W-boson physics with proton beams at Relativistic Heavy Ion Collider

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- Probing flavor dependence of unpolarized and spin-dependent parton distribution functions (PDF's)
- Lepton-level asymmetries $A_L(y_\ell)$ in W boson production
- Implementation of NLO $A_L(y_\ell)$ in the global fits

Probing parton distributions in W boson production at RHIC

- Intermediate energies ($\sqrt{S}$=200-500 GeV); sizeable luminosities ($\mathcal{L} = 100 - 800$ pb^{-1})
- pp collider: good sensitivity to quark sea at scales of order M_W
  - complements Tevatron and low-energy Drell-Yan measurements
- Flavor sensitivity through the CKM matrix
  - analysis of flavor dependence of valence and sea quark PDF's
- Beam polarization option
  - first measurements of $\Delta q_{\text{sea}}(x, Q)$ at large Q

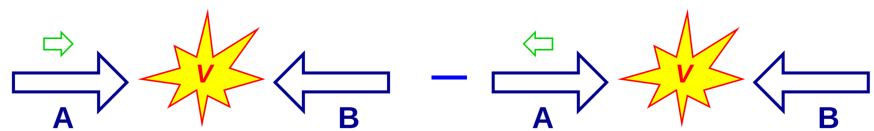
Convenient combinations of helicity cross sections

❖ Unpolarized cross section

$$\sigma = \frac{1}{4}\left(\sigma^{++} + \sigma^{+-} + \sigma^{-+} + \sigma^{--}\right)$$

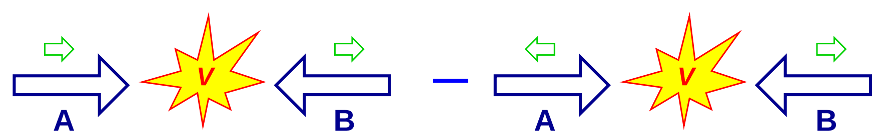
❖ Single-spin (parity-violating) cross section

$$\Delta_L \sigma = \frac{1}{4}\left(\sigma^{++} - \sigma^{-+} + \sigma^{+-} - \sigma^{--}\right)$$



❖ Double-spin cross section

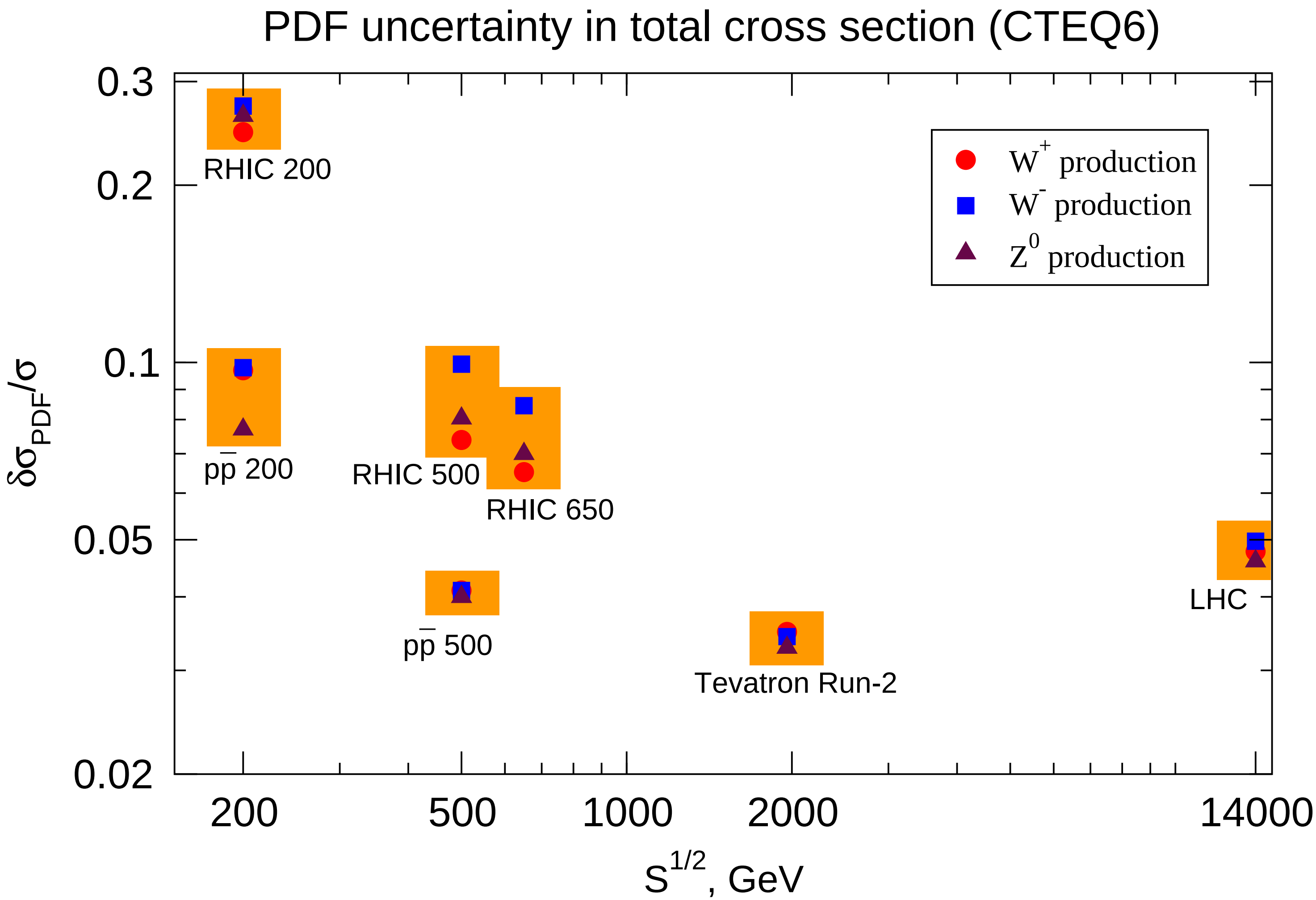
$$\Delta_{LL} \sigma = \frac{1}{4}\left(\sigma^{++} - \sigma^{+-} - \sigma^{-+} + \sigma^{--}\right)$$



Spin asymmetries (as functions of a kinematical variable $p = p_T, y...$):

$$A_L(p) \equiv \frac{d\Delta_L\sigma/dp}{d\sigma/dp}, \qquad A_{LL}(p) \equiv \frac{d\Delta_{LL}\sigma/dp}{d\sigma/dp}$$

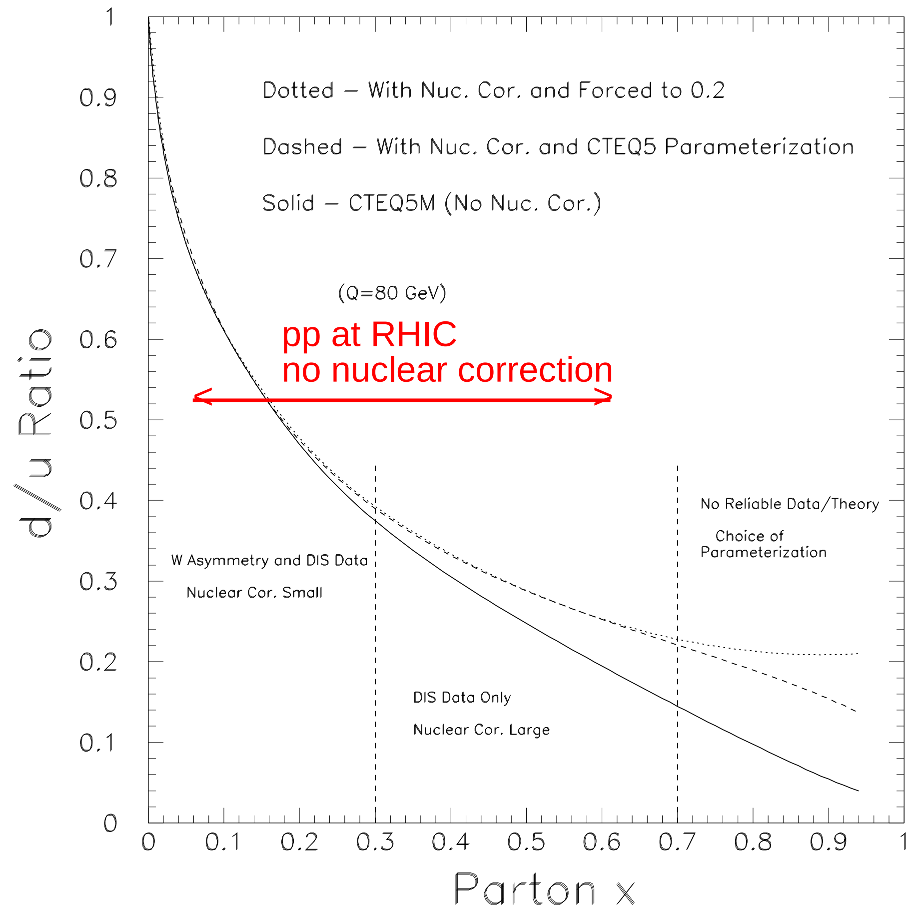
Unpolarized cross sections



RHIC probes sea quark PDF's and $d(x)$ at $x > 0.1$, where these PDF's are not well constrained

Measurements of $d(x)/u(x)$: RHIC vs. other experiments

- DIS and lower-energy Drell-Yan
  - $x > 0.2$, $Q \lesssim 10$ GeV
  - possible nuclear corrections
- Tevatron Run-2
  - $x \lesssim 0.3$, $Q \sim M_W$
- RHIC
  - $x \gtrsim 0.3$, $Q \sim M_W$



- $\sqrt{S_{RHIC}} < \sqrt{S_{Tevatron}}$
  $\therefore$ For same y_W, $x_{RHIC} > x_{Tevatron}$: RHIC has better access to the large-x region

Charge lepton asymmetry of unpolarized cross sections at a $p\bar{p}$ collider

$$A_{ch}^{p\bar{p}}(y_e) \equiv \frac{\frac{d\sigma^{W^+}}{dy_e} - \frac{d\sigma^{W^-}}{dy_e}}{\frac{d\sigma^{W^+}}{dy_e} + \frac{d\sigma^{W^-}}{dy_e}}$$

- related to the boson Born-level asymmetry (y_W=rapidity of W)

$$A_{ch}^{p\bar{p}}(y_W) \stackrel{y_W \to y_{max}}{\longrightarrow} \frac{r(x_b) - r(x_a)}{r(x_b) + r(x_a)}, \; r(x) \equiv \frac{d(x, M_W)}{u(x, M_W)}$$

- PDF analyses use $A_{ch}^{p\bar{p}}(y_e)$ for electrons with large y and $p_{Te} > p_{Te}^{min}$ to constrain $d(x, M_W)/u(x, M_W)$ at $x \to 1$

- At a pp collider RHIC, an analogous quantity is $\left(d\sigma^{W^+}/dy_e\right) / \left(d\sigma^{W^-}/dy_e\right)$

Probing valence quark PDF's at large x (forward region)

Neglecting strange and heavy flavors, at Born level,
for $\sqrt{S} = 500$ GeV:

$$\frac{d\sigma^{W^+}}{dy_W} \propto u(x_A)\bar{d}(x_B) + \bar{d}(x_A)u(x_B) \underset{y_W \to y_{\max}}{\longrightarrow} u(x_A \sim 1)\bar{d}(x_B \sim \tau)$$

$$\frac{d\sigma^{W^-}}{dy_W} \propto d(x_A)\bar{u}(x_B) + \bar{u}(x_A)d(x_B) \underset{y_W \to y_{\max}}{\longrightarrow} d(x_A \sim 1)\bar{u}(x_B \sim \tau)$$

$$\left.\frac{d\sigma^{W^-}/dy_W}{d\sigma^{W^+}/dy_W}\right|_{y_W \to y_{\max}} \sim \frac{d(x_A \sim 1)}{u(x_A \sim 1)} \times \frac{\bar{u}(x_B \sim \tau)}{\bar{d}(x_B \sim \tau)}$$

$\tau \equiv M_W^2/S \sim 0.03$; $y_{\max} \equiv -\frac{1}{2}\ln\tau \sim 1.82$

☞ A correlated constraint on flavor symmetry breaking in d/u at $x \to 1$ and $\bar{u}/\bar{d}$ at $x \to 0.03$ (Gottfried sum rule violation)

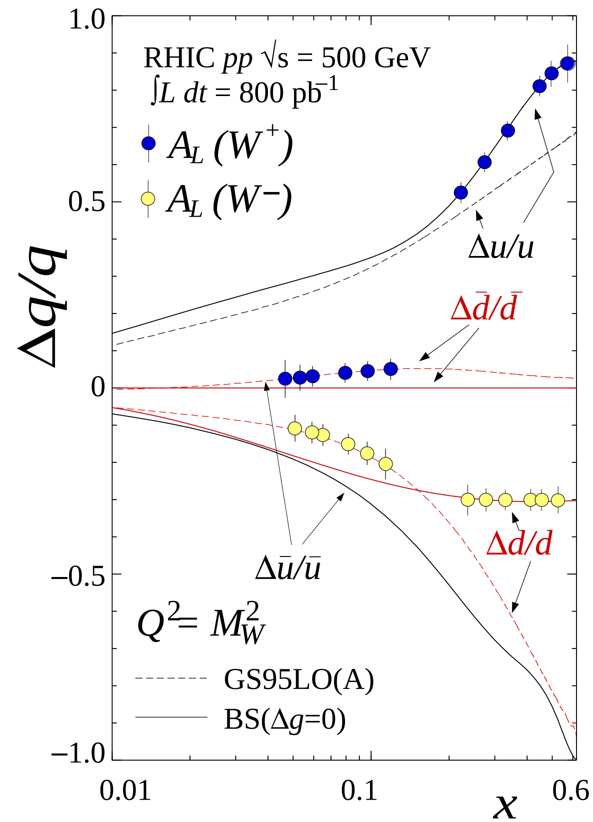
Single-spin asymmetries in a QCD resummation calculation

(P. N., C.-P. Yuan, Nucl. Phys. B666, 3 (2003);
Nucl. Phys. B666, 35 (2003))

Leading order single-spin asymmetries for W boson rapidity distributions

$$A_L^{W^+}(y_W) = \frac{-\Delta u(x_a)\bar{d}(x_b) + \Delta\bar{d}(x_a)u(x_b)}{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)u(x_b)} = \begin{cases} -\Delta u(x_a)/u(x_a), & x_a \to 1 \\ \Delta\bar{d}(x_a)/d(x_a), & x_b \to 1 \end{cases}$$

$$A_L^{W^-}(y_W) = \frac{-\Delta d(x_a)\bar{u}(x_b) + \Delta\bar{u}(x_a)d(x_b)}{d(x_a)\bar{u}(x_b) + \bar{u}(x_a)d(x_b)} = \begin{cases} -\Delta d(x_a)/d(x_a), & x_a \to 1 \\ \Delta\bar{u}(x_a)/\bar{u}(x_a), & x_b \to 1 \end{cases}$$



Source: G. Bunce et al., hep-ph/0007218

Would be a convenient test of $\Delta q/q$ and $\Delta\bar{q}/\bar{q}$ if $W^{\pm}$-bosons were observed directly

Interest in fully differential cross sections at the lepton level

1. Partial angular coverage of PHENIX and STAR detectors

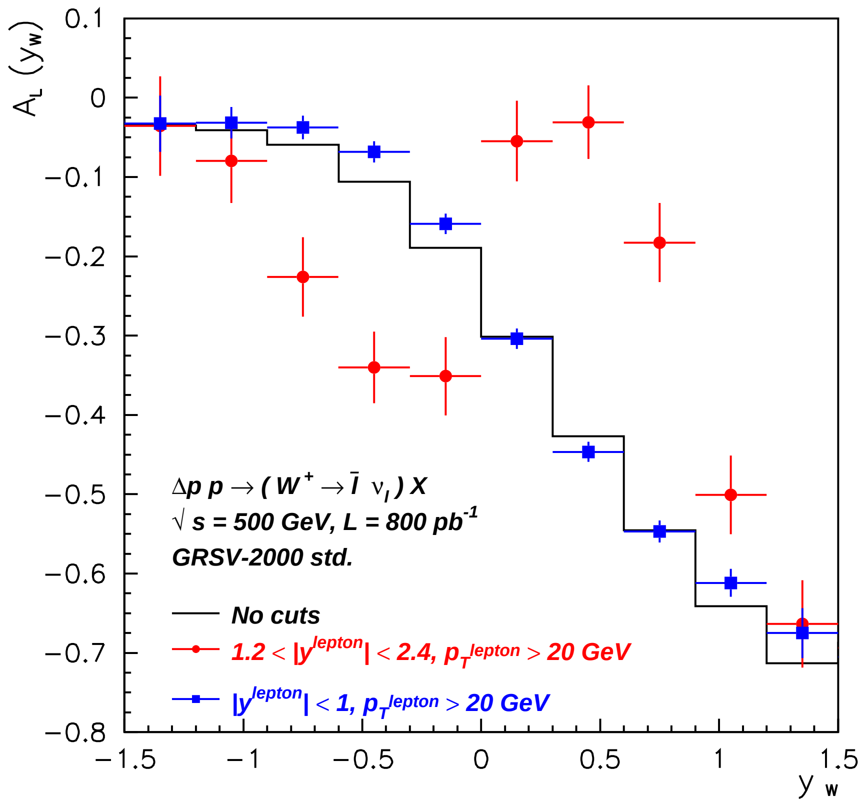
2. y_W can be approximately reconstructed in a limited event sample and only if dynamics is well understood

   - The correct solution for y_W can be chosen based on the knowledge of the rate if $|y_\ell| \gg 0$ and $p_{T\ell}$ not large

3. Due to the spin-1 of $W^\pm$ boson, cuts affect the numerator and denominator of $A_L(y_W)$ differently

$$A_L(y_W)\Big|_{\substack{with\\ lepton\\ cuts}} \neq A_L(y_W)\Big|_{\substack{without\\ lepton\\ cuts}}$$

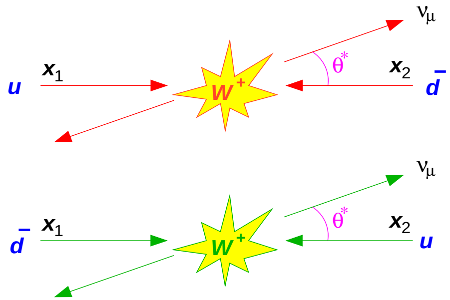
Impact of leptonic cuts on the measurement of $A_L(y_W)$



Due to the spin-1 of $W^{\pm}$ boson, cuts affect the numerator and denominator of $A_L(y_W)$ differently

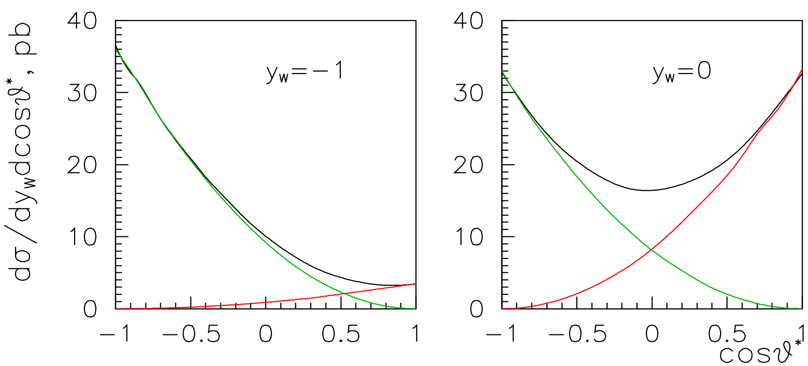
Correlation between y_W and leptonic angle $\cos\theta$

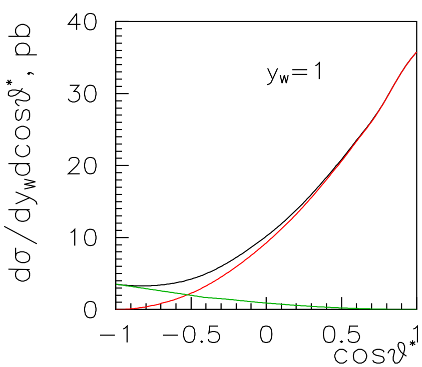
Angular distributions in the W rest frame: LO analysis



$\propto u(x_1)\bar{d}(x_2)(1+cos\theta^*)^2$

$\propto \bar{d}(x_1)u(x_2)(1-cos\theta^*)^2$





Unpolarized case

$p\,p \to (W^+ \to \bar{l}\,\nu_l)\,X$

$\sqrt{s}$ = 500 GeV, L = 800 pb^{-1}

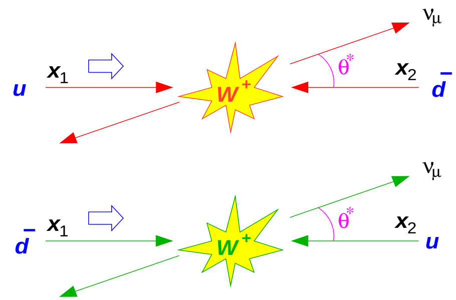
GRV-98, leading order

Total

$(1 + \cos\theta^*)^2$

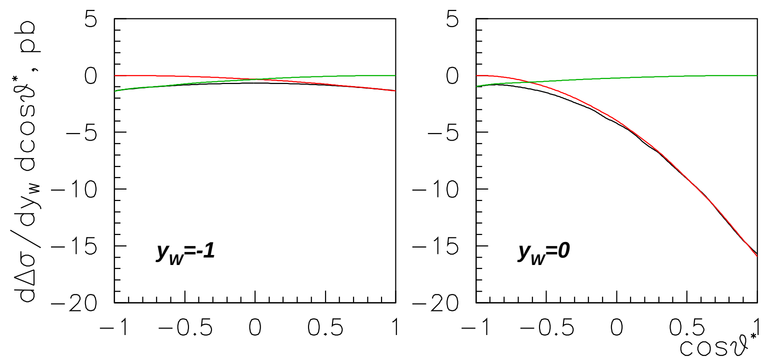
$(1 - \cos\theta^*)^2$

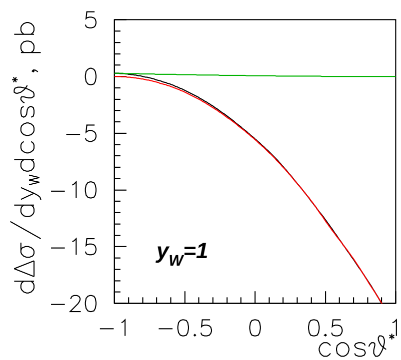
Angular distributions in the W rest frame: LO single−spin cross sections



$\propto -\Delta u(x_1)\bar{d}(x_2)(1+cos\theta^*)^2$

$\propto \Delta\bar{d}(x_1)u(x_2)(1-cos\theta^*)^2$





Single-polarized case

$\Delta p\,p \to (W^+ \to \bar{l}\,\nu_l)\,X$

$\sqrt{s}$ = 500 GeV, L = 800 pb^{-1}

GRSVs-2000, leading order

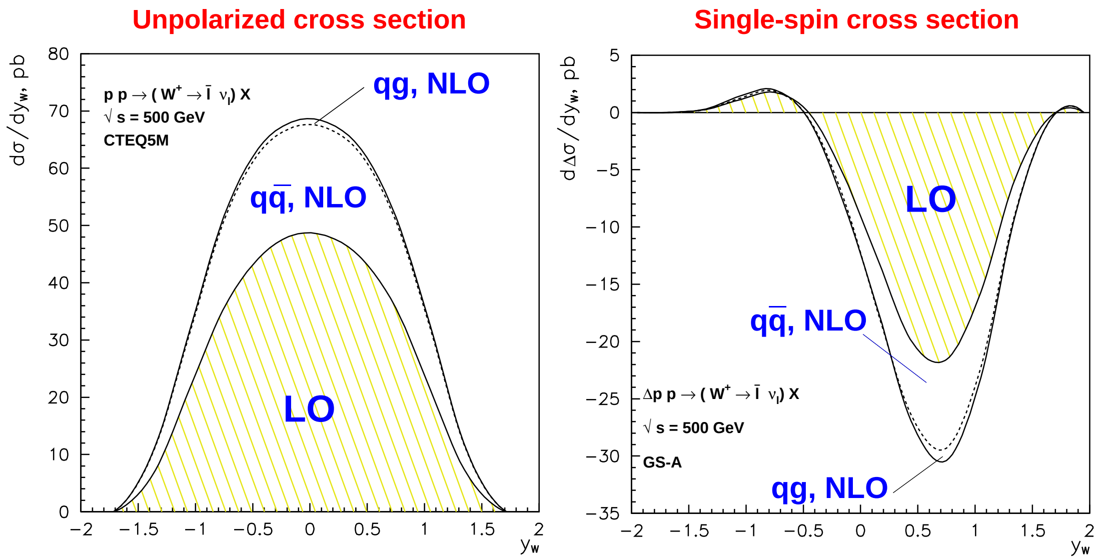
Total

$(1 + \cos\theta^*)^2$

$(1 - \cos\theta^*)^2$

Interest in NLO fully differential cross sections

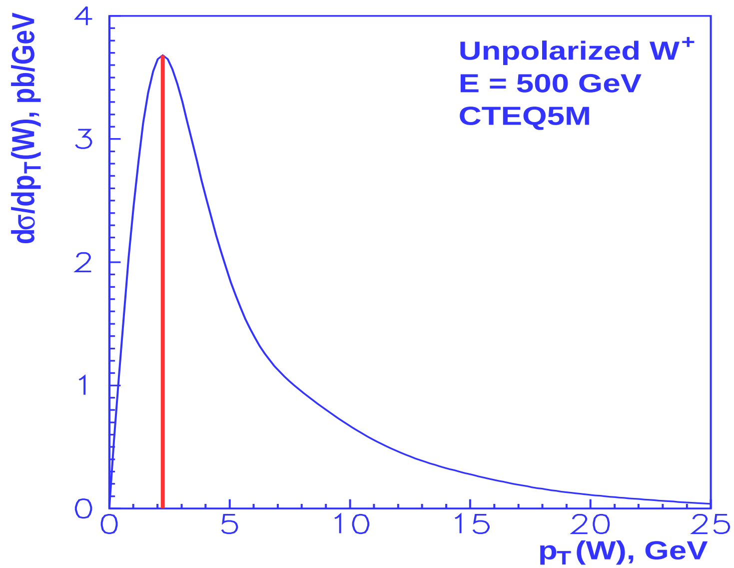
✲ Sizeable NLO corrections (30%) to the numerator and denominator



◇ NLO accuracy required by the global analysis of polarized PDF's

◇ Indefinite sign of $\Delta f(x) \Rightarrow$ possible radiation zeros at the LO

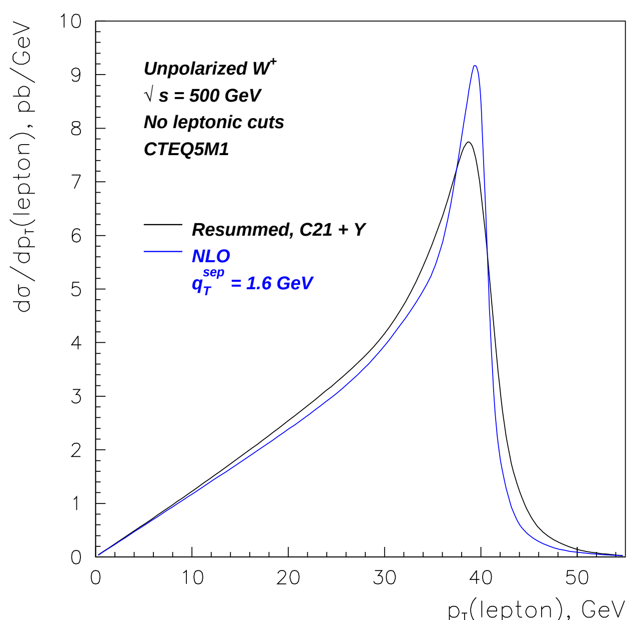
Transverse momentum distributions

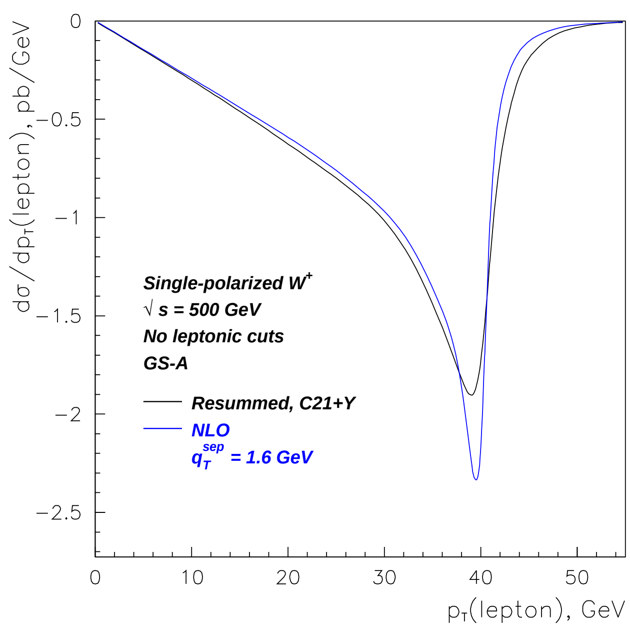


$p_{TW} \neq 0$! The shape of $d\sigma/dp_{TW}$ at $p_{TW} \to 0$ cannot be described at a finite order of PQCD: calculation of the sum

$$\frac{1}{p_{TW}^2}\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \sum_{m=0}^{2n-1} v_{mn}\left(\ln^m \frac{Q^2}{p_{TW}^2} \quad \text{or} \quad \delta(\vec{p}_{TW})\right)$$

is needed





Similar multiple parton radiation effects in lepton p_T distributions

$\Delta pp \to (W^{\pm} \to l\nu)X$: asymmetry $A_L(y_\ell)$ with respect to the rapidity y_ℓ of the decay charged lepton

$$A_L(y_\ell) \equiv \frac{\frac{d\sigma^{\vec{p}p}}{dy_\ell} - \frac{d\sigma^{\overleftarrow{p}p}}{dy_\ell}}{\frac{d\sigma^{\vec{p}p}}{dy_\ell} + \frac{d\sigma^{\overleftarrow{p}p}}{dy_\ell}}$$

A better alternative to the commonly discussed single-spin asymmetry $A_L(y)$ with respect to the rapidity y_W of the W boson

- Directly measurable
- Not distorted by limited acceptance of RHIC detectors (while $A_L(y_W)$ is strongly distorted)
- Sensitive to different polarized parton distributions
- A fully differential $\mathcal{O}(\alpha_S)$ calculation with inclusion of W-boson decay and transverse momentum resummation exists in the form of a Monte-Carlo code

(available at http://hep.pa.msu.edu/~nadolsky/RhicBos)

Lepton-level resummation calculation

- $\mathcal{O}(\alpha_S)$ fully differential cross section

$$\frac{d\sigma}{d^3\vec{p}_l d^3\vec{p}_{\nu_l}}[\, pp \rightarrow (\gamma^*, W^\pm, Z^0) X\,]$$

for arbitrary longitudinal polarizations of the beams $\Rightarrow$ A_L, A_{LL} at the lepton level

- γ_5 matrices from the axial current and spin projectors; the t'Hooft-Veltman and dimensional reduction schemes used
- In the region $p_T^W \rightarrow 0, \frac{d\sigma}{d^3\vec{p}_l d^3\vec{p}_{\nu_l}}$ is dominated by large terms

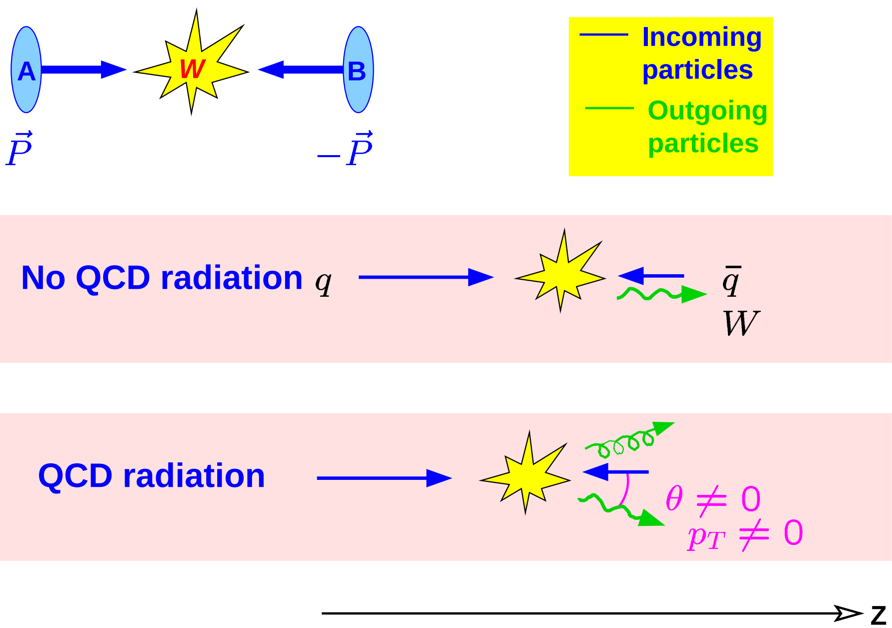
$$\alpha_S^n \left(\frac{1}{p_T^2} \ln^m \frac{Q^2}{p_T^2} \quad \text{or} \quad \delta(\vec{p}_T) \right),$$
$$n = 0, \ldots \infty; m = 0, \ldots, 2n-1$$

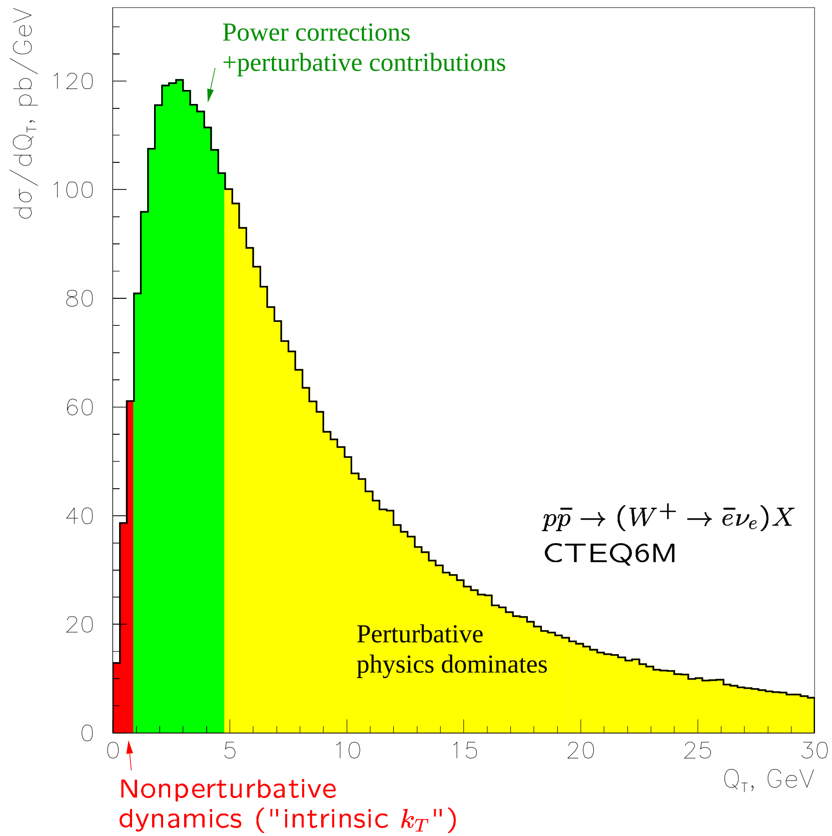
We found the sum of these terms with the help the impact parameter space resummation formalism *(Collins, Soper, Sterman, 1985)*

$$\frac{d\sigma_{h_A h_B}}{dQ^2 dy dp_T^2 d\Omega_l} \approx \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} \widetilde{W}_{h_A h_B}(b, \ldots)$$

q_T resummation for vector boson production at the Tevatron

Resummation: W boson production at the Tevatron





Needed to precisely measure W-boson mass

Different q_T ranges $\Leftrightarrow$ different dynamical mechanisms

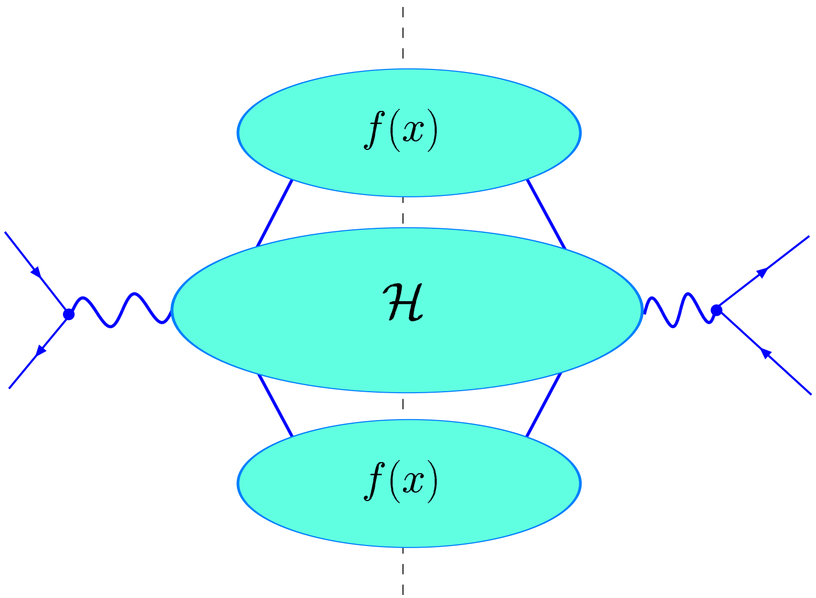
Resummation describes all q_T range in one unified framework

RhicBos:
correct NLO normalization
for low-Q Drell-Yan pairs, W, and Z!

QCD factorization in hard and soft regions

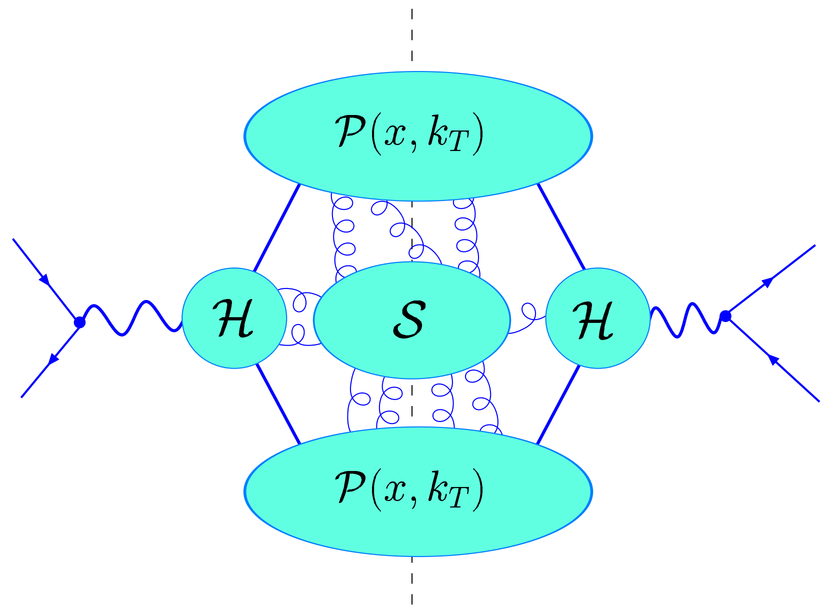
Finite-order (FO) factorization

$\Lambda^2_{QCD} \ll p^2_{TW} \sim Q^2$

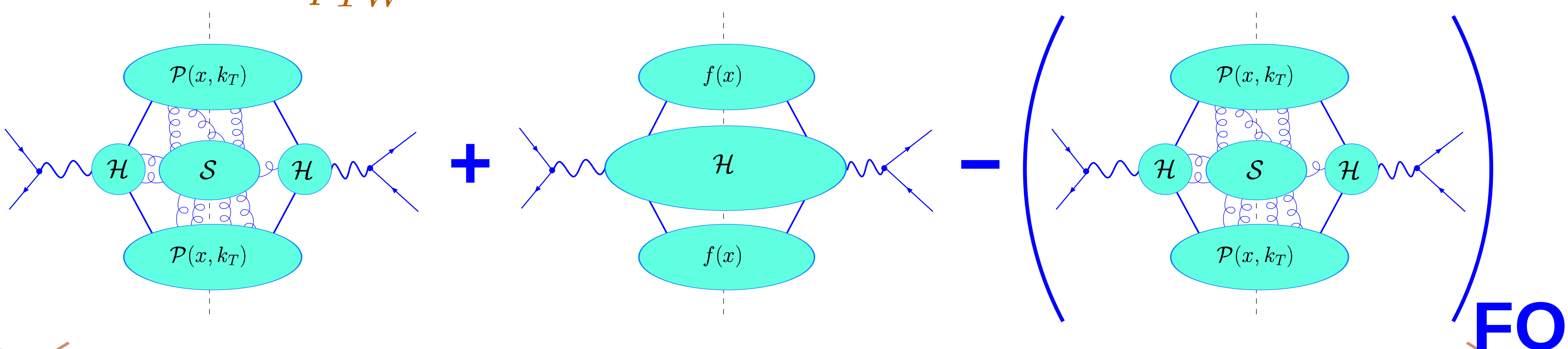


Small-p_T factorization

$\Lambda^2_{QCD} \ll p^2_{TW} \ll Q^2$

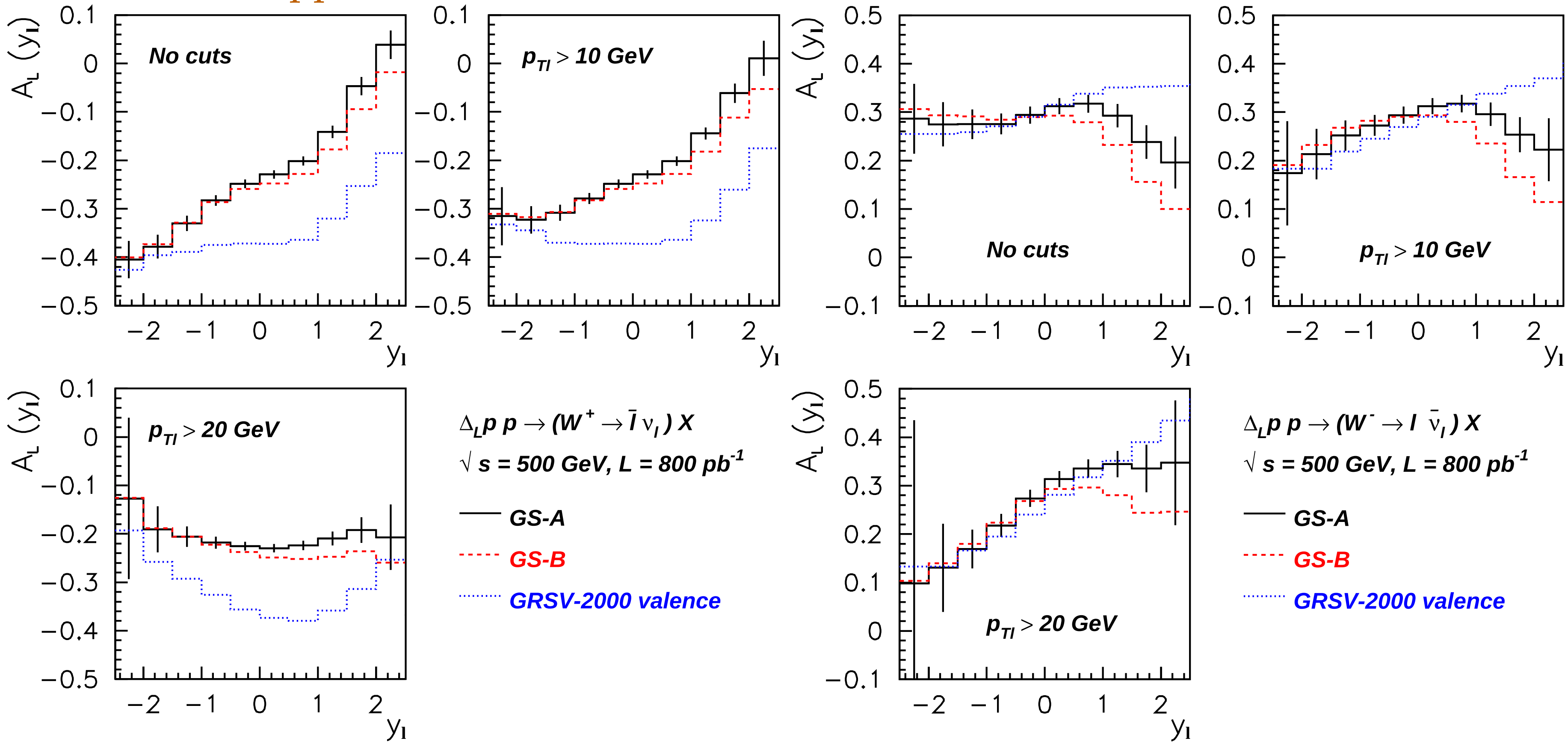


Solution for all p_{TW} :



$A_L(y_\ell)$ for different choices of $\min p_{T\ell}$

$\Delta pp \to W^+ X$ $\Delta pp \to W^- X$



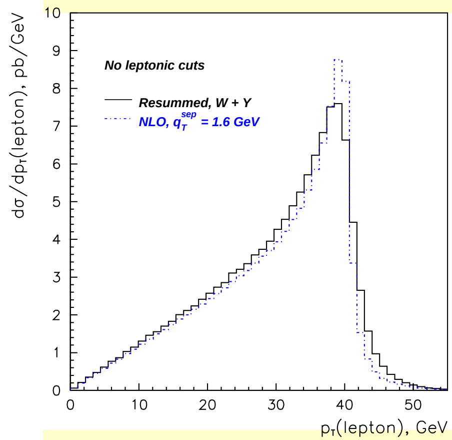
The direct observable is $A_L(y_\ell)$ with $p_{T\ell} \geq p_{T\ell}^{\text{min}}$

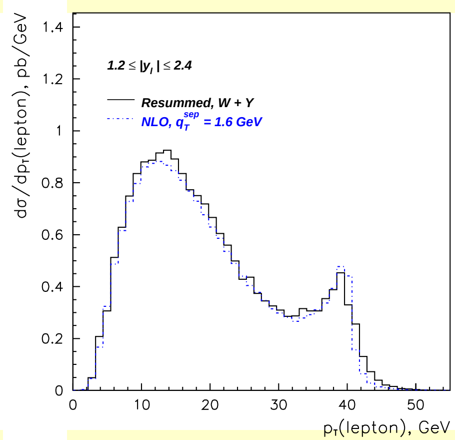
Predicted statistical errors are for $\mathcal{L} = 800$ pb^{-1}

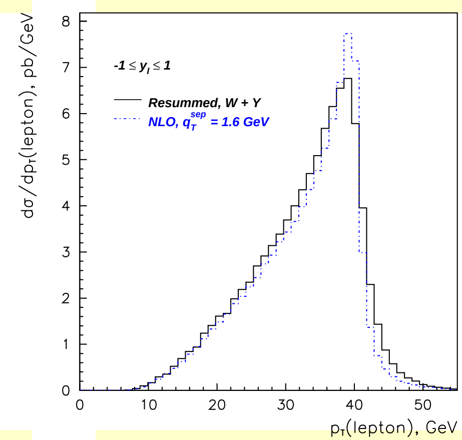
Central region (with higher rate) is as sensitive to PDF's as the forward region

W^+ production: p_T^{lepton} distributions with experimental rapidity cuts

$$\frac{d\sigma(pp \to W^+ X)}{dp_T^{lepton}}:$$







No cuts ↕

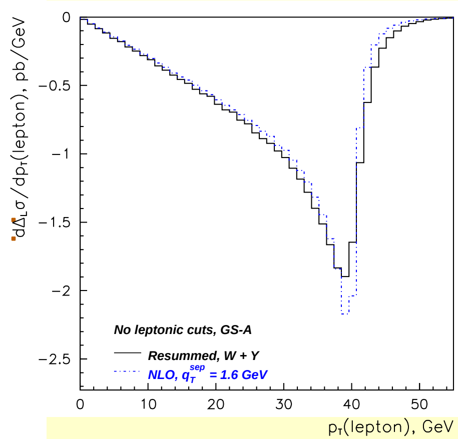
PHENIX ↕

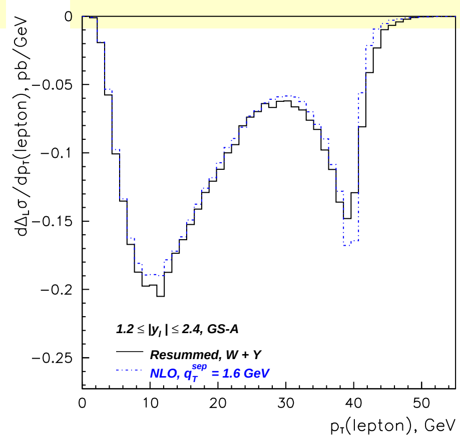
$(1.2 \le |y_l| \le 2.4)$

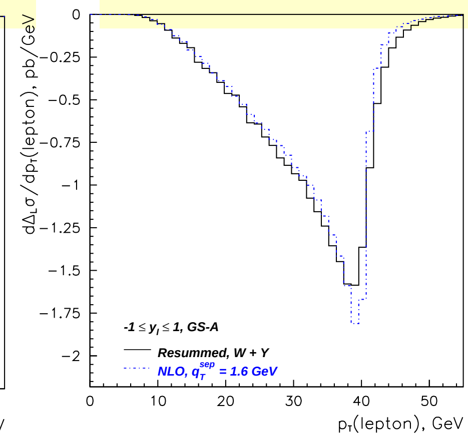
STAR ↕

$(-1 \le y_l \le 1)$

$$\frac{d\Delta_L\sigma(\vec{p}p \to W^+ X)}{dp_T^{lepton}}:$$

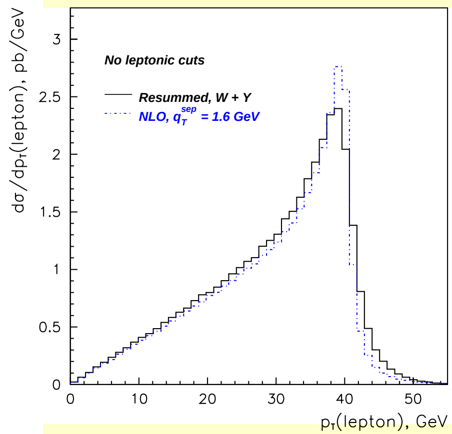


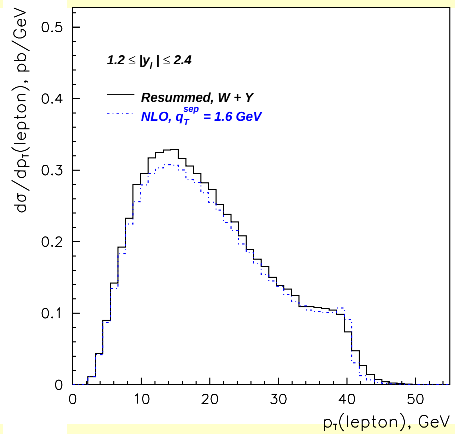


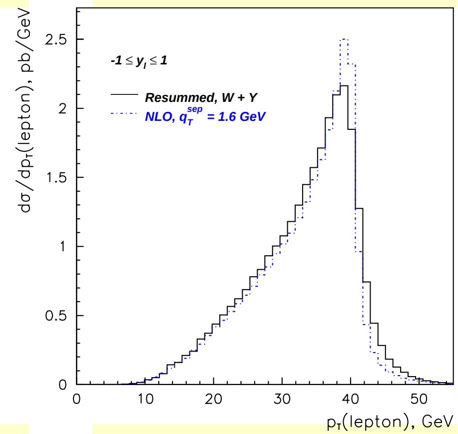


W^- production: p_T^{lepton} distributions with experimental rapidity cuts

$$\frac{d\sigma(pp \to W^- X)}{dp_T^{lepton}}:$$





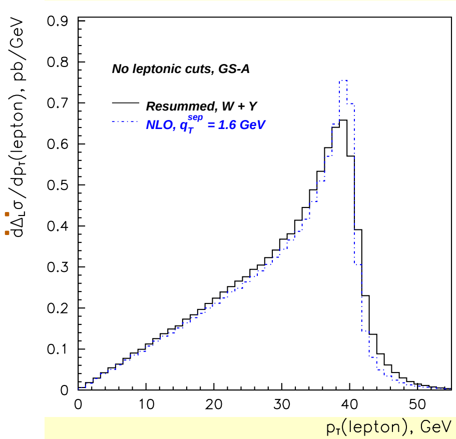


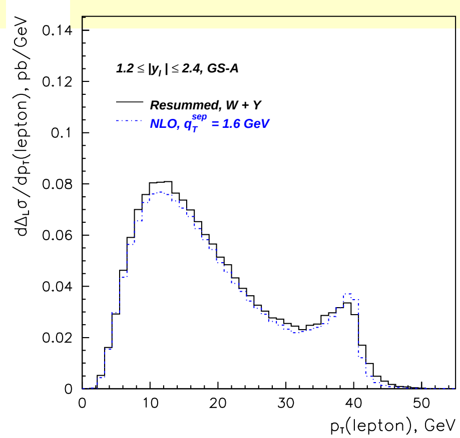
No cuts ↕

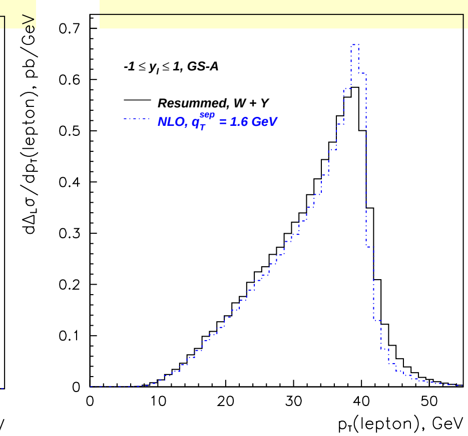
PHENIX ↕ $(1.2 \leq |y_l| \leq 2.4)$

STAR ↕ $(-1 \leq y_l \leq 1)$

$$\frac{d\Delta_L\sigma(\vec{p}p \to W^- X)}{dp_T^{lepton}}:$$



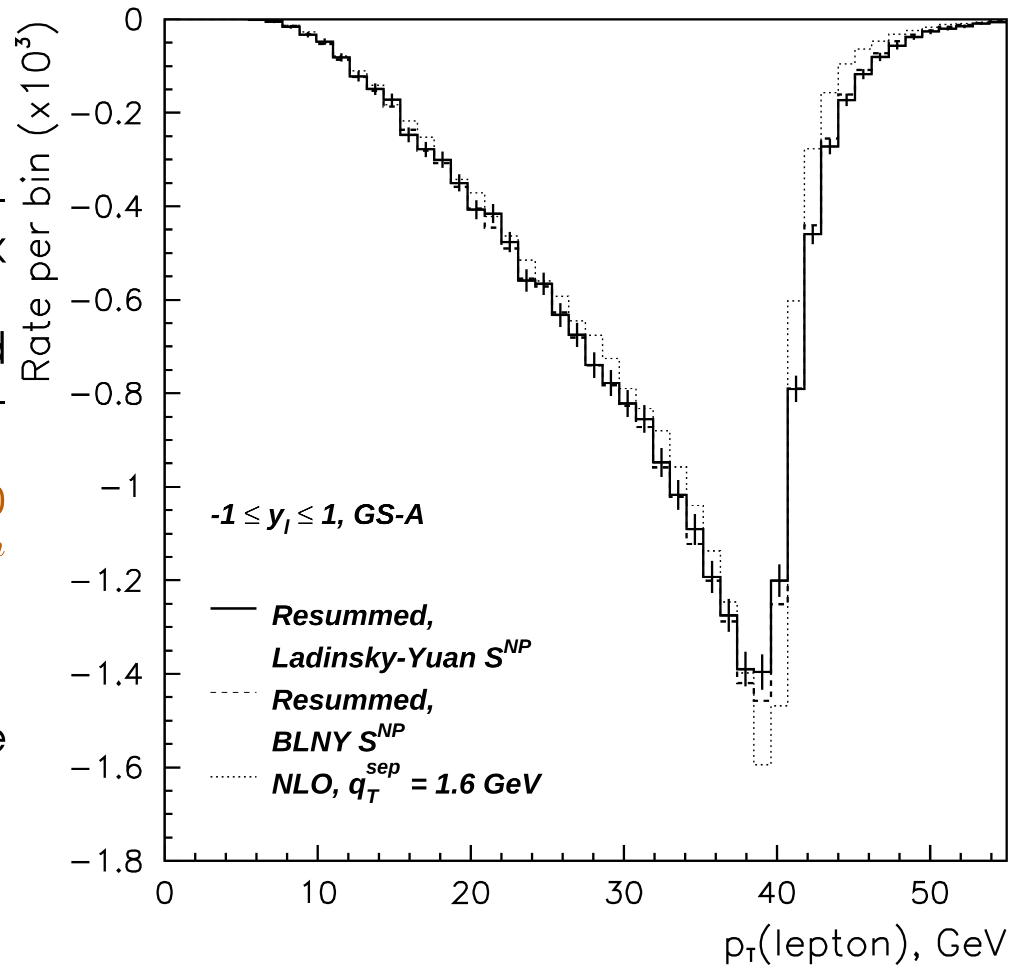




"Spin independence" of the Jacobian peak

Due to

- the spin independence of the perturbative Sudakov factor (quark helicity conservation)
- reduced importance and probable universality of non-perturbative contributions
- the shape of $d\sigma/dp_T^W$ at $p_T^W \to 0$ and Jacobian peak in $d\sigma/dp_T^{lepton}$ can be predicted based on
- the unpolarized measurements
- measurements of $d\Delta\sigma/dp_T$ in the polarized γ^*, Z^0 production



This consequence of the factorization picture must be tested at RHIC for all types of vector bosons

Lepton-level spin asymmetries in the global PDF fit

Unpolarized W-boson charge asymmetry at the Tevatron

$$A_{charge}(y_\ell) \equiv \frac{\frac{d\sigma^{W^+}}{dy_\ell} - \frac{d\sigma^{W^-}}{dy_\ell}}{\frac{d\sigma^{W^+}}{dy_\ell} + \frac{d\sigma^{W^-}}{dy_\ell}}$$

- analog of $A_L(y_\ell)$ in the unpolarized case; related to

$$A_{charge}(y_W) = \frac{u(x_a)d(x_b) - d(x_a)u(x_b)}{u(x_a)d(x_b) + d(x_a)u(x_b)}$$

- constrains $d(x, M_W)/u(x, M_W)$ in CTEQ and MRST analyses

- published data is implemented in the global fit with the selection cut $p_{T\ell} \geq p_{T\ell}^{\text{min}} = 25$ GeV

$d\sigma/dy_\ell$ at the Born level

$$\frac{d\sigma(p\bar{p}\to W^+X)}{dy_\ell} = \frac{2\pi\sigma_0}{S}\int_{y_{\rm min}(p_{T\ell}^{\rm min})}^{y_{\rm max}(p_{T\ell}^{\rm min})} dy_W \,\sin^2\theta$$
$$\times\left\{u(x_a)d(x_b)(1+\cos\theta)^2 + d(x_a)u(x_b)(1-\cos\theta)^2\right\},$$

with $x_{a,b}=\frac{Q}{\sqrt{S}}e^{\pm y_W}$, $\cos\theta=\tanh(y_\ell-y_W)$

- ✲ Simple kinematics due to $p_{TW}=0$
- ✲ Only 2 structure functions $\propto (1\pm\cos\theta)^2$ in the $W^\pm$ rest frame
- ✲ $p_{T\ell}^{\rm min}$ appears only in the limits of the integration $y_{\rm min}$, $y_{\rm max}$

Similarly,

for $d\Delta\sigma/dy_\ell$:

$$\frac{d\Delta\sigma(pp\to W^+X)}{dy_\ell} = \frac{2\pi\sigma_0}{S}\int_{y_{\rm min}(p_{T\ell}^{\rm min})}^{y_{\rm max}(p_{T\ell}^{\rm min})} dy_W \,\sin^2\theta$$
$$\times\left\{-\Delta u(x_a)\bar{d}(x_b)(1+\cos\theta)^2 + \Delta\bar{d}(x_a)u(x_b)(1-\cos\theta)^2\right\}$$

NLO calculation of $d\sigma/dy_\ell$ is much more complex

- many structure functions, complicated phase space, initial-state gluons, resummation effects, ...

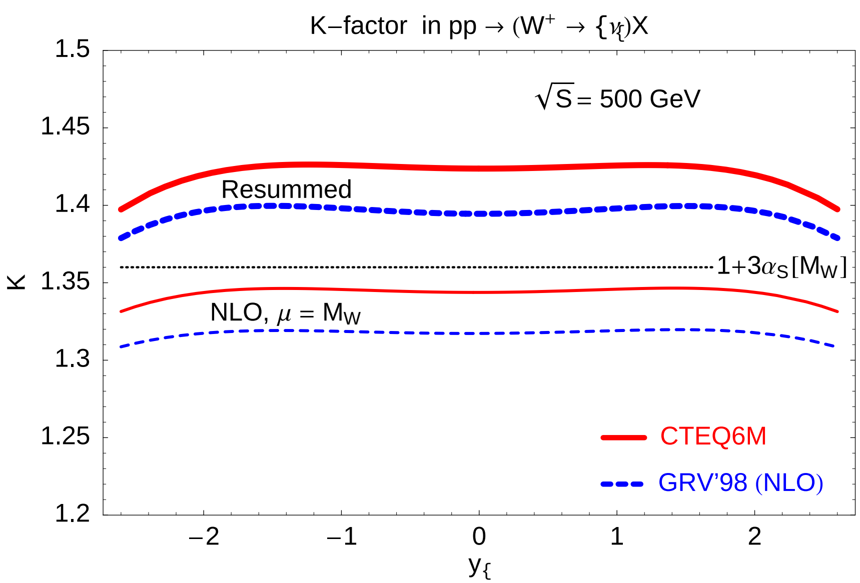
- is implemented in unpolarized PDF analyses using an effective K-factor

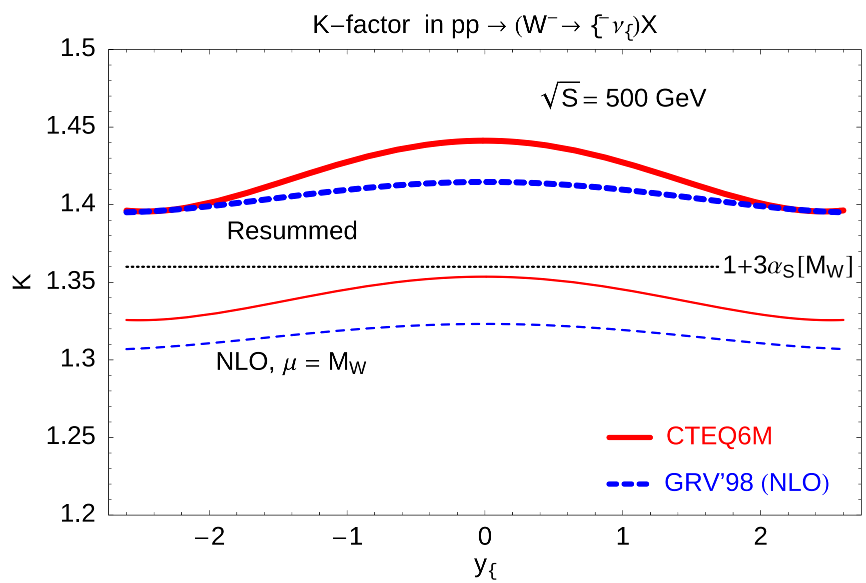
K-factor for $d\sigma/dy_\ell$

(Barger & Phillips, Collider Physics, ch. 7.11)

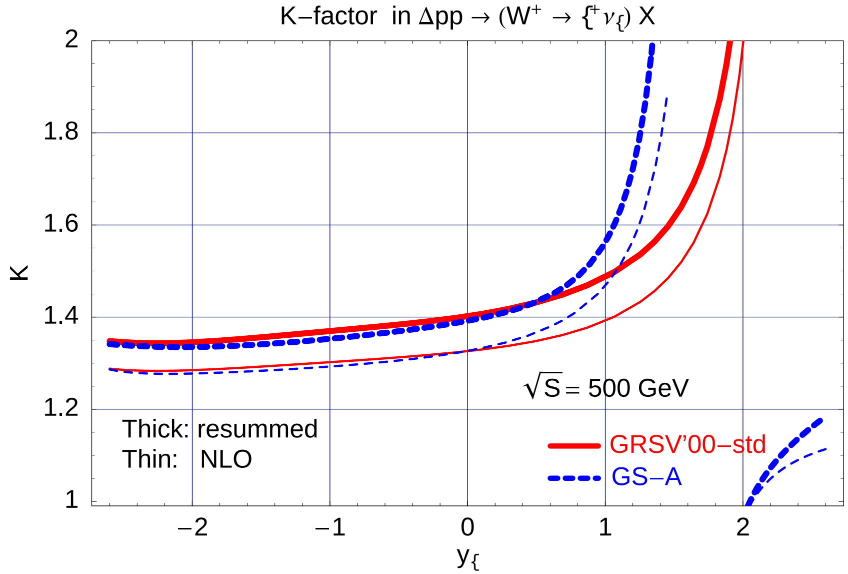
$$\frac{\frac{d(\Delta)\sigma_{NLO}}{dy_\ell}}{\frac{d(\Delta)\sigma_{LO}}{dy_\ell}} \approx \left(\underbrace{1+3\alpha_S(Q)}_{K_0 \approx 1.36} + \text{extra terms} \right)$$

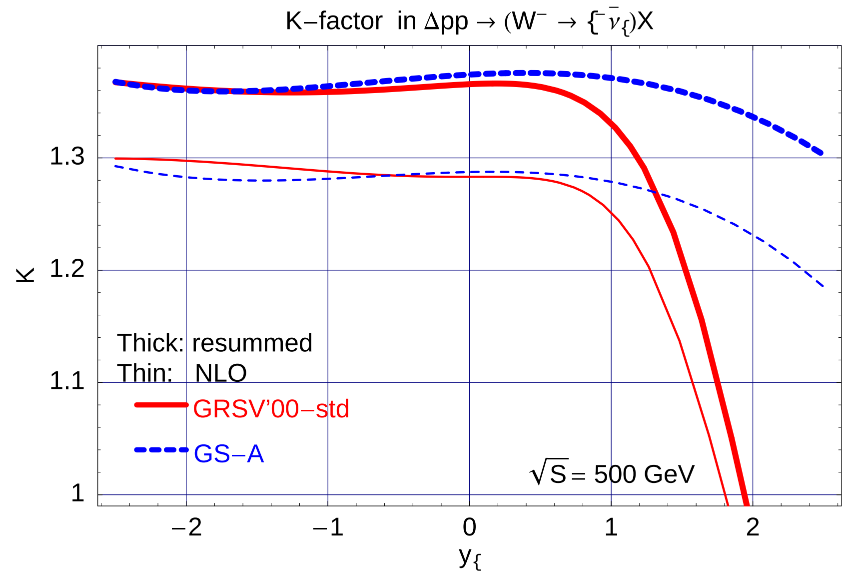
No $p_{T\ell}$ cuts:

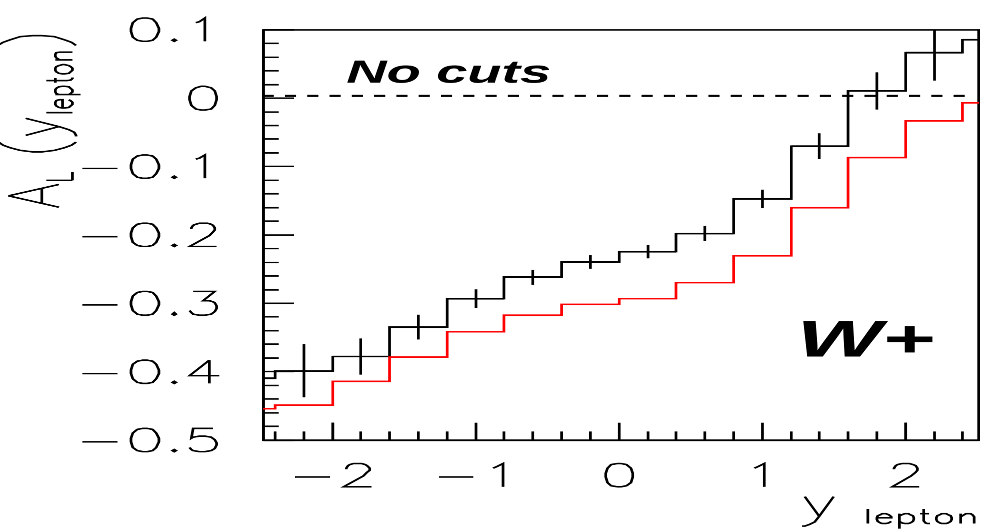


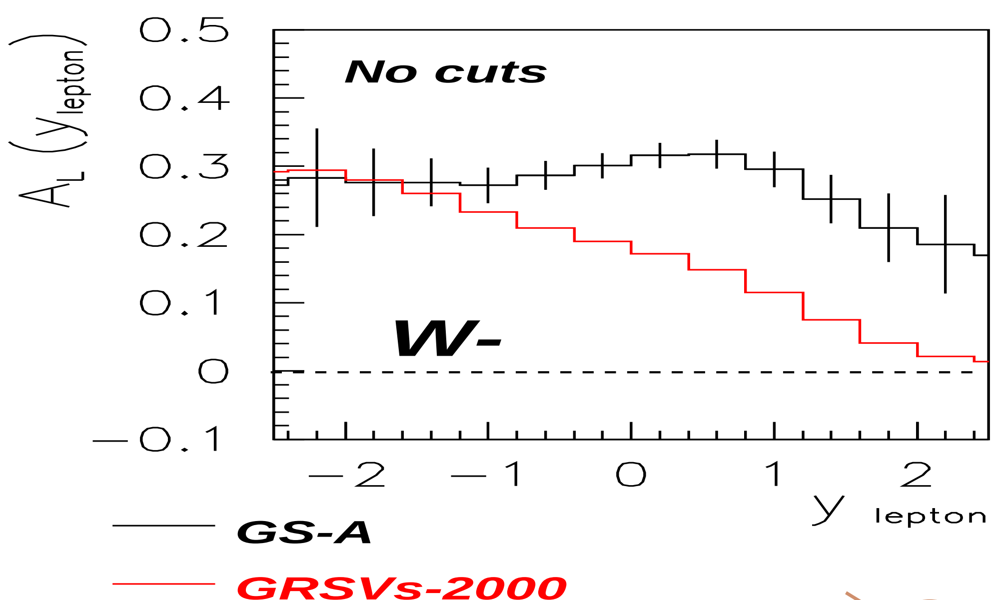


K-factors for $d\Delta\sigma/dy_\ell$



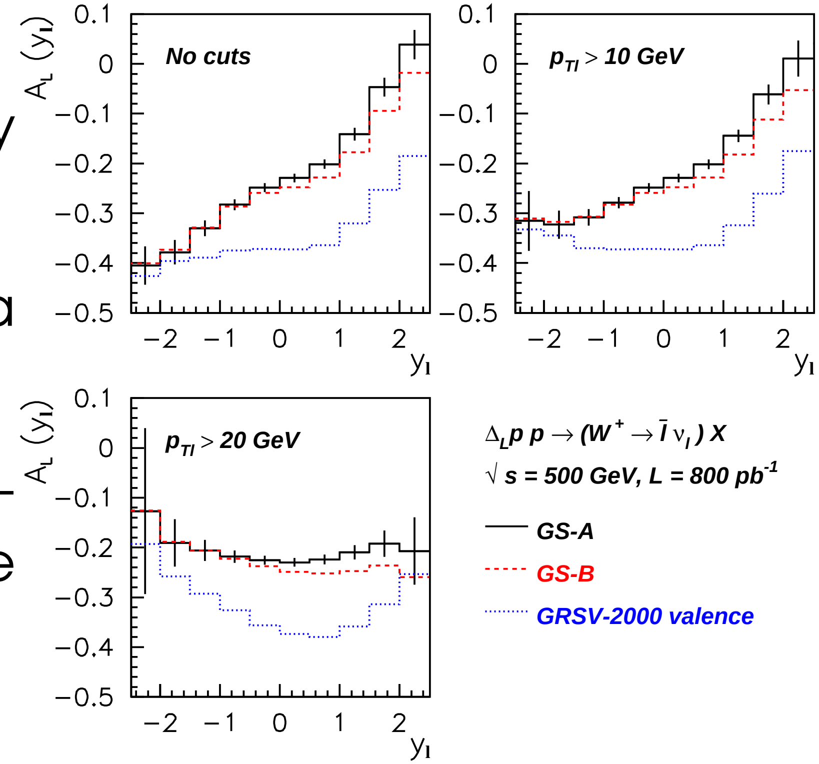






Radiation zeros...

- ...are easily identifiable in the data ($|A_L(y_\ell)| \lesssim 0.1$)
- ...are smeared by experimental resolution and statistical errors
- ...can be removed from the data by $p_{T\ell}$ cuts
- ...can be excluded from the fit by data selection cuts
- ...can be included in the fit, with the direct NLO calculation used only in the vicinity of the radiation zero



Summary

1. Measurement of forward leptons from unpolarized W boson production will provide important information about $u(x)$, $d(x)$ as $x \to 1$ (complementary to the Tevatron and low-energy Drell-Yan data)

2. The lepton single-spin asymmetry $A_L(y_\ell)$ provides a theoretically clean and direct observable in polarized W-boson production

3. $\mathcal{O}(\alpha_S)$ resummation calculation exists for fully differential lepton cross sections

4. Next-to-leading order $A_L(y_\ell)$ (with $p_{T\ell}$ cuts) can be easily implemented in the global fits using an effective K factor $K = (\Delta)\sigma_{NLO}/(\Delta)\sigma_{LO}$ and a simple procedure to deal with radiation zeros

Backup slides

$p\,p \to (W^{\pm}, Z^0 \to \ell_1\,\bar{\ell}_2)\,X$ at RHIC: expected total cross sections σ and numbers of events N for 1 lepton generation

		$\sqrt{S} = 200$ GeV, $\mathcal{L} = 320$ pb^{-1}	$\sqrt{S} = 500$GeV, $\mathcal{L} = 800$ pb^{-1}
W^+	$x\vert_{y_W=0}$	0.4	0.16
	$\sigma \pm \delta\sigma_{PDF}\left(\frac{\delta\sigma_{PDF}}{\sigma}\right)$	$1.38 \pm 0.34\,(0.25)$	$124 \pm 9\,(0.07)$
	$N \pm \sqrt{N}(1/\sqrt{N})$	$440 \pm 20\,(0.05)$	$99200 \pm 300\,(0.003)$
W^-	$x\vert_{y_W=0}$	0.4	0.16
	$\sigma \pm \delta\sigma_{PDF}\left(\frac{\delta\sigma_{PDF}}{\sigma}\right)$	$0.43 \pm 0.12\,(0.27)$	$41 \pm 4\,(0.10)$
	$N \pm \sqrt{N}(1/\sqrt{N})$	$142 \pm 12\,(0.09)$	$32800 \pm 200\,(0.006)$
Z^0	$x\vert_{y_Z=0}$	0.46	0.18
	$\sigma \pm \delta\sigma_{PDF}\left(\frac{\delta\sigma_{PDF}}{\sigma}\right)$	$0.07 \pm 0.02\,(0.26)$	$10.0 \pm 0.8\,(0.08)$
	$N \pm \sqrt{N}(1/\sqrt{N})$	$21 \pm 5\,(0.22)$	$8010 \pm 90\,(0.01)$

The unpolarized cross sections are estimated using CTEQ6 parton distribution functions (PDF's)

$\delta\sigma_{PDF}$ are due to experimental uncertainties in PDF's ($\sim 90\%$ c.l.)

$x_{A,B} \equiv (M_V/\sqrt{S})e^{\pm y_V}$ are Born-level momentum fractions for incoming partons; max $y_W = 0.92\,(1.82)$ for $\sqrt{S} = 200\,(500)$ GeV

What is the spin of the proton made of?

The interest in polarized hadronic reactions originates in the “spin crisis” (1989)

$$\begin{aligned}\Delta\Sigma &\equiv \sum_{flavors} \int_0^1 d\xi \left(\Delta f_{q/p}(\xi) + \Delta f_{\bar{q}/p}(\xi)\right) \\ &= 0.27 \pm 0.04\end{aligned}$$

Here $\Delta f_{a/A}(\xi, \mu_F)$ are PDFs for a longitudinally polarized nucleon

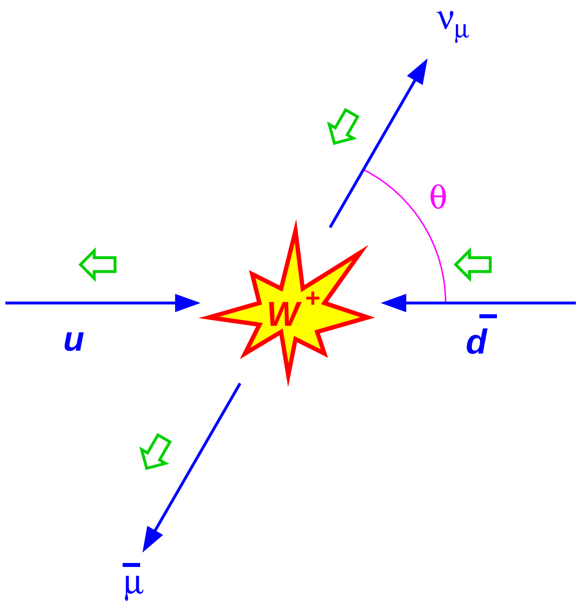
$$\begin{aligned}f_{a/A}(\xi, \mu_F) &\equiv f_{+/+}(\xi, \mu_F) + f_{-/+}(\xi, \mu_F) \\ \Delta f_{a/A}(\xi, \mu_F) &\equiv f_{+/+}(\xi, \mu_F) - f_{-/+}(\xi, \mu_F)\end{aligned}$$

Proton spin sum rule:

$$\int_0^1 d\xi \left[\frac{1}{2}\Delta\Sigma(\xi) + \Delta f_{G/p}(\xi)\right] + \langle L_q \rangle + \langle L_{\bar{q}} \rangle + \langle L_G \rangle = \frac{1}{2}$$

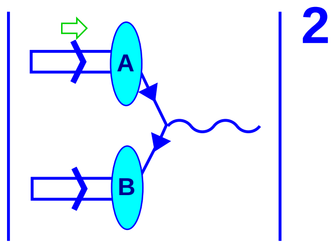
$\langle L_q \rangle$ is the orbital momentum of quarks, etc.

$W^{\pm}$-bosons as ideal polarimeters

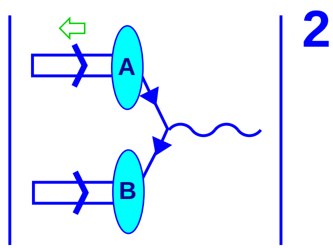


At the Born level:

$$\frac{d\Delta_L\sigma(pp \xrightarrow{W^+} \ell^+\nu_\ell X)}{dx_a dx_b d\cos\theta d\varphi} \propto$$
$$-\Delta u(x_a)\bar{d}(x_b)\,(1+\cos\theta)^2 +$$
$$+\Delta\bar{d}(x_a)u(x_b)\,(1-\cos\theta)^2$$



$\sim u_{-/+}(x_a)\bar{d}(x_b)$

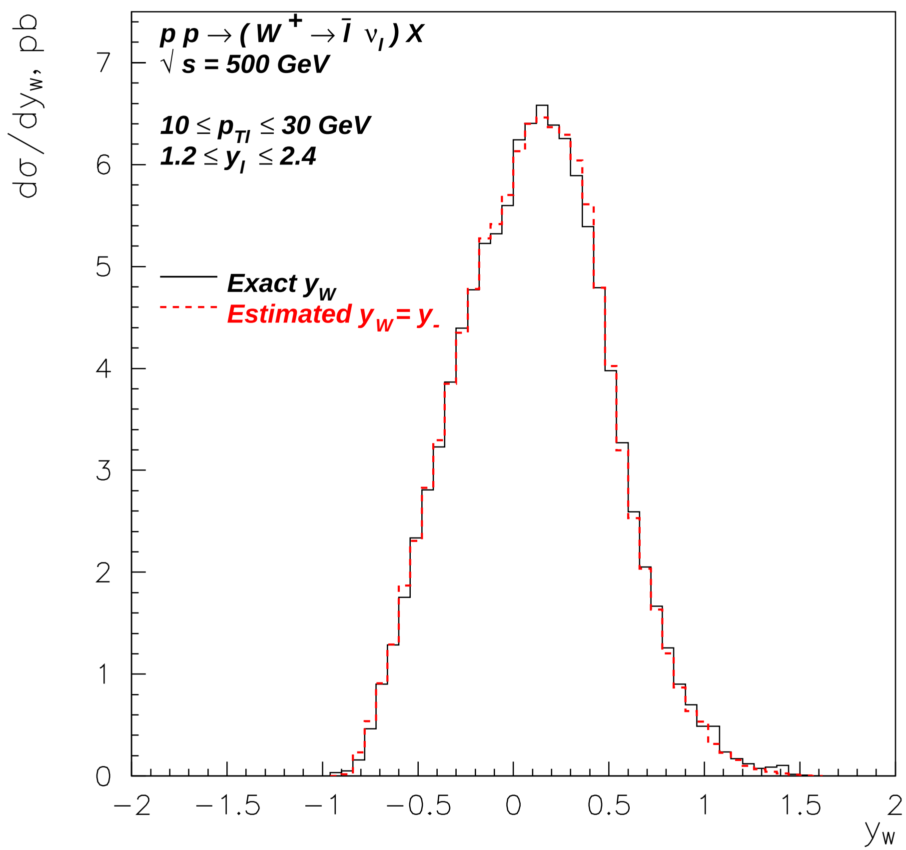


$\sim u_{-/-}(x_a)\bar{d}(x_b)$

Spin asymmetries in $W^{\pm}$ production are sensitive to the flavor structure of the polarized quark sea

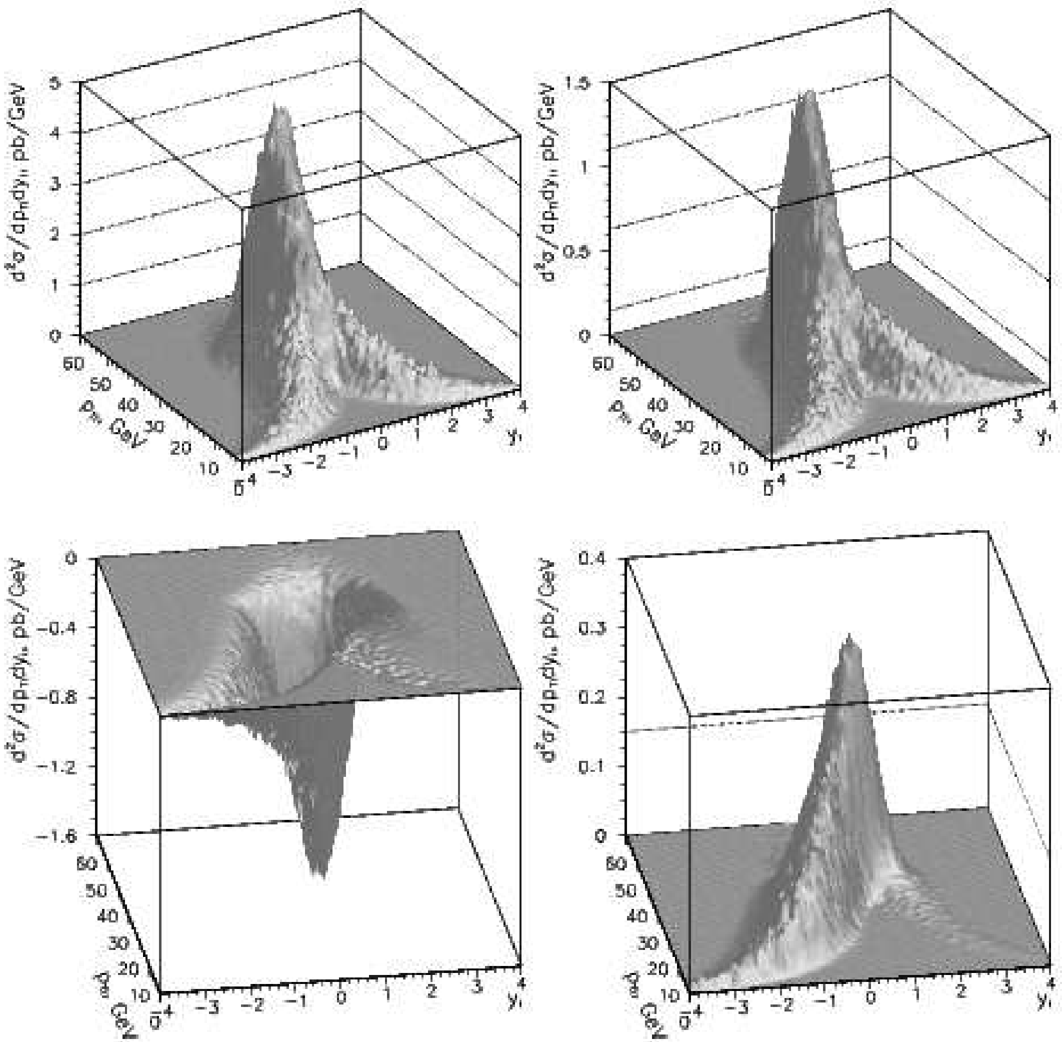
Signature of W boson events: high-p_T charged leptons and $\not{E}_T$

Comparison of $d\sigma/d(y_W^{exact})$ and $d\sigma/d(y_W^{estimated})$ at large lepton rapidities



$d\sigma/dy_W$ are resummed cross sections described below

Unpolarized (top) and single-spin (bottom) distributions $d^2\sigma/(dp_{T\ell}dy_\ell)$ in W^+ (left) and W^- (right) boson production



Most of the rate is at $y_\ell \sim 0$ and $p_T^{lepton} \sim M_W/2$

Resummation effects must be included to describe that region

Next-to-leading order (NLO) corrections in the analysis of parton distributions

- Required to achieve acceptable accuracy
- Would drastically slow calculations if straightforwardly implemented in the fit

Common solution: calculate the NLO cross section as

$$\sigma_{NLO} = K\sigma_{LO},$$

where

- the LO cross section σ_{LO} is updated in each call of the minimization subroutine
- the more complicated factor $K \equiv \sigma_{NLO}/\sigma_{LO}$ is updated every n calls (where n is a large number, e.g., $n \sim 10^3$)

K-factors in the spin-dependent fit

In the polarized case, the convergence of such procedure is questioned due to

- larger flexibility of spin-dependent distributions $\Delta f(x, Q)$
- possible presence of radiation zeros ($\sigma_{LO} = 0$) in spin-dependent cross sections

An alternative method involves a complete calculation of σ_{NLO} in each call of minimization using Mellin transform *(M. Stratmann, W. Vogelsang, Phys. Rev. D64, 114007)*

I will argue that the K-factors provide an efficient way to implement NLO corrections in polarized W boson production

Double Mellin transform

$$\sigma = \frac{1}{(2\pi i)^2} \int_{C_n} dn \int_{C_m} dm \Delta f_n \Delta f_m \widetilde{\sigma}(m,n),$$

where

$$\Delta f_m \equiv \int_0^1 dx\, x^{n-1} \Delta f(x)$$

is the n-th moment of $\Delta f(x)$,

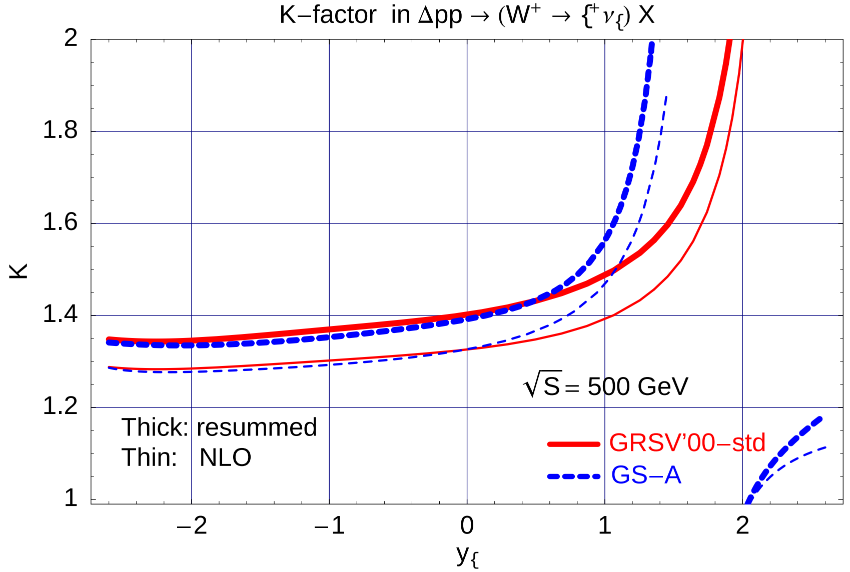
$$\widetilde{\sigma}(m,n) = \int d\{P.S.\} \int_0^1 dx_a \int_0^1 dx_b\, x_a^{-n} x_b^{-m} \frac{d\widehat{\sigma}(x_a,x_b)}{d\{P.S.\}}$$

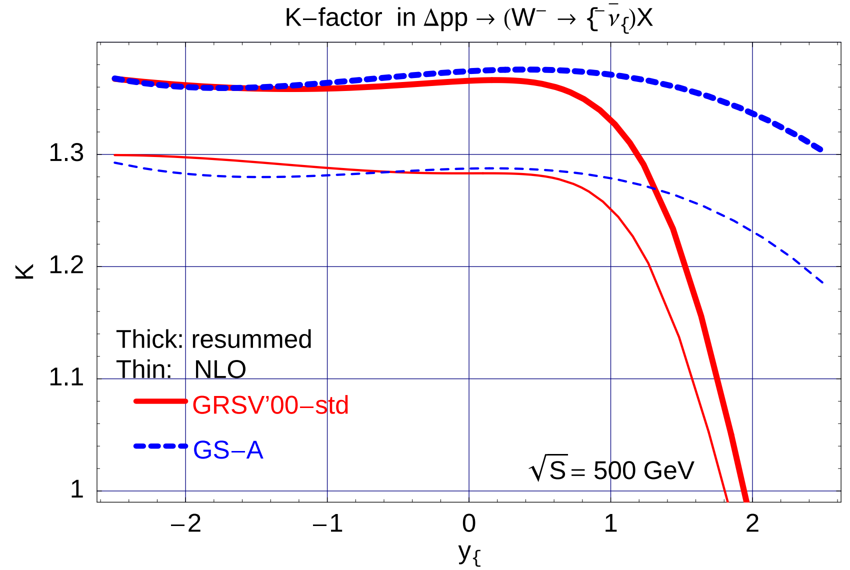
is the convolution of the cross section $\sigma(x_a, x_b)$ (integrated over phase space P.S.) with the "eigenvector PDFs" x_a^{-m}, x_b^{-n}

$\sigma(m,n)$ can be calculated at the full NLO before the fitting

It is not obvious that $\int d\{P.S.\}$ can be evaluated using Monte-Carlo methods for complex m and n

K factors for $d\Delta_L\sigma/dy_\ell$, no $p_{T\ell}$ cuts:





$p_{T\ell} > 20$ GeV: