# Expanding Universes

Intro Cosmology Short Course

Lecture 2

Paul Stankus, ORNL

# The original Hubble Diagram

"A Relation Between Distance and Radial Velocity Among Extra-Galactic Nebulae" E.Hubble (1929)

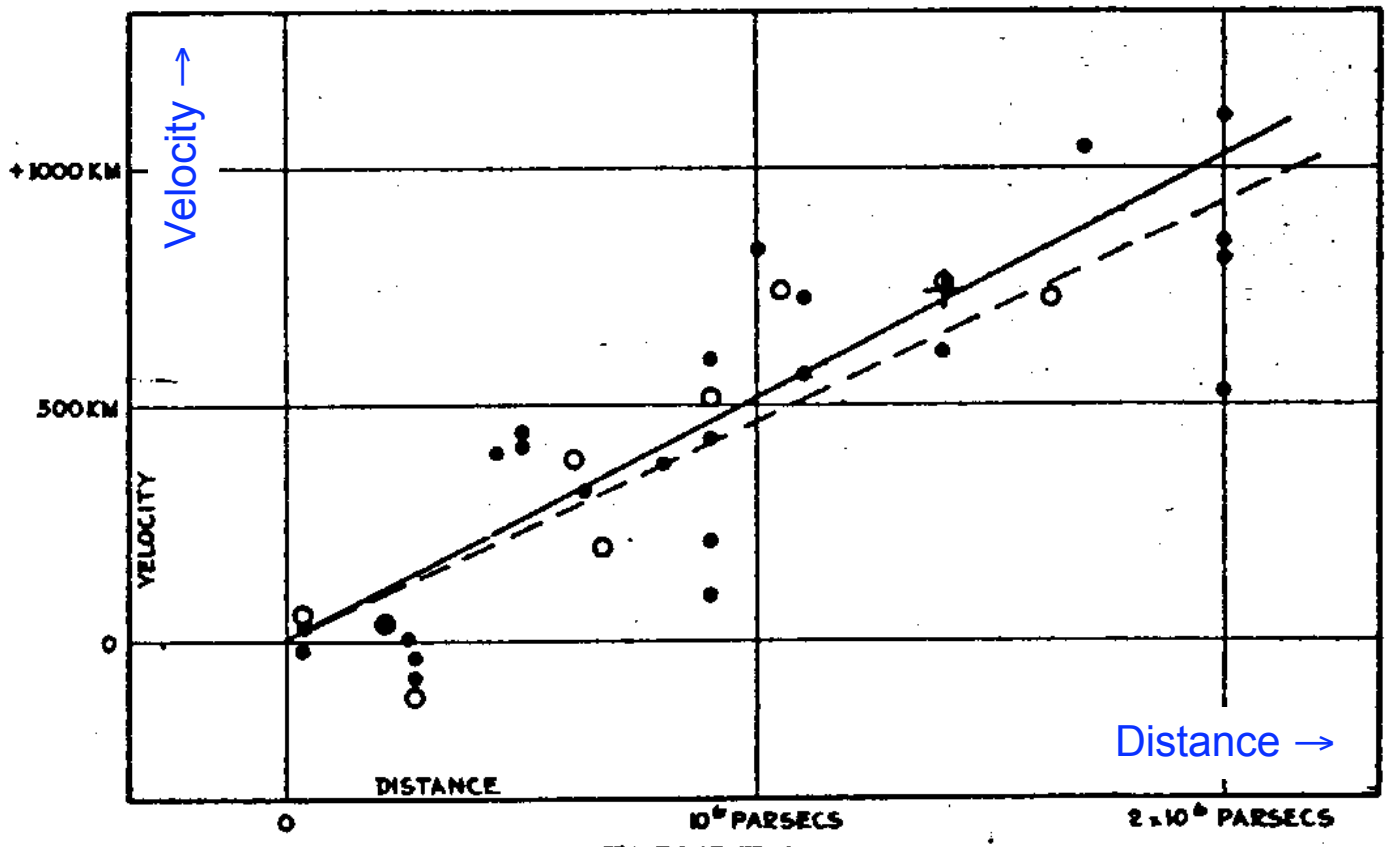
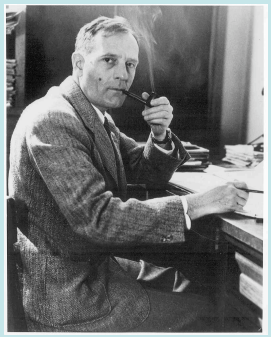


FIGURE 1



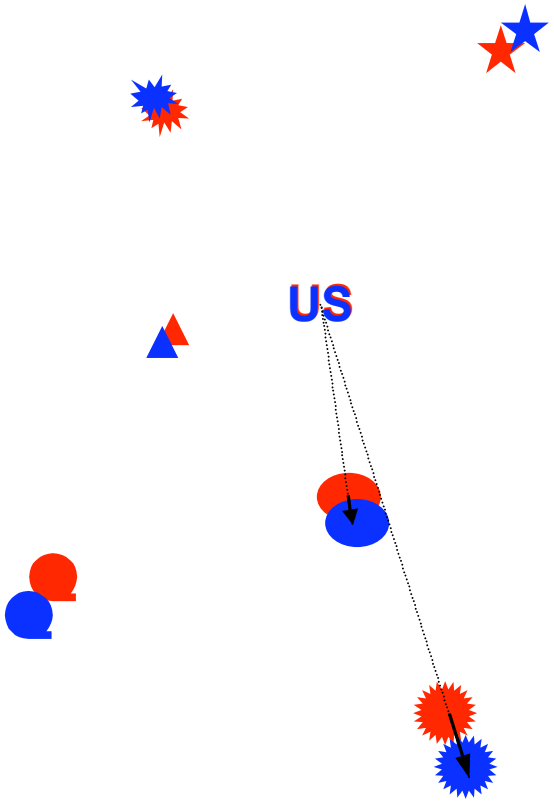
**Edwin Hubble**
American

Galaxies outside Milky Way

**Henrietta Leavitt**
American

Distances via variable stars

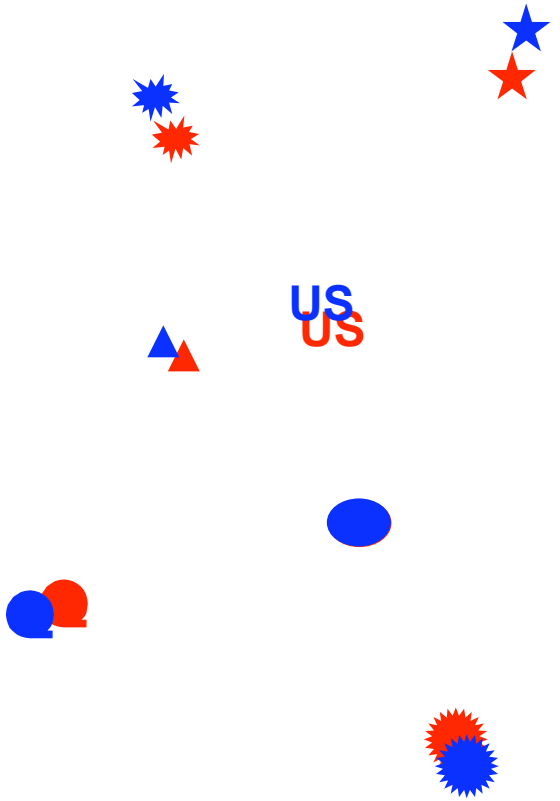
As seen from our position:



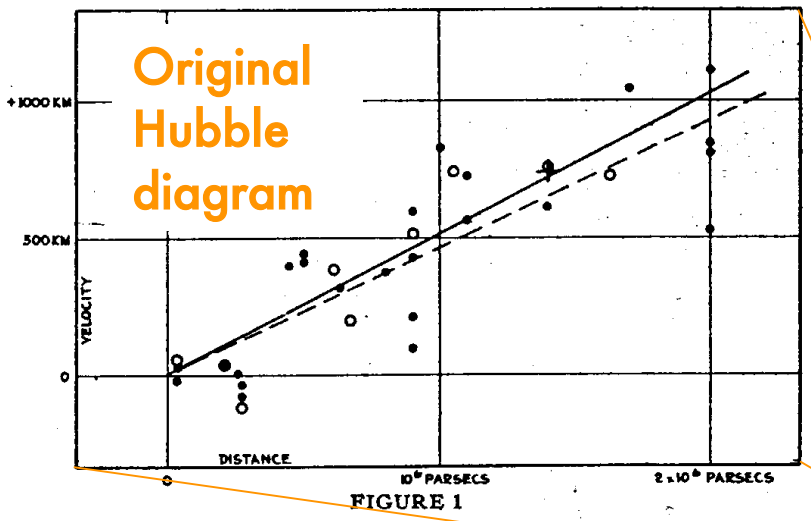

Now

Slightly Earlier

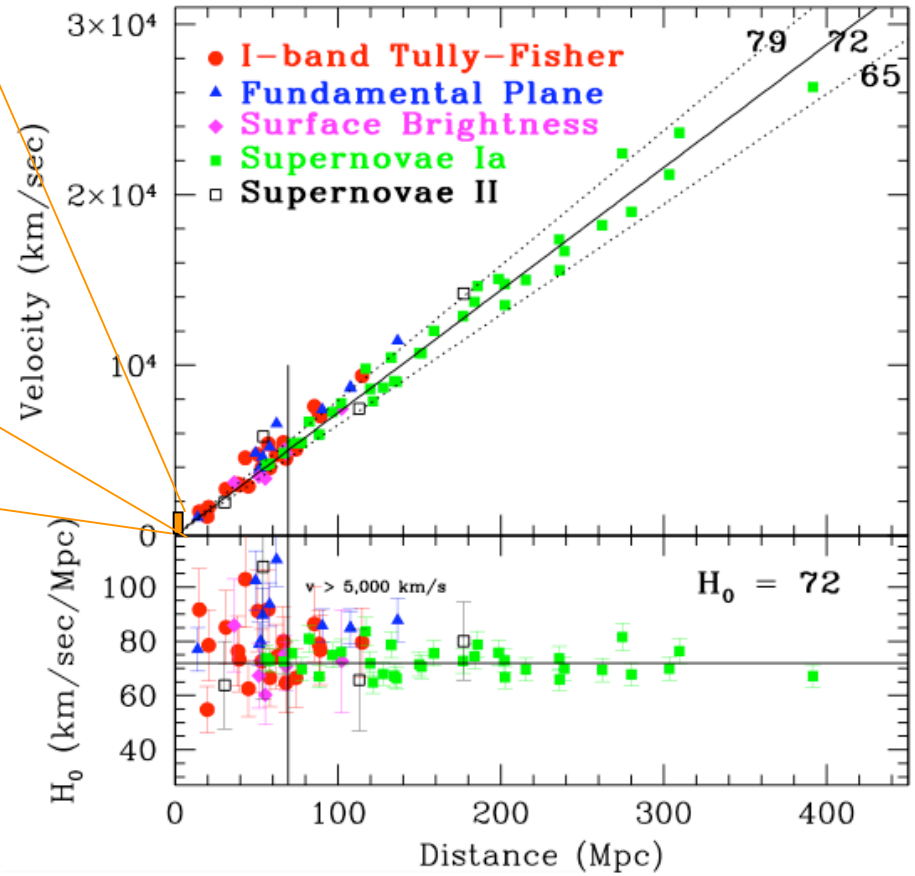
As seen from another position:




Same pattern seen by all observers!




## Modern Hubble Diagram




Freedman, et al. Astrophys. J. **553**, 47 (2001)

Hubble relation:

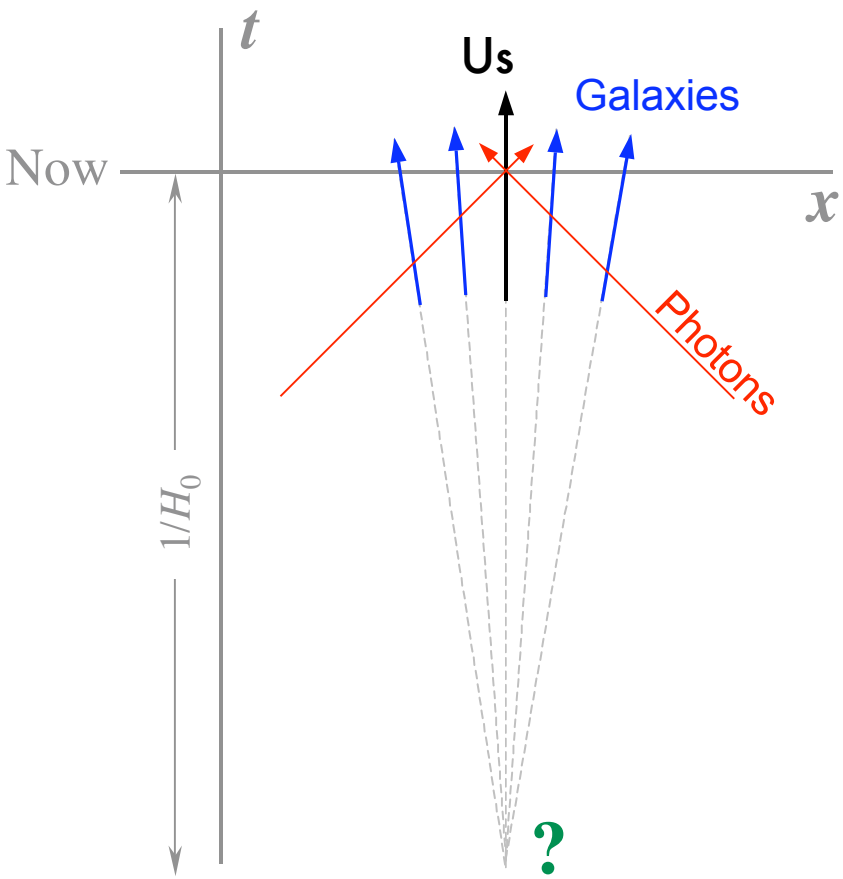
$$v_{\text{Recession}} = H_0\, d$$

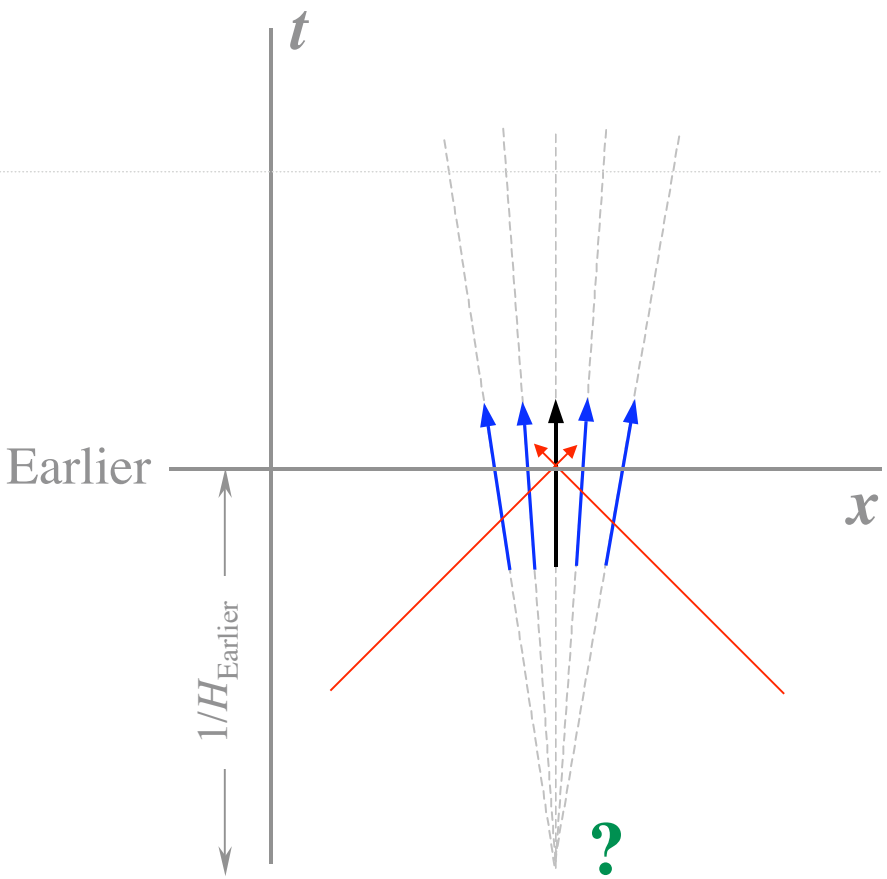
1929: $H_0$ ~500 km/sec/Mpc

2001: $H_0 = 72 \pm 7$ km/sec/Mpc

**W. Freedman**
Canadian

Modern Hubble constant (2001)




$$v_{\text{Recession}} = H_0\, d$$

$1/H_0 \sim 10^{10}$ year ~ Age of the Universe?

$H(\text{Earlier}) > H(\text{Now})$

$H(\text{Earlier}) > H_0$

# Friedmann-LeMatire Cosmologies

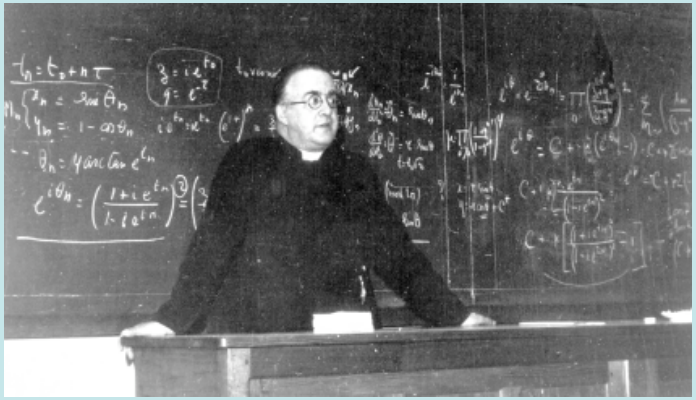
1. Isotropy Same in all directions
2. Homogeneity Same at all locations

"Cosmological Principle"
"Copernican Principle"

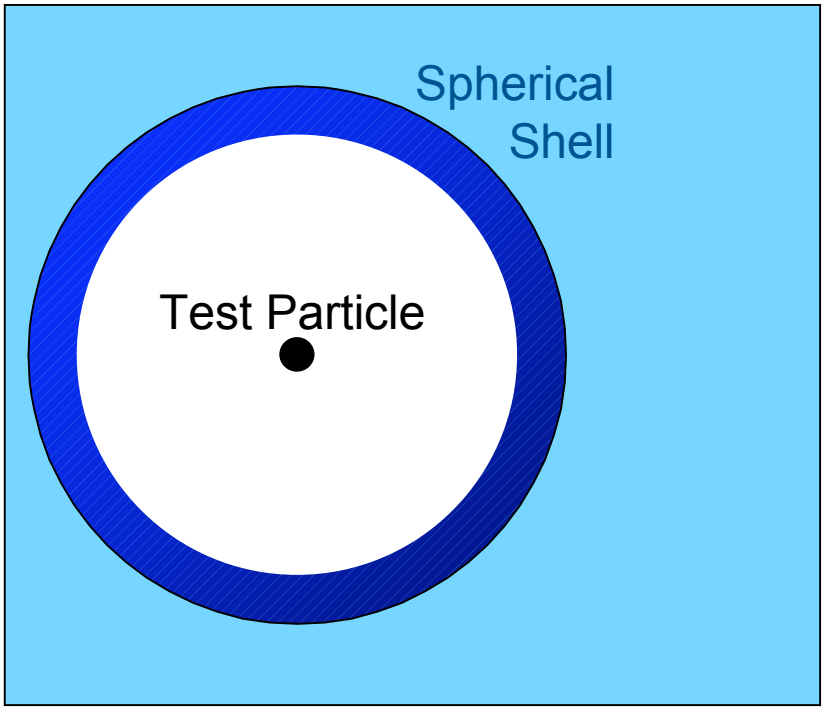
**A. Friedmann**
Russian

**G. LeMaitre**
Belgian

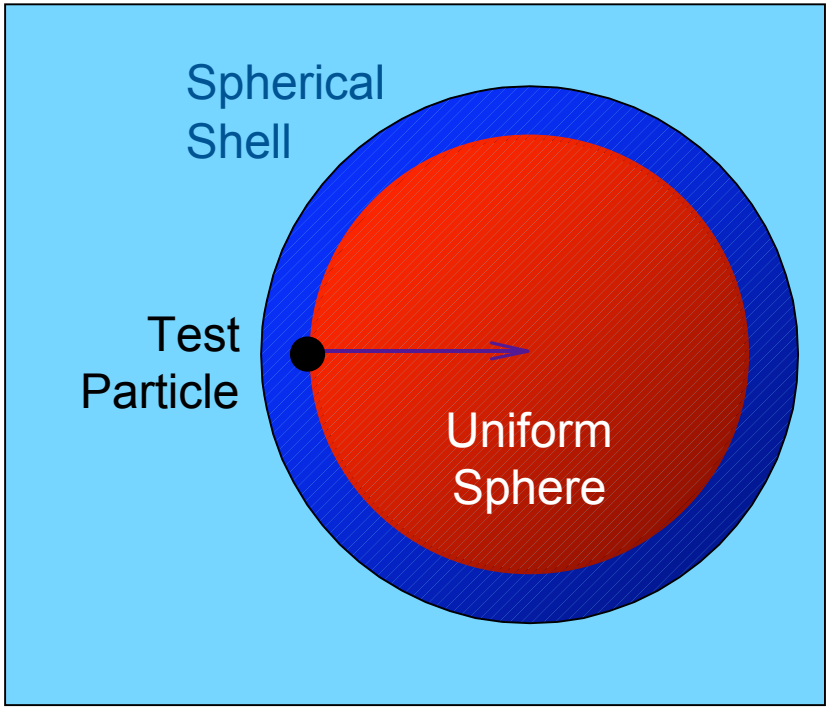
Evolution of homogeneous, non-static (expanding) universes
"Friedmann models" (1922, 1927)



# Warning: *No* Newtonian Solution
## (Get used to it, get over it)




$\boldsymbol{F}, \boldsymbol{a} = 0$




$\boldsymbol{F}, \boldsymbol{a} \neq 0$

## Free-fall Paths at constant $\chi$

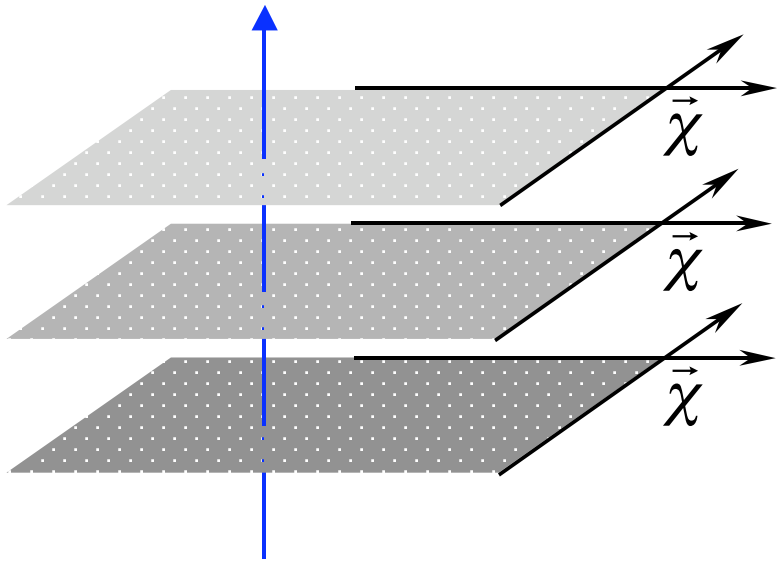
Surfaces $t$ are simultaneous to these observers

Choose $t$ such that $d\tau = dt$ along these paths

Space-like Hyper-surfaces

Each at constant $t$

Energy density, spatial curvature, and pressure are all uniform across each surface




Spatial $\chi$ Coordinate Systems

Distances within each space-like hyper-surface all obey $d = a(t)\Delta\chi$

**H.P. Robertson** American

**A.G. Walker** British

Formalized most general form of isotropic and homogeneous universe in GR "Robertson-Walker metric" (1935-6)

# Minkowski Metric

$$d\tau^2 = dt^2 - dx^2/c^2$$

# Robertson-Walker Metric

$$d\tau^2 = dt^2 - [a(t)]^2\, d\chi^2/c^2$$

1. The $\chi$ coordinate system is uniform (not necessarily Euclidean)
2. $\chi$ has units of length
3. $a$ is dimensionless
4. By convention $a(\text{now}) \equiv a_0 = 1$
5. Physical distance for $\Delta\chi$ on surface $t$ is $d = a(t)\,\Delta\chi$

# Three basic (Euclidean) solutions for $a(t)$:

1. Relativistic gas, "radiation dominated"

   $P/\rho = 1/3 \quad \rho \propto a^{-4} \quad \boxed{a(t) \propto t^{1/2}}$

2. Non-relativistic gas, "matter dominated"

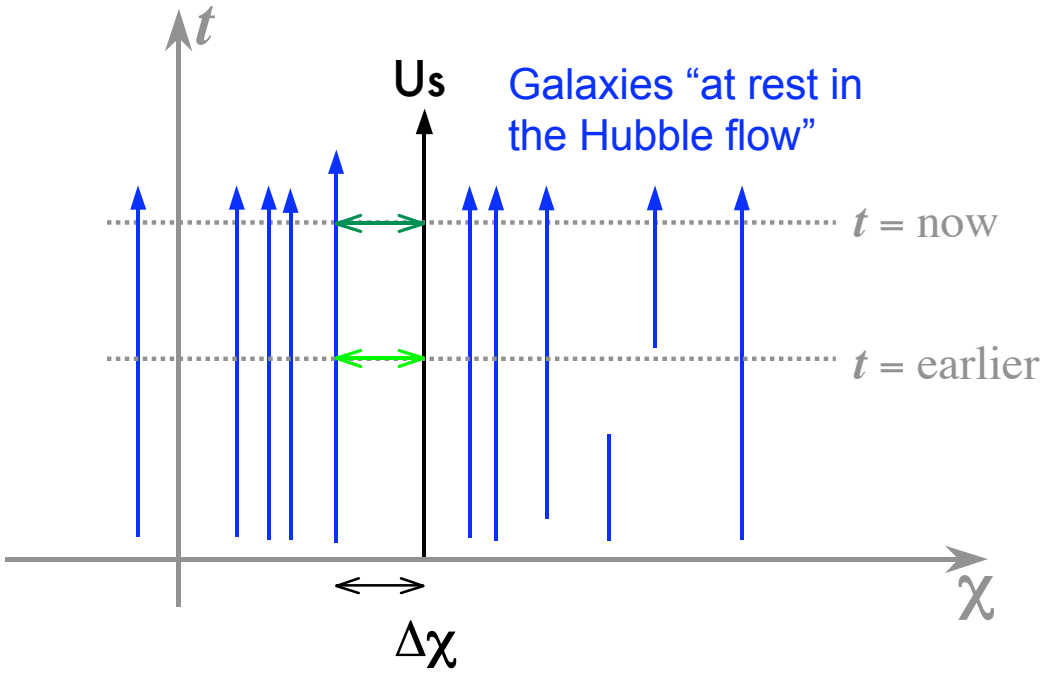
   $P/\rho = 0 \quad \rho \propto a^{-3} \quad \boxed{a(t) \propto t^{2/3}}$

3. "Cosmological-constant-dominated" or "vacuum-energy-dominated"

   $P/\rho = -1 \quad \rho \propto \text{constant} \quad \boxed{a(t) \propto e^{Ht}}$ "de Sitter space"

# Galaxies

Robertson-Walker Coordinates




$$d\tau^2 = dt^2 - \left[a(t)\right]^2 d\chi^2 / c^2$$

Galaxies

$\chi$ = constant

"At rest in the Hubble flow"

Hubble "Constant"

$$H(t) = \frac{\text{velocity}}{\text{distance}} = \frac{\frac{d}{dt}\left[a(t)\Delta\chi\right]}{a(t)\Delta\chi} = \frac{\dot{a}(t)}{a(t)}$$

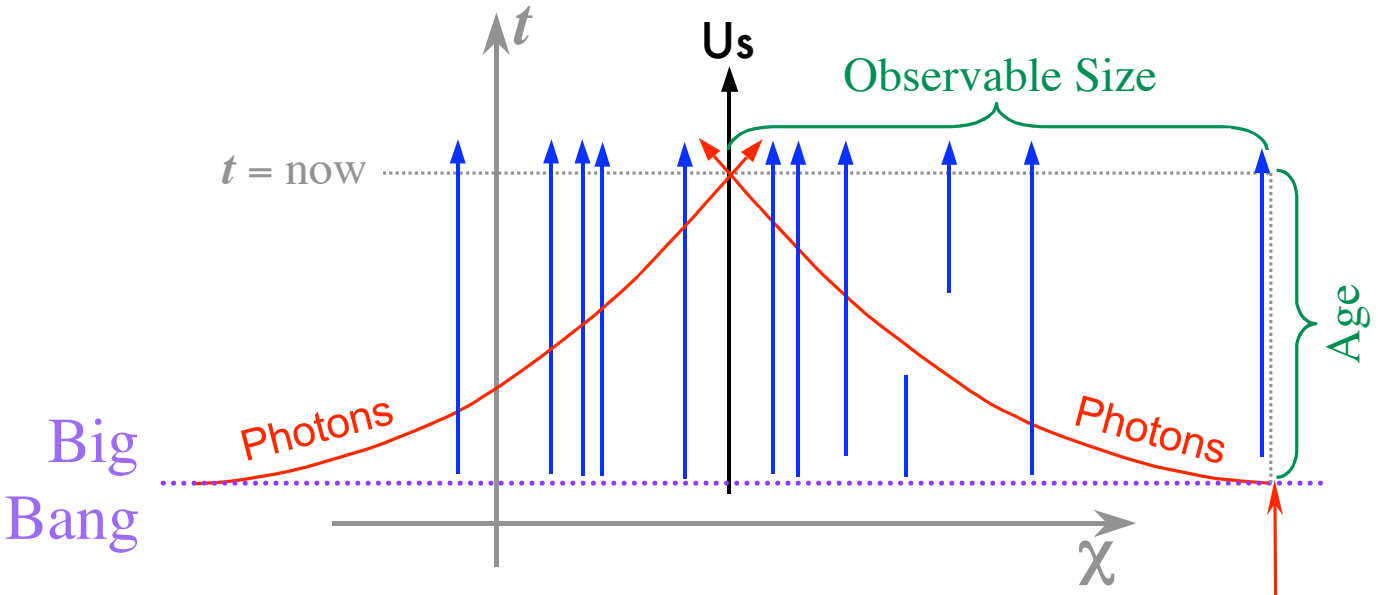
⟷ Current separation $\boldsymbol{d}$ (now) = $\boldsymbol{a}$(now) $\Delta\chi = \boldsymbol{a}_0\,\Delta\chi$

⟷ Earlier separation $\boldsymbol{d}$ (earlier) = $\boldsymbol{a}$(earlier) $\Delta\chi < \boldsymbol{d}$ (now)

# Photons

$$d\tau^2 = dt^2 - \left[a(t)\right]^2 d\chi^2 / c^2$$

Robertson-Walker Coordinates




Photons

$$d\tau = 0$$

$$\frac{dt}{d\chi} = \pm a(t)/c$$

$$\frac{d\chi}{dt} = \pm \frac{c}{a(t)}$$

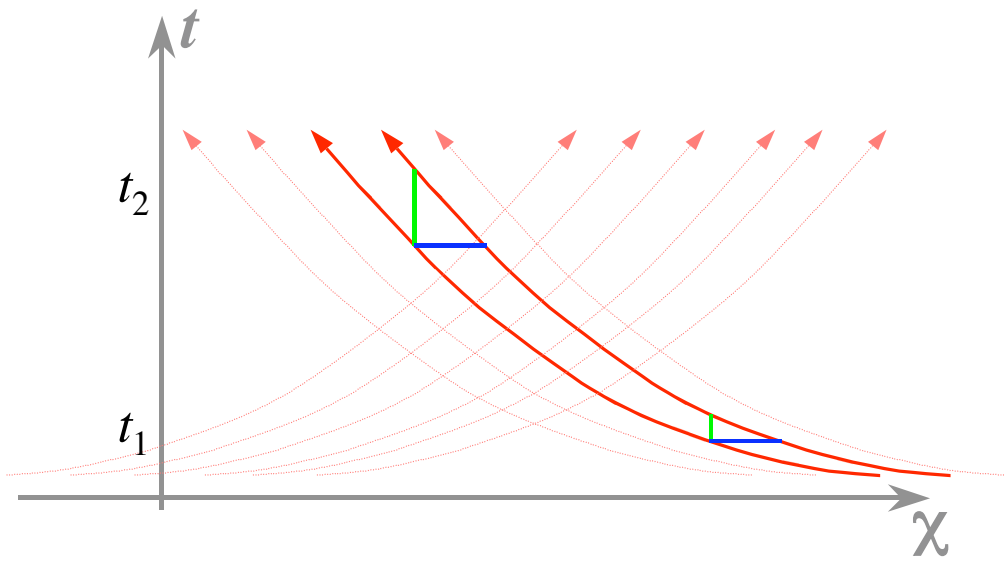
Slope = 0

$$a(t) = 0$$

# Cosmological Red-Shift

$$d\tau^2 = dt^2 - \left[a(t)\right]^2 d\chi^2 / c^2$$

Robertson-Walker Coordinates




Photons

$$\frac{d\chi^{\gamma}(t)}{dt} = \pm\frac{c}{a(t)}$$

$$\chi^{\gamma}(t) = \pm\int \frac{c}{a(t')}dt' + \text{Const}$$

Period

Wavelength

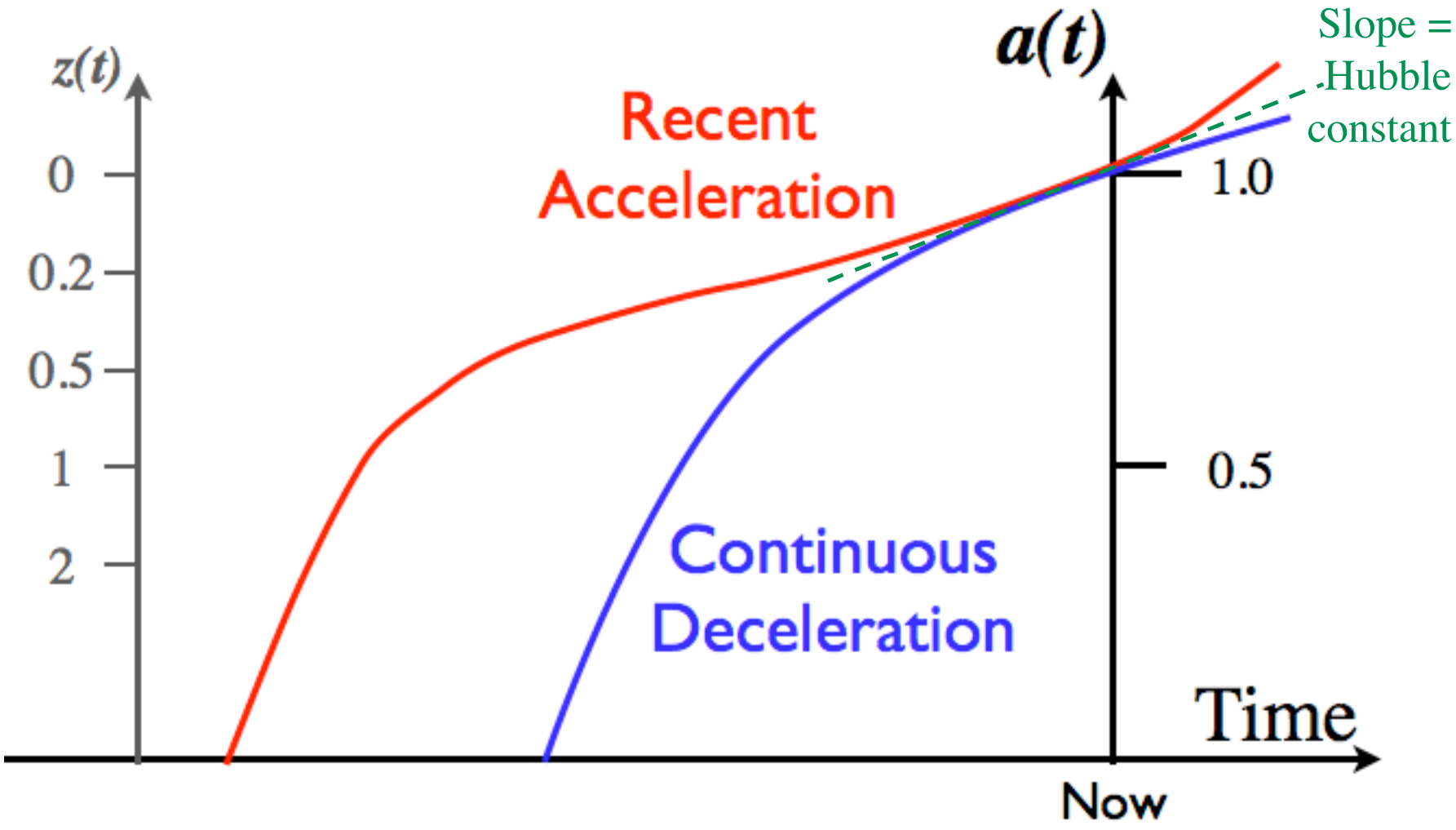
Cosmological Red Shift

A photon's period grows $\propto a(t)$

Its coordinate wavelength $\Delta\chi$ is constant; its physical wavelength $a(t)\Delta\chi$ grows $\propto a(t)$

$\lambda(t_2)/\lambda(t_1) = a(t_2)/a(t_1) \equiv 1+z$

# Today's big question:



# Points to take home

- Hubble expansion:  recession velocity $\propto$ distance away
- Uniform expansion pattern
- Describe in GR by Robertson-Walker metric
- Galaxies “at rest in the flow,” but physically separating
- Photon paths simple, derive cosmological red-shift