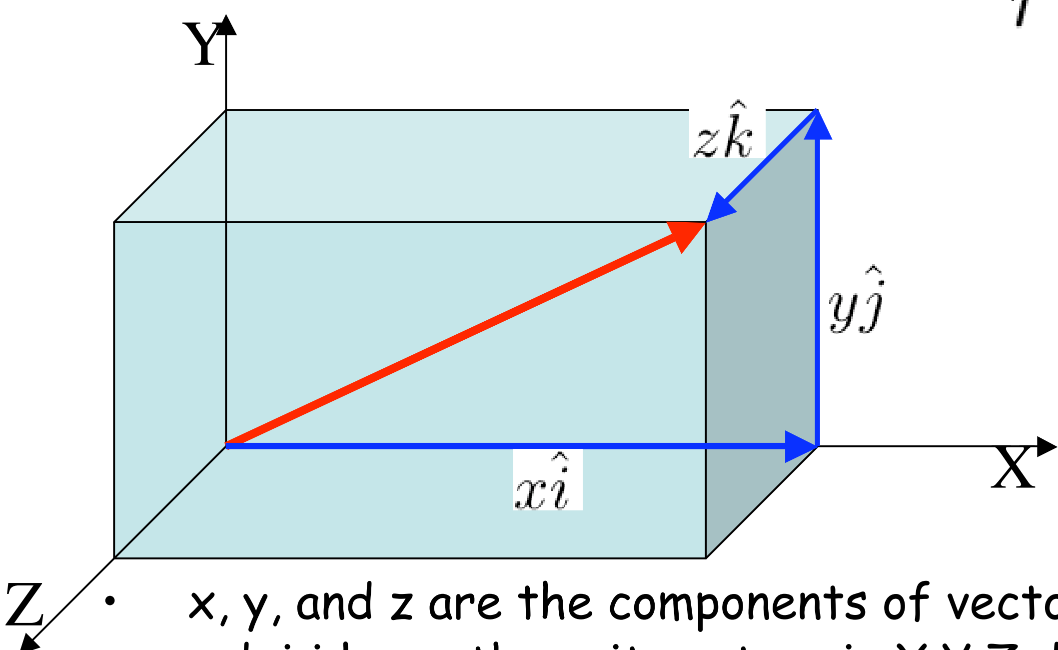
Position Vector

- Last time we saw motion in one dimension. What happens when you add the other two dimensions y and z to these problems.
  - Nothing profound! Only complicates things. But that is reality and so we deal with it in this lecture.
- Position vector r:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



- x, y, and z are the components of vector r in three orthogonal axis and i,j,k are the unit vectors in X,Y,Z directions

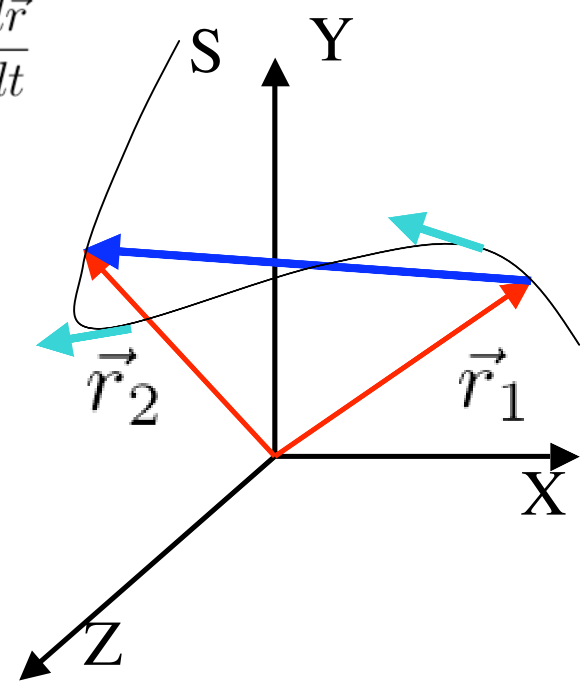
Velocity: Instantaneous & Average

- Rate of change of position is velocity: $\vec{v} = \frac{d\vec{r}}{dt}$
- A particle is at r1 at time t1 and at position r2 at time t2 as it passes along the path S
- The average velocity is:

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

- The limit of average velocity as the time interval between measurements goes to zero:

$$\vec{v} = \lim \Delta t \rightarrow 0 \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



At every point along the path, the instantaneous Velocity is tangential to the path at that point

Vectors in components:

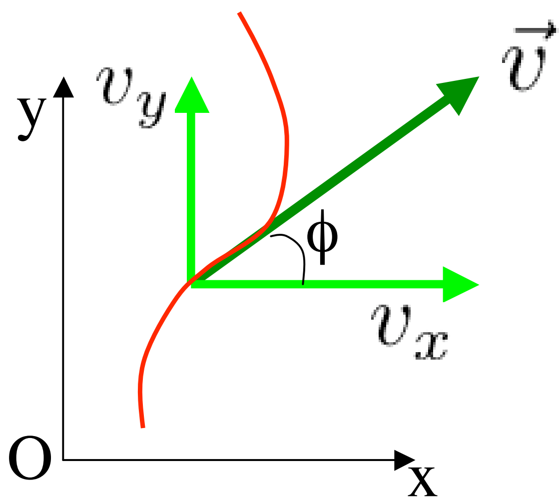
- Components:

$$v_x = \frac{dx}{dt}, \qquad v_y = \frac{dy}{dt}, \qquad v_z = \frac{dz}{dt}$$

- Vector:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

- Magnitude of the vector: $|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2}$



$$v = \sqrt{v_x^2 + v_y^2}$$

$$\tan\phi = \frac{v_y}{v_x}$$

Identical formulae for Acceleration

- Re-write all formulae for velocity: except replace v by a (including for its components)

Robot in Motion

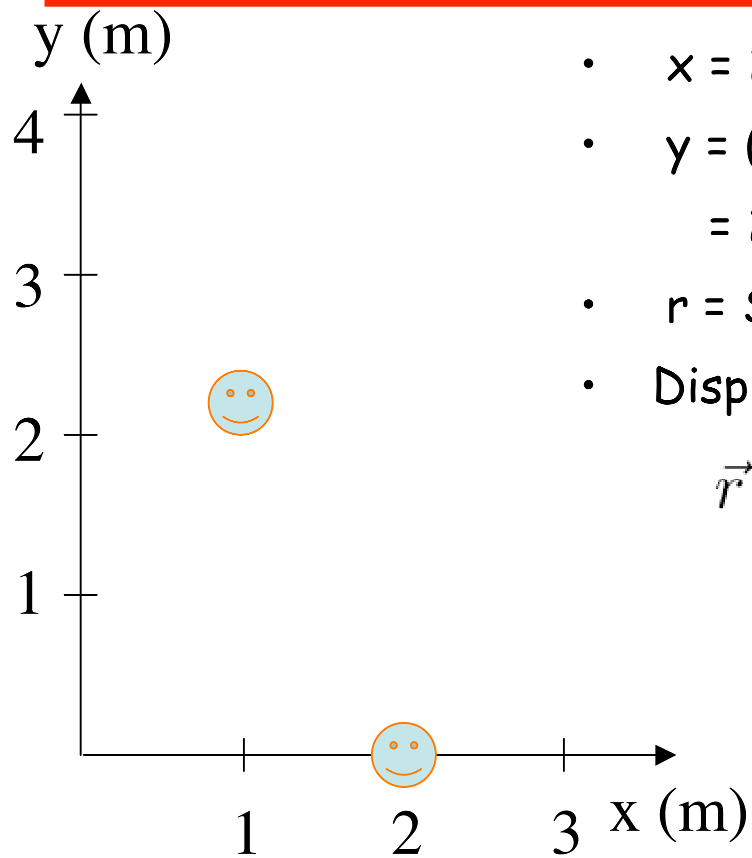
- A robot lands on Mars. Its x and y coordinates are given by equations:

  $x(t) = 2.0\ m - (0.25\ m/s^2)\ t^2$

  $y(t) = (1.0\ m/s)\ t + (0.025\ m/s^3)$

- What is the position and distance from landing at t = 2.0 seconds?
- What is the displacement and average velocity during the interval from t = 0.0 s to t = 2.0 s
- Derive a general expression for its instantaneous velocity vector and find its instantaneous velocity at t = 2.0 s. Express the instantaneous velocity in its component form as well as in terms of its magnitude.
- Realize that the problem is 2 dimensional, its best to start drawing a figure as we go step by step to get an overall view of what is going on. ---NASA scientists in the control mission do similar things to keep track of where a robot might be after it landed!

Robot in Motion: Solution



- $x = 2.0\text{m} - (0.25\ \text{m/s}^2)(2.0\ \text{s})^2 = 1.0\ \text{m}$
- $y = (1.0\ \text{m/s})(2.0\ \text{s}) + (0.025\ \text{m/s}^3)(2.0\ \text{s})^3$
  $= 2.2\ \text{m}$
- $r = \text{Sqrt}(x^2 + y^2) = 2.4\ \text{m}$
- Displacement and Average Velocity:

$$\begin{aligned}\vec{r} &= x\hat{i} + y\hat{j} \\ &= \left[2.0m - (0.25m/s^2)t^2\right]\hat{i} \\ &+ \left[(1.0m/s)t + (0.025m/s^3)t^3\right]\hat{j}\end{aligned}$$

$$\begin{aligned}\vec{r_0} &= (2.0m)\hat{i} + (0.0m)\hat{j} \\ \vec{r_2} &= (1.0m)\hat{i} + (2.2m)\hat{j} \\ \Delta\vec{r} &= \vec{r_2} = \vec{r_0} \\ &= (-1.0m)\hat{i} + (2.2m)\hat{j}\end{aligned}$$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(-1.0m)\hat{i} + (2.2m)\hat{j}}{2.0s - 0.0s}$$

Robot in Motion: Solution II

- Instantaneous velocity components are:

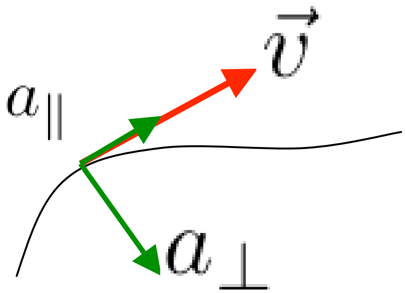
$$v_x = \frac{dx}{dt} = (-0.25m/s^2)(2t)$$
$$v_y = \frac{dy}{dt} = 1.0m/s + (0.025m/s^2)(3t^2)$$

- Instantaneous velocity vector: $\vec{v} = v_x\hat{i} + v_y\hat{j}$ at time t = 2.0 seconds, substitute the values

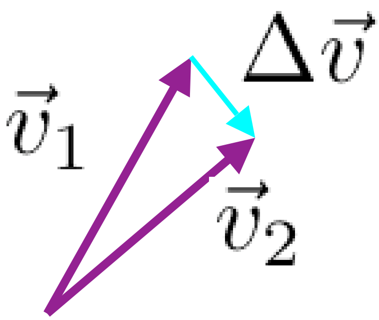
- Magnitude of instantaneous velocity: $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

- Direction is given by angle ϕ : $\tan\phi = \frac{v_y}{v_x}$

Parallel & Perpendicular Acceleration



- A_para is parallel to vector v
- A_perp is perpendicular to vector v
- V(t) = v(0) + a_para * t where Delta v = a_para * t



$$\Delta\vec{v} \rightarrow 0 \quad \text{as} \quad \Delta t \rightarrow 0$$

- If Delta-t is infinitesimally small, $\vec{v}_1 = \vec{v}_2$
- Magnitudes of vectors v1,v2 same but angle chances.

- When acceleration vector acts on a velocity vector perpendicular to it, its magnitude remains the same, but its direction changes.

Acceleration (II)

- When a is parallel to v effect is change in velocity
- When a is perpendicular to v effect is change of direction

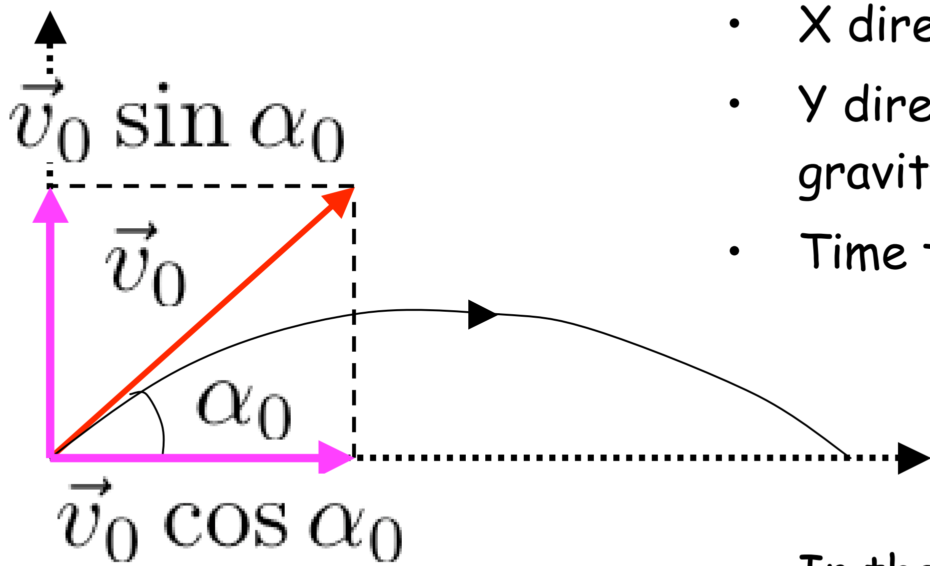
- To think about: Are the following two quantities different?

$$\frac{d|\vec{v}|}{dt} \quad \text{and} \quad \left|\frac{d\vec{v}}{dt}\right|$$

Projectile Motion

- When a body is given an initial velocity and if later it moves purely under the influence of gravity, then the motion is called projectile motion
- In this class we will ignore two aspects of "real life" projectile motions (for simplicity):
  - Air resistance
  - Curvature of earth
- Key to solving projectile motion problem is to separate its x and y components and treat them separately
  - Horizontal motion is with constant velocity
  - Vertical motion is is with constant gravitational acceleration

Projectile Motion (II)



- X direction constant velocity
- Y direction constant acceleration due to gravity (-g)
- Time taken to reach maximum height?

$$y = y_0 + v_0 \sin \alpha_0 t_1 - \frac{1}{2} g t_1^2$$

- In the same time, what distance will be traversed in x direction?

$$x = x_0 + v_x t_1 = x_0 + v_0 \cos \alpha_0 t_1$$

- Substitute for t and you get: (parabola)

$$\begin{aligned} y &= v_0 \tan \alpha_0 \cdot x - \frac{g}{2 v_0^2 \cos^2 \alpha_0} x^2 \\ &= ax - bx^2 \end{aligned}$$

What will air resistance do to this trajectory?