Rapid thermalization and thermal radiation: From CGC to QGP

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Nuclei as heavy as bulls through collision generate new states of matter

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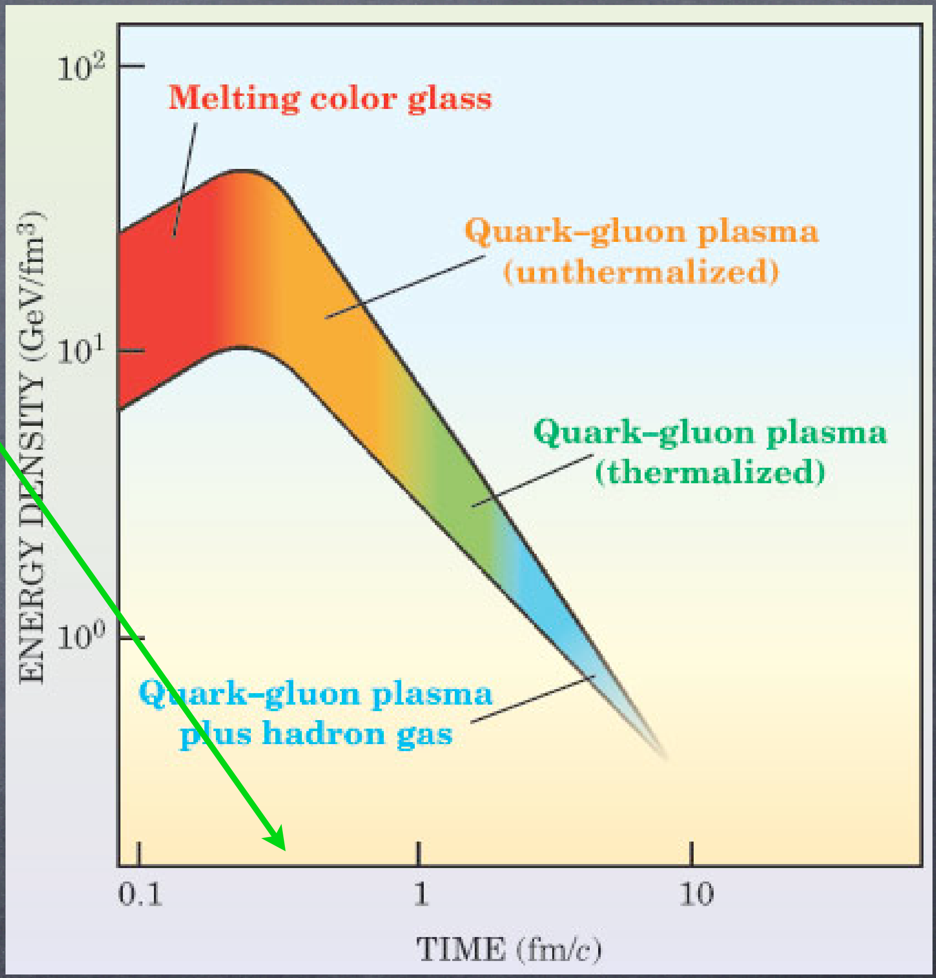
The main puzzle at RHIC

- Hydrodynamic models are successful if thermalization time is Δt≈0.5 fm
- Naively, $\Delta t \sim 1/(n\sigma) \sim \mathrm{fm}/\alpha_s$

$$\Delta t \sim 1/\alpha_s^{13/5} Q_s$$

(Baier, Mueller, Son, Schiff)



Why thermalization is so fast?

Possible solutions

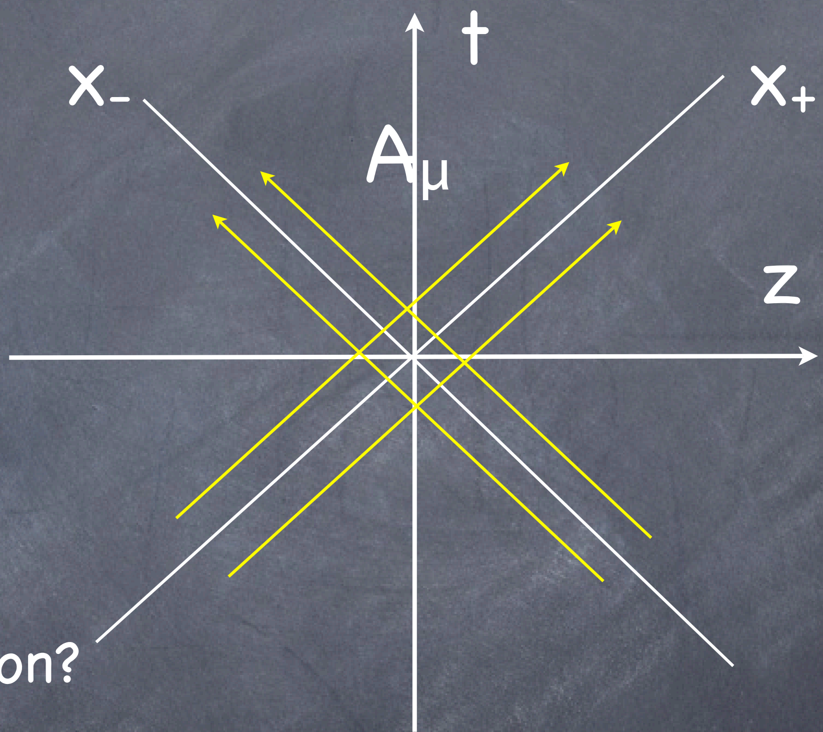
- Plasma instabilities (Mrowczynski; Arnold, Lenaghan, Moore).
- Early isotropization may help explain v_2 (Arnold, Lenaghan, Moore; Rebhan, Romatschke, Strickland).
- No thermalization in pQCD (Kovchegov).
- AdS/CFT ☺

Macroscopic picture

Space-time structure of the HIC

- Upon collision heavy ions experience very large deceleration.

  Indeed, most of matter is produced in the transverse plane!



- How large is acceleration?

$$a \sim eE/m \sim Q_s^2/m$$

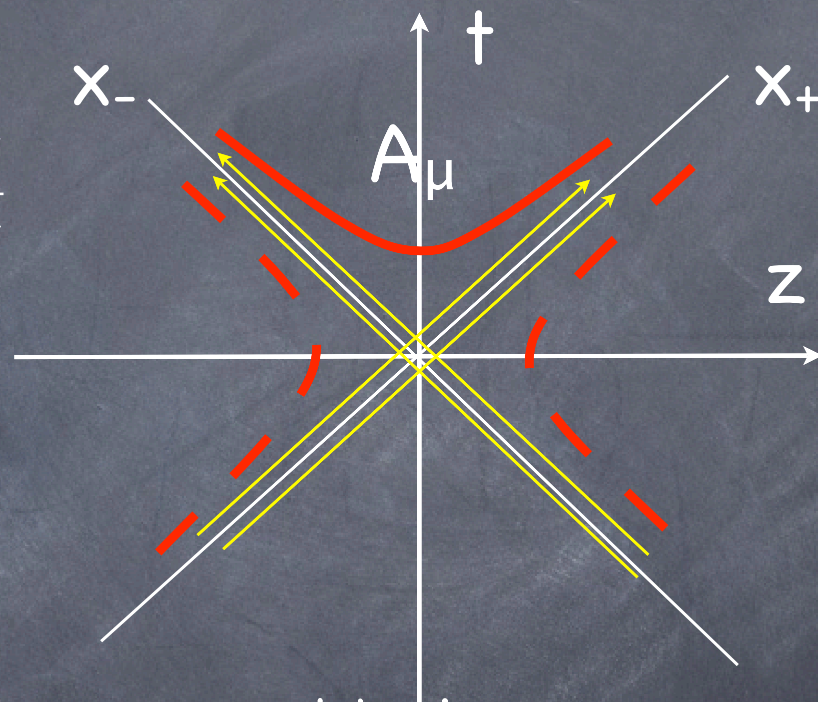
- It depends only on the strength of the nuclear field ⇒ Limiting fragmentation.

Comoving frame

- Comoving frame:

$$\rho^2 = z^2 - t^2\,, \quad \eta = \frac{1}{2}\ln\frac{t+z}{t-z}$$

- ρ, η define the Rindler space in which accelerated particle is at rest.



- In the Rindler space x_- and x_+ are event horizons. Therefore, the produced particle can carry information only about conserved quantities. This is realized in the thermal distribution with $T=a/(2\pi)$ (Unruh,Hawking).

Hagedorn argument

- Consider breakdown of a high energy hadron of mass m into a final hadronic state of mass M. Transition probability:

$$P(m \to M) = 2\pi |T(m \to M)|^2 \, \rho(M)$$

- In dual resonance model density of states is

$$\rho(M) \approx \exp(4\pi b^{1/2} M / 6^{1/2})$$

- Transition amplitude: $|T(m \to M)|^2 \simeq \exp(-2\pi M/a)$

- Probability conservation requires that Σ_M P(M) = 1.

Limiting temperature

- Therefore the string tension cannot accelerate particles beyond

$$a_{cr} = \sqrt{3/2}\, b^{-1/2}$$

which exactly corresponds to the Hagedorn temperature:

$$T_H = \frac{a}{2\pi} = \frac{6^{1/2}}{4\pi b^{1/2}}$$

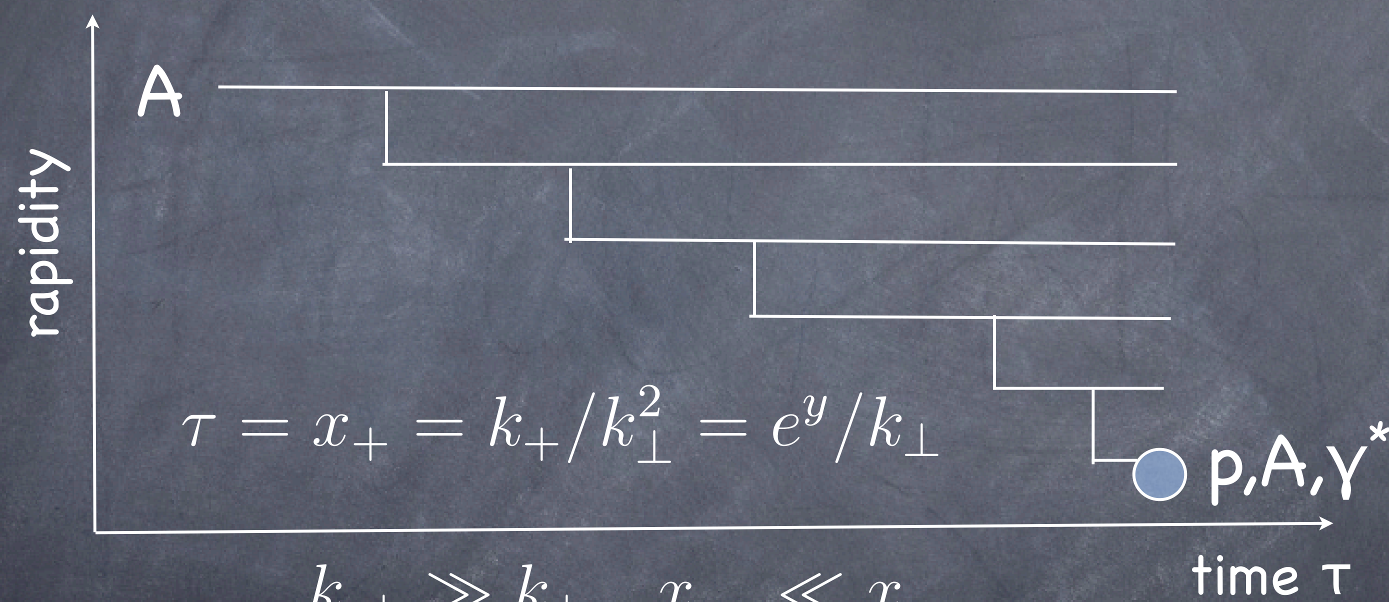
- One needs stronger fields to get $T > T_H$; e.g. CGC

- In this macroscopic quasiclassical picture one can also study phase transitions. (Ohsaku)

Microscopic picture

Particle production at high energies

- Structure of the partonic cascade at high energies:



$$\tau = x_+ = k_+/k_\perp^2 = e^y/k_\perp$$

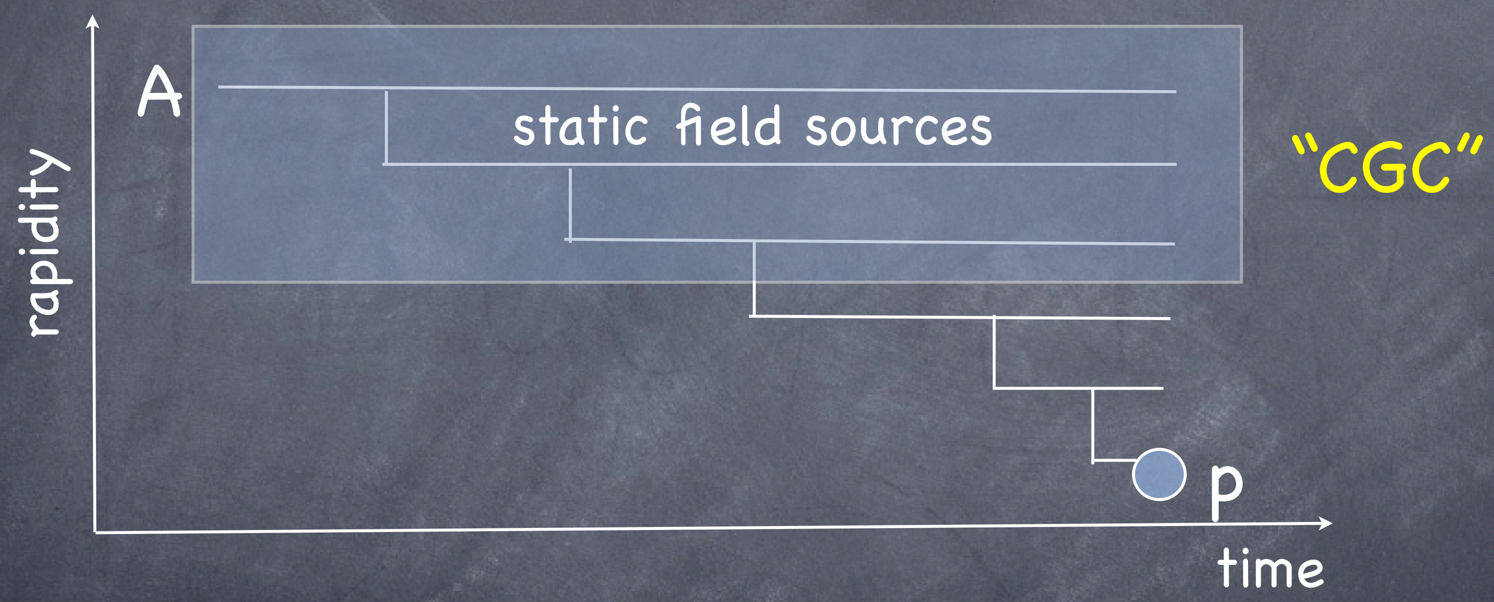
$$k_{s+} \gg k_+ \quad x_{s-} \ll x_-$$

$$k_{s-} \ll k_- \quad x_{s+} \gg x_+$$

$$k_{s\perp} \ll k_\perp \quad x_{s\perp} \gg x_\perp$$

Background field method

- Structure of the partonic cascade at high energies:



- Strength of the field is determined by the density of color charges in the transverse plane, i.e. $Q_s(y)$

Field configuration at low x (I)

Kharzeev, KT hep-ph/051234;
Kharzeev, Levin, KT hep-ph/0602063

- (Chromo) Electric and Magnetic fields of a high energy hadron/nucleus are transverse (plane wave) and stable.
- However, this is true only for a non-interacting hadron. Boundary conditions at the interaction point generate the longitudinal fields.
  - Recall reflection of light in Electrodynamics!

$$A_+(x_-, x_+, x_\perp) = 0\,, \quad x_+ \le x_-$$

Field configuration at low x (II)

$$A(\mathbf{x},t) = \int_S d^3x' \, \frac{\rho(\mathbf{x}', t - |\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}$$

- Since $Q_s(y)$ increases down the cascade, the parton transverse size decreases. Therefore, $\mathbf{x}_\perp \ll \mathbf{x}'_\perp$
- and A(x,t) does not depend on $\mathbf{x}_\perp$

$$E_\parallel \gg E_\perp \simeq B_\perp \gg B_\parallel$$

- Fries, Kapusta and Li argue that non-Abelian interactions can produce both longitudinal chromoelectric and chromomagnetic fields.
  - implications: see paper by Lappi & McLerran

Supercritical fields

- The work done by the external chromo-electric field E accelerating a virtual qq pair apart by a Compton wavelength λ_c=h/mc is W=gEh/mc.
- If W>2mc^2 the pair becomes real.

$$E_{\rm cr} = \frac{m^2 c^3}{g\hbar}$$

- In QED E_{cr}=10^{16} V/cm - beyond the current lab frontier.
- In QCD g~1, m~Q_s or Λ, thus E_{cr}~(1 GeV)^2 : pair production is a common phenomenon.

Vacuum rearrangement (I)

- Example: charged scalar field in time dependent electromagnetic background

$$(\partial_\mu + ieA_\mu)^2\phi(x) + m^2\phi(x) = 0$$

- Eigenstates: $$\phi_p^\pm(x) = \frac{1}{(2\pi)^{3/2}\sqrt{2\omega_-}} e^{i\vec{p}\vec{r}} g^\pm(\vec{p}, t)$$

  where

$$\ddot{g}(p,t) + \omega^2(t) g(p,t) = 0\,, \quad \omega^2(t) = p^2 - 2ep_z A_z + e^2 A_z^2 + m^2$$

- Note, that $\phi_{in} = \phi(t\rightarrow-\infty)$ are different from $\phi_{out} = \phi(t\rightarrow+\infty)$

Vacuum rearrangement (II)

- Second quantization $\phi(x) = \sum_i a_{i,in}\phi(x)a_{i,in}(x) + a^\dagger_{i,in}\phi^*_{i,in}(x)$

  where $[a_{i,in}, a^\dagger_{j,in}] = \delta_{ij}\ , \quad a_{i,in}|0\rangle_{in} = 0$

- Equivalently $\phi(x) = \sum_i a_{i,out}\phi(x)a_{i,out}(x) + a^\dagger_{i,out}\phi^*_{i,out}(x)$

$$[a_{i,out}, a^\dagger_{j,out}] = \delta_{ij}\ , \quad a_{i,out}|0\rangle_{out} = 0$$

- Unitarity implies $a_{i,in} = \sum_j (\alpha_{ij} a_{j,out} + \beta^*_{ji} a^\dagger_{j,out})$

- Therefore, even if $\langle 0|a_{i,in}a^\dagger_{i,in}|0\rangle_{in} = 0$

$$\langle 0|a_{i,in}a^\dagger_{i,in}|0\rangle_{out} = \sum_j |\beta_{ij}|^2$$

Particle production by black hole

- The same problem of quantization in external field

$$D^\mu D^\nu g_{\mu\nu}\phi(x) + m^2\phi(x) = 0$$

- In Schwartzschild metric

$$\phi_{\omega lm}(x) = r^{-1} R_{\omega l}(r) Y_{lm}(\theta, \varphi)\exp(\pm i\omega t)$$

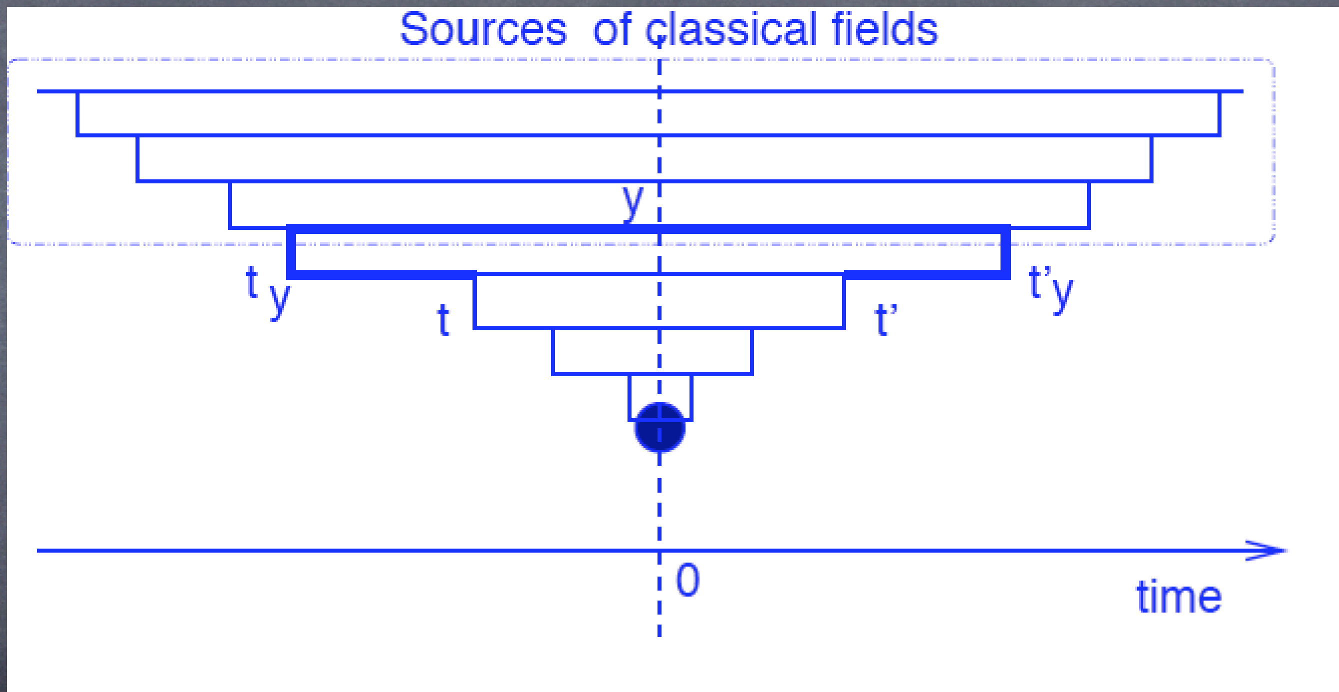
$$R_{\omega l}(r) \approx \begin{cases} \exp(i\omega r^*) + \alpha_{\omega l}\exp(-i\omega r^*)\,, & r^* \to -\infty \\ \beta_{\omega l}\exp(i\omega r^*)\,, & r^* \to +\infty \end{cases} \qquad r^* = r + r_g \ln(r/r_g - 1)$$

- Black hole emission rate $|\beta_{\omega l}|^2 \propto \exp(-2\pi\omega/\kappa)$ Hawking, 75

where the surface gravity is $\kappa = 1/4GM$

Propagator in external field

- Pair production rate = imaginary part of the propagator
- WKB approximation is quite useful for calculating tunneling probabilities.



- see T. Lappi's numerical solution of Dirac equation in external field.

WKB approximation

- In the pair production process electron's energy changes from ϵ_- to ϵ_+ where

$$\epsilon_{\pm} = \pm\sqrt{p^2(z) + m^2} + eEz$$

- Pair production rate $\Gamma \approx e^{-2\mathrm{Im}S}$. Imaginary part of action ImS can be found by integrating phase over states with imaginary momentum:

$$\Gamma = \exp\{-\int_{z_a}^{z_b} dz\, |p(z)|\} = \exp\{-\pi m_\perp^2 / gE\}$$

where the turning points of the linear potential are

$$z_{a,b} = (\epsilon \pm m)/gE$$

Canonical approach

- Gauge boson propagator in SU(2) can be found from equation of motion

$$((-\partial_\mu - igA_\mu)^2 \pm 2eE_z)\, W_\pm = 0$$

- In the quasiclassical approximation we seek solution in form W=exp(-iS-i$\mathbf{p}\cdot\mathbf{x}$) such that $S'' \ll (S')^2$

$$-2\partial_+ S\,(\partial_- S - g\, A_+(x_-)) + p_\perp^2 + 2g\sigma\, E_z = 0\,; \quad x_+ > x_-$$

- Introduce canonical momenta $P_\mu = -\partial_\mu S + gA_\mu$. Then S can be found along the classical particle trajectories.

Action as a function of coordinates

- Quasiclassical trajectory

$$x_- = \frac{p_\perp^2}{2}\int \frac{dx_+}{(p_+ + gA_+(x_+))^2}$$

- Action

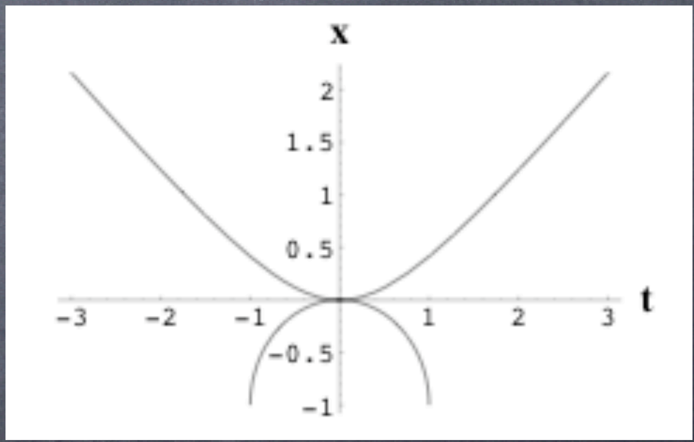
$$S = p_- x_+ + \int dx_+ \frac{p_\perp^2}{p_+ + gA_+(x_+)} - \int dx_- gA_+(x_-)$$

- Introduce new variables

$$\tau = x_+\omega \quad A_+ = \frac{E_0}{\omega} f(\tau)\,, \quad \gamma = \frac{p_+\omega}{gE_0}$$

- Imaginary part of action $\mathrm{Im} S = \dfrac{p_\perp^2}{2gE_0}\dfrac{\pi}{f'(f^{-1}(\gamma))}$

- Trajectory of a particle moving with constant acceleration coincides with the that of particle in a constant field E



Schwinger vs Unruh

- ImS corresponds to instable, classically forbidden motion. Vacuum decay probability is Γ=exp(-2ImS).
- In the constant field $\Gamma = e^{-\frac{\pi p_\perp^2}{gE}}$

□ How to reconcile this with Unruh thermal emission?

- In Schwiner formula we average over produced pairs while in Unruh one over the excited states of the classical detector. The relative occupation number of two excited states is

$$\frac{N_1}{N_2} = \exp\left\{-\frac{\pi}{gE}(m_1^2 - m_2^2)\right\} \approx \exp\left\{-\frac{\pi \Delta m}{gE}2\bar{m}\right\} = \exp\left\{-\frac{2\pi\Delta m}{\bar{a}}\right\}$$

Adiabaticity parameter

- The adiabaticity parameter measures how rapidly the external field change

$$\gamma = \frac{p_+ \omega}{gE_0}$$

- At early proper time $\tau \ll 1/Q_s$ it follows from the parton model that $\gamma \ll 1$ since

$$\omega = p_{s-} = \frac{p_{s\perp}^2}{p_{s+}} \ll p_{s+}$$

- Then the spectrum is: $\phi(p_\perp) \propto S_\perp e^{-\frac{2\pi m_\perp^2}{gE}} = S_\perp e^{-\frac{p_\perp^2}{Q_s^2}}$ as it should be in E=const

Time evolution of pair production

- At later time $\tau \sim 1/Q_s$ the parton cascade decays and $\gamma \sim 1$
- The field $E \propto e^{-\omega\tau}$ due to the screening of the original field by the field of produced pairs.

$$\phi(p_\perp) \propto S_\perp e^{-\frac{4\pi p_-}{\omega}} = S_\perp e^{-\frac{p_-}{T}}$$

- Momentum conservation (momentum lost by the fields equals momentum gain by produced particles) implies

$$T \simeq \frac{1}{2\sqrt{2\pi} Q_s}$$

Statistical interpretation

- Consider the relative probability of single pair production

$$w_1(\sigma, p) = e^{-2\mathrm{Im}S}$$

- The probability that no pairs is produced w_0 satisfies

$$w_0 \sum_{n=0}^{\infty} w_1^n = \frac{w_0}{1 - w_1} = 1$$

- Total probability that vacuum is unchanged is

$$W_0 = \prod_{\sigma,p} w_0(\sigma, p) = e^{\sum \ln(1-w_1)} = \left|e^{i\mathcal{L}Vt}\right|^2 = e^{-2\mathrm{Im}\mathcal{L}Vt}$$

$$\mathrm{Im}\mathcal{L}Vt = -\frac{1}{2}\sum \ln(1 - w_1) = -d\frac{V}{2(2\pi)^3}\int d^3p \ln(1 - w_1)$$

Statistical interpretation

- Narozhny and Nikishov demonstrated in 1970 that the imaginary part of the effective action for multi-pair production has the same form as the thermodynamic potential:

$$\mathrm{Im}\mathcal{S} = -\frac{dV}{2(2\pi)^3}\int d^3p \ln(1 - w_1(\sigma, \mathbf{p})) = -\frac{\Omega}{2T}$$

- We can follow the produced system till times $\tau \sim 1/T$.

- However, we cannot follow it all the way to the equilibrium until we solve the back-reaction problem.

see Cooper, Eisenberg, Kluger, Mottola, Svetitsky, hep-ph/9212206

Summary

- We have started to reexamine the role of the vacuum instability in high energy QCD.
- Due to existence of a hard scale Q_s we hope to be able to analytically address many interesting problems.
- Already now we can see that these new ideas can considerably modify our physical picture of particle production in high energy.