PHENIX results on the Lévy analysis of Bose-Einstein correlation functions

Sándor Lökös for the PHENIX Collaboration

Eötvös University, Budapest, Hungary
Outline

1. The PHENIX experiment
2. Bose-Einstein correlations
3. Lévy-type HBT and the critical point
4. PHENIX Lévy HBT results: $\sqrt{s_{NN}} = 200$ GeV, MinBias
5. PHENIX Lévy HBT results: $\sqrt{s_{NN}}$ & centrality dependence
6. Summary
The PHENIX Experiment

- Versatile detector, operating until 2016
- Tracking via Drift Chambers and Pad Chambers
- Charged pion ID with TOF, from \( \sim 0.2 \) to 2 GeV/c
- This analysis: PID also with EMCal
The PHENIX Experiment

- Versatile detector, operating until 2016
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PHENIX runs at a glance

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○ p+p  ○ Au+Au  ○ d+Au  ○ Cu+Cu  ○ U+U  ○ Cu+Au  ○ He+Au  ○ p+Au  ○ p+Al

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1. The PHENIX experiment
2. Bose-Einstein correlations
3. Lévy-type HBT and the critical point
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6. Summary
Bose-Einstein correlations in heavy ion physics

- Quantum statistics connects spatial and momentum space distributions
- Spatial source $S(x)$ versus momentum correlation function $C_2(q)$:
  \[
  C_2(q) \approx 1 + \left| \frac{\tilde{S}(q)}{\tilde{S}(0)} \right|^2, \quad \tilde{S}(q) = \int S(x) e^{i q x} d^4 x, \quad q = p_1 - p_2
  \]
- Final state interactions distort the simple Bose-Einstein picture
- Coulomb interaction important, handled via two-particle wave function
- Resonance pions: Halo around primordial Core

The out-side-long system, HBT radii

- $C(q)$ usually measured in the Bertsch-Pratt pair coordinate-system
  - out: direction of the average transverse momentum ($K_t$)
  - long: beam direction
  - side: orthogonal to the latter two
- $R_{out}, R_{side}, R_{long}$: HBT radii
- Out-side difference $\rightarrow \Delta \tau$ emission duration
- From a simple hydro calculation:
  
  $R_{out}^2 = \frac{R^2}{1 + u_T^2 m_T / T_0} + \beta_T^2 \Delta \tau^2$

  $R_{side}^2 = \frac{R^2}{1 + u_T^2 m_T / T_0}$

- RHIC: ratio is near one $\rightarrow$ no strong 1st order phase transition
- Plus lots of other details (pre-eq flow, initial state, EoS, ...)

Example: recent PHENIX HBT measurements

- Corr. func. in Bertsch-Pratt system, radii from Gaussian fit
- Linear $1/\sqrt{m_T}$ scaling of HBT radii for all systems and energies
- Interpolation to common $m_T$, PHENIX and STAR consistent

HBT radii and the search for the critical endpoint

- Signals of QCD CEP: softest point, long emission
- $R_o^2 - R_s^2$: related to emission duration
- $(R_s - \sqrt{2} \cdot \bar{R})/R_l$: related to expansion velocity
- Non-monotonic patterns
- Indication of the CEP?
- Further detailed studies done
- Maybe Levy exponent $\alpha$ gives further insight?
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Lévy distributions in heavy ion physics

- Expanding medium, increasing mean free path: anomalous diffusion
  

- Lévy-stable distribution: \( \mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2}|qR|^{\alpha}} \)
  
  - Generalized Gaussian from generalized central limit theorem
  - \( \alpha = 2 \) Gaussian, \( \alpha = 1 \) Cauchy

- Shape of the correlation functions with Levy source:

  \[
  C_2(q) = 1 + \lambda \cdot e^{-(Rq)^\alpha}
  \]
  
  \( \alpha = 2 \): Gaussian
  \( \alpha = 1 \): Exponential

- Critical behaviour \( \rightarrow \) described by critical exponents

- Spatial corr. \( \propto r^{-(d-2+\eta)} \rightarrow \) defines \( \eta \) exponent

- Symmetric stable distributions (Levy) \( \rightarrow \) spatial corr. \( \propto r^{-1-\alpha} \)

- \( \alpha \) associated to critical exponent \( \eta \)

A possible way of finding the critical point

- QCD universality class ↔ 3D Ising

- At the critical point:
  - random field 3D Ising: $\eta = 0.50 \pm 0.05$
  - 3D Ising: $\eta = 0.03631(3)$

- Modulo finite size effects
- Distance from the critical point?
- Motivation for precise Levy HBT!
- Change in $\alpha_{\text{Levy}}$ ↔ proximity of CEP?
- Non-static system, finite size effects...
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Eötvös University
PHENIX Lévy HBT analysis

- Dataset used for the analysis:
  - Run-10, Au+Au, $\sqrt{s_{NN}} = 200$ GeV, $7.3 \cdot 10^9$ events
  - Minimum bias results so far

- Additional offline requirements:
  - Collision vertex position less than $\pm 30$ cm

- Particle identification:
  - time-of-flight data from PbSc e/w, TOF e/w, momentum, flight length
  - $2 \sigma$ cuts on $m^2$ distribution

- Single track cuts:
  - $2\sigma$ matching cuts in TOF & PbSc for pions

- Pair-cuts:
  - A random member of pairs assoc. with hits on same tower were removed
  - customary shaped cuts on $\Delta \varphi - \Delta z$ plane for PbSc e/w, TOF e/w

- 1D corr. func. as a function of $|k|_{LCMS}$ in various $m_T$ bins
  - Levy fits for 31 $m_T$ bins with Coulomb effect incorporated
  - Coulomb effect incorporated in fit function
Example $C(|k|_{\text{LCMS}})$ measurement result

Measured in 31 $m_T^2 = m^2 + p_T^2$ bins for $\pi^+\pi^+$ and $\pi^-\pi^-$ pairs

MinBias Au+Au @ $\sqrt{s_{\text{NN}}} = 200$ GeV, $\pi^+\pi^+$, $p_T = 0.2-0.22$ GeV/c

- $\lambda = 0.72 \pm 0.02$
- $R = 8.74$ fm $\pm 0.24$ fm
- $\alpha = 1.16 \pm 0.03$
- $\varepsilon = -0.102 \pm 0.005$
- $N = 1.0095 \pm 0.0005$

$\chi^2/\text{NDF} = 93/97$

Conf. level = 0.5909

Physical parameters: $R, \lambda, \alpha$; measured versus pair $m_T$
Example \( C(\|k\|_{\text{LCMS}}) \) measurement result

Measured in 31 \( m_T^2 = m^2 + p_T^2 \) bins for \( \pi^+\pi^+ \) and \( \pi^-\pi^- \) pairs

Physical parameters: \( R, \lambda, \alpha \); measured versus pair \( m_T \)
Levy scale parameter $R$

- Similar decreasing trend as Gaussian HBT radii
- Hydro predicts $1/R^2_{\text{Gauss}} = a + bm_T$
- Hydro behaviour not invalid for $R_{\text{Levy}}$
- The linear scaling of $1/R^2$, breaks for high $m_T$
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- The linear scaling of $1/R^2$, breaks for high $m_T$
Correlation strength $\lambda$

- From the Core-Halo model: $\lambda = \left( \frac{N_C}{N_C + N_H} \right)^2$
- Observed decrease ("hole") at small $m_T \rightarrow$ increase of halo fraction
- Different effects can cause change in $\lambda$
  - Resonance effects, partially coherent pion production
- $\frac{\lambda}{\lambda_{\text{max}}}$ with smaller systematic uncertainties
- Precise measurement may help extract physics info

Sándor Lőkös for PHENIX

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A possible (?) interpretation of $\lambda(m_T)$

- $\lambda(m_T)$ measures core/(core+halo) fraction
- May be connected to mass modifications (c.f. chiral restoration)
  - Decreased $\eta'$ mass $\rightarrow$ $\eta'$ enhancement $\rightarrow$ halo enhancement
  - Kinematics: $\eta'$ decay pions will have low $m_T$ $\rightarrow$ decreased $\lambda$ at small $m_T$
- Incompatibility with unmodified in-medium $\eta'$ mass?

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$\minbias$ Au+Au @ $\sqrt{s_{NN}} = 200$ GeV

$\lambda/\lambda_{\max}$ vs. $m_T$ [GeV/c$^2$]

$\lambda_{\max} = \langle \lambda \rangle_{(0.5-0.7)}$ GeV/c$^2$

$m_{\eta'} = 958$ MeV
$m_{\eta'} = 900$ MeV
$m_{\eta'} = 700$ MeV
$m_{\eta'} = 500$ MeV
$m_{\eta'} = 250$ MeV
$m_{\eta'} = 50$ MeV
Levy exponent $\alpha$

The measured value is far from Gaussian ($\alpha = 2$) and expo. ($\alpha = 1$)

Also far from the rfd.3D Ising value at CEP ($\alpha = 0.5$)

More or less constant (at least within systematic errors)

Motivation to do fits with fixed $\alpha = 1.134$

Note: $\alpha(m_T) = \text{const.}$ fit statistically not acceptable (only with syst.)
Levy scale parameter $R$ with fixed $\alpha = 1.134$

- More smooth trend
- Remarkable linearity of $1/R^2$
- Hydro behavior valid, despite $\alpha < 2$
Levy scale parameter $R$ with fixed $\alpha = 1.134$

- More smooth trend
- Remarkable linearity of $1/R^2$
- Hydro behavior valid, despite $\alpha < 2$
Newly discovered scaling parameter $\hat{R}$

- $R(m_T)$
- $\lambda(m_T)$
- $\alpha(m_T)$

MinBias Au+Au @ $1.96\ TeV = 200$ GeV

**PHENIX results**

$\sqrt{s_{NN}}$ & centrality dep.

Summary
Newly discovered scaling parameter $\hat{R}$

- Empirically found scaling parameter
- Linear in $m_T$
- Physical interpretation $→$ open question

$\minBias Au+Au @ \sqrt{s_{NN}} = 200$ GeV

$R(m_T)$

$\lambda(m_T)$

$\alpha(m_T)$

$\frac{1}{\hat{R}[1/fm]} = \frac{\lambda \cdot (1 + \alpha)}{R}$
Newly discovered scaling parameter $\hat{R}$

- $\alpha = 1.134$ fixed
- Empirically found scaling parameter
- Linear in $m_T$
- Physical interpretation $\rightarrow$ open question
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Lévy exponent $\alpha$ at 200 GeV

- Slightly non-monotonic behavior as a function of $m_T$
- Average $\langle \alpha \rangle$ non-monotonic behavior versus $N_{\text{part}}$
- $\alpha = \langle \alpha \rangle$ constant fits were performed
Lévy exponent $\alpha$ at 62 and 39 GeV

- Lévy exponent $\alpha$: no significant change vs $\sqrt{s_{NN}}$ at 39-62-200 GeV
- Usual values between 1 and 1.5
- Non-monotonicity in $m_T$
Lévy scale \( R \): similar trends for all \( \sqrt{s_{NN}} \) and cent.
“Hole” in $\lambda$: all energies and centralities

\begin{align*}
\text{PHENIX Au+Au & $\sqrt{s_{_{\text{NN}}}} = 200$ GeV} \\
\lambda & \text{ vs. } m_T \text{ [GeV/c}^2] \\
\end{align*}

\begin{align*}
\text{PHENIX Au+Au & $\sqrt{s_{_{\text{NN}}}} = 62$ GeV, $\pi^+\pi^+\pi^+$} \\
\end{align*}

\begin{align*}
\text{PHENIX Au+Au & $\sqrt{s_{_{\text{NN}}}} = 39$ GeV, $\pi^+\pi^+\pi^+$} \\
\end{align*}

\begin{align*}
\text{rel. syst. uncertainties} \\
\end{align*}
$\hat{R}$ scaling for all energies & centralities

**PHENIX Au+Au $\sqrt{s_{NN}} = 200$ GeV**

- $0-10\%$,
- $10-20\%$,
- $20-30\%$,
- $30-40\%$,
- $40-50\%$,
- $50-60\%$,

Linear fit: $\hat{R} = \frac{R}{\lambda(1+\alpha)}$

**PHENIX Au+Au $\sqrt{s_{NN}} = 62$ GeV, $\pi\pi+\pi^+\pi^+$**

- $0-10\%$,
- $10-20\%$,
- $20-30\%$,
- $30-40\%$

Linear fit: $\hat{R} = \frac{R}{\lambda(1+\alpha)}$

**PHENIX Au+Au $\sqrt{s_{NN}} = 39$ GeV, $\pi\pi+\pi^+\pi^+$**

- $0-20\%$,
- $20-40\%$

Linear fit: $\hat{R} = \frac{R}{\lambda(1+\alpha)}$

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Summary

- B-E correlation functions, run-10 200 GeV Au+Au, \( \sim 7 \) billion evts.
- Levy fits yield statistically acceptable description
- Fine \( m_T \) binned Levy source parameters (\( R, \lambda, \alpha \))
  - Nearly constant \( \alpha \), away from 2, 1 and 0.5 \( \leftrightarrow \) distance to CEP?
  - Linear scaling of \( 1/R^2(m_T) \) \( \leftrightarrow \) hydro?
  - Low-\( m_T \) decrease in \( \lambda(m_T) \) \( \leftrightarrow \) resonances, \( \eta' \) in-medium mass?
- New empirically found scaling parameter \( \hat{R} = R/(\lambda \cdot (1 + \alpha)) \)
- Centrality and \( \sqrt{s_{NN}} \) dependence also explored
  - No \( \alpha \) decrease down to \( \sqrt{s_{NN}} = 39 \) GeV
  - Non-monotonic \( \alpha \) vs \( N_{\text{part}} \) dependence
  - “Hole” in \( \lambda(m_T) \) present down to \( \sqrt{s_{NN}} = 39 \) GeV (c.f. SPS result!)
  - No change in \( 1/R^2 \) and \( \hat{R} \) scaling

Let me invite you to 17th Zimanyi-COST Winter School
Budapest, Hungary, Dec. 4. - Dec. 8. 2017
http://zimanyischool.kfki.hu/17/
Outline

7 Backup
Run4 preliminary & Gauss → Run10 preliminary & Lévy

\( \lambda (\pi^+ \pi^+) \) \text{ RUN4 200GeV Au+Au }
Correlation strength $\lambda$ with fixed $\alpha = 1.134$

- More smooth trend
- Smaller systematic errors
- Saturation at large $m_T$
- Decrease ("hole") for smaller $m_T$ values
STAR centrality dependent results (left) and the comparison of STAR results in different energy with NA44 data (right)
Lévy source function and kinematic variables

Basic two-particle variables

\[ K^\mu = \frac{p_1^\mu + p_2^\mu}{2}, \quad q^\mu = p_1^\mu - p_2^\mu, \quad q_{\text{inv}} = \sqrt{-q^\mu q_\mu} \]

- \( C_2(q_{\text{inv}}) \) - Lorentz invariant 1 dimensional function
- \( |k| = \frac{1}{2} \sqrt{q_{\text{out}}^2 + q_{\text{side}}^2 + q_{\text{long}}^2} \) instead of \( q_{\text{inv}} \) - better
- \( C_2(|k|) \) - 1 dim. function
- Generalized Gaussian - Levy-distribution
  - Anomalous diffusion
  - Generalized limit theorem

\[ \mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2} |qR|^\alpha} \]

\[ S(r) = (1 - \sqrt{\lambda}) \mathcal{L}(\alpha, R_H, r) + \sqrt{\lambda} \cdot \mathcal{L}(\alpha, R_C, r) \] (1)

- Shape of the correlation functions with Levy source \((R_H \to \infty)\):

\[ C_2(|k|) = 1 + \lambda \cdot e^{-(2R|k|)^\alpha} \] (2)