

## Monopolium: the key to monopoles

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### Abstract

Dirac showed that the existence of one magnetic pole in the universe could offer an explanation for the discrete nature of the electric charge. Magnetic poles appear naturally in most Grand Unified Theories. Their discovery would be of greatest importance for particle physics and cosmology. The intense experimental search carried thus far has not met with success. A way out of this impasse could be that the monopoles are dynamically confined forming monopolium, a monopole- anti-monopole bound state. Even if the poles are very massive, the binding could be so strong, that monopolium could have a relatively small mass. We study in this scenario the feasibility of detecting monopolium in present and future accelerators.

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# 1 Introduction

The theoretical justification for the existence of classical magnetic poles, hereafter called monopoles, is that they add symmetry to Maxwell's equations and explain charge quantization [1, 2]. Dirac formulated his theory of monopoles considering them basically point like particles and quantum mechanical consistency conditions lead to the so called Dirac Quantization Condition (DQC),

$$\frac{eg}{\hbar c} = \frac{N}{2}, \quad N = 1, 2, \dots, \quad (1)$$

where  $e$  is the electron charge and  $g$  the monopole magnetic charge. In this theory the monopole mass,  $m$ , is a parameter, limited only by classical reasonings to be  $m > 2$  GeV [3]. In non-Abelian gauge theories monopoles arise as topologically stable solutions through spontaneous breaking via the Kibble mechanism [4]. They are allowed by most Grand Unified Theory (GUT) models, have finite size and come out extremely massive  $m > 10^{16}$  GeV. There are also models based on other mechanisms with masses between those two extremes [3, 5, 6].

All the attempts to discover monopoles have met with failure. [3, 5, 7, 8, 9]. This lack of experimental confirmation has led many physicist to abandon the hope in their existence. A way out of this impasse is the old idea of Dirac [1, 10], namely, monopoles are not seen freely because they are confined by their strong magnetic forces. Recently this idea has been reformulated in a scenario which incorporates standard model unification [11]. Monopoles and anti-monopoles bind immediately after creation forming monopolium [12]. This state is not stable since, once the orbital motion of the poles is strongly overlapping, it annihilates into conventional elementary particles. By studying this annihilation we might get a chance to establish the existence of monopoles.

This phenomenon is not a novel feature of physics. Quark-gluon confinement describes the strong limit of Quantum Chromodynamics, the theory of the hadronic interactions and their existence is proven by the detection of jets, showers of conventional hadrons. There is however a main difference between the two scenarios. In the monopolium case, the elementary constituents may be separated asymptotically, when they are orbiting far from each other, if the energy provided to the system is high enough, while in the quark-gluon case this is not possible. In practice, however, there is no big difference, since due to the high binding energies of monopolium, asymptotic monopoles might only be found, for short periods of time, in the center of galaxies, or clusters of galaxies.

The aim of our work is to analyze the possibility to observe monopole physics in accelerators, therefore at relatively small energies, by cosmological standards, by studying the formation of a low mass monopolium as an intermediate state dominating subsequent decays.

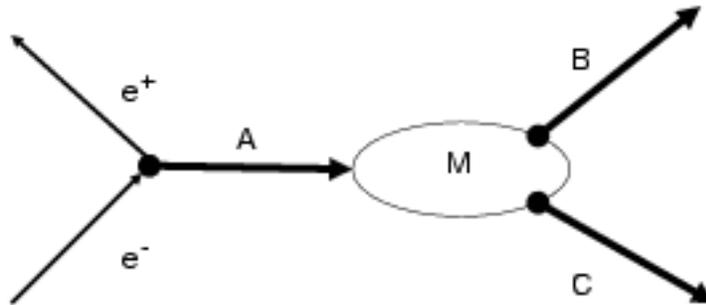


Figure 1: Diagrammatic description of the reactions studied.

## 2 Monopolium detection

We proceed to discuss signatures of monopolium, the monopole-anti-monopole bound state, when produced in  $e^+e^-$  annihilation<sup>1</sup>. We use to describe the interaction the low energy effective theory of Ginzburg and Schiller [13]. This theory is based on the standard electroweak theory and in order to couple the monopoles to the photon and weak bosons one considers that  $m \gg m_{Z_0}$  and that the monopole interacts with the fundamental fields of the  $SU(2) \otimes U(1)$  theory before symmetry breaking, i.e., with the isoscalar field  $B$ , in the conventional notation of the standard model [14]. In this way  $\gamma$  and  $Z_0$  have the same coupling except for an additional  $\tan \theta_W$ , where  $\theta_W$  is the Weinberg angle, for the latter. The effective description is based on the one loop approximation of the fundamental theory and therefore the coupling becomes

$$g_{eff} \sim \frac{g}{m}, \quad (2)$$

below the monopole production threshold, thus rendering the theory perturbative.

The process under consideration is (see Fig. 1)

$$e^+e^- \rightarrow A \rightarrow M \rightarrow B + C$$

where A,B,C are  $\gamma$ 's or  $Z_0$ 's, in all allowed combinations and  $M$  represents the monopolium state.

The dynamical scheme proposed by Ginzburg and Schiller leads to effective couplings in a vector like theory between the monopole and the photon [13], given by

$$g_{eff}^\gamma = C(J_m) g \frac{\omega}{m} = C(J_m) \frac{\hbar c}{2e} \frac{\omega}{m} N \quad (3)$$

with  $C(J_m) \sim 1$ ,  $\omega$  the photon energy,  $m$  the monopole mass and  $N$  the monopole charge. The effective interaction between the monopole and the  $Z_0$  becomes

$$g_{eff}^Z = \tan(\theta_W) g_{eff}^\gamma \quad (4)$$

<sup>1</sup>The description in terms of quark-anti-quark annihilation is straightforward although complicated by the partonic description of the real experimental probes which are hadrons.

where  $\theta_W$  is the Weinberg angle and naturally here  $\omega$  refers to the  $Z_0$  energy. We have used the Dirac quantization condition Eq.(1) to express the coupling in terms of the electron charge.

The standard expression for the cross section in these cases results in

$$\sigma = \frac{\pi}{E_e^2} G \frac{4 M^2 \Gamma_{ee} \Gamma_{BC}}{(E^2 - M^2) + M^2 \Gamma_M^2} \quad (5)$$

Here  $M$  stands for the monopodium mass and  $G$  is defined by

$$G = \frac{2 J_M + 1}{(2 s_e + 1) \sqrt{(2 s_B + 1) (2 s_C + 1)}}$$

The interesting physical situation occurs when  $M \ll m$  and consequently, from (3) and the fact that  $\omega \sim M$ , one gets  $g_{eff} \sim (M/m) \ll 1$ , which grants validity to the perturbative approach.

We enter now the computation of the widths  $\Gamma$ . Let us start with

$$\Gamma_{ee} = 2 \pi |M|^2 \rho(E_e)$$

where

$$\rho(E_e) = \frac{4 \pi E_e^2}{(2 \pi)^3}$$

and

$$|M|^2 = \left| \langle e^+ e^- | \frac{1}{(q^2 - m_A^2)} | M \rangle \right|^2 = 4 \pi \alpha g_A^2 \frac{1}{(q^2 - m_A^2)^2} |\psi_M(0)|^2$$

Here  $q = (2 E_e, \vec{0})$ . One obtains,

$$\Gamma_{ee} = 4 \alpha g_A^2 \frac{E_e}{(4 E_e^2 - m_A^2)^2} |\psi_M(0)|^2. \quad (6)$$

We now proceed to calculate

$$\Gamma_{BC} = 2 \pi |M|^2 \rho(E_{BC})$$

Using the standard result [15] we obtain

$$\Gamma_{BC} = \frac{8 \pi \alpha_B \alpha_C}{m^2} |\psi_M(0)|^2 \quad (7)$$

where the approximation  $m \gg m_B, m_C$  has been used.

The cross section therefore becomes

$$\sigma = \frac{2 G}{(16)^3 \pi^4} \frac{(\tan \theta_W)^{2N Z_0}}{\alpha^2} \frac{M^2 c^4}{m^8 c^{16}} (\hbar c)^8 |\psi_M(0)|^4 \frac{E_B^2 E_C^2 (2 E_e)^2}{[(4 E_e^2 - m_A^2 c^4)^2 + m_A^2 \Gamma_A^2 c^4] [(4 E_e^2 - M^2 c^4)^2 + M^2 \Gamma_M^2 c^4]} \quad (8)$$

where  $N Z_0$  indicates the number of  $Z_0$  present among  $A, B$  and  $C$ .

In order to go ahead with the calculation, one has to obtain the wave function corresponding to monopodium. This is done and analyzed in the next sections.

### 3 Monopolium Potential

We restrict our calculation to the lowest charge monopole, i.e.  $N = 1$  in the Dirac condition Eq.(1). We regard the monopole as possessing some spatial extension in line with the arguments of Schiff and Goebel [16, 17]. This assumption makes the potential energy of the monopole-anti-monopole interaction non-singular when the relative separation goes to zero. Mathematically we describe this feature by means of an exponential cut-off in the interaction potential,

$$V(r) = -g^2 \left( \frac{1 - \exp(-\mu r/\hbar c)}{r} \right). \quad (9)$$

Our notations provide  $g^2$  with dimensions of  $[\hbar c]$ .

We fix the cut-off parameter  $\mu$  by physical arguments. Eq.(9) has the following properties,

i)  $r \rightarrow \infty$

$$V(r) \rightarrow -\frac{g^2}{r} \quad (10)$$

ii)  $r \rightarrow 0$

$$V(r) \rightarrow -g^2 \frac{\mu}{\hbar c} + \dots \quad (11)$$

When the monopole-anti-monopole are closest to each other the distance between the corresponding centers  $O$  and  $O'$  is

$$r_{OO'} = 2r_m \quad (12)$$

where  $r_m$  is the pole radius. Consequently, the potential energy at the "contact" region  $C$ , i.e. the most attractive possible potential energy, is

$$V_C \simeq -\frac{2g^2}{r_m}.$$

Let us represent this fact by choosing  $\mu$  in Eq.(9) such that

$$\frac{\mu}{\hbar c} = \frac{2}{r_m},$$

an approximation valid since in general  $r_m$  is small. Moreover, for our purposes the following choice

$$r_m = r_{classical}$$

seems suitable, and therefore

$$\mu = 2 \frac{mc^2}{g^2/\hbar c}. \quad (13)$$

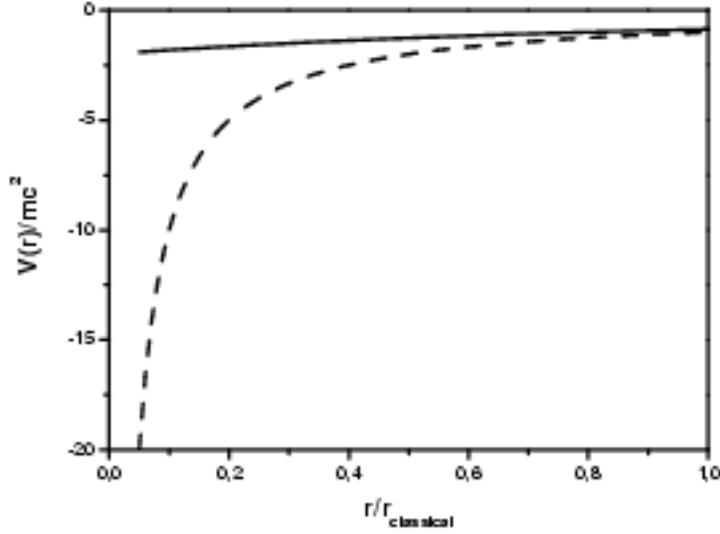


Figure 2: Comparison between the Coulomb potential and the Coulomb with exponential cut-off potential.

Consequently, the effective potential finally becomes

$$V(r) = -g^2 \frac{1 - \exp\left(-2 \frac{r}{r_{classical}}\right)}{r}. \quad (14)$$

Note that with our choice, for  $r \rightarrow 0$ ,  $V(r) \rightarrow -2mc^2$ . Thus, the mass of the bound state becomes the energy over the minimum

$$M c^2 = 2mc^2 + E_{binding}. \quad (15)$$

Summarizing, our analysis shows that the cut-off potential is quite close to the Coulomb potential as long as the monopole radius,  $r_m$  is greater than the classical monopole radius  $r_{classical}$ . Thus, we shall use the "magnetic" Coulomb potential (Fig. 2) as our interaction in what follows.

Solving the non-relativistic Schrödinger equation for monopolum we obtain its mass [18]

$$M c^2 = 2 m c^2 - \left(\frac{1}{8\alpha}\right)^2 \frac{m c^2}{n^2}. \quad (16)$$

where  $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$  and  $n$  is the principal quantum number. We see that we can reach zero mass for  $n \sim 12$  and therefore for  $n > 12$  the formula is well defined and describes all values of  $M$

$$0 \leq M \leq 2m$$

The monopolum radius is given by

$$\frac{r_M}{r_{classical}} = 48\alpha^2 n^2. \quad (17)$$

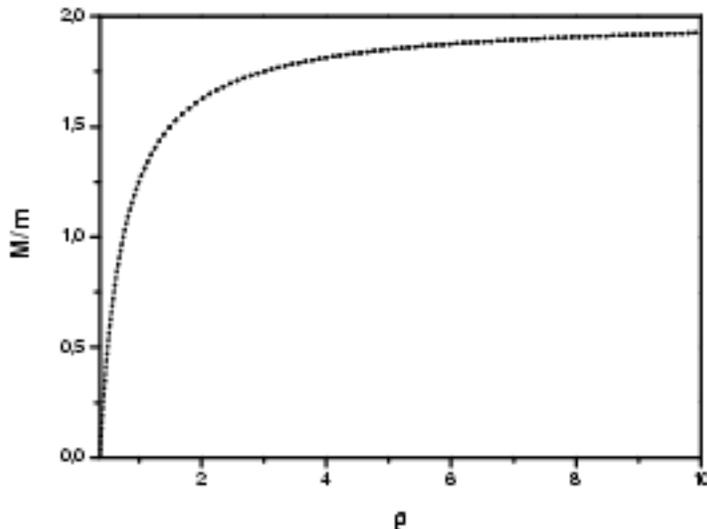


Figure 3: Mass of the monopolum as a function of the size parameter.

Now we introduce the size parameter be  $\rho = \frac{r_M}{r_{classical}}$ .

By substituting  $n^2$  from Eq.(17) into Eq.(16), we obtain an equation for the monopolum mass as a function of its size, namely

$$Mc^2 = mc^2(2 - \frac{3}{4\rho}), \quad (18)$$

which is plotted in Fig.(3). Although for low values of  $\rho$  our approximation becomes worse, we expect, that the soft behavior of the wave function at the origin, allows for order of magnitude estimates.

Before we continue, a discussion on the validity of the non-relativistic approximation is convenient. Let us define the relativistic factor  $\beta = v/c$  through the equation

$$\beta^2 = \langle n l m_l m_s | \frac{p^2}{m^2 c^2} | n l m_l m_s \rangle. \quad (19)$$

It can be easily calculated using the exact expectation value to give [18],

$$\beta = \sqrt{\frac{3}{4\rho}}. \quad (20)$$

This result, which coincides with the semiclassical treatment and the use of Ehrenfest's theorems, namely equating the centrifugal and Coulomb forces

$$m \frac{v^2}{r} = \frac{e^2}{r^2} \Rightarrow \frac{p^2}{2m} = \frac{1}{2} \frac{e^2}{r}$$

leads to

$$E = \frac{p^2}{2m} - \frac{e^2}{r} = \frac{p^2}{2m} - \frac{p^2}{m} = -\frac{p^2}{2m}.$$

This corresponds to equating the absolute values of the kinetic and the binding energies,

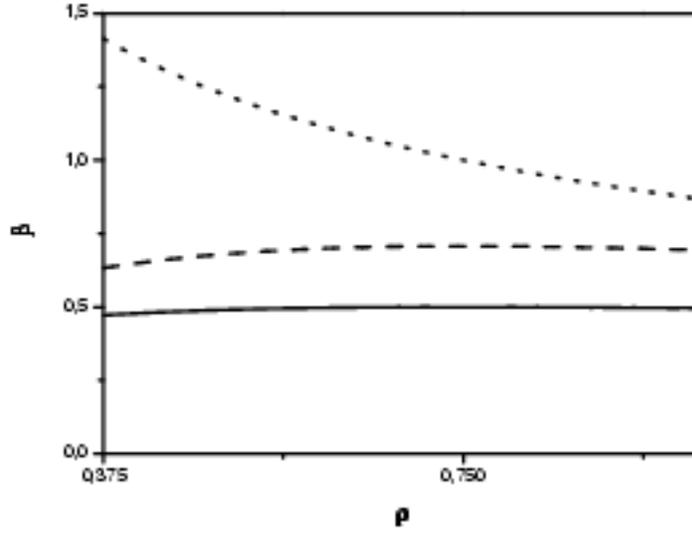


Figure 4: Different ways of calculating the  $\beta$ : non relativistic (dotted), relativistic bosonic (dashed) and relativistic fermionic (full).

$$Kinetic\ Energy = |Binding\ Energy|, \quad (21)$$

which leads to

$$\frac{p^2}{m} = \left(\frac{1}{8\alpha}\right)^2 \frac{m c^2}{n^2},$$

that gives rise, using Eq.(17), to

$$\frac{p^2 c^2}{m^2 c^4} = \frac{3}{4} \frac{1}{\rho}.$$

Thus, the non-relativistic calculation is only truly valid for  $\rho \gg 3/4$ , i.e.  $M \gg m$ .

Let us perform, however, a relativistic calculation for  $\beta$  defined as  $\beta = \frac{pc}{E_{Total}}$  and use the two conventional forms for the total energy

$$i) E_{Total} = m c^2 + |E_{binding}|,$$

$$ii) E_{Total} = \sqrt{m^2 c^4 + |E_{binding}|^2}.$$

These give rise, using the semiclassical expression, Eq.(21) to define the momentum, to the following expressions for  $\beta$ ,

$$i) \beta = \frac{\sqrt{\frac{3}{4\rho}}}{1 + \frac{3}{4\rho}},$$

$$ii) \beta = \frac{\sqrt{\frac{3}{4\rho}}}{\sqrt{1 + \left(\frac{3}{4\rho}\right)^2}}.$$

We draw in Fig.(4) these values to show that the correct calculation gives relatively low values of the velocity for very bound systems since the maximum occurs for  $\beta = \frac{3}{4}$  which never goes above  $\frac{1}{\sqrt{2}}$ .

Since our potential is cut off for small values of  $r$  we expect a slow down of the particles with respect to the conventional Coulomb potential and therefore a non-relativistic treatment more accurate than in the Coulomb case. Moreover, the calculation for the wave function at the origin is less sensitive to the short range behavior of the potential, than the velocity, which depends on the slope of the wave function.

## 4 Cross section estimates

We found that the analysis of the cross section that follows is physically appealing because

- i) The mass of the monopolum may be chosen small (much smaller than the monopole mass) in the formalism just develop, which allows to study monopolum detection at relatively low energies.
- ii) The monopolum production will be accompanied by a radiation spectrum, which is also qualitatively described by the Coulomb spectrum.
- iii) The calculation is easy to perform and physically understandable.

It seems therefore safe to go ahead to calculate the monopolum decay probability as a function of  $\rho$ . The range of values of  $\rho : 3/8 < \rho < \infty$ . Moreover,

$$n = \frac{1}{4\alpha} \sqrt{\frac{\rho}{3}}$$

and therefore, given a value of  $\rho$ , one can determine  $n$  and this fixes  $|\psi(0)|^2$ , which is what one needs for computing the decay probability. In summary, the calculation seems to be feasible in terms of only one mass scale, the mass of the monopole,  $m$ , and one parameter,  $\rho$ .

Let us take the wave function for  $\ell = 0$ . Consequently [18]

$$\psi_{n,0,0} = \frac{1}{a^{3/2}} N_{n,0} F_{n,0} \left( \frac{2r}{na} \right) Y_0^0(\Omega) \quad (22)$$

with

$$a = \frac{\hbar^2}{m e^2} ; \quad N_{n,0} = \frac{2}{n^2} \sqrt{\frac{(n-1)!}{(n!)^3}}$$

and

$$F_{n,0}(x) = e^{-1/2x} L_{n-1}^1(x) ; \quad L_{n-1}^1(x) = \sum_{s=0}^{n-1} (-1)^s \frac{(n!)^2}{(n-s-1)!(s+1)!s!} x^s$$

We need  $|\psi_{n,0,0}(0)|$ . Then, taking into account that

$$\lim_{x \rightarrow 0} L_{n-1}^1(x) = n n! ; \quad \lim_{x \rightarrow 0} F_{n,0}(x) = n n!$$

one has

$$|\psi_{n,0,0}(0)| = \frac{1}{a^{3/2}} \frac{2}{n} \frac{1}{\sqrt{n}} \quad (23)$$

The reduced mass of the monopodium system is  $m/2$  and the Dirac condition Eq.(1) can be written as

$$\frac{g^2}{\hbar c} \frac{e^2}{\hbar c} = \frac{1}{4} \Rightarrow \alpha_g \alpha_e = \frac{1}{4}$$

one gets

$$|\psi_{n,0,0}(0)| = \frac{1}{4} \left( \frac{m c^2}{2 \alpha_e n \hbar c} \right)^{3/2} \quad (24)$$

Notice that  $|\psi_{n,0,0}(0)|$  has the correct dimensions, namely it is measured in  $(fm)^{3/2}$  as it should be. Finally one can write the wave function in terms of the variable  $\rho$  to obtain

$$|\psi_{n,0,0}(0)| = \frac{1}{4} \left( \frac{2\sqrt{3} m c^2}{\hbar c \sqrt{\rho}} \right)^{3/2} \quad (25)$$

This is the main ingredient to be included in the expression for the cross section that was computed before.

Without making any assumption about the spin of monopoles, and being particularly interested in  $n$  large and  $\ell$  small, one has  $G \sim 1$ . Moreover, we neglect for the time being  $\Gamma_M$ , which will be introduced later on when required. Replacing the value of the wave function given in (25), the cross section, Eq.(8), becomes

$$\sigma = \frac{1}{4^4} \frac{27}{32 \pi^4} \frac{(\tan \theta_W)^{2NZ_0}}{\alpha^2} (\hbar c)^2 \left( 2 - \frac{3}{4} \frac{1}{\rho} \right)^2 \frac{1}{\rho^3} \frac{1}{m^2 c^4} \frac{E_B^2 E_C^2 \frac{4 E_e^2}{m^2 c^4}}{\left[ (4 E_e^2 - m_A^2 c^4)^2 + m_A^2 \Gamma_A^2 c^4 \right] \left[ \frac{4 E_e^2}{m^2 c^4} - \left( 2 - \frac{3}{4} \frac{1}{\rho} \right)^2 \right]^2} \quad (26)$$

Taking into account the kinematics

$$p_A = (2 E_e, \vec{0}) ; \vec{p}_B + \vec{p}_C = \vec{0} ; E_B + E_C = 2 E_e$$

one can study the different cases with photons and  $Z_0$ .

Let us describe here the three photons case, namely:  $A = \gamma$ ,  $B = \gamma$  and  $C = \gamma$ . In this case one has  $E_B^2 = E_C^2 = E_e^2$ . The corresponding cross section results in

$$\sigma_{3\gamma} = \frac{1}{4^5} \frac{27}{32 \pi^4} \frac{1}{\alpha^2} (\hbar c)^2 \left( 2 - \frac{3}{4} \frac{1}{\rho} \right)^2 \frac{1}{\rho^3} \frac{E_e^2}{(m^2 c^4)^2} \left[ \frac{4 E_e^2}{m^2 c^4} - \left( 2 - \frac{3}{4} \frac{1}{\rho} \right)^2 \right]^{-2}. \quad (27)$$

Clearly  $A = Z_0$ ,  $B = \gamma$  and  $C = \gamma$  contributes also to the  $2\gamma$  cross-section. We omit it here for simplicity, since we are here just estimating the observability of the process, not its precise magnitude.

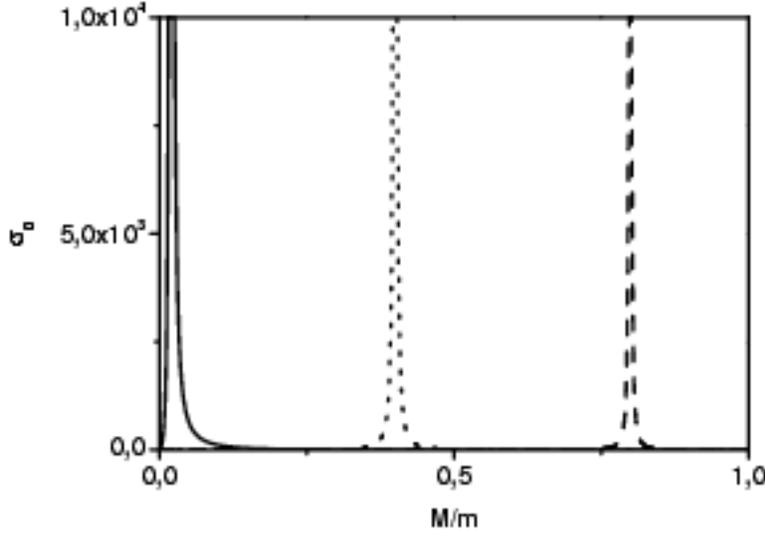


Figure 5:  $\sigma_0$ , as defined in Eq.(29), as a function of the monopole mass ( $M/m$ ) for various values of the beam energy  $E_e/mc^2 = 0.01$  (solid), 0.2(dotted), 0.4(dashed). The figure shows the motion of the resonance structure towards the lower endpoint.

Introducing the constant values in appropriate units, one gets

$$\left[ \frac{\sigma_{3\gamma}}{fb} \right] = 0.064 \left( 2 - \frac{3}{4} \frac{1}{\rho} \right)^2 \frac{1}{\rho^3} \left[ \frac{4E_e^2}{m^2 c^4} - \left( 2 - \frac{3}{4} \frac{1}{\rho} \right)^2 \right]^{-2} \left[ \frac{E_e}{GeV} \right]^2 \left[ \frac{TeV}{mc^2} \right]^4. \quad (28)$$

In order to simplify the analysis we write it in terms of the monopole to monopole mass ratio  $\frac{M}{m} = 2 - \frac{3}{4\rho}$  and becomes,

$$\left[ \frac{\sigma_{3\gamma}}{fb} \right] = 0.152 \left( \frac{M}{m} \right)^2 \left( 2 - \frac{M}{m} \right)^3 \left[ \frac{4E_e^2}{m^2 c^4} - \frac{M^2}{m^2} \right]^{-2} \left[ \frac{E_e}{GeV} \right]^2 \left[ \frac{TeV}{mc^2} \right]^4 \quad (29)$$

$$= \sigma_0 \left[ \frac{E_e}{GeV} \right]^2 \left[ \frac{TeV}{mc^2} \right]^4, \quad (30)$$

where  $\sigma_0$  is dimensionless and contains the resonance structure of the cross section. Recall that the  $M/m$  mass ratio has the range  $0 \leq M/m \leq 2$ .

The cross section (29) has the monopole resonance pole at

$$Mc^2 = 2E_e.$$

For  $E_e \ll mc^2$  the resonance structure appears for low monopole masses as shown in Fig. (5).

We may extract from Eq.(29) the resonant structure which is  $m$  (scale) independent,

$$\sigma_{scaling} = \left[ \frac{4E_e^2}{M^2 c^4} - 1 \right]^{-2} \quad (31)$$

and which is shown in Fig. (6).

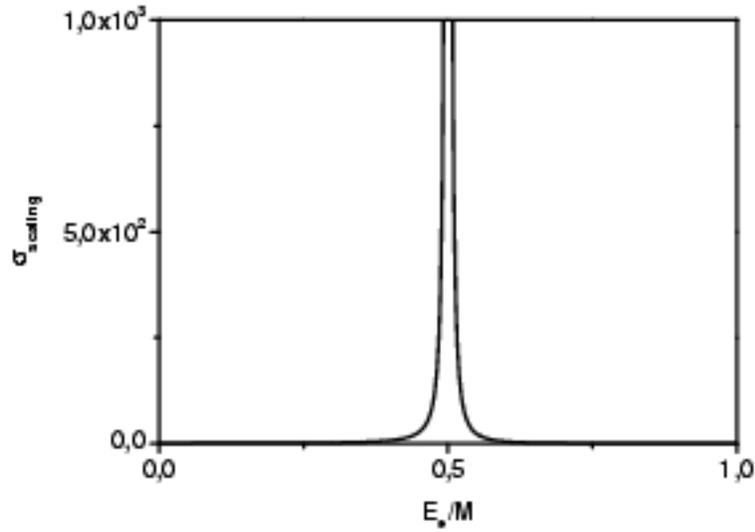


Figure 6: Isolated resonance structure of the cross-section in Eq.(29) as defined by Eq.(31).

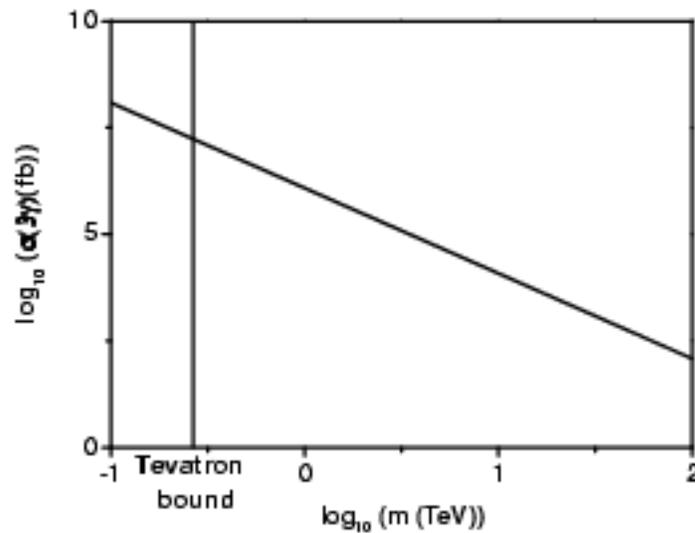


Figure 7: Logarithmic plot of the cross section (in  $fb$ ) as a function of the monopole mass ( $TeV$ ). The vertical line represents the Tevatron bound of  $m > 265$  GeV [19].

The scenario we want to investigate is one in which the scales associated with monopolium will be low energy scales and reachable by accelerators,  $M c^2 \sim E_e$ , while the monopole scales will be high energy (short distance) scales,  $m c^2 \gg E_e$ .

For a  $M/m \ll 1$ , Eq.(29) becomes,

$$\left[ \frac{\sigma_{3\gamma}}{fb} \right] = 1.216 \frac{m^2}{M^2} \sigma_{scaling} = 1.216 \frac{m^2 M^2 c^8}{[4 E_e^2 - M^2 c^4]^2} \left[ \frac{E_e}{GeV} \right]^2 \left[ \frac{TeV}{m c^2} \right]^4. \quad (32)$$

We have a clear structure of a resonant peak at the monopolium mass governed by a low scale  $M c^2$ .

It becomes clear from the above discussion that the best place to search for monopolium is at the resonance. At resonance, if we reintroduce the monopolium width  $\Gamma_M$ , the cross section, Eq.(32), becomes,

$$\left[ \frac{\sigma_{3\gamma}}{fb} \right] = 1.216 \frac{m^2}{\Gamma_M^2} \left[ \frac{E_e}{GeV} \right]^2 \left[ \frac{TeV}{m c^2} \right]^4 = 1.216 10^6 \left[ \frac{GeV}{\Gamma_M} \right]^2 \left[ \frac{E_e}{GeV} \right]^2 \left[ \frac{TeV}{m c^2} \right]^2. \quad (33)$$

In correspondence with our two scale physical scenario we take  $\Gamma_M \sim E_e$  and therefore we get a very simple relation between the expected cross section and the mass of the monopole,

$$\left[ \frac{\sigma_{3\gamma}}{fb} \right] \sim 1.216 10^6 \left[ \frac{TeV}{m c^2} \right]^2. \quad (34)$$

The result is plotted in Fig.(7) together with the last bound of the Tevatron [19].

Let us summarize our findings. In our two scale scenario,

- i) the studied cross section has a resonant peak (see Figs. (5) and (6)) at the monopolium mass  $M$ ;
- ii) the order of magnitude of the cross section is "almost" beam energy independent and is consistent with observability in present day machines [13, 20, 21] for monopole masses of up to 100 TeV (see Figs. (7));
- iii) a similar analysis can be carried out for  $\gamma Z_0$  and  $2Z_0$  decays;
- iv) a similar analysis can be carried out for hadronic production, complicated by the inclusion of the sub-structure of the intervening hadrons.

## 5 Conclusions

We have performed an investigation looking for hints of the so far not seen monopolies. Our working assumption is that monopolies appear strongly bound forming monopolium, a monopole-anti-monopole bound state, due to their strong electromagnetic interaction.

We develop a scenario in which monopolium is produced and desintegrates into  $2\gamma$ ,  $\gamma Z_0$  and  $2Z_0$ 's. We detail the structure and magnitude of the first of this processes to determine observability. We develop a two energy scale scenario, whose

- i) low scale is governed by monopolium and we consider for quantitative purposes that it is reachable by present day machines

$$M c^2 \sim \Gamma_M \sim E_e;$$

- ii) and whose high energy scale is governed by the monopole mass and arises through the structure of monopolium

$$m c^2 \gg E_e$$

Under these circumstances we can estimate the the cross section as a function of monopole mass. Cosmological GUT monopoles are not observable, however, the intermediate scenarios for which monopoles are heavy, but not tremendously heavy, up to hundreds of TeV, the cross section is large enough for the studied processes to be feasible of detection.

Since at present we can not calculate the monopolium parameters,  $M$  and  $\Gamma_M$ , the experimental endeavor is not easy. There are however some features which might simplify the task,

- i) the resonance peak of the monopolium can be found in three exit channels  $2\gamma$ ,  $\gamma Z_0$  and  $2Z_0$ 's;
- ii) monopolium can be produced in an excited state before it annihilates, thus the annihilation process will be accompanied by a Rydberg radiation spectrum;
- iii) the same processes can be studied hadronically, the only complication arising from the inclusion of the hadron sub-structure.

The fact that the values of the calculated cross sections are not extremely small, for reasonable monopole mass scenarios, render our calculation interesting and this line of research worth pursuing.

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## References

- [1] P.A.M. Dirac, Proc. Roy. Soc. **A133** (1931) 60, Phys. Rev. **74** (1940) 817.
- [2] J. D. Jackson, *Classical Electrodynamics*, de Gruyter, N.Y. (1982).

- [3] N. Craigie, G. Giacomelli, W. Nahern and Q. Shafi, *Theory and detection of magnetic monopoles in gauge theories*, World Scientific, Singapore 1986.
- [4] T.W.B. Kibble, J. Phys. **A 9** (1976) 1387, Phys. Rep. **67** (1980) 183; A. Vilenkin, Phys. Rep. **121** (1985) 263.
- [5] G. Giacomelli and L. Patrizzii, hep-ex/0506014.
- [6] A. De Rújula, Nucl. Phys. **B435** (1995) 257.
- [7] K. A. Milton, hep-ex/0602040.
- [8] Review of Particle Physics, S. Eidelman et al. Physics Letters b592 2004 1.
- [9] M. J. Mulhearn, Ph.D. Thesis MIT 2004.
- [10] Ya.B Zeldovich and M. Yu. Khlopov, Phys. Lett. **79** (1978) 239.
- [11] V. Vento, Are monopoles hiding in monoplium? astro-ph/0511764.
- [12] C.T. Hill, Nucl. Phys. **B224** (1983) 469
- [13] I.F. Ginzburg and A. Schiller, Phys. Rev. **D60**, 075016 (1999).
- [14] S. Weinberg, *The Quantum theory of fields* (Cambridge University Press, 1996); T.P. Cheng and L.F. Lee, *Theory of elementary particles* (Oxford University Press, 1982); M.E. Peskin and D.V. Schroeder, *An introduction to quantum field theory*, by (HarperCollins, 1995).
- [15] J.M. Jauch and F. Rohrlich, *The theory of electrons and photons* (Springer 1975).
- [16] L.I. Schiff **160** (1967) 1257.
- [17] C.J. Goebel, *Quanta, Essays in Theoretical Physics*, eds. P.G.O. Freund, C.J. Goebel and Y. Nambu (Chicago, 1970).
- [18] A. Galindo and P. Pascual, *Quantum Mechanics* (Springer, 1991).
- [19] Jour. of Phys. **G33** (2006) 1103.
- [20] I.F. Ginzburg, G.L. Kotkin, V.G. Serbo and V. I. Telnov, Nucl. Inst. and Methods Phys. Res. **205** (1983) 47; I.F. Ginzburg, G.L. Kotkin, S.L. Panfil, V.G. Serbo and V. I. Telnov, Nucl. Inst. and Methods Phys. Res. **219** (1984) 5.
- [21] R. Brinkmann et al., Nucl. Inst. and Methods Phys. Res. **A 406** (1998) 13.