Why Brooklyn Is Not Expanding

Intro Cosmology Short Course
Addendum to Lecture 2

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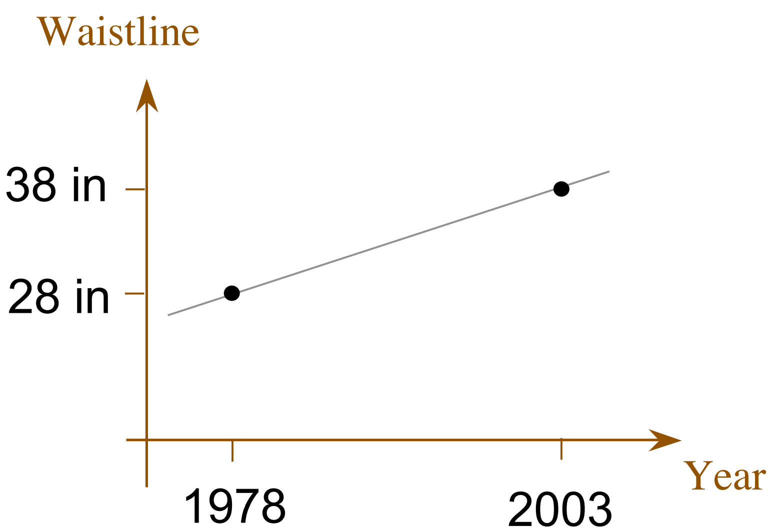
It's really Hubble's fault….



How much of this can I blame on the Hubble expansion?

$$\frac{\Delta d_{\mathrm{AB}}}{d_{\mathrm{AB}}} = \frac{(v_A - v_B)\Delta t}{d_{\mathrm{AB}}} = \frac{H_0\, d_{\mathrm{AB}} \Delta t}{d_{\mathrm{AB}}} = \frac{\Delta t}{H_0^{-1}}$$

$$28\mathrm{in} \times 25\mathrm{y}/10\mathrm{Gyr} \approx 18\text{Å}$$

(I'll take what I can get)

A man once asked me:

"If space itself is expanding, then why isn't the distance from the Earth to the Moon increasing?"

If Earth and Moon were both in the Hubble flow:

$$\Delta d_{\text{Earth-Moon}} = \frac{\Delta t}{H_0^{-1}} d_{\text{Earth-Moon}}$$

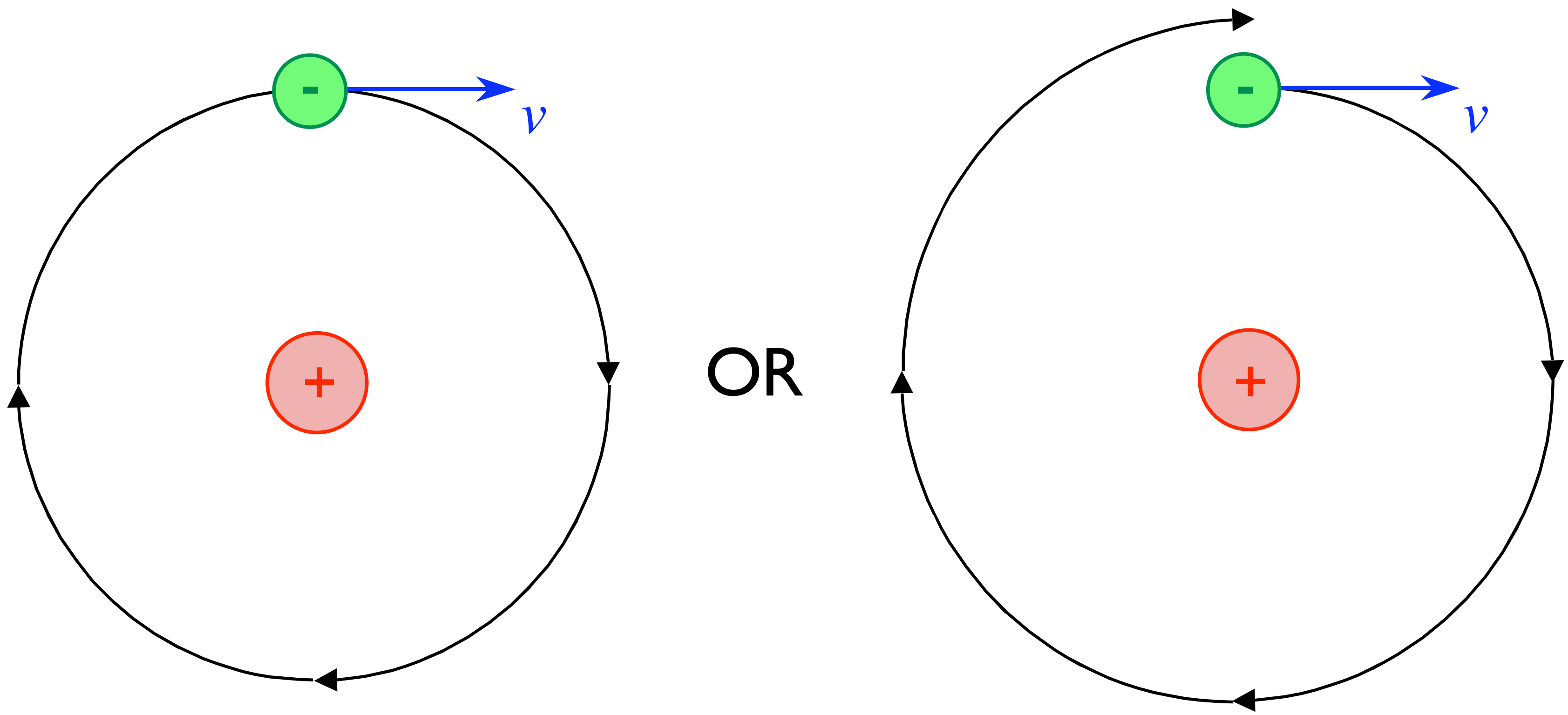
$$= \frac{35\text{yr} \times 384{,}000\text{km}}{10 \times 10^9 \text{yr}} \approx 1.3\text{m}$$

Retroreflector on the Moon

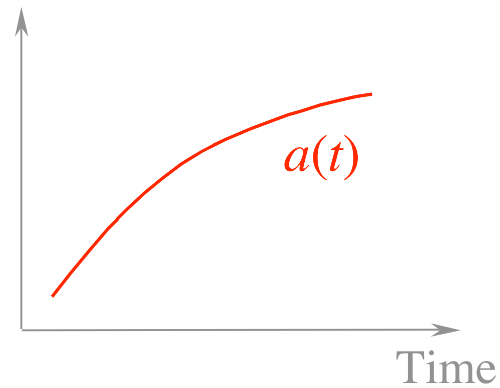
Laser ranging good to ~3cm

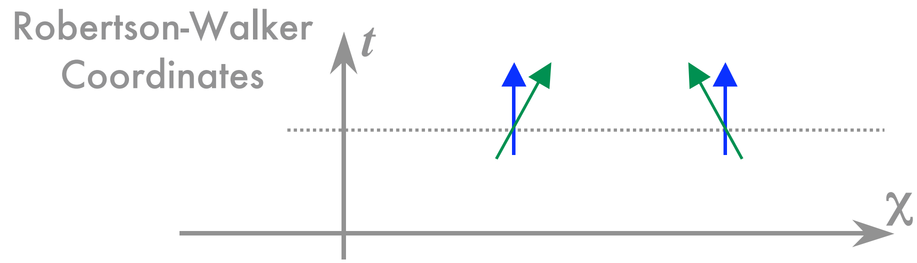
An initial value problem

Assume that the Robertson-Walker metric is valid down to very small scales. How does the cosmic expansion $a(t)$ <u>change</u> the relative trajectory of two particles?

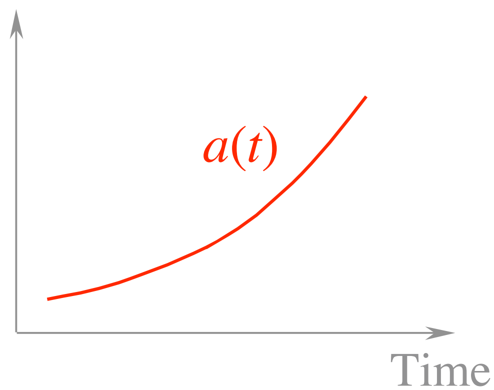


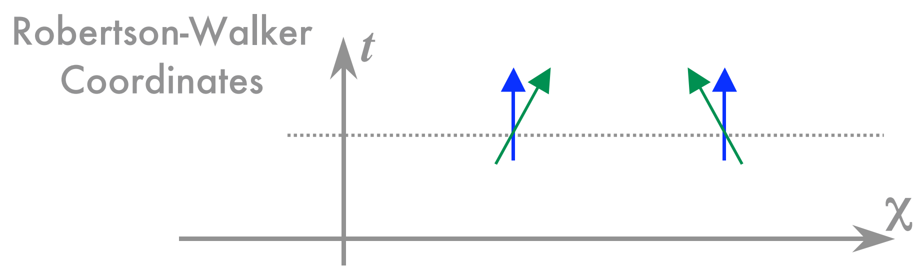
A -- very small -- apparent force





Velocity: Apart   Acceleration: Together
Velocity: Zero   Acceleration: Together





Velocity: Apart   Acceleration: Apart
Velocity: Zero   Acceleration: Apart

OK, but how small?

(Ans: ~~*damn*~~ *darn* small)

$$\text{Apparent acceleration} \sim \pm \left(H_0\right)^2 \times \text{separation}$$

Normal $a_{\text{Moon-Earth}} \sim d_{\text{Moon-Earth}} /(1 \text{ month})^2$ (up to 2π)

Extra $a_{\text{Moon-Earth}} \sim d_{\text{Moon-Earth}} (H_0)^2$ (up to 8π)

$d_{\text{Moon-Earth}}/(10 \text{ Gyr})^2$

$$F_{\text{Coulomb}} = \frac{q_1 q_2}{r^2} \pm H_0{}^2 r$$