## Shadowing and Small $x$ Physics on $J / \psi$ Production at RHIC

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## Outline

- Color Evaporation Model
- Setting the Parameters from Heavy Flavor Production
- $J / \psi$ Production in $p p, \mathrm{~d}+\mathrm{Au}$ and $A A$ Collisions at RHIC
- Initial and Final State Effects


## Calculating Heavy Flavors in Perturbative QCD

'Hard' processes have a large scale in the calculation that makes perturbative QCD applicable: high momentum transfer, $\mu^{2}$, high mass, $m$, high transverse momentum, $p_{T}$, since $m \neq 0$, heavy quark production is a 'hard' process
Asymptotic freedom assumed to calculate the interactions between two hadrons on the quark/gluon level but the confinement scale determines the probability of finding the interacting parton in the initial hadron
Factorization assumed between the perturbative hard part and the universal, nonperturbative parton distribution functions
The hadronic cross section in an $A B$ collision where $A B=p p, p A$ or nucleus-nucleus is

$$
\begin{aligned}
\sigma_{A B}\left(S, m^{2}\right)= & \sum_{i, j=q, \bar{q}, g} \int_{4 m_{Q}^{2} / s}^{1} \frac{d \tau}{\tau} \int d x_{1} d x_{2} \delta\left(x_{1} x_{2}-\tau\right) \\
& \times f_{i}^{A}\left(x_{1}, \mu_{F}^{2}\right) f_{j}^{B}\left(x_{2}, \mu_{F}^{2}\right) \widehat{\sigma}_{i j}\left(s, m^{2}, \mu_{F}^{2}, \mu_{R}^{2}\right)
\end{aligned}
$$

$f_{i}^{A}$ are the nonperturbative parton distributions, determined from fits to data, $x_{1}$ and $x_{2}$ are the fractional momentum of hadrons $A$ and $B$ carried by partons $i$ and $j, \tau=s / S$
$\widehat{\sigma}_{i j}\left(s, m^{2}, \mu_{F}^{2}, \mu_{R}^{2}\right)$ is hard partonic cross section calculable in QCD in powers of $\alpha_{s}^{2+n}$ : leading order (LO), $n=0$; next-to-leading order (NLO), $n=1 \ldots$
Results depend strongly on quark mass, $m$, factorization scale, $\mu_{F}$, in the parton densities and renormalization scale, $\mu_{R}$, in $\alpha_{s}$

## Quarkonium Production: Color Evaporation Model (CEM)

Gavai et al., G. Schuler and R.V.

All quarkonium states are treated like $Q \bar{Q}$ below $H \bar{H}$ threshold
Distributions $\left(x_{F}, p_{T}, \sqrt{s}, A\right)$ for all quarkonium family members identical - leads to constant ratios At LO, $g g \rightarrow Q \bar{Q}$ and $q \bar{q} \rightarrow Q \bar{Q}$; NLO add $g q \rightarrow Q \bar{Q} q$

$$
\sigma_{C}^{\mathrm{CEM}}=F_{C} \sum_{i, j} \int_{4 m^{2}}^{4 m_{H}^{2}} d \hat{s} \int d x_{1} d x_{2} f_{i / p}\left(x_{1}, \mu^{2}\right) f_{j / p}\left(x_{2}, \mu^{2}\right) \hat{\sigma}_{i j}(\hat{s}) \delta\left(\hat{s}-x_{1} x_{2} s\right)
$$

$F_{C}$ fixed at NLO from total cross section data as a function of $\sqrt{s}, \sigma\left(x_{F}>0\right)$ for inclusive $J / \psi$ and $B_{\mu \mu} d \sigma\left(\Upsilon+\Upsilon^{\prime}+\Upsilon^{\prime \prime}\right)_{y=0} / d y$
Values of $m$ and $\mu$ (here $\mu \propto \sqrt{\left(p_{T Q}^{2}+p_{T \bar{Q}}^{2}\right) / 2+m_{Q}^{2}}=m_{T Q \bar{Q}} \equiv m_{T}$ in the exclusive $Q \bar{Q}$ code) for several parton densities fixed from $Q \bar{Q}$ production, as described in the following

## $\chi_{c} / J / \psi$ Ratio Energy Independent

HERA-B plot comparing $R_{\chi_{c}}=\sigma\left(\chi_{c}\right) / \sigma(J / \psi)$ with $\pi A$ and $p A$ data
Color singlet model (CSM) result is high since direct $J / \psi$ only comes from $g g \rightarrow J / \psi g$, a 4-point vertex at $\mathcal{O}\left(\alpha_{s}^{3}\right)$, while $\chi_{c}$ is produced at $\mathcal{O}\left(\alpha_{s}^{2}\right)$ by $g g \rightarrow \chi_{0}$
Nonrelativistic QCD (NRQCD) is lower, especially at low energies, because of greater importance of $q \bar{q} \rightarrow \chi_{c} X$ channel, the octet contribution where octet $J / \psi$ matrix element is much bigger Result is most consistent with $R_{\chi_{c}}$ independent of $\sqrt{S}$, as predicted by CEM CDF result, $R_{\chi_{c}}=0.297 \pm 0.017 \pm 0.057$, consistent with fixed-target results


Figure 1: Ratio of $\chi_{c}$ to $J / \psi$ cross sections as a function of $\sqrt{S}$ for $\pi A$ and $p A$ fixed-target measurements. The CSM and NRQCD curves are obtained from Monte Carlo while the 'average' is the average value of all measurements. From I. Abt et al. (HERA-B Collab.), Phys. Lett. 561 (2003) 61.

## $\psi^{\prime} / J / \psi$ Ratio Also Energy Independent

Data are from $p p$ and $p A$ interactions
Solid curve corresponds to CEM


Figure 2: Ratio of $\psi^{\prime}$ to $J / \psi$ cross sections with lepton pair branching ratios included as a function of $\sqrt{S}$ for $p p$ and $p A$ measurements. Adapted from R.V., Phys. Rept. 310 (1999) 197.

## Production and Feeddown Fractions

Data and branching ratios can be used to separate out the $F_{C}$ 's for each state in quarkonium family

| Resonance | $\sigma_{i}^{\text {dir }} / \sigma_{H}$ | $f_{i}$ |
| :---: | :---: | :---: |
| $J / \psi$ | 0.62 | 0.62 |
| $\psi^{\prime}$ | 0.14 | 0.08 |
| $\chi_{c 1}$ | 0.6 | 0.16 |
| $\chi_{c 2}$ | 0.99 | 0.14 |
| $\Upsilon$ | 0.52 | 0.52 |
| $\Upsilon^{\prime}$ | 0.33 | 0.10 |
| $\Upsilon^{\prime \prime}$ | 0.20 | 0.02 |
| $\chi_{b}(1 P)$ | 1.08 | 0.26 |
| $\chi_{b}(2 P)$ | 0.84 | 0.10 |

Table 1: The ratios of the direct quarkonium production cross sections, $\sigma_{i}^{\text {dir }}$, to the inclusive $J / \psi$ and $\Upsilon$ cross sections, denoted $\sigma_{H}$, and the feed down contributions of all states to the $J / \psi$ and $\Upsilon$ cross sections, $f_{i}$.

## Calculating Heavy Flavor Total Cross Sections

Partonic total cross section only depends on quark mass $m$, not kinematic quantities To NLO

$$
\begin{aligned}
\widehat{\sigma}_{i j}\left(s, m, \mu_{F}^{2}, \mu_{R}^{2}\right) & =\frac{\alpha_{s}^{2}\left(\mu_{R}^{2}\right)}{m^{2}}\left\{f_{i j}^{(0,0)}(\rho)\right. \\
& \left.+4 \pi \alpha_{s}\left(\mu_{R}^{2}\right)\left[f_{i j}^{(1,0)}(\rho)+f_{i j}^{(1,1)}(\rho) \ln \left(\mu_{F}^{2} / m^{2}\right)\right]+\mathcal{O}\left(\alpha_{s}^{2}\right)\right\}
\end{aligned}
$$

$\rho=4 \mathrm{~m}^{2} / \mathrm{s}, \mathrm{s}$ is partonic center of mass energy squared
$\mu_{F}$ is factorization scale, separates hard part from nonperturbative part
$\mu_{R}$ is renormalization scale, scale at which strong coupling constant $\alpha_{s}$ is evaluated
$\mu_{F}=\mu_{R}$ in evaluations of parton densities
$f_{i j}^{(a, b)}$ are dimensionless, $\mu$-independent scaling functions, $a=0, b=0$ and $i j=q \bar{q}, g g$ for LO, $a=1$, $b=0,1$ and $i j=q \bar{q}, g g$ and $q g, \bar{q} g$ for NLO
$f_{i j}^{(0,0)}$ are always positive, $f_{i j}^{(1, b)}$ can be negative also
Note that if $\mu_{F}^{2}=m^{2}, f_{i j}^{(1,1)}$ does not contribute

## Scaling Functions to NLO

Near threshold, $\sqrt{s} / 2 m \rightarrow 1$, Born contribution is large but dies away for $\sqrt{s} / 2 m \rightarrow \infty$
At large $\sqrt{s} / 2 m, g g$ channel is dominant, then $q g$
High energy behavior of the cross sections due to phase space and low $x$ behavior of parton densities


Figure 3: Scaling functions needed to calculate the total partonic $Q \bar{Q}$ cross section. The solid curves are the Born results, $f_{i j}^{(0,0)}$, the dashed and dot-dashed curves are NLO contributions, $f_{i j}^{(1,1)}$ and $f_{i j}^{(1,0)}$ respectively.

## Comparison of $c \bar{c}$ Calculations to Data

Two ways to evaluate total cross sections and make predictions for higher energies
There are only 2 important parameters at fixed target energies: the quark mass $m$ and the scale $\mu$ at higher energies, the low $x$, low $\mu$ behavior of the parton densities plays an important role in the asymptotic result
The scale is usually chosen so that $\mu_{F}=\mu_{R}$, as in parton density fits although there is no strict reason for doing so for heavy flavors
First way (RV, Hard Probes Collaboration): fix $m$ and $\mu \equiv \mu_{F}=\mu_{R} \geq m$ to data at lower energies and extrapolate to unknown regions - tends to favor lower masses
Second way (Cacciari, Nason and RV): determine an uncertainty band within $1.3<m<1.7 \mathrm{GeV}$ for charm and $4.5<m<5 \mathrm{GeV}$ for bottom with $\left(\mu_{F} / m, \mu_{R} / m\right)=(1,1),(2,2),(0.5,0.5),(0.5,1)$, $(1,0.5),(1,2),(2,1)$
We have to be careful with the resulting total charm cross sections for $\mu_{F} \leq m$ with the CTEQ6M parton densities since the minimum $\mu$ is 1.3 GeV , giving us big $K$ factors for the lower scales and making the use of $\mu_{F} \leq m$ problematic, to say the least!
Densities like GRV98 have a lower starting scale, making their behavior for low $x$, low $\mu$ charm production less problematic
Note also that even the two-loop evaluation of $\alpha_{s}$ is big for low scales, for $m=1.5 \mathrm{GeV}$ :
$\alpha_{s}(m / 2=0.75)=0.648, \alpha_{s}(m=1.5)=0.348$ and $\alpha_{s}(2 m=3)=0.246$

## CTEQ6M Densities at $\mu=m / 2, m$ and $2 m$

CTEQ6M densities extrapolate to $\mu<\mu_{\min }=1.3 \mathrm{GeV}$
When backwards extrapolation leads to $x g(x, \mu)<0$, then $x g(x, \mu) \equiv 0$




Figure 4: The CTEQ6M parton densities as a function of $x$ for $\mu=m / 2$ (left), $\mu=m$ (middle) and $\mu=2 m$ (right) for $m=1.5 \mathrm{GeV}$.

## Fixing $m$ and $\mu^{2}$ to All Data: Method 1

Difficult to obtain a large calculated $c \bar{c}$ cross section with $\mu_{F}^{2}=\mu_{R}^{2}$, as in parton density fits Data favors lower masses - lowest mass used here is 1.2 GeV but much lower masses than allowed in pQCD needed to agree with largest cross sections



Figure 5: Total $c \bar{c}$ cross sections in $p p$ and $p A$ interactions up to ISR energies as a function of the charm quark mass using the CTEQ6M parton densities. The left-hand plot shows the results with $\mu_{F}=\mu_{R}=m$ while in the right-hand plot $\mu_{F}=\mu_{R}=2 m$. From top to bottom the curves are $m=1.2,1.3,1.4,1.5,1.6,1.7$, and 1.8 GeV .

## Extrapolation to Higher Energies

We have kept only the most recent measurements, including the PHENIX $\sqrt{S}=130 \mathrm{GeV}$ result from $\mathrm{Au}+\mathrm{Au}$, lowest $\sqrt{S}=200 \mathrm{GeV}$ point is from PHENIX $p p$
Note the $\mu=m$ behavior at high energy: the cross section grows slower with $\sqrt{s}$ due to the small $x$ behavior of $x g(x, \mu)$ for $\mu$ close to $\mu_{\text {min }}$


Figure 6: Same as previous but the energy range extended to LHC energies.

## K Factors Using Method 1

$K$ factors defined here as the ratio of the NLO to LO cross sections, both calculated with NLO parton densities and two loop evaluation of $\alpha_{s}$
Note the $\mu=m$ behavior at high energy - $K$ factors grow at low mass and then turn over due to both the low $x$ parton densities and the fact that the LO cross section gets small far from threshold The larger the value of $\mu$, the better behaved the $K$ factors


Figure 7: The $K$ factors over the full $\sqrt{s}$ range.

## Theoretical Uncertainty Band: Method 2

Curves with $\left(\mu_{F} / m, \mu_{R} / m\right)=(1,0.5)$ and $(0.5,0.5)$ have large total cross sections since $\alpha_{s}(m / 2)=$ 0.648 , really too high to get a convergent result

Curves with $\mu_{F} \leq m$ turn over for $\sqrt{s}>100 \mathrm{GeV}$ due to low $x$, low $\mu$ behavior of parton densities


Figure 8: Total $\bar{c} \bar{c}$ cross sections calculated using CTEQ6M. The solid curve is the central value $\left(\mu_{F} / m, \mu_{R} / m\right)=(1,1)$ with $m=1.5$ GeV . The upper and lower dashed curves are $m=1.3$ and 1.7 GeV with ( 1,1 ) respectively. The upper and lower dot-dashed curves correspond to $(0.5,0.5)$ and $(2,2)$ while the upper and lower dotted curves are with $(1,0.5)$ and $(0.5,1)$ with $m=1.5 \mathrm{GeV}$.

## Theoretical Uncertainty Band: K Factors

Results with $\left(\mu_{F} / m, \mu_{R} / m\right)=(1,0.5)$ and $(0.5,0.5)$ have largest $K$ factors
Results with $(1,1),(2,2),(2,1)$ and $(1,2)$ with $m=1.5 \mathrm{GeV}$ and $(1,1)$ with $m=1.7 \mathrm{GeV}$ give $K<10$ at highest energies


Figure 9: The $c \bar{c} K$ factors calculated using CTEQ6M. The solid curve is the central value ( $\mu_{F} / m, \mu_{R} / m$ ) $=(1,1)$ with $m=1.5 \mathrm{GeV}$. The upper and lower dashed curves are $m=1.3$ and 1.7 GeV with $(1,1)$ respectively. The upper and lower dot-dashed curves correspond to $(0.5,0.5)$ and $(2,2)$ while the upper and lower dotted curves are with $(1,0.5)$ and $(0.5,1)$ with $m=1.5 \mathrm{GeV}$.

## 'Best' Agreement Found for Several Cases



Figure 10: Total $c \bar{c}$ cross sections in $p p$ and $\pi^{-} p$ interactions compared to data. All calculations are fully NLO. The curves are: MRST HO (central gluon) with $\mu=m=1.4 \mathrm{GeV}$ (solid) and $\mu=2 m=2.4 \mathrm{GeV}$ (dashed); CTEQ 5M with $\mu=m=1.4 \mathrm{GeV}$ (dot-dashed) and $\mu=2 m=2.4 \mathrm{GeV}$ (dotted); and GRV98 HO with $\mu=m=1.3 \mathrm{GeV}$.

## Fitted Fractions and $J / \psi$ Cross Sections in CEM

| Case | PDF | $m(\mathrm{GeV})$ | $\mu / m_{T}$ | $\sigma_{J / \psi} / \sigma_{C}^{\mathrm{CEM}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\psi 1$ | MRST HO | 1.2 | 2 | 0.0144 |
| $\psi 2$ | MRST HO | 1.4 | 1 | 0.0248 |
| $\psi 3$ | CTEQ 5M | 1.2 | 2 | 0.0155 |
| $\psi 4$ | GRV 98 HO | 1.3 | 1 | 0.0229 |

Table 2: The production fractions obtained from simultaneously fitting $F_{C}$ to the $J / \psi$ total cross sections and $y=0$ cross sections as a function of energy. The PDF, charm quark mass, and scales used are obtained from comparison of the $c \bar{c}$ cross section to data.

| Case | $\sigma_{J / \psi}^{\mathrm{inc}}$ | $\sigma_{J / \psi}^{\mathrm{dir}}$ | $\sigma_{\chi_{c 1}}$ | $\sigma_{\chi_{c 2}}$ | $\sigma_{\psi^{\prime}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\psi 1$ | 2.35 | 1.46 | 1.41 | 2.33 | 0.33 |
| $\psi 2$ | 1.76 | 1.09 | 1.06 | 1.74 | 0.25 |
| $\psi 3$ | 2.84 | 1.76 | 1.70 | 2.81 | 0.40 |
| $\psi 4$ | 2.10 | 1.31 | 1.26 | 2.08 | 0.29 |

Table 3: The charmonium cross sections (in $\mu \mathrm{b}$ ) for $200 \mathrm{GeV} p p$ collisions. The inclusive and direct $J / \psi$ cross sections are both given.

## Extrapolated $J / \psi$ Total Cross Sections

Total forward $J / \psi$ cross sections extrapolated to higher energy
Energy dependence obtained from NLO CEM
Factor of $\sim 1.6-2$ between results at 200 GeV and at 5.5 TeV


Figure 11: NLO $J / \psi$ forward cross sections. The solid curve employs the MRST HO distributions with $m=1.2 \mathrm{GeV} \mu / m_{T}=2$, the dashed, MRST HO with $m=1.4 \mathrm{GeV} \mu / m_{T}=1$, the dot-dashed, CTEQ 5 M with $m=1.2 \mathrm{GeV} \mu / m_{T}=2$, and the dotted, GRV 98 HO with $m=1.3 \mathrm{GeV} \mu / m_{T}=1$.

## From Total Cross Sections to Distributions

Distributions as a function of kinematic variables can provide more information than the total cross section
In total cross section, the quark mass is the only relevant scale
When considering kinematic observables like $x_{F}$ or $p_{T}$, the momentum scale is also relevant so that, instead of $\mu^{2} \propto m^{2}$, one usually uses $\mu^{2} \propto m_{T}^{2}$ - this difference makes the $p_{T}$-integrated total cross section decrease a bit relative to that calculated using the dimensionless scaling functions Quarkonium $p_{T}$ distributions at LO for $\mathcal{O}\left(\alpha_{s}^{3}\right)$ since pair $p_{T}$ is zero at $\mathcal{O}\left(\alpha_{s}^{2}\right)$ (LO in total cross section) Intrinsic transverse momentum added to calculation to reduce steepness of $p_{T}$ distribution

## The Quarkonium $p_{T}$ Distribution in the $Q \bar{Q}$ NLO Code

Gaussian $k_{T}$ smearing, $\left\langle k_{T}^{2}\right\rangle_{p}=1 \mathrm{GeV}^{2}$ for fixed target $p p$ and $\pi p$, broadened for $p A$ and $A A$, NLO code adds in final state:

$$
g_{p}\left(k_{T}\right)=\frac{1}{\pi\left\langle k_{T}^{2}\right\rangle_{p}} \exp \left(-k_{T}^{2} /\left\langle k_{T}^{2}\right\rangle_{p}\right)
$$

Comparison with $J / \psi$ and $\Upsilon$ Tevatron data at 1.8 TeV shows that the broadening should increase with energy, to $\left\langle k_{T}^{2}\right\rangle_{p} \approx 2.5 \mathrm{GeV}^{2}$
Fits of increase of $\left\langle p_{T}^{2}\right\rangle$ to old data are inadequate to explain this increase so we make a simple linear extrapolation to obtain

$$
\left\langle k_{T}^{2}\right\rangle_{p}=1+\frac{1}{6} \ln \left(\frac{s}{s_{0}}\right) \mathrm{GeV}^{2}
$$

Thus at RHIC energies $\left\langle k_{T}^{2}\right\rangle_{p}=1.77 \mathrm{GeV}^{2}$ for 200 GeV and $1.38 \mathrm{GeV}^{2}$ for $62 \mathrm{GeV} p p$ collisions

## Comparison with Tevatron $J / \psi p_{T}$ Distributions



Figure 12: The $p_{T}$ distributions of direct $J / \psi$ as well as $J / \psi^{\prime}$ 's from $\psi^{\prime}$ and $\chi_{c}$ decays calculated for cases $\psi 1$ (solid) and $\psi 4$ (dashed) are compared to the CDF data. We use $\left\langle k_{T}^{2}\right\rangle_{p}=2.5 \mathrm{GeV}^{2}$.

## Nuclear Effects on $p_{T}$ Broadening

Additional broadening - beyond the intrinsic broadening - assumed to arise from multiple parton scattering in the target before hard interaction
$J / \psi, \Upsilon$ and Drell-Yan show effects of broadening in $p A$, parameterized as

$$
\left\langle k_{T}^{2}\right\rangle_{i A}=\left\langle k_{T}^{2}\right\rangle_{p}+(\langle\nu\rangle-1) \Delta^{2}(\mu)
$$

The broadening is proportional to the average number of collisions of the projectile parton in the target,

$$
\langle\nu\rangle=\sigma_{N N} \frac{\int d^{2} b T_{A}^{2}(b)}{\int d^{2} b T_{A}(b)}=\frac{3}{2} \sigma_{N N} \rho_{0} R_{A}
$$

$T_{A}(b)$ is the nuclear profile function
The second equality is average over impact parameter assuming a spherical nucleus, $\rho_{0}=0.16 \mathrm{fm}^{-3}$ is the central nuclear density and $R_{A}$ is the nuclear radius $\Delta^{2}(\mu=2 m)$, the strength of the broadening, depends on the scale of the interactions

$$
\begin{array}{r}
\Delta^{2}(\mu)=0.225 \frac{\ln ^{2}(\mu / \mathrm{GeV})}{1+\ln (\mu / \mathrm{GeV})} \mathrm{GeV}^{2} \\
\begin{array}{ccc}
(\langle\nu\rangle & -1) \Delta^{2}(\mu)\left(\mathrm{GeV}^{2}\right) \\
\hline Q \bar{Q} & p A & \text { central } A A \\
c \bar{c} & 0.35 & 0.7 \\
b \bar{b} & 1.57 & 3.14 \\
\hline
\end{array}
\end{array}
$$

## $J / \psi p_{T}$ Distributions at RHIC

At RHIC, $\left\langle k_{T}^{2}\right\rangle_{p}=1.77 \mathrm{GeV}^{2}$
Agreement with normalized total cross section good, data somewhat steeper in forward direction than calculation


Figure 13: The inclusive $J / \psi p_{T}$ distributions, calculated for case $\psi 1$, compared to the preliminary PHENIX data. From left to right: $e^{+} e^{-} p p$ and $\mathrm{d}+\mathrm{Au} ; \mu^{+} \mu^{-} p p$; and $\mu^{+} \mu^{-} \mathrm{d}+\mathrm{Au}$. Thanks to Mike Leitch for making these plots!

## $J / \psi p_{T}$ Distributions in $A A$ Collisions

Broadening more effective at 62 GeV since $\left\langle k_{T}^{2}\right\rangle_{p}$ is smaller, $1.38 \mathrm{GeV}^{2}$ instead of $1.77 \mathrm{GeV}^{2}$ at 200 GeV and average $\left\langle p_{T}\right\rangle$ also smaller (note reduced range of $p_{T}$ distribution at 62 GeV )

Little difference between $p p$ and $A A$ at 200 GeV


Figure 14: The inclusive $J / \psi p_{T}$ distributions, calculated for case $\psi 1$ for $|y|<0.75$ (top) and $1.2<y<2.2$ (bottom) at $\sqrt{S}=200$ GeV (left) and 62 GeV (right). At 200 GeV the results show $p p$ (solid), $\mathrm{d}+\mathrm{Au}$ (dashed), $\mathrm{Au}+\mathrm{Au}$ (dot-dashed) and $\mathrm{Cu}+\mathrm{Cu}$ (dotted) interactions. At 62 GeV , we show $p p$ (solid) and $\mathrm{Cu}+\mathrm{Cu}$ (dashed) results.

## Prediction of $J / \psi$ Rapidity Distributions at RHIC

## Agreement of CEM calculation with overall normalization of Run 3 data good

Shape has right trend for $\mathrm{d}+\mathrm{Au}$ with EKS98 shadowing


Figure 15: The inclusive $J / \psi y$ distributions in $\sqrt{s}=200 p p$ (left-hand side for $\psi 1$ (solid), $\psi 2$ (dashed), $\psi 3$ (dot-dashed) and $\psi 4$ (dotted)) and $\mathrm{d}+\mathrm{Au}$ (right-hand side with $\psi 1$ and EKS98). The rapidity distributions are unaffected by broadening. Thanks to Mike Leitch for making the plots!

## $J / \psi$ Rapidity Distributions in $A A$ Collisions

## Note significant antishadowing in $\mathrm{Cu}+\mathrm{Cu}$ relative to $p p$ at 62 GeV

At $200 \mathrm{GeV}, A A$ distributions also symmetric around $y=0$


Figure 16: The inclusive $J / \psi y$ distributions, calculated for case $\psi 1$ at $\sqrt{S}=200 \mathrm{GeV}$ (left) and 62 GeV (right). At 200 GeV the results show $p p$ (solid), $\mathrm{d}+\mathrm{Au}$ (dashed), $\mathrm{Au}+\mathrm{Au}$ (dot-dashed) and $\mathrm{Cu}+\mathrm{Cu}$ (dotted) interactions. At 62 GeV , we show pp (solid) and $\mathrm{Cu}+\mathrm{Cu}$ (dashed) results.

## In $A B$ Interactions, Nuclear Effects on Rapidity Distributions Become Important

Nuclear effects seen to be important in charmonium production at fixed target energies In extrapolated $p A$ cross sections, the exponent $\alpha$ was shown to be a function of both $x_{F}$ and $p_{T}$ Several mechanisms affect $A$ dependence in cold matter, we consider two here:

- Nuclear Shadowing - initial-state effect on the parton distributions affecting the level of production, important as a function of rapidity $/ x_{F}$
- Absorption - final-state effect, after $c \bar{c}$ that forms the $J / \psi$ has been produced, pair breaks up in matter due to interactions with nucleons

Here we only show effects on charmonium, $\Upsilon$ studies not done yet

## Nuclear Parton Distributions

Nuclear parton densities

$$
\begin{aligned}
F_{i}^{A}\left(x, Q^{2}, \vec{r}, z\right) & =\rho_{A}(s) S^{i}\left(A, x, Q^{2}, \vec{r}, z\right) f_{i}^{N}\left(x, Q^{2}\right) s=\sqrt{b^{2}+z^{2}} \\
\rho_{A}(s) & =\rho_{0} \frac{1+\omega\left(s / R_{A}\right)^{2}}{1+\exp \left[\left(s-R_{A}\right) / d\right]}
\end{aligned}
$$

We use EKS98 and Frankfurt, Guzey and Strikman (FGS) parameterizations: original, FGSo, high, FGSh, and low, FGSl, gluon shadowing
EKS98 has no spatial dependence, two FGS inhomogeneous parameterization recently made available - compare our spatial parameterizations with those of FGS

With no nuclear modifications, $S^{i}\left(A, x, Q^{2}, \vec{r}, z\right) \equiv 1$.
Spatial dependence of shadowing
Proportional to local nuclear density:

$$
S_{\mathrm{WS}}^{i}=S^{i}\left(A, x, Q^{2}, \vec{r}, z\right)=1+N_{\mathrm{WS}}\left[S^{i}\left(A, x, Q^{2}\right)-1\right] \frac{\rho(s)}{\rho_{0}}
$$

Proportional to nuclear path length:

$$
S_{\rho}^{i}\left(A, x, Q^{2}, \vec{r}, z\right)=1+N_{\rho}\left(S^{i}\left(A, x, Q^{2}\right)-1\right) \frac{\int d z \rho_{A}(\vec{r}, z)}{\int d \rho_{A}(0, z)} .
$$

Normalization: $(1 / A) \int d^{2} r d z \rho_{A}(s) S_{\mathrm{WS}, \rho}^{i} \equiv S^{i}$. Larger than average modifications for $b=0$. Nucleons like free protons when $s \gg R_{A}$. Similar normalization for FGS inhomogeneous parameterizations.

## Comparing Shadowing Parameterizations: $x$ Dependence

Recent parameterizations by Frankfurt et al use EKS98 for valence shadowing, stronger gluon shadowing at low $x$, cuts off modification above $x=0.25$ for sea, 0.03 for gluon
Newer FGS parameterizations have lower gluon antishadowing, smoother $x$ dependence over $10^{-4}<x<0.02$


Figure 17: The EKS98 and FGS shadowing parameterizations are compared at the scale $\mu=2 m=2.4 \mathrm{GeV}$. The solid curves are the EKS98 parameterization, the dashed, FGSo, dot-dashed, FGSh, dotted, FGSl.

## Average $x_{2}$ as a Function of Energy and Rapidity

We calculate $\left\langle x_{2}\right\rangle$ as a function of rapidity in the CEM (N.B. $\left\langle x_{1}\right\rangle$ is mirror imagine of $\left\langle x_{2}\right\rangle$ ) Increasing $\sqrt{S}$ broadens $y$ range and decreases $x_{2}$, at RHIC, $\sqrt{S}=62 \mathrm{GeV}$ range is narrower and $x_{2}$ values higher than 200 GeV
In PHENIX muon arms, it is possible to reach lower $\left\langle x_{2}\right\rangle$ than with leading hadrons at similar rapidities: $g g$ dominates and scale is relatively lower


Figure 18: We give the average value of the nucleon momentum fraction, $x_{2}$, in $p p$ collisions as a function of rapidity for (a) the CERN SPS with $\sqrt{S}=19.4 \mathrm{GeV}$, (b) RHIC with $\sqrt{S}=200$ (solid) and 62 (dashed) GeV and (c) the LHC with $\sqrt{S}=6.2 \mathrm{TeV}$.

## $J / \psi$ Absorption by Nucleons

Woods-Saxon nuclear density profiles typically used

$$
\begin{aligned}
\sigma_{p A} & =\sigma_{p N} \int d^{2} b \int_{-\infty}^{\infty} d z \rho_{A}(b, z) S_{A}^{\mathrm{abs}}(b) \\
& =\sigma_{p N} \int d^{2} b \int_{-\infty}^{\infty} d z \rho_{A}(b, z) \exp \left\{-\int_{z}^{\infty} d z^{\prime} \rho_{A}\left(b, z^{\prime}\right) \sigma_{\mathrm{abs}}\left(z^{\prime}-z\right)\right\}
\end{aligned}
$$

Note that if $\rho_{A}=\rho_{0}, \alpha=1-9 \sigma_{\text {abs }} /\left(16 \pi r_{0}^{2}\right)$

## Absorption models

singlet Individual charmonium cross sections grow quadratically with proper time until formation time; only effective when state can form in target
octet $\left|(c \bar{c})_{8} g\right\rangle$ state travels through nucleus, only forms charmonium outside; assume either "constant" over all $y$ or "growing", allowing octet to singlet conversion inside target at negative $y-$ little difference at collider energy

NRQCD Nonrelativistic QCD approach differs from CEM in that states are produced with fixed singlet and octet contributions

We show results for absorption of color singlet and color octet states separately in the CEM and a combination of the two in NRQCD

## Singlet Absorption Model

All $c \bar{c}$ pairs assumed to be produced in small color singlet states
Assume quadratic growth of cross section with proper time until formation time $\tau_{F}$ (Blaizot and Ollitrault)
Strongest at low to negative $x_{F}$ where $J / \psi$ can form in the target
Asymptotic $\psi^{\prime}$ and $\chi_{c}$ cross sections proportional to the final state meson size, e.g. $\sigma_{\psi^{\prime} N}^{\mathrm{s}}=\sigma_{J / \psi N}^{\mathrm{s}}\left(r_{\psi^{\prime}} / r_{J / \psi}\right)^{2}($ Povh and Hüfner $)$

$$
\begin{aligned}
& \sigma_{\mathrm{abs}}\left(z^{\prime}-z\right)= \begin{cases}\sigma_{C N}^{\mathrm{s}}\left(\frac{\tau}{\tau_{F}^{C}}\right)^{2} & \text { if } \tau<\tau_{F}^{C} \\
\sigma_{C N}^{\mathrm{s}} & \text { otherwise }\end{cases} \\
& \tau_{F}^{J / \psi}=0.92 \mathrm{fm} \quad \sigma_{J / \psi N}^{\mathrm{s}} \sim 2.5 \mathrm{mb} \\
& \tau_{F}^{\psi^{\prime}}=1.5 \mathrm{fm} \\
& \tau_{F}^{\chi c}=2 \mathrm{fm}
\end{aligned} \quad \sigma_{\psi^{\prime} N}^{\mathrm{s}}=3.7 \sigma_{J / \psi N}^{\mathrm{s}} . \sigma_{\chi_{c} N}^{\mathrm{s}}=2.4 \sigma_{J / \psi N}^{\mathrm{s}} .
$$

## Octet Absorption Model

Pre-resonant $c \bar{c}$ pairs travel through the nucleus as $\left|(c \bar{c})_{8} g\right\rangle$ color octet states Characteristic octet lifetime $\tau_{8} \sim 0.25 \mathrm{fm}$
For $x_{F} \geq-0.1$, path length of $\left|(c \bar{c})_{8} g\right\rangle$ through the target from its production point is greater than maximum path length

These fast states pass through nucleus in color octets so that the pre-resonant $A$ dependence is the same for $J / \psi, \psi^{\prime}$ and $\chi_{c}$ (Kharzeev and Satz) - $\sigma_{\mathrm{abs}}^{0}=3 \mathrm{mb}$ agrees with E866 forward $A$ dependence
Universal constant absorption cross section usually assumed for nuclear collision studies (NA38, NA50) where $0<x_{F}<0.18$

At negative $x_{F}$, path length is shorter and octet state can neutralize its color inside target and be absorbed as color singlet with 2.5 mb cross section
Only $J / \psi$ likely to be fully formed inside target even though color neutralization may occur for all states
We compare results with no octet to singlet conversion (constant octet) and with conversion (growing octet)

## Singlet + Octet Absorption

Relative contributions of singlet and octet production set by NRQCD (Zhang et al.)
Equal absorption cross sections for all octet states
Singlet cross sections set by final state size

$$
\begin{aligned}
& \frac{d \sigma_{p A}^{\psi}}{d x_{F}}=\int d^{2} b\left[\frac{d \sigma_{p p}^{\psi, \text { oct }}}{d x_{F}} T_{A}^{\psi, \text { eff }(\text { oct })}(b)+\frac{d \sigma_{p p}^{\psi, \text { sing }}}{d x_{F}} T_{A}^{\psi, \text { eff (sing) }}(b)\right], \\
& \frac{d \sigma_{p A}^{\chi_{c J} \rightarrow J / \psi X}}{d x_{F}}=\int d^{2} b \sum_{J=0}^{2} B\left(\chi_{c J} \rightarrow J / \psi X\right)\left[\frac{d \sigma_{p p}^{\chi_{c J}, \text { oct }}}{d x_{F}} T_{A}^{\chi_{c J J}, \text { eff (oct) }}(b)+\frac{d \sigma_{p p}^{\chi_{c J}, \text { sing }}}{d x_{F}} T_{A}^{\chi_{c J}, \text { eff (sing) }}(b)\right], \\
& \frac{d \sigma_{p A}^{J / \psi, \text { tot }}}{d x_{F}}=\int d^{2} b\left\{\left[\frac{d \sigma_{p p}^{J / \psi, \text { dir, oct }}}{d x_{F}} T_{A}^{J / /, \text {,eff (oct) }}(b)\right.\right. \\
& \left.+\sum_{J=0}^{2} B\left(\chi_{c J} \rightarrow J / \psi X\right) \frac{d \sigma_{p p}^{\chi_{c J}, \text { oct }}}{d x_{F}} T_{A}^{\chi_{c J,}, \text { eff }(\mathrm{oct})}(b)+B\left(\psi^{\prime} \rightarrow \psi X\right) \frac{d \sigma_{p p}^{\psi^{\prime}, \text { oct }}}{d x_{F}} T_{A}^{\chi_{c J}, \text { eff }(o c t)}(b)\right] \\
& +\left[\frac{d \sigma_{p p}^{J / \psi, \text { dir, sing }}}{d x_{F}} T_{A}^{J / \psi, \text { dir, eff (sing) }}(b)+\sum_{J=0}^{2} B\left(\chi_{c J} \rightarrow \psi X\right) \frac{d \sigma_{p p}^{\chi_{c J}, \text { sing }}}{d x_{F}} T_{A}^{\chi_{c J}, \text { eff (sing) }}(b)\right. \\
& \left.\left.+B\left(\psi^{\prime} \rightarrow \psi X\right) \frac{d \sigma_{p p}^{\psi^{\prime}, \text { sing }}}{d x_{F}} T_{A}^{\psi^{\prime}, \text { eff }(\operatorname{sing})}(b)\right]\right\} \\
& T_{A}^{\mathrm{eff}}(b)=\int_{-\infty}^{\infty} d z \rho_{A}(b, z) \exp \left\{-\int_{z}^{\infty} d z^{\prime} \rho_{A}\left(b, z^{\prime}\right) \sigma_{\mathrm{abs}}\left(z^{\prime}-z\right)\right\}
\end{aligned}
$$

## Rapidity Dependence of Homogeneous Absorption

Results shown for different charmonium states: inclusive and direct $J / \psi, \psi^{\prime}$ and $\chi_{c}$ Constant and growing octet indistinguishable in detector range, singlet absorption only effective for $y<-1$, NRQCD also shows little rapidity dependence

Only small effect of growing octet when octet cross section similar to or larger than singlet


Figure 19: The $J / \psi \mathrm{dAu} / p p$ ratio at 200 GeV as a function of rapidity for absorption alone. We show (a) constant octet with 3 mb , (b) growing octet with 3 mb asymptotic cross section for all states, (c) singlet with $2.5 \mathrm{mb} J / \psi$ absorption cross section, all calculated in the CEM and (d) NRQCD with a combination of octet and singlet matrix elements. The curves show total $J / \psi$ (solid), direct $J / \psi$ (dashed), $\psi^{\prime}$ (dot-dashed) and $\chi_{c}$ (dotted).

## Effect of Inhomogeneous Absorption

Example of impact parameter dependence of absorption
Solid curve is 3 mb constant octet cross section, all rapidities, dashed is at $y=-2$, singlet


Figure 20: The $J / \psi \mathrm{dAu} / p p$ ratio as a function of $b$ for absorption alone with $\sigma_{\text {abs }}=3 \mathrm{mb}$ for a constant octet (all $y$ ), solid, and singlet ( $y=-2$ ), dashed. The homogeneous results are indicated by the dotted lines.

## Absorption and Shadowing in d+Au: Absorption Models






Figure 21: The $J / \psi \mathrm{dAu} / p p$ ratio at 200 GeV with EKS98 for: (top left) constant octet, (top right) growing octet, (bottom left) singlet and (bottom right) NRQCD. In the first three, we show $\sigma_{\mathrm{abs}}=0$ (solid), 1 (dashed), 3 (dot-dashed) and 5 mb (dotted). For NRQCD, we show no absorption (solid), 1 mb octet/singlet (dashed), 3 mb octet/singlet (dot-dashed), and 5 mb octet $/ 3 \mathrm{mb}$ singlet (dotted).

## Absorption and Shadowing in d+Au: Shadowing Parameterizations



Figure 22: Comparison of the results for a 3 mb growing octet absorption cross section with the EKS98 (solid), FGSo (dashed), FGSh (dot-dashed) and FGSl (dotted) shadowing parameterizations. Thanks to Mike Leitch for making this plot and all the previous ones for RHIC!

## Absorption and Shadowing in $200 \mathrm{GeV} \mathrm{Au}+\mathrm{Au}$

Effect of shadowing alone symmetric around $y=0$, similar to multiplying $\mathrm{d}+\mathrm{Au} / p p$ ratio by its mirror image, antishadowing peaks at $y \approx \pm 1.5$ are not above unity
Absorption effects stronger when both beams are nuclei, not symmetric around $y=0$ except for constant octet


Figure 23: In (a)-(c) we show the $J / \psi \mathrm{AuAu} / p p$ ratio at 200 GeV with the EKS98 shadowing parameterization as a function of rapidity for our absorption models: (a) constant octet, (b) growing octet and (c) singlet. The curves are no absorption (solid), $\sigma_{\text {abs }}=1$ (dashed), 3 (dot-dashed) and 5 mb (dotted). In (d), we compare the results for a 3 mb growing octet absorption cross section with the EKS98 (solid), FGSo (dashed), FGSh (dot-dashed) and FGSl (dotted) shadowing parameterizations.

## Absorption and Shadowing in $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$

Effects similar to $\mathrm{Au}+\mathrm{Au}$ but weaker due to smaller $A$
FGS parameterizations not available for Cu , used $A=40$ for these


Figure 24: In (a)-(c) we show the $J / \psi \mathrm{CuCu} / p p$ ratio at 200 GeV with the EKS98 shadowing parameterization as a function of rapidity for our absorption models: (a) constant octet, (b) growing octet and (c) singlet. The curves are no absorption (solid), $\sigma_{\text {abs }}=1$ (dashed), 3 (dot-dashed) and 5 mb (dotted). In (d), we compare the results for a 3 mb growing octet absorption cross section with the EKS98 (solid), FGSo (dashed), FGSh (dot-dashed) and FGSl (dotted) shadowing parameterizations.

## Absorption and Shadowing in $62 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$

Lower energy has $x \approx 0.05$ at $y=0$, in antishadowing region, gives shadowing peak at $y=0$
Decrease for $|y|>0$ due to combination of EMC effect and beginning of shadowing region
Upward turn at large $y$ due to rise after EMC region at large $x$
Growing octet and singlet absorption effective over larger rapidity range


Figure 25: In (a)-(c) we show the $J / \psi \mathrm{AuAu} / p p$ ratio at 200 GeV with the EKS98 shadowing parameterization as a function of rapidity for our absorption models: (a) constant octet, (b) growing octet and (c) singlet. The curves are no absorption (solid), $\sigma_{\text {abs }}=1$ (dashed), 3 (dot-dashed) and 5 mb (dotted). In (d), we compare the results for a 3 mb growing octet absorption cross section with the EKS98 (solid), FGSo (dashed), FGSh (dot-dashed) and FGSl (dotted) shadowing parameterizations.

## Inhomogeneous Shadowing and Absorption in $\mathrm{d}+\mathrm{Au}$

PHENIX results presented as a function of $N_{\text {coll }}$, the convolution of the nuclear profile functions multiplied by the inelastic $N N$ cross section, 42 mb at RHIC

$$
N_{\text {coll }}(b)=\sigma_{N N}^{\mathrm{in}} \int d^{2} s T_{A}(s) T_{B}(|\vec{b}-\vec{s}|)
$$

Results with EKS98 compared at $y=-2$ (antishadowing), 0 (transition region) 2 (shadowing)


Figure 26: Left-hand side: The $J / \psi$ ratio $(\mathrm{dAu}(b) / p p) /(\mathrm{dAu}(\mathrm{ave}) / p p)$ as a function of $b / R_{A}$. Right-hand side: The ratio $\mathrm{dAu} / p p$ as a function of $N_{\text {coll }}$. Results are shown for $y=-2$ (dot-dashed), $y=0$ (dashed) and $y=2$ (solid) at 200 GeV for a growing octet with $\sigma_{\mathrm{abs}}=3 \mathrm{mb}$ and the EKS98 parameterization.

## Comparison of $N_{\text {coll }}$ Dependence in d+Au

Path length parameterization, $S_{\mathrm{P}, \rho}$, with EKS98 and FGSo gives linear $N_{\text {coll }}$ dependence due to long tails of density distributions
FGSh and FGSl forced to $S=1$ at $b=10 \mathrm{fm}$ so that as $N_{\text {coll }} \rightarrow 1$, shadowing disappears and only residual absorption remains


Figure 27: The ratio $\mathrm{dAu} / p p$ as a function of $N_{\text {coll }}$ for the EKS98 (a), FGSo (b), FGSh (c) and FGSl (d) shadowing parameterizations. The calculations with EKS98 and FGSo use the inhomogeneous path length parameterization while that obtained by FGS is used with FGSh and FGSl. Results are given for $y=-2$ (dot-dashed), $y=0$ (dashed) and $y=2$ (solid) at 200 GeV for a growing octet with $\sigma_{\mathrm{abs}}=3 \mathrm{mb}$.

## Prediction of $N_{\text {coll }}$ Dependence in $\mathrm{Au}+\mathrm{Au}$

In nucleus-nucleus collisions, we only show $y=0$ and 2 since, for a growing octet, the results at $y=2$ and -2 are symmetric
$N_{\text {coll }}>1$ for $b<2 R_{A}$ so FGSh and FGSl does not change as fast with $N_{\text {coll }}$ as in $\mathrm{d}+\mathrm{Au}$
Result with $y=0$ lower than for $y=2$ due to "dip" between peaks seen in homogeneous calculations


Figure 28: The ratio $\mathrm{AuAu} / p p$ as a function of $N_{\text {coll }}$ for the EKS98 (a), FGSo (b), FGSh (c) and FGSl (d) shadowing parameterizations. The calculations with EKS98 and FGSo use the inhomogeneous path length parameterization while that obtained by FGS is used with FGSh and FGSl. Results are given for $y=0$ (dashed) and $y=2$ (solid) at 200 GeV for a growing octet with $\sigma_{\mathrm{abs}}=3 \mathrm{mb}$.

## Prediction of $N_{\text {coll }}$ Dependence in $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ Collisions

$N_{\text {coll }}$ lower for $\mathrm{Cu}+\mathrm{Cu}$ due to smaller nuclear size


Figure 29: The ratio $\mathrm{CuCu} / p p$ as a function of $N_{\text {coll }}$ for the EKS98 (a), FGSo (b), FGSh (c) and FGSl (d) shadowing parameterizations. The calculations with EKS98 and FGSo use the inhomogeneous path length parameterization while that obtained by FGS is used with FGSh and FGSl. Results are given for $y=0$ (dashed) and $y=2$ (solid) at 200 GeV for a growing octet with $\sigma_{\mathrm{abs}}=3 \mathrm{mb}$.

## Prediction of $N_{\text {coll }}$ Dependence in $62 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ Collisions

At $\sqrt{S}=62 \mathrm{GeV}$, number of collisions reduced because $\sigma_{N N}^{\text {in }}$ is smaller at the lower energy Now, due to different shapes of the shadowing results with energy, the $y=0$ curve is again above that of $y=2$


Figure 30: The ratio $\mathrm{CuCu} / p p$ as a function of $N_{\text {coll }}$ for the EKS98 (a), FGSo (b), FGSh (c) and FGSl (d) shadowing parameterizations. The calculations with EKS98 and FGSo use the inhomogeneous path length parameterization while that obtained by FGS is used with FGSh and FGSl. Results are given for $y=0$ (dashed) and $y=2$ (solid) at 62 GeV for a growing octet with $\sigma_{\text {abs }}=3 \mathrm{mb}$.

## Summary

- CEM gives good agreement with hadroproduction results, even for $p_{T}$ distributions, without $K$ factors
- We used d+Au calculations at 200 GeV to extrapolate shadowing and absorption baseline for $A A$ collisions
- Intrinsic $k_{T}$ broadening important for $p_{T}$ distributions, nuclear effects on broadening are larger at 62 GeV than at 200 GeV
- $c \bar{c}$ production could be used as baseline for $J / \psi$ effects in $A A$ but care must be taken in comparing total charm cross sections - need to have better measurements to higher $p_{T}$

