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Basic Properties of the Higgs Boson in the Standard Model (SM)

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1. Basics

1-1 Review of Lagrangians

L $\equiv$ K.E. - V

Classical		Field theory
$L(q_i, \dot{q}_i, t)$	$\Leftrightarrow$	$\mathcal{L}(\phi, \frac{\partial \phi}{\partial x_\mu}, x_\mu)$
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$	$\Leftrightarrow$	$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial x_\mu)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$

$\mathcal{L}$ = K.E. + Mass + Interaction (F-B, B-B)

Fields	K.E.(and B-B)	Mass
Fermion	$i\bar{\psi}\partial_\mu \gamma^\mu \psi$	$-m\bar{\psi}\psi$
Complex Scalar	$(\partial^\mu\phi)^\star(\partial_\mu\phi)$ $\phi \equiv \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$	$-m^2\phi^\star\phi$
Aberian Boson	$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ $F^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu$	$+\frac{1}{2}m^2 B^\mu B_\mu$
Non-Aberian Boson	$-\frac{1}{4}W_{\mu\nu a}W_a^{\mu\nu}$ $W_a^{\mu\nu} \equiv \partial^\mu W^\nu - \partial^\nu W^\mu$ $+g f_{abc} W_b^\mu W_c^\nu$	$+\frac{1}{2}m^2 W_a^\mu W_{a\mu}$

Intercation F-B is described as $Q\bar{\psi}\gamma^\mu\psi A_\mu$

1-2 Local Gauge Invariance

Under the transformation, $\psi^{'}(x) \equiv e^{i\alpha(x)}\psi(x)$

$\psi^{'}(x) = \psi(x)$ holds.

For a massive fermion in a free potential

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

mass term is invariant

$$\psi \to e^{i\alpha(x)}\psi(x),\ \bar{\psi} \to e^{-i\alpha(x)}\psi(x)$$

K.E. term is not invariant

$$\partial\psi \to e^{i\alpha(x)}\partial_{\mu}\psi(x) + ie^{i\alpha(x)}\partial_{\mu}\alpha(x)$$

Introduce covariant derivative D_{μ} which transforms in the same way as ψ itself

$$D_{\mu} \to e^{i\alpha(x)}D_{\mu}$$

$$D_{\mu} \equiv \partial_{\mu} - ieA_{\mu}$$

$$A_{\mu} \to A_{\mu} + \frac{1}{e}\partial_{\mu}\alpha(x)$$

$$\begin{aligned}\mathcal{L} &= i\bar{\psi}\gamma^{\mu}D_{\mu}\psi - m\bar{\psi}\psi \\ &= i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + e\bar{\psi}\gamma^{\mu}\psi A_{\mu}\end{aligned}$$

By requiring the local gauge invariance, a new vector field A_{μ} emerges !!!

1-3 Lagrangian of mass-less unphysical fields in SM

Fermions in SM

$$\mathbf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

[redacted] e_R

$\times$ 3 generations

$$\mathbf{Q} = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R, d_R$$

Charges

$$Q = T_3 + \frac{Y}{2}$$

Q is electric charge
T is isospin
Y is week-hypercharge

fermion	Q	T	T_3	Y
ν_{eL}	0	$1/2$	$1/2$	-1
e_L	-1	$1/2$	$-1/2$	-1
u_L	$2/3$	$1/2$	$1/2$	$1/3$
d_L	$-1/3$	$1/2$	$-1/2$	$1/3$
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
e_R	-1	0	0	-2
u_R	$2/3$	0	0	$4/3$
d_R	$-1/3$	0	0	$-2/3$

$$f_{SM} \equiv \mathbf{L}, e_R, \mathbf{Q}, u_R, d_R$$

$$\mathcal{L}_{Ferm} = \sum_{f_{SM}} i\bar{f}\gamma^\mu D_\mu f$$

$$D_\mu = \partial_\mu - ig_1\frac{Y}{2}B_\mu - ig_2\frac{\tau^i}{2}W_\mu^i - ig_3\frac{\lambda^a}{2}G_\mu^a$$

where $\tau^i W_\mu^i \equiv \tau^1 W_\mu^1 + \tau^2 W_\mu^2 + \tau^3 W_\mu^3$

$$\lambda^a G_\mu^a \equiv \sum_{a=1}^{8} \lambda^a G_\mu^a$$

U(1)

$Y =$ constant

SU(2)

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SU(3)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

1-4 Lagrangian of mass-less physical fields in SM

$$\mathcal{L}_{Ferm} = \sum_{f=\nu_e,e,u,d} g_1 Q_f \bar{f}\gamma^\mu f A_\mu$$

$$+ \sum_{f=\nu_e,e,u,d} \frac{g_2}{\cos\theta_w}(T_f^3 - Q_f \sin^2\theta_w)\bar{f}_L\gamma^\mu f_L Z_\mu$$

$$+ \sum_{f=e,u,d} \frac{g_2}{\cos\theta_w}(-Q_f \sin^2\theta_w)\bar{f}_R\gamma^\mu f_R Z_\mu$$

$$+\frac{g_2}{\sqrt{2}}(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_{eL}\gamma^\mu e_L)W_\mu^+$$

$$+\frac{g_2}{\sqrt{2}}(\bar{d}_L\gamma^\mu u_L + \bar{e}_L\gamma^\mu \nu_{eL})W^{\mu-}$$

$$+ \sum_{q=u,d} \frac{g_3}{2}\lambda_{\alpha\beta}^a \bar{q}_\alpha\gamma^\mu q_\beta G_\mu^a$$

where

$$\begin{aligned} W^+ &= (-W^1 + iW^2)/\sqrt{2} \\ W^- &= (-W^1 - iW^2)/\sqrt{2} \\ W^0 &= -W^3 \end{aligned}$$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^0 \end{pmatrix}$$

$$\sin\theta_w = \frac{g_1}{\sqrt{g_1^2+g_2^2}},\ \cos\theta_w = \frac{g_2}{\sqrt{g_1^2+g_2^2}},\ e = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}},\ \sin^2\theta_w = 0.23$$

2. Why we need Higgs in SM

2-1 Where is mass ?

If mass terms were added by hand.....

In U(1) gauge transformation, $A^\mu \to A^\mu + \frac{1}{e}\partial_\mu \alpha(x)$

the mass term $\frac{1}{2}m^2 A^\mu A_\mu$ is not invariant clearly.

In the case of a SU(2) fermion, current term is

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R \ .$$

On the other hand the mass term is

$$m\bar{\psi}\psi = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$$

which contains only doublet but not singlet,

therefore breaks SU(2) invariant Lagrangian.

We need a gauge invariant Lagrangian
which generates masses of fermions
and physical gauge bosons !!!

From hep-ph/9708417 20 Aug. 1997

Higgs Boson: Keyboards

Higgs Boson is a classically trained Jazz pianist with a taste for rock and pop music. His influences are far and wide, ranging from Debussy to Pink Floyd and Dave Grusin to Chick Corea.

Instruments:- Ensoniq VFX SD11 / Roland JV 1080 - 880 - U220 - JD 800 / Korg i3 - Wavestation AD / Alesis D4 / E111 Sampler / Akai S1000 / TG100 / Emu Proteus 1 / Juno 6Instruments:- Ensoniq VFX SD11 / Roland JV 1080 - 880 - U220 - JD 800 / Korg i3 - Wavestation AD / Alesis D4 / E111 Sampler / Akai S1000 / TG100 / Emu Proteus 1 / Juno 6.

Email Higgs Boson:- hboson@enterprise.net

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2-2 Introduce Higgs Lagrangian

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$$

where $V(\phi) \equiv \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \begin{array}{ll} \phi^+ & = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \phi^0 & = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{array}$$

$$\phi^\dagger \phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$$

In $-\mu^2 < 0$, find $\phi^\dagger \phi$ where $V(\phi)$ becomes minimum

$$\frac{\partial V}{\partial(\phi^\dagger \phi)} = \mu^2 + 2\lambda \phi^\dagger \phi = 0, \ \phi_0^\dagger \phi_0 = -\frac{\mu^2}{2\lambda}$$

Choose ϕ_0 so that it satisfies electric charge conservation $\phi_1 = \phi_2 = \phi_3 = 0, \phi_4 = v$

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \text{ then } \phi_0^\dagger \phi_0 = \frac{v^2}{2} = -\frac{\mu^2}{2\lambda}$$

Expand around ϕ_0 so that Higgs field, $H(x)$ and week gauge bosons can have correct mass terms

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

2-3 Generate week boson mass

Out of $(D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$, relevant terms are

$$\phi^\dagger \left(ig_1\frac{Y}{2}B_\mu + ig_2\frac{\tau^i}{2}W^i_\mu\right)^\dagger \left(ig_1\frac{Y}{2}B_\mu + ig_2\frac{\tau^i}{2}W^i_\mu\right)\phi$$

$$= \left(\frac{1}{2}vg_2\right)^2 W^+_\mu W^{\mu-} + \frac{v^2}{8}(g_1^2+g_2^2)Z_\mu Z^\mu$$

$$+\frac{g_2^2 v}{2}HW^+_\mu W^{\mu-} + \frac{1}{4}(g_1^2+g_2^2)vHZ_\mu Z^\mu$$

$$+\frac{g_2^2}{4}HHW^+_\mu W^{\mu-} + \frac{1}{8}(g_1^2+g_2^2)HHZ_\mu Z^\mu$$

$$= M_W^2 W^+_\mu W^{\mu-} + M_Z^2 Z_\mu Z^\mu$$

$$+g_2 M_W HW^+_\mu W^{\mu-} + \frac{g_2 M_z^2}{2M_W}HZ_\mu Z^\mu$$

$$+\frac{g_2^2}{4}HHW^+_\mu W^{\mu-} + \frac{g_2^2}{8\cos^2\theta_w}HHZ_\mu Z^\mu$$

$$HWW > HZZ >> HHWW > HHZZ$$

2-4 Generate fermion mass

Mass of lepton

$$\mathcal{L}_{Lep} = g_e(\bar{\mathbf{L}}\phi e_R + \bar{e_R}\phi^\dagger \mathbf{L})$$

$$= \frac{g_e v}{\sqrt{2}}(\bar{e_L}e_R + \bar{e_R}e_L) + \frac{g_e}{\sqrt{2}}(\bar{e_L}e_R + \bar{e_R}e_L)H$$

$$= m_e\bar{e}e + \frac{m_e}{v}\bar{e}eH$$

Mass of quark

u_R must have mass, thus we need another Higgs doublet, ϕ_c which is isospin conjugate of ϕ

$$\phi_c = e^{-i\pi\tau^2/2}\phi^\star = -i\tau^2\phi^\star = \frac{1}{\sqrt{2}}\begin{pmatrix} -\phi^{0\star} \\ \phi^- \end{pmatrix} \to \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

$$\mathcal{L}_Q = g_d\bar{\mathbf{Q}}_L\phi d_R + g_u\bar{\mathbf{Q}}_L\phi_c u_R + g_d\bar{d}_R\phi^\dagger\mathbf{Q}_L + g_u\bar{u}_R\phi_c^\dagger\mathbf{Q}_L$$

$$= \frac{g_d}{\sqrt{2}}(v+H)(\bar{d_L}d_R + \bar{d_R}d_L) + \frac{g_u}{\sqrt{2}}(v+H)(\bar{u_L}u_R + \bar{u_R}u_L)$$

$$= \frac{g_d v}{\sqrt{2}}\bar{d}d + \frac{g_d}{\sqrt{2}}\bar{d}dH + \frac{g_u v}{\sqrt{2}}\bar{u}u + \frac{g_u}{\sqrt{2}}\bar{u}uH$$

$$= m_d\bar{d}d + \frac{m_d}{v}\bar{d}dH + m_u\bar{u}u + \frac{m_u}{v}\bar{u}uH$$

2-5 Summary of SM and Higgs

$$\mathcal{L}_{SM} = \mathcal{L}_{Ferm} + \mathcal{L}_{Higgs} + \mathcal{L}_{Lep} + \mathcal{L}_{Q} - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

Scenario toward Higgs

- To guarantee the gauge invariance, the covariant derivatives must be defined which contains gauge bosons consequently.
- We constructed Lagrangians for the SM fermions requiring the gauge invariance.
- The mass-less fermion Lagrangian can fit the physical particles, if theoretical fields are interpreted correctly.
- However, if one adds masses to physical particles by hand, it breaks the gauge invariance.
- We need a gauge invariant potential so that it can give masses to fermions and gauge bosons by breaking symmetry. That is Higgs!!!.

What SM can not predict

- The number of generations
- Coupling strength
- Masses of fermions
- Mass of Higgs

3. Properties of SM Higgs

3-1 Decay of SM Higgs

Points !!!

$$g_f = \frac{g_2}{2M_W} M_f$$

$$\Gamma_{f\bar{f}} \propto m_f^2 M_H$$

$$\Gamma_{WW or ZZ} \propto M_H^3$$

$$\Gamma_{gg} \propto \alpha_s^2 M_H^3$$

For low mass region $M_H < 130$ GeV

$$H \to b\bar{b}, c\bar{c}, \tau\bar{\tau}$$

$$H \to gg$$

$$H \to \gamma\gamma$$

$$H \to Z\gamma$$

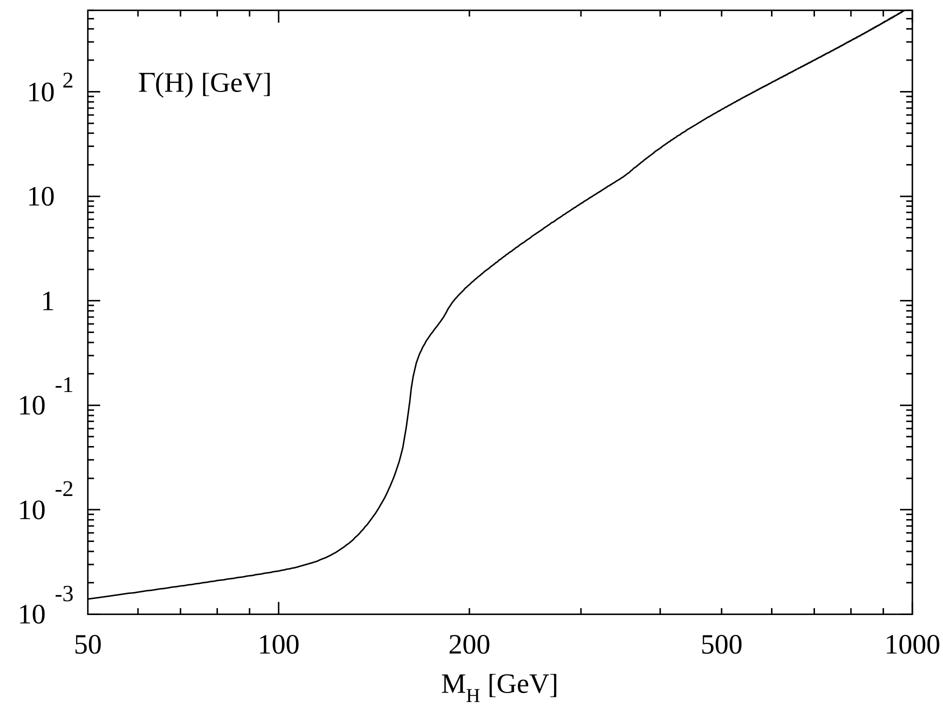
For high mass region $M_H > 130$ GeV

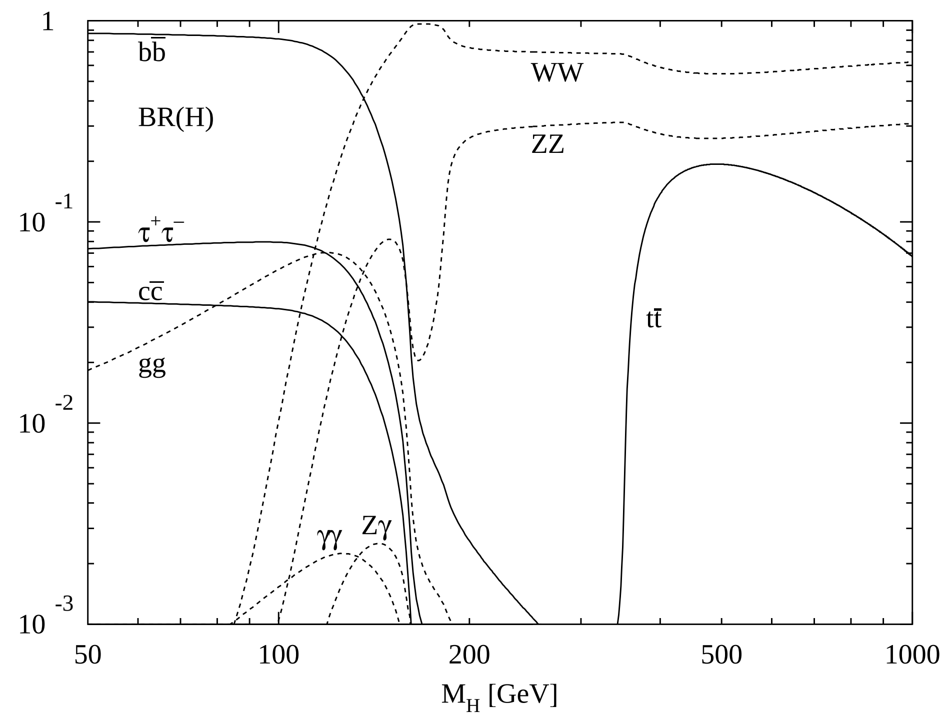
$$H \to WW$$

$$H \to ZZ$$

$$H \to t\bar{t}$$

From hep-ph/9712334 10 Dec. 1997





3-2 Production of SM Higgs

At e^+e^- colliders

$e^+e^- \to (Z) \to Z + H$

$e^+e^- \to \nu\bar{\nu}(WW) \to \nu\bar{\nu} + H$

$e^+e^- \to e^+e^-(ZZ) \to e^+e^- + H$

$e^+e^- \to (\gamma, Z) \to t\bar{t} + H$

At pp colliders

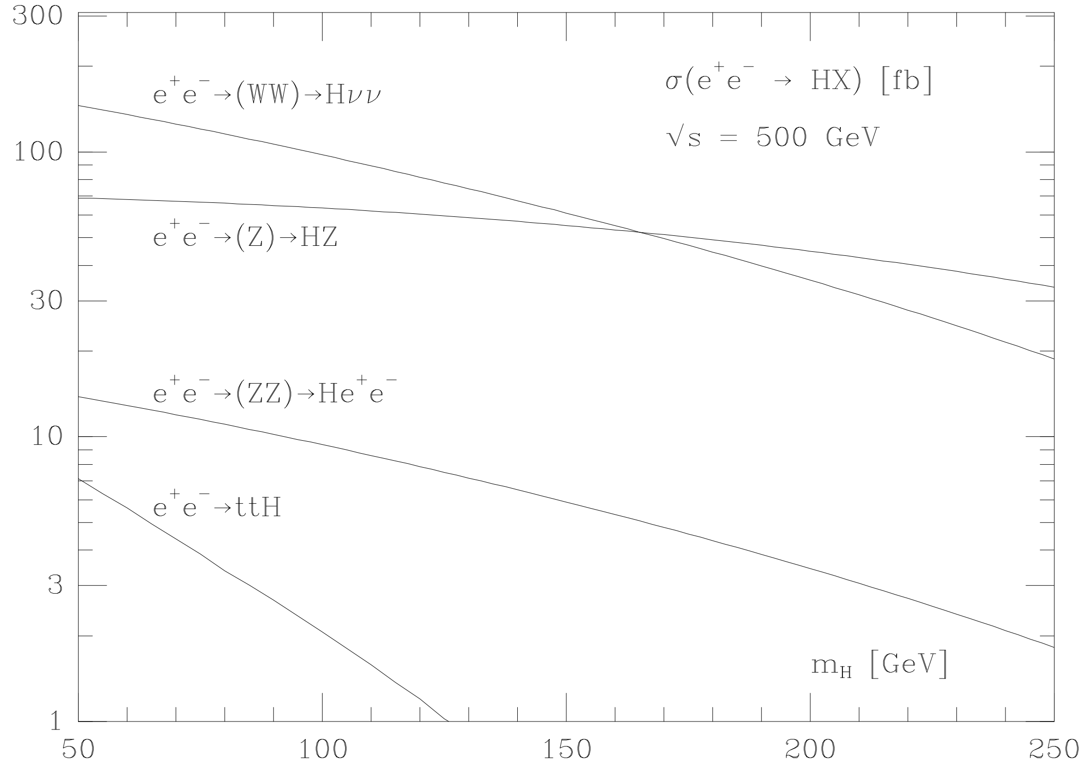
$gg \to H$

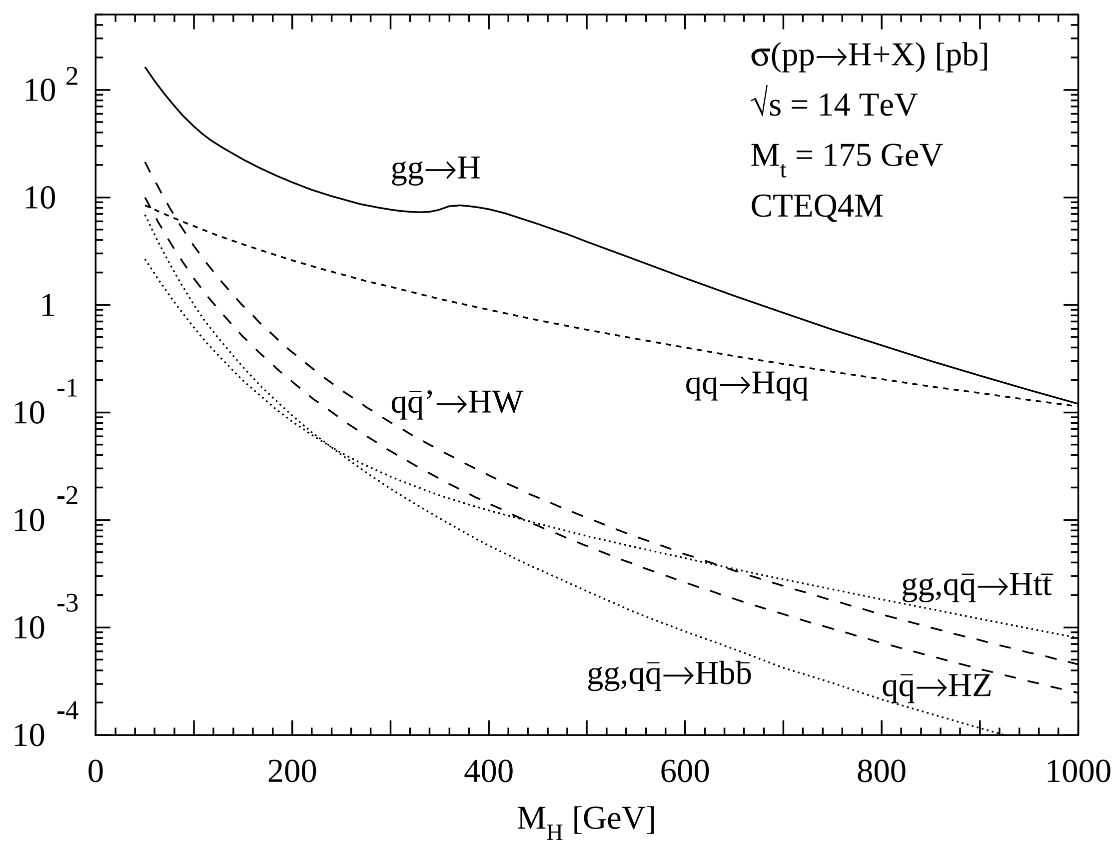
$WW/ZZ \to H$

$q\bar{q} \to W/Z + H$

$gg, q\bar{q} \to t\bar{t} + H$

From hep-ph/9710439 22 Oct. 1997





3-3 Mass bound on SM Higgs

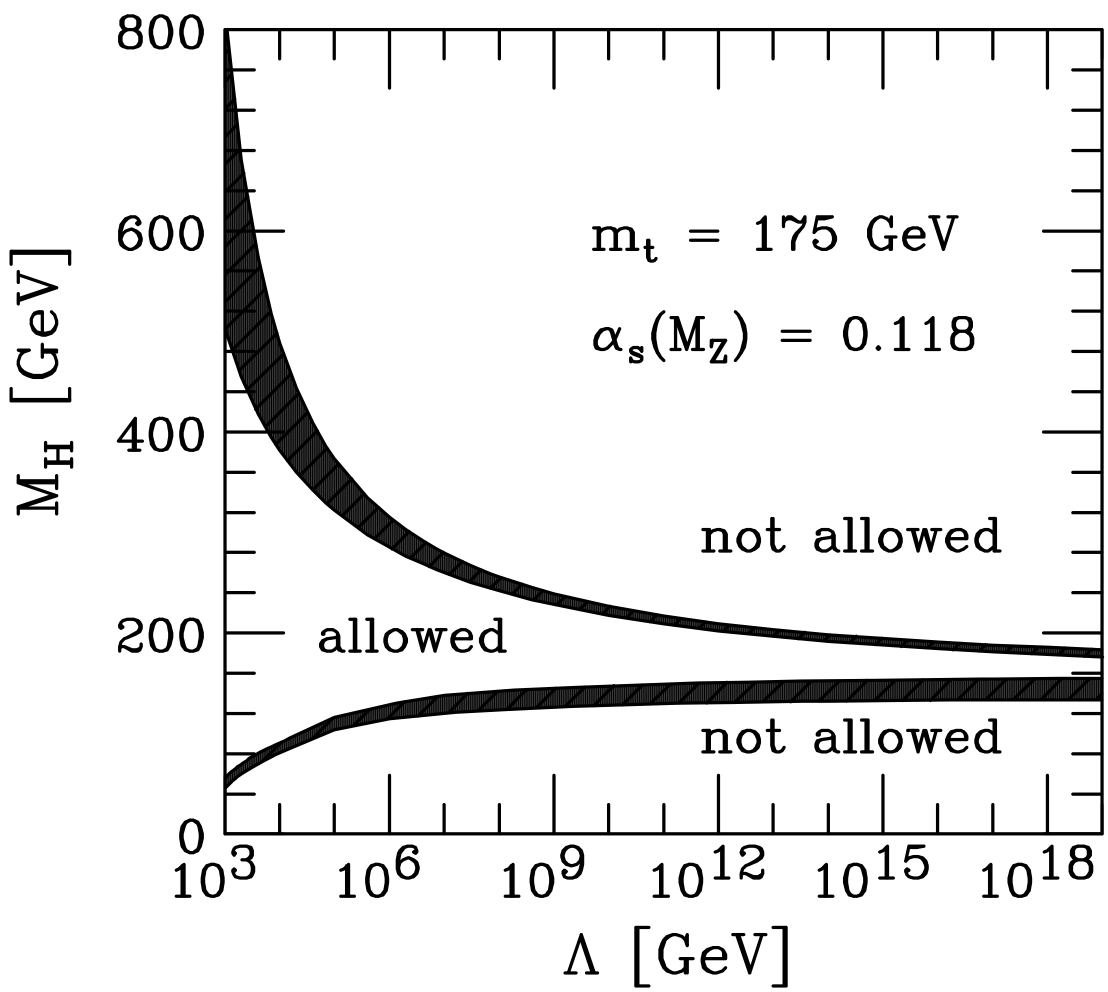
$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$v = \sqrt{-\mu^2/\lambda}, \qquad \mu^2 < 0$$

- Upper bound can be constrained by requiring that perturbative theory is valid up to the GUT scale.

- Lower bound can be determined by requiring $\lambda > 0$ up to the GUT scale which corresponds to the requirement of vacuum stability.

From hep-ph/9708416 20 Aug. 1997



If $160 < M_H < 170$ GeV, the perturbative treatment is valid up to GUT scale, $\Lambda \sim 10^{19}$GeV.