Pair Azimuthal Correlations in a Two-Source “Jet/Flow” Model

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Correlation Functions: Full and Reduced

$$\frac{1}{N_{event}}\frac{d^6N^{AB}}{d^3\mathbf{P}_A d^3\mathbf{P}_B}$$

The six-differential per-event yield for two types of particles, A and B, is the container of all information on (A,B) pair production in an event sample.

$$C(\mathbf{P}_A,\mathbf{P}_B) = \frac{\dfrac{1}{N_{event}}\dfrac{d^6N^{AB}}{d^3\mathbf{P}_A d^3\mathbf{P}_B}}{\left(\dfrac{1}{N_{event}}\dfrac{d^3N^A}{d^3\mathbf{P}_A}\right)\left(\dfrac{1}{N_{event}}\dfrac{d^3N^B}{d^3\mathbf{P}_B}\right)}$$

The Full Correlation Function measures the degree to which A and B production are correlated (*ie* not independent) at any point in six-dimensional $(\mathbf{P}_A,\mathbf{P}_B)$ phase space.

$$C(\Omega(\xi)) = \frac{\int_{\Omega(\xi)}\left(\dfrac{1}{N_{ev}}\dfrac{d^6N^{AB}}{d^3\mathbf{P}_A d^3\mathbf{P}_B}\right)d^3\mathbf{P}_A d^3\mathbf{P}_B}{\int_{\Omega(\xi)}\left(\dfrac{1}{N_{ev}}\dfrac{d^3N^A}{d^3\mathbf{P}_A}\;\dfrac{1}{N_{ev}}\dfrac{d^3N^B}{d^3\mathbf{P}_B}\right)d^3\mathbf{P}_A d^3\mathbf{P}_B}$$

A Reduced Correlation Function measures the correlation between A and B production in some 6-D phase space volume Ω, which is often parameterized; here the parameter(s) is(are) generically named ξ.

Residual Multiplicity Correlations

When the event sample can be divided into sub-samples such that A and B production are *uncorrelated* within a sub-sample, then A and B are said to show only a residual correlation. The simplest example in heavy-ion collisions would be residual multiplicity correlations. Suppose A and B production are, at all points in phase space, proportional to N_{part}, the number of participants in a Glauber model:

Centrality parameter N_{part}; event sample defines distribution $p(N_{\text{part}})$, width $\equiv \sigma(N_{\text{part}})$,mean $\equiv \langle N_{\text{part}} \rangle$

Assume: N^A and N^B proportional to N_{part}, but otherwise uncorrelated

$$\text{Singles: } \frac{1}{N_{evt}} \frac{d^3N^A}{d^3\mathbf{P}_A} = \sum_{N_{\text{part}}} p(N_{\text{part}}) N_{\text{part}} F_A(\mathbf{P}_A); \quad \frac{1}{N_{evt}} \frac{d^3N^B}{d^3\mathbf{P}_B} = \sum_{N_{\text{part}}} p(N_{\text{part}}) N_{\text{part}} F_B(\mathbf{P}_B)$$

$$\text{Joint: } \frac{1}{N_{evt}} \frac{d^6N^{AB}}{d^3\mathbf{P}_A d^3\mathbf{P}_B} = \sum_{N_{\text{part}}} p(N_{\text{part}}) \left(N_{\text{part}}\right)^2 F_A(\mathbf{P}_A) F_B(\mathbf{P}_B)$$

$$C(\mathbf{P}_A,\mathbf{P}_B) = \frac{\sum p(N_{\text{part}}) \left(N_{\text{part}}\right)^2 F_A(\mathbf{P}_A) F_B(\mathbf{P}_B)}{\left(\sum p(N_{\text{part}}) N_{\text{part}} F_A(\mathbf{P}_A)\right)\left(\sum p(N_{\text{part}}) N_{\text{part}} F_B(\mathbf{P}_B)\right)}$$

$$= \frac{\sum p(N_{\text{part}}) \left(N_{\text{part}}\right)^2}{\left(\sum p(N_{\text{part}}) N_{\text{part}}\right)^2} = 1 + \left(\frac{\sigma(N_{\text{part}})}{\langle N_{\text{part}} \rangle}\right)^2 \geq 1 \text{ for all } (\mathbf{P}_A,\mathbf{P}_B)$$

In the example of N_{part} scaling the correlation function is always greater than one, by an amount which increases with the width of the N_{part} distribution.

(Relative-Azimuthal-)Angular Correlations

A widely-used reduced correlation function is the relative-azimuthal-angle CF, often called the Angular Correlation Function for short. The phase space volume Ω is parameterized by $\Delta\phi$ and counts the number of pairs at fixed $\Delta\phi=\phi_A-\phi_B$;“binA” and “binB” denote some arbitrary ranges in (P_A,η_A) and (P_B,η_B):

$$C(\Delta\phi)=\frac{\int d\phi_A d\phi_B \delta((\phi_A-\phi_B)-\Delta\phi)\int_{\mathrm{binA}} dP_A d\eta_A \int_{\mathrm{binB}} dP_B d\eta_B \frac{1}{N_{ev}}\frac{d^6N^{AB}}{d^3\mathbf{P}_A d^3\mathbf{P}_B}}{\int d\phi_A d\phi_B \delta((\phi_A-\phi_B)-\Delta\phi)\left[\int_{\mathrm{binA}} dP_A d\eta_A \frac{1}{N_{ev}}\frac{d^3N^A}{d^3\mathbf{P}_A}\right]\left[\int_{\mathrm{binB}} dP_B d\eta_B \frac{1}{N_{ev}}\frac{d^3N^B}{d^3\mathbf{P}_B}\right]}$$

$$=\frac{2\pi}{n^A n^B}\int d\phi_A d\phi_B \delta((\phi_A-\phi_B)-\Delta\phi)\frac{d^2n^{AB}}{d\phi_A d\phi_B} \qquad \text{if we define } n^X \equiv \frac{1}{N_{ev}}N^X$$

$$\int_0^{2\pi} d(\Delta\phi)\, C(\Delta\phi)=2\pi\frac{n^{AB}}{n^A n^B}=2\pi\frac{\text{rate of AB pairs}}{\text{(rate of A singles)(rate of B singles)}}$$

The integral of the angular CF satisfies a simple sum rule with the pair rate and singles rates.

Worked Example: Elliptic Flow

Angular CF's are often used to investigate elliptic flow, defined here as a residual correlation in which the singles distributions follow a quadrupole pattern relative to a reaction plane direction $\psi_{\rm RP}$ but are otherwise uncorrelated. We can then write the single and joint distributions, and the angular correlation function follows immediately:

$$\frac{dn^A(\psi_{\rm RP})}{d\phi_A} = \frac{n^A}{2\pi}\left[1+2v_2^A\cos(2(\phi_A - \psi_{\rm RP}))\right] \;\; ; \;\; \frac{dn^B(\psi_{\rm RP})}{d\phi_B} = \frac{n^B}{2\pi}\left[1+2v_2^B\cos(2(\phi_B - \psi_{\rm RP}))\right]$$

$$\frac{d^2n^{AB}}{d\phi_A d\phi_B} = \int \frac{d\psi_{RP}}{2\pi}\frac{n^A n^B}{(2\pi)^2}\left[1+2v_2^A\cos(2(\phi_A - \psi_{\rm RP}))\right]\left[1+2v_2^B\cos(2(\phi_B - \psi_{\rm RP}))\right]$$

$$= \frac{n^A n^B}{(2\pi)^2}\left[1+2v_2^A v_2^B\cos(2(\phi_A - \phi_B))\right] \quad \text{after some non-trivial (!) work}$$

The angular correlation function then becomes simply $C(\Delta\phi) = 1 + 2v_2^A v_2^B\cos(2\Delta\phi)$

Measurement of the angular correlation function between two types of particles can reveal the product of the quadrupole strengths $v_2^A v_2^B$ without a measurement of $\psi_{\rm RP}$.

(In heavy-ion collisions the angular CF will also show the effects of residual multiplicity correlations, but if *only* elliptic flow is present then the effect is just an overall multiplicative factor.)

The Two-Source Model

Each particle is assumed to come from one of two sources, "Flow" or "Jet". The Flow source is multi-collisional, possibly thermalized, and its particles exhibit elliptic flow relative to the reaction plane ϕ_{RP}. The Jet source is fragmentation from prompt jets (and dijets). We allow for the possibility that jets "feel" the collision geometry by giving the jet rate (before fragmentation) a quadrupole modulation. The singles distributions are controlled by the parameters ϕ_{RP}, and for the jet source the jet axis ϕ_{Jet} :

The quadrupole strength v_2^{FlowA} is specific to particles of type A from the Flow source; n^A_{Flow} is the rate of A-type singles per event from the Flow source.

$$\frac{dn^A_{\mathrm{Flow}}(\phi_{\mathrm{RP}})}{d\phi_A} = \frac{n^A_{\mathrm{Flow}}}{2\pi}\left[1 + 2v_2^{FlowA}\cos(2(\phi_A - \phi_{\mathrm{RP}}))\right]$$

The function $J^A()$ is peaked at 0, normalized to 1, and describes fragmentation. The constant $\langle v_2^{JetA}\rangle$ is an average ellipticity for all jets which produce A-type particles into binA.

$$\frac{dn^A_{\mathrm{Jet}}(\phi_{\mathrm{RP}},\phi_{\mathrm{Jet}})}{d\phi_A} = \frac{n^A_{\mathrm{Jet}}}{2\pi} J^A(\phi_A - \phi_{\mathrm{Jet}})\left[1 + 2\left\langle v_2^{JetA}\right\rangle\cos(2(\phi_{\mathrm{Jet}} - \phi_{\mathrm{RP}}))\right]$$

(The corresponding definitions hold for B-type particles, of course.)

Sum Over Pair Types

Since the rate of all pairs can be partitioned into distinct types of pairs, the angular correlation function can be written as a sum over pair types:

$$C(\Delta\phi) = \frac{2\pi}{n^A n^B} \int d\phi_A d\phi_B \delta((\phi_A - \phi_B) - \Delta\phi) \frac{d^2 n^{AB}}{d\phi_A d\phi_B} = \frac{2\pi}{n^A n^B} \sum_{\text{Pair Types}} \int d\phi_A d\phi_B \delta((\phi_A - \phi_B) - \Delta\phi) \frac{d^2 n^{AB}_{\text{Pair Type}}}{d\phi_A d\phi_B}$$

Within the two-source model we can identify five distinct and disjoint pair types, and we will write the angular correlation function as a sum over these types:

Flow-Flow: Each particle A and B area from the Flow source.

Flow-Jet: A is from the Flow source and B from Jet source, plus the reverse.

Jet-Other Jet: A and B are both from jets, but not the same hard scattering process

Jet-Same Jet: A and B are both fragments from the same jet.

Di-Jet: A and B are fragments from a back-to-back pair of jets.

Pair Types I: Flow-Flow, Flow-Jet, and Jet-Other Jet

These are the terms in the angular CF for the first three types of pairs. The term for the Flow-Flow type is exactly the same as in the elliptic flow example:

$$\frac{2\pi}{n^A n^B}\int d\phi_A d\phi_B \delta((\phi_A - \phi_B) - \Delta\phi)\int \frac{d\phi_{\mathrm{RP}}}{2\pi}\frac{dn^A_{\mathrm{Flow}}(\phi_{\mathrm{RP}})}{d\phi_A}\frac{dn^B_{\mathrm{Flow}}(\phi_{\mathrm{RP}})}{d\phi_B}$$
$$= \frac{n^A_{\mathrm{Flow}} n^B_{\mathrm{Flow}}}{n^A n^B}\left[1 + 2v_2^{FlowA} v_2^{FlowB}\cos(2\Delta\phi)\right]$$

The JetA-FlowB term is similar (we need to remember to add its reverse also):

$$\frac{2\pi}{n^A n^B}\int d\phi_A d\phi_B \delta((\phi_A - \phi_B) - \Delta\phi)\int \frac{d\phi_{\mathrm{RP}}}{2\pi}\frac{d\phi_{\mathrm{Jet}}}{2\pi}\frac{dn^A_{\mathrm{Jet}}(\phi_{\mathrm{RP}},\phi_{\mathrm{Jet}})}{d\phi_A}\frac{dn^B_{\mathrm{Flow}}(\phi_{\mathrm{RP}})}{d\phi_B}$$
$$= \frac{n^A_{\mathrm{Jet}} n^B_{\mathrm{Flow}}}{n^A n^B}\left[1 + 2 j_2^A \left\langle v_2^{JetA}\right\rangle v_2^{FlowB}\cos(2\Delta\phi)\right] \quad \text{where} \quad j_2^A \equiv \int J^A(x)\cos(2x)dx$$

The Jet-Other Jet term also has a quadrupole shape in the end:

$$\frac{2\pi}{n^A n^B}\int d\phi_A d\phi_B \delta((\phi_A - \phi_B) - \Delta\phi)\int \frac{d\phi_{\mathrm{RP}}}{2\pi}\frac{d\phi_{\mathrm{JetA}}}{2\pi}\frac{d\phi_{\mathrm{JetB}}}{2\pi}\frac{dn^A_{\mathrm{Jet}}(\phi_{\mathrm{RP}},\phi_{\mathrm{JetA}})}{d\phi_A}\frac{dn^B_{\mathrm{Jet}}(\phi_{\mathrm{RP}},\phi_{\mathrm{JetB}})}{d\phi_B}$$
$$= \frac{n^A_{\mathrm{Jet}} n^B_{\mathrm{Jet}}}{n^A n^B}\left[1 + 2 j_2^A \left\langle v_2^{JetA}\right\rangle j_2^B \left\langle v_2^{JetB}\right\rangle \cos(2\Delta\phi)\right]$$

Pair Types II: Same Jet, Same Dijet

For (A,B) pairs which fragment from the same jet the joint distribution includes two angular fragmentation functions $J^A()$ and $J^B()$; and the result for the corresponding term in the correlation function involves their convolution:

$$\frac{2\pi}{n^A n^B}\int d\phi_A d\phi_B \delta((\phi_A - \phi_B) - \Delta\phi)$$

$$\times \int \frac{d\phi_{\mathrm{RP}}}{2\pi}\frac{d\phi_{\mathrm{Jet}}}{2\pi}\; n^{AB}_{SameJet} J^A(\phi_A - \phi_{\mathrm{Jet}}) J^B(\phi_B - \phi_{\mathrm{Jet}})[1+2\langle v_2^{JetAB}\rangle \cos(2(\phi_{\mathrm{Jet}} - \phi_{\mathrm{RP}}))]$$

$$= \frac{2\pi\, n^{AB}_{SameJet}}{n^A n^B}\; J^A \circ J^B\;(\Delta\phi)$$

For pairs from opposite sides of a dijet the two jet axes ϕ_{JetA} and ϕ_{JetB} are not independent, but their acoplanarity Δ^{AB} has a distribution $D^{AB}()$:

$$\Delta^{AB} \equiv \phi_{\mathrm{JetA}} - \phi_{\mathrm{JetB}} - \pi$$

$$\frac{dn}{d\Delta^{AB}} \equiv D^{AB}(\Delta^{AB})$$

After a great of algebra and calculus (here we spare the reader) the final result for the dijet term in the correlation function is:

$$\frac{2\pi\, n^{AB}_{DiJet}}{n^A n^B}\; J^A \circ J^B \circ E^{AB}(\Delta\phi - \pi)$$

where $E^{AB}()$ is a function almost identical to $D^{AB}()$

$E^{AB}(\Delta^{AB}) \propto D^{AB}(\Delta^{AB})[1+2\langle v_2^{DiJetA}\rangle\langle v_2^{DiJetB}\rangle \cos(2\Delta^{AB})]$ and $E()$ is normalized to 1

Result for Angular Correlation Function

Summing up all the terms for the different pair types we have

$$C(\Delta\phi) = \frac{n^A_{\text{Flow}} n^B_{\text{Flow}}}{n^A n^B}\left[1 + 2 v_2^{FlowA} v_2^{FlowB}\cos(2\Delta\phi)\right] + \frac{n^A_{\text{Jet}} n^B_{\text{Flow}}}{n^A n^B}\left[1 + 2 j_2^A \left\langle v_2^{JetA}\right\rangle v_2^{FlowB}\cos(2\Delta\phi)\right]$$

$$+ \text{ JetB/FlowA term } + \frac{n^A_{\text{Jet}} n^B_{\text{Jet}}}{n^A n^B}\left[1 + 2 j_2^A \left\langle v_2^{JetA}\right\rangle j_2^B \left\langle v_2^{JetB}\right\rangle \cos(2\Delta\phi)\right]$$

$$+ \frac{2\pi\, n^{AB}_{SameJet}}{n^A n^B} \; J^A \circ J^B \;(\Delta\phi) \;+\; \frac{2\pi\, n^{AB}_{DiJet}}{n^A n^B} \; J^A \circ J^B \circ E^{AB}(\Delta\phi - \pi)$$

With $n^A = n^A_{\text{Flow}} + n^A_{\text{Jet}}$ by definition (and the same for B) this simplifies to

$$C(\Delta\phi) = 1 + 2 V_2^A V_2^B \cos(2\Delta\phi) \;+\; \frac{2\pi\, n^{AB}_{SameJet}}{n^A n^B} \; J^A \circ J^B \;(\Delta\phi) + \frac{2\pi\, n^{AB}_{DiJet}}{n^A n^B} \; J^A \circ J^B \circ E^{AB}(\Delta\phi - \pi)$$

where $V_2^A \equiv \frac{n^A_{\text{Flow}}}{n^A} v_2^{FlowA} + \frac{n^A_{\text{Jet}}}{n^A} j_2^A \left\langle v_2^{JetA}\right\rangle$ is the ellipticity of the A singles distribution (same for B)

Since the singles rates n^A and n^B are easily measured, a decomposition of the angular correlation function can extract the rates of jet- and dijet-induced pairs n^{AB}_{SameJet} and n^{AB}_{Dijet} directly, as well as the true singles ellipticities V_2^A and V_2^B.

(**Caveat**: The effects of residual multiplicity correlations, not taken into account here, will raise the non-jet terms in the CF by a constant factor, which should be very close to 1 for central event samples but could approach ~2 for wide peripheral or p/d+A samples.)

A Word About Conditional Yields

A related pair quantity of interest is the conditional yield of one type of particle, say type *B*, *conditioned* on the presence of another type, say *A*. This amounts to counting the rate of (*A*,*B*) pairs compared to the rate of *A* singles:

$$\text{Yield of B conditioned on A} = \frac{1}{N^A}\frac{dN^{AB}}{d(\Delta\phi)} = \frac{1}{N^A}\int d\phi_A\, d\phi_B\, \delta(\Delta\phi - (\phi_A - \phi_B))\frac{d^2N^{AB}}{d\phi_A\, d\phi_B}$$

With this definition, it is clear that the azimuthal distribution of the conditional yield is closely related to the correlation function:

$$\frac{1}{N^A}\frac{dN^{AB}}{d(\Delta\phi)} = \frac{n^B}{2\pi}C(\Delta\phi)$$

So the conditional yield of, say, all same-jet-induced B particles per A particle can be calculated easily once the correlation function is decomposed:

$$\int d(\Delta\phi)\frac{1}{N^A}\frac{dN^{AB}_{SameJet}}{d(\Delta\phi)} = \frac{n^B}{2\pi}\frac{2\pi\, n^{AB}_{SameJet}}{n^A n^B} = \frac{n^{AB}_{SameJet}}{n^A}$$

Conclusions

In the two-source model the angular correlation function has a straightforward interpretation in terms of the rates of different kinds of pairs, including jet- and dijet-induced pairs.

The amplitude of the quadrupole term follows the product of the quadrupole strengths of the two singles distributions, even when jets show a dependence relative to the reaction plane.

Residual multiplicity correlations can increase the rate of non-jet/non-dijet pairs, especially in peripheral event samples.

The relative-angle conditional yield distribution is related to the angular correlation function by a simple constant scaling.