PHENIX results on the Lévy analysis of Bose-Einstein correlation functions

BGL 17 – 10th Bolyai-Gauss-Lobachevsky Conference

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Outline

1. The PHENIX experiment
2. Bose-Einstein correlations
3. Lévy-type HBT and the critical point
4. PHENIX Lévy HBT results: $\sqrt{s_{NN}} = 200$ GeV, MinBias
5. Summary
The PHENIX Experiment

- Versatile detector, operating until 2016
- Tracking via Drift Chambers and Pad Chambers
- Charged pion ID with TOF, from $\sim 0.2$ to 2 GeV/c
- This analysis: PID also with EMCal
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### PHENIX runs at a glance

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- ○ Cu+Cu
- ○ U+U
- ○ Cu+Au
- ○ He+Au
- ○ p+Au
- ○ p+Al

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Bose-Einstein correlations in heavy ion physics

- Quantum statistics connects spatial and momentum space distributions
- Spatial source $S(x)$ versus momentum correlation function $C_2(q)$:
  \[
  C_2(q) \approx 1 + \left| \frac{\tilde{S}(q)}{\tilde{S}(0)} \right|^2, \quad \tilde{S}(q) = \int S(x)e^{iqx}d^4x, \quad q = p_1 - p_2
  \]
  - Final state interactions distort the simple Bose-Einstein picture
  - Coulomb interaction important, handled via two-particle wave function
  - Resonance pions: Halo around primordial Core

The out-side-long system, HBT radii

- $C(q)$ usually measured in the Bertsch-Pratt pair coordinate-system
  - out: direction of the average transverse momentum ($K_t$)
  - long: beam direction
  - side: orthogonal to the latter two

- $R_{\text{out}}, R_{\text{side}}, R_{\text{long}}$: HBT radii

- Out-side difference $\rightarrow \Delta \tau$ emission duration

- From a simple hydro calculation:

  $$R_{\text{out}}^2 = \frac{R^2}{1 + u_T^2 m_T / T_0} + \beta_T^2 \Delta \tau^2$$

  $$R_{\text{side}}^2 = \frac{R^2}{1 + u_T^2 m_T / T_0}$$

- RHIC: ratio is near one $\rightarrow$ no strong 1st order phase transition

- Plus lots of other details (pre-eq flow, initial state, EoS, ...)

Example: recent PHENIX HBT measurements

- Corr. func. in Bertsch-Pratt system, radii from Gaussian fit
- Linear $1/\sqrt{m_T}$ scaling of HBT radii for all systems and energies
- Interpolation to common $m_T$, PHENIX and STAR consistent

HBT radii and the search for the critical endpoint

- Signals of QCD CEP: softest point, long emission
- $R_o^2 - R_s^2$: related to emission duration
- $(R_s - \sqrt{2} \cdot \bar{R})/R_l$: related to expansion velocity
- Non-monotonic patterns
- Indication of the CEP?
- Further detailed studies done
- Maybe Levy exponent $\alpha$ gives further insight?
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Lévy distributions in heavy ion physics

- Expanding medium, increasing mean free path: anomalous diffusion

- Lévy-stable distribution: \( L(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha} \)
  - Generalized Gaussian from generalized central limit theorem
  - \( \alpha = 2 \) Gaussian, \( \alpha = 1 \) Cauchy

- Shape of the correlation functions with Levy source:
  \[
  C_2(q) = 1 + \lambda \cdot e^{-(Rq)^\alpha} \quad \alpha = 2 : \text{Gaussian} \\
  \alpha = 1 : \text{Exponential}
  \]

- Critical behaviour \( \rightarrow \) described by critical exponents
- Spatial corr. \( \propto r^{-(d-2+\eta)} \rightarrow \) defines \( \eta \) exponent
- Symmetric stable distributions (Lévy) \( \rightarrow \) spatial corr. \( \propto r^{-1-\alpha} \)
- \( \alpha \) associated to critical exponent \( \eta \)

A possible way of finding the critical point

- QCD universality class ↔ 3D Ising

- At the critical point:
  - random field 3D Ising: \( \eta = 0.50 \pm 0.05 \)
  - 3D Ising: \( \eta = 0.03631(3) \)

- Modulo finite size effects

- Distance from the critical point?

- Motivation for precise Levy HBT!

- Change in \( \alpha_{\text{Levy}} \) ↔ proximity of CEP?

- Non-static system, finite size effects...
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PHENIX Lévy HBT analysis

- Dataset used for the analysis:
  - Run-10, Au+Au, $\sqrt{s_{NN}} = 200$ GeV, $7.3 \cdot 10^9$ events
- Additional offline requirements:
  - Collision vertex position less than $\pm 30$ cm
- Particle identification:
  - time-of-flight data from PbSc e/w, TOF e/w, momentum, flight length
  - $2 \sigma$ cuts on $m^2$ distribution
- Correlation variable $|k|_{\text{LCMS}}$: $|p_1 - p_2|$ in longitudinally comoving frame
- Single track cuts:
  - $2\sigma$ matching cuts in TOF & PbSc for pions
- Pair-cuts:
  - A random member of pairs assoc. with hits on same tower were removed
  - customary shaped cuts on $\Delta \varphi - \Delta z$ plane for PbSc e/w, TOF e/w
- 1D corr. func. as a function of $|k|_{\text{LCMS}}$ in various $m_T$ bins
  - Levy fits for 31 $m_T$ bins ($m_T$ in $[0.228, 0.871]$ GeV/$c^2$)
  - Coulomb effect incorporated in fit function
Example $C(|k|_{\text{LCMS}})$ measurement result

Measured in 31 $m_T^2 = m^2 + p_T^2$ bins for $\pi^+\pi^+$ and $\pi^-\pi^-$ pairs

MinBias Au+Au @ $\sqrt{s_{\text{NN}}} = 200$ GeV, $\pi^+\pi^+$, $p_T = 0.2-0.22$ GeV/c

- $\lambda = 0.72 \pm 0.02$
- $R = 8.74$ fm $\pm 0.24$ fm
- $\alpha = 1.16 \pm 0.03$
- $\varepsilon = -0.102 \pm 0.005$
- $N = 1.0095 \pm 0.0005$
- $\chi^2/\text{NDF} = 93/97$
- conf. level = 0.5909

Physical parameters: $R, \lambda, \alpha$; measured versus pair $m_T$
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Confidence level $= 0.5909$

Physical parameters: $R, \lambda, \alpha$; measured versus pair $m_T$
Similar decreasing trend as Gaussian HBT radii

Hydro predicts $1/R^2_{Gauss} = a + bm_T$

Hydro behaviour not invalid for $R_{Levy}$!

The linear scaling of $1/R^2$, breaks for high $m_T$
Correlation strength $\lambda$

From the Core-Halo model: $\lambda = \left( \frac{N_C}{N_C+N_H} \right)^2$

- Observed decrease (“hole”) at small $m_T \rightarrow$ increase of halo fraction
- Different effects can cause change in $\lambda$
  - Resonance effects, partially coherent pion production
- $\lambda/\lambda_{\text{max}}$ with smaller systematic uncertainties
- Precise measurement may help extract physics info
A possible (?) interpretation of $\lambda(m_T)$

- $\lambda(m_T)$ measures core/(core+halo) fraction
- May be connected to mass modifications (c.f. chiral restoration)
  - Decreased $\eta'$ mass $\rightarrow$ $\eta'$ enhancement $\rightarrow$ halo enhancement
  - Kinematics: $\eta'$ decay pions will have low $m_T$ $\rightarrow$ decreased $\lambda$ at small $m_T$
- Incompatibility with unmodified in-medium $\eta'$ mass?

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Levy exponent $\alpha$

- Measured values far from Gaussian ($\alpha = 2$), also not expo. ($\alpha = 1$)
- Also far from the random field 3D Ising value at CEP ($\alpha = 0.5$)
- More or less constant (at least within systematic errors)
- Motivation to do fits with fixed $\alpha = 1.134$
- Note: $\alpha(m_T) = \text{const.}$ fit statistically not acceptable (only with syst.)
Levy scale parameter $R$ with fixed $\alpha = 1.134$

- More smooth trend
- Remarkable linearity of $1/R^2$
- Hydro behavior valid, despite $\alpha < 2$
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Newly discovered scaling parameter $\hat{R}$
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- Empirically found scaling parameter
- Linear in $m_T$
- Physical interpretation $\rightarrow$ open question

PHENIX results

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Newly discovered scaling parameter $\hat{R}$

- $\alpha = 1.134$ fixed

- Empirically found scaling parameter

- Linear in $m_T$

- Physical interpretation $\rightarrow$ open question
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Summary

- B-E correlation functions, run-10 200 GeV Au+Au, \( \sim 7 \) billion evts.
- Levy fits yield statistically acceptable description
- Fine \( m_T \) binned Levy source parameters (\( R, \lambda, \alpha \))
  - Nearly constant \( \alpha \), away from 2, 1 and 0.5 \( \leftrightarrow \) distance to CEP?
  - Linear scaling of \( 1/R^2(m_T) \) \( \leftrightarrow \) hydro?
  - Low-\( m_T \) decrease in \( \lambda(m_T) \) \( \leftrightarrow \) resonances, \( \eta' \) in-medium mass?
- New empirically found scaling parameter \( \hat{R} = R/(\lambda \cdot (1 + \alpha)) \)

Thank you for your attention!

If you are interested in these subjects: come to the 17th Zimanyi-COST Winter School
Budapest, Hungary, Dec. 4. - Dec. 8. 2017
http://zimanyischool.kfki.hu/17/
Outline

6 Backup
Run4 preliminary & Gauss → Run10 preliminary & Lévy

\[ \lambda (\pi^+ \pi^+) \text{ RUN4 200GeV Au+Au} \]

\[ m^*_{\eta'} = \begin{array}{c} 958 \text{ MeV} \\ 600 \text{ MeV} \\ 500 \text{ MeV} \\ 400 \text{ MeV} \\ 300 \text{ MeV} \\ 200 \text{ MeV} \end{array} \]

Gauss fit

MinBias Au+Au @ \( \sqrt{s_{NN}} = 200 \text{ GeV} \)

\[ \lambda_{max} = \langle \lambda \rangle_{(0.5-0.7) \text{ GeV/c}^2} \]

PRL105:182301(2010),
PRC83:054903(2011)

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Correlation strength $\lambda$ with fixed $\alpha = 1.134$

- More smooth trend
- Smaller systematic errors
- Saturation at large $m_T$
- Decrease ("hole") for smaller $m_T$ values
Low energy comparison

STAR centrality dependent results (left) and the comparison of STAR results in different energy with NA44 data (right)
Lévy source function and kinematic variables

- Basic two-particle variables
  \[ K^\mu = \frac{p_1^\mu + p_2^\mu}{2}, \quad q^\mu = p_1^\mu - p_2^\mu, \quad q_{inv} = \sqrt{-q^\mu q_\mu} \]  
  \[ |k| = \frac{1}{2} \sqrt{q_{out}^2 + q_{side}^2 + q_{long}^2} \text{ instead of } q_{inv} \text{ - better} \]

- \( C_2(q_{inv}) \) - Lorentz invariant 1 dimensional function

- Anomalous diffusion, generalized central limit theorem: Levy

  \[ \mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2} |qR|^\alpha} \]  
  \[ S(r) = (1 - \sqrt{\lambda})\mathcal{L}(\alpha, R_H, r) + \sqrt{\lambda} \cdot \mathcal{L}(\alpha, R_C, r) \]

- Shape of the correlation functions with Levy source \( (R_H \to \infty) \):
  \[ C_2(|k|) = 1 + \lambda \cdot e^{-(2R|k|)^\alpha} \]
Lévy exponent $\alpha$ at 200 GeV

- Slightly non-monotonic behavior as a function of $m_T$
- Average $\langle \alpha \rangle$ non-monotonic behavior versus $N_{\text{part}}$
- $\alpha = \langle \alpha \rangle$ constant fits were performed
Lévy exponent $\alpha$ at 62 and 39 GeV

- Lévy exponent $\alpha$: no significant change vs $\sqrt{s_{NN}}$ at 39-62-200 GeV
- Usual values between 1 and 1.5
- Non-monotonicity in $m_T$
Lévy scale $R$: similar trends for all $\sqrt{s_{NN}}$ and cent.

- For $\sqrt{s_{NN}} = 200$ GeV:
  - Linear fit $\pi^-\pi^+\pi^+$
  - Data points for different centrality classes (0-10%, 10-20%, 20-30%, 30-40%, 40-50%, 50-60%)
  - $\alpha$ free

- For $\sqrt{s_{NN}} = 62$ GeV, $\pi^-\pi^+\pi^+$:
  - Similar trends for all $\sqrt{s_{NN}}$

- For $\sqrt{s_{NN}} = 39$ GeV, $\pi^-\pi^+\pi^+$:
  - $\alpha$ fixed

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“Hole” in $\lambda$: all energies and centralities

**PHENIX Au+Au $\sqrt{s_{NN}} = 200$ GeV**

- $0\% - 10\%$
- $10\% - 20\%$
- $20\% - 30\%$
- $30\% - 40\%$
- $40\% - 50\%$
- $50\% - 60\%$

**PHENIX Au+Au $\sqrt{s_{NN}} = 39$ GeV**

- $0\% - 10\%$
- $10\% - 20\%$

**PHENIX Au+Au $\sqrt{s_{NN}} = 62$ GeV**

- $0\% - 10\%$
- $10\% - 20\%$
$\hat{R}$ scaling for all energies & centralities

$\hat{R} = \frac{R}{\lambda(1+\alpha)}$

$\pi^-\pi^+\pi^\pm\pi^\mp$

$\tau = \frac{R}{\lambda(1+\alpha)}$

$\pi^-\pi^+\pi^\pm\pi^\mp$

PHENIX Au+Au $\sqrt{s_{NN}} = 200$ GeV

$\alpha$ free linear fit

PHENIX Au+Au $\sqrt{s_{NN}} = 200$ GeV

$\alpha$ fixed linear fit

$\delta R^1/R^1$

$\delta R^2/R^2$

rel. syst. uncertainties

rel. syst. uncertainties

PHENIX preliminary

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