Relativistic Boltzmann equation

$$
\begin{aligned}
p^\mu \partial_\mu f_a(\vec{x},\vec{p},t) \quad = \quad & \sum_m \sum_{b_1,b_2,\cdots,b_m} \int \prod_{i=1}^{m} \frac{d^3 p_{b_i}}{(2\pi)^3 2E_{b_i}} f_{b_i}(\vec{x},\vec{p_{b_i}},t) \\
& \sum_n \sum_{c_1,c_2,\cdots,c_n} \int \prod_{j=1}^{n} \frac{d^3 p_{c_j}}{(2\pi)^3 2E_{c_j}} |M_{m\to n}|^2 \\
& (2\pi)^4 \delta^4 \left(\sum_{k=1}^{m} p_{b_k} - \sum_{l=1}^{n} p_{c_l} \right) \\
& \left[-\sum_{q=1}^{m} \delta_{ab_q} \delta^3(\vec{p}-\vec{p_{b_q}}) + \sum_{r=1}^{n} \delta_{ac_r} \delta^3(\vec{p}-\vec{p_{c_r}}) \right]. \\
\sim \quad & C(2\leftrightarrow 2) + C(2\leftrightarrow 1) + C(1\to 1,2,3,\cdots,n).
\end{aligned}
$$

- $a = p, n, \pi, K, \rho, \Delta, \Delta^*, N^*, Y, Y^*, \ldots.$

All particles from Particle Data Book.

Inputs

- hadronic cross sections
- space time picture (formation time..)

Hadronic Cross Sections in JAM

$$\begin{aligned}\sigma_{tot}(s) &= \sigma_{el}(s) + \sigma_{ch}(s) + \sigma_{ann}(s) \\ &+ \sigma_{t-R}(s) + \sigma_{s-R}(s) \quad : \text{Resonance} \\ &+ \sigma_{t-S}(s) + \sigma_{s-S}(s) \quad : \text{String}\end{aligned}$$

- Resonance productions (absorption)

$$\begin{aligned}\sigma_{t-R}(s) &: \quad NN \leftrightarrow N\Delta, \quad NN \leftrightarrow N^*\Delta^*, \cdots \\ \sigma_{s-R}(s) &: \quad \pi N \leftrightarrow \Delta, \quad \bar{K}N \leftrightarrow Y^*, \cdots\end{aligned}$$

- String formation.

$$\begin{aligned}\sigma_{t-S}(s) &: \quad NN \leftrightarrow \text{String} + \text{String}, \\ \sigma_{s-S}(s) &: \quad \pi N \leftrightarrow \text{String}\end{aligned}$$

- Eikonal formalism for pQCD:HIJING

$$\sigma_{t-S}(s) = 2\pi \int_0^\infty db^2 \left[1 - \exp(\chi(b,s))\right],$$

$$\chi(b,s) = \frac{1}{2}\left(\sigma_{jet}(s) + \sigma_{soft}(s)\right) A(b,s).$$

Simulation procedure

1 Initial state: The nucleons are sampled with Fermi distribution, Fermi momentum $p_f = \hbar(3/2\pi^2\rho(r))^{1/3}$, and boost

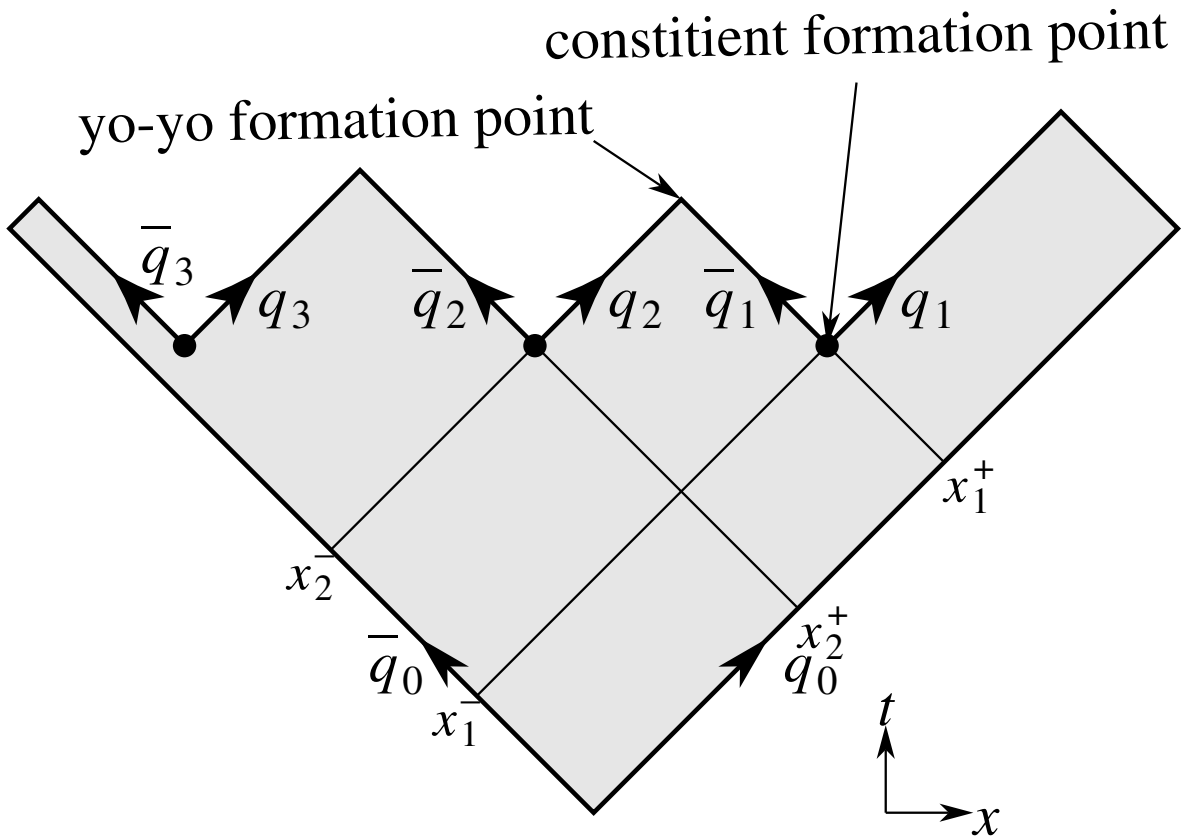
2 Straight line trajectories until they ineract. $\vec{x}_i = x\vec{(0)}_i + \frac{\vec{p}_i}{\vec{E}_i}t$.

3 Collision at closest approach: $d < \sqrt{\sigma_{tot}/\pi}$
collision or resonance decay

4 Outgoing channel selection:
Prob. of elastic$=\frac{\sigma_{el}}{\sigma tot}$,

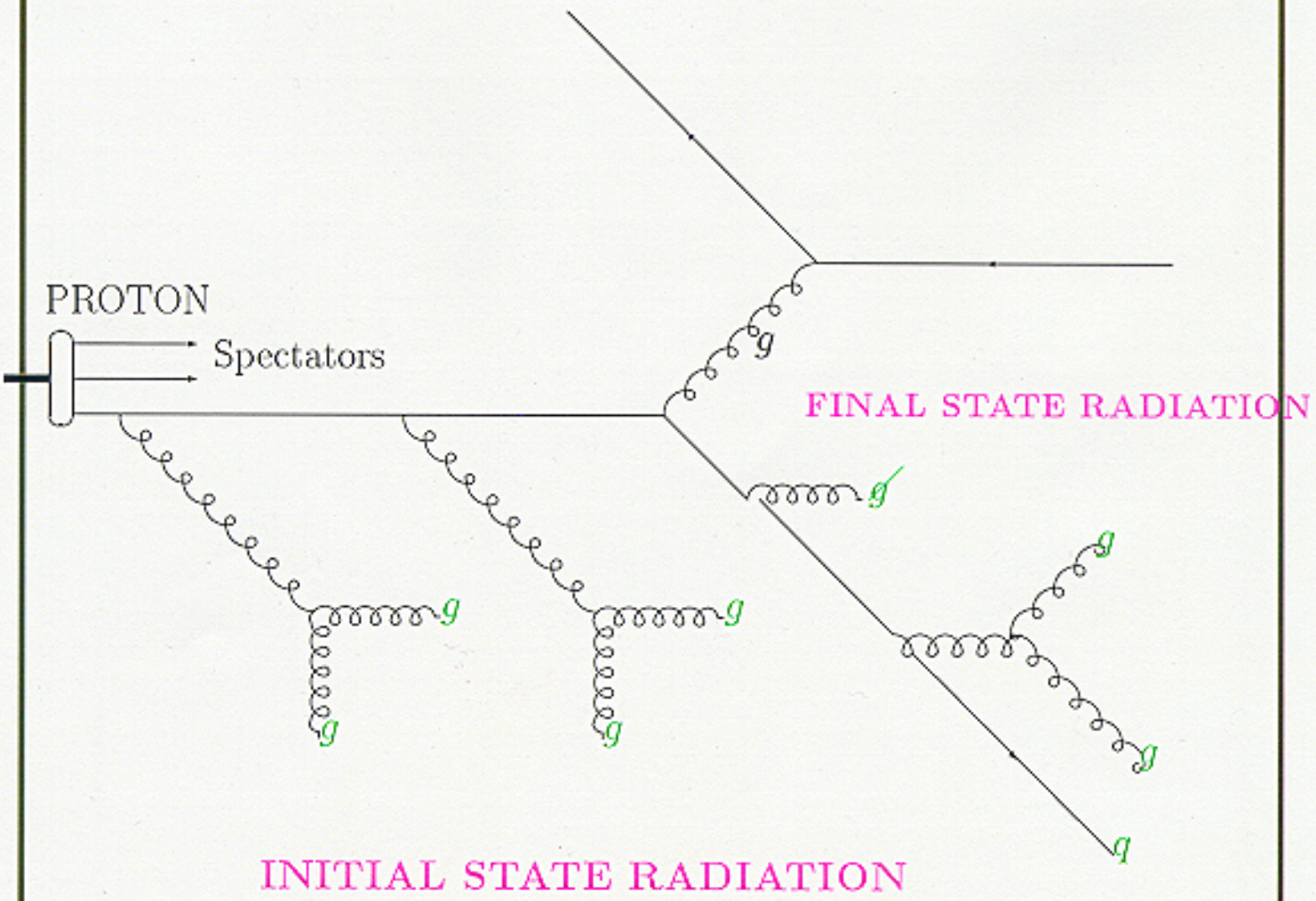
- soft or hard?
- If soft: resonance production or string formation.
  string formation (DPM type excitation law),
  string fragmentation (Lund string model).
- If hard: Pythia

Hadron Formation Point



Space-time picture of the motion in a simple $q\bar{q}$ system and how two breakups of the string result in the production of a hadron.
Ref: A. Bialas and M. Gyulassy, N.P. B291 (1986) 793

Hard scattering and the associated branching trees

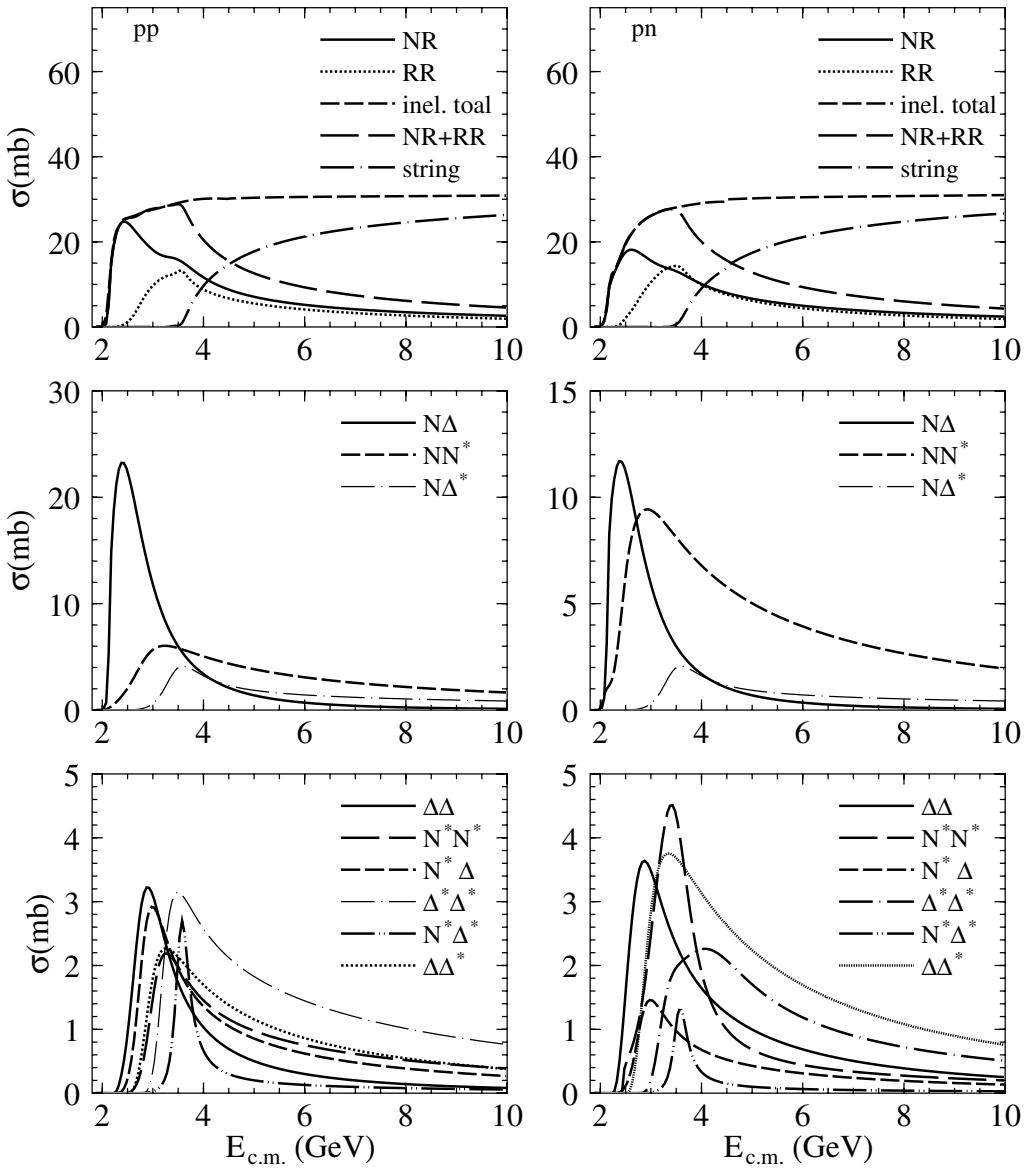


g: on shell (produced partons)

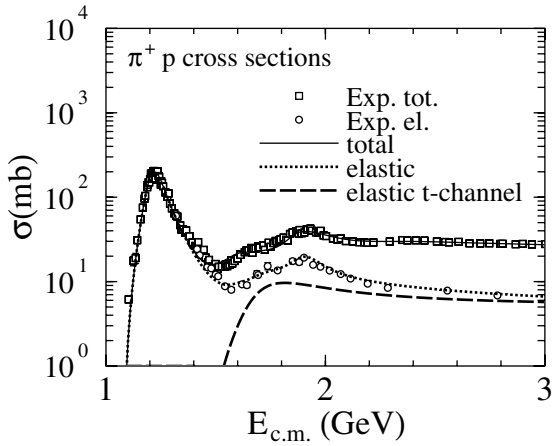
off shell parton with virtuality $Q \Longrightarrow \Delta t \sim E/Q^2$

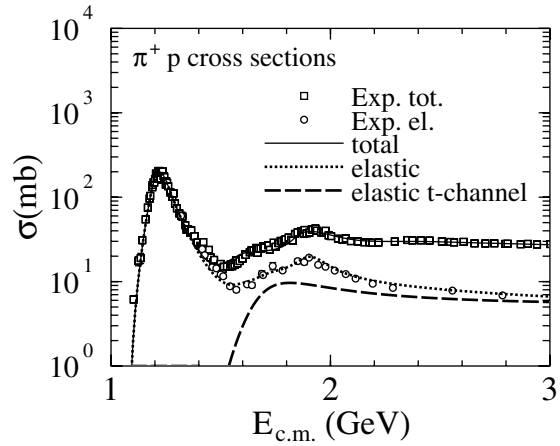
K. J. Eskola and X. N. Wan, P. R. D49(1994)1284

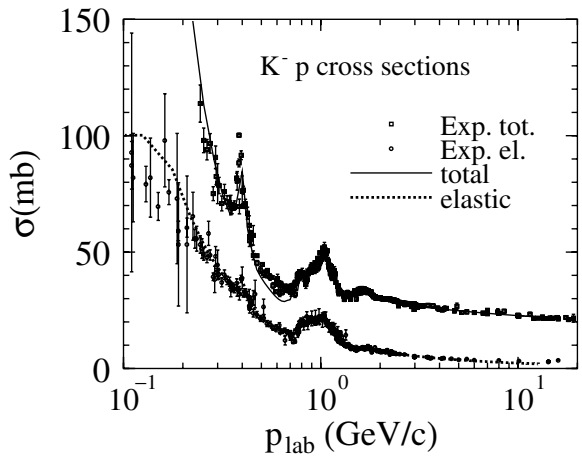
JAM: resonance production cross sections

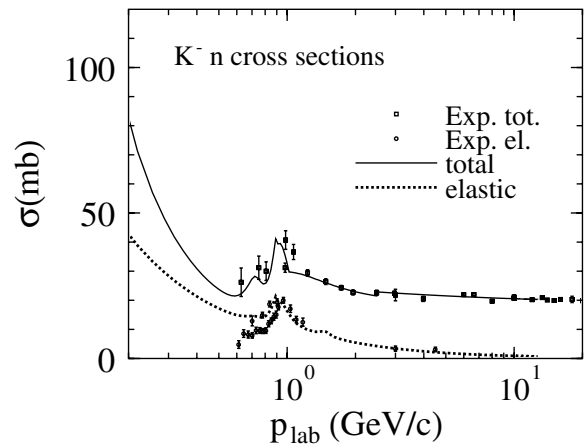


JAM: Total and Elastic cross sections

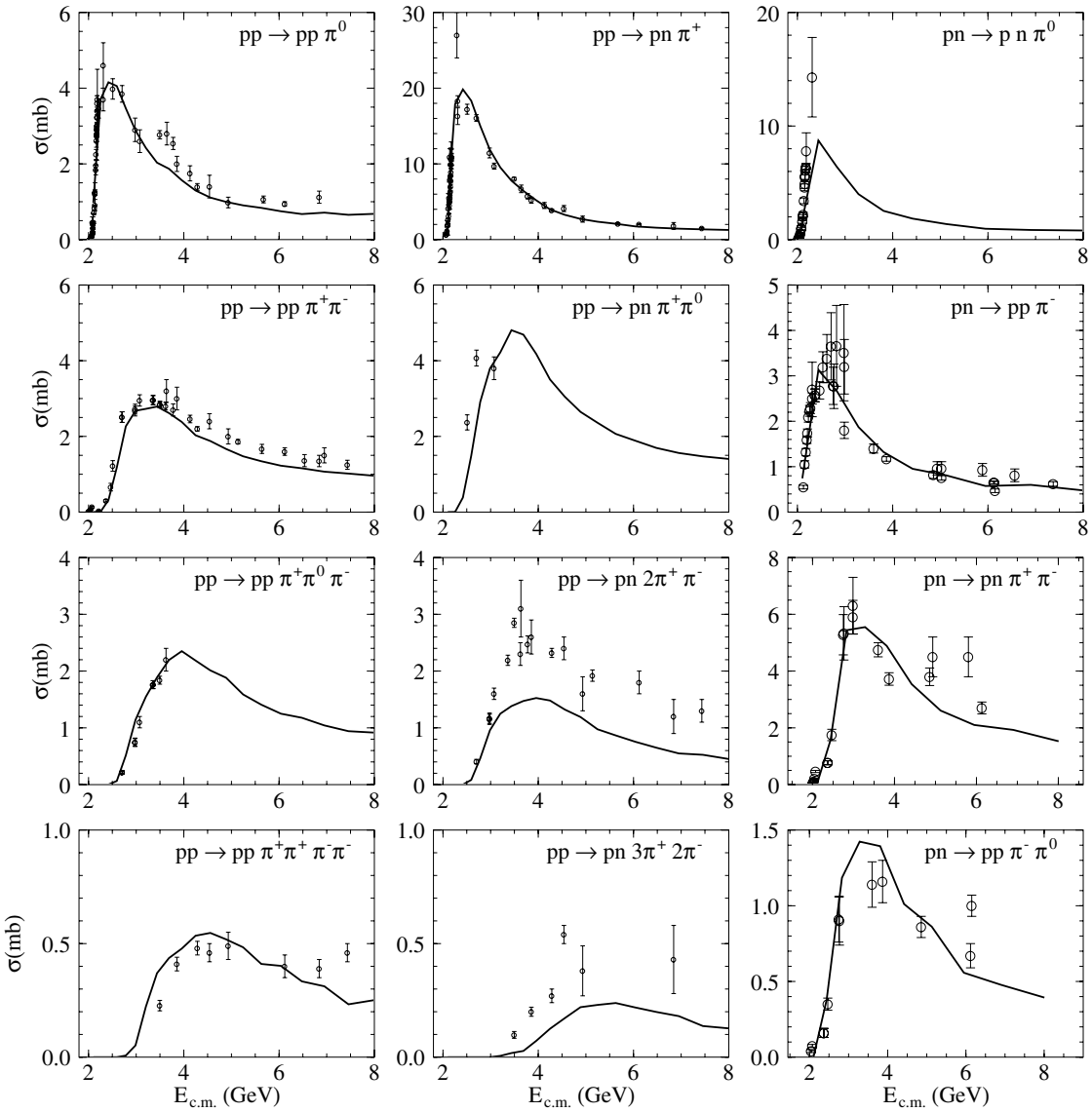




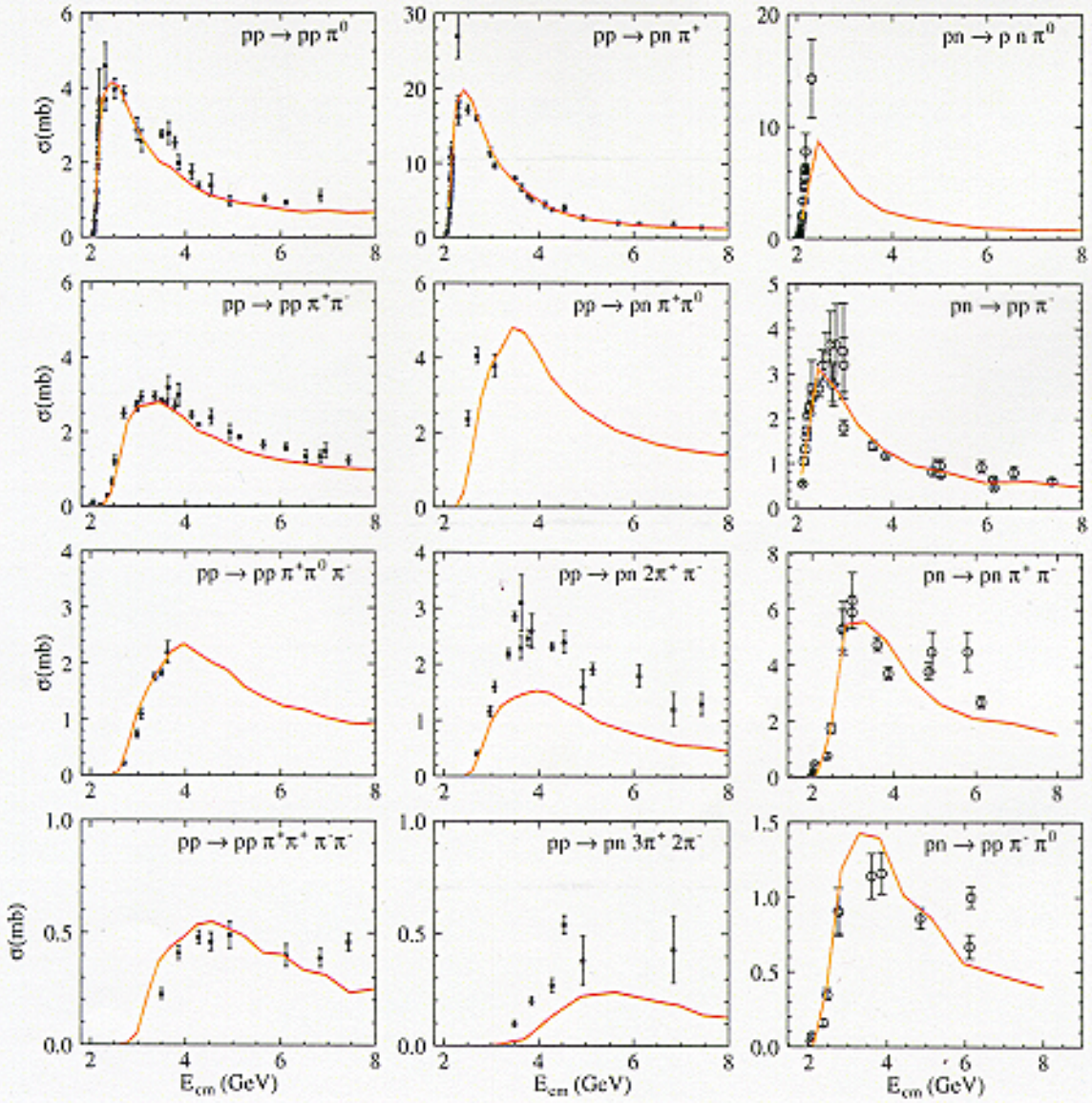


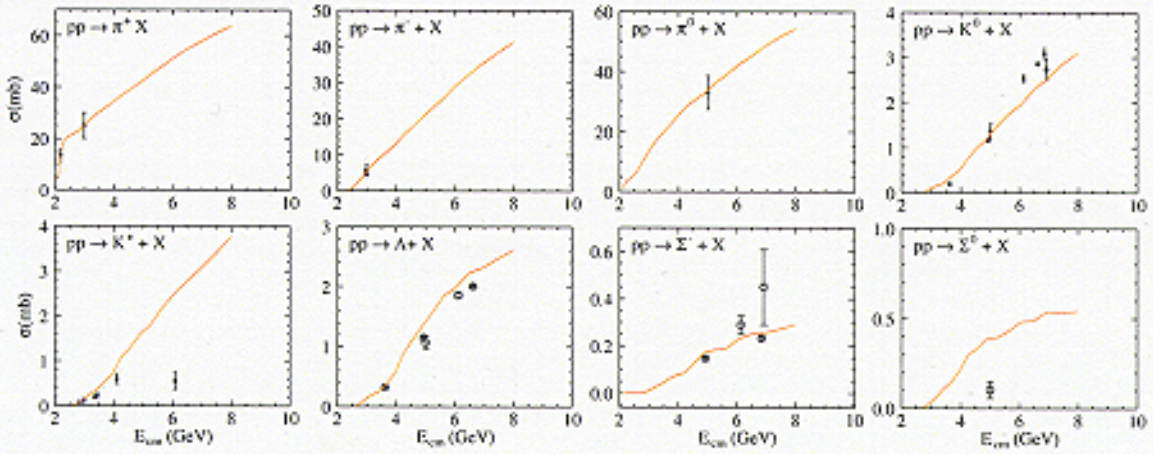


pion production data fitted by JAM

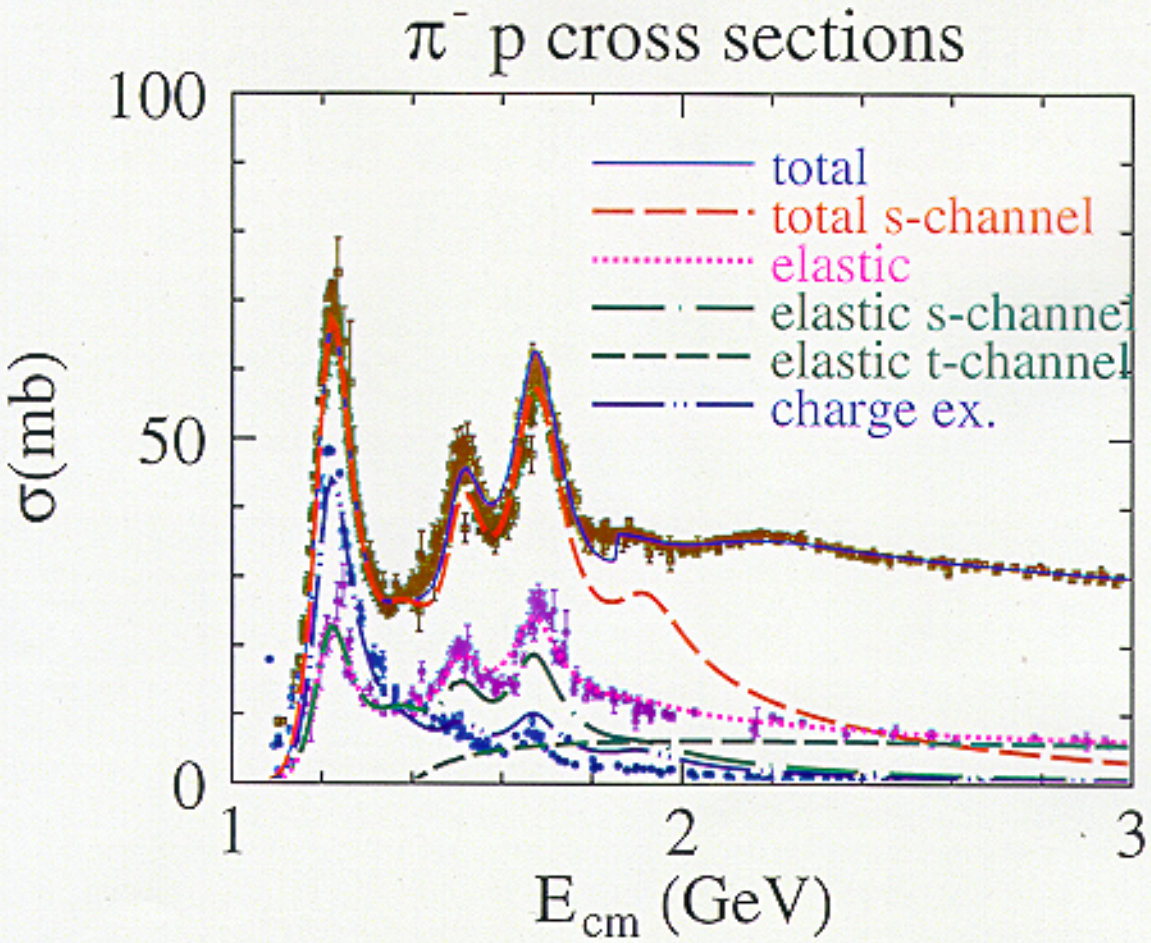


Resonance production cross sections are determiend to fit the pion data.


pp → pp π0
pp → pn π+
pn → p n π0
pp → pp π+π-
pp → pn π+π0
pn → pp π-
pp → pp π+π0 π-
pp → pn 2π+ π-
pn → pn π+ π-
pp → pp π+π+ π-π-
pp → pn 3π+ 2π-
pn → pp π- π0
σ(mb)
Ecm (GeV)


pp → π+ X
pp → π- + X
pp → π0 + X
pp → K0 + X
pp → K+ + X
pp → Λ+ X
pp → Σ- + X
pp → Σ0 + X
σ(mb)
Ecm (GeV)

Modeling low energy $\pi^- p$ cross sections



$R = N(1440) \sim N(1990),\ \Delta(1232) \sim \Delta(1950)$

- s-channel inelastic:

  $\pi^- p \to$ Resonance (string)

- t-channel inelastic:

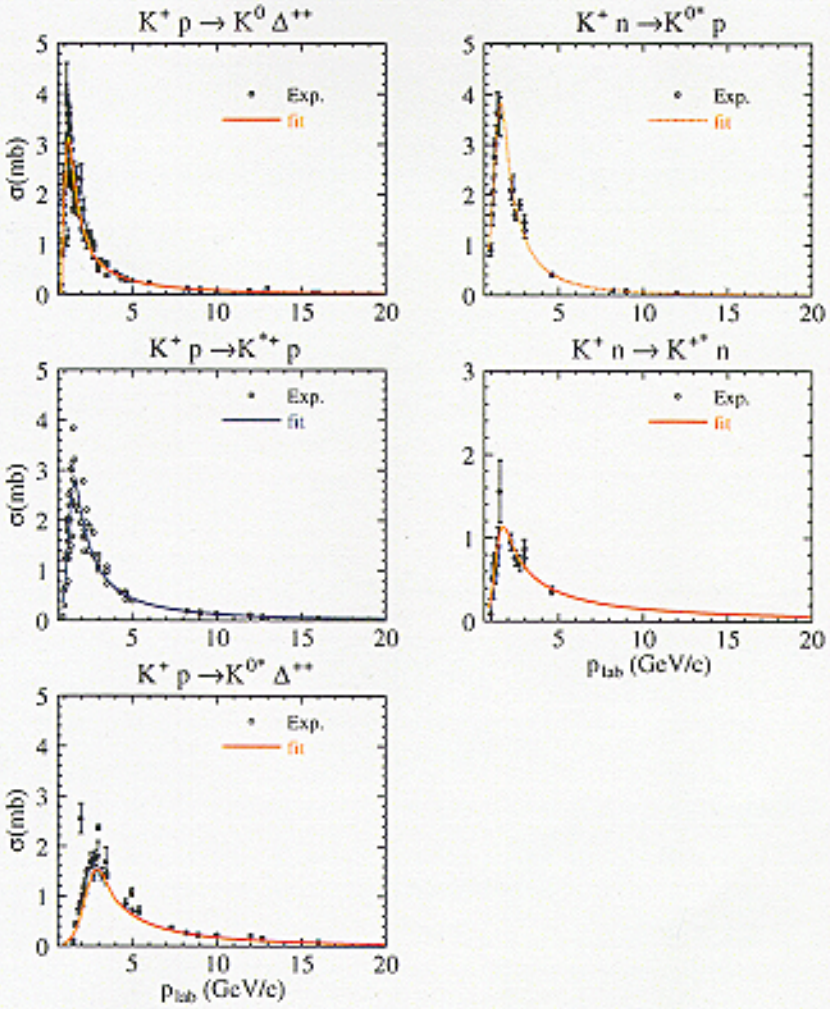
  $\pi^- p \to$ Resonance (string) + Resonance (string)

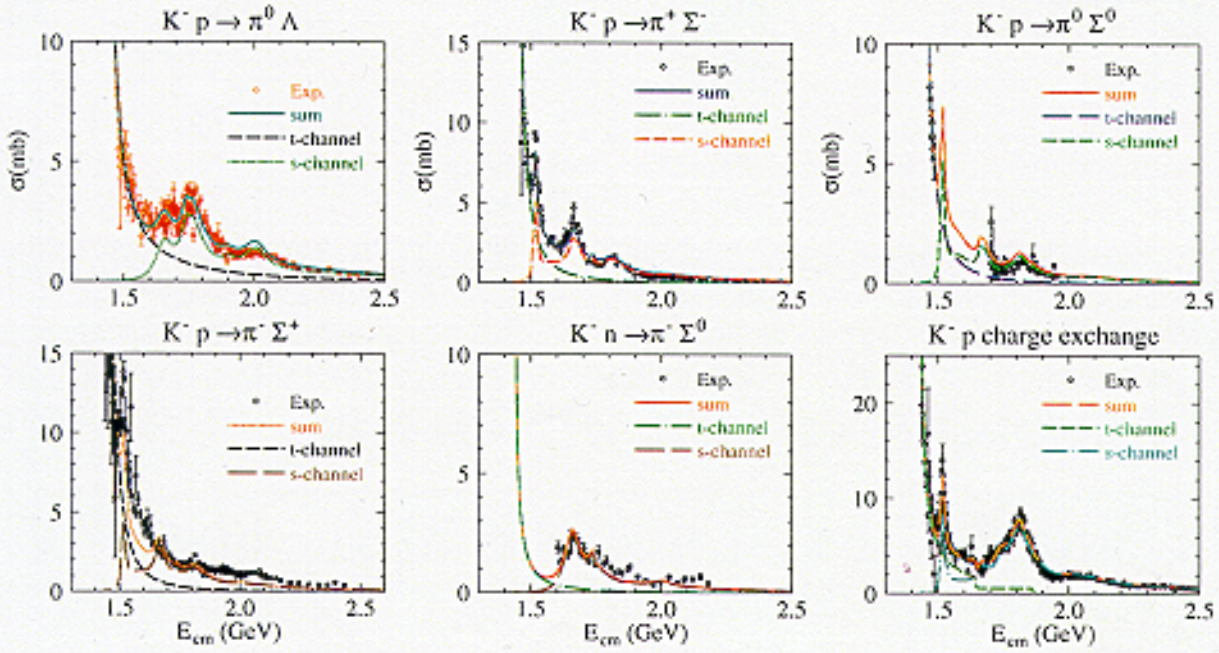
- elastic:

  s-channel: $\pi^- p \to R \to \pi^- p$, t-channel:$\pi^- p \to \pi^- p$

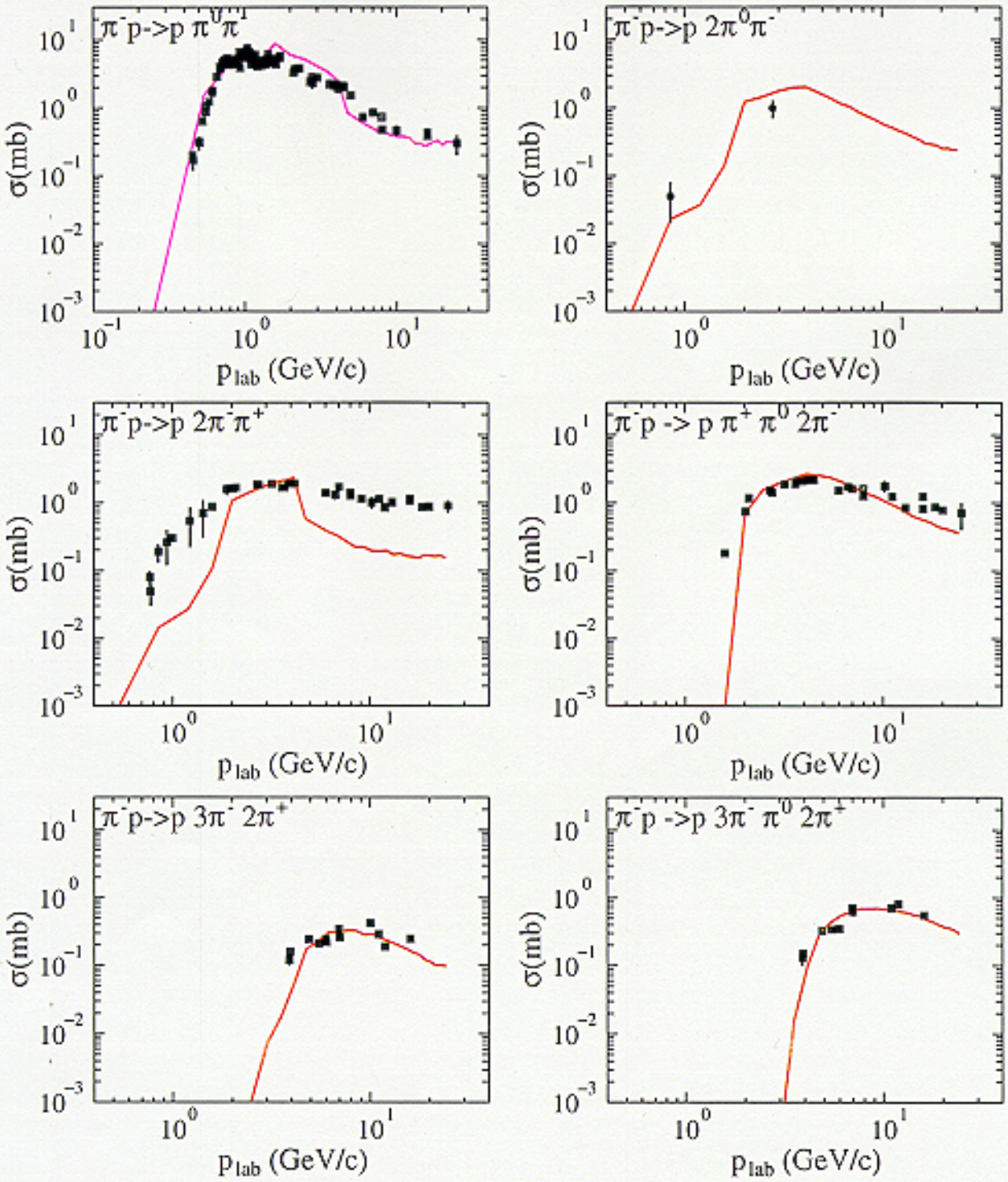
- charge exchange:

  s-channel: $\pi^- p \to R \to \pi^0 n$

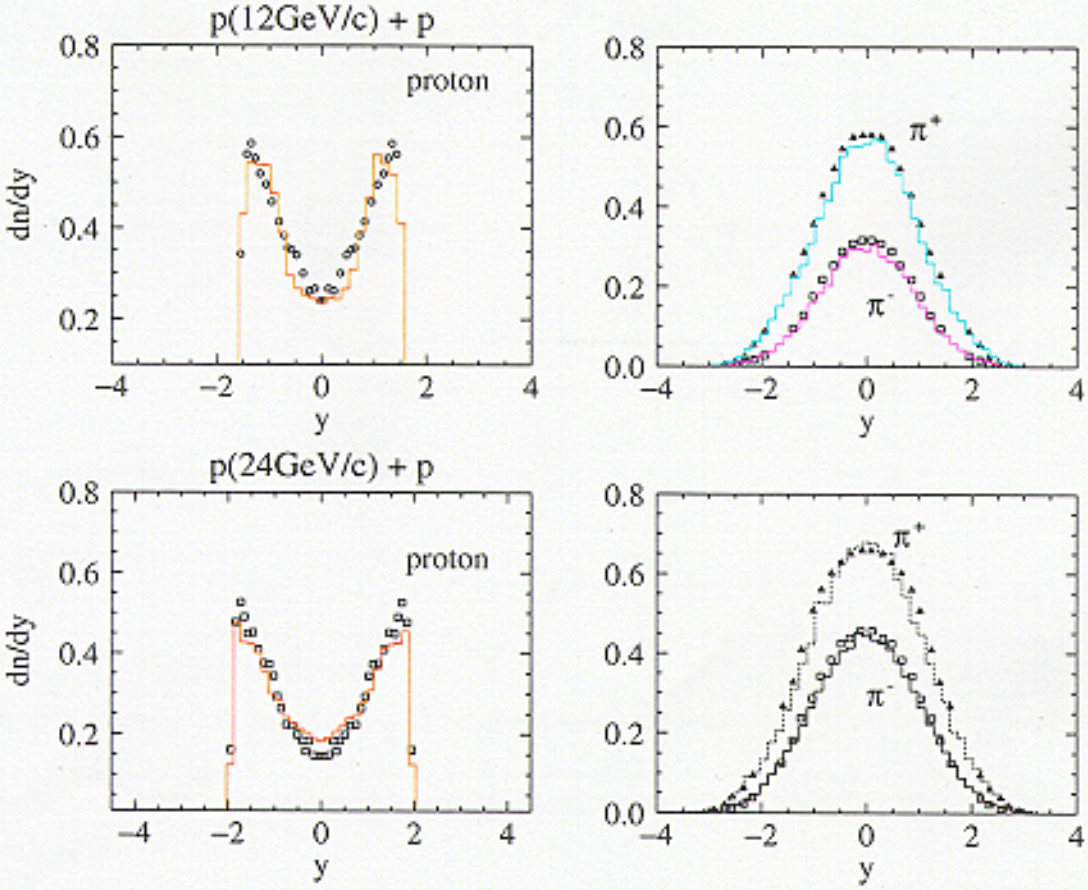

K+ p → K0 Δ++
K+ n → K0* p
K+ p → K*+ p
K+ n → K+* n
K+ p → K0* Δ++
Exp.
fit
σ(mb)
plab (GeV/c)

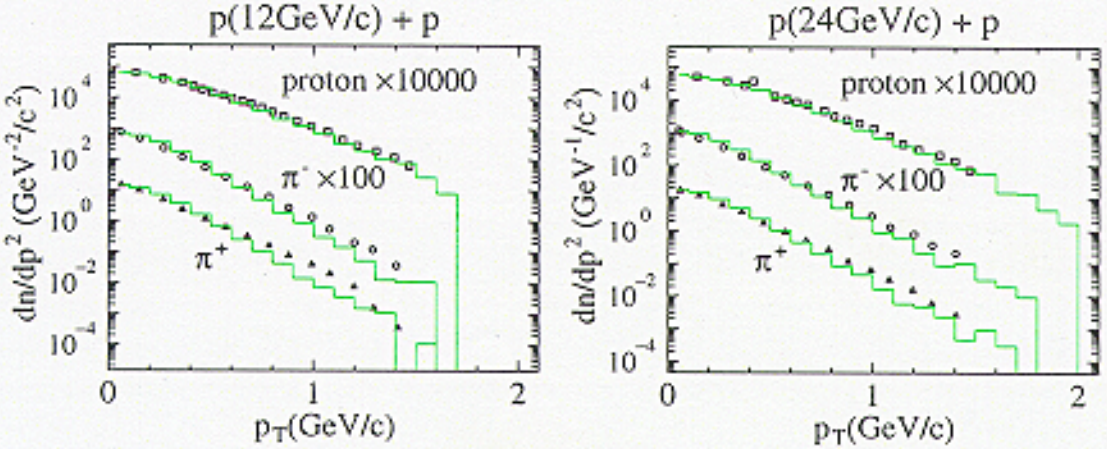

K- p → π0 Λ
K- p → π+ Σ-
K- p → π0 Σ0
K- p → π- Σ+
K- n → π- Σ0
K- p charge exchange
Exp.
sum
t-channel
s-channel
σ(mb)
Ecm (GeV)

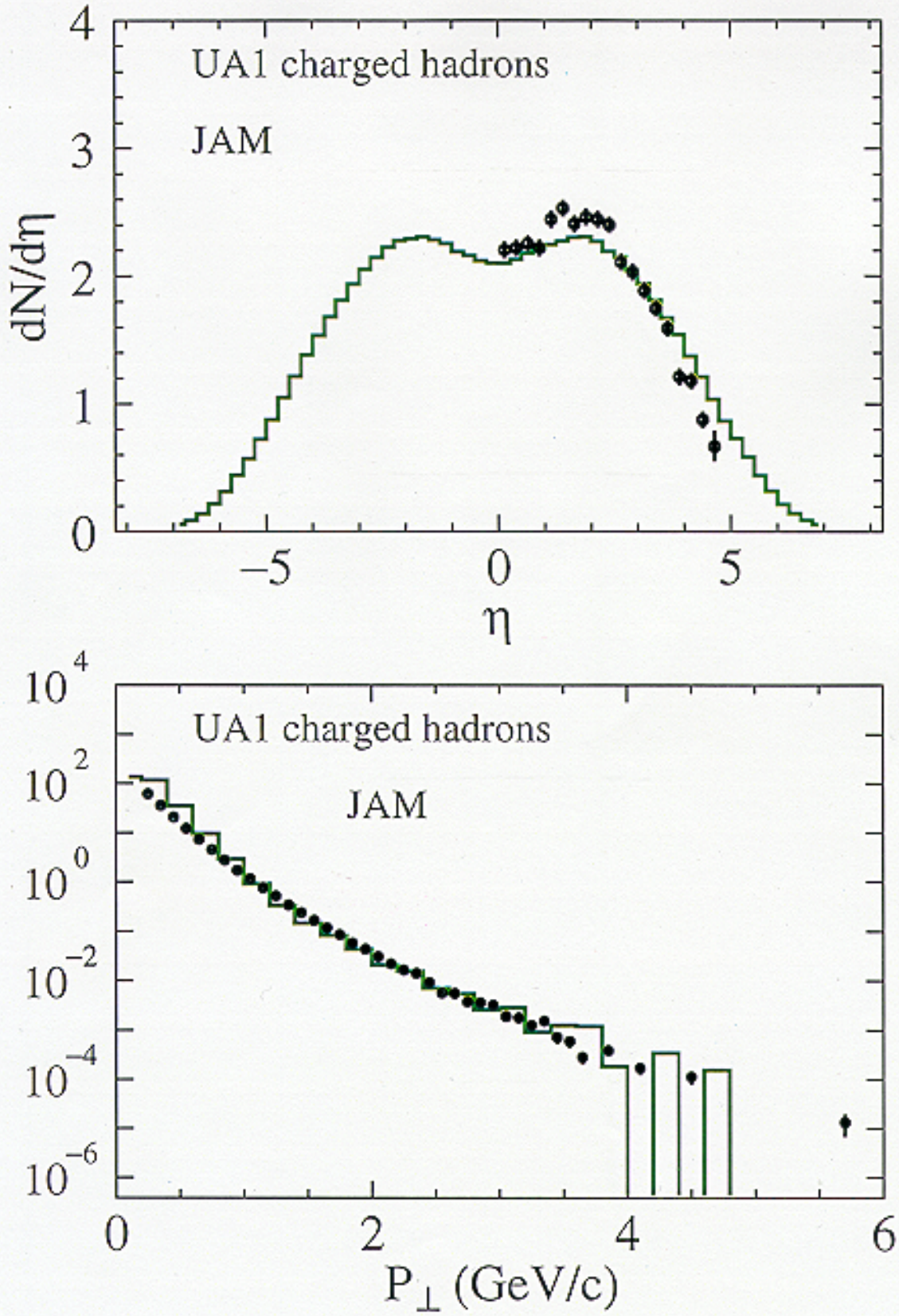
JAM1.0

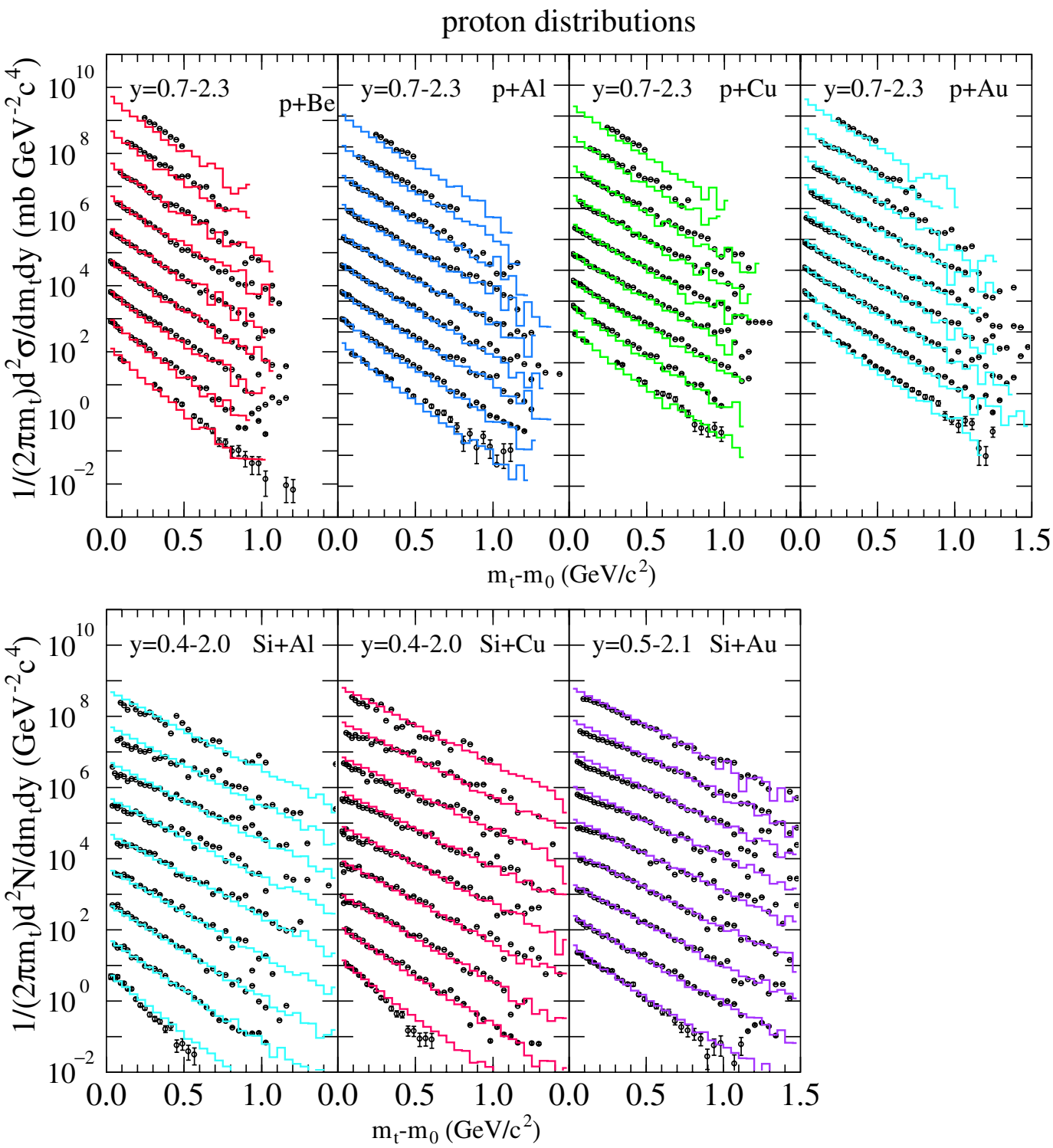

π⁻p->p π⁰π⁻
π⁻p->p 2π⁰π⁻
π⁻p->p 2π⁻π⁺
π⁻p -> p π⁺ π⁰ 2π⁻
π⁻p->p 3π⁻ 2π⁺
π⁻p ->p 3π⁻ π⁰ 2π⁺
σ(mb)
p_lab (GeV/c)

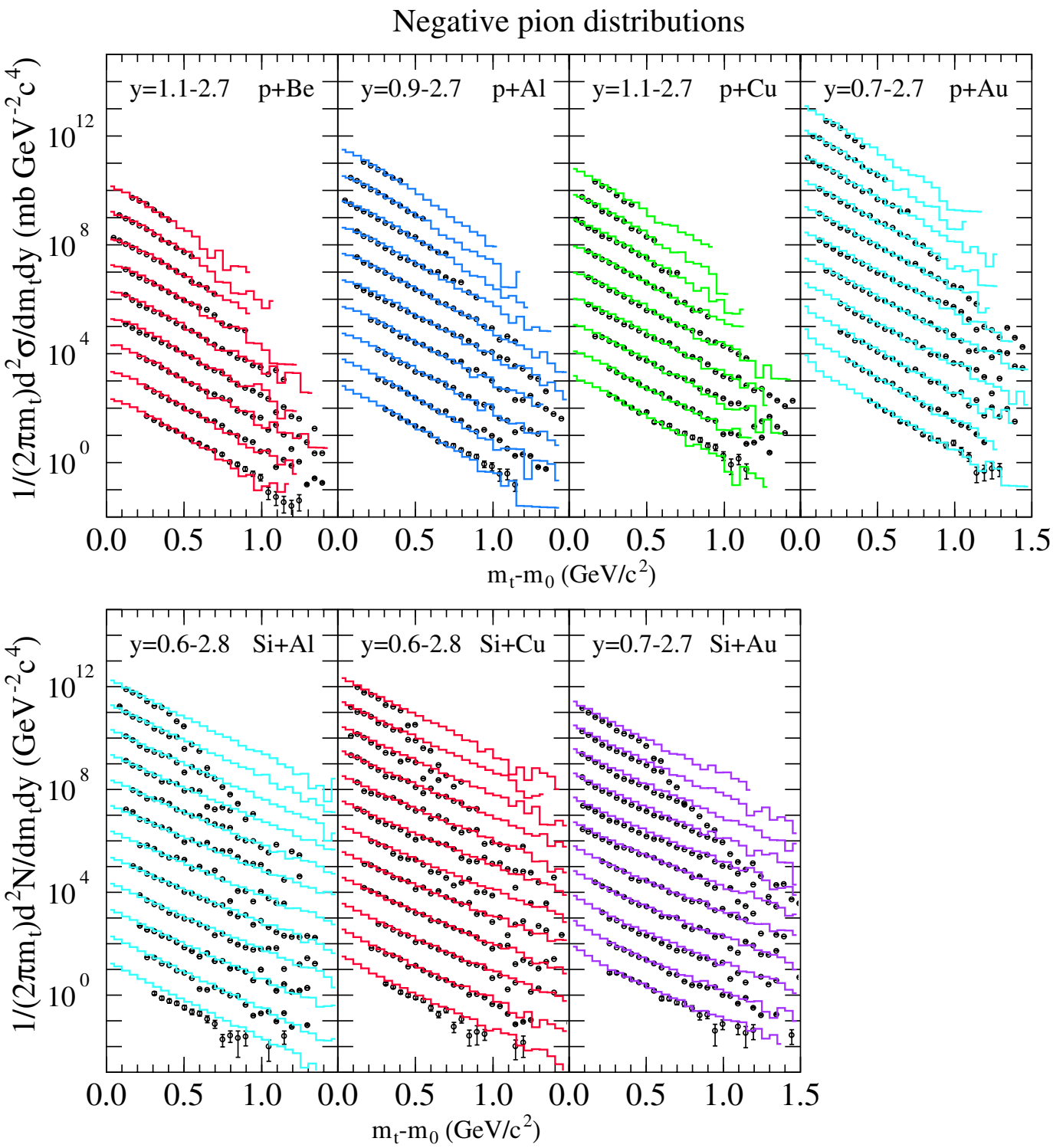
Rapidity/transverse distributions for p+p collision at AGS energies

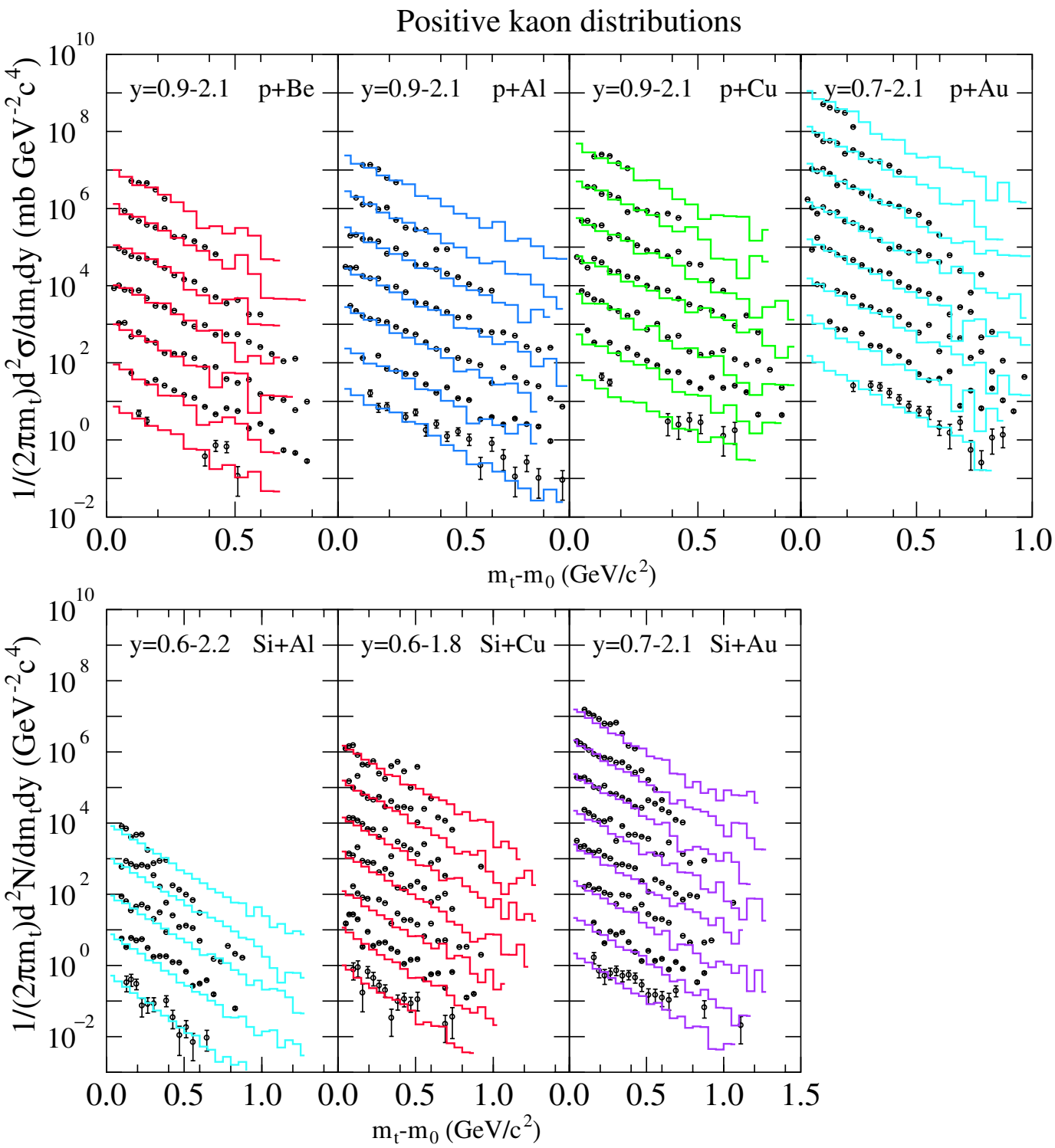


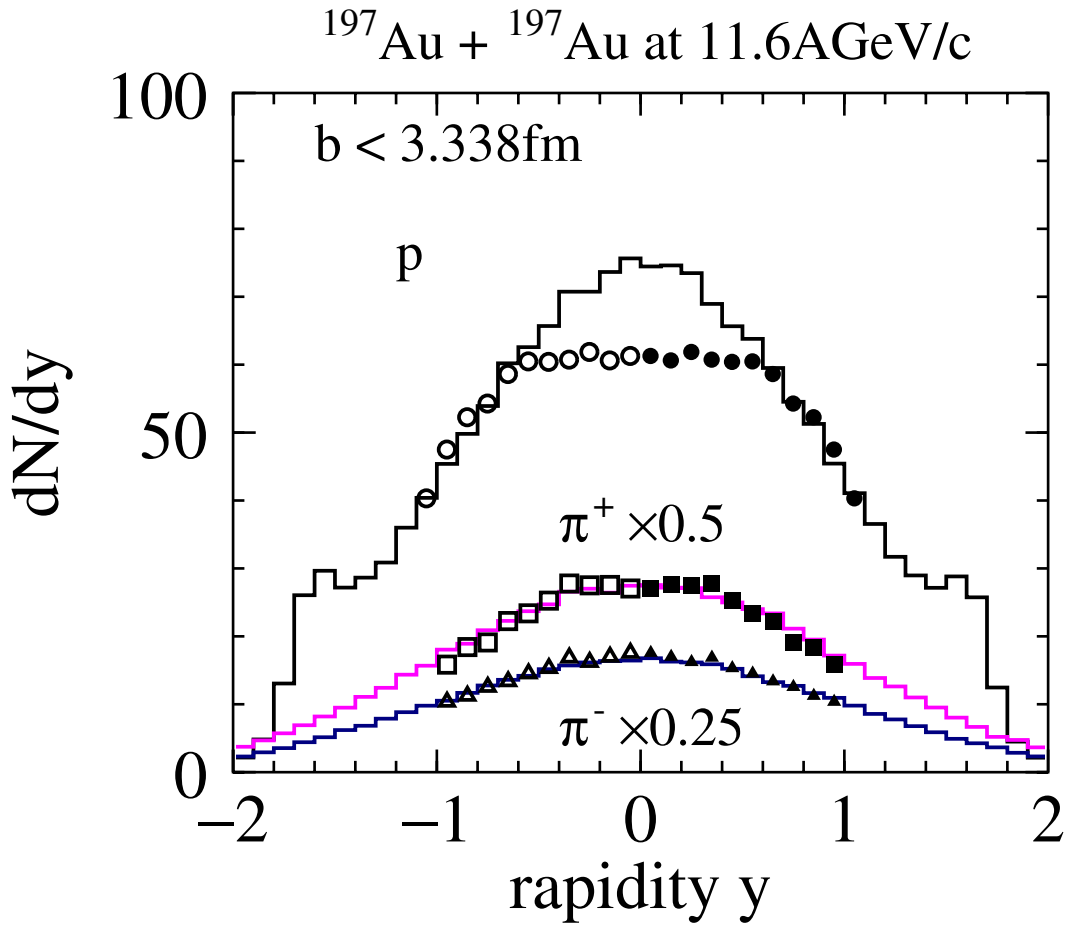


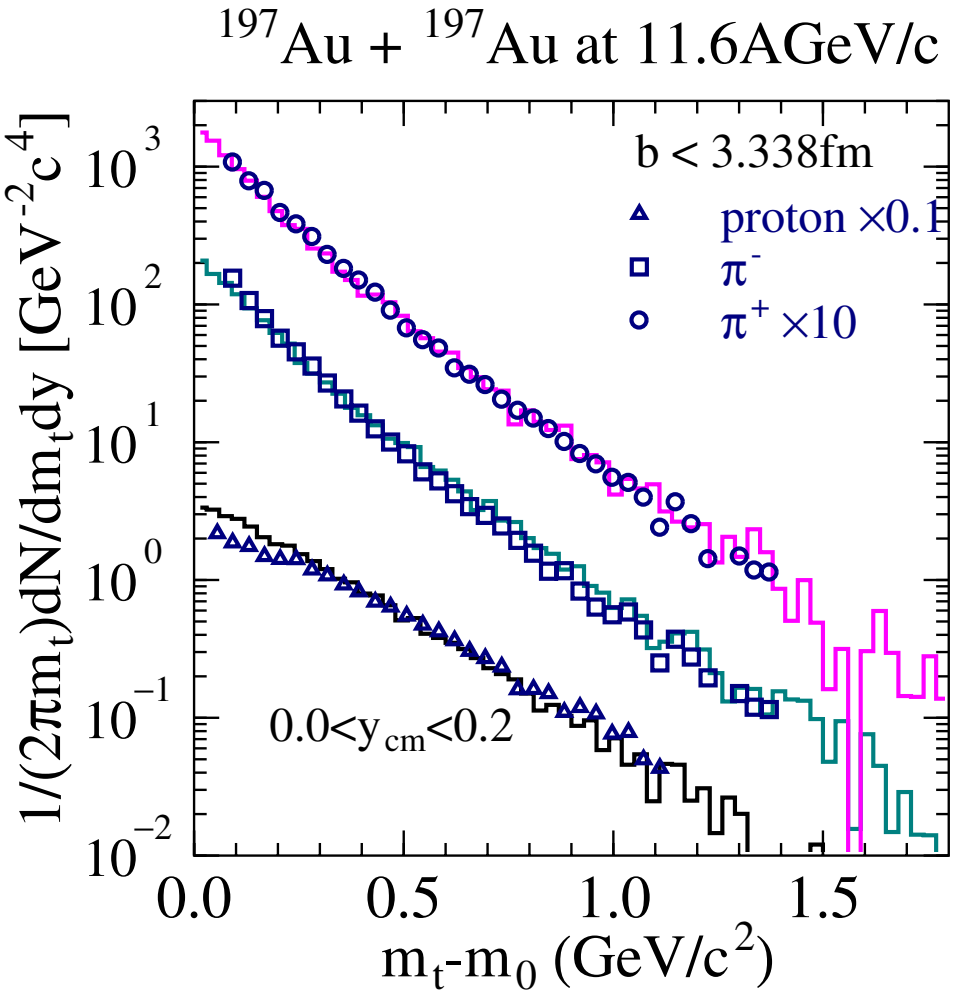

UA1 charged hadrons
JAM
dN/dη
η
UA1 charged hadrons
JAM
P⊥ (GeV/c)

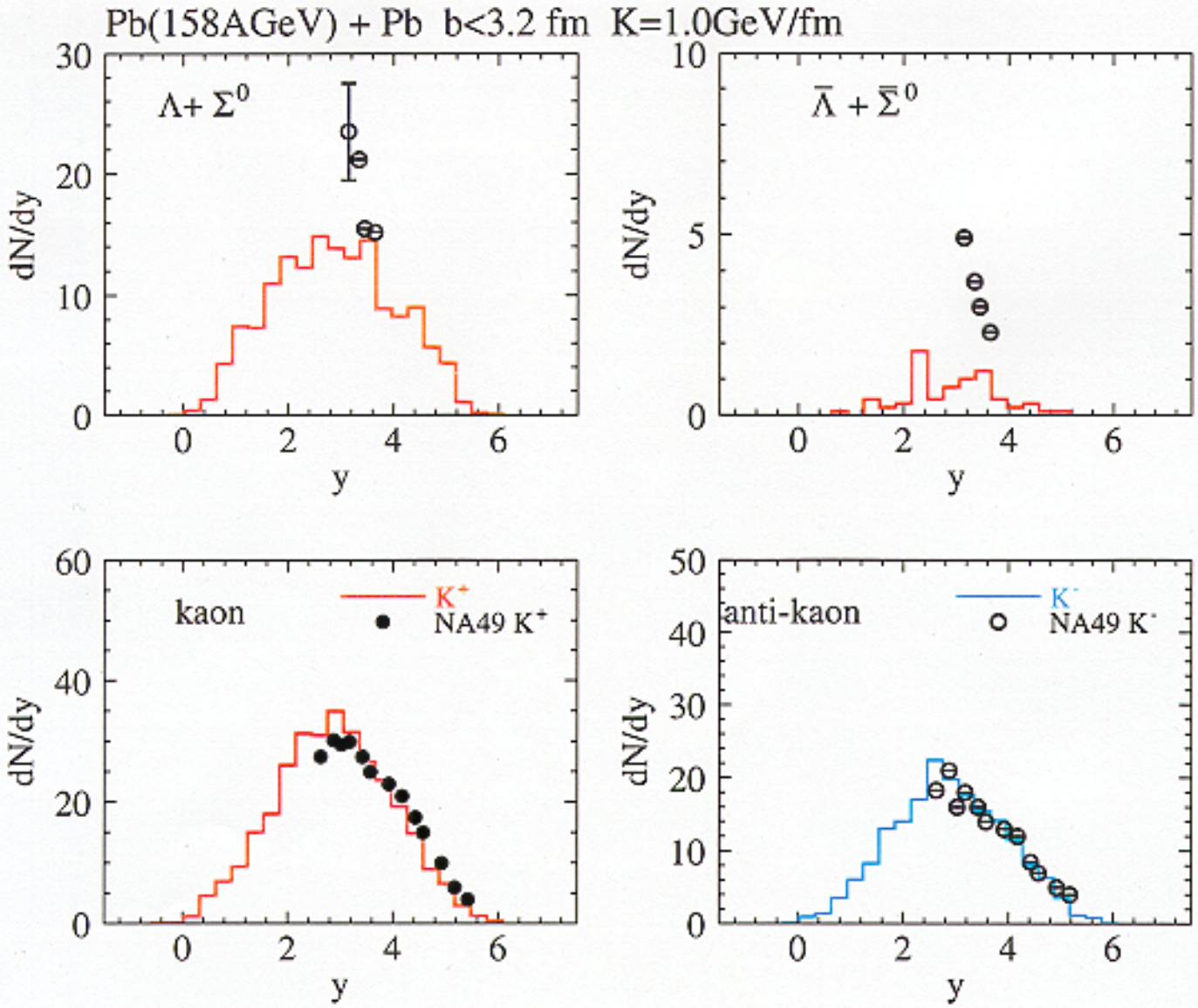

proton distributions
y=0.7-2.3
p+Be
y=0.7-2.3
p+Al
y=0.7-2.3
p+Cu
y=0.7-2.3
p+Au
1/(2πm_t)d^2σ/dm_tdy (mb GeV^-2c^4)
m_t-m_0 (GeV/c^2)
y=0.4-2.0
Si+Al
y=0.4-2.0
Si+Cu
y=0.5-2.1
Si+Au
1/(2πm_t)d^2N/dm_tdy (GeV^-2c^4)
m_t-m_0 (GeV/c^2)

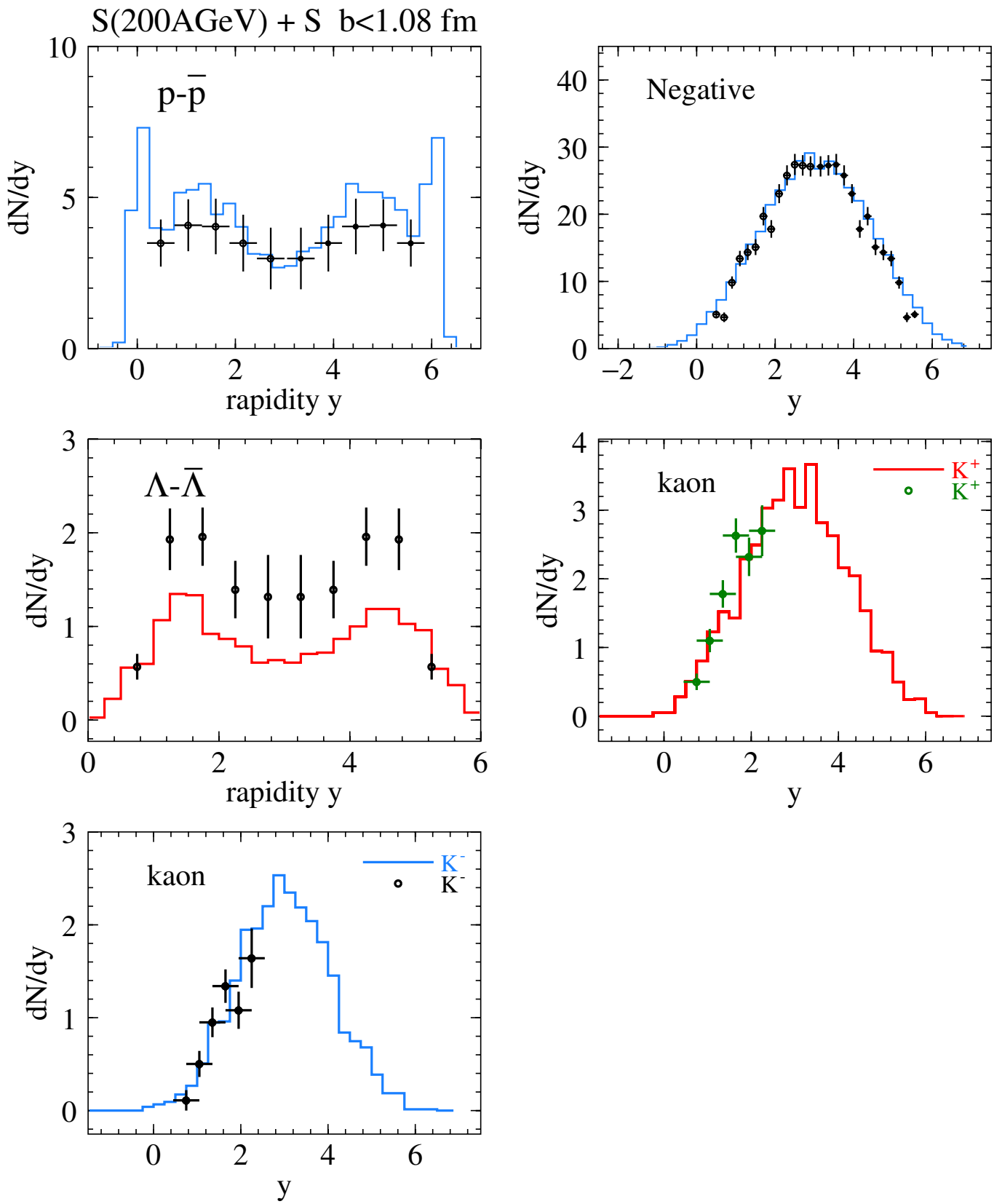

Negative pion distributions
y=1.1-2.7 p+Be
y=0.9-2.7 p+Al
y=1.1-2.7 p+Cu
y=0.7-2.7 p+Au
1/(2πm_t)d²σ/dm_tdy (mb GeV⁻²c⁴)
m_t-m_0 (GeV/c²)
y=0.6-2.8 Si+Al
y=0.6-2.8 Si+Cu
y=0.7-2.7 Si+Au
1/(2πm_t)d²N/dm_tdy (GeV⁻²c⁴)
m_t-m_0 (GeV/c²)

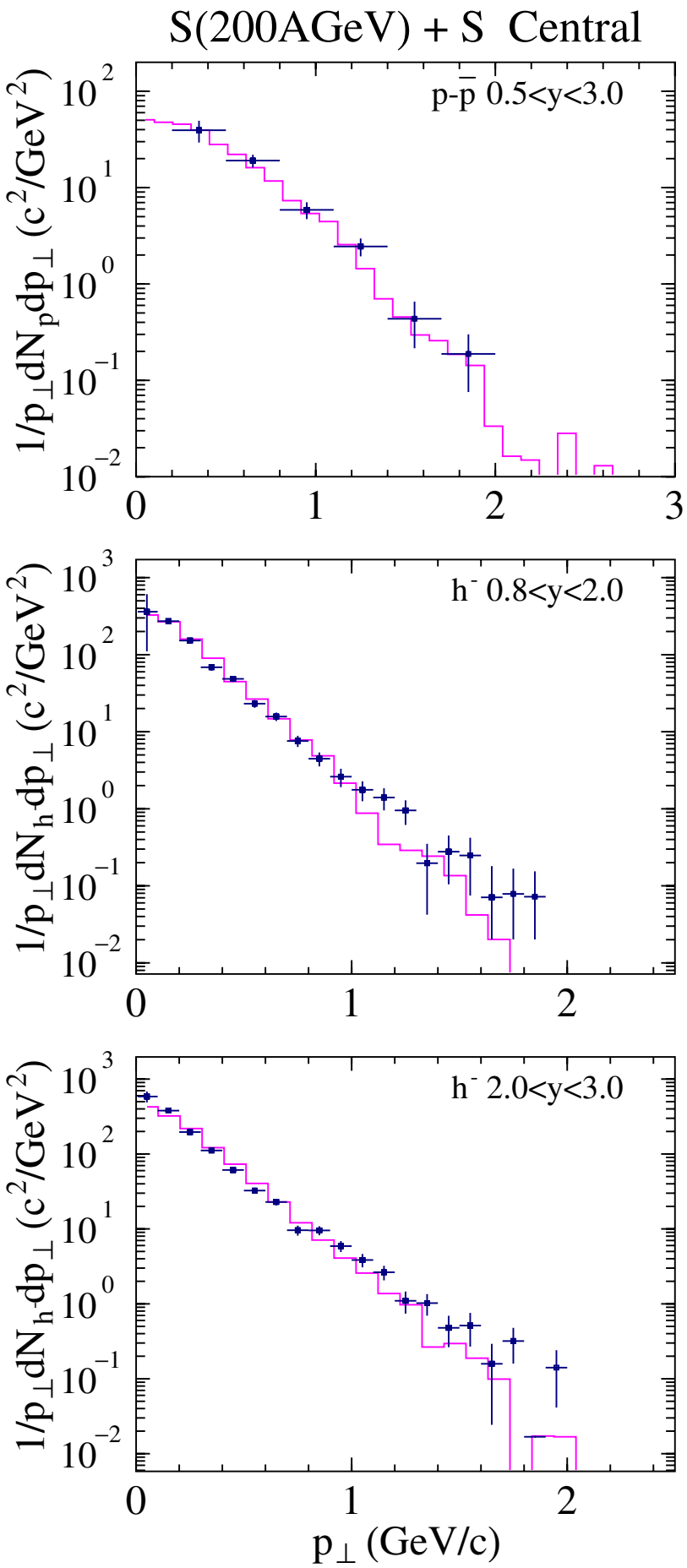

Positive kaon distributions
y=0.9-2.1 p+Be
y=0.9-2.1 p+Al
y=0.9-2.1 p+Cu
y=0.7-2.1 p+Au
1/(2πm_t)d²σ/dm_tdy (mb GeV⁻²c⁴)
m_t-m_0 (GeV/c²)
y=0.6-2.2 Si+Al
y=0.6-1.8 Si+Cu
y=0.7-2.1 Si+Au
1/(2πm_t)d²N/dm_tdy (GeV⁻²c⁴)
m_t-m_0 (GeV/c²)


^{197}Au + ^{197}Au at 11.6AGeV/c
b < 3.338fm
p
$\pi^+ \times 0.5$
$\pi^- \times 0.25$
dN/dy
rapidity y
100
50
0
−2
−1
0
1
2

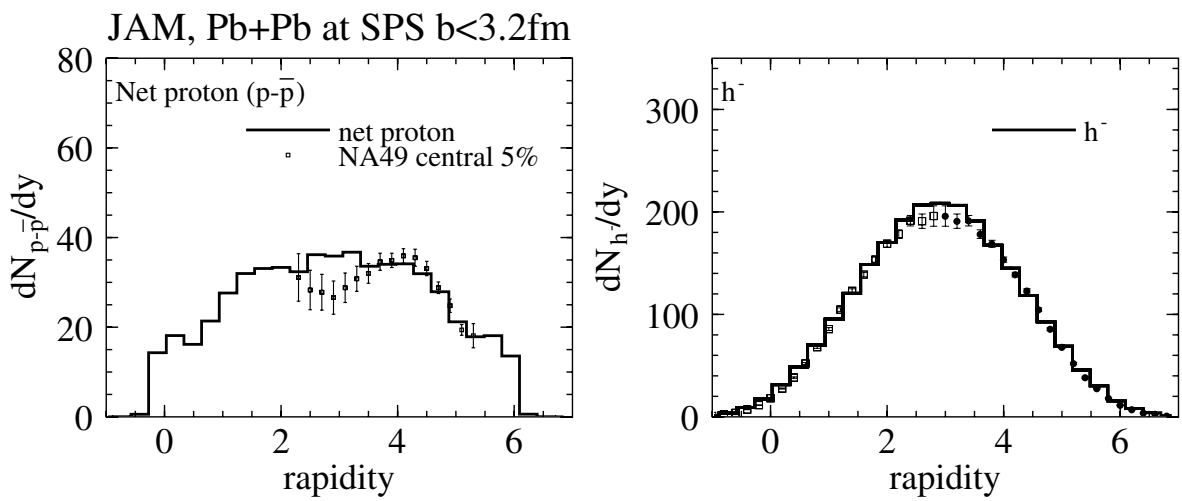

^{197}Au + ^{197}Au at 11.6AGeV/c
b < 3.338fm
proton ×0.1
π^-
π^+ ×10
0.0<y_{cm}<0.2
$1/(2\pi m_t)dN/dm_t dy$ [GeV^{-2}c^4]
m_t-m_0 (GeV/c^2)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0.0
0.5
1.0
1.5

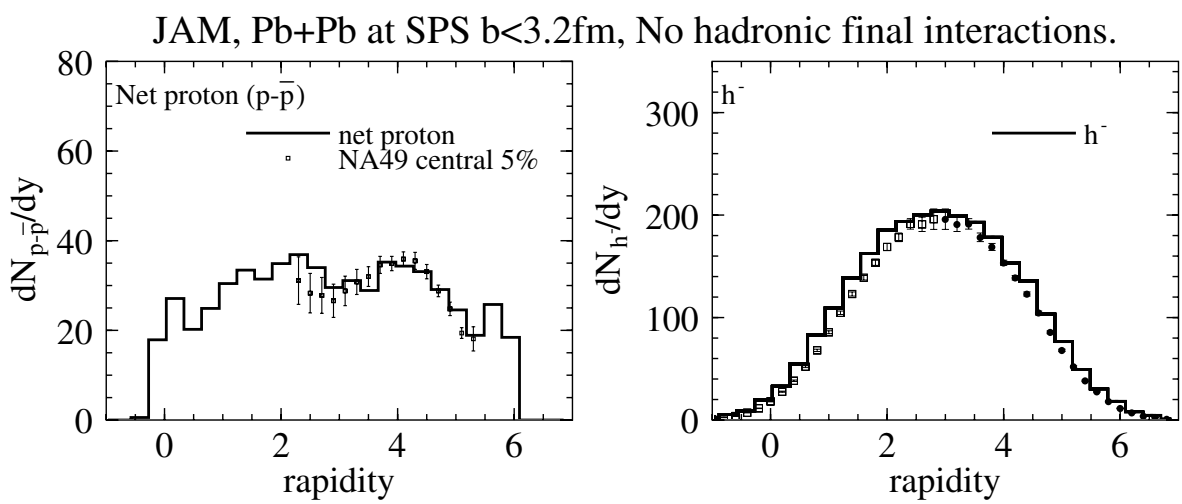

Pb(158AGeV) + Pb b<3.2 fm K=1.0GeV/fm
Λ+ Σ0
Λ̄ + Σ̄0
kaon
K+
NA49 K+
anti-kaon
K-
NA49 K-
dN/dy
y


S(200AGeV) + S b<1.08 fm
p-p̄
dN/dy
rapidity y
Negative
dN/dy
y
Λ-Λ̄
dN/dy
rapidity y
kaon
K+
K+
dN/dy
y
kaon
K-
K-
dN/dy
y


S(200AGeV) + S Central
p-p̄ 0.5<y<3.0
h⁻ 0.8<y<2.0
h⁻ 2.0<y<3.0
1/p⊥dNpdp⊥ (c²/GeV²)
1/p⊥dNh-dp⊥ (c²/GeV²)
p⊥ (GeV/c)

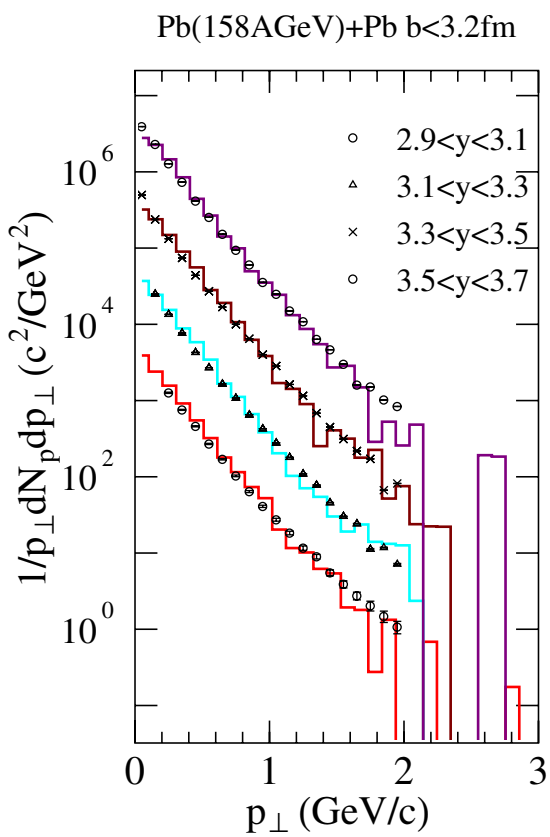
JAM calculaton of rapidity spectra

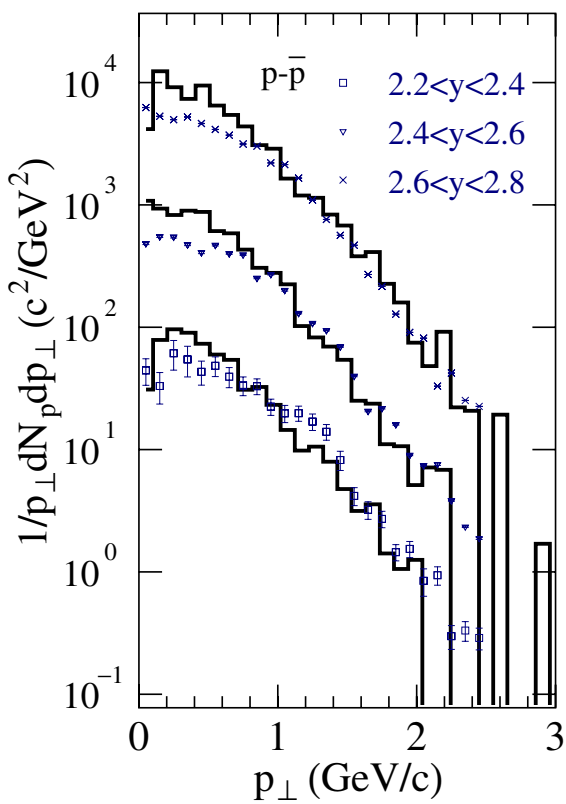




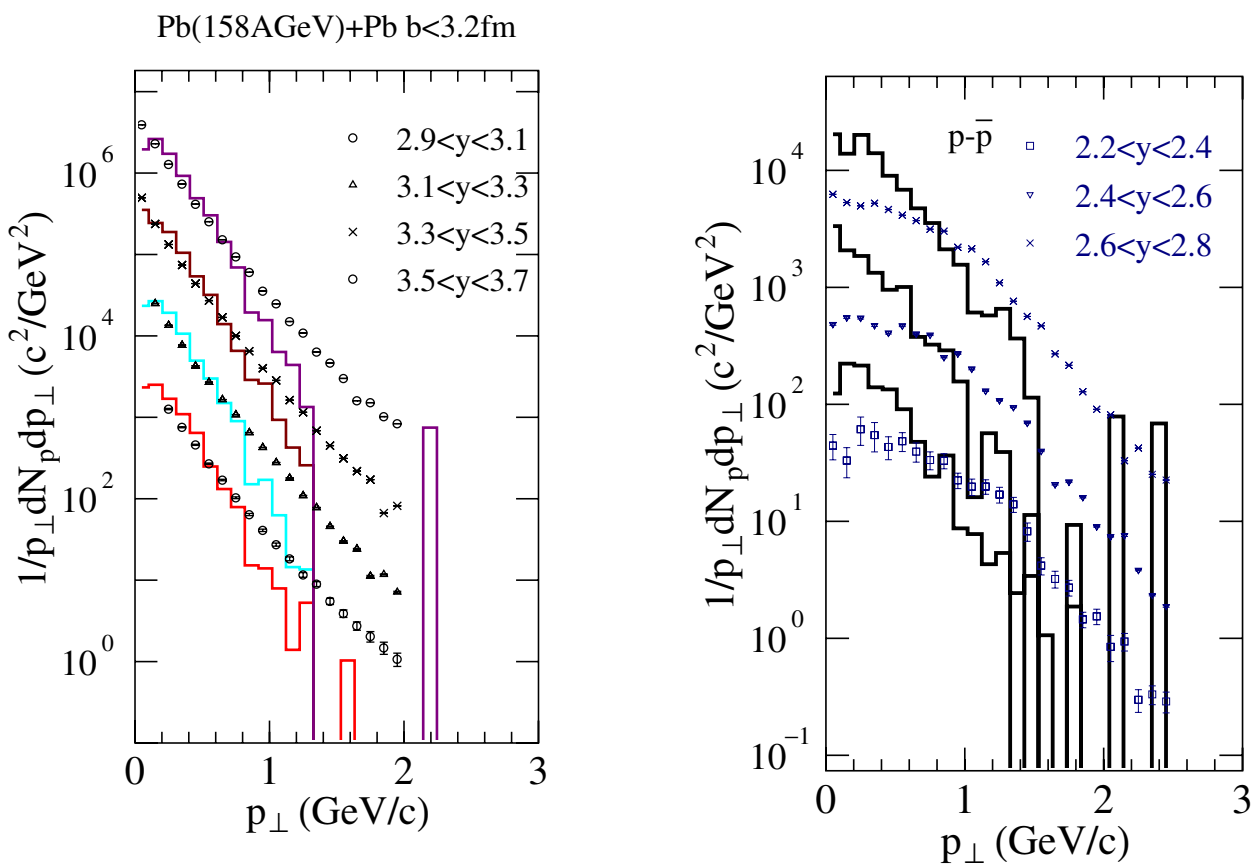
More text.

JAM calc. of $p_\perp$ spectra for Pb(158GeV)+Pb

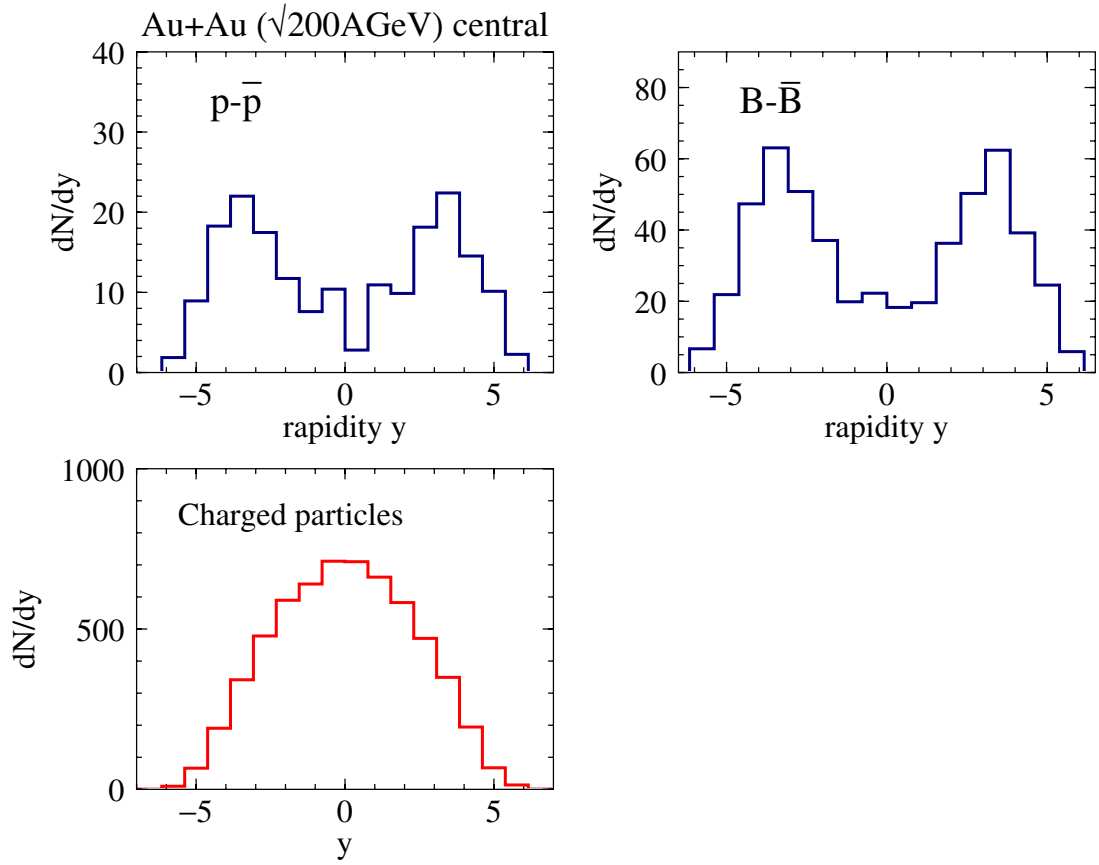




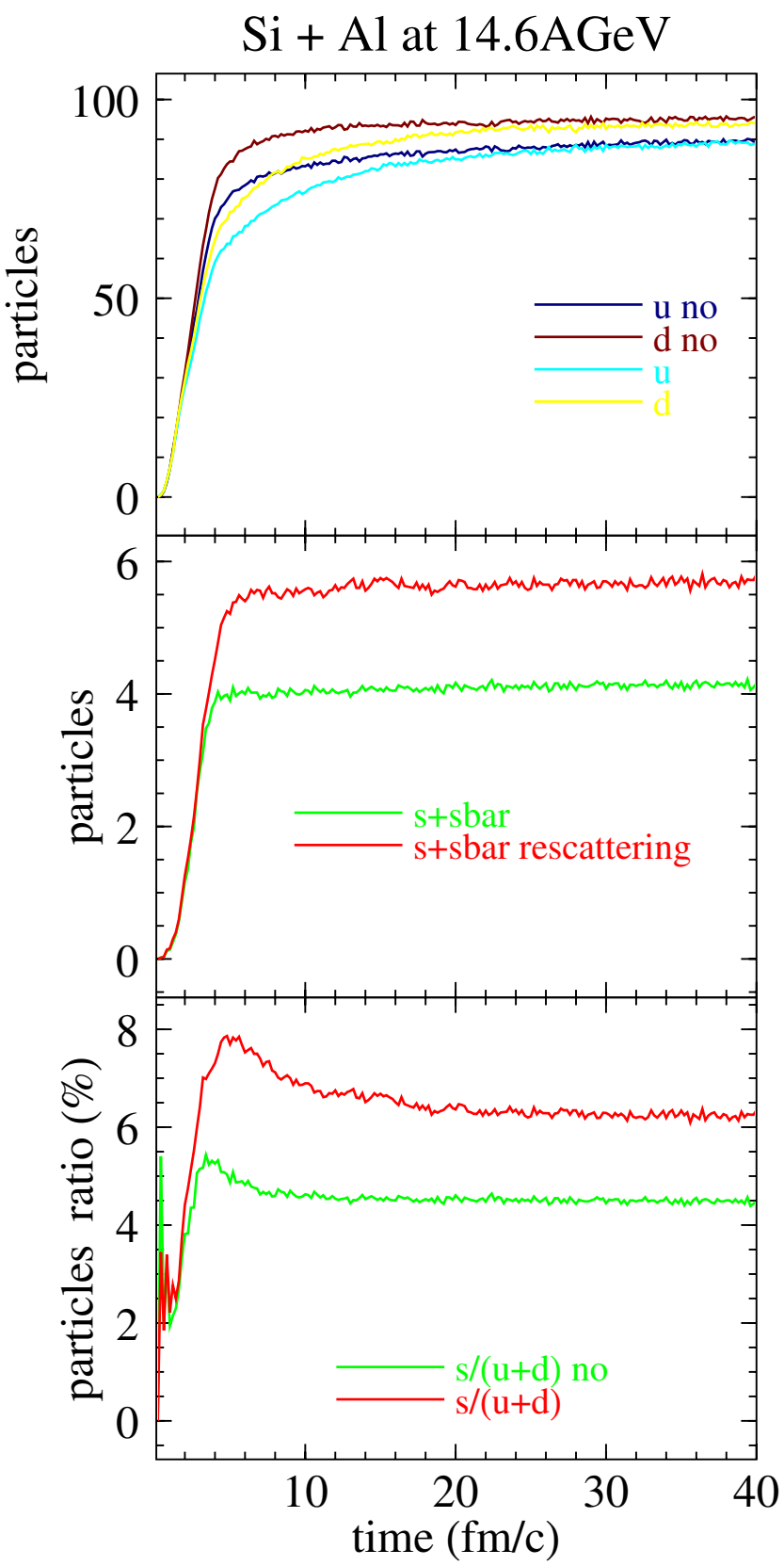
JAM calc. of $p_\perp$ spectra for Pb(158GeV)+Pb ,No hadronic final state ineractions.

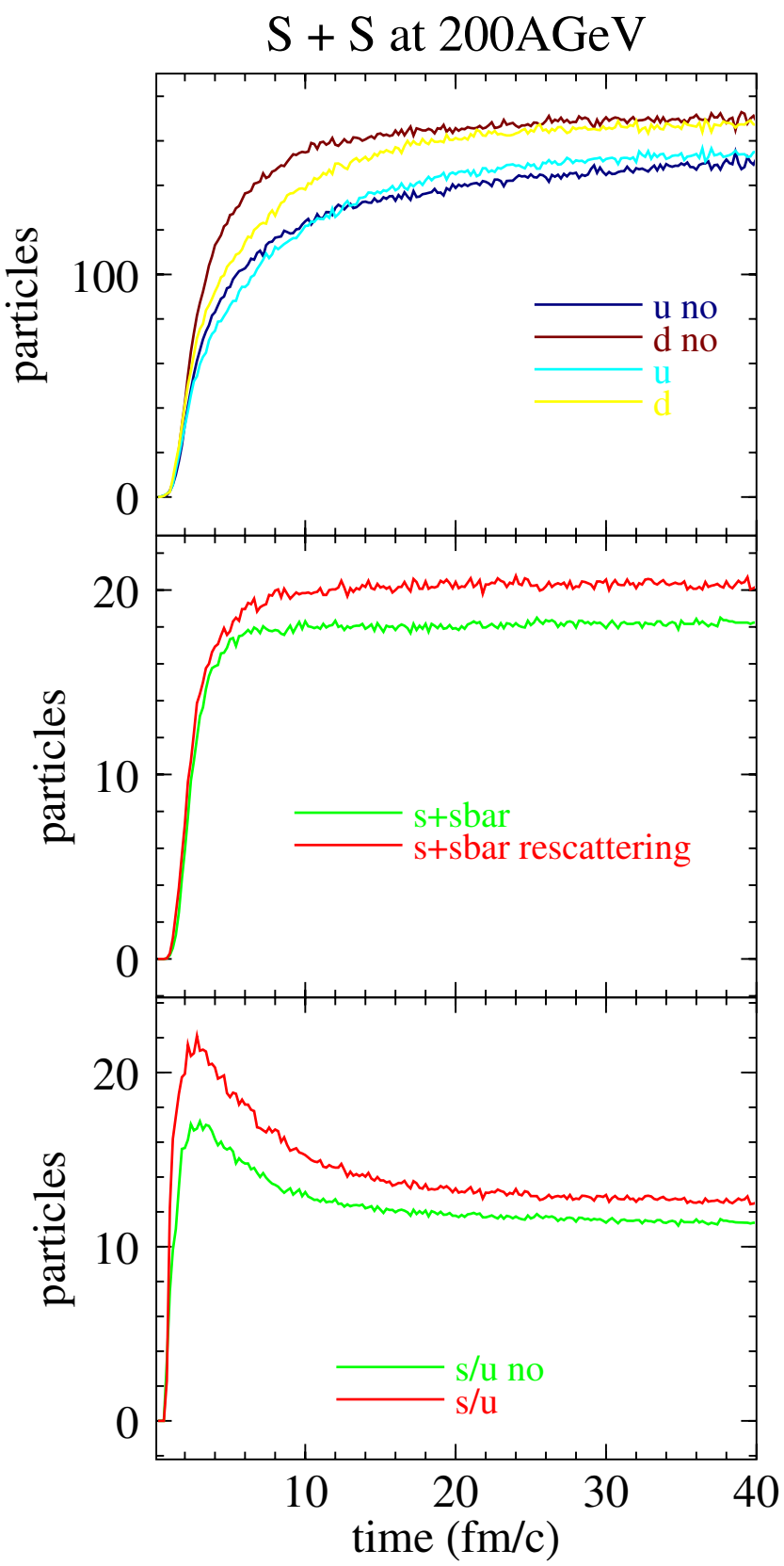


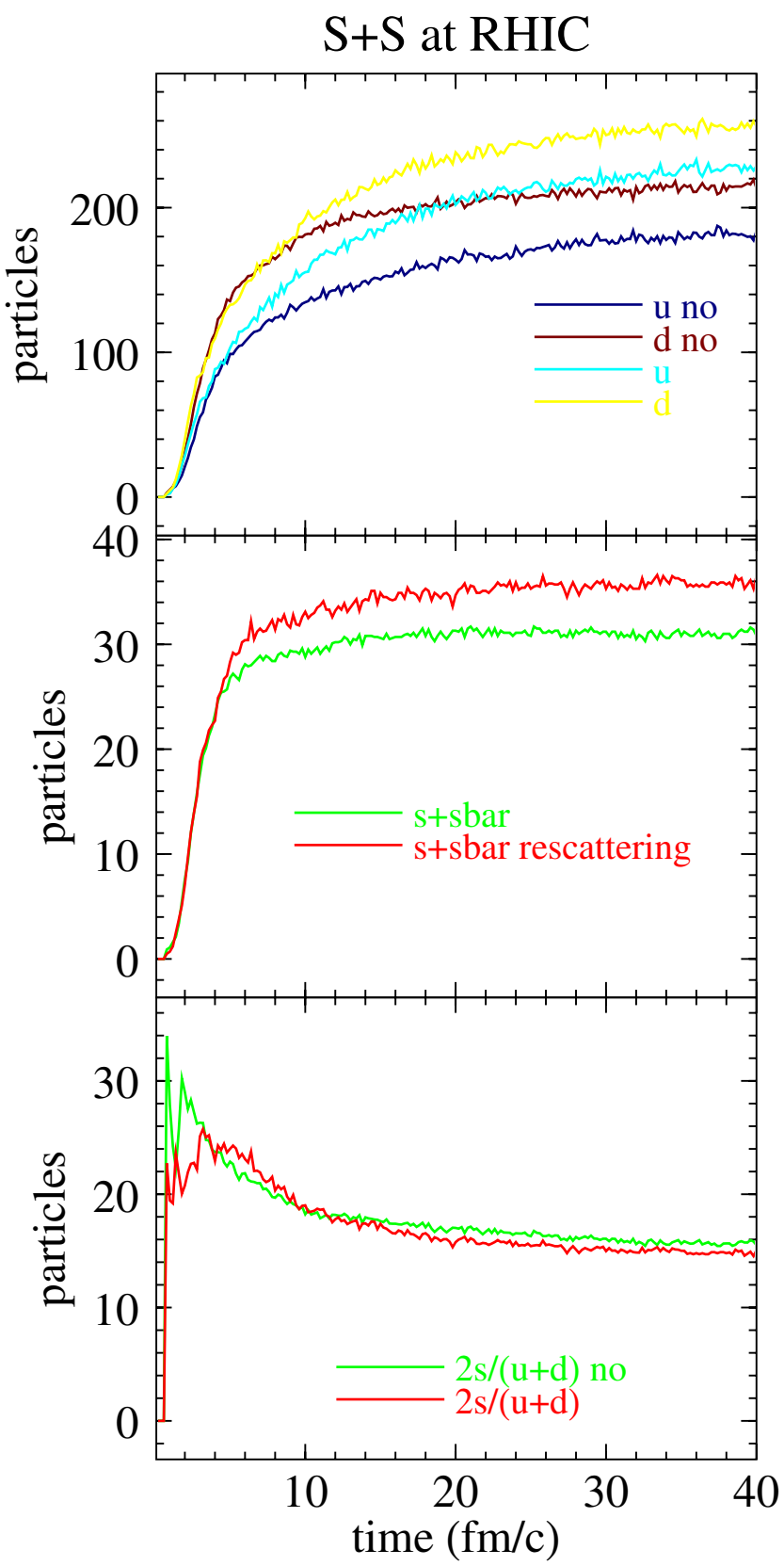
JAM:rapidity dist. for RHIC

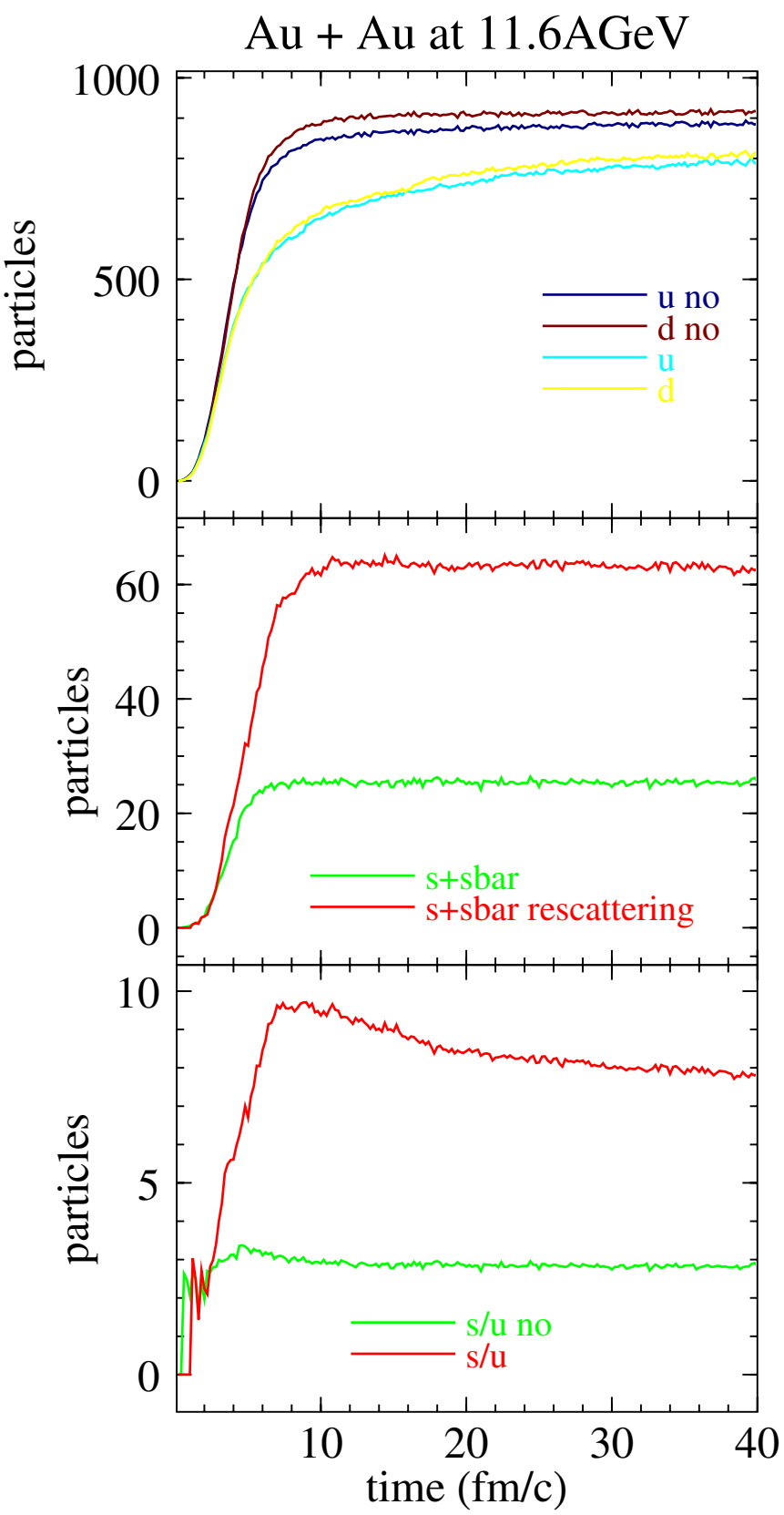


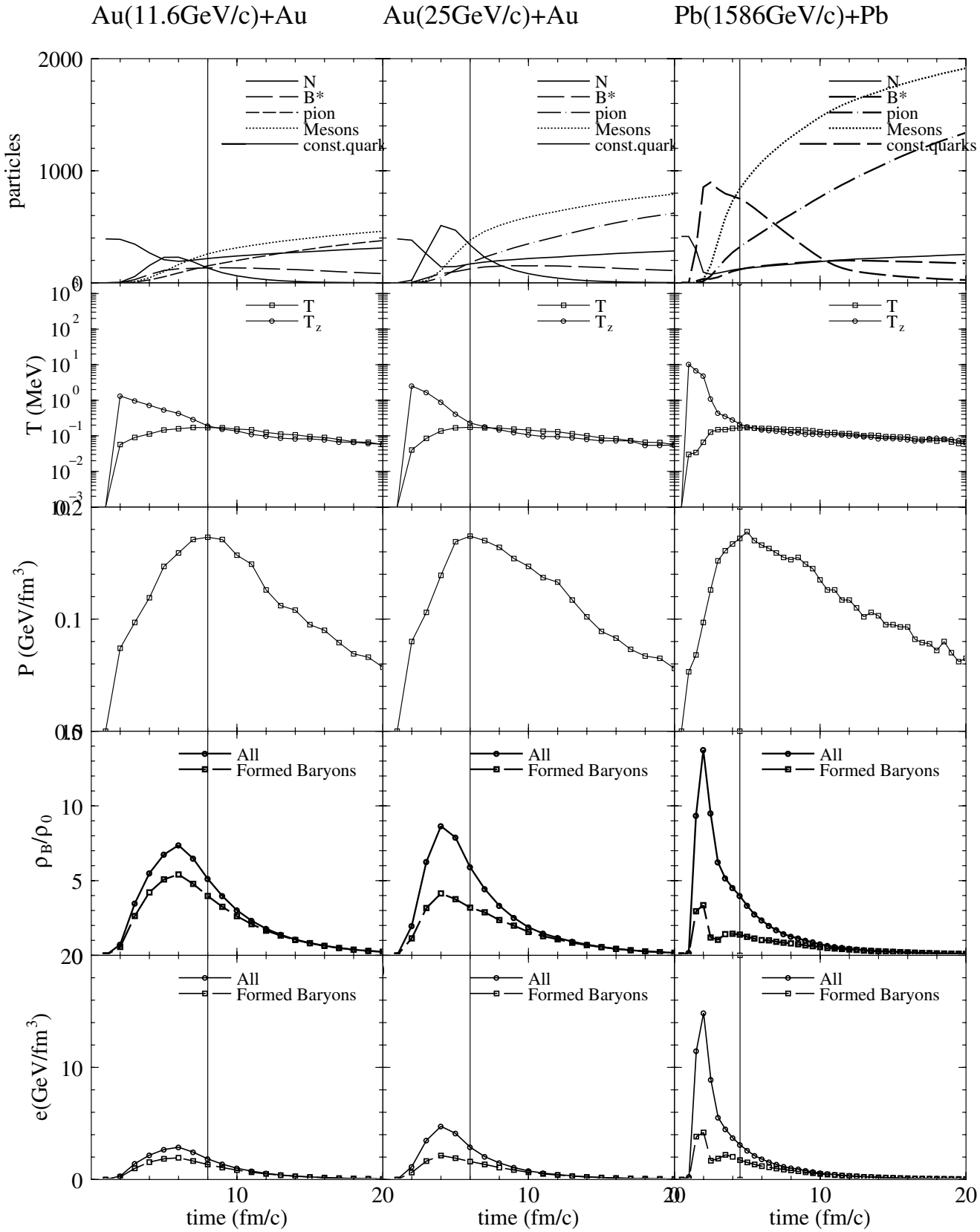
- Transport model based on hadron degree of freedom (JAM)
- hadronic resonances + strings + hard scattering
- No parton-parton collisions.

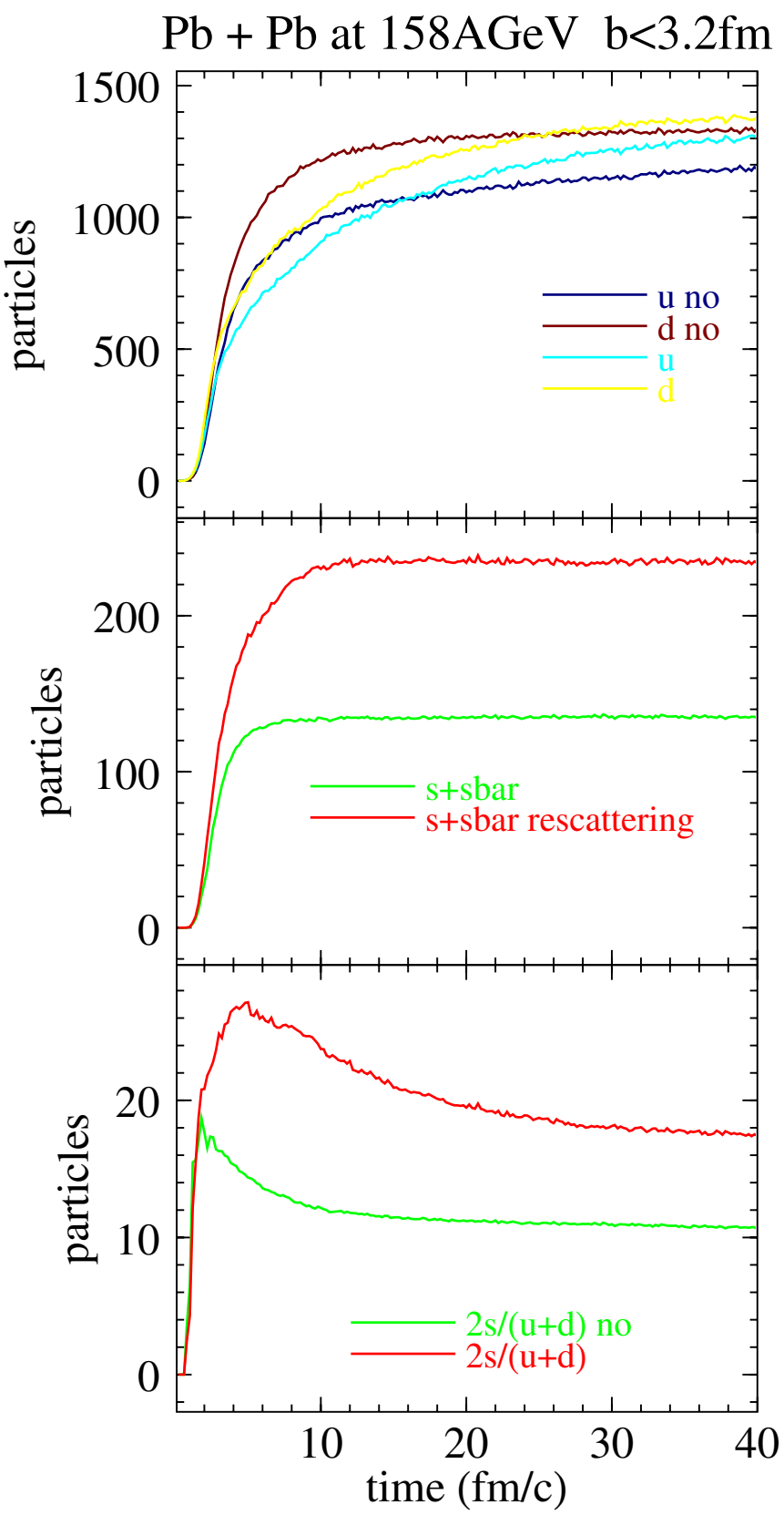

Si + Al at 14.6AGeV
particles
100
50
0
u no
d no
u
d
particles
6
4
2
0
s+sbar
s+sbar rescattering
particles ratio (%)
8
6
4
2
0
s/(u+d) no
s/(u+d)
10
20
30
40
time (fm/c)

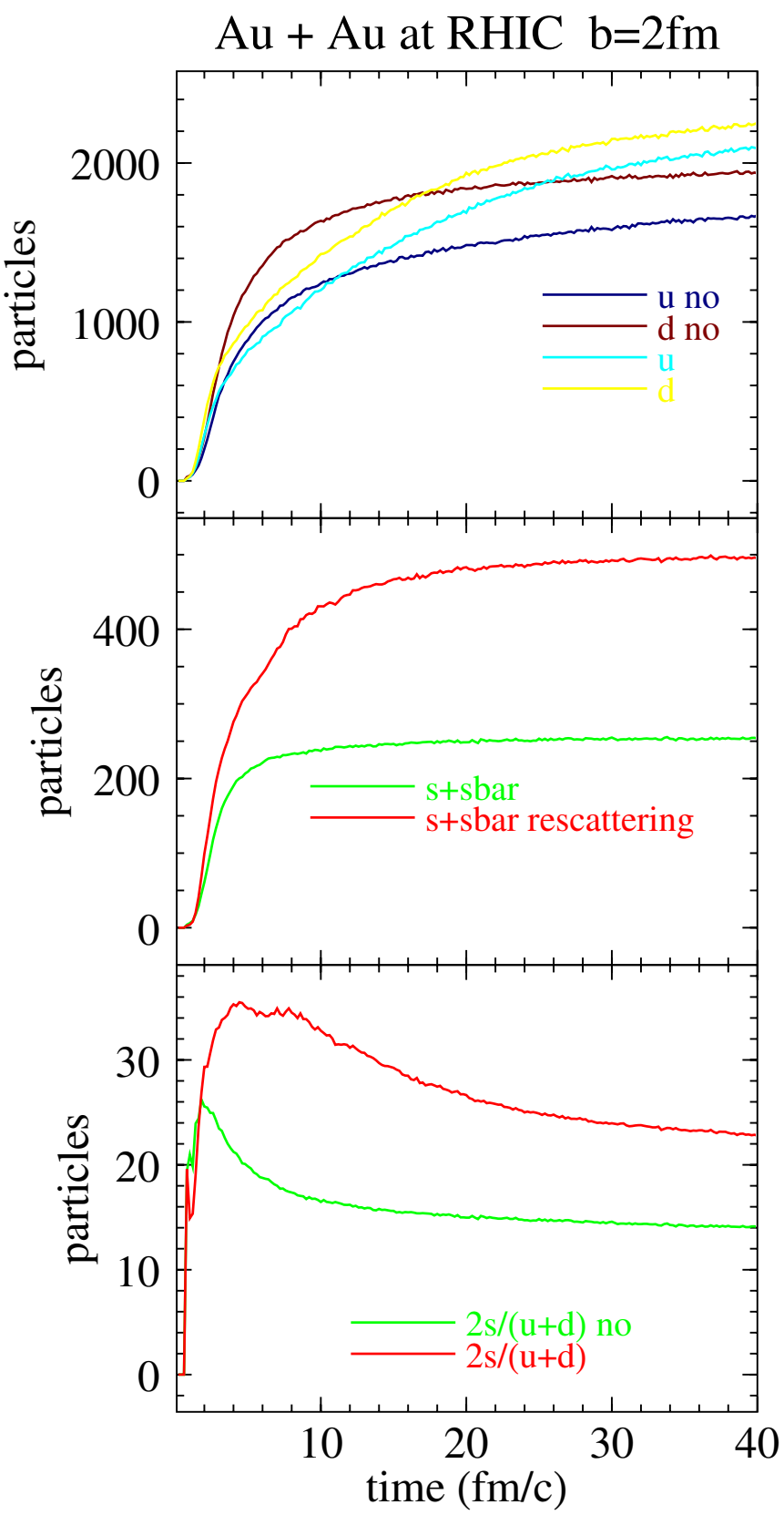

S + S at 200AGeV
particles
100
0
u no
d no
u
d
20
10
0
s+sbar
s+sbar rescattering
s/u no
s/u
10
20
30
40
time (fm/c)

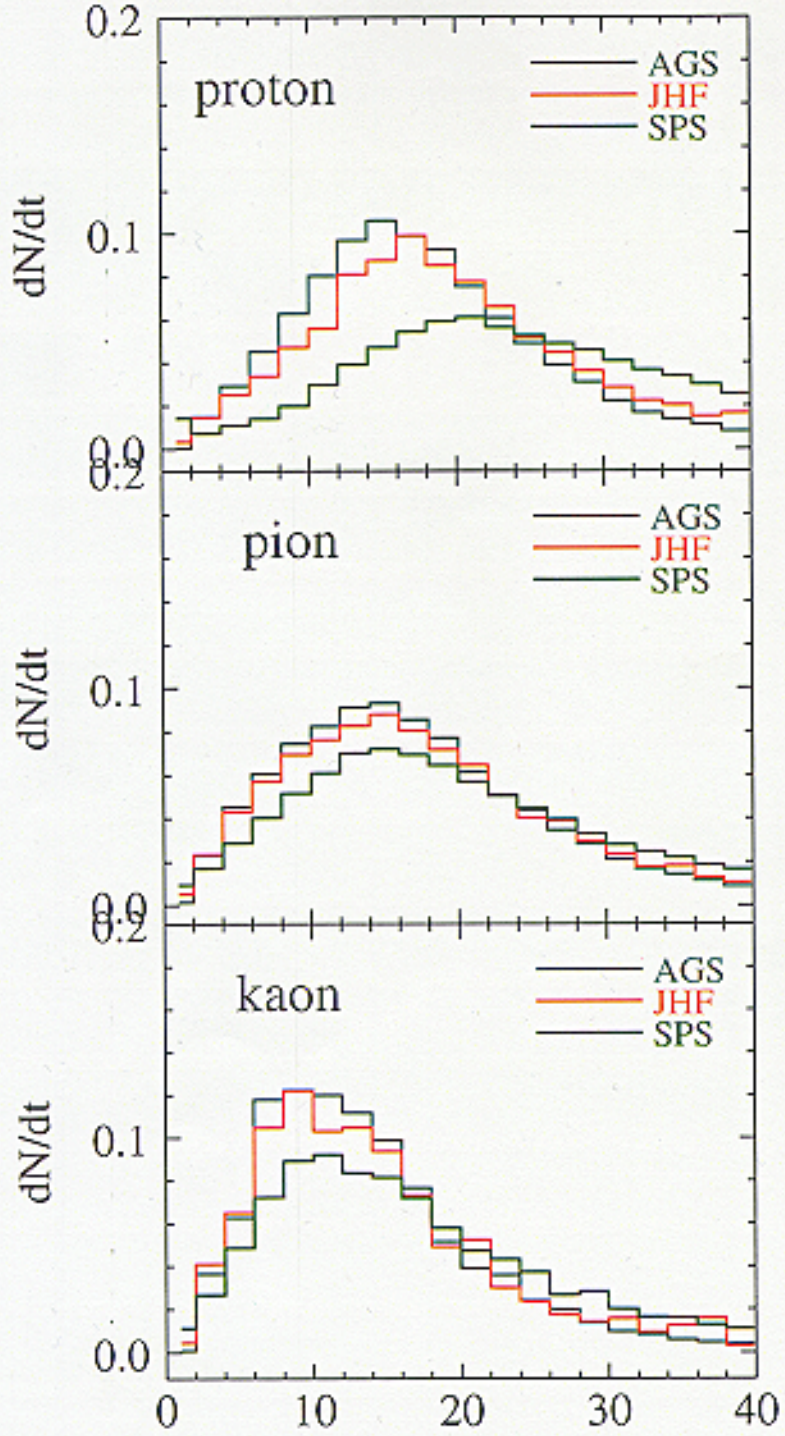

S+S at RHIC
particles
200
100
0
u no
d no
u
d
40
30
20
10
0
particles
s+sbar
s+sbar rescattering
30
20
10
0
particles
2s/(u+d) no
2s/(u+d)
10
20
30
40
time (fm/c)

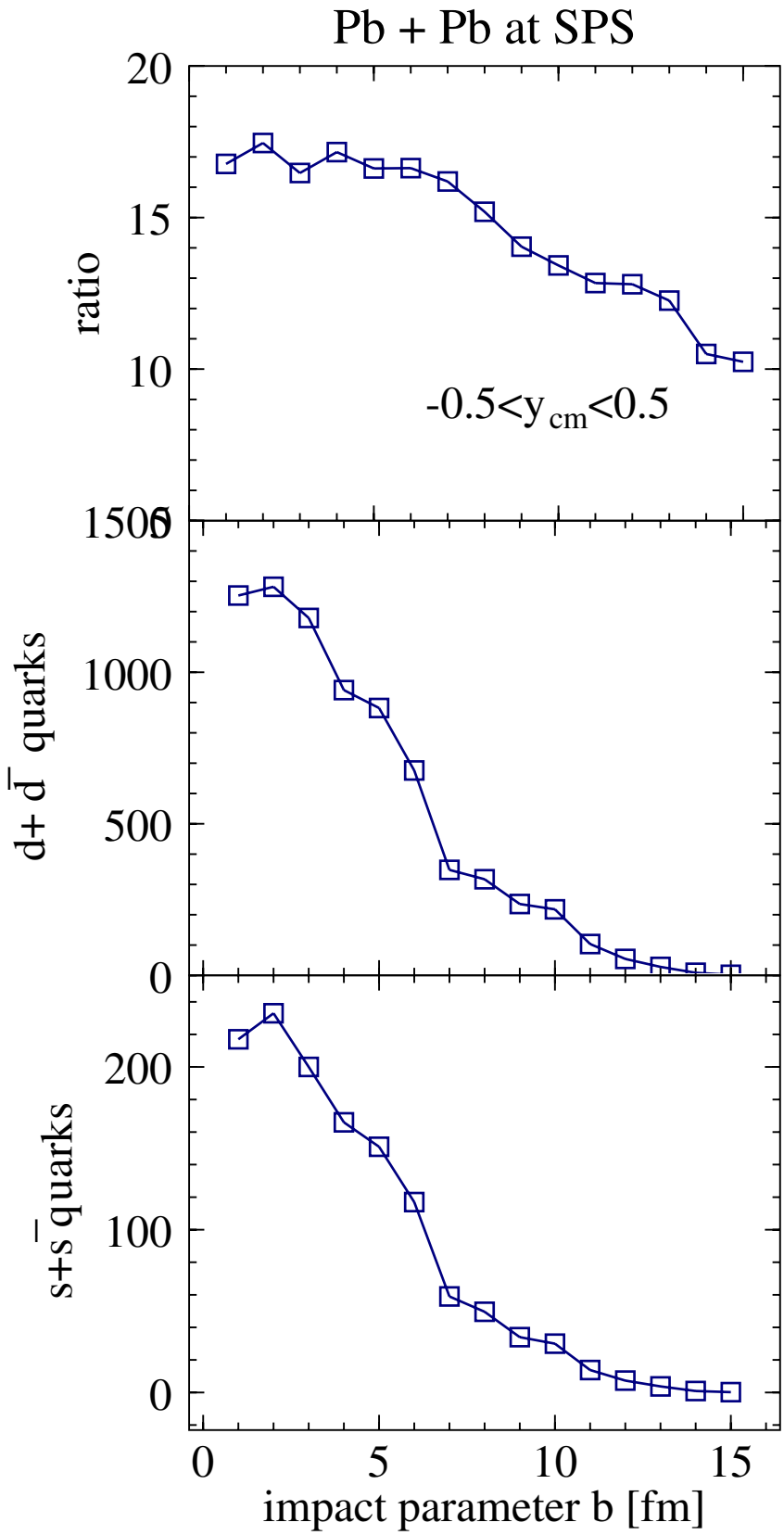

Au + Au at 11.6AGeV
1000
500
0
particles
u no
d no
u
d
60
40
20
0
particles
s+sbar
s+sbar rescattering
10
5
0
particles
s/u no
s/u
10
20
30
40
time (fm/c)


Au(11.6GeV/c)+Au
Au(25GeV/c)+Au
Pb(1586GeV/c)+Pb
particles
N
B*
pion
Mesons
const.quark
const.quarks
T (MeV)
T
T_z
P (GeV/fm^3)
ρ_B/ρ_0
All
Formed Baryons
e(GeV/fm^3)
time (fm/c)

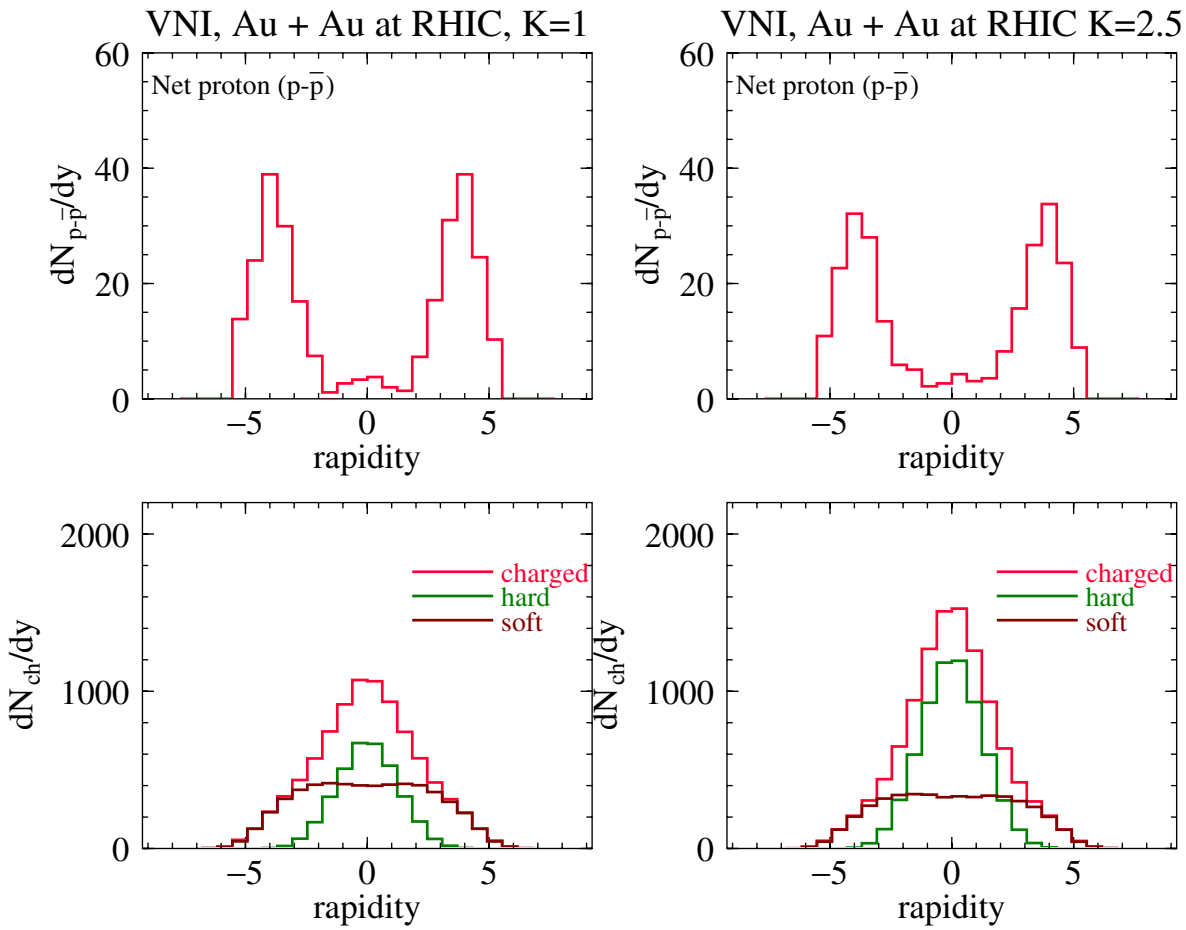

Pb + Pb at 158AGeV b<3.2fm
particles
1500
1000
500
0
u no
d no
u
d
particles
200
100
0
s+sbar
s+sbar rescattering
particles
20
10
0
2s/(u+d) no
2s/(u+d)
10
20
30
40
time (fm/c)


Au + Au at RHIC b=2fm
particles
2000
1000
0
u no
d no
u
d
400
200
0
s+sbar
s+sbar rescattering
30
20
10
0
2s/(u+d) no
2s/(u+d)
10
20
30
40
time (fm/c)

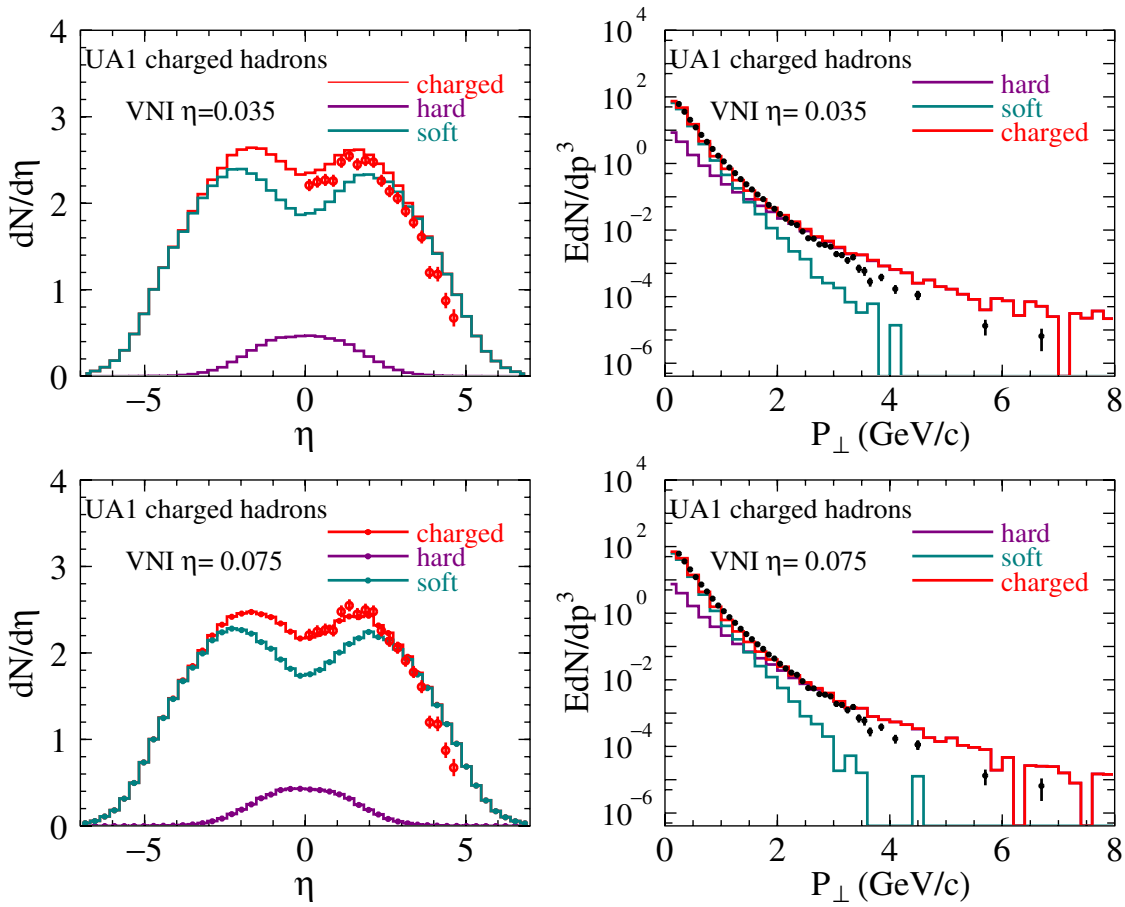

proton
pion
kaon
AGS
JHF
SPS
dN/dt
0.0
0.1
0.2
0
10
20
30
40


Pb + Pb at SPS
ratio
-0.5<y cm<0.5
d+ d̄ quarks
s+s̄ quarks
impact parameter b [fm]
20
15
10
5
1500
1000
500
0
200
100
0
0
5
10
15

Au+Au at RHIC from VNI



$p+\bar{p}$ at $\sqrt{s} = 200$GeV from VNI, η

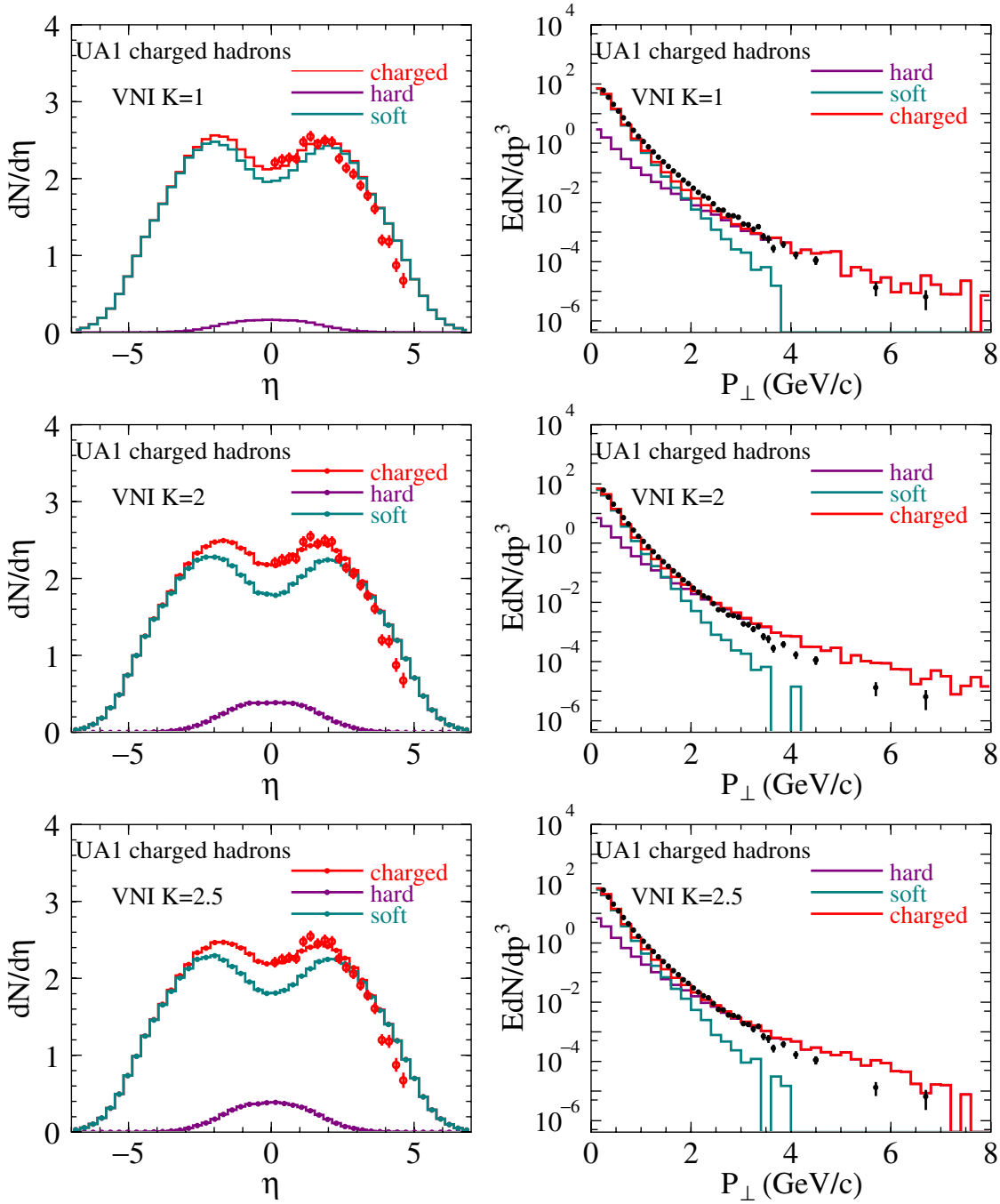


Higher order correation in pQCD

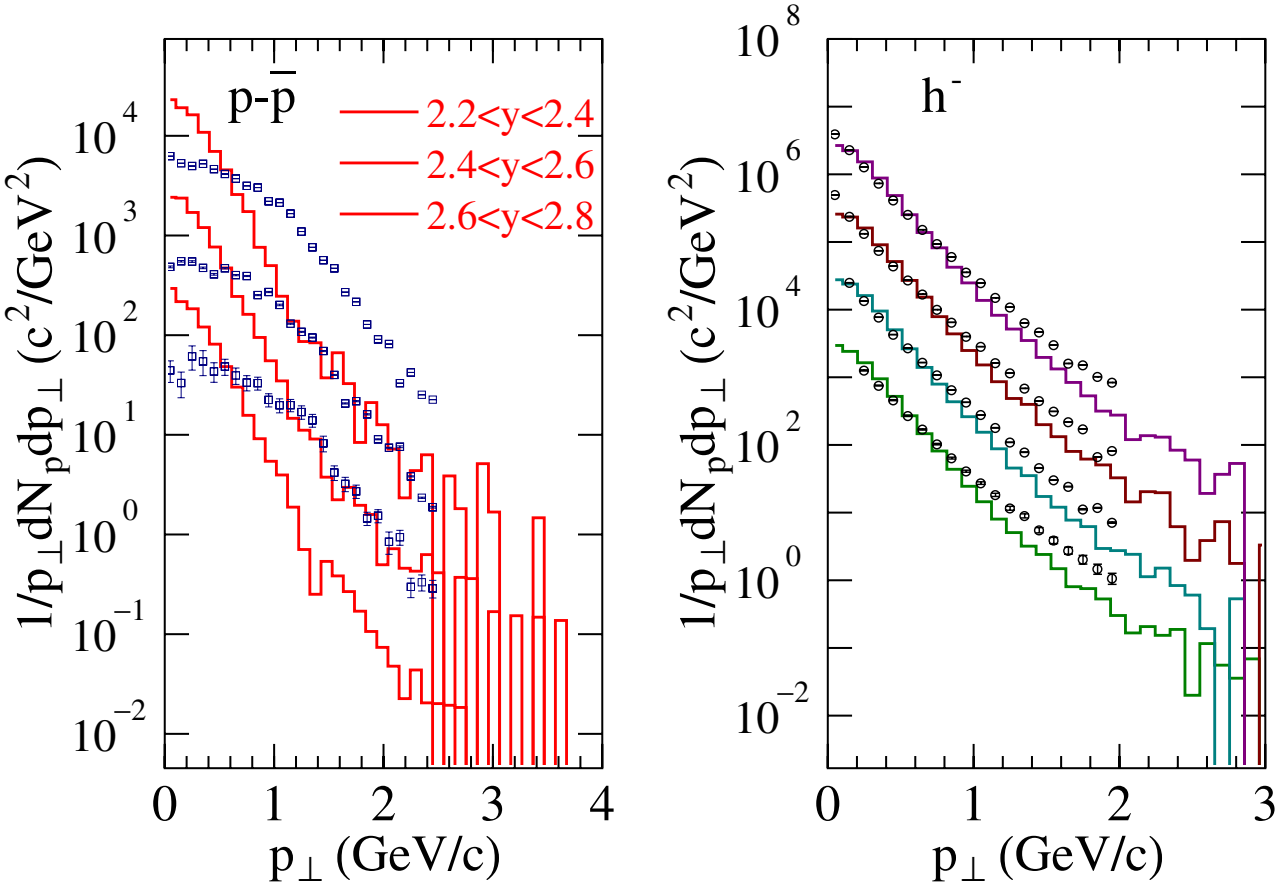
1) $\sigma = \sigma^{LO}(Q(\eta\alpha_s)), \qquad \eta = 0.035 \sim 0.075$

2) $\sigma = K\sigma^{LO}, \qquad K = 1.5 \sim 2.5$

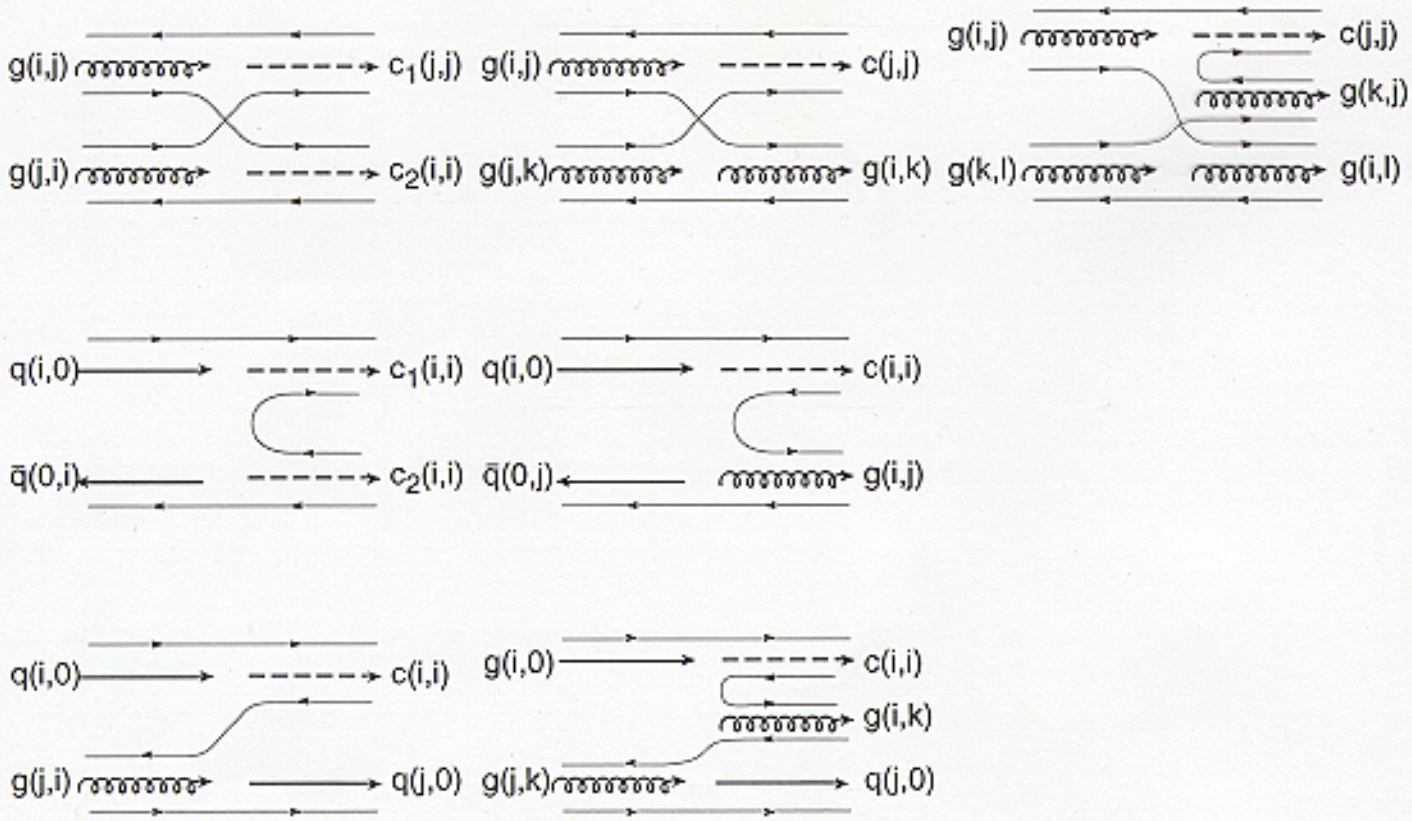
$p+\bar{p}$ at $\sqrt{s} = 200$GeV from VNI, K factor



VNI calculaton of $p_\perp$ spectra for Pb(158GeV)+Pb)



Cluster formation processes in the parton cascade model



$g + g \to C + C,$

$g + g \to C + g,$

$g + g \to C + g + g,$

$q + \bar{q} \to C + C,$

$q + \bar{q} \to C + g,$

$q + \bar{g} \to C + q,$

$q + \bar{g} \to C + q + g.$

$qq + q \to C$

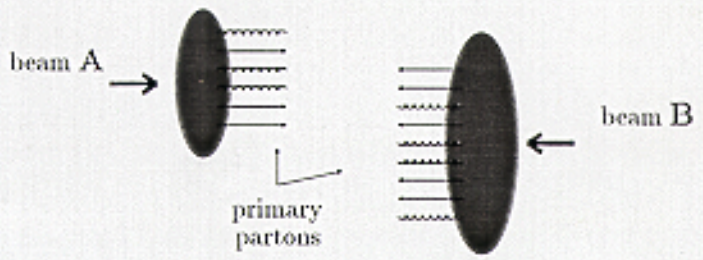
$\bar{q}\bar{q} + \bar{q} \to C$

$q_1 q_2 + \bar{q}_3 \to q_1 \bar{q}_3 + q_2$

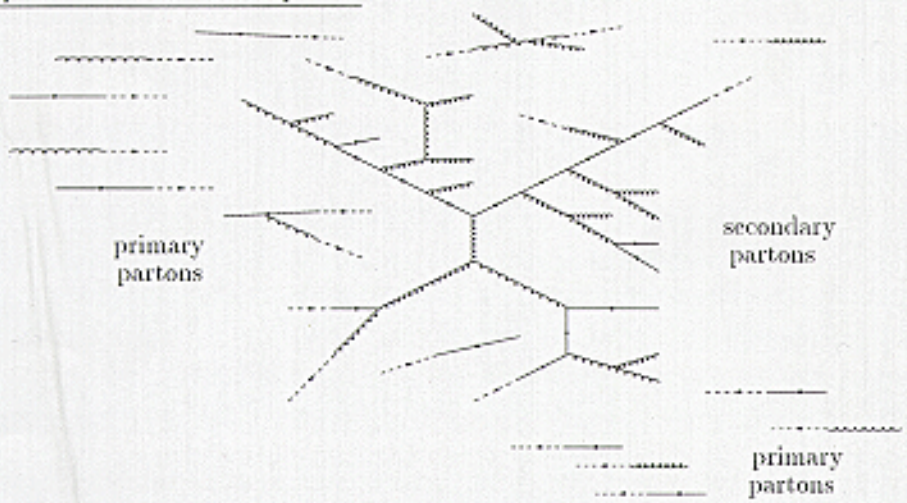
$q_1 q_2 + g \to q_1 q_2 q_3 + \bar{q}_3$

The components of the parton cascade model

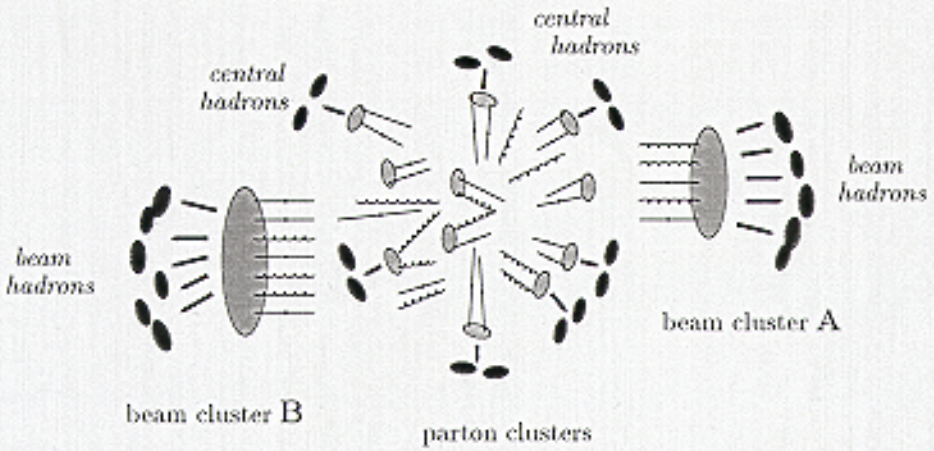
a) initial state

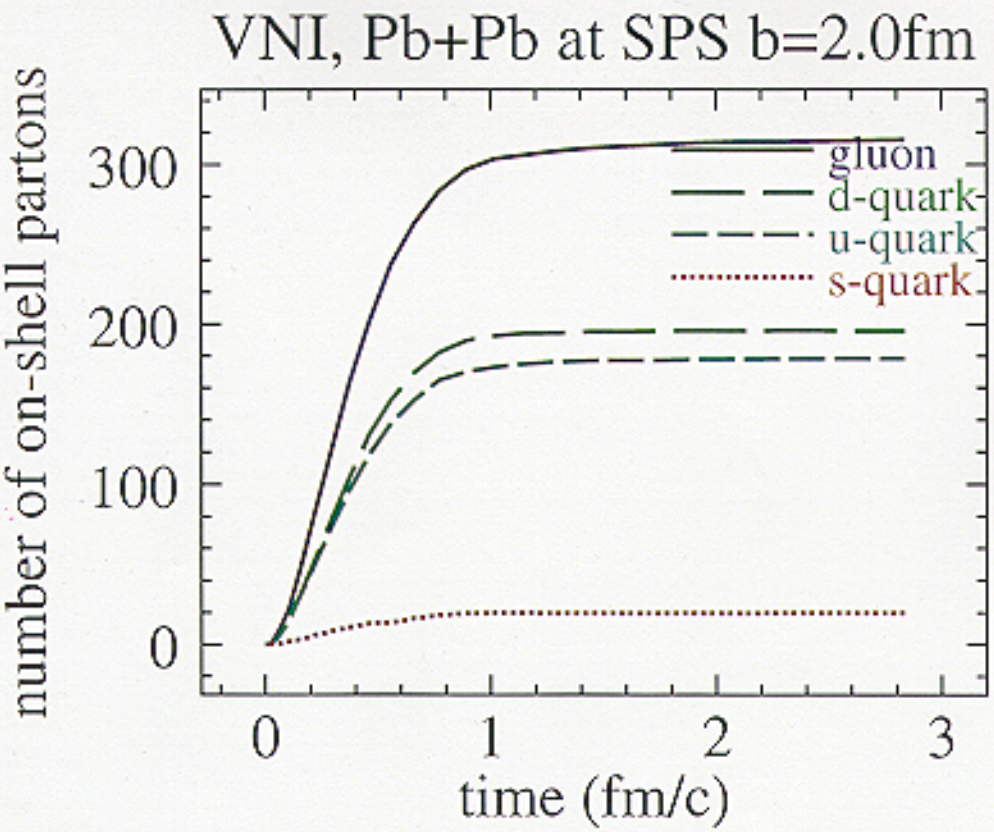


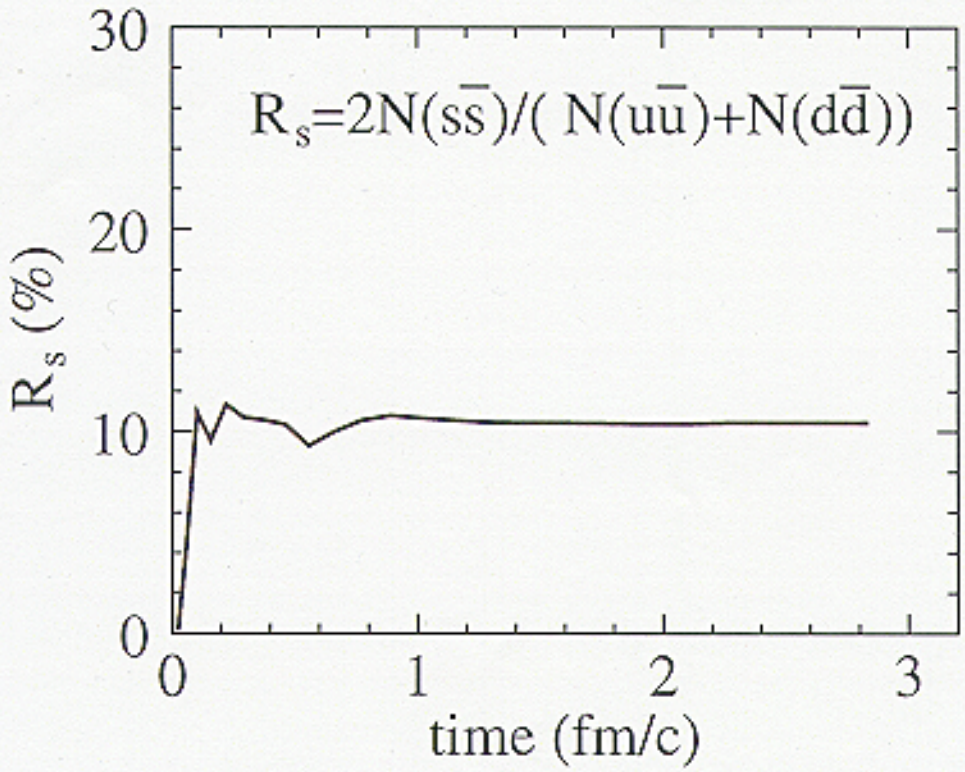
b) parton cascade development

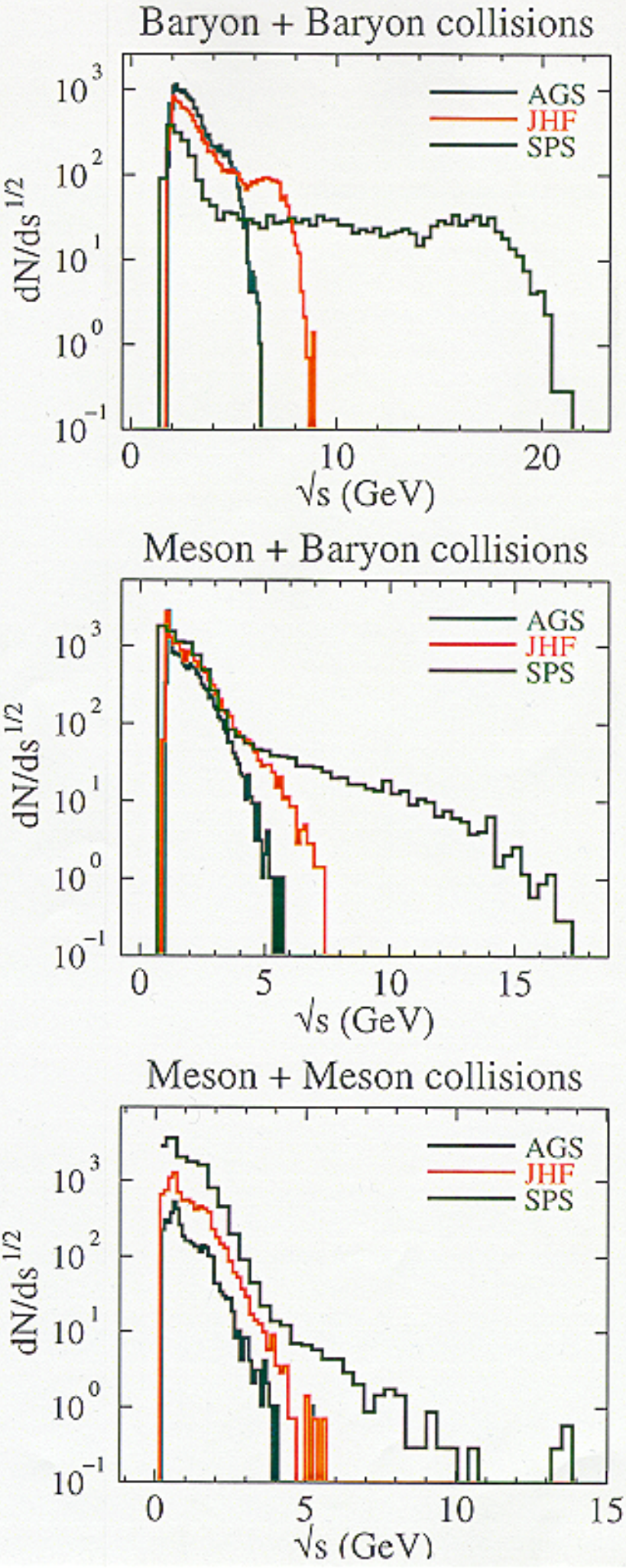


c) parton-cluster formation, hadronization, hadron cascade development




VNI, Pb+Pb at SPS b=2.0fm
number of on-shell partons
300
200
100
0
0
1
2
3
time (fm/c)
gluon
d-quark
u-quark
s-quark


$R_s=2N(s\bar{s})/(N(u\bar{u})+N(d\bar{d}))$
R_s (%)
30
20
10
0
0
1
2
3
time (fm/c)


Baryon + Baryon collisions
AGS
JHF
SPS
dN/ds^1/2
√s (GeV)
Meson + Baryon collisions
AGS
JHF
SPS
dN/ds^1/2
√s (GeV)
Meson + Meson collisions
AGS
JHF
SPS
dN/ds^1/2
√s (GeV)