Physics 2C (Seto)

LECTURE: 08:10-09:00 AM, MWF, Physics 2000

INSTRUCTOR: Richard Seto

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Office Location: Physics 3029A
Office hours: M 9-10 W 11-12 and by appt.

TEACHING ASSISTANT:

E-mail:
Office Hours:

TEXTBOOK: Principles of Physics, 4th Edition, by R.A. Serway and J.W. Jewett Jr

PROBLEM SETS: Webassign – get an access code

CLASS PARTICIPATION: You will need clickers by Friday. Don't forget to register them. clickers.ucr.edu

CLASS WEB SITE: Look at **http://iLearn.ucr.edu** and select **GENERAL PHYSICS 002C (001) (SPRING 2006)**

look under Course material for general info(this stuff)

- Grading
  - to be determined but will something like
    - ~13% Homework
    - ~12% Clickers in class
    - ~20% midterm
    - ~55% Final

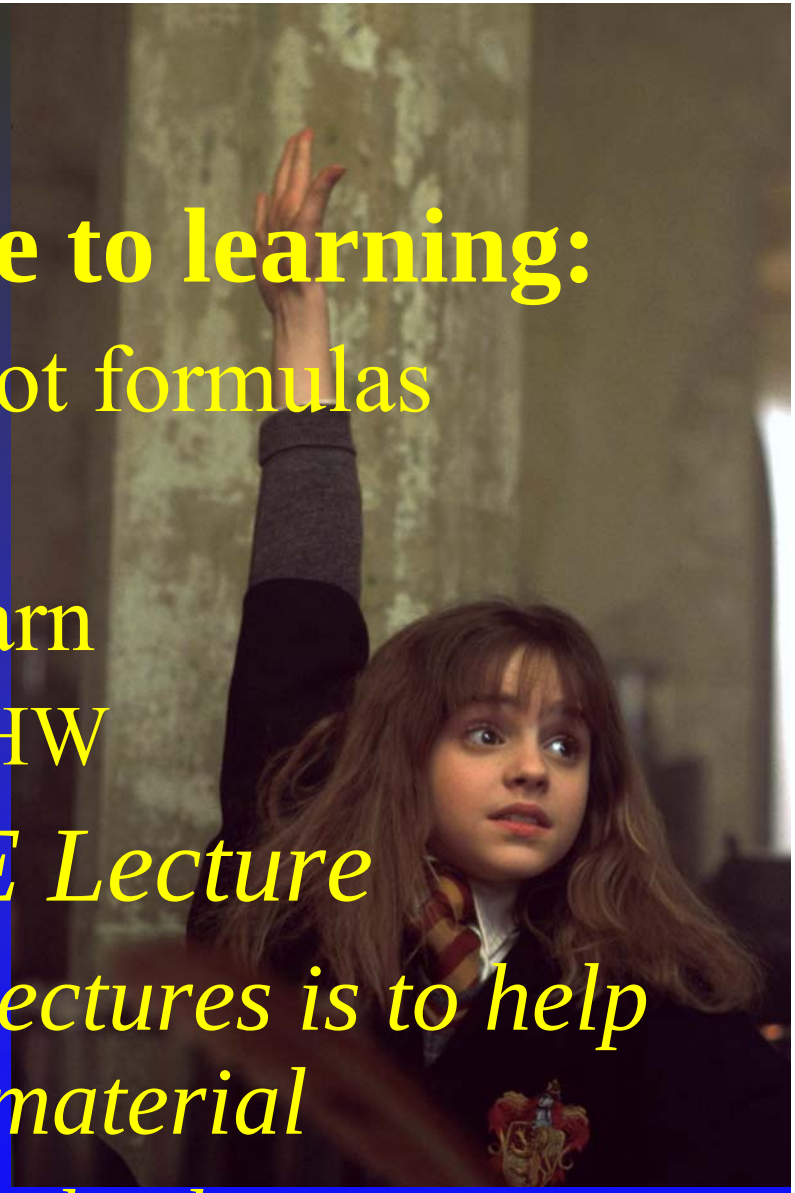
HOMEWORK

- ***Webassign***
  - From ilearn – go to “STUDENT TOOLS” then “WEBASSIGN”
- due: Monday by 8:00 AM).
  - Multiple submissions (up to 5) are allowed
  - No late work. No exceptions.

Laboratory – separate class

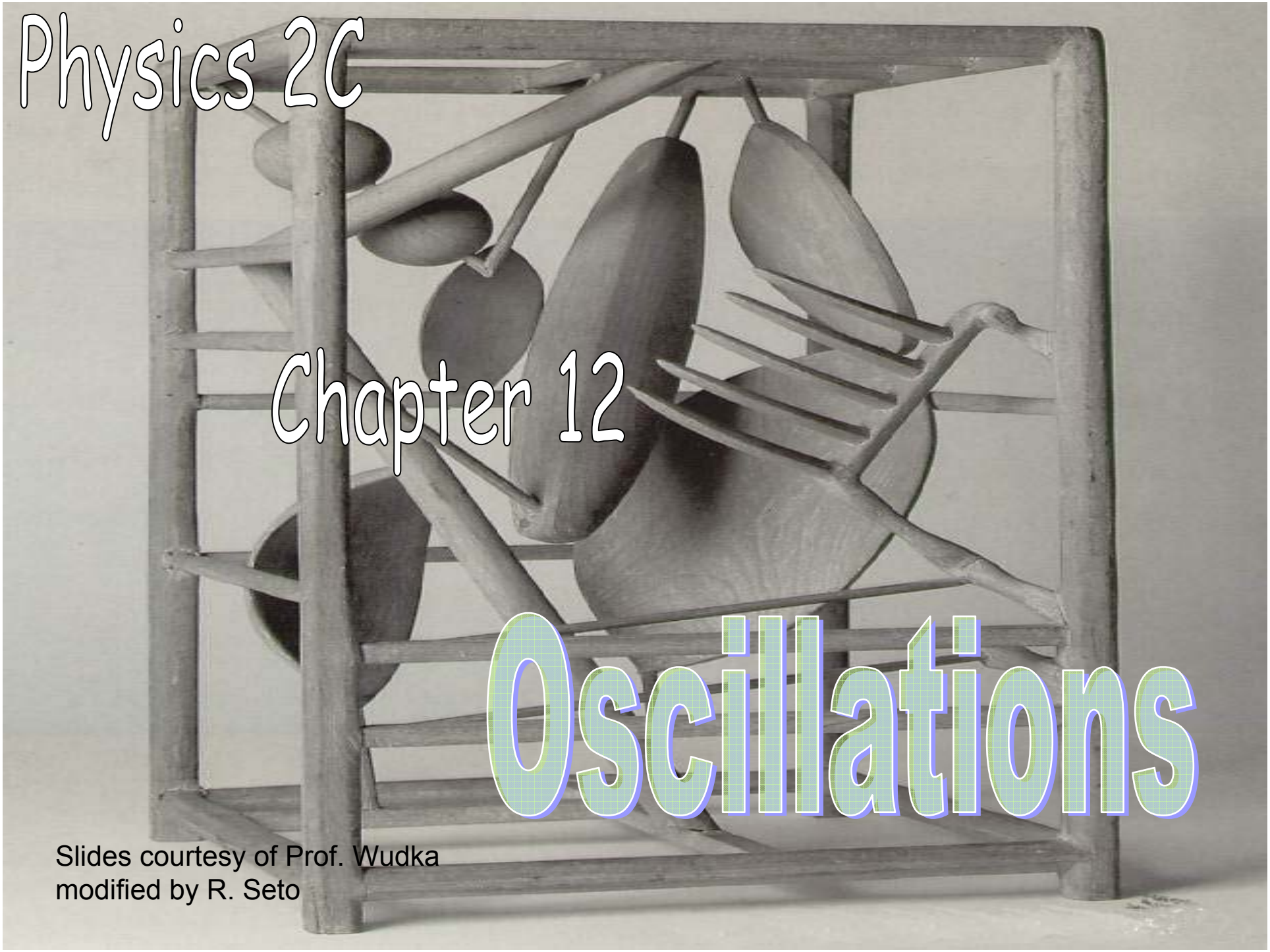
- Laboratory sessions begin the week of Monday, April 10th.
- Personnel: The faculty person in charge of the lower division undergraduate labs is Prof. Lau (jeanie.lau@ucr.edu)
- Web site: The lab write-ups and other pertinent information, including grades, will be available on the Blackboard web site at http://ilearn.ucr.edu/. Make sure you can access the web site early, so that you can download the write-ups when needed.
- Lab notebooks: Get one. See infor on idearn.

- **My general attitude to learning:**
  - a way of thinking, not formulas
    - notation
  - "cheat sheet" on ilearn
    - use for exams and HW
- *Read book BEFORE Lecture*
  - *The purpose of the lectures is to help you understand the material*
    - *not to regurgitate the book*



Waves and Modern Physics

- Topics
  - Oscillations and mechanical waves
    - Your heart, day and night, walking
  - Electromagnetic waves
    - Light
  - Geometric optics (lenses and mirrors)
    - your eye
  - Wave optics (Interference and diffraction)
    - How you measure stuff, imaging, hearing
  - Relativity
    - Moving fast
  - Quantum Mechanics
    - How the chemistry in your body runs
  - Atomics and Nuclear Physics
    - How to treat diseases_



Physics 2C

Chapter 12

Oscillations

Slides courtesy of Prof. Wudka
modified by R. Seto

Examples of oscillating systems

The Sun

Many things move

Back and fort

A spring

A pendulum

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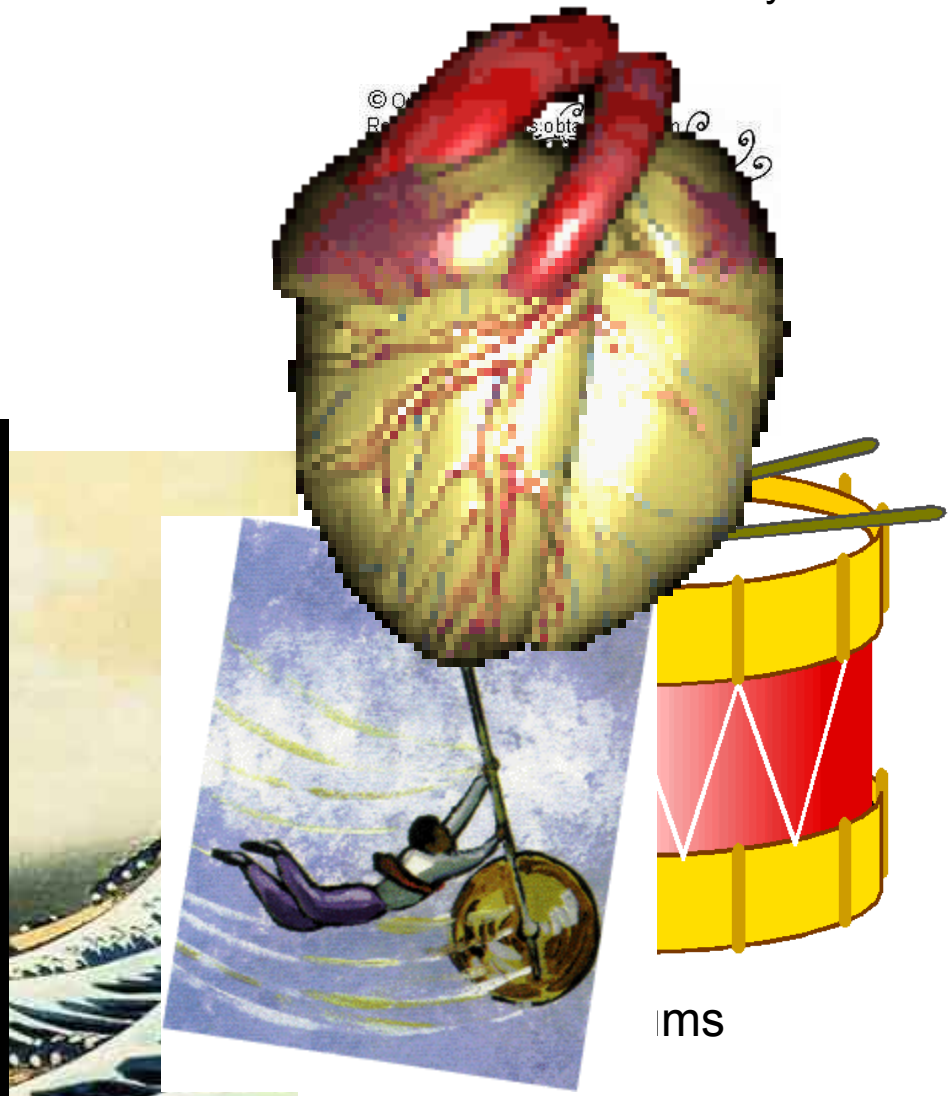
Examples of oscillating systems

demo: keys on string
keys on rubber band

Many things move

Back and forth

A spring



A pendulum

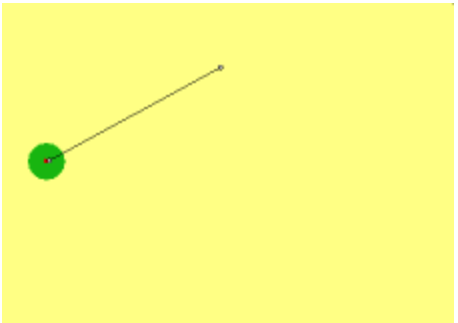
ms

Quantities describing oscillating systems

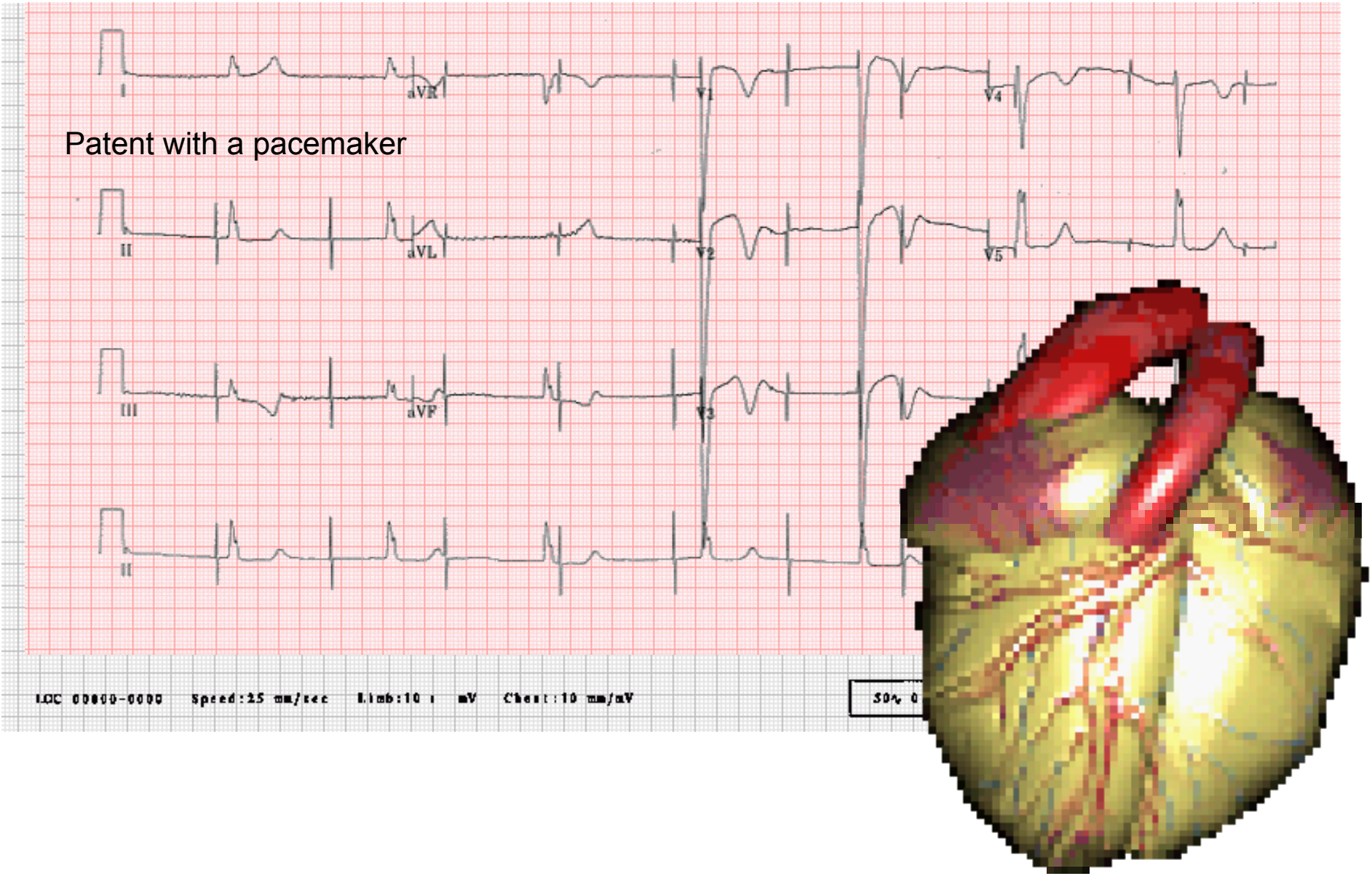
Some oscillating systems are irregular…

… for example, ocean waves.

But other are quite regular…



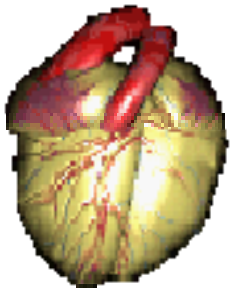
… for example, a pendulum.



ECG

http://mcdb.colorado.edu/courses/2115/units/Other/heartbeat%20animation.html

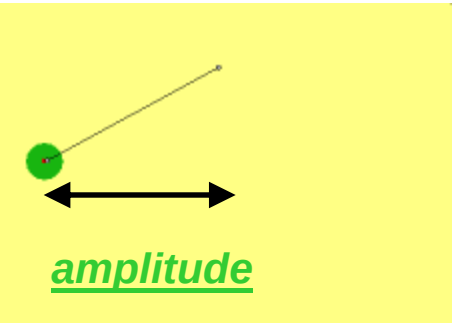
For regular motion we can define:



period T of oscillation … how long it takes to make a full trip

frequency f=2πω of oscillation … how many full trips per second

amplitude A of oscillation … how far does the oscillating thing move.



This takes (about) 4 seconds per full trip

T=4 s (period)

It makes 1/4 trips per second

f=1/4=0.25 Hz (frequency)

Why does it oscillate?

Look at a pendulum:

Not moving at constant speed in a straight line: there's a force at work

For the pendulum the force is always

The force making the bob move is always opposite to the displcement!

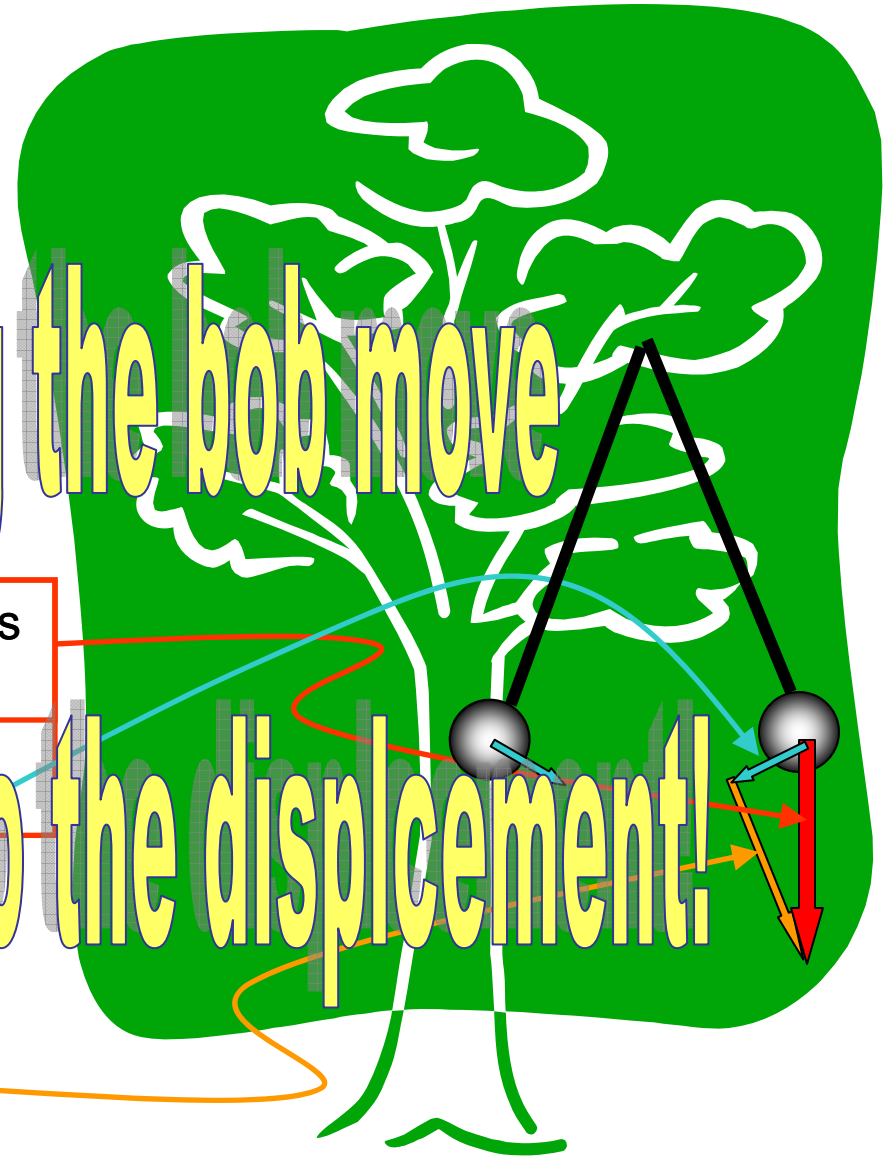
When the bob is to the right, the perpendicular force points left

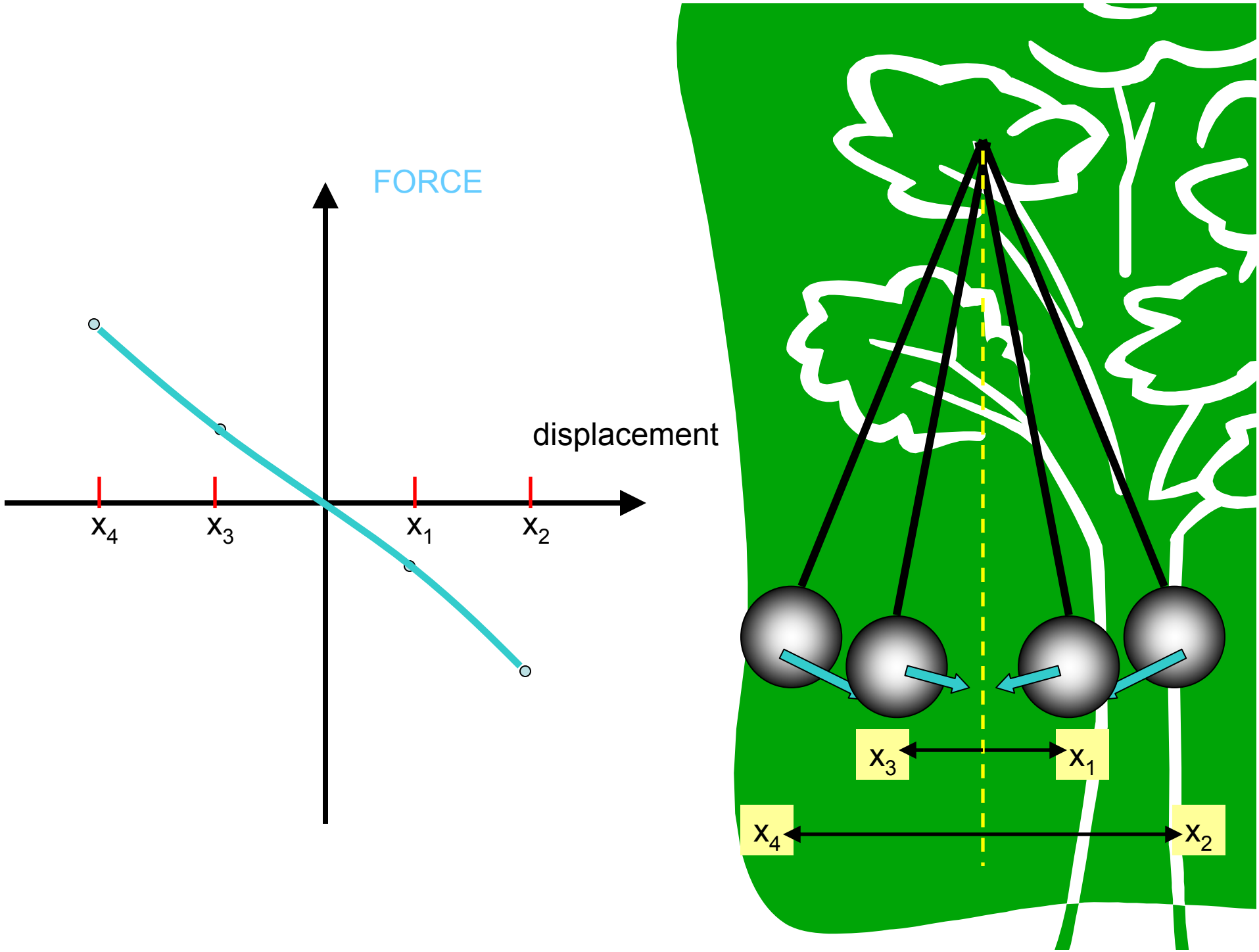
When the bob is to the left, the perpendicular force points right

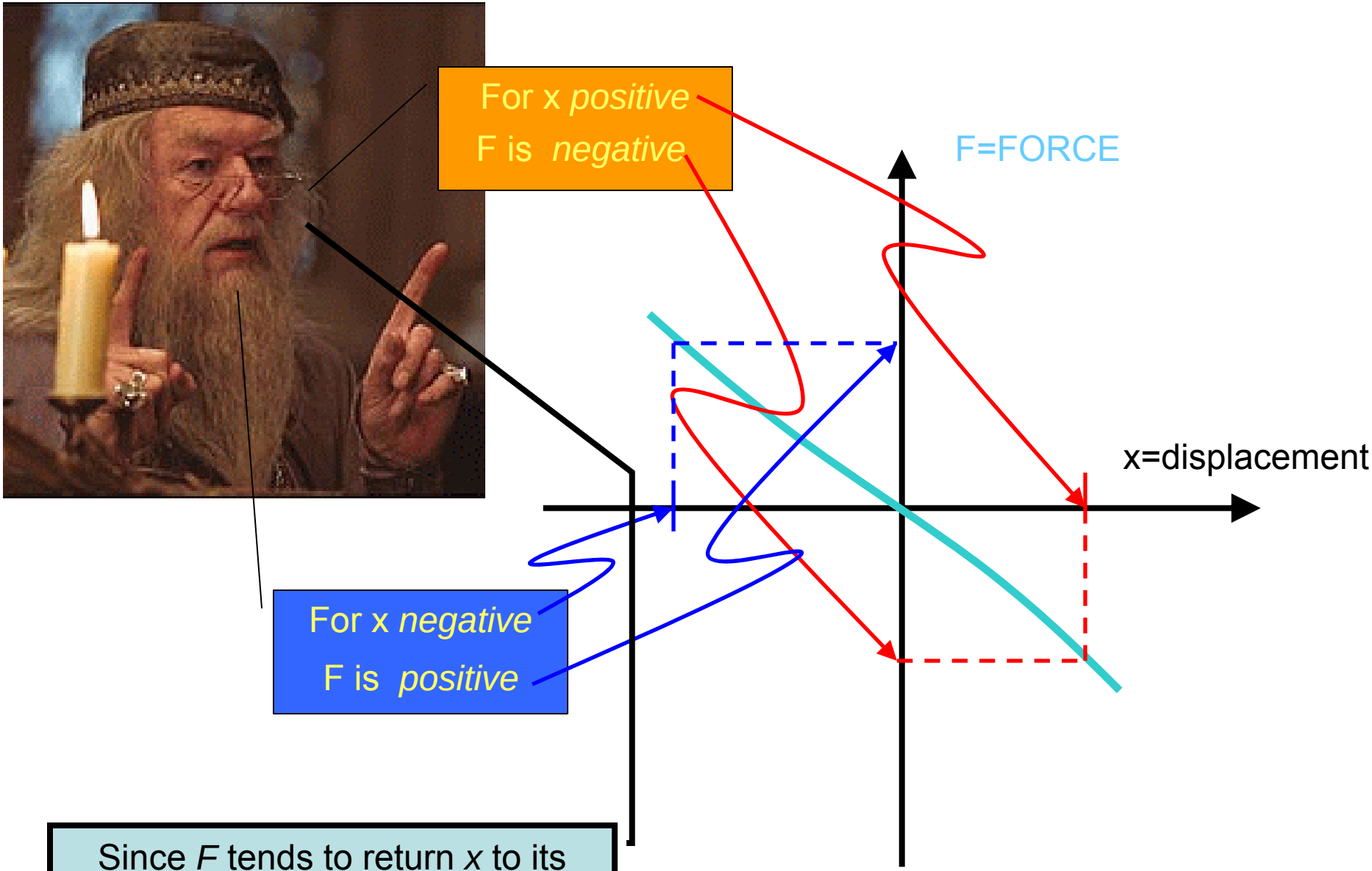
Force of gravity points straight down…

… but can be written as a component along the string

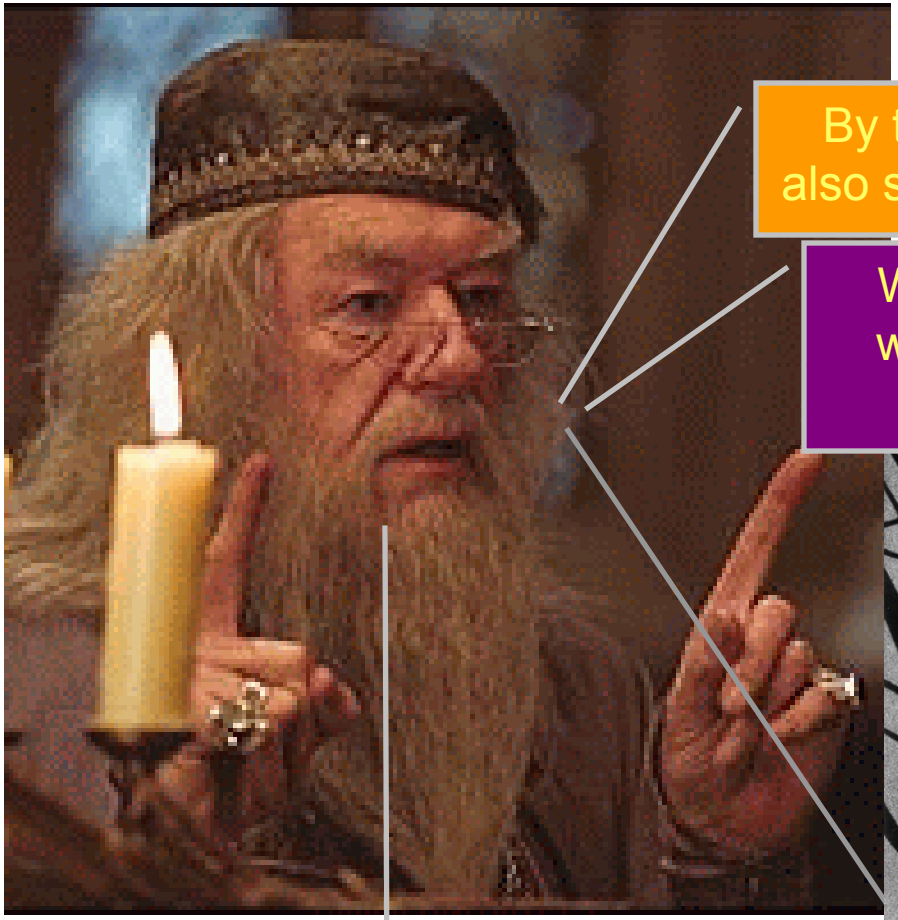
The other along the string keeps the string straight




FORCE
displacement
x_4
x_3
x_1
x_2
x_3
x_1
x_4
x_2



Since *F* tends to return *x* to its previous value it is called a ***RESTORING FORCE***



By the way, water also show oscillations

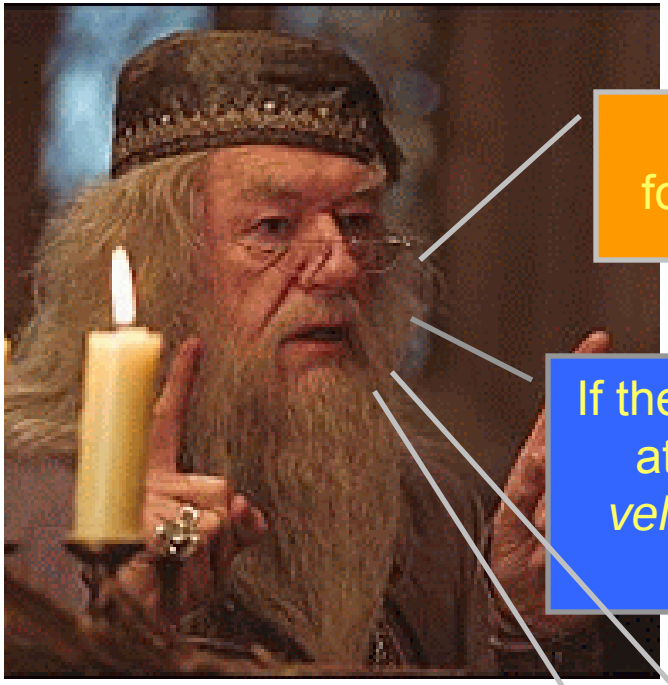
Water oscillations **propagate…** what about oscillations on other things like a spring?

The restoring force is generated by pressure and gravity

next section -

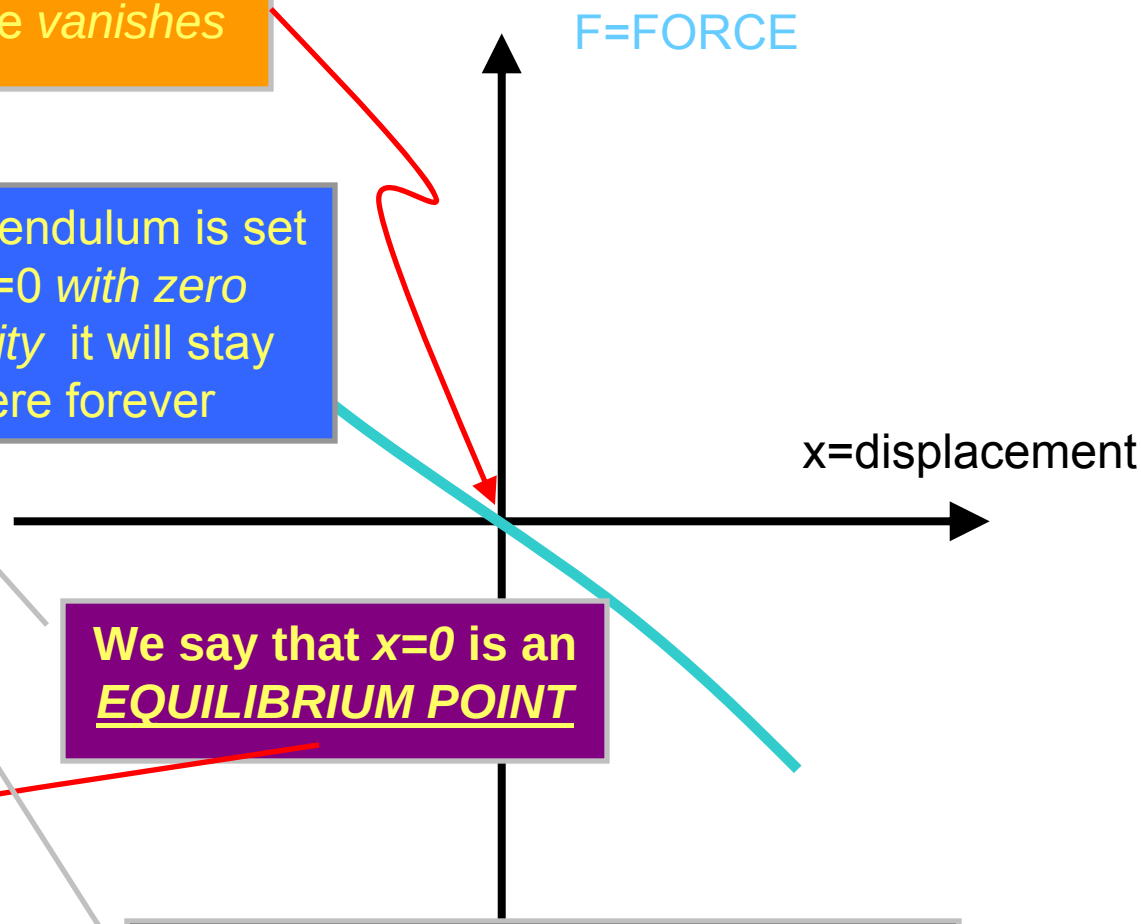
If a bit of water goes down the pressure increases and pushes it up
If a bit of water goes up gravity pulls it down
So the bit of water moves up and down, up and down…

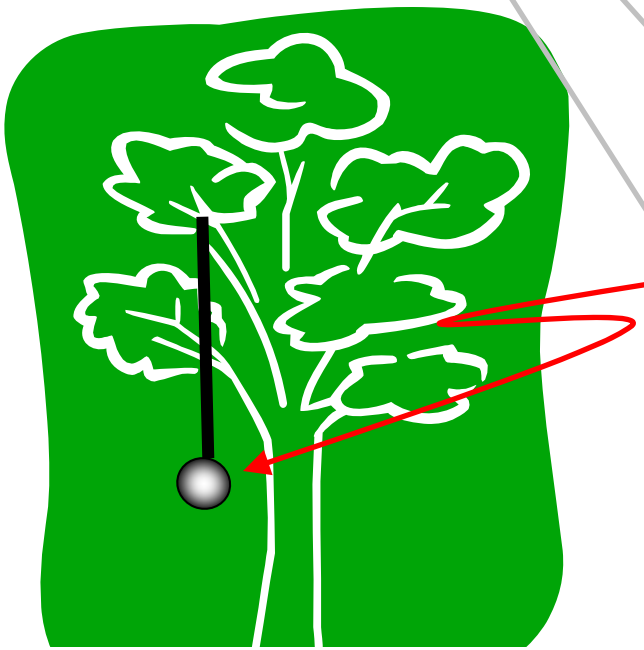
Description of the oscillations.



For x=0 the force *vanishes*

If the pendulum is set at x=0 *with zero velocity* it will stay there forever



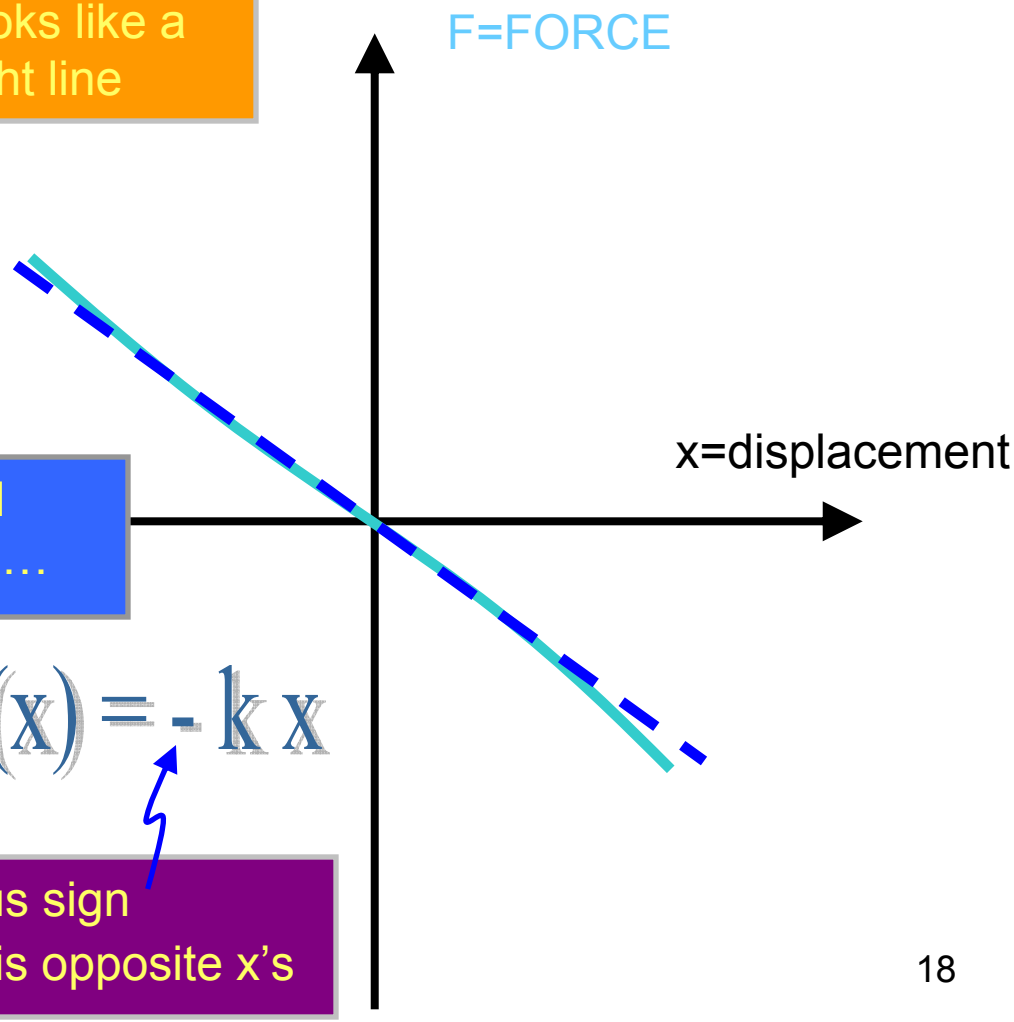


We say that *x=0* is an *EQUILIBRIUM POINT*

At x=0 the restoring force vanishes
But the total force *does not*

What does the restoring force look like? (A spring)

Near x=0 the force graph looks like a straight line



So, for small displacements…

$$F(x) = -k\,x$$

The minus sign
Insures F's sign is opposite x's

By the way, if the **sign** is different…

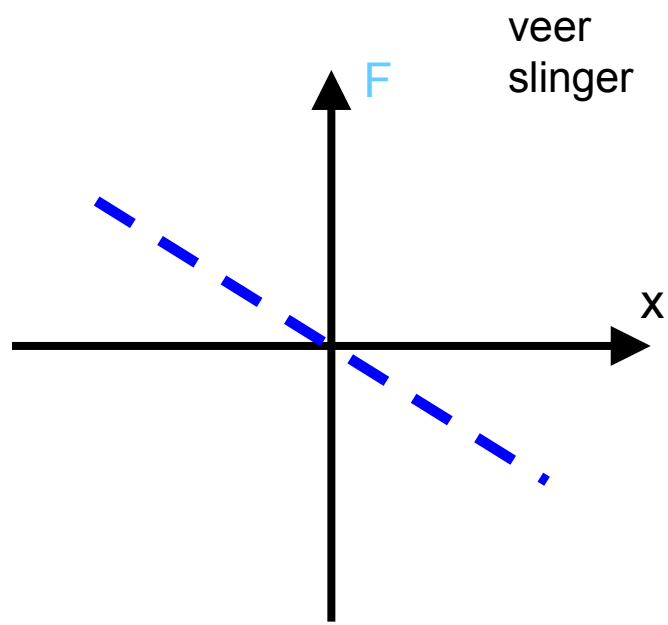
$$F(x) = + k\,x$$

The motion is ***completely*** different..

… since if x is positive the force will make it larger. This is ***not a restoring force!!***

For
$F(x) = - k x$
Newton's equations give

$$m a = - k x$$



veer
slinger

With solution…

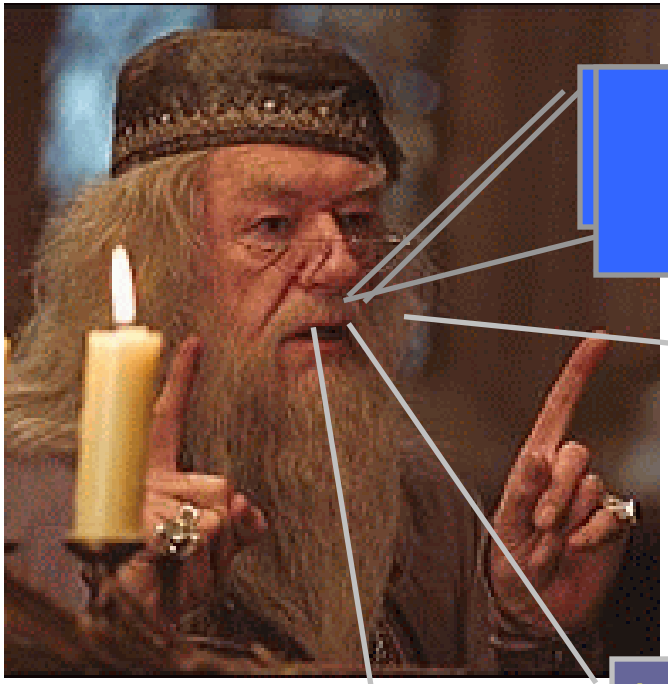
$$x = A \cos(\omega t + \phi)$$

Where A and ϕ are constants and $\omega = \sqrt{k/m}$

Frequency

Amplitude

Note also that
- When |x| is largest |a| is also largest, but v=0.
- When x=0, a=0 also, but |v| is largest



Lets start here

ma= -kx

can we solve this to get a nice formula for the position of the mass, x(t)?

first let's rewrite the egn

$$m\frac{d^2x}{dt^2}+kx=0$$

Lets guess and answer for x(t)
What gives negative itself on taking a 2nd derivative?

The COSINE! so *guess* $x(t)=A\cos(\omega t+\phi)$

Now let's check it

$$\frac{\mathrm{d}}{\mathrm{dt}}A\cos(\omega t+\phi)=-A\omega\sin(\omega t+\phi)$$

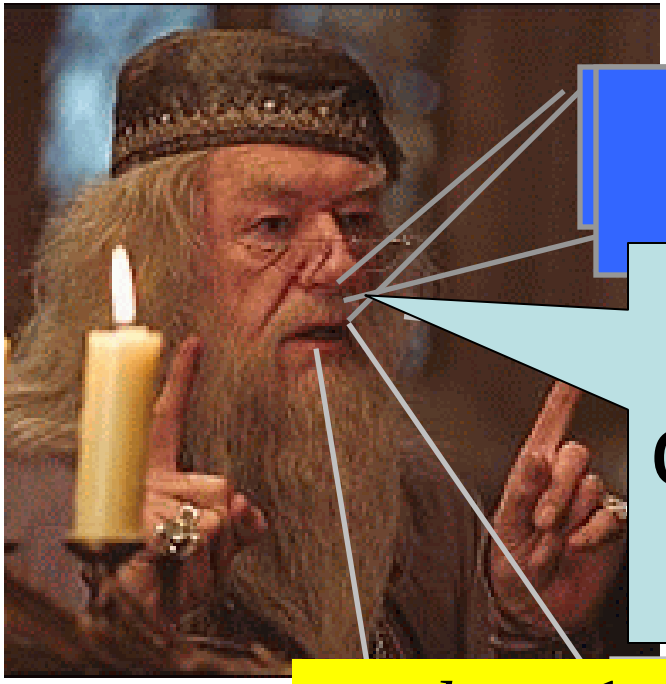
$$\frac{\mathrm{d}}{\mathrm{dt}}[-A\omega\sin(\omega t+\phi)]=-A\omega^2\cos(\omega t+\phi)$$

Plug it back in

$$-mA\omega^2\cos(\omega t+\phi)+kA\cos(\omega t+\phi)=0$$

$$k=m\omega^2 \qquad \Rightarrow \qquad \omega=\sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$$

IT WORKS!



Lets start here

ma= -kx

can we solve this to get a nice formula for the position of the mass, x(t)?

Incidentally we just calculated the velocity and acceleration

$$v = \frac{dx}{dt} = \frac{\mathrm{d}}{\mathrm{dt}} A\cos(\omega t + \phi) = -A\omega \sin(\omega t + \phi)$$

$$a = \frac{dv}{dt} = \frac{\mathrm{d}}{\mathrm{dt}}[-A\omega \sin(\omega t + \phi)] = -A\omega^2 \cos(\omega t + \phi)$$

Now let's check it

$$\frac{\mathrm{d}}{\mathrm{dt}} A\cos(\omega t + \phi) = -A\omega \sin(\omega t + \phi)$$

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Plug it back in

$$-mA\omega^2 \cos(\omega t + \phi) + kA\cos(\omega t + \phi) = 0$$

$$k = m\omega^2 \qquad \Rightarrow \qquad \omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$$

IT WORKS!

announcements

- TA's
  - Myra Tovar
  - Victor Chen

I am taking this class

A. So I can become a doctor
B. So I can become a dentist or pharmacist
C. So I can go into the medical profession (other than doctor, dentist, pharmacist)
D. So I can go into the life sciences (other than the medical profession)
E. Other

Lecture 2
Ok so where are we?

- We were looking at oscillating systems.
  - To get an oscillating system we needed a restoring force F=-kx (def of SHM)
  - Using F=ma we found that

$$x(t)=A\cos(\omega t+\phi) \quad \text{where} \quad \omega=\sqrt{\frac{k}{m}}$$

- ***SPRING***
  - *Definition* of spring constant! F=-kx
  - So for a spring

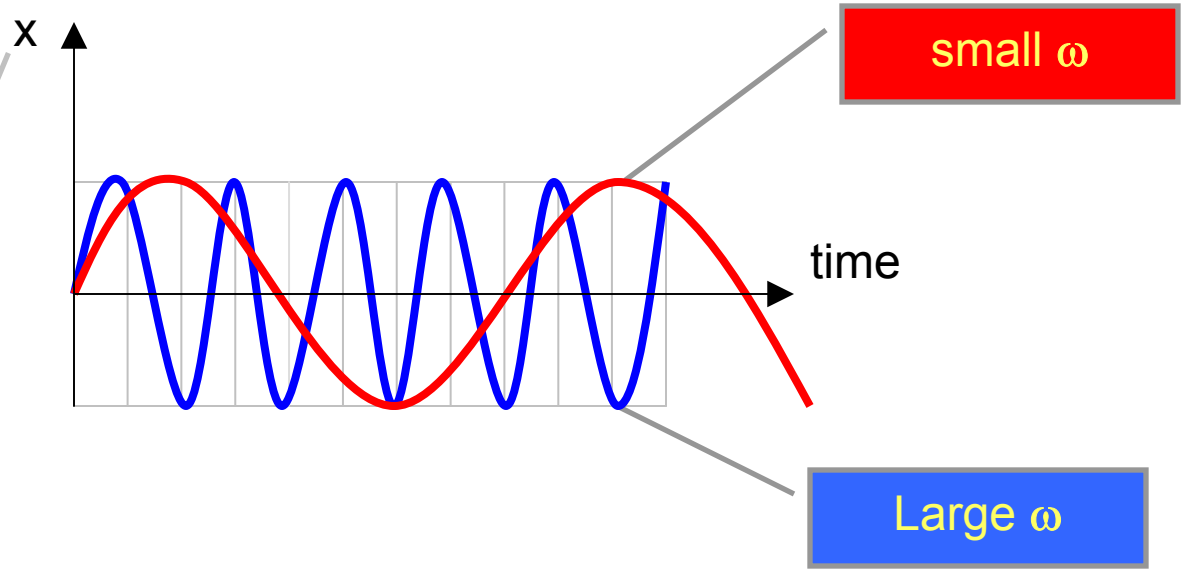
$$\omega=\sqrt{\frac{k}{m}}$$

Let us look at what $\omega=2\pi f$ means

$x=A\cos(\omega t+\phi)$

The frequency determines how often oscillations are repeated

x measures the displacement form the equilibrium position



Remember:

$-1 \leqslant \cos(u) \leqslant 1$

$\cos(u) = \cos(u+2\pi)$

Spring

applet veer.htm

$$x(t)=A\cos(\omega t+\phi) \quad \text{where} \quad \omega=\sqrt{\frac{k}{m}}$$

- what are A, ω and ϕ ?

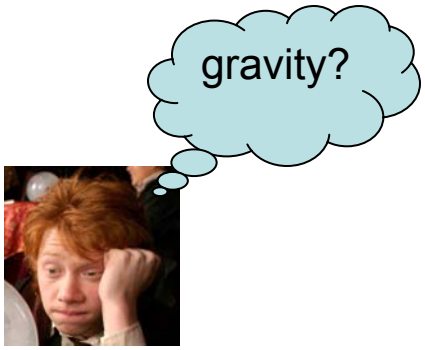
  (remember we want $2\pi f=\omega$ and $f=1/T$ where T is the period)

  – max of cos=1 A= amplitude

  – full period of cos=2π ➔ $\omega T=2\pi$

  ➔$T=2\pi/\omega=1/f$

  ➔so f is a frequency (since ~1/T)



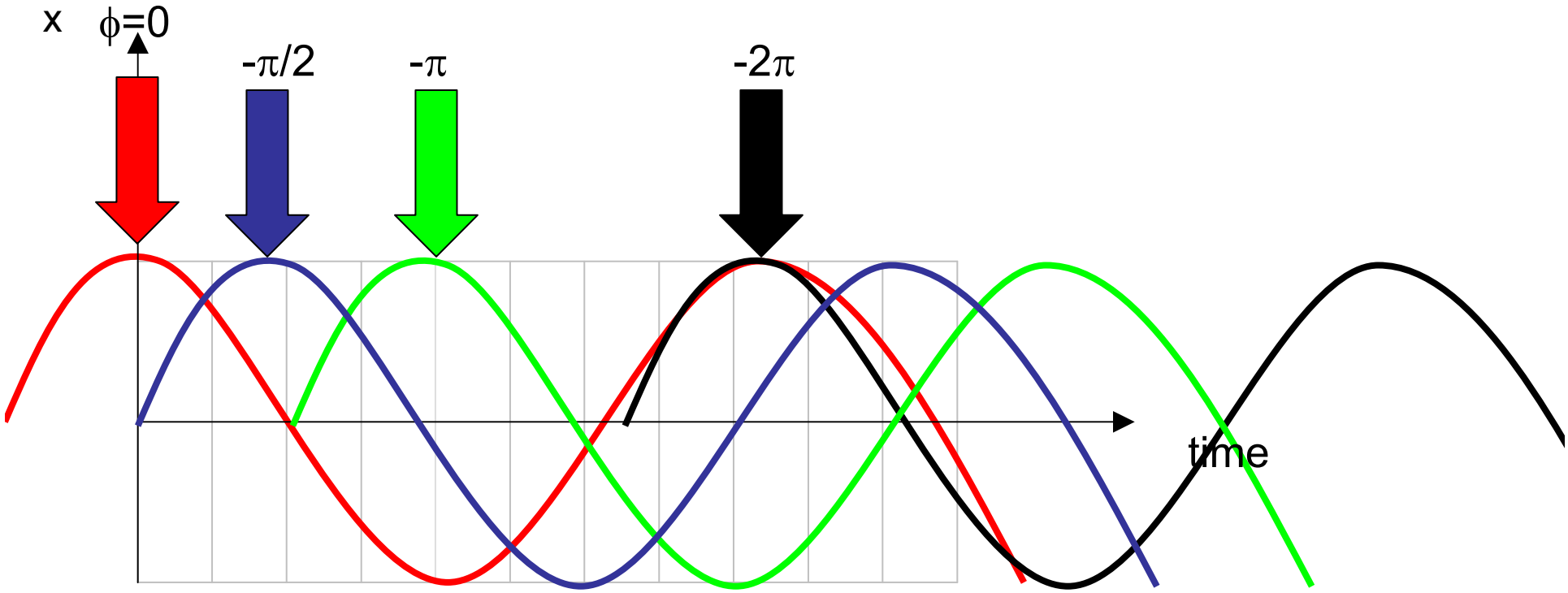
In the applet k=20N/m, m=5kg, A=.05m

➔$\omega=2\pi f$

$$\omega=\sqrt{\frac{20N/m}{5kg}}=\sqrt{\frac{20kg\cdot m/s^2/m}{5kg}}=2s^{-1} \qquad f=\frac{2s^{-1}}{2\pi}=\frac{1}{\pi}s^{-1} \qquad T=\pi\ s$$

ϕ ??

$$x(t)=A\cos(\omega t+\phi)$$



- ϕ is a measure of how much we push up the oscillation, in units of ωt. Its called a phase
- usually set by initial condition

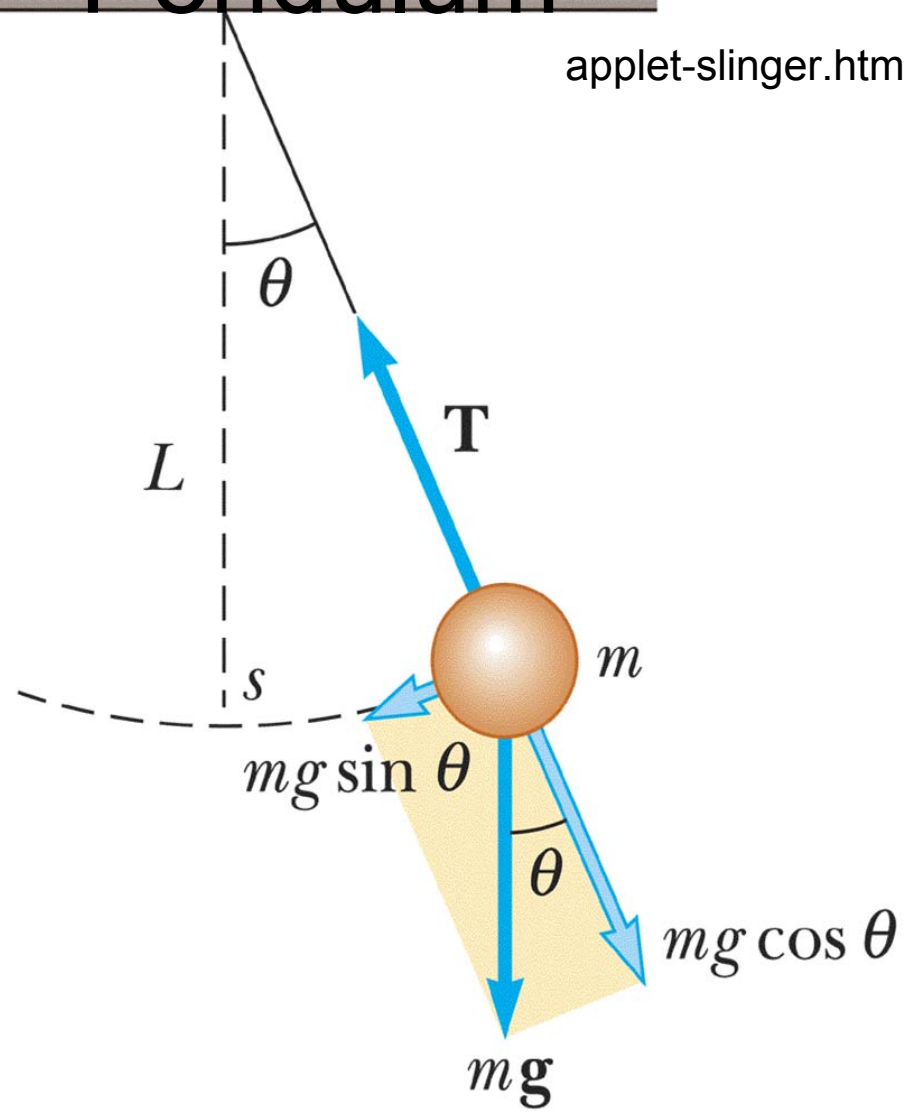
applet- shm

angular frequency

- In fact we can think of ωt as just an angle since it is inside the cosine! Often ω is called an angular frequency
- Got it!???

Pendulum

applet-slinger.htm



$$F = -mg\sin\theta \approx -mg\theta$$

$$x = L\theta \quad \Rightarrow \quad F = -\frac{mg}{L}L\theta$$

$$\Rightarrow \; k = \frac{mg}{L}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$$

ω depends on L and g

ω independent of m!

Lecture 3

- We were looking at oscillating systems
- There had a period T=1/f where f=frequency
- We used the angular frequency ω=2πf so T=2π/ ω
  - To get an oscillating system we needed a restoring force F=-kx (def of SHM)
  - Using F=ma we found that $x(t)=A\cos(\omega t+\phi)$

$$v=\frac{dx}{dt}=\frac{\mathrm{d}}{\mathrm{dt}}A\cos(\omega t+\phi)=-A\omega\sin(\omega t+\phi)$$

$$a=\frac{dv}{dt}=\frac{\mathrm{d}}{\mathrm{dt}}[-A\omega\sin(\omega t+\phi)]=-A\omega^{2}\cos(\omega t+\phi)$$

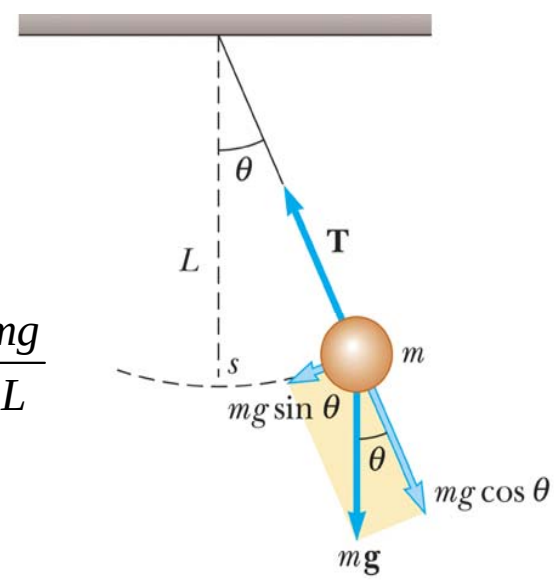
- ***SPRING***
  - *Definition* of spring constant! F=-kx
  - for a spring

$$\omega=\sqrt{\frac{k}{m}}$$

- ***Pendulum*** $F=-mg\sin\theta\approx-mg\theta$

$$x=L\theta \quad\Rightarrow\quad F=-\frac{mg}{L}L\theta\Rightarrow\ k=\frac{mg}{L}$$

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{mg/L}{m}}=\sqrt{\frac{g}{L}}$$



C1) I am taking this class

A. So I can become a doctor
B. So I can become a dentist or pharmacist
C. So I can go into the medical profession (other than doctor, dentist, pharmacist)
D. So I can go into the life sciences (other than the medical profession)
E. Other

c2) Assuming $x(t)=A\cos(\omega t+\phi)$ for a simple harmonic oscillator, which of the following is true?

A. $v(t)= - A \sin(\omega t+\phi)$

B. $v(t)= A \omega \sin(\omega t+\phi)$

C. $v(t)= - A \omega \sin(\omega t+\phi)$

D. $a(t)= - A \omega \cos(\omega t+\phi)$

E. $a(t)= - A \omega^2 \sin(\omega t+\phi)$

c3)In terms of the angular frequency ω, the period T is

a. $\frac{2\pi}{\omega}$.

b. $\frac{\omega}{2\pi}$.

c. $2\pi\omega$.

d. $\pi\omega$.

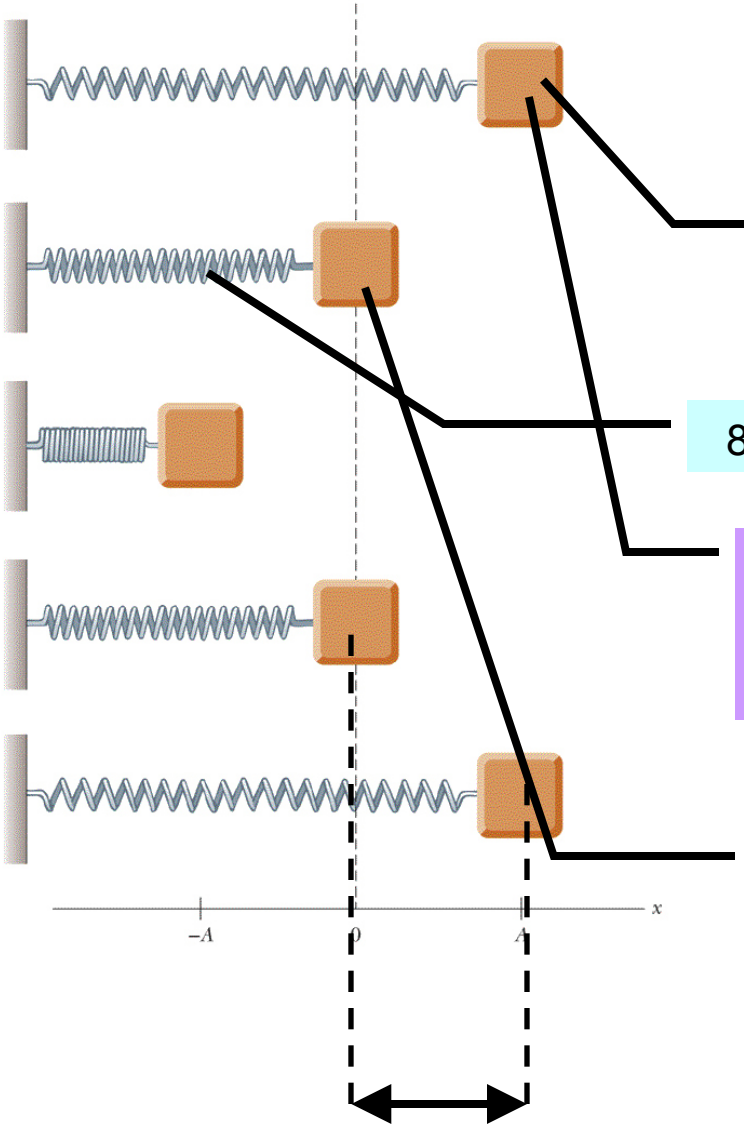
e. $\frac{\pi}{\omega}$.

Example 2

- A 0.5 kg object attached to a spring with a force constant of 8N/m vibrates in simple harmonic motion with an amplitude of 10 cm. Calculate (a) the maximum value of its speed and acceleration (b) the speed and acceleration when the object is 6cm from equilibrium and (c) the time it takes the object to move from x=0 to x=8 cm.

Example 2

- There is no friction
- The only force is produced by the spring
- Acceleration is determined by Newton's 2nd



0.5 kg

8 N/m

- Here the displacement is **largest**
- Then the force is **largest**
- Then the acceleration is **largest**

- Here the displacement is smallest
- The speed is smaller before and after
- Then here the velocity is **largest**

10 cm=0.1 m

Example 2

- m=0.5 kg k=8N/m A=0.1m
  - calculate max speed, accel
  - calculate speed, accel when x=6cm
  - time for x=0 to x=8 cm

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = -A\omega \sin(\omega t + \phi)$$

$$a(t) = -A\omega^2 \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8kg \cdot m \cdot s^{-2} / m}{0.5kg}} = 4s^{-1}$$

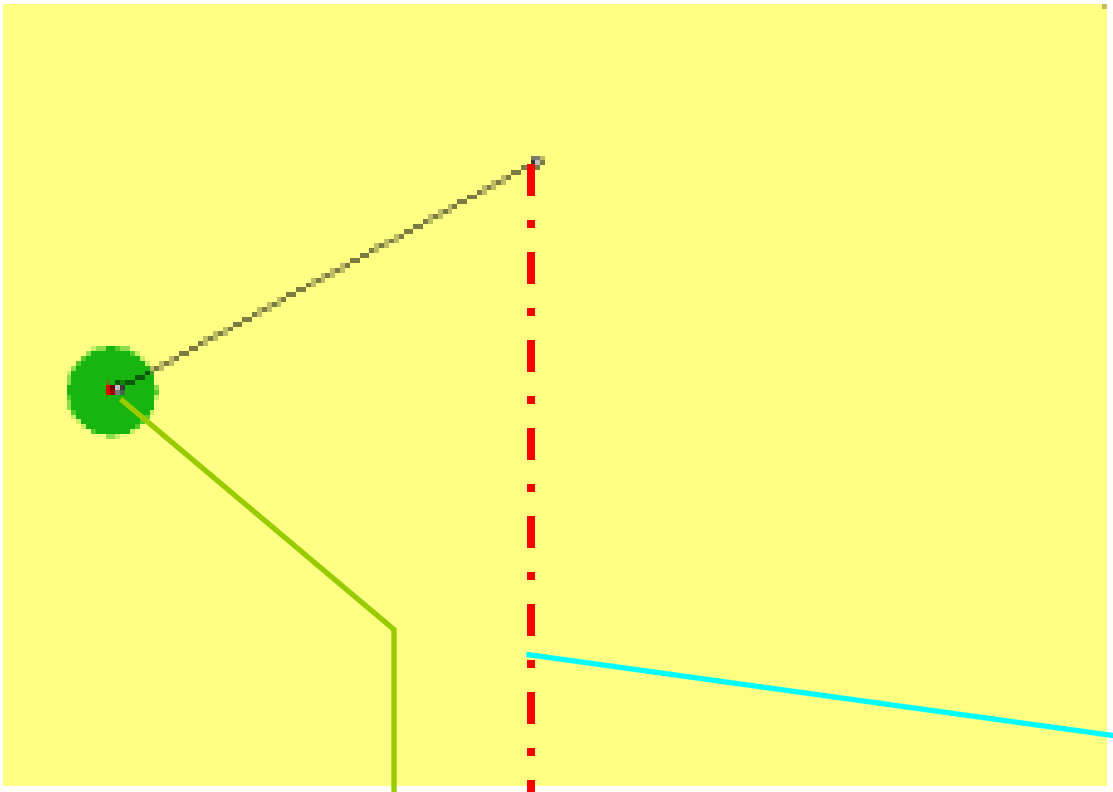
- set ϕ=0
- max speed =Aω=.4 m/s max accel=Aω^2=1.6 m/s^2
- when x=.06m we have .06m=0.1m cos(4t)

  t(x=0.6m) =¼ cos^{-1}(.06/.1)=0.23 s

  v= -Aωsin(ωt)=-0.1m 4s^{-1} sin((4)(0.23))=-.32 m/s

  etc

The flow of energy



Here the bob st
(soon to turn ba

No kinetic energy

Energy=potential energy

As the pendulum oscillates there energy flows:

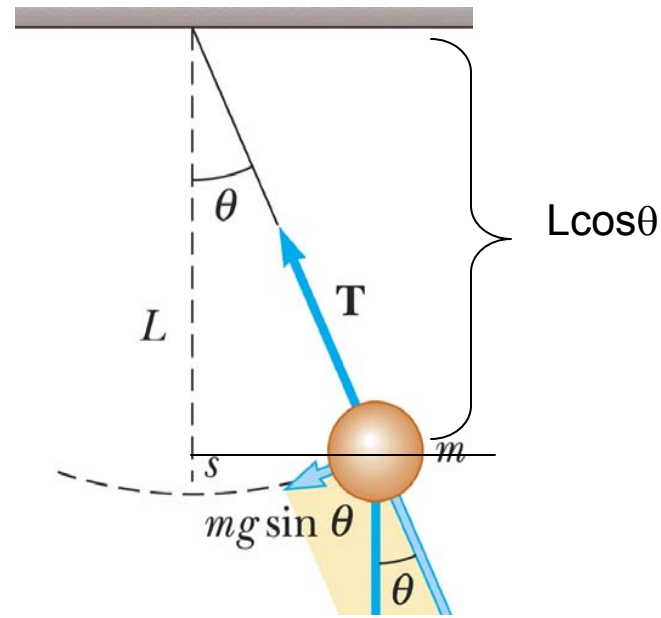
Kinetic → Potential → Kinetic → Potential …

its

Energy=kinetic energy

No potential energy

Flow of energy



- Potential energy

  $U_{spring}=\frac{1}{2} kx^2$

  $U_{pendulum}=mgh=mgL(1-\cos\theta)$
- Kinetic energy

  $\frac{1}{2} mv^2$
- Total energy E must be conserved so

  $E_{spring} = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$ is constant

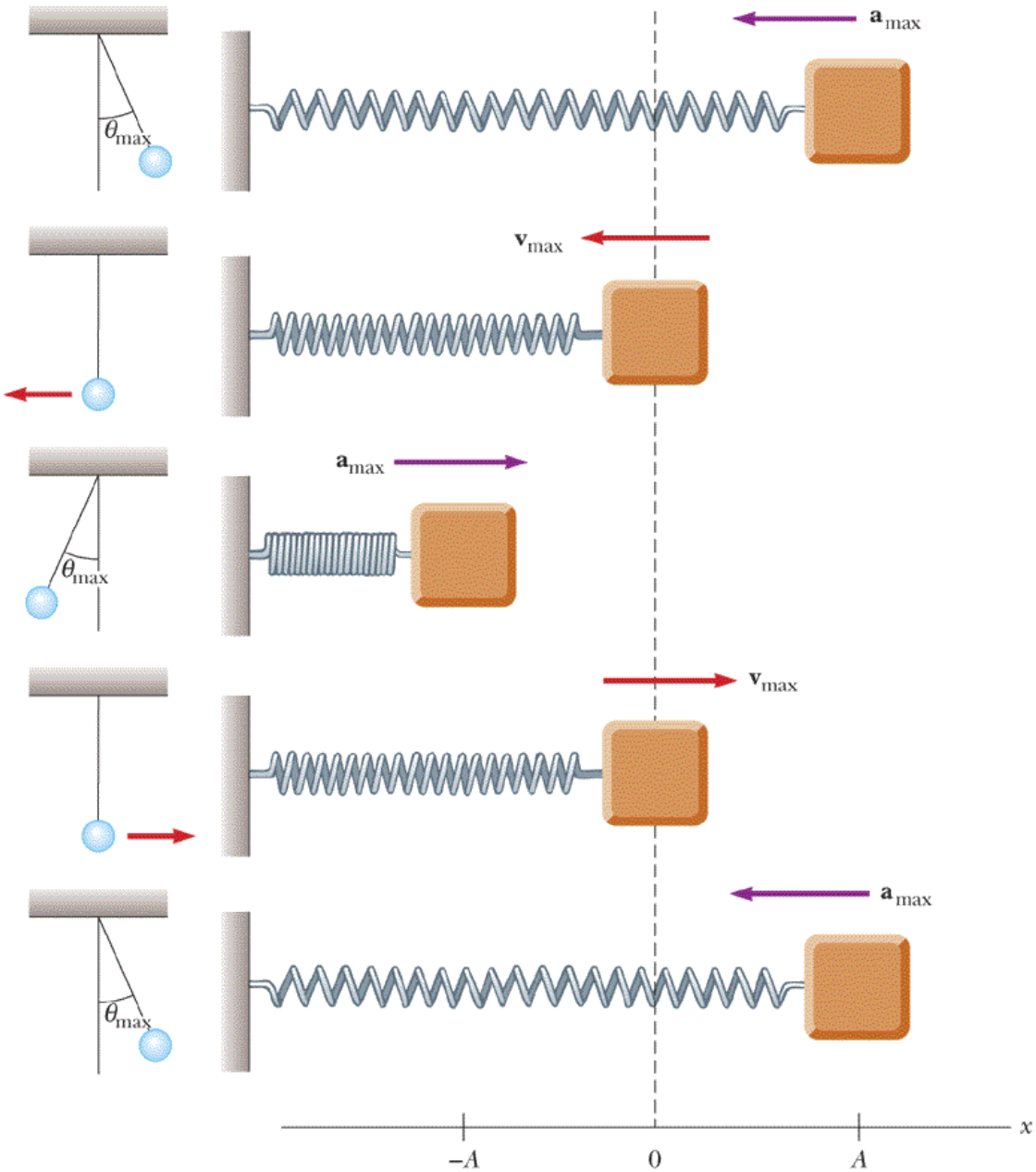
  $E_{pendulum} = \frac{1}{2} mv^2 +=mgL(1-\cos\theta)$ is constant

demo: bowling ball

veer slinger

applet

A pendulum and a spring have similar behaviors



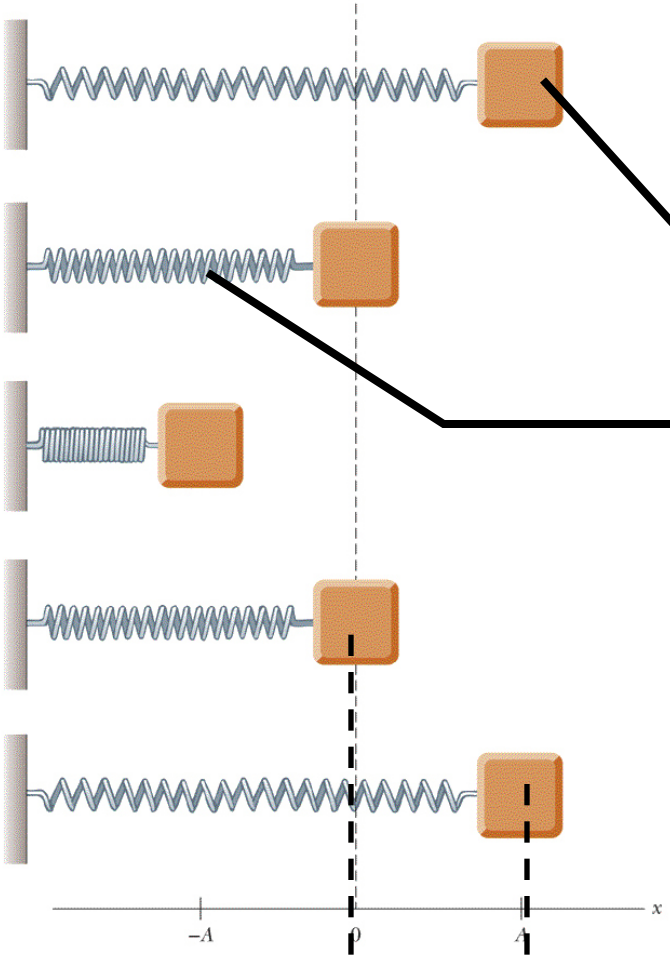
t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$T/4$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$T/2$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$3T/4$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$

Example 3

- A 50 g block connected to a spring with a force constant of 35 N/m oscillates on a horizontal, frictionless surface with an amplitude of 4 cm. find a) the total energy of the system and b) the speed of the block when the displacement is 1 cm. Find c) the kinetic energy and d) the potential energy when the displacement is 3 cm

Example 3

- There is no friction
- The only force is produced by the spring
- Acceleration is determined by Newton's 2nd

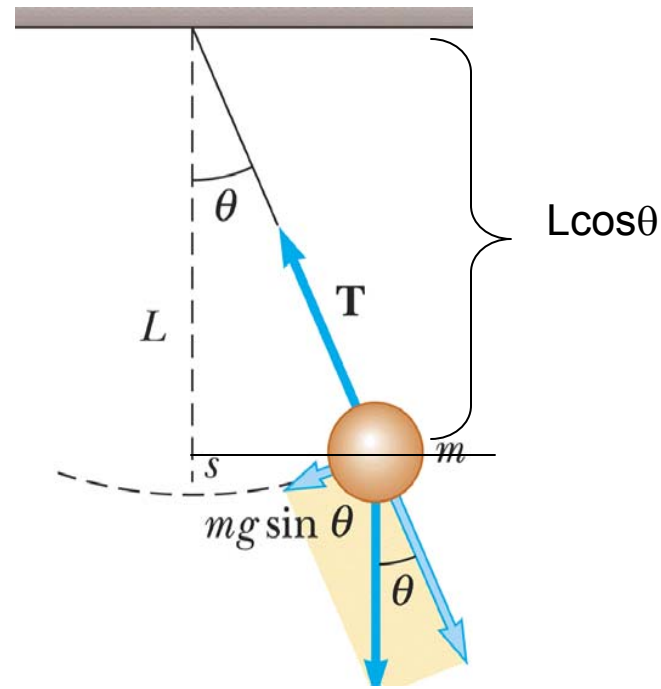


50 g=0.05 kg

35 N/m

4 cm=0.04m

Lecture 4



- Review- flow of energy
- Potential energy

  $U_{spring}=\frac{1}{2} kx^2$

  $U_{pendulum}=mgh=mgL(1-\cos\theta)$
- Kinetic energy

  $\frac{1}{2} mv^2$
- Total energy E must be conserved so

  $E_{spring} = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$ is constant

  $E_{pendulum} = \frac{1}{2} mv^2 +=mgL(1-\cos\theta)$ is constant

applet

L4 C1) For a simple harmonic oscillator without friction the total energy

A. remains constant
B. Goes up and down depending on the speed
C. is zero
D. slowly decreases
E. other

Example 3

- A 50 g block connected to a spring with a force constant of 35 N/m oscillates on a horizontal, frictionless surface with an amplitude of 4 cm. find a) the total energy of the system and b) the speed of the block when the displacement is 1 cm. Find c) the kinetic energy and d) the potential energy when the displacement is 3 cm

Ex 3

$$m = 0.05kg \quad k = 35N/m \quad A = 0.04m \quad \omega = \sqrt{\frac{k}{m}} = 26.5s$$

$$a)\ E = \frac{1}{2}kA^2 = \frac{1}{2}(35)(0.04)^2$$

$$b)\ \textit{Two ways: we are given } x' = 0.01m, \textit{ and we want } v'$$

$$i)\ \textit{easy way} - \textit{Energy Conservation}:\ E = \frac{1}{2}kA^2 = \frac{1}{2}kx'^2 + \frac{1}{2}mv'^2$$

$$v'^2 = \frac{k}{m}A^2 - \frac{k}{m}x'^2 \qquad v' = \omega\sqrt{A^2 - x'^2} = 1.02m/s$$

$$\textit{Actually we get that } v' = \pm 1.02m/s$$

$$ii)\textit{harder way: use } x' = A\cos(\omega t')\ \ v' = -A\omega\sin(\omega t')\ \ \textit{where we have set } \phi = 0$$

$$t' = \frac{1}{\omega}\cos^{-1}\left(\frac{x'}{A}\right)$$

$$v' = -A\omega\sin\left[\omega\frac{1}{\omega}\cos^{-1}\left(\frac{x'}{A}\right)\right] = -A\omega\sin\left[\cos^{-1}\left(\frac{x'}{A}\right)\right] = -(0.04)(26.5)\sin\left[\cos^{-1}\left(\frac{0.01}{0.04}\right)\right] = -1.02m/s$$

$$\textit{In reality here too the answer can be} \pm \textit{because of the } \cos^{-1}$$

$$c)\ x' = 0.03m$$

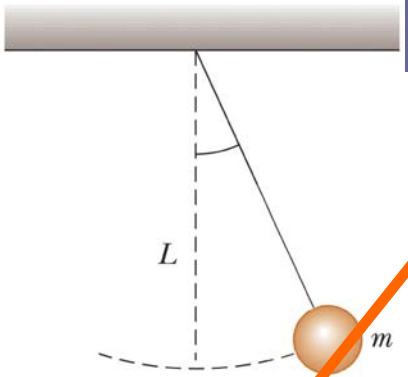
$$PE = \frac{1}{2}kx'^2 \qquad K = E - PE = \frac{1}{2}kA^2 - \frac{1}{2}kx'^2 \qquad \textit{plug it in}$$

Example 4

- A “seconds pendulum is one that moves through its equilibrium point once each second. (i.e. T=2s) The length of the seconds pendulum is 0.9927 m in Tokyo and 0.9942 m in Cambridge, England. What is the ratio of free fall accelerations (g) at these two locations?

Example 4

- We are told both pendulums have T=1 s
- We are told the lengths are *different*
- The frequency depends only on L and g
- Then $g_{Cambridge} \neq g_{Tokyo}$

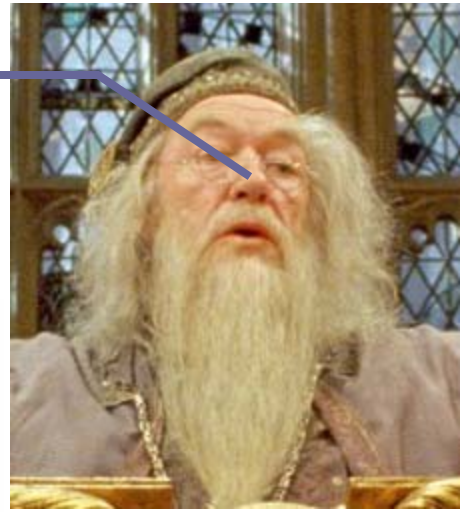


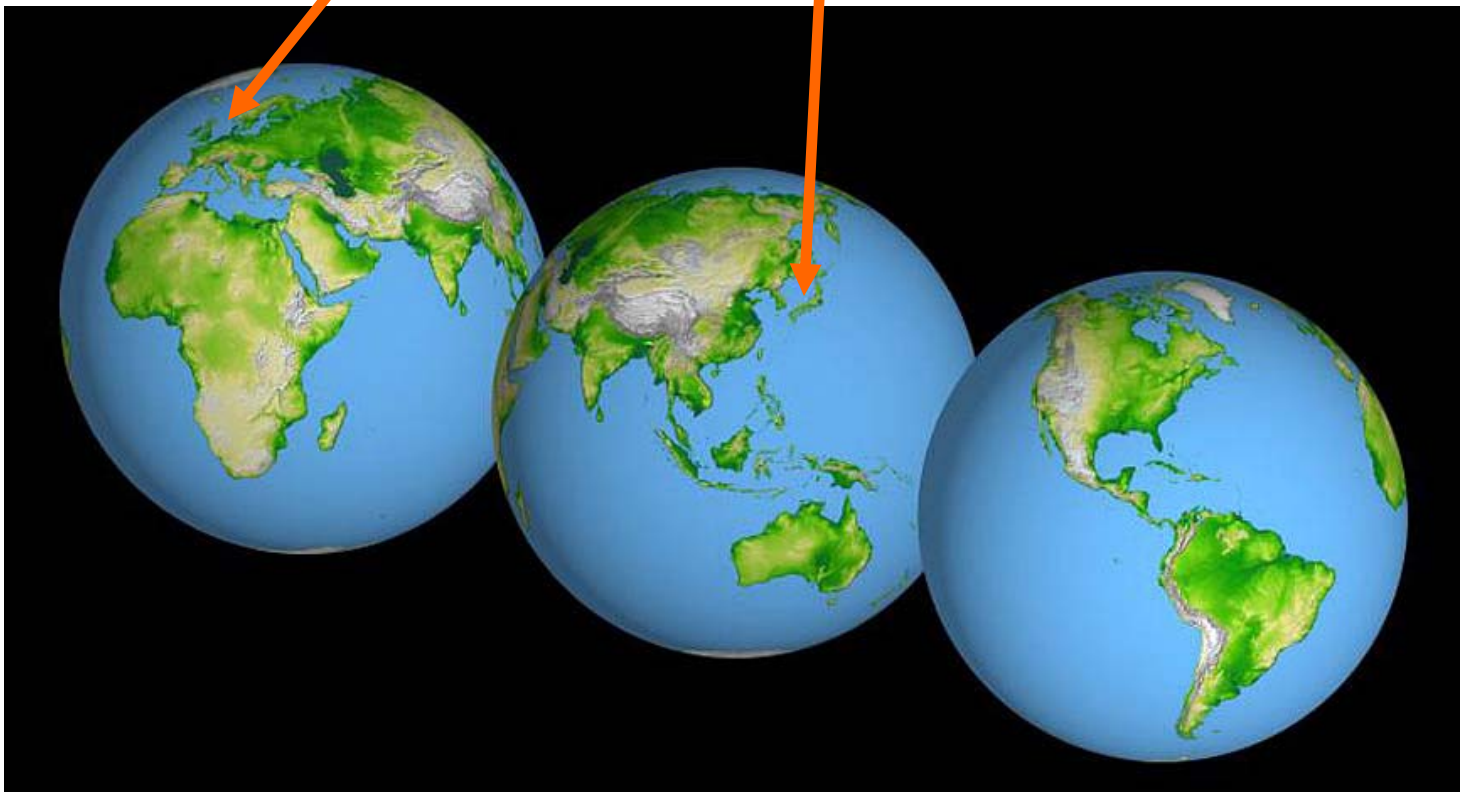
$$\text{Tokyo } \omega_T = \sqrt{\frac{g_T}{L_T}}$$

$$\text{Cambridge } \omega_C = \sqrt{\frac{g_C}{L_C}}$$

$$\text{we want } \omega_T = \omega_C$$

$$\frac{g_T}{g_C} = \frac{L_T}{L_C} = \frac{0.9927}{0.9924}$$





- L4 c2) A simple pendulum is moving with simple harmonic motion and is at its maximum displacement from equilibrium. Which of the following is also at its maximum? (absolute value)

a. Speed
b. Acceleration
c. Period
d. Frequency
e. Kinetic energy

Example 5 – spring with gravity

- F=mg-kx

$$m\frac{d^2x}{dt^2} + kx - mg = 0$$

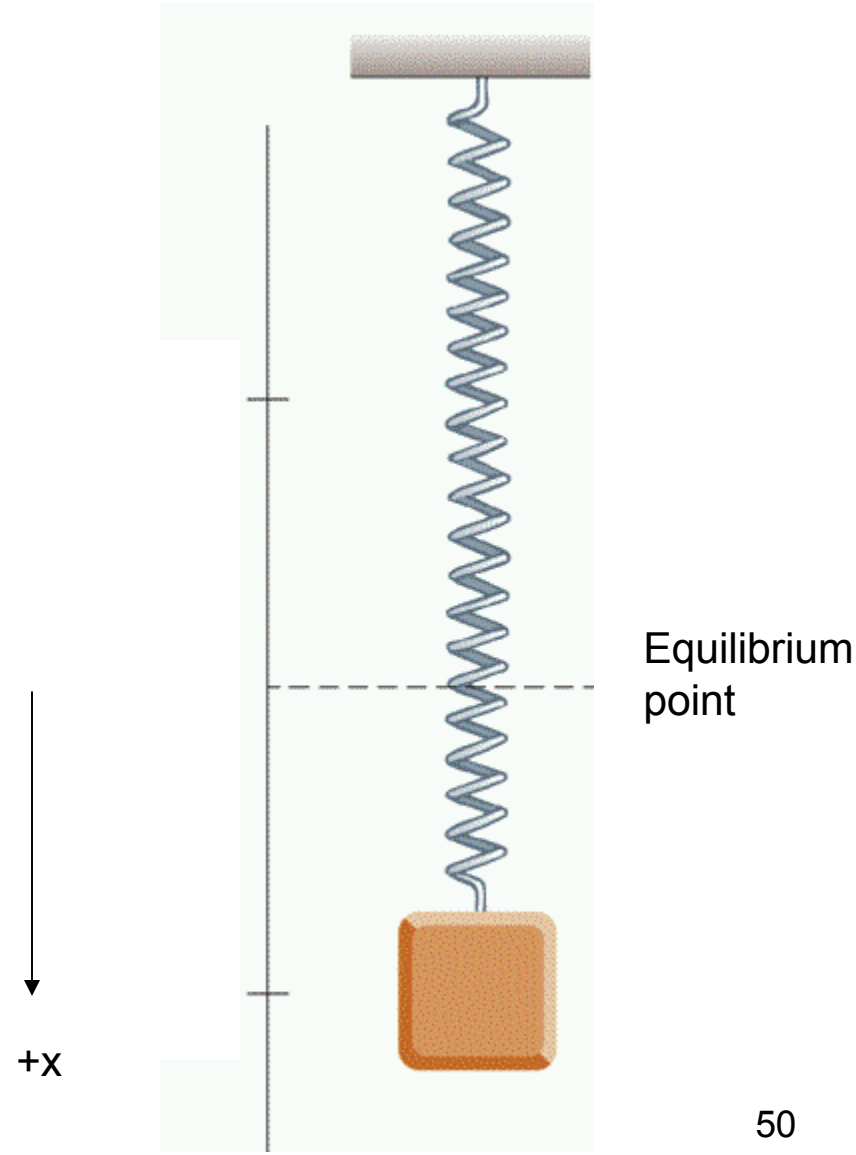
$$guess \quad x(t) = A\cos(\omega t + \phi) + x_0$$

we will get that $kx_0 = mg$

$$x_0 = \frac{mg}{k}$$

- This is nice – its just the static solution, i.e. when a=0

But the oscillation frequency remains the same!

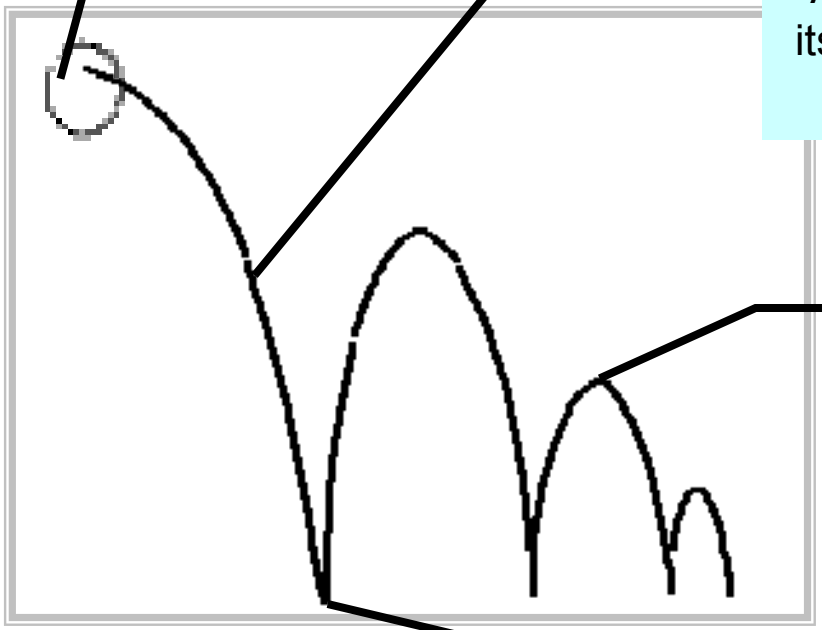


Damping

When objects come into contact - in almost all cases – some energy is transformed into heat

Initially we have only potential energy

As the ball falls its kinetic energy increases



Then it reaches a lower height after the bounce since it has less energy

When the ball bounces heat is generated

What does friction do?

In other cases ***friction*** is important

Friction:
- Opposes motion
- For a moving object it increases when the speed increases

So an approximate expression is

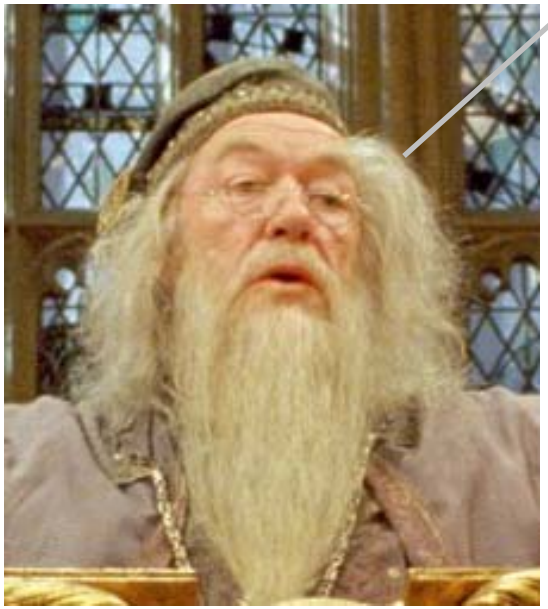
$$R = -bv$$

The frictional force ***R***

And the minus sign tells us ***R*** tends to decrease ***v***

The proportionality constant is *b.* We assume *b>0*

Is proportional to the velocity ***v***



So in realistic cases oscillators experience *two* forces:

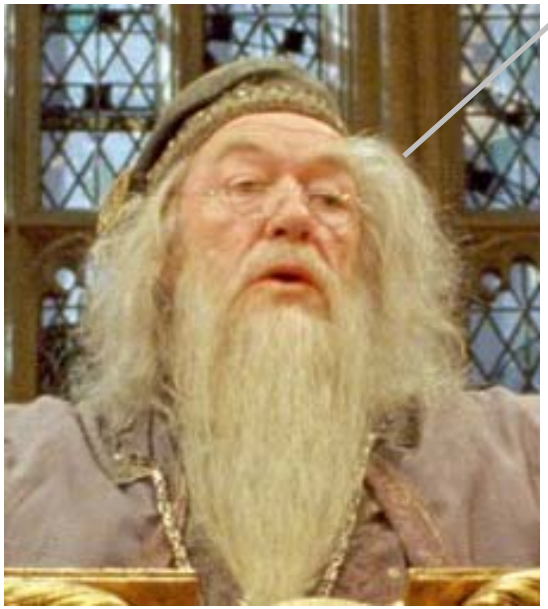
- The restoring force $F = -k\,x$
- The frictional force $R = -b\,v$

Newton's 2nd law implies

$$ma = F + R$$

The acceleration is the time derivative of the velocity

The velocity is the time derivative of the position



When friction is present energy is *not* conserved

$$E = K + V = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

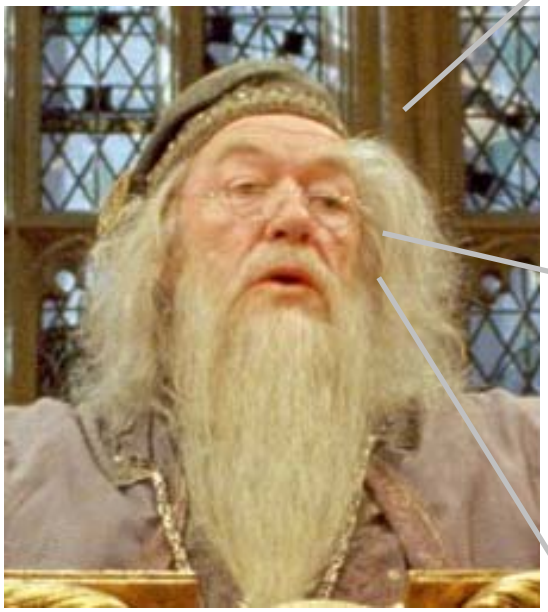
The total energy

$$ma = -kx - b$$

If there is no friction *b=0* and *E* remains constant.

The minus sign tells us that *E* decreases

The larger the velocity *v* the faster friction “eats up” *E*



Solutions of
are of two types
(only one of which
must know)

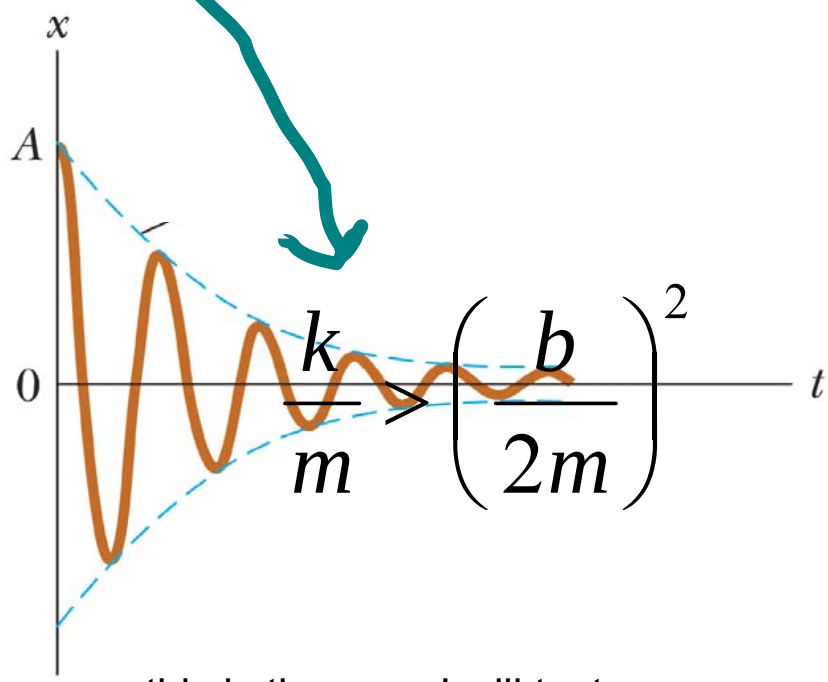
$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

For *b* small the damping is small
So we expect the thing to oscillate
… but will slowly lose energy
So it will eventually stop

The solution is

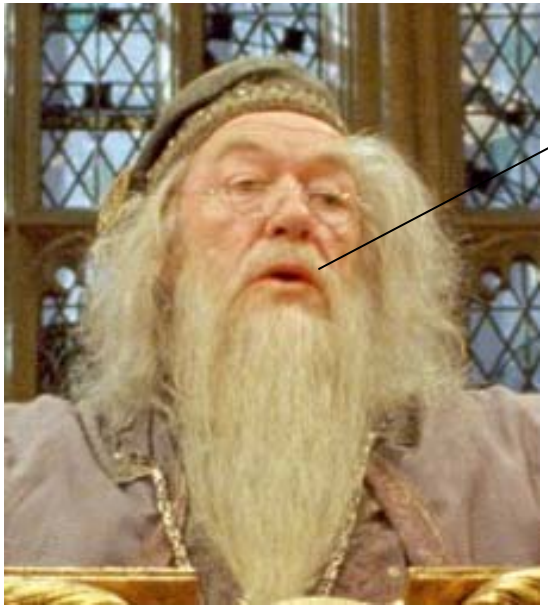
$$x = Ae^{-(b/2m)t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

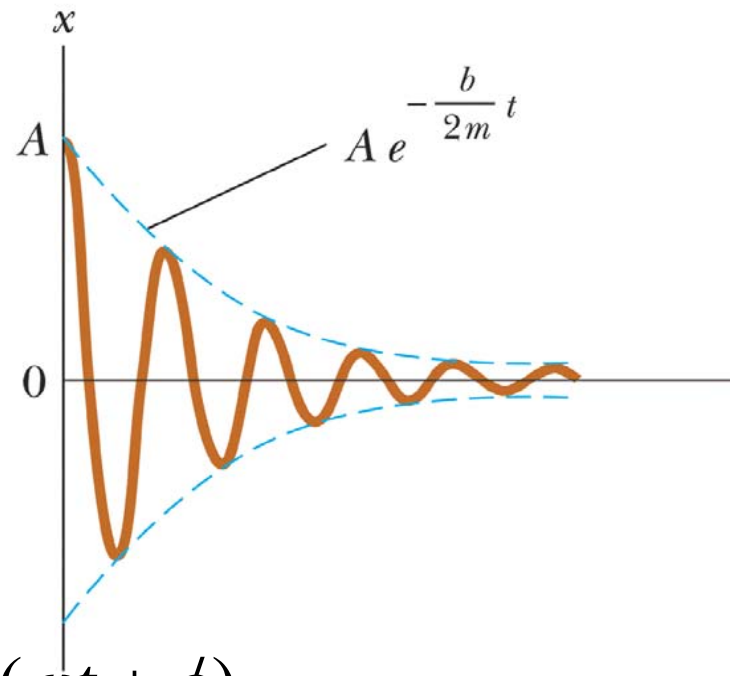


$$\frac{k}{m} > \left(\frac{b}{2m}\right)^2$$

this is the case I will test you on



We can use this solution and $E=\frac{1}{2}kA^2$ to see how the energy changes with time

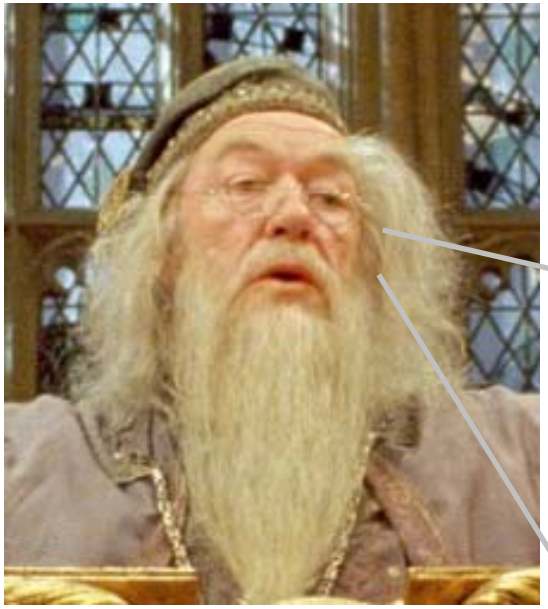


$$x = Ae^{-(b/2m)t}\cos(\omega t + \phi)$$

Think of this as a SHO with it amplitude changing like $Amp = Ae^{-(b/2m)t}$

$$E = \frac{1}{2}kAmp^2 = \frac{1}{2}kA^2 e^{-\left(\frac{b}{m}\right)t} = E_0 e^{-t/\tau}$$

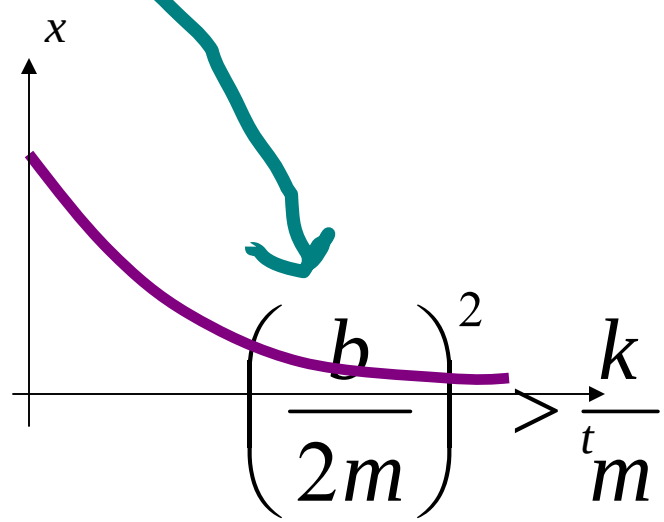
$$where \quad E_0 = \frac{1}{2}kA^2 \quad and \quad \tau = \frac{m}{b}$$



$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

For *b* large the damping will eat energy before the thing can oscillate and the amplitude simply decays to zero.
This is an ***overdamped*** oscillator

The solution is



$$x = Ae^{-(b/2m+u)t} + Be^{-(b/2m-u)t}$$

$$u = \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$

I will not test you on this case

dampled.shm

L4 c3) For a simple harmonic oscillator with friction, which statement is NOT true

A. The amplitude decreases
B. the kinetic energy decreases
C. The energy is turned into heat
D. The energy – (potential + kinetic) is conserved
E. The frequency is decreased

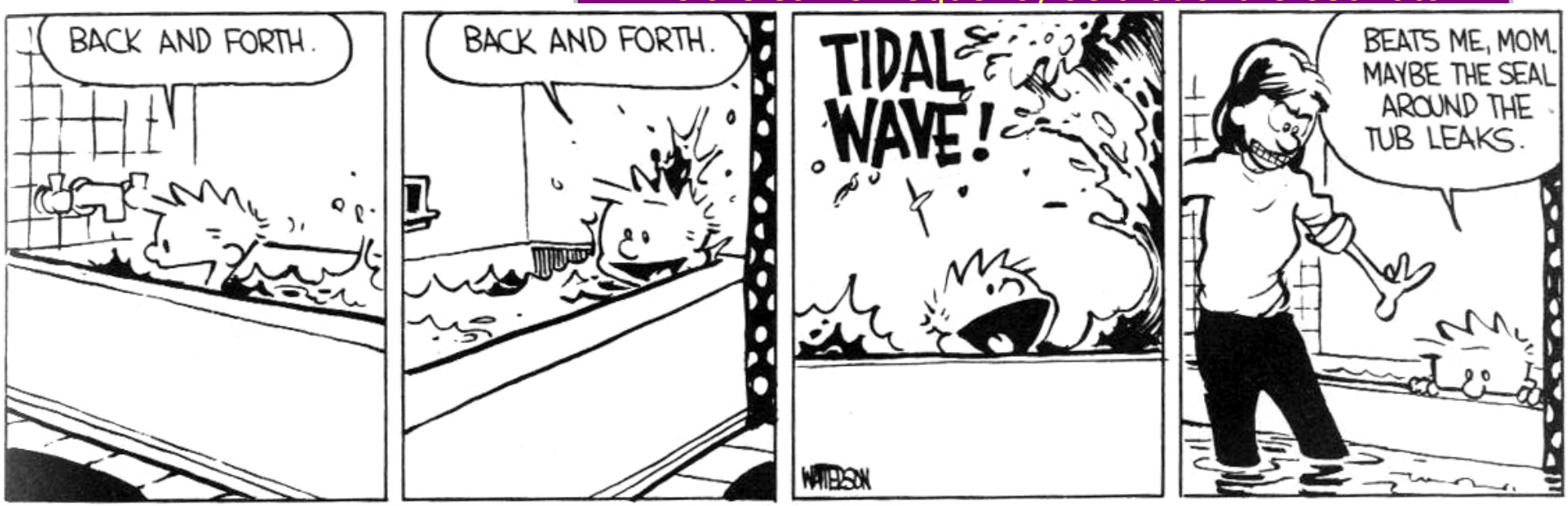
What do external forces do?

In some cases the oscillators are pushed/pulled by external forces

A simple case is

$$F_{\text{external}} = F_0 \sin(\omega t)$$

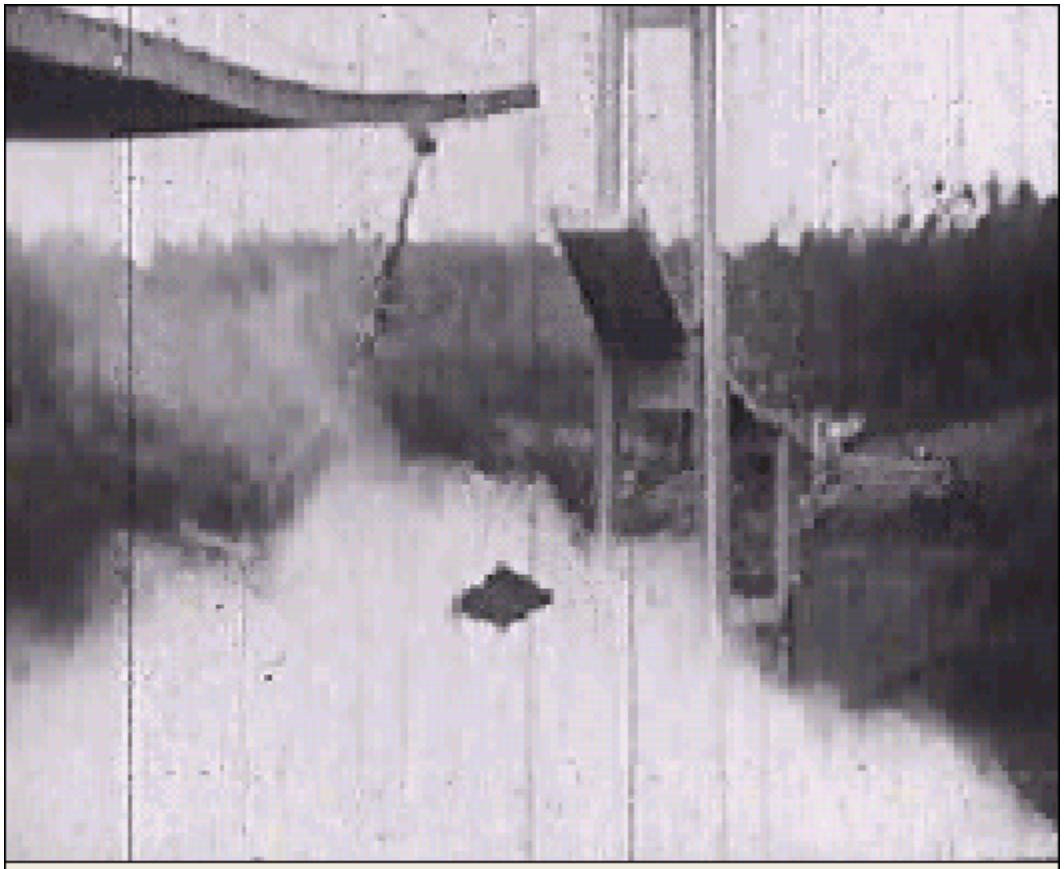
If ω is the same frequency as that of the oscillator



film:tacoma narrows

Things can get pretty rough…

Really rough…



suspen

Putting it all together

In more realistic cases there are external forces *and* friction

$$ma = F_{\text{total}} = -kx - bv + F_0 \sin(\omega t)$$

$$\sqrt{k/m} = \omega_0$$

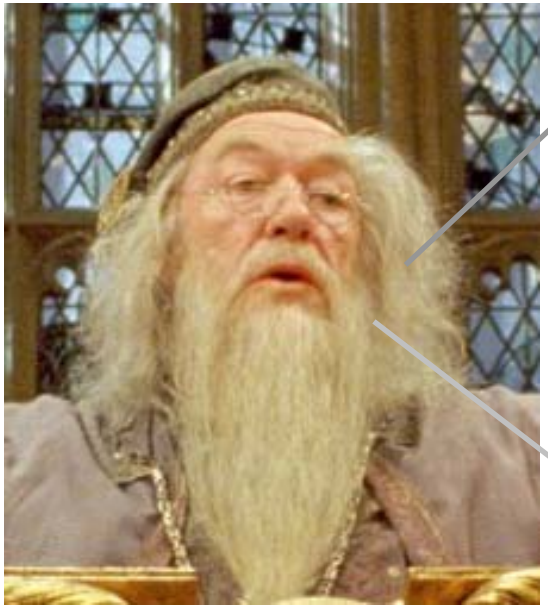
ω can be equal to, or different from ω_0

$$x(t) = A\sin(\omega t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(b\omega/m)^2 + (\omega^2 - \omega_0^2)^2}}$$

$$\tan\phi = \frac{b\omega/m}{\omega^2 - \omega_0^2}$$

Interesting case: when the driving frequency is ω_0.



On resonance $\omega = \omega_0$:

$$x(t) = A\sin(\omega t + \phi)$$

$$A = F_0 / (b\omega)$$

$$\tan\phi = \pm\infty \rightarrow \phi = \pm\pi / 2$$

So if *b* is small, *A* can get very large

The complete solution to the equations is more complicated

$$x(t) = A\sin(\omega t + \phi) + Ce^{-(b/2m)t}\cos(\omega_1 t + \alpha)$$

$$A = \frac{F_0/m}{\sqrt{(b\omega/m)^2 + (\omega^2 - \omega_0^2)^2}}$$

$$\tan\phi = \frac{b\omega/m}{\omega^2 - \omega_0^2}$$

$$\omega_0^2 = k/m$$

$$\omega_1^2 = \omega_0^2 - \left(\frac{b}{2m}\right)^2$$

Determined by the initial conditions

Same as when F_0=0

Fixed by the external force

Same as before

driven_damp.shm

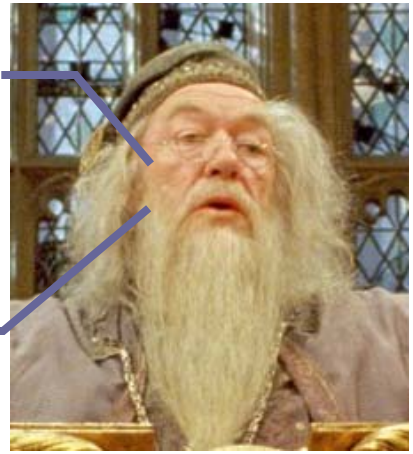
Example 6

- A 2kg object attached to a spring is driven by an external force given by $F=(3N)\cos(2\pi t)$. If the force constant of the spring is 20N/m, determine a) the period and b) the amplitude of the motion. Assume there is no damping.

Example 6

- There is no damping
- This looks lika “plug & chug” problem

Yep, plug & chug



$$F = 3\cos(2\pi t) \quad \text{then} \quad \omega = 2\pi$$

$$\omega_0^2 = k/m = 10/\mathrm{s}^2 \quad \text{then} \quad T = \frac{2\pi}{\sqrt{10}} = 1.99\mathrm{s}$$

$$\text{For } b = 0, \quad A = \frac{F_0/m}{|\omega_0{}^2 - 4\pi^2|} = \frac{3/2}{|10 - 4\pi^2|} = 0.05\mathrm{m}$$

Example 7

- You are a doctor on call and you take your emergency pager to a fine restaurant. You switch the pager, whose battery is low, to vibrate, instead of beep and you put it in the side pocket of your suit coat. The arm of your chair presses the light cloth against your body at one spot. Fabric with a length of 8.2 cm hangs freely below the spot with the pager at the bottom. You get a call. The motion of the pager makes the hanging part of your coat swing back and forth with a remarkably large amplitude. The pager weighs 200g. Find the frequency at which your pager vibrates.

Example 7

The problem is cute, but the bare facts are

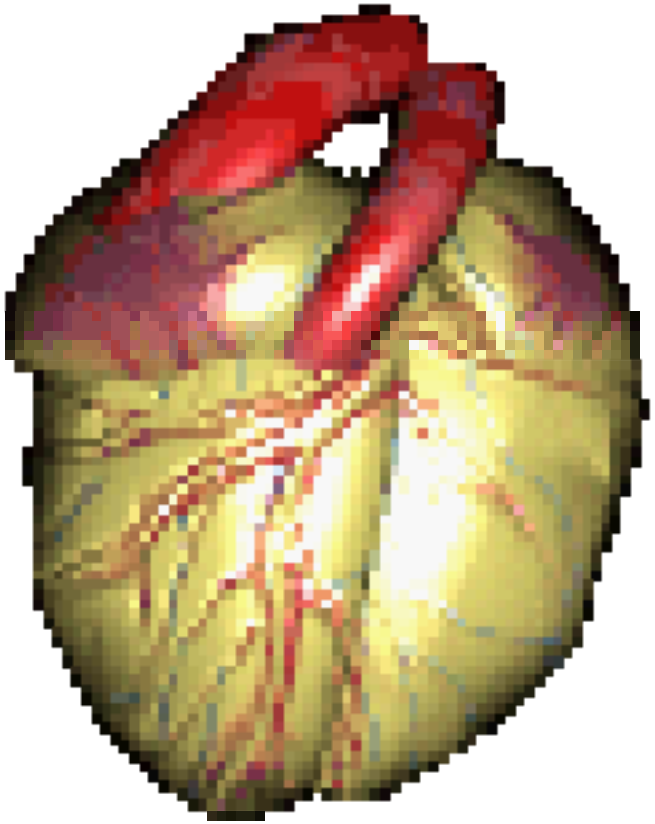
- The pager vibrates, so it generates a driving force
- I'll assume this is a harmonic force
- The hanging cloth is like a pendulum
- The reason the amplitude is so large is that its resonating
- The pager's frequency must be the same as the pendulum's

$$\text{Pendulum} \quad \omega = \sqrt{g/L} = \sqrt{9.8/0.0821} = 10.93/\text{s}$$

$$f = \frac{\omega}{2\pi} = 1.74/\text{s}$$

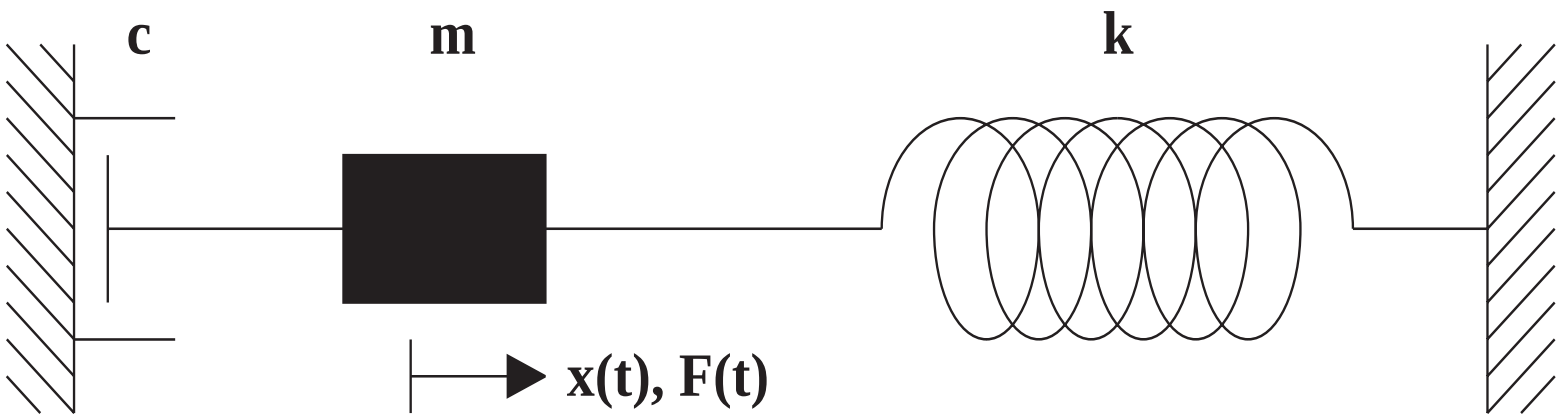
The heart – a special oscillator

- Damp
- driven_damp



Discovering (predicting) new cardiac physiology/function from cardiac imaging, mathematical modeling and first principles

To go from correlation to causality devise a kinematic model of suction initiated filling:



Newton's Law: $m\, d^2x/dt^2 + c\, dx/dt + k\, x = 0$

Initial conditions: $x(0) = x_o \Rightarrow$ stored elastic strain to power suction
$v(0) = 0 \Rightarrow$ no flow prior to valve opening

Recall SHO has 3 regimes of motion, underdamped $c^2-4mk<0$, critically damped $c^2=4mk$, overdamped $c^2 - 4mk>0$.

VALIDATION: Compare model-predicted velocity of oscillator to velocity of blood entering the ventricle through mitral valve.

beginning of “cheat sheet”

$$\omega = 2\pi f \qquad f = \frac{1}{T}$$

$$SHO \quad \text{F=-kx} \qquad \text{x=Acos}(\omega\text{t}+\phi) \qquad \omega = \sqrt{\frac{k}{m}}$$

$$Pendulum\ \omega = \sqrt{\frac{g}{L}}$$

$$PE_{spring} = \frac{1}{2}kx^2 \qquad Energy = PE + KE$$

$$x = Ae^{-(b/2m)t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$E = E_0 e^{-t/\tau} \qquad where \quad \tau = \frac{m}{b}$$

$$ma = F_{\text{total}} = -kx - bv + F_0 \sin(\omega t)$$

$$A = \frac{F_0 / m}{\sqrt{(b\omega / m)^2 + (\omega^2 - \omega_0^2)^2}}$$

$$\tan\phi = \frac{b\omega / m}{\omega^2 - \omega_0^2}$$