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Effects of virtual monopoles

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Abstract

Electromagnetism would be a “more unified” theory if there were elementary magnetic monopoles and/or particles with both electric and magnetic charges (dyons). I discuss the simplest possibilities for the addition of these entities onto the Standard Model, and their empirical consequences. Lower limits on the masses of monopoles and dyons stemming from their quantum effects on current observables turn out to be much stronger than the existing limits from direct searches. Anomalies in the three-photon decay of the Z constitute good specific signatures for monopoles or dyons. T -odd observables in the $e^+e^- \rightarrow W^+W^-$ process are signatures for dyons, but they are severely constrained by existing data. The subjects of monopolium, monopole cosmology and non-elementary monopoles are also discussed.

1. Introduction

The quest for symmetries in the laws of Nature, sometimes exact, occasionally broken in more or less elegant ways, has been crowned by a long list of successes. This, and not only aesthetical pleasure, is a good reason to pursue the quest. Often quoted as the first, or second, “grand unification” is that of electricity and magnetism. But Maxwell’s or Nature’s feat is only partial: the conceivable symmetry between electric and magnetic charges is broken by the non-existence of (relatively light) magnetic monopoles.

A higher symmetry was one of Dirac’s motivations to introduce the notion of magnetic charges [1]. But he found that the elementary electric and magnetic charges e and g ought to be quantified so that $eg = 2\pi n$, with n an integer (units are such that $\alpha_e \equiv e^2/4\pi \sim 1/137$, $\alpha_g \equiv g^2/4\pi \sim 34n^2$). Thus, in a sense, the addition of Dirac’s monopoles to QED would not constitute a truly “grand” unification of electricity and magnetism, since $e \neq g$. Perhaps this asymmetry can

be cured as in conventional Grand-Unified Theories, by recalling that, in a quantum theory, couplings are energy-dependent and it is only at a scale wherein masses are negligible that the couplings merge into a single value (the Dirac condition $e(q^2)g(q^2) = 2\pi n$ has been argued to hold [2] for the one-loop renormalized quantities at any given momentum scale q^2 , so that as e increases with q^2 , e and g do tend to merge).

Interest in monopoles revived in the 70's, following the discovery by 't Hooft and Polyakov [3] that there exist monopole solutions to the field equations of theories in which a semi-simple unifying gauge group is broken to the U(1) of QED. If the unification scale is M_{GUT} , the monopole mass [4] is $m \geq M_{\text{GUT}}/\bar{\alpha}$, with $\bar{\alpha}$ the grand-unified fine-structure constant. A consequence of these theoretical developments is that the feeling has permeated the community that monopoles are extremely heavy objects, desperately out of the reach of accelerator experiments. I perceive this circumstance as negative and I contend that monopoles much lighter than the "grand-unified" ones may simply *happen* to exist, and should be looked for.

Grand-unified monopoles are "topological" entities with a nontrivial internal structure. Another consequence of the theoretical discovery of these objects is that interest in the theory of elementary point-like monopoles has waned. The belief that there is no consistent way of adding point-like monopoles to the standard lore has gained followers, though, to my knowledge, there is no general proof of this statement, or of its denial.

Almost everywhere, I assume monopoles to be spin-1/2 *point-like* particles whose radius, R , is negligible relative to the inverse of their mass, in contrast to grand-unified monopoles, extensive objects with $R \sim 1/m\bar{\alpha}$.

The production of grand-unified monopole–antimonopole pairs has been argued to be exponentially damped by form-factor effects, to a level wherein observability would be out of the question [5]. In Section 7, I discuss monopoles that are not point-like, but are unconventional in being much lighter than the grand-unified ones. I extend the results of [5] to the virtual effects of extensive monopoles, and argue that they too are negligible in practice. In a euclidean lattice approach [6], the *coupling strength* of monopoles has also been claimed to be exponentially suppressed, independent of the monopole's substructure. The continuum counterpart of this surprising result (that would seem to conflict with the classical limit for the monopole's charge) is admittedly unclear [6], and I choose to proceed in the disbelief that the magnetic charge of elementary monopoles is damped.

The extension of QED to a theory containing both electric and magnetic elementary charges is nontrivial and, in the case of point-like monopoles, it has not reached the theoretically satisfactory point wherein consistency and renormalizability have been demonstrated to all orders of perturbation theory (even if one were to finesse the problem that a perturbative expansion cannot be simultaneously convergent in e and g). The Feynman rules have been derived [7–10] and are consistently usable at the one-loop level, in the case of monopoles. For dyons the situation is more obscure (these points are discussed in Section 2).

For the purposes of this paper, to estimate the current limits on monopole masses and to suggest concrete strategies for indirect monopole searches, our current understanding of monopole theory is (almost) all one needs.

The standard $SU(2) \otimes U(1)_Y$ model is sufficiently well established for it to be necessary, when considering monopoles, to discuss the gauge-group representation to which they might belong. To avoid anomalies and other potential strictures, one is most economically led to introduce “vector-like” spin-1/2 monopoles, whose left- and right-handed projections belong to the same group representation. I consider only the two simplest cases, that of electrically neutral $SU(2)$ -singlet monopoles $M_{L,R}^0$ and that of “dyons” $(D^0, D^-)_{L,R}$, doublets with (ν^0, e^-) -like electrical charges¹. In what follows I refer to these two choices simply as monopoles and dyons, or to both as monopoles when no distinction is necessary. In both cases the magnetic charge is commensurate à la Dirac: $g = 2\pi n/e$. To economize parameters, I generally let the dyons be (quasi-)degenerate, $|m(D^-) - m(D^0)| \ll m(D^i)$.

The current to which the standard Z^0 -boson couples has an electromagnetic component, so that not only photons, but also Z 's must couple to monopoles, with a large amplitude of $O(g)$. If monopoles are not confined, as they might [11] by the strong monopole–antimonopole forces, the fact that LEP detectors have not been swamped by monopoles immediately implies a “direct” limit $m > M_z/2$.

We show in Section 3 how the trumpeted success of the Standard Model in accommodating all current data implies a lower limit on the monopole mass of order 1 TeV, a much stronger result than the direct one². In Section 4 we discuss how the Z -decays into odd numbers of photons constitute a similarly sensitive but much more specific limiting (or discovery) test. The masses of our dyons are constrained by current data in the same way as those of monopoles; in Section 6 we discuss how dyons, which couple not only to γ 's and Z 's, but also to W 's, may induce very characteristic T -odd observables in the $e^+e^- \rightarrow W^+W^-$ process. But the available data constrain these effects below the level of observability at LEP-II. I do not discuss monopole signatures at higher-energy e^+e^- colliders.

The monopole charge is too large for a perturbative expansion in g to be trustworthy. This unsurmounted caveat will unavoidably haunt all our numerical considerations: an argument is needed to judge the credibility of the results.

Throughout the paper, we discuss how monopoles may modify the standard relations between observables measured at energy scales below the monopole mass, $q^2 < m^2$. The incidence of monopoles can thus be characterized in the customary fashion, by grafting onto the standard lagrangian a series of dimension $d > 4$ effective operators induced by virtual monopoles, whose degrees of freedom have been integrated out, or, more pictorially, integrated “in”. In each case we

¹ I refrain here from discussing representations containing objects of different magnetic charge; this would lead to the introduction of magnetically charged gauge bosons and an enlarged gauge group.

² This is so in spite of the fact that, with our choice of monopole properties, the “oblique” vector-boson propagator corrections S , T and U , at least in their original disguise [12], vanish automatically.

characterize the effect of monopoles by the lowest-dimension relevant operator (this is tantamount to keeping the first non-vanishing term in an expansion in q^2/m^2). All we assume is that the *order of magnitude* of the coefficient of each of these operators can be estimated by computing it to lowest non-vanishing order in g . This is indeed an assumption: in every case the corrections (to the next order in g) are $O(g^2/16\pi^2) \sim 1$.

Equivalent to the above is the approach used in other strongly interacting realms, such as the chiral lagrangians describing the low-energy interactions of pseudoscalar mesons. There, the size of the coefficient of a particular operator is set by demanding that its estimates to successive orders of perturbation give results of the same magnitude [13]. The coefficients of the effective operators we deal with involve powers of g/m . In a “chiral-like” approach one would simply substitute a multiple of g/m by the analogue of f_π^{-1} . In a sense, computing diagrams explicitly, as we do, is simply a way to keep straight the powers of 4π .

Monopole–antimonopole bound states may occur below the threshold for “open” monopole production, and need to be discussed. I argue in Section 5 that these states are so wide that they should not be considered explicitly. Their collective effects should already be included in the “partonic” description in terms of virtual-monopole loops.

Arbitrarily heavy elementary monopoles or dyons would antagonize the standard cosmological lore. The ensuing upper mass limits are discussed in Section 8. A summary and the conclusions are offered in Section 9.

Since the subject of monopoles has not been investigated at length in the spirit of this paper, I have taken the liberty to report on the results of the exploration of various alleys which do not take us to fertile destinations. Consequently, a positive-thinking reader may consider skipping Sections 5 to 7, which deal with monopolia, W-pair production and extended monopoles, and to concentrate on Section 4, that deals with the phenomenologically relevant subject of the multiphoton decays of the Z.

2. Electrodynamics with electric and magnetic poles

It is not possible to write a theory of point-like electric *and* magnetic charges wherein the electromagnetic field is described exclusively by a local vector potential $A^\mu(x)$. Whence the necessity of introducing the Dirac string [1] or a multivalued potential [14]. The string, or its surrogates, can be regarded as gauge-artefacts and are unobservable if the Dirac charge-quantization condition is satisfied. Yet, they entail a measure of non-locality in the action and in probability amplitudes that must not make its way to the observables.

An action describing electric and magnetic poles can be translated with the customary methods into a set of Feynman rules. Very little attention has been paid to these rules, the conventional wisdom (that I have challenged in the introduction) being that the large value of g makes them entirely useless. These calcula-

tional rules are of delicate handling in the case of monopoles, and are worse than delicate for dyons.

Let n be a space-like four-vector tangent to the Dirac string, $\epsilon_{\mu\nu\rho\sigma}$ the fully antisymmetric four-index symbol, and ϵ_A^ρ a photon-polarization vector. The monopole counterparts to the $\mathcal{L}_e = -ieA \cdot J$ coupling of a photon to a charged object, the configuration- and momentum-space vertex “Feynman rules” for the coupling to a monopole, are [8–10]

$$\mathcal{L}_g = -ig \frac{\epsilon(n, \partial_x, A(x), J(x))}{n \cdot \partial_x} \leftrightarrow -ig \frac{\epsilon_{\mu\nu\rho\sigma} n^\mu q^\nu \epsilon_A^\rho J^\sigma}{n \cdot q + i\epsilon}, \tag{1}$$

with $J_\sigma = (p_\sigma + p'_\sigma)$ or $\bar{\psi}(p')\gamma_\sigma\psi(p)$, for spinless and spin-1/2 monopoles. The ϵ tensor is needed to describe a “dual” coupling, and the vector n must be contracted with it, since there are only two independent four-vectors in the set q, p, p' . The terrifying denominators in Eq. (1) embody the non-local character of the Dirac string.

Our $M_{L,R}^0$ monopoles and $(D^0, D^-)_{L,R}$ dyons are “magnetic” weak-SU(2) singlets, but they carry $U(1)_Y$ magnetic charge. Their couplings to Z 's and γ 's are the dual or magnetic counterparts to the standard couplings of a singlet with non-vanishing hypercharge:

$$\mathcal{L}_D = -i \frac{g}{c} \frac{\epsilon(n, \partial_x, B(x), J(x))}{n \cdot \partial_x}, \tag{2}$$

where

$$B_\mu \equiv cA_\mu - sZ_\mu, \tag{3}$$

is the $U(1)_Y$ potential, and $(s, c) = (\sin \theta_W, \cos \theta_W)$. Our electrically neutral monopoles, M^0 , have no gauge couplings other than the ones of Eq. (2), while dyons also sport standard “electric” vector-like couplings to W 's, to Z 's and, in the case of D^- , to γ 's.

To illustrate the use of Eq. (1), consider the photon-mediated processes of e^+e^- annihilation into lepton (L), monopole (M) or dyon (D) pairs. The matrix element squared for L^+L^- production is of the form $|J_e^\dagger \cdot J_L|^2/q^4$. The calculation of the cross section for $e^+e^- \rightarrow M\bar{M}$ production with casual use of the rule (1) leads to an n -dependent result, even after the erasure of all terms that vanish because of current conservation ($q \cdot J_e = q \cdot J_M = 0$). The somewhat debonair recipe [10] of dropping all terms proportional to q^2 (that would cancel the photon's propagator pole) results in a squared matrix element proportional to $(|J_e^\dagger \cdot J_M|^2 - |J_e|^2 |J_M|^2)/q^4$, a sensible answer³ at last. The total cross sections for the production of magnetically and electrically charged pairs of a given spin have the

³ The crossed process of electron scattering on a monopole has the same squared matrix element, from which one can reproduce the classical results for the scattering on a static monopole of electrons that do not hit the string.

same functional form, as expected; while their angular distributions are different, also as expected.

So far, so good. The troubles arise as one attempts to extend these considerations to the e^+e^- production of dyon pairs. The Feynman rules are simply obtained [9] by adding to Eq. (1) a conventional electric-charge coupling of the D^- -dyon. The interference term between the “electric” and “magnetic” dyon amplitudes does not cancel in the expression for the cross section and is proportional to $\epsilon(n, p, p', k)$ with k one of the incoming momenta and n , alas, the vector defining the Dirac string. I have neither solved this problem nor found it discussed in the literature. Clearly, the bugaboo is in the treatment of superimposed singularities at “the end of the string”. Presumably a study of the effective vertex operators describing the long-wavelength limit of the photon couplings to (not point-like) topological dyons would help clarify these issues.

The slippery character of all these grounds emanates from the necessity of using vector potentials, rather than E and B fields, in describing photonic couplings to monopolar electric and magnetic charges. There is no difficulty in translating a magnetic-dipole coupling such as $F_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi$ into its dual electric-dipole counterpart, suffice it to trade $F_{\mu\nu}$ for $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. This simple remark will allow us, in discussing dyons in Section 6, to bypass the problem of dealing with inconsistent Feynman rules.

3. Current limits on the masses of monopoles and dyons

It has become a flourishing industry to set limits on novel effects by exploiting the accurate success of the Standard Model in relating precise electroweak data at energies up to M_Z (see [15] for a clear review). Typically, three of the best measured quantities (α , M_Z and the Fermi constant as extracted from muon beta decay) are used as inputs to specify the parameters and predict other quantities. The predictions depend significantly on the mass of the top quark, m_t , and to a lesser extent on the Higgs boson mass, M_H .

The CDF group has announced [16] evidence for the t-quark, at $m_t = 175 \pm 15$ GeV, a result that we take into account where appropriate. The uncertainty in M_H can be conveniently dealt with by allowing it to vary from its experimental lower limit (~ 50 GeV) to some theoretically sensible upper limit (~ 1 TeV) and adding the effect linearly as a “theoretical error” in the prediction at hand. Comparison with measured values of the predicted quantities then results in limits on the parameters of some non-standard effect, as functions of m_t . In dealing with limits on the mass of monopoles, we follow this traditional procedure.

The most relevant predicted observables in constraining monopole masses turn out to be g_A and g_V/g_A , the vector and axial couplings of the Z to charged leptons (extracted from the Z leptonic widths and asymmetries) and M_W/M_Z . For ease of reference, I collect below the standard results (the data and analysis are as in [15], and the m_t -dependences are my own simplistic fits to the full predictions of the Standard Model, sufficiently accurate over the relevant m_t range; the 1σ

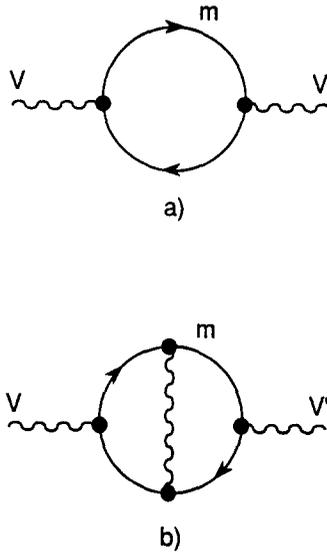


Fig. 1. (a) A one-loop virtual-monopole correction to the propagator of neutral vector bosons $V = \{\gamma, Z\}$. (b) A two-loop correction.

errors in the predictions are given in parenthesis, the theoretical Higgs-related error is labelled $[M_H]$):

$$\frac{g_V}{g_A} = 0.0753(12) + 0.00345 \left(\frac{m_t^2 - (176 \text{ GeV})^2}{M_Z^2} \pm 1.0[M_H] \right), \quad (4)$$

$$-g_A = 0.5(0) + 0.00065 \left(\frac{m_t^2 - (111 \text{ GeV})^2}{M_Z^2} \pm 0.4[M_H] \right), \quad (5)$$

$$\frac{M_W}{M_Z} = 0.8768(2) + 0.00163 \left(4.25 \frac{m_t - 100 \text{ GeV}}{M_Z} \pm 0.6[M_H] \right). \quad (6)$$

These are to be compared with the experimental results:

$$\frac{g_V}{g_A} = 0.0728(28), \quad -g_A = 0.4999(9), \quad \frac{M_W}{M_Z} = 0.8798(29). \quad (7)$$

Let $V = \{\gamma, Z\}$ represent the pair of neutral gauge bosons. Virtual monopoles contribute to the V -propagator via diagrams such as those in Fig. 1, and modify the standard predictions for the observables mentioned in the previous paragraph. As discussed in the introduction, we estimate the effect of monopoles by focusing on the pertinent lowest-dimension effective operator (induced by integrating “in” the monopole degrees of freedom) and computing its coefficient to leading order in g^2 . In the case of the V -propagator, this is equivalent to computing, to $O(m^{-2})$,

the $O(g^2)$ diagram of Fig. 1a and neglecting corrections of $O(g^2/16\pi^2)$ or higher, such as those represented by the diagram of Fig. 1b.

The results for the putative departures (denoted Δ) from the standard predictions, are

$$\Delta \frac{g_V}{g_A} = \frac{16s^4c^2}{c^2 - s^2} \frac{g^2}{4\pi^2} \frac{M_Z^2}{15m^2}, \quad (8)$$

$$\Delta g_A = \frac{s^2}{c^2} \frac{g^2}{4\pi^2} \frac{M_Z^2}{15m^2}, \quad (9)$$

$$\Delta \frac{M_W}{M_Z} = \frac{cs^2}{c^2 - s^2} \frac{g^2}{4\pi^2} \frac{M_Z^2}{15m^2}, \quad (10)$$

where $s^2 \equiv 1 - c^2 \equiv \sin^2(\theta_W) \sim 0.231$ ⁴.

The various 1σ and 3σ limits on the monopole mass m as functions of m_t are respectively displayed in Figs. 2a and 2b, for a monopole of minimum magnetic charge, $n = 1$ (since the monopole-induced corrections “ Δ ” are all quadratic in g/m , the figure can also be interpreted as the limits on m/n , for a monopole of charge $g = 2\pi n/e$, with $n \geq 1$). The combined lower limits are

$$m > n \times 1.0 \text{ TeV}, \quad \text{at } m_t = 147 \text{ GeV} \quad (1\sigma), \quad (11)$$

$$m > n \times 0.7 \text{ TeV}, \quad \text{at } m_t = 137 \text{ GeV} \quad (3\sigma), \quad (12)$$

where in the numerics I have used the value of g that corresponds to $\alpha_e(M_Z^2) \sim 1/128$. As can be seen in Fig. 2, the recent CDF result [16] on m_t modifies Eq. (11) to

$$m > n \times 1.2 \text{ TeV}, \quad \text{at } m_t = 165 \text{ GeV} \quad (1\sigma), \quad (13)$$

while the 3σ result of Eq. (12) stays put.

For our mass-degenerate dyons, the D^0 - and the D^- -loop give contributions to the various Δ 's identical to those of a singlet monopole, and the above lower mass limits are to be multiplied by $\sqrt{2}$. At first sight, the dealer in the art of setting limits on physics beyond the Standard Model may be surprised that, in spite of their strong couplings, monopoles and dyons are not more constrained than in Eqs. (11)–(13). The reason for the relative weakness of these strictures is that vector-like monopoles and quasi-degenerate dyons are very particularly guileful at avoiding all low-energy constraints.

Given our perilous use of a perturbative expansion in g , Eqs. (11)–(13) are only estimates of monopole-mass limits. But these limits are so much larger than M_Z that it is fair to conclude that *direct* searches for monopoles at LEP are unlikely to be successful⁵.

⁴ Notice that with our conventions for e and g and the corresponding currents, these corrections are a factor of two larger than the ones that would be obtained by first computing the effect of a singlet vector-like heavy lepton E^- of mass m and then substituting e for g in the result.

⁵ To a lesser extent, this grim conclusion could also have been reached from a previous analysis, along similar lines, of the monopole contributions to the anomalous magnetic moment of the muon [17], that result in a limit $m > 120 \text{ GeV}$.

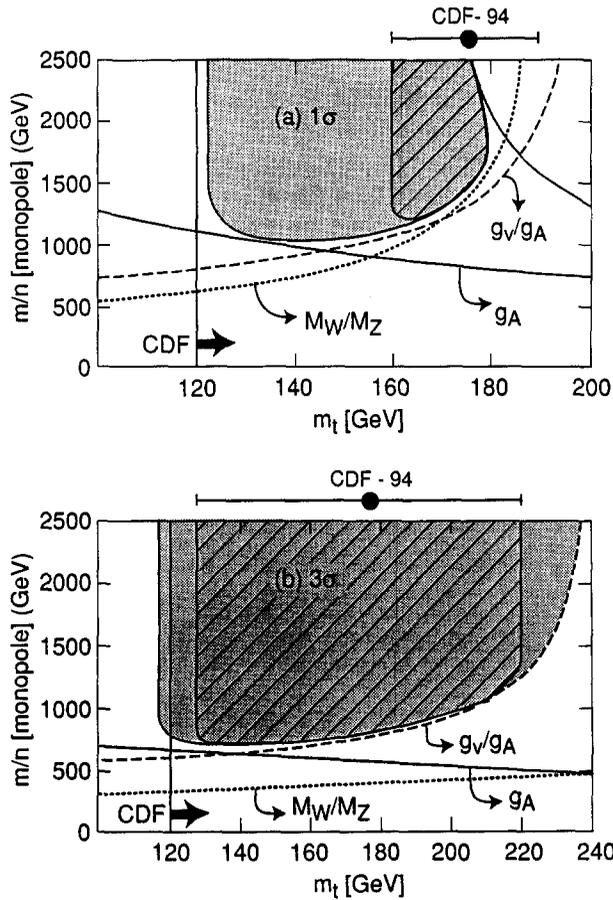


Fig. 2. Lower limits on m/n as a function of m_t , with m the monopole mass and n its charge in elementary units, (a) at the 1σ confidence level, (b) at 3σ . The shaded areas are allowed. The recently published CDF result on m_t narrows the allowed domain down to the barred regions.

4. Z-decays into multiple photons

There are ways to search for virtual-monopole effects more specific than the ones discussed in the previous section. Given the very large couplings of monopoles to Z's and γ 's, the most obvious candidate is the monopole-mediated decay of a Z into an odd number of photons, l , as depicted in Fig. 3a for $l = 3$. Because of the naturally large couplings of monopoles to the gauge bosons, it is difficult to imagine a sensible theory that, for a fixed scale of its new dynamics, could modify $Z \rightarrow l\gamma$ decays as much as monopoles would.

Even for the dominant ($l = 3$) decay, the standard $Z \rightarrow l\gamma$ transition, induced by all real and virtual charged-particle intermediate states, has a totally negligible branching ratio $\sim 2.8 \times 10^{-10}$ [19]. The main conventional source of 3γ final states

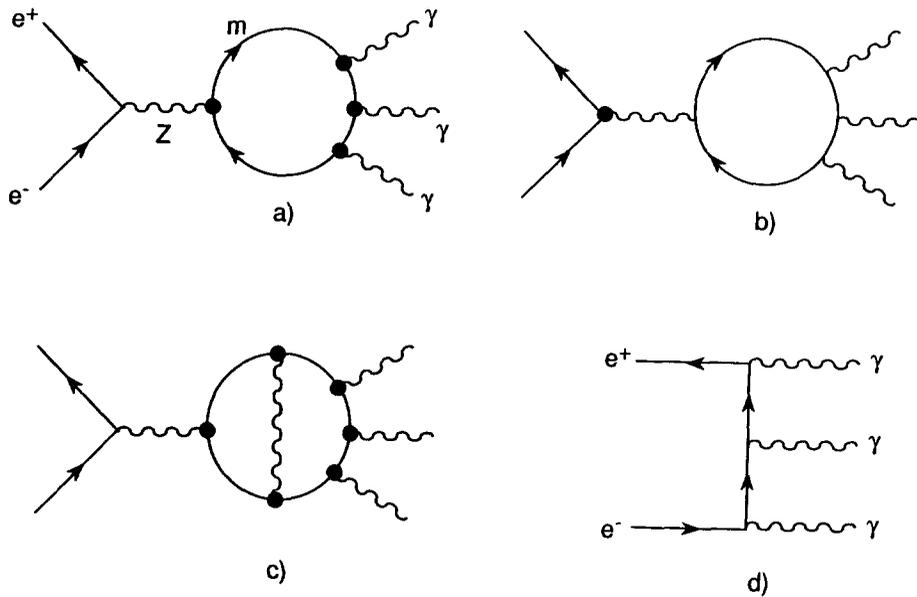


Fig. 3. Z -production in e^+e^- annihilation and monopole-induced $Z \rightarrow 3\gamma$ decay. (a) With the dual photon vertices (denoted by a heavy dot) hooked to the monopole line and the standard vertex on the electron line. (b) A dual vice versa. (c) A higher-order correction to (a). (d) The QED background.

at $\sqrt{s} = M_Z$ is the pure QED e^+e^- annihilation depicted in Fig. 3d. This process has a characteristic photon-bremsstrahlung behaviour, with its cross section peaking when one of the photons is soft, or collinear to one of the colliding particles. For photons sufficiently hard and acollinear to be observable in LEP detectors, $\sigma(e^+e^- \rightarrow 3\gamma)/\sigma_{\text{TOT}} \sim \text{a few } 10^{-6}$ at $\sqrt{s} = M_Z$ [20]. This means that for millions of produced Z 's, the QED process is marginally observable⁶.

Since the standard $e^+e^- \rightarrow 3\gamma$ process is not negligible, in studying a possible deviation from the expectations one ought to compute the interference of the QED amplitude of Fig. 3d with that for a monopole-induced transition, Fig. 3a. Prior to the discovery of a significant excess of 3γ final states (or of a departure from the QED-predicted angular and energy distributions), it appears reasonable to refrain from this laborious task. For our current purposes, it suffices to deal with the pure monopole-induced $Z \rightarrow l\gamma$ partial widths.

We estimate the virtual-monopole-mediated $Z \rightarrow l\gamma$ amplitudes by computing to leading order in g the corresponding lowest-dimension effective operators. This involves a calculation of Feynman diagrams such as that of Fig. 3a, and the neglect of higher-order effects like the one illustrated by Fig. 3c. In the figure we emphasize the distinction between the conventional Z - e vertex and the dual

⁶There are clearly other detector-related backgrounds, such as π^0 's or even jets masquerading as single photons, but these are not for a theorist to deal with.

{ γ , Z }-monopole couplings of Eqs. (2),(3) by denoting the latter with a dot. The simplest way to present the results is to trade the calculation of Fig. 3a for that of Fig. 3b, that is, to use the theory's duality to treat the electron as a monopole and the monopole as a conventional QED charged lepton, while keeping straight the powers of e and g and substituting $\bar{\psi}_e \gamma_\sigma (g_V + g_A \gamma_5) \psi_e$ for J_σ in Eq. (1).

But for a rescaling of coupling constants, we have reduced our task to the calculation of the effective interactions describing the couplings of an even number $(l + 1)$ of vector bosons, induced at the one-loop level by a massive charged spinor field to which they couple as in QED. This is precisely the definition of the Euler–Heisenberg lagrangian [18] describing light-by-light scattering (and its generalizations) at scales below m_e . The effective interactions relevant to monopole-induced $Z \rightarrow 3\gamma$ and $Z \rightarrow 5\gamma$ transitions are

$$\mathcal{L}_3 = \frac{\alpha_g^2}{360m^4} \frac{s}{c} \left[4(F \cdot F)^2 + 7(F \cdot \tilde{F})^2 \right], \tag{14}$$

$$\mathcal{L}_5 = \frac{\pi \alpha_g^3}{22680m^8} \frac{s}{c} \left[8(F \cdot F)^3 + 13F \cdot F (F \cdot \tilde{F})^2 \right], \tag{15}$$

where $F \cdot F = F^{\mu\nu} F_{\mu\nu}$, to be properly symmetrized over all photons. Notice the factor $s/c = \tan \theta_W$ arising from the substitution of one photon by a vectorially-coupled Z in the conventional multiphoton effective lagrangian. Given these results, it is only a matter of some toil to compute the rates and the energy and angular distributions of the final-state photons.

It is convenient to discuss first the total cross sections $\sigma(e^+e^- \rightarrow Z \rightarrow l\gamma)$, and to re-express them as Z partial widths. For $l = 3$:

$$\Gamma(Z \rightarrow 3\gamma) = M_Z \frac{17\pi s^2}{972000c^2} \left(\frac{g^2}{4\pi^2} \right)^4 \left(\frac{M_Z}{m} \right)^8, \tag{16}$$

while, for $l = 5$ ⁷:

$$\Gamma(Z \rightarrow 5\gamma) = M_Z \frac{\pi s^2}{2^{25} 3^{11} 5^2 7^2 c^2} \left(\frac{g^2}{4\pi^2} \right)^6 \left(\frac{M_Z}{m} \right)^{16} (3.92 \times 10^{-4}). \tag{17}$$

Notice that the RHS of Eq. (16) is a function of m/n , with n the monopole charge in elementary units, and recall that the other monopole-induced effects of Eqs. (8)–(10) also depend on m/n . Thus, in a search for virtual-monopole signatures, the relative merits of the indirect limits of Section 3 and of the more specific $Z \rightarrow 3\gamma$ rate are independent of n .

⁷ Here an overall factor $1/2^{30} 3^3 \pi^7$ is phase space, the result for an $\mathcal{L}_5 = \phi^6/M_Z^2$ coupling between distinguishable scalars. The quantity in curly brackets is an estimate (of better than 1% precision) of the average over the eight-dimensional phase space of the normalized matrix element squared, i.e. the result of substituting the six F 's within the square brackets of Eq. (15) by all permutations of $F_1 \dots F_6$, adding, dividing by $6! M_Z^6$, squaring, and summing over polarizations; the smallness of this number reflects the derivative nature of the coupling to the five photons that share the available energy.

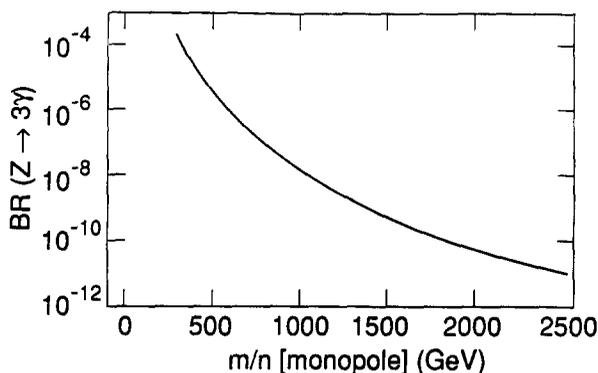


Fig. 4. Branching ratio for monopole-induced $Z \rightarrow 3\gamma$ decay, as a function of m/n , in the notation of Fig. 2.

Fig. 4 shows Eq. (16) converted into a branching ratio with use of $\Gamma_Z^{\text{TOT}} = 2.487$ GeV, and plotted as a function of m/n . An anomaly in this branching ratio of $O(10^{-6})$ is currently observable at LEP-I, and corresponds to $m = n \times 0.60$ TeV. This mass value is close to the limit of Eq. (12) and, given the uncertainties inherent in a perturbative expansion in g , it is fair to conclude that the search for anomalies in $Z \rightarrow 3\gamma$ transitions should be pursued with vigour. Consequently, I give more details on these decays.

Let $x_i \equiv 2E_i/M_Z$ be the thrust of the individual photons, with $0 \leq x_i \leq 1$ and $\sum_i x_i = 2$. The Dalitz plot density is

$$\frac{d\Gamma}{\Gamma dx_1 dx_2 dx_3} = \frac{5}{34} \left(139 \sum_i x_i^2 (1-x_i)^2 - 18 \prod_i (1-x_i) \right) \delta\left(\sum_i x_i - 2\right). \quad (18)$$

The single-photon energy distribution is

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx} = \frac{x^3(3430 - 6210x + 2919x^2)}{102}. \quad (19)$$

The photons' angular distribution relative to the beam axis (three entries per event) is

$$\frac{d\sigma}{d \cos \theta} \propto 1 - \frac{77}{479} \cos^2 \theta. \quad (20)$$

The angular distribution of the most energetic photon relative to the beam direction is

$$\frac{d\sigma}{d \cos \theta} \propto 1 - 0.1762 \cos^2 \theta. \quad (21)$$

The distribution in the angle between the normal to the three-photon plane and the beam axis is

$$\frac{d\sigma}{d \cos \alpha} \propto 1 + \frac{163}{671} \cos^2 \alpha. \quad (22)$$

All of these angular distributions are fairly uniform.

For a monopolar mass above the limits discussed in Section 3, the ratio of five- to three-photon decay widths of the Z , as obtained from Eqs. (16),(17), is a tiny number; the enhancement due to the extra powers of g is compensated by phase-space and derivative-coupling factors. The three-photon decay of the Z constitutes the best hope for a specific monopole-induced signature.

5. A digression on monopolium

Given the strength of the coupling of monopoles to Z 's and γ 's (and of dyons to W 's), *monopolia*, tightly bound states of a monopole and its antiparticle, should exist. Are they the signatures to look for in e^+e^- annihilation? Do they affect our previous considerations? I argue that both questions ought to be answered negatively.

The monopolium bound state is not tractable by analogy with positronium, or even charmonium. With a “fine-structure” constant $\alpha_g > 1$, one expects the constituent monopoles to be tightly bound and highly non-relativistic.

Various authors [21] have computed monopolium masses by “softening” the potential with the introduction of a non-vanishing extension of the monopoles, in ways premonitory of the properties of topological monopoles. They obtain mass estimates for the ground-state monopolium some 200 times (!) lighter than the sum of masses of the constituents. While these results are to be taken *cum grano salis*, they convey the presumably correct feeling that monopolium states should appear, in e^+e^- annihilations, well below the “open monopole” threshold. Why are monopolia not the objects to hunt for?

Another calculation that cannot be confidently performed in analogy with positronium is that of the lifetime of a monopolium state against its decay into vector bosons. Recall that, for the 3γ decay of the ($J^P = 1^-$) orthopositronium ground state:

$$\Gamma = 2m_e \times \frac{\pi^2 - 9}{9\pi} \alpha_e^6. \quad (23)$$

Upon substitution of α_e for α_g one obtains a forcible hint of a foretold conclusion: monopolia ought to have a width much larger than their mass. If so, they are not prominent peaks.

I expect the collective effect of wide monopolia to be describable at the “partonic” level by the monopole–antimonopole loops we have dealt with, in analogy with the QCD description of the energy-smearred cross section for e^+e^-

annihilation into hadrons [22]. This completes the argument whereby monopoles ought to be irrelevant in practice.

6. W-pair production in e^+e^- collisions

Dyons have a potential incidence on the ZW^+W^- and γW^+W^- Triple Gauge-Boson Vertices (TGVs) to be thoroughly studied at LEP-II. To discuss these effects, it may be useful to revamp one's souvenir of some known properties of the Standard TGVs.

In the Standard Model, the tree-level TGVs are described by a lagrangian

$$\mathcal{L}_{\text{SM}} = -i\frac{e}{s} \left[\kappa_e W_3^\nu (W_{\mu\nu}^\dagger W^\mu - W_{\mu\nu} W^{\dagger\mu}) + \kappa_m W_3^{\mu\nu} W_\mu^\dagger W_\nu \right], \quad (24)$$

$$W_3^\mu \equiv sA^\mu + cZ^\mu, \quad (25)$$

with W_3^μ the electrically neutral isovector potential, and $W_i^{\mu\nu} \equiv \partial^\mu W_i^\nu - \partial^\nu W_i^\mu$. In Eq. (24), $\kappa_e = \kappa_m = 1$ are the non-anomalous “electric” and “magnetic” couplings of W 's to γ and Z . Radiative effects modify the coefficients κ_e , κ_m of the (dimension $d = 4$) couplings of \mathcal{L}_{SM} ; the dispersive parts of these corrections are akin to the charge form factor and magnetic anomaly ($g - 2$) of an elementary fermion.

Also in analogy with $g - 2$, vertex corrections generate TGVs that are describable by effective interactions not appearing in \mathcal{L}_{SM} . The lowest dimension ($d = 6$) of these TGVs is

$$\mathcal{O}_W \equiv W_3^{\mu\nu} W_{\nu\rho}^\dagger W_\mu^\rho. \quad (26)$$

The corresponding effective lagrangian is $\mathcal{L}_W = a\mathcal{O}_W$, with a of order $e\alpha/\pi M^2$ and M representative of the mass of virtual particles. Effective operators such as \mathcal{O}_W are particularly useful in discussing physics [23,24] beyond the Standard Model.

As an example of non-standard effects, take a vector-like doublet of conventional leptons $(E^0, E^-)_{L,R}$ of degenerate mass $M > M_Z/2$. This appendage affects the couplings of Eq. (24), entailing modifications that (after renormalization) are of the form $\Delta\kappa_e$, $\Delta\kappa_m \propto e^3 q^2/M^2$. The vertex correction illustrated in Fig. 5a also induces effects that, to leading order in $1/M$, are described by the addition to \mathcal{L}_{SM} of the effective interaction:

$$\mathcal{L}_E = -\frac{i}{240\pi^2 M^2} \left(\frac{e}{s}\right)^3 \mathcal{O}_W. \quad (27)$$

Non-standard radiative effects may also induce CP -violating TGVs not included in \mathcal{L}_{SM} , two of which, of dimension $d = 6$, have been discussed in the literature ⁸,

⁸ The low-energy limits of couplings of the form $\tilde{\mathcal{O}}_k$ and $\tilde{\mathcal{O}}_W$ describe electric-dipole and magnetic-quadrupole moments of the W 's, and their electroweak generalizations. Their standard values are of very high order of perturbation theory, and negligible in practice.

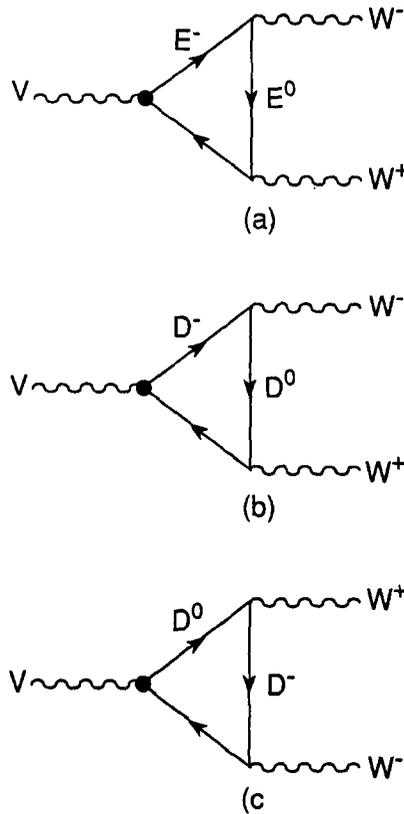


Fig. 5. Corrections to the VW^+W^- vertex, with $V = \{\gamma, Z\}$. (a) A lepton loop. (b),(c) Two dyon loops. Only the heavy dots are magnetic couplings.

$\tilde{\mathcal{O}}_\kappa \equiv \tilde{W}_3^{\mu\nu} W_\mu^\dagger W_\nu$, the dual sibling of the Standard “magnetic” coupling of Eq. (24), and $\tilde{\mathcal{O}}_W \equiv \tilde{W}_3^{\mu\nu} W_\nu^\dagger W_\mu^\rho$, the dual partner of \mathcal{O}_W . A third possibility, a dual counterpart of the “electric” (κ_e) coupling of Eq. (24), has not been discussed; and this for a good reason: its coefficient at $q^2 = 0$ would be the magnetic-monopole charge of the W .

Consider, at long last, the $O(ge^2)$ vertex corrections of Figs. 5b,c, induced by our hypothetical dyon doublet $(D^0, D^-)_{L,R}$. We are interested in the lowest-dimension TGVs characterizing this radiative effect or, more precisely, in the interference between the standard $e^+e^- \rightarrow W^+W^-$ amplitude and the amplitude induced by this correction. For mass-degenerate dyons, the amplitudes of Figs. 5b,c add to zero, since D^- and \bar{D}^0 have opposite magnetic charges (the “Furry theorem” stating the vanishing of this TGV is in fact a consequence of the magnetic analogue of charge-conjugation symmetry, and applies to all orders of perturbation theory). To explore a possible non-vanishing effect, we momentarily lift our restriction to mass-degenerate dyons.

We saw in Section 2 that the dyon Feynman rules are not without blemish and, indeed, their use to compute the amplitudes of Figs. 5b,c leads to troubles identical to the ones we discussed there. To get out of this *cul de sac*, I simply conjecture that a consistent set of computational rules for dyons would result in an effective interaction analogous to the unequal-mass generalization of Eq. (27) but for the trading of \mathcal{O}_W for its dual, $\bar{\mathcal{O}}_W$, and the substitutions of electric for magnetic coupling constants. In support of this conjecture, recall that there is no difficulty in “taking the dual” of an interaction involving \mathbf{E} and \mathbf{B} fields rather than the vector potentials, essentially all we suggest is to make the replacement $W_{\mu\nu} \rightarrow \bar{W}_{\mu\nu}$ in Eqs. (26),(27) while substituting leptons for dyons⁹.

Let the split dyon masses be $m_-^2 = m^2 + \mu^2$, $m_0^2 = m^2 - \mu^2$. Following our conjecture and recalling that $eg = 2\pi n$, we may compute the dyon-induced TGV, for small μ^2/m^2 , as

$$\mathcal{L}_D \approx \frac{ien}{720\pi cs^2} \frac{\mu^2}{m^4} \bar{B}^{\mu\nu} W_{\nu\rho}^\dagger W_\mu^\rho, \quad (28)$$

with $B^{\mu\nu}$ the derivative fields of the $U(1)_Y$ potential of Eq. (3). Behold! the operator in Eq. (28) is not of the $\epsilon^{ijk} W_i W_j W_k$ form of Eqs. (24),(26); sacrosanct on grounds of the gauge symmetry. The reason is that this operator is of dimension $d = 8$, in spite of its $d = 6$ disguise. The mass m , common to both (vector-like) dyons can be directly ascribed to an invariant term in the lagrangian. The mass μ , which splits them, must be proportional to a term such as $\bar{D}M^0\langle\Phi\rangle$, and the Higgs field Φ accounts for the extra field-dimensions.

The observability at LEP-II of “ T -odd” effects¹⁰ induced by a CP -violating interaction such as Eq. (28) has been carefully investigated [25]. Even for the most optimistic case (μ comparable to m , $m \sim 1$ TeV) the coefficient in Eq. (28) is three orders of magnitude below the level required to observe a 1σ effect in the combined results of four LEP-II experiments, each stockpiling some 10^4 W-pairs [25]. And, to make matters worse, the lower limits on the mass of non-degenerate dyons are much stronger than the limits we discussed in Section 3, for the same reason (a breaking of the “custodial” $SU(2)$ symmetry) that a split (t, b) quark pair contributes so significantly to radiative corrections.

The moral of this section is that there appears to be no hope for LEP-II, concerning monopoles and/or dyons, to improve on the exclusion or discovery capabilities of LEP-I. This is an example of a general result regarding the measurement of triple gauge-boson vertices [24]. I have not explored the potential of higher-energy e^+e^- colliders.

⁹ Dyons also induce effects proportional to $\bar{\mathcal{O}}_\kappa$, and a non-vanishing “magnetic-charge radius” of the W. We do not dwell on these effects, which are of the same order of magnitude as the ones we do discuss, and do not modify the conclusions.

¹⁰ An observable is T -odd if it changes sign as the spin and momenta of all particles are reversed.

7. Monopoles that are not point-like

The very heavy composite monopoles that arise in Grand-Unified Theories are certainly impossible to produce in current terrestrial laboratories. It is conceivable, however, that much lighter yet non-point-like monopoles exist, either for reasons that we have not at all fathomed, or because something like a grand unification actually takes place at an unsuspectedly low energy. It is therefore interesting to ponder what the low-energy signatures of composite monopoles might be.

The grand-unified monopoles are classical or semi-classical solutions to a gauge theory. They do not correspond to a field appearing in the lagrangian and, strictly speaking, they do not exist as virtual particles to be included in a quantum loop expansion, wherein only gauge bosons and other point-like particles should play a role. Analogously, only quarks and gluons, and perhaps their condensates, but not hadrons, ought to be used in pure-QCD perturbation theory (though useful alternative schemes do exist, such as the chiral lagrangians).

Recall the foregone times when the quark model and QCD were not established. Imagine that the anomalous magnetic moment of muons and electrons had been measured, by then, to a sufficient precision for the hadronic contribution to the photon's vacuum polarization, $\Pi(q^2)$, to play a significant role. To have the theory agree with experiment one would need to estimate this contribution. The use of a dispersion relation for $\Pi(q^2)$, and of $\pi^+\pi^-$ intermediate states with a form factor, $F_\pi(q^2)$, describing the ρ -resonance, would result in an answer of the correct magnitude, even though one would have violated the dictum of the last paragraph: thou shalt not use but fundamental particles in a quantum correction¹¹.

For the rest of this section, I imitate the procedure of the previous paragraph, trading pions for extended monopoles. I also trade a pion-like form factor for a proton-like one, since the monopole threshold should be above the monopolium states, like the $p\bar{p}$ threshold is above the ρ -mass. For the conventionally defined electric and magnetic form factors at threshold ($q^2 = 4m^2$) I set a normalization $G_e = G_m \sim 0.5$, the measured value for protons. Above threshold I let the form factors decrease as in a dipole fit, with a slope that corresponds to a mean square radius $\langle r^2 \rangle = (\bar{\alpha}m)^{-2}$, with $\bar{\alpha} = 1/40$.

With the use of conventional dispersion-relation techniques, one can redo our previous calculations of monopole-mass limits and the $Z \rightarrow 3\gamma$ width. The mass limits of Eqs. (11)–(13) are weakened by a factor ~ 3 , and the 3γ branching ratio of Fig. 4 now reaches the interesting level of 1 ppm also for monopoles that are ~ 3 times lighter than the point-like ones. These masses and mass limits are very low, some 200 GeV. Consistency of the extended-monopole weak-coupling picture demands that the monopole-constituent gauge fields be much lighter than the monopole. For monopoles close to their lower mass limit, the novel gauge fields should already have been found!

¹¹ I am indebted to Andy Cohen for insisting on this commandment, and on its exemptions.

The moral of this section's bantam exercise is not unexpected [5]. Not only are composite monopoles particularly difficult to produce, but it should be much easier to find their "constituents" than any of their indirect signatures.

8. Cosmological considerations

Fossil remnants of any stable particle species may survive from the time when our Universe was hot enough to sustain a thermal population of its specimens, and monopoles are no exception. In a monopole–antimonopole symmetric Universe, conventional Big-Bang theory fixes the present relic monopole (and antimonopole) abundance in terms of the monopole mass and $\sigma \cdot v$, the velocity-weighted \overline{MM} annihilation cross section¹². To an admittedly suspicious lowest order in α_g , the total $\sigma \cdot v$ for annihilation into ZZ , $Z\gamma$ and $\gamma\gamma$ is

$$\sigma \cdot v \simeq \frac{4\pi\alpha_g^2}{m^2c^4}, \quad (29)$$

where we have used $m \gg M_Z$, to be justified a posteriori. This result also applies to D^+D^- or $D^0\overline{D}^0$ (dyon) annihilation, since the additional annihilation of dyons into W -pairs proceeds with a conventional electroweak strength, and may be neglected.

Let g and g_{SM} be the number of degrees of freedom of a spin-1/2 monopole and of the complete Standard-Model zoo above the weak scale ($g = 4$, $g_{\text{SM}} = 427/4$). Let $\Omega = \rho/\rho_c$ be the contribution of monopoles to the current universal energy density, in current critical units $\rho_c \sim 2h^2 \times 10^{-29}$ g/cm³, with the Hubble-constant controversy still raging between $h \sim \frac{1}{2}$ and $h \sim 1$. For "cold relics" such as monopoles, a conventional calculation [27] results in the following estimates of the decoupling temperature T_d , and of Ω :

$$\frac{T_d}{m} \simeq (A - \frac{1}{2}\ln(A))^{-1} \quad (A \equiv 0.038 g g_{\text{SM}}^{-1/2} M_P m \sigma \cdot v), \quad (30)$$

$$\Omega h^2 \simeq \frac{1.1 \times 10^{12}}{1 \text{ TeV}} \frac{m}{T_d} \frac{1}{\sqrt{g_{\text{SM}}} M_P \sigma \cdot v}, \quad (31)$$

with M_P the Planck mass.

Eqs. (29)–(31) imply that, for monopoles or dyons to avert "overclosing" the Universe (contributing an $\Omega > 1$), their mass must obey the restrictions

$$m < 8.7 nh \text{ PeV} \quad (\text{monopoles}), \quad (32)$$

$$m < 6.1 nh \text{ PeV} \quad (\text{dyons}). \quad (33)$$

¹² An excessive abundance of topological monopoles was one of the original motivations for inflationary models [26]. We are exclusively dealing in this section with point-like monopoles and assuming inflation to have taken place at a temperature above the monopole mass.

A primordial monopole–antimonopole asymmetry would make the mass limits more stringent. Recall Eqs. (11), (12) (and that $1 \text{ PeV} = 10^3 \text{ TeV}$) to conclude that there is room for point-like monopoles only in a mass interval of some four to five octaves.

9. Conclusions

Magnetic monopoles are some of the most interesting particles that we have not found.

The “official” non-elementary topological monopoles are too heavy to be made in the laboratory. Unsuspectedly light extensive monopoles are also difficult to produce, much as a nucleus of anti-helium, due to a severe “form-factor” suppression. Neither are their indirect signatures, as we have discussed, a good way to look for them.

The virtual effects of point-like monopoles and dyons are observable in experiments conducted at energies below their production threshold. We have exploited the current data to set lower limits of the order of $1/2$ of a TeV on the masses of monopoles and degenerate dyons, and the standard Big-Bang cosmology to set upper limits of the order of 5000 TeV.

In the allowed mass interval, an unsuspectedly large $Z \rightarrow \gamma\gamma\gamma$ branching ratio would be the most specific currently accessible signature for the existence of magnetic poles.

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