Cosmological Distances

Intro Cosmology Short Course

Lecture 3

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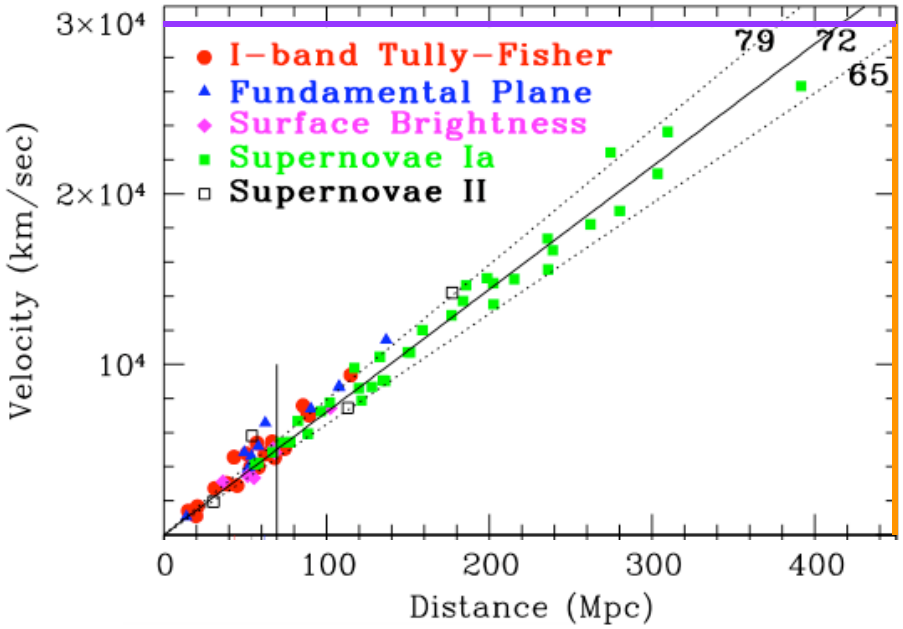
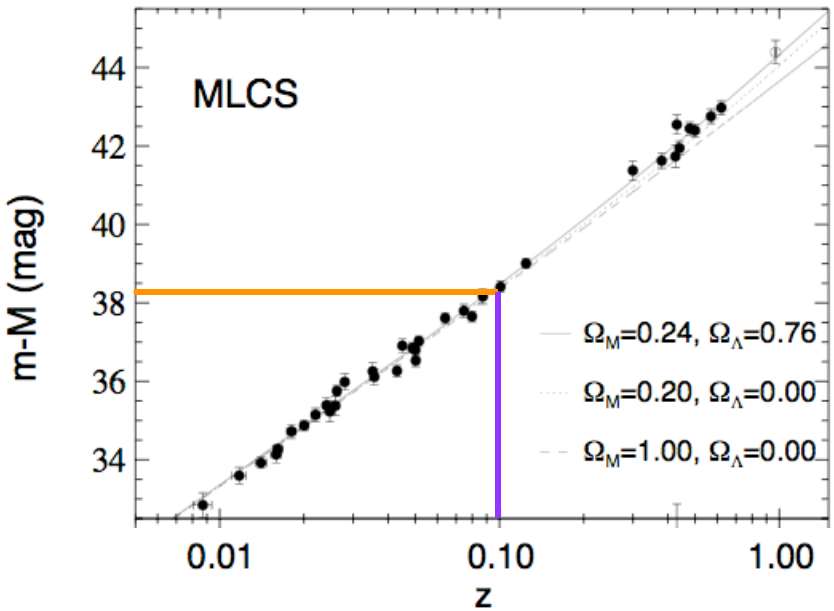
Two Modern Hubble Diagrams



Freedman, *et al.* Astrophys. J. **553**, 47 (2001)



Riess, *et al.* (High-Z) Astron. J. **116** (1998)

W. Freedman
Canadian

Modern Hubble constant (2001)

Generalize

velocity ⇒ redshift

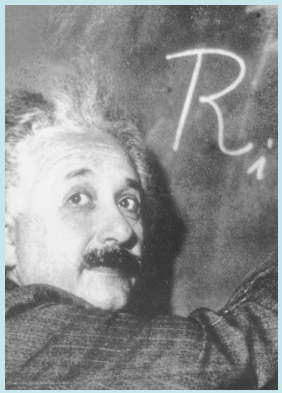
distance ⇒ magnitude

A. Riess
American

Supernovae cosmology (1998)

Einstein Equivalence Principle

Any small patch of GR reduces to SR

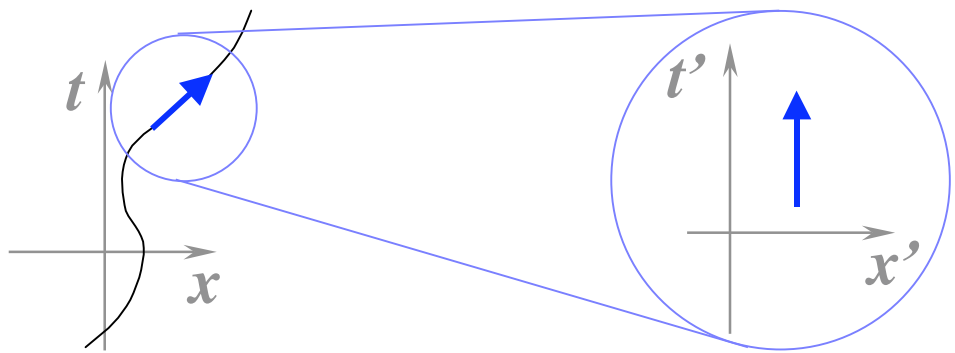


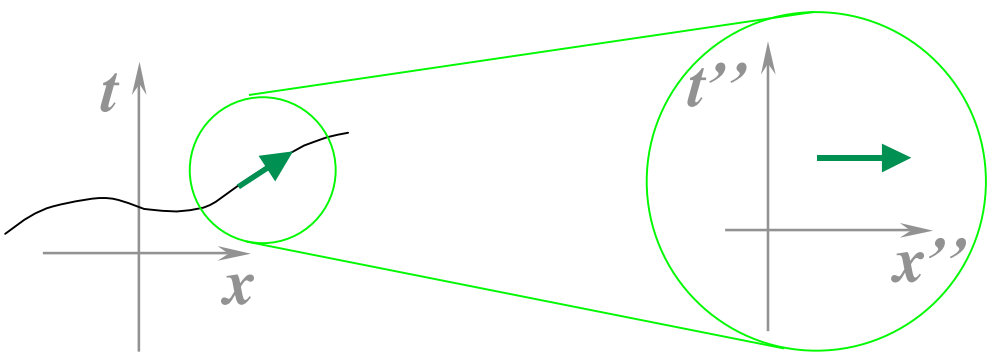
Albert Einstein
German

General Theory of Relativity (1915)

General Curved Space

Minkowski Space





$$d\tau^2 = \sum_{\mu\nu} dx_\mu \, g_{\mu\nu} \, dx_\nu$$

$$d\tau^2 = (dt')^2 - (dx'/c)^2$$

$$d\tau = dt'$$

For x' constant
"Proper time"

$$d\tau^2 = (dt'')^2 - (dx''/c)^2 \quad ?$$

$$ds^2 \equiv -c^2 d\tau^2 = -(c\,dt'')^2 + (dx'')^2$$

$$ds = dx''$$

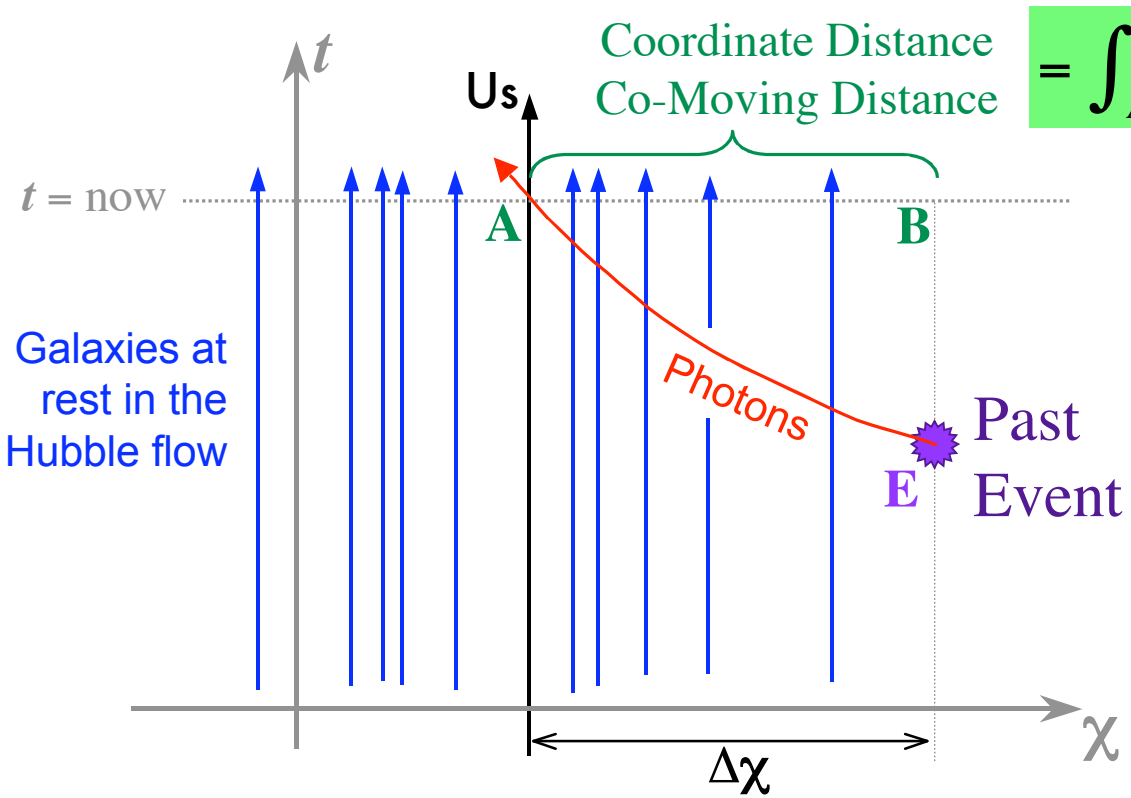
For t'' constant
Proper distance

What was the distance to that event?

$$d\tau^2 = dt^2 - \left[a(t)\right]^2 d\chi^2/c^2 \quad \text{if } d\tau^2 > 0$$

$$ds^2 = -c^2 dt^2 + \left[a(t)\right]^2 d\chi^2 \quad \text{if } d\tau^2 < 0$$

Robertson-Walker Coordinates



$$= \int_A^B ds = a(\text{now})\Delta\chi = \Delta\chi$$

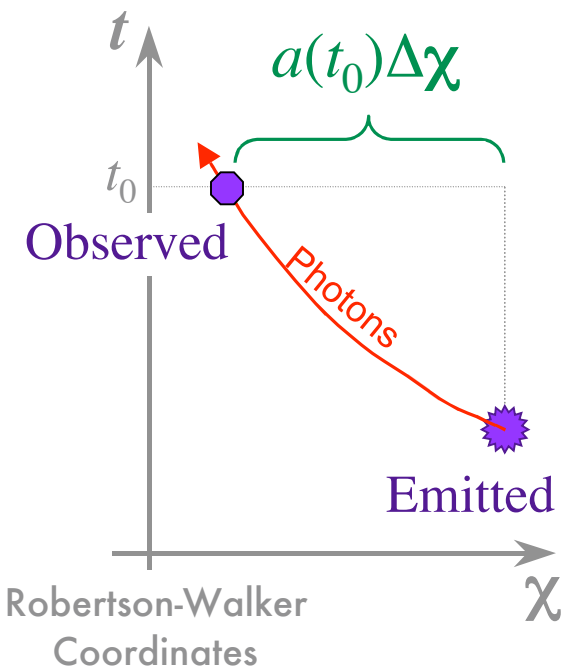
Well-defined… but is it *meaningful*?

NB: Distance along photon line

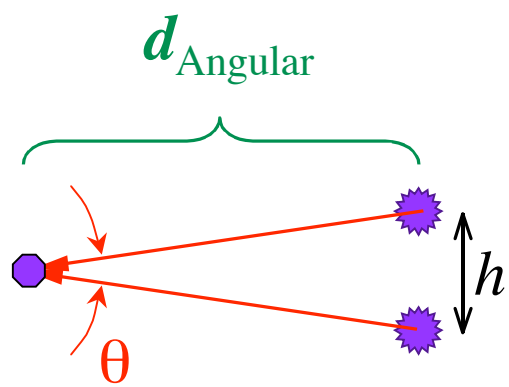
$$\int_A^E ds = 0$$

Three definitions of distance

Co-Moving/ Coordinate



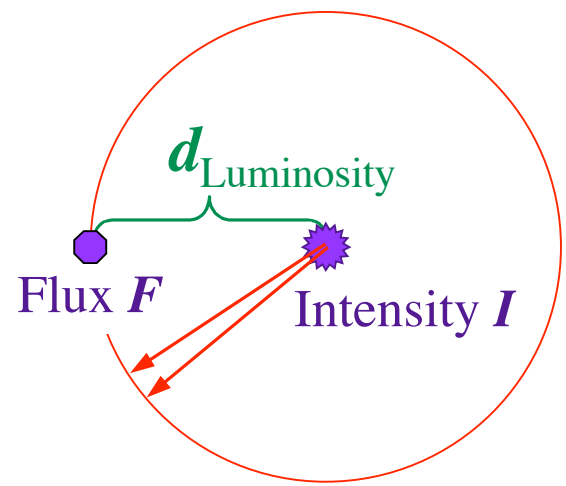
Angular



Euclidean: $h = d\theta$

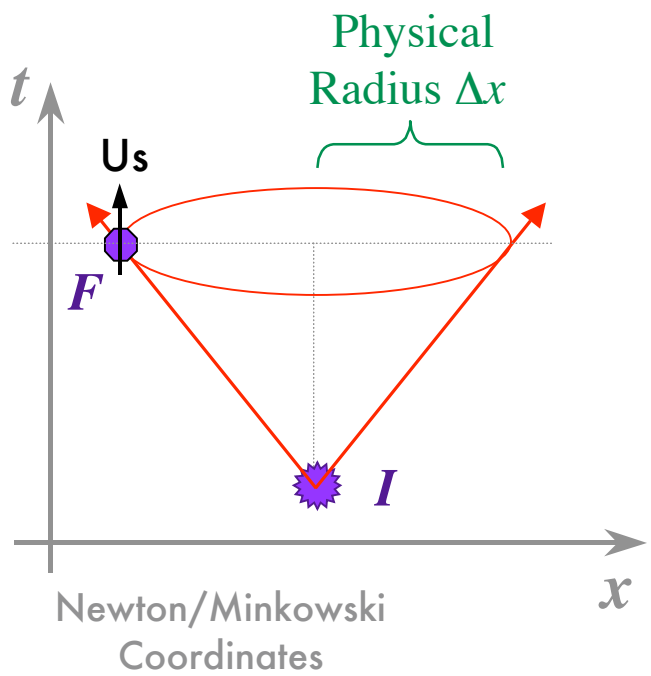
Define: $d_{\text{Angular}} \equiv h/\theta$

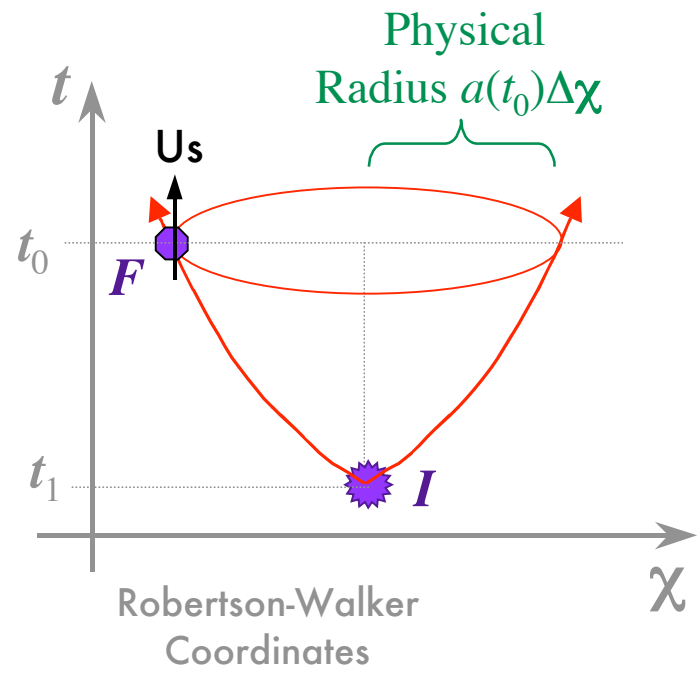
Luminosity



Euclidean: $F = I/4\pi d^2$

Define: $d_{\text{Lum}}{}^2 \equiv I/4\pi F$





$$F = \frac{I}{4\pi(\Delta x)^2}$$

$$d_{\text{Lum}} \equiv \sqrt{\frac{I}{4\pi F}} = \Delta x$$

Flat

$$F^{\text{Observed}}_{\text{Energy/Area/Time}} = \frac{I^{\text{Emitted}}_{\text{Energy/Time}}}{\underbrace{4\pi\left[a(t_0)\Delta\chi\right]^2}_{\text{Area of sphere}}} \underbrace{\frac{1}{1+z(t_1,t_0)}}_{\text{Rate of photons}} \underbrace{\frac{1}{1+z(t_1,t_0)}}_{\text{Redshift of photons}}$$

$$d_{\text{Lum}} \equiv \sqrt{\frac{I^{\text{Emit}}}{4\pi F^{\text{Obs}}}} = \underbrace{a(t_0)\Delta\chi}_{\text{Physical Radius}} \underbrace{\left(1+z(t_1,t_0)\right)}_{\text{Effect of Cosmic Expansion}}$$

Astronomical Magnitudes

Rule 1: Apparent Magnitude $m \sim \log(\text{Flux}^{\text{Observed}}$ a.k.a. Brightness)

Rule 2: More positive $\Rightarrow$ Dimmer

Rule 3: Change 5 magnitudes $\Rightarrow$ $\times 100$ change in brightness

$$m^{\text{Bolometric}} = -\log_{\sqrt[5]{100}}\left(F^{\text{Observed}}_{\text{Energy/Area/Time}}\right) + \text{Constant}$$

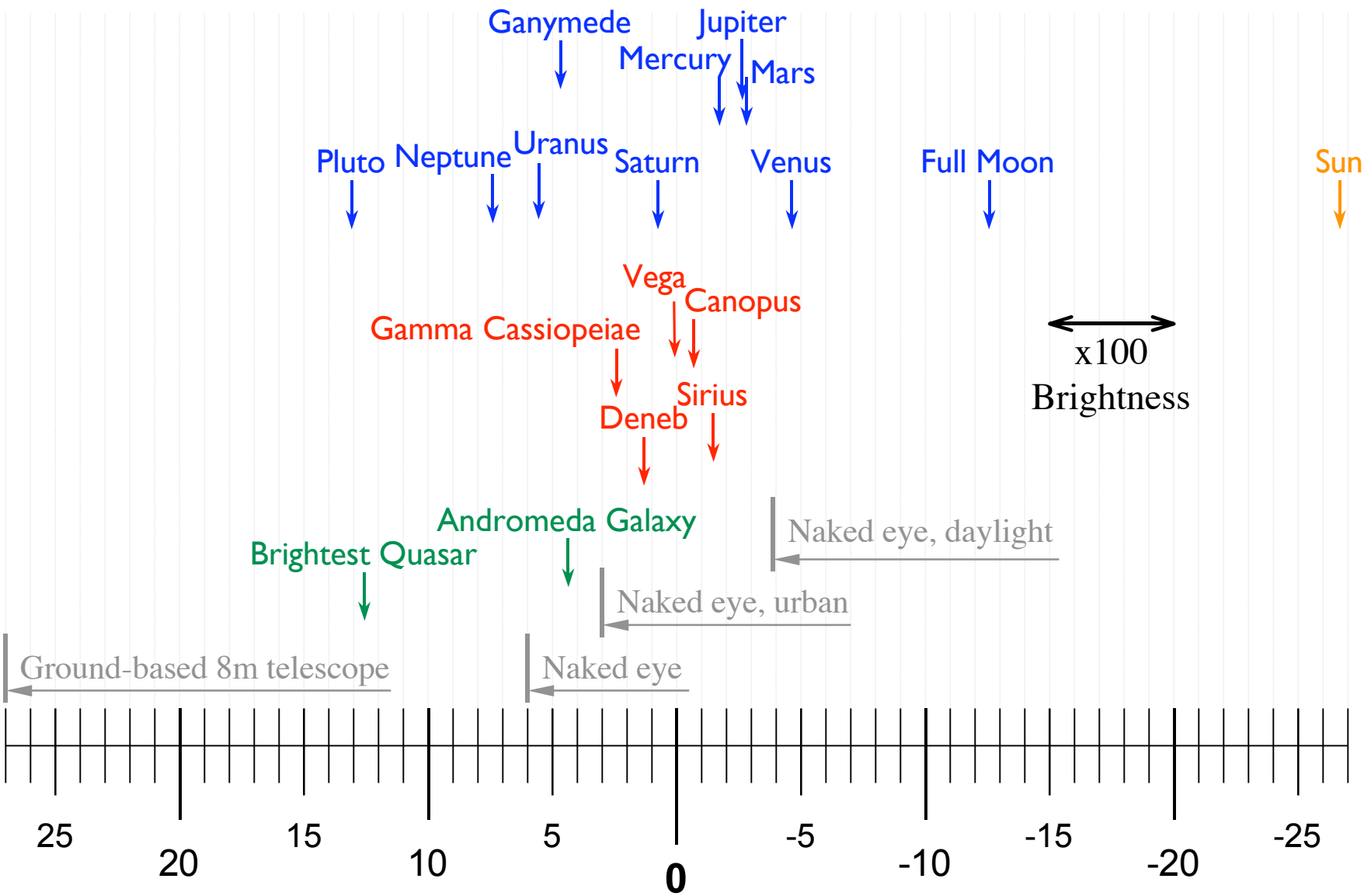
$$= -\frac{5}{2}\log_{10}\left(F^{\text{Observed}}_{\text{Energy/Area/Time}}\right) + \text{Constant}$$

Rule 4: Absolute Magnitude M = Apparent Magnitude if the source were at a standard distance (10 parsec = 32.6 light-year)

$$M^{\text{Bolometric}} = -\frac{5}{2}\log_{10}\left(I^{\text{Emitted}}_{\text{Energy/Time}}\right) + \text{Constant}$$

Some apparent magnitudes

(Source: Wikipedia)



Magnitudes and Luminosity Distances

$$m^{\text{Bolometric}} = -\frac{5}{2}\log_{10}\left(F^{\text{Observed}}_{\text{Energy/Area/Time}}\right) + C$$

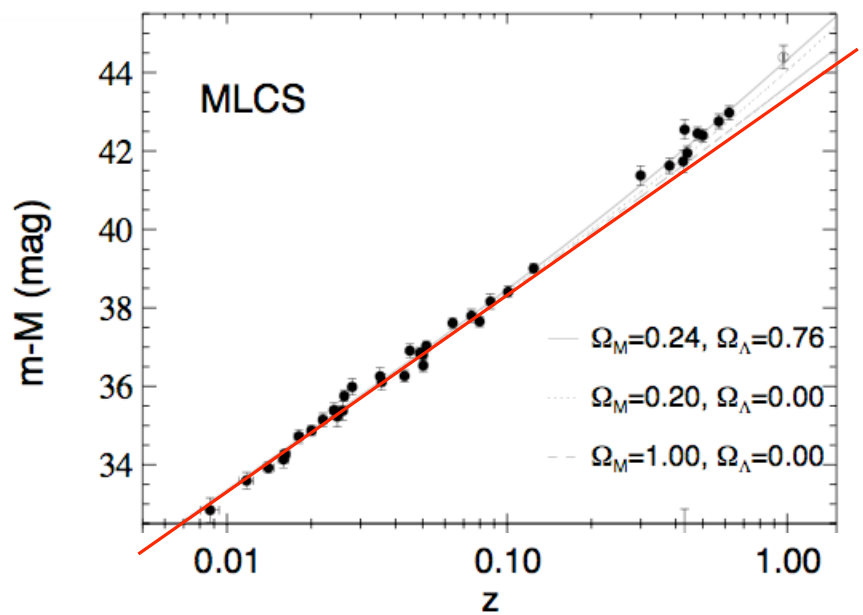
$$M^{\text{Bolometric}} = -\frac{5}{2}\log_{10}\left(I^{\text{Emitted}}_{\text{Energy/Time}}\right) + C'$$

$$m - M = -\frac{5}{2}\log_{10}\left(F^{\text{Obs}}\right) + \frac{5}{2}\log_{10}\left(I^{\text{Emit}}\right) + C''$$

$$= 5\log_{10}\left(\sqrt{I^{\text{Emit}}/F^{\text{Obs}}}\right) + C''$$

$$= 5\log_{10}\left(d_{\text{Luminosity}}\right) + C'''$$

Proxy for $\log(d_{\text{Lum}})$



Extended Hubble Relation $d_{\text{Lum}} \propto z$

If $d_{\text{Lum}} \propto z$ then

$$5\log_{10}(d_{\text{Lum}}) = 5\log_{10}(z) + C$$

$$m - M = 5\log_{10}(z) + C'$$

$t=0 \Rightarrow$ Big Bang
$t=t_0 \Rightarrow$ Now

$$a(t) = \left(t/t_0\right)^{2/3}$$

Flat

$$H_0 \equiv \frac{\dot{a}(t_0)}{a(t_0)} = \frac{2}{3t_0} \quad \text{Hubble constant}$$

$$q_0 \equiv -\frac{\ddot{a}(t_0)}{a(t_0)H_0^{\,2}} = \frac{1}{2} \quad \text{Deceleration parameter}$$

$$1 + z(t_1,t_0) = t_0^{\,2/3} t_1^{\,-2/3} \quad \text{Redshift from } t_1 \text{ to } t_0$$

Recall from Lec 2

$$\frac{d\chi^\gamma(t)}{dt} = \pm\frac{c}{a(t)}$$

$$\chi^\gamma(t) = \pm\int \frac{c}{a(t')}dt' + \text{Const}$$

$$d_{\text{Lum}}(t_1,t_0) = a(t_0)\Delta\chi^\gamma(t_1,t_0)(1+z) = \underbrace{\left(\int_{t_1}^{t_0} \frac{c}{a(t)}dt\right)}_{\text{Cosmology!}}(1+z)$$

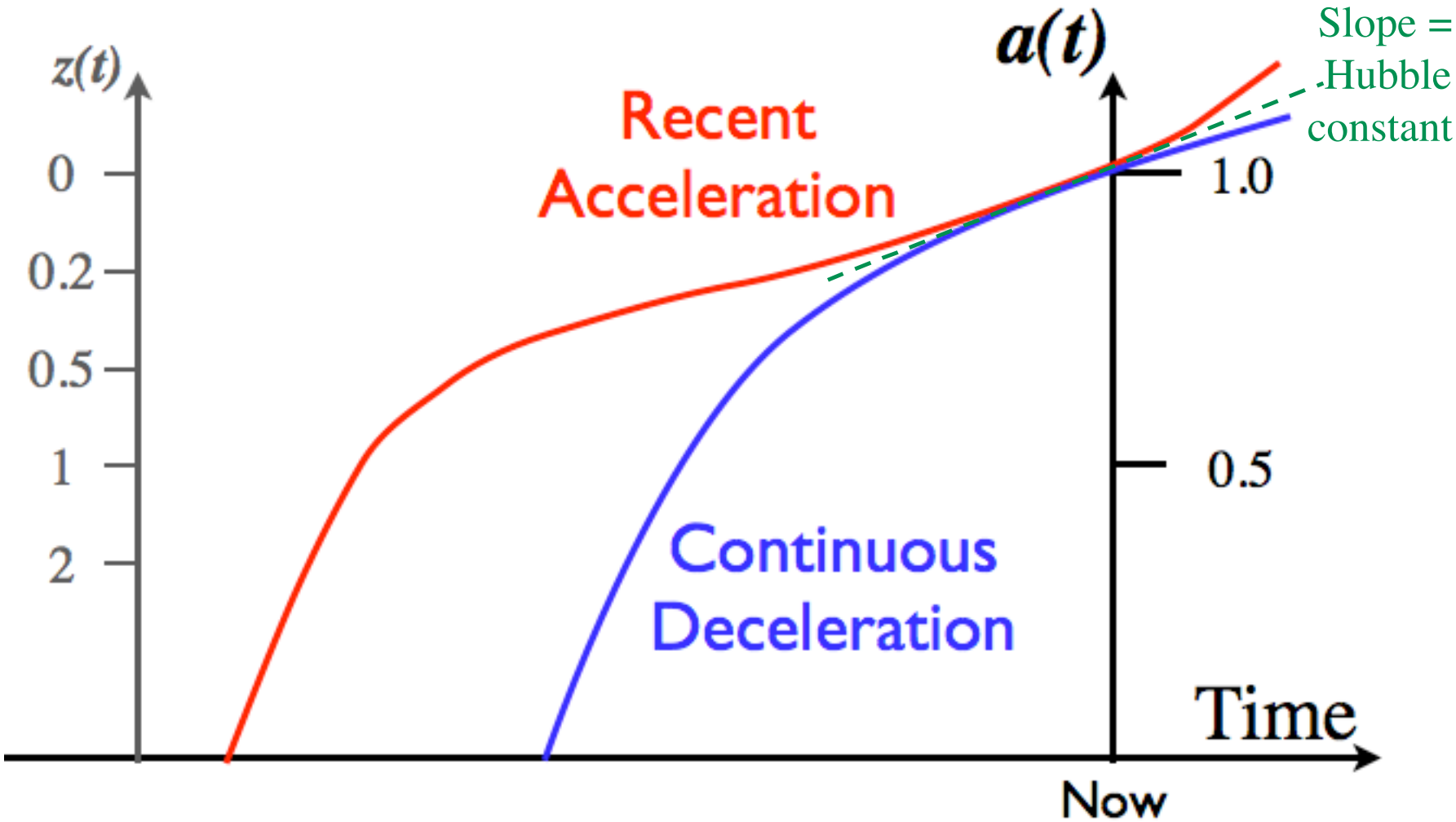
$$d_{\text{Lum}}(t_1,t_0) = \frac{c}{H_0}\left[z + z^2/4 + O(z^3)\right]$$

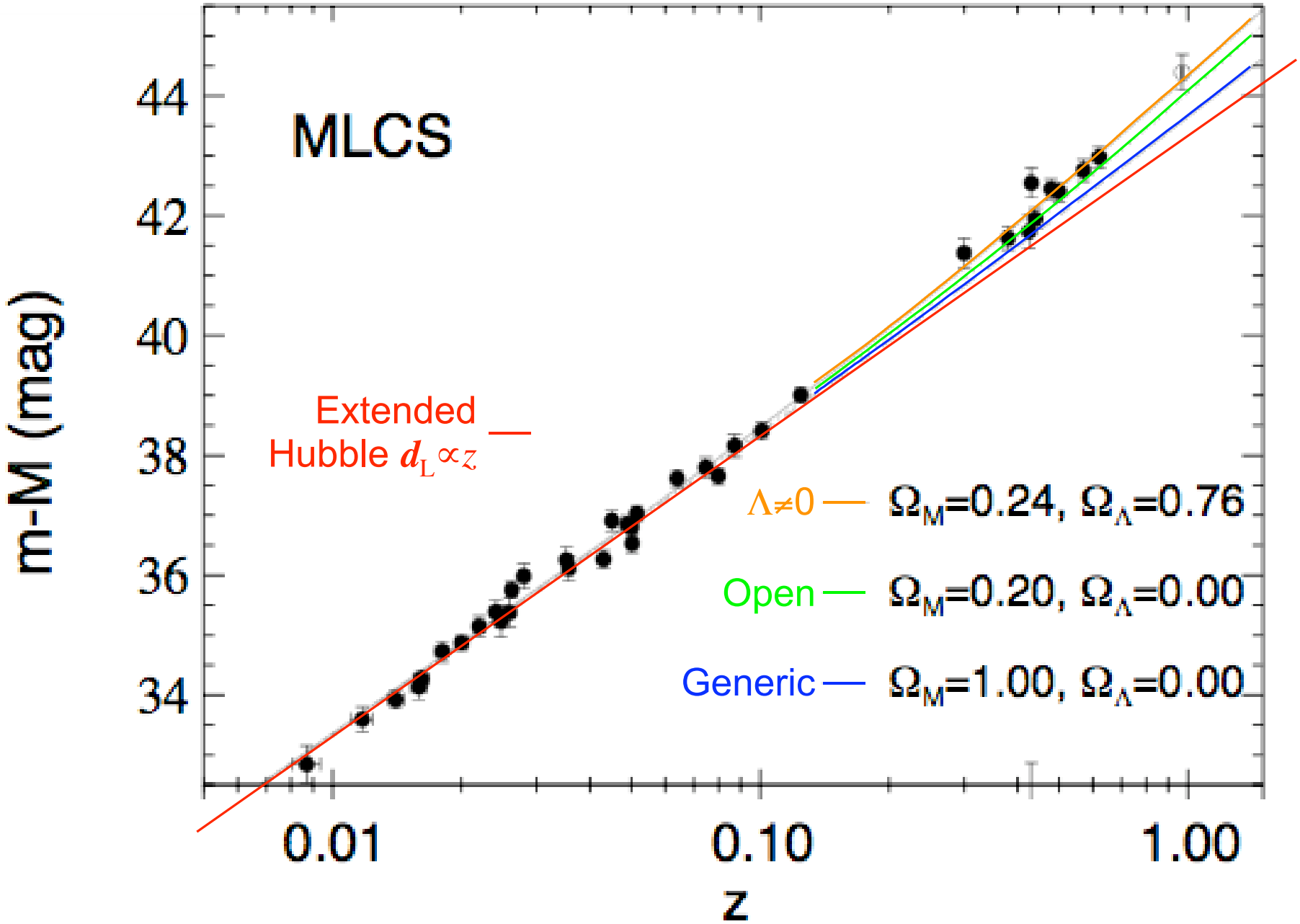
More generally (all cosmologies):

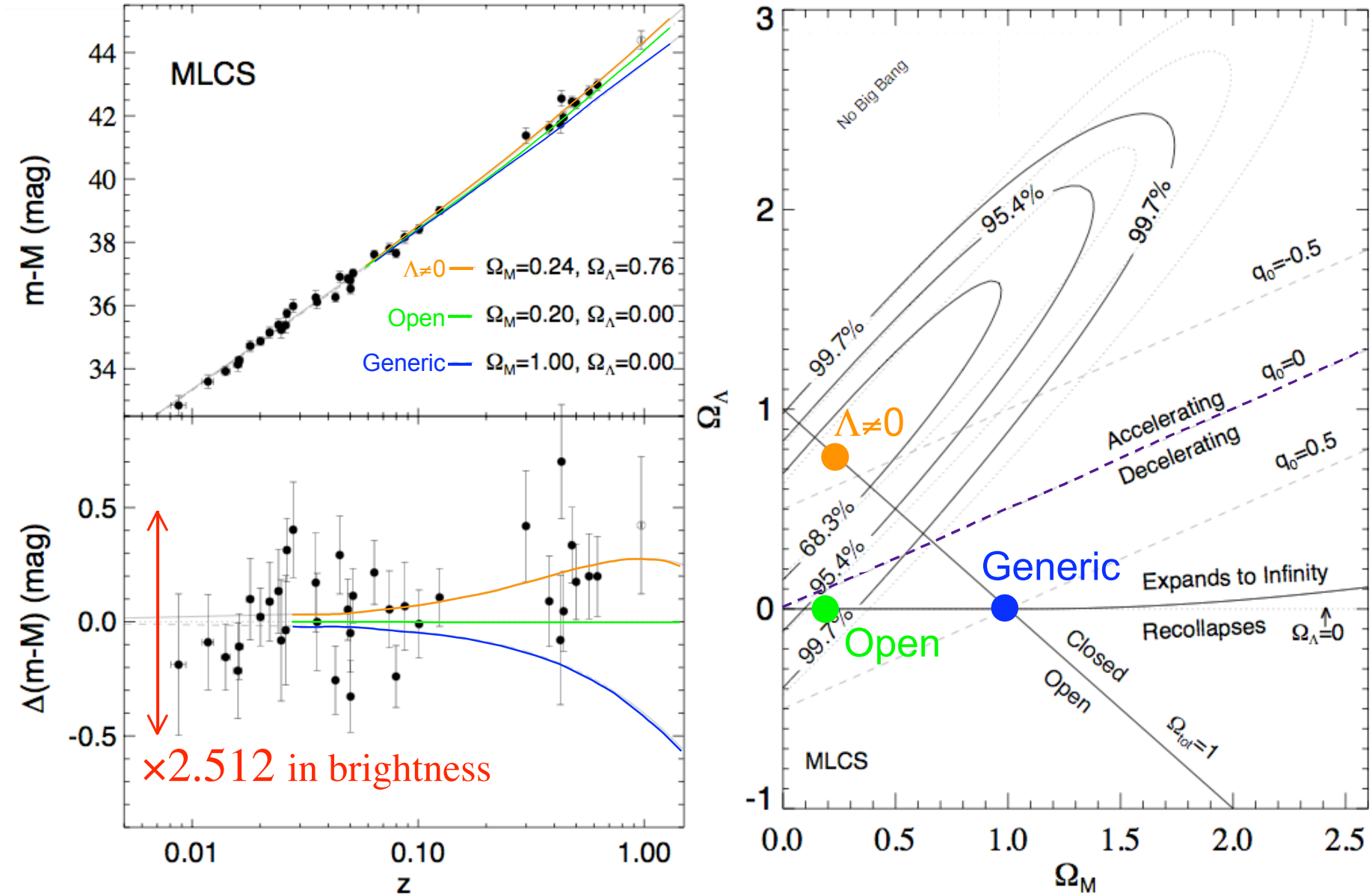
Weinberg 14.6.8

$$d_{\text{Lum}}(t_1,t_0) = \frac{c}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + O(z^3)\right]$$

Today's big question:




MLCS
Extended Hubble $d_L \propto z$
$\Lambda \neq 0$ — Ω_M=0.24, Ω_Λ=0.76
Open — Ω_M=0.20, Ω_Λ=0.00
Generic — Ω_M=1.00, Ω_Λ=0.00
m-M (mag)
44
42
40
38
36
34
0.01
0.10
1.00
z


MLCS
m-M (mag)
Λ≠0 — Ω_M=0.24, Ω_Λ=0.76
Open — Ω_M=0.20, Ω_Λ=0.00
Generic — Ω_M=1.00, Ω_Λ=0.00
Δ(m-M) (mag)
×2.512 in brightness
z
No Big Bang
95.4%
99.7%
68.3%
q_0=-0.5
q_0=0
q_0=0.5
Accelerating
Decelerating
Λ≠0
Generic
Open
Expands to Infinity
Recollapses
Ω_Λ=0
Closed
Open
Ω_tot=1
MLCS
Ω_Λ
Ω_M

Points to take home

- Extend the Hubble relation to “see” cosmology
- Distance to past events somewhat ambiguous
- Operational definitions: luminosity and angular distances
- Observational units: red-shifts and magnitudes
- Second derivative of d_{Lum} vs z for distant/ancient supernovae reveals cosmic acceleration