Transversity, Sivers Function and Collins Fragmentation Functions: Towards a New Global Analysis

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The fundamental distributions of partons inside a nucleon

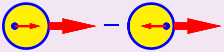
Unpolarised Distribution

$f_1(x)$ or $q(x)$

Distribution of unpolarised partons in an unpolarised nucleon.
Well known

Helicity Distribution

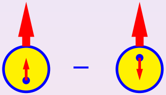
$g_1(x)$ or $\Delta q(x)$



Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.
Known

Transversity Distribution

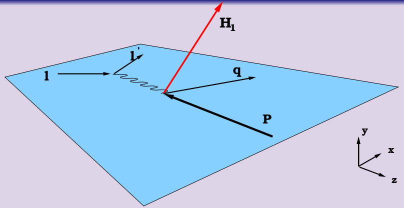
$h_1(x)$ or $\Delta_T q(x)$



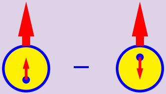
Distribution of transversely polarised quarks in a transversely polarised nucleon.
Little known!
HERMES and COMPASS experimental measurements

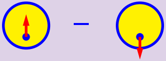
SIDIS and e^+e^- annihilation

SIDIS $lN \to l' H_1 X$



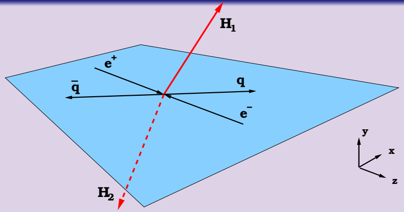
Collins effect gives rise to azimuthal Single Spin Asymmetry

 $= \Delta_T q(x, Q^2)$

 $= \Delta^N D_{h/q^\uparrow}(z, Q^2)$

J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

$e^+e^- \to H_1 H_2 X$



Collins effect gives rise to azimuthal asymmetry, q and $\bar{q}$ Collins functions are present in the process:

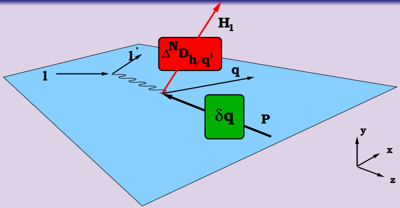
$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$

$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$

D. Boer, R.Jacob and P. J. Mulders *Nucl. Phys.* **B504** (1997) 345

SIDIS and e^+e^- annihilation

SIDIS $lN \to l' H_1 X$

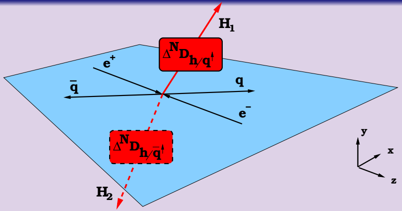


Cross Section $\sim sin(\phi_H + \phi_S)\cdot$
$\Delta_T q(x, Q^2) \otimes \Delta^N D_{h/q^\uparrow}(z, Q^2)$

We extract (PRD75:054032,2007)

$\Delta_T q(x, Q^2)$, $\Delta^N D_{h/q^\uparrow}(z, Q^2)$

$e^+e^- \to H_1 H_2 X$



Cross Section $\sim cos(\phi_{H_1} + \phi_{H_2})\cdot$
$\Delta^N D_{h/q^\uparrow}(z_1) \otimes \Delta^N D_{h/\bar{q}^\uparrow}(z_2)$

We extract (PRD75:054032,2007)

$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$,
$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$

Collins function and transversity distribution

Model for Collins FF

For $\Delta^N D_{h/q^\uparrow}(z, |p_\perp|) = \frac{2|p_\perp|}{zM_\pi} H_1^{\perp q}(z, |p_\perp|)$ we use factorization of z and $p_\perp$ and Gaussian dependence on $p_\perp$, $\Delta^N D_{h/q^\uparrow} \propto z^\gamma (1-z)^\delta$, positivity constraint $|\Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp)| \leq 2 D_{h/q}(z, \mathbf{p}_\perp)$ is fulfilled.

Model for Transversity distribution

$$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) \left[f_{q/p}(x) + \Delta q(x) \right] \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T},$$

$\Delta_T q(x) \propto x^\alpha (1-x)^\beta$, Soffer bound

$$|\Delta_T q(x)| \leq \frac{1}{2} \left[f_{q/p}(x) + \Delta q(x) \right]$$

is fulfilled.

Statistical errors of the fit

Numerical Recipes, Cambridge University Press, Third Edition (2007)

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - F(x_i; \mathbf{a})}{\sigma_i} \right)^2$$

for a set of N of experimental measurements y_i, σ_i. We estimate the values of M unknown parameters $\mathbf{a} = \{a_1, ..., a_M\}$,χ^2_{min} yields $\mathbf{a}_0$.

In order to estimate *statistical* error of our theoretical function $F(x; \mathbf{a})$ one uses all sets of parameters $\hat{\mathbf{a}}$ which satisfy:

$$\chi^2(\hat{\mathbf{a}}) - \chi^2(\mathbf{a}_0) < \Delta\chi^2 \ ,$$

where $\Delta\chi^2 = 1$ (ideal situation)

Statistical errors of the fit

Numerical Recipes, Cambridge University Press, Third Edition (2007)

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In order to estimate *statistical* error of our theoretical function $F(x; \mathbf{a})$ one uses all sets of parameters $\hat{\mathbf{a}}$ which satisfy:

$$\chi^2(\hat{\mathbf{a}}) - \chi^2(\mathbf{a}_0) < \Delta\chi^2 \ ,$$

or $\Delta\chi^2 = 2\div5\%$ of χ^2_{min} due to presence of unknown correlated experimental errors among different sets of experimental data. (see CTEQ pdf extraction Phys.Rev.D65:014012,2002, Phys.Rev.D65:014013,2002 or DSS FF analysis Phys.Rev.D75:114010,2007 or helicity distribution extraction De Florian, Sassot, Stratmann, Vogelsang Phys.Rev.Lett.101:072001,2008

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$$\chi^2(\hat{\mathbf{a}}) - \chi^2(\mathbf{a}_0) < \Delta\chi^2 \ ,$$

another method (used in our previous analysis PRD75:054032,2007) connects $\Delta\chi^2$ to 95.45% CL of coverage probability (see arXiv:0805.2677)

$$P = \int_0^{\Delta\chi^2} \frac{1}{2\Gamma(M/2)} \left(\frac{\chi^2}{2} \right)^{(M/2)-1} \exp\left(-\frac{\chi^2}{2} \right) \mathrm{d}\chi^2 \, .$$

Statistical errors of the fit

Numerical Recipes, Cambridge University Press, Third Edition (2007)

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - F(x_i; \mathbf{a})}{\sigma_i} \right)^2$$

for a set of N of experimental measurements y_i, σ_i. We estimate the values of M unknown parameters $\mathbf{a} = \{a_1, ..., a_M\}$,χ^2_{min} yields $\mathbf{a}_0$.

In order to estimate *statistical* error of our theoretical function $F(x; \mathbf{a})$ one uses all sets of parameters $\hat{\mathbf{a}}$ which satisfy:

$$\chi^2(\hat{\mathbf{a}}) - \chi^2(\mathbf{a}_0) < \Delta\chi^2 \ ,$$

In this analysis for simplicity we present statistical errors of the fit calculated with

$$\Delta\chi^2 = 1$$

Description of $A_{UT}^{sin(\phi_h+\phi_S)}$

We use HERMES and COMPASS data sets on $A_{UT}^{sin(\phi_h+\phi_S)}$ in the fitting procedure, we use one of the two sets of data from BELLE corresponding to either $\cos(\varphi_1+\varphi_2)$ or $\cos(2\varphi_0)$ extraction method.
Favored and unfavored fragmentation functions are defined as follows:

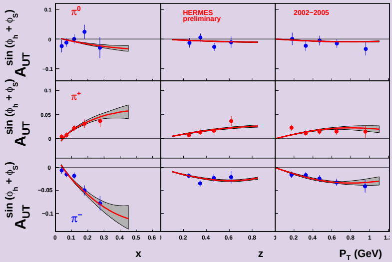
$$D^{fav}(z) \equiv D^{u\to\pi^+}(z) = D^{d\to\pi^-}(z) = D^{\bar{u}\to\pi^-}(z) = D^{\bar{d}\to\pi^+}(z)$$
$$D^{unfav}(z) \equiv D^{u\to\pi^-}(z) = D^{d\to\pi^+}(z) = D^{\bar{u}\to\pi^+}(z) = D^{\bar{d}\to\pi^-}(z)$$

For simplicity we assume that Collins FFs have universal z behaviour and transversity for u and d quarks have universal x behaviour:

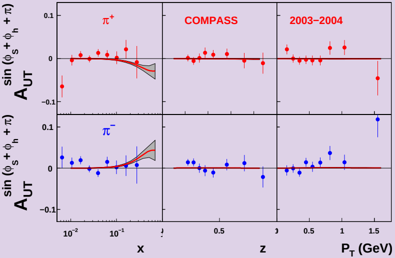
$$\alpha_u = \alpha_d \equiv \alpha, \beta_u = \beta_d \equiv \beta$$
$$\gamma_{fav} = \gamma_{unfav} \equiv \gamma, \delta_{fav} = \delta_{unfav} \equiv \delta$$

Preliminary results

HERMES $A_{UT}^{sin(\phi_h+\phi_S)}$



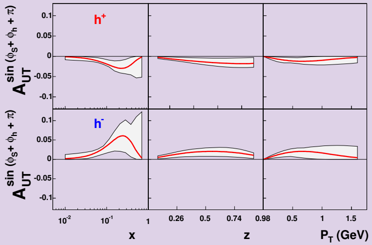
COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$



Preliminary results

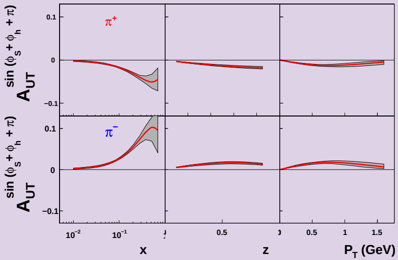
Predictions for COMPASS operating on PROTON target

COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$



Anselmino et al, Phys. Rev. D **75** (2007) 054032

COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$

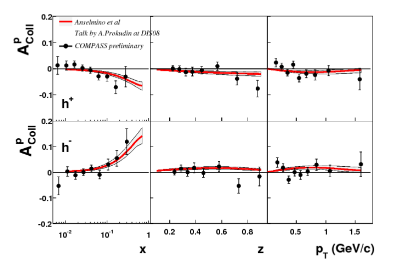


This extraction

Preliminary results

Predictions for COMPASS operating on PROTON target

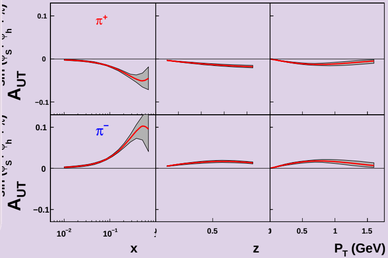
Comparison with preliminary COMPASS data arXiv:0808.0086



C

Anselmino et al, Phys. Rev. D **75** (2007) 054032

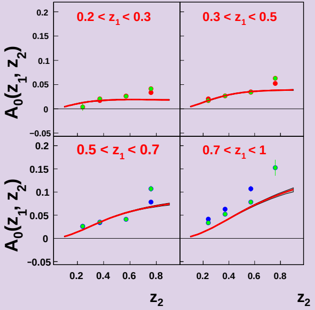
OMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$



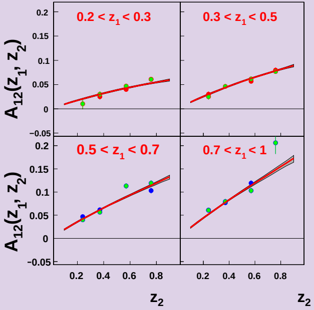
This extraction

Preliminary results

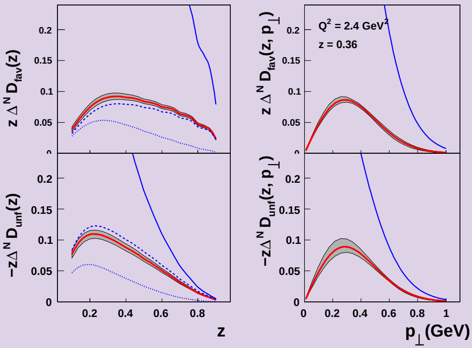
BELLE $\cos(2\varphi_0)$



BELLE $\cos(\varphi_1 + \varphi_2)$



Collins fragmentation function

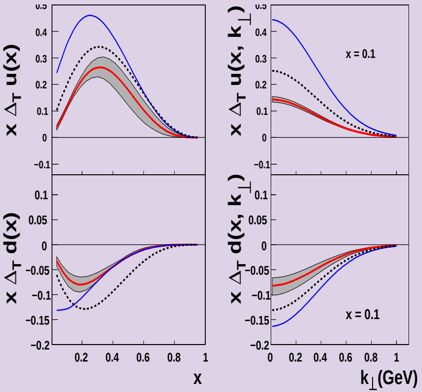


compared to Ref. [1] (dashed line), Ref. [2] (dotted line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).

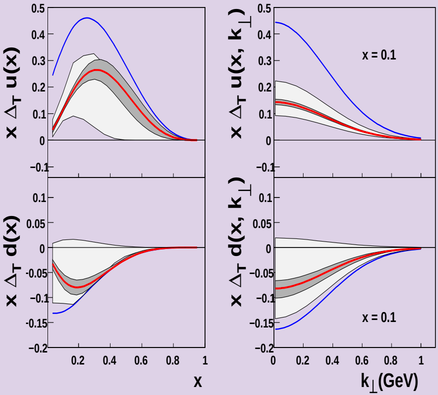
[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).

Transversity



- This is the extraction of transversity from new experimental data.
- Compared to previous extraction PRD75:054032,2007
- Compared to $\Delta\chi^2 = 1$ error estimate of PRD75:054032,2007
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$ The errors are diminished significantly.
- $\Delta_T u(x)$ became larger than that of the previous fit.

Transversity



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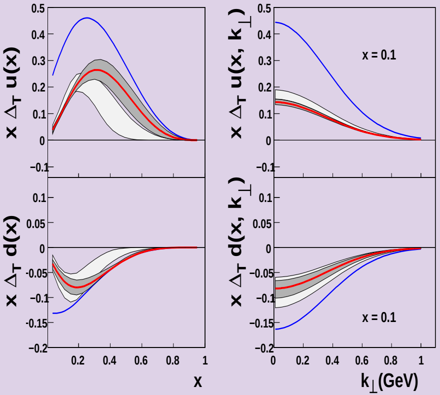

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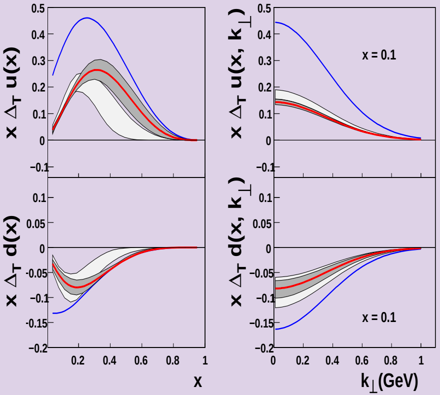

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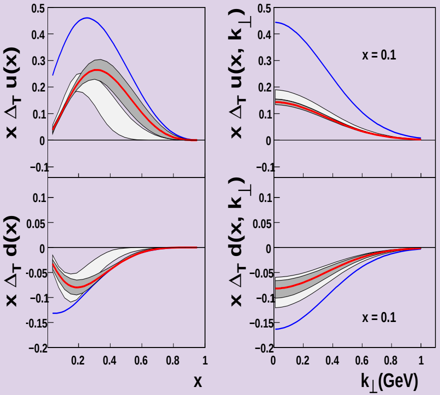
Transversity



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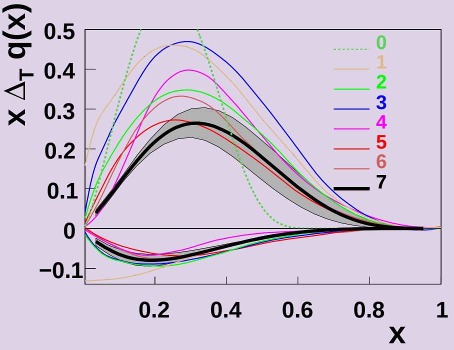
- Compared to previous extraction PRD75:054032,2007
- Compared to $\Delta\chi^2 = 1$ error estimate of PRD75:054032,2007
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$ The errors are diminished significantly.
- $\Delta_T u(x)$ became larger than that of the previous fit.

Transversity, comparison with models

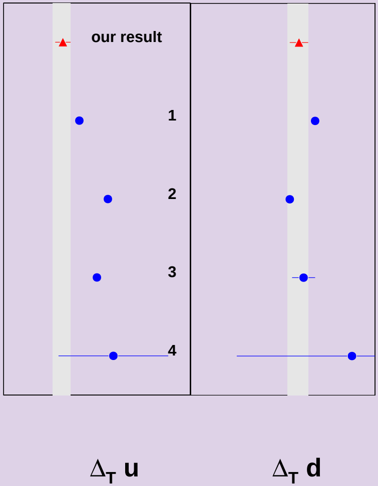
New extraction is close to most models.



0. Barone, Calarco, Drago PLB 390 287 (97)
1. Soffer et al. PRD 65 (02)
2. Korotkov et al. EPJC 18 (01)
3. Schweitzer et al. PRD 64 (01)
4. Wakamatsu, PLB B653 (07)
5. Pasquini et al., PRD 72 (05)
6. Cloet, Bentz and Thomas PLB 659 (08)
7. This analysis.

Tensor charges

$$\Delta_T u = 0.54^{+0.07}_{-0.09},\ \Delta_T d = -0.23^{+0.04}_{-0.05} \text{ at } Q^2 = 0.8 \text{ GeV}^2$$



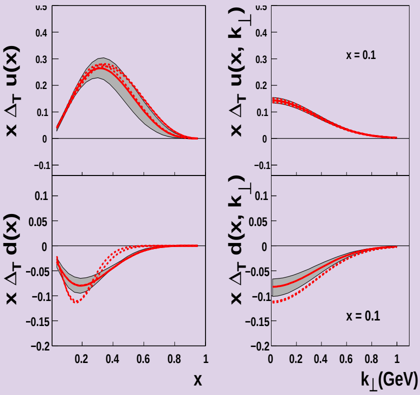
1. Quark-diquark model:
   Cloet, Bentz and Thomas
   PLB **659**, 214 (2008), $Q^2 = 0.4$ GeV^2
2. CQSM:
   M. Wakamatsu, PLB B **653** (2007) 398
   $Q^2 = 0.3$ GeV^2
3. Lattice QCD:
   M. Gockeler et al.,
   Phys.Lett.B627:113-123,2005 , Q^2 =
   GeV^2
4. QCD sum rules:
   Han-xin He, Xiang-Dong Ji,
   PRD 52:2960-2963,1995, $Q^2 \sim 1$ GeV^2

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$

$\chi^2/\mathrm{d.o.f} = 1.31$



Different parameterizations result in ≈10% change of $\chi^2/\mathrm{d.o.f}$ that give us an idea of uncertainty due to parameterization choice.

$\alpha_u \neq \alpha_d$, $\beta_u \neq \beta_d$
$\chi^2/\mathrm{d.o.f} = 1.33$

$\gamma_{fav} \neq \gamma_{unfav}$, $\delta_{fav} \neq \delta_{unfav}$,
$\chi^2/\mathrm{d.o.f} = 1.24$

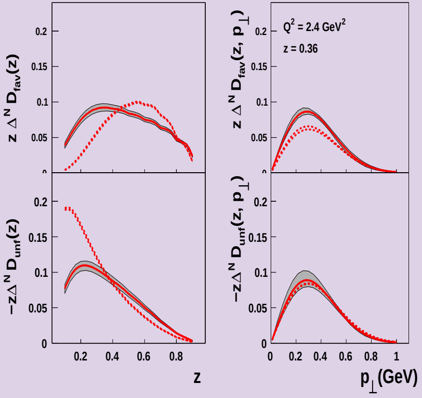
$\alpha_u \neq \alpha_d$, $\beta_u \neq \beta_d$, $\gamma_{fav} \neq \gamma_{unfav}$, $\delta_{fav} \neq \delta_{unfav}$
$\chi^2/\mathrm{d.o.f} = 1.25$.

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$

$\chi^2/\mathrm{d.o.f} = 1.31$



Different parameterizations result in ≈10% change of $\chi^2/\mathrm{d.o.f}$ that give us an idea of uncertainty due to parameterization choice.

Note that such an uncertainty is big for both favoured and unfavoured Collins FF.

Sivers effect

The azimuthal asymmetry $A_{UT}^{sin(\phi_h-\phi_S)}$ arises due to Sivers function

$$f_{q/p^\uparrow}(x,\mathbf{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\mathbf{S}_T\cdot(\hat{\mathbf{P}}\times\hat{\mathbf{k}}_\perp)$$
$$= f_{q/p}(x,k_\perp) - f_{1T}^{\perp q}(x,k_\perp)\,\frac{\mathbf{S}_T\cdot(\hat{\mathbf{P}}\times\mathbf{k}_\perp)}{m_p}\,,$$

Relation

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = -\frac{2|k_\perp|}{m_p}\,f_{1T}^{\perp q}(x,k_\perp)\,.$$

Trento conventions: A. Bacchetta, U. D'Alesio, M. Diehl, and C. A. Miller, Phys. Rev. **D70**, 117504 (2004).

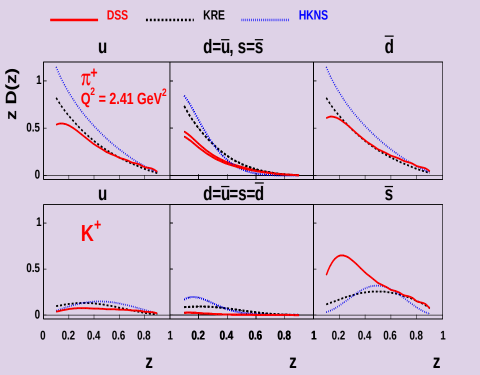
Choice of FF set, arXiv:0805.2677

Choice of FF set is important especially for Kaon asymmetry. Among existing FF sets we compared three of them:

Kretzer Phys. Rev. D62, 054001 (2000)

DSS D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D75, 114010 (2007)

HKNS M. Hirai, S. Kumano, T. H. Nagai, and K. Sudoh, Phys. Rev. D75, 094009 (2007)

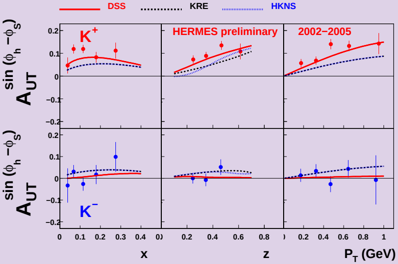


The only set capable of describing HERMES data on K production is DSS. $K^+(u\bar{s})$, $\pi^+(u\bar{d})$ knowledge of $\bar{s} \to K^+$ FF is very important.

KAON HERMES DATA

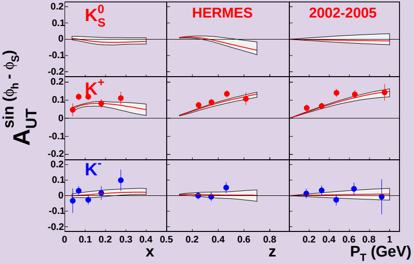
HERMES

$ep \to eKX$, $p_{lab} = 27.57$ GeV.



HERMES

$ep \to eKX$, $p_{lab} = 27.57$ GeV.



Kaon FF as given by De Florian *et al.* in Ref.

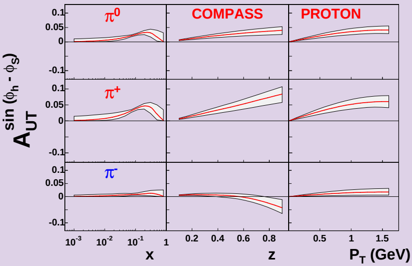
de Florian D., Sassot R., and Stratmann M. Phys. Rev. **D75** 114010 (2007)

(right panel) are compared the Kretzer (dotted lines) and HKNS set (dashed lines) of fragmentation functions (left panel).

PREDICTIONS

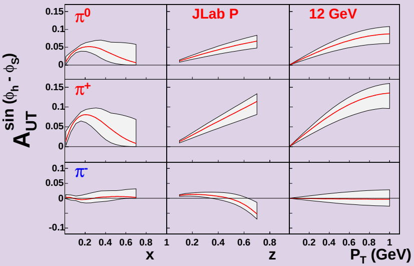
COMPASS on PROTON

$\mu p \rightarrow \mu \pi X$, $p_{lab} = 160$ GeV.



JLAB12

$ep \rightarrow e\pi X$, $p_{lab} = 12$ GeV.



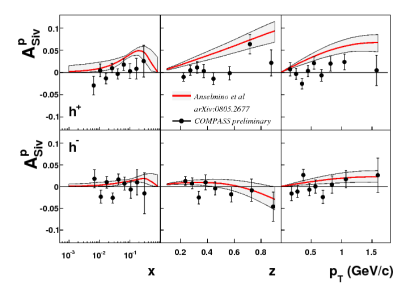
JLab can improve our knowledge of Sivers function in high x region. COMPASS operating on proton target is expected to measure 5% asymmetry for h^+.

PREDICTIONS

COMPASS on PROTON

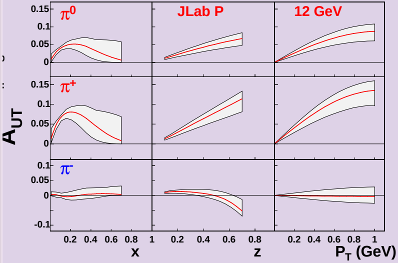
$\mu p \to \mu \pi X$, $p_{lab} = 160$ GeV

Comparison with preliminary COMPASS data arXiv:0808.0086



JLAB12

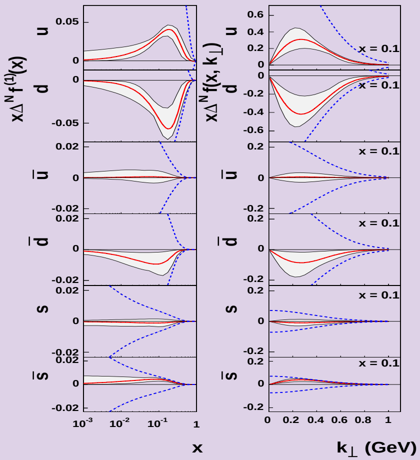
$ep \to e\pi X$, $p_{lab} = 12$ GeV.



JLab can improve our knowledge of Sivers function in high x region. COMPASS operating on proton target is expected to measure 5% asymmetry for h^+. Not supported by the preliminary data.

Sivers functions

$$\Delta^N f_q^{(1)}(x) \equiv \int d^2\mathbf{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x) \, .$$



Sivers functions for u, d and *sea* quarks are extracted from HERMES and COMPASS data.

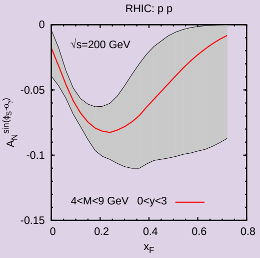
$\Delta^N f_u > 0$, $\Delta^N f_d < 0$, first hints on nonzero sea quark Sivers functions.

Sivers effect in Drell-Yan processe $AB \to l^+ l^- X$

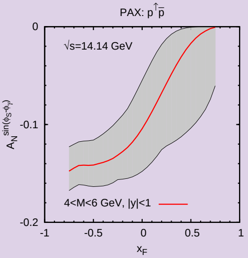
Sivers SSA (using $\Delta^N f_{q/p^\uparrow}(x, k_\perp)_{D-Y} = -\Delta^N f_{q/p^\uparrow}(x, k_\perp)_{SIDIS}$)

$$A_N^{\sin(\phi_S-\phi_\gamma)} = \frac{\int d\phi_\gamma [\sum_q e_q^2 \int d^2\mathbf{k}_{\perp q}\, d^2\mathbf{k}_{\perp \bar{q}}\, \delta^2(\mathbf{k}_{\perp q} + \mathbf{k}_{\perp \bar{q}} - \mathbf{q}_T)\, \Delta^N f_{q/p^\uparrow}(x_q, \mathbf{k}_{\perp q})\, f_{\bar{q}/p}(x_{\bar{q}}, \mathbf{k}_{\perp \bar{q}}) \hat{\sigma}_0^{q\bar{q}}] \sin(\phi_S - \phi_\gamma)}{\int d\phi_\gamma [\sum_q e_q^2 \int d^2\mathbf{k}_{\perp q}\, d^2\mathbf{k}_{\perp \bar{q}}\, \delta^2(\mathbf{k}_{\perp q} + \mathbf{k}_{\perp \bar{q}} - \mathbf{q}_T)\, f_{q/p}(x_q, \mathbf{k}_{\perp q})\, f_{\bar{q}/p}(x_{\bar{q}}, \mathbf{k}_{\perp \bar{q}}) \hat{\sigma}_0^{q\bar{q}}]},$$

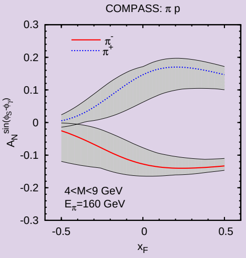
RHIC, $p^\uparrow p \to \ell^+ \ell^- X$, 200 GeV



GSI, $p^\uparrow \bar{p} \to \ell^+ \ell^- X$, 14.14 GeV



COMPASS, $\pi p^\uparrow \to \ell^+ \ell^- X$, 17.4 GeV



CONCLUSIONS

- Extraction of transversity for u and d quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- New data from HERMES, COMPASS and BELLE improve significantly quality of the fit.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$ and much closer to most model predictions.
- The Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained. $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for u, d and sea quarks are extracted from HERMES and COMPASS data.
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THANK YOU!

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BACKUP SLIDES

One word of caution

This extraction is done at tree level.

$$\sigma \propto \sum_q \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_\perp + \mathbf{k}_\perp - \mathbf{q}_T) D_1^q(z_1, \mathbf{k}_\perp) D_1^{\bar{q}}(z_2, \mathbf{p}_\perp)$$

Daniel Boer (arXiv:0808.2886) argues that beyond tree level due to presence of Sudakov factor

$$\delta^2(\mathbf{p}_\perp + \mathbf{k}_\perp - \mathbf{q}_T) \to \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-\mathbf{b}(\mathbf{p}_\perp + \mathbf{k}_\perp - \mathbf{q}_T)} U e^{-S}$$

the asymmetry acquires strong Q^2 behaviour that leads to suppression up to a factor 5 when Q^2 varies from 2.4 to 100 GeV^2. Such a scenario should be studied both theoretically and experimentally.
Data on Q_T behaviour of cross section from Belle are needed to reveal importance of Sudakov factors.

Collins function

Model for Collins FF

For $\Delta^N D_{h/q^\uparrow}(z,|p_\perp|) = \frac{2|p_\perp|}{zM_\pi} H_1^{\perp q}(z,|p_\perp|)$ we use factorization of z and $p_\perp$ and Gaussian dependence on $p_\perp$

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = 2\,\mathcal{N}_q^C(z)\;D_{h/q}(z)\;h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}\,,$$

with

$$\mathcal{N}_q^C(z) = N_q^C\,z^\gamma(1-z)^\delta\,\frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma\delta^\delta}$$

$$h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M}\,e^{-p_\perp^2/M^2},$$

where N_q^C, γ, δ, and M are parameters.

Collins function

Model for Collins FF

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$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = 2\,\mathcal{N}_q^C(z)\; D_{h/q}(z)\; h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}\,,$$

with

$$\mathcal{N}_q^C(z) \le 1$$
$$h(p_\perp) \le 1$$

positivity constraint $|\Delta^N D_{h/q^\uparrow}(z,\mathbf{p}_\perp)| \le 2D_{h/q}(z,\mathbf{p}_\perp)$ is fulfilled.

Transversity

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\ \left[f_{q/p}(x) + \Delta q(x)\right]\ \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T},$$

where

$$\mathcal{N}_q^T(x) = N_q^T\, x^\alpha (1-x)^\beta\, \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta},$$

N_q^T, α, β and $\langle k_\perp^2\rangle_T$ are parameters.

$$\mathcal{N}_q^T(x) \le 1$$

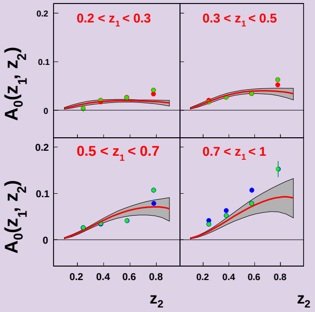
thus Soffer bound

$$|\Delta_T q(x)| \le \frac{1}{2}\left[f_{q/p}(x) + \Delta q(x)\right]$$

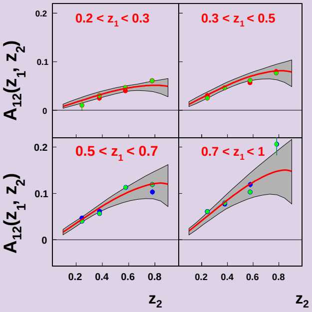
is fulfilled.

Description of BELLE data PRD75:054032,2007

BELLE $\cos(2\varphi_0)$



BELLE $\cos(\varphi_1 + \varphi_2)$

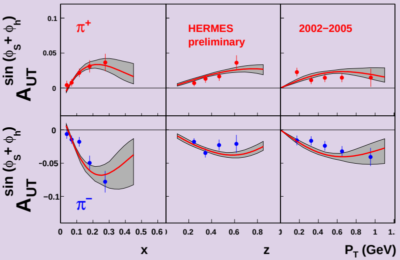


The results of PRD75:054032,2007 compared to NEW BELLE data sets

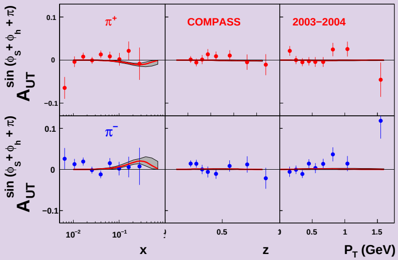
Belle Collaboration (R. Seidl et al.) Phys.Rev.D78:032011,2008

Description of SIDIS data $A_{UT}^{sin(\phi_h+\phi_S)}$

HERMES $A_{UT}^{sin(\phi_h+\phi_S)}$



COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$

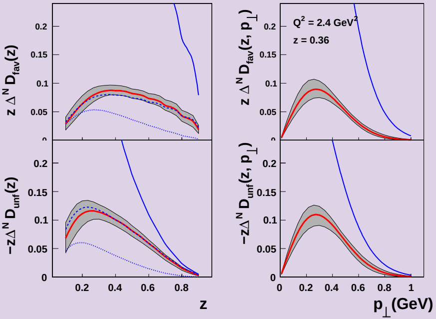


The results of PRD75:054032,2007 compared to NEW HERMES and COMPASS data sets

HERMES results for Collins Asymmetries, M. Diefenthaler, DIS 2007, Munich arXiv:0706.2242

COMPASS Collaboration, M. Alekseev *et al.*, arXiv:0802.2160

Collins fragmentation function PRD75:054032,2007



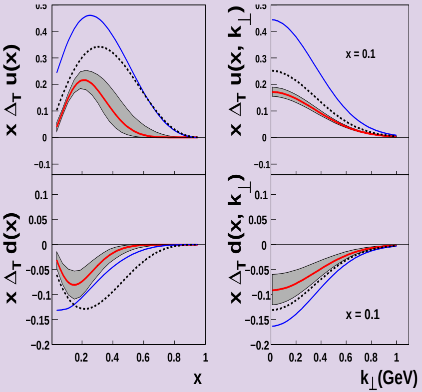
compared to Ref. [1] (dashed line) and Ref. [2] (dotted line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).

[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).

Transversity

M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. P., C. Turk, Phys.Rev.D75:054032,2007



- This is the first extraction of transversity from experimental data with $\Delta\chi^2 = 1$ error estimate applied.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- Neither $\Delta_T u(x)$ nor $\Delta_T d(x)$ saturates Soffer bound.

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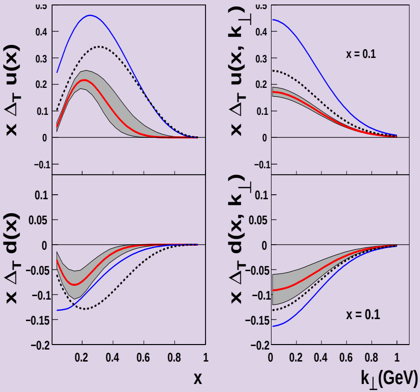

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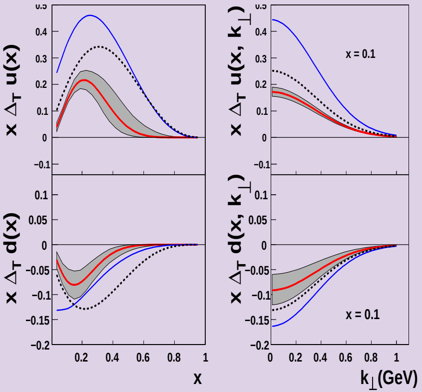

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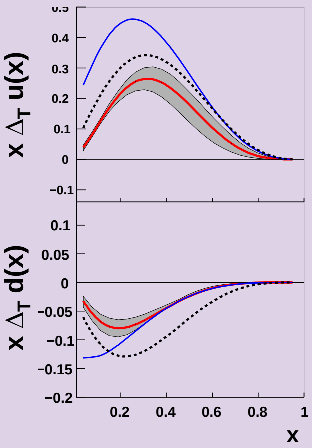
Transversity vs. helicity



1. Solid red line – transversity distribution
$$\Delta_T q(x)$$
this analysis at $Q^2 = 2.4$ GeV^2.
2. Solid blue line – Soffer bound
$$\frac{q(x) + \Delta q(x)}{2}$$
GRV98LO + GRSV98LO
3. Dashed line – helicity distribution
$$\Delta q(x)$$
GRSV98LO

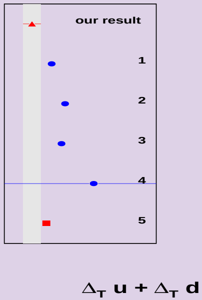
Transversity vs. helicity

$\Delta_T u = 0.54^{+0.07}_{-0.09}$, $\Delta_T d = -0.23^{+0.04}_{-0.05}$ at $Q^2 = 0.8$ GeV^2

$\Delta u = 0.87$, $\Delta d = -0.39$ at $Q^2 = 0.8$ GeV^2

The contribution to the spin:

$\Delta u + \Delta d = 0.47$, $\Delta_T u + \Delta_T d = 0.31^{+0.1}_{-0.1}$



1. Quark-diquark model:
   Cloet, Bentz and Thomas
   PLB **659**, 214 (2008), $Q^2 = 0.4$ GeV^2
2. CQSM:
   M. Wakamatsu, PLB B **653** (2007) 398.
   $Q^2 = 0.3$ GeV^2
3. Lattice QCD:
   M. Gockeler et al.,
   Phys.Lett.B627:113-123,2005 , $Q^2 = 4$ GeV^2
4. QCD sum rules:
   Han-xin He, Xiang-Dong Ji,
   PRD 52:2960-2963,1995, $Q^2 \sim 1$ GeV^2
5. $\Delta u + \Delta d = 0.47$

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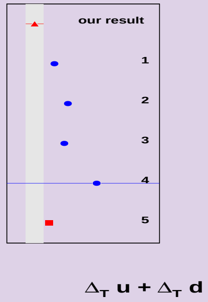


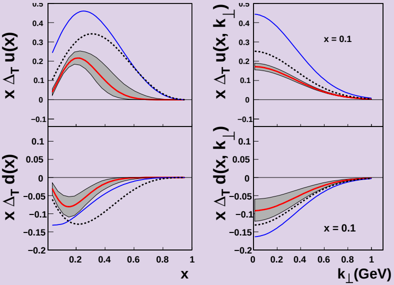
Phenomenological implementation of spin sum rules?

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + < L_z^{q,\bar{q}} > + < L_z^G >$$

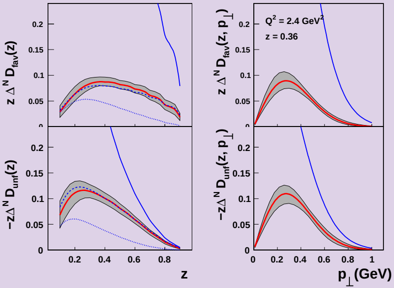
$$\frac{1}{2} = \frac{1}{2}\sum_{q,\bar{q}} \Delta_T q + < L_{s_T}^{q,\bar{q}} > + < L_{s_T}^G >$$

Uncertainties of the fit due to new data sets

$\chi^2/n.d.p. \simeq 0.8$



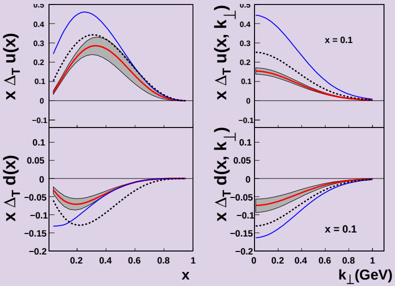
$\chi^2/n.d.p. \simeq 0.7$



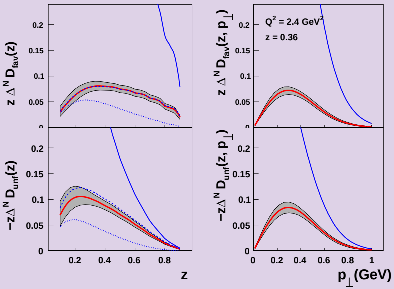
The results of PRD75:054032,2007. OLD COMPASS and HERMES sets and OLD BELLE data

Uncertainties of the fit due to new data sets

$\chi^2/n.d.p. \simeq 1.$



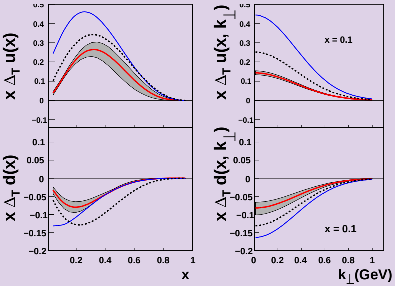
$\chi^2/n.d.p. \simeq 0.6$



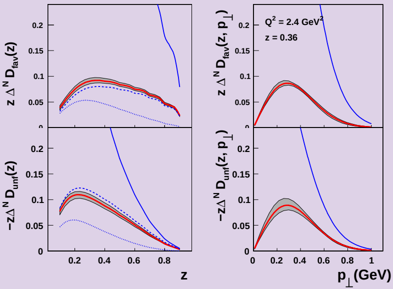
Usage of NEW COMPASS and HERMES sets and OLD BELLE data

Uncertainties of the fit due to new data sets

$\chi^2/n.d.p. \simeq 1.3$



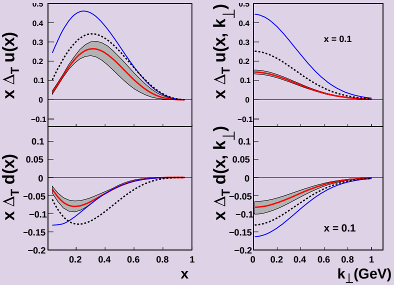
$\chi^2/n.d.p. \simeq 2.3$



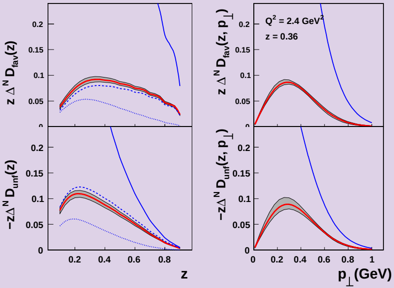
Usage of NEW COMPASS and HERMES sets and NEW BELLE data

Uncertainties of the fit due to new data sets

$\chi^2/n.d.p. \simeq 1.3$



$\chi^2/n.d.p. \simeq 2.3$

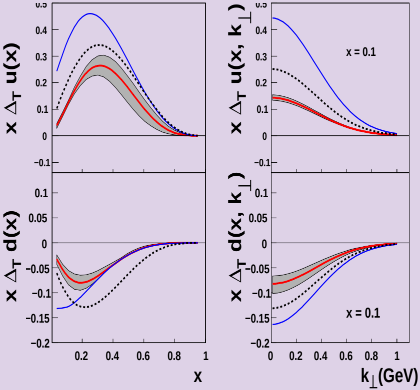


Usage of BELLE data results in better determination of Collins FF (with bad $\chi^2 \sim 2$ though), while errors of transversity distribution are mainly due to experimental errors of COMPASS and HERMES data sets

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$

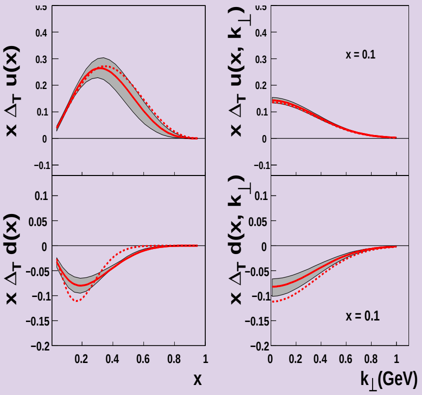


- $\chi^2/\text{d.o.f} = 1.31$
- $\alpha_u \neq \alpha_d$, $\beta_u \neq \beta_d$, γ, δ
  $\chi^2/\text{d.o.f} = 1.33$
- α, β, $\gamma_{fav} \neq \gamma_{unfav}$,
  $\delta_{fav} \neq \delta_{unfav}$
  $\chi^2/\text{d.o.f} = 1.24$
- $\alpha_u \neq \alpha_d$, $\beta_u \neq \beta_d$,
  $\gamma_{fav} \neq \gamma_{unfav}$,
  $\delta_{fav} \neq \delta_{unfav}$
  $\chi^2/\text{d.o.f} = 1.25$

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$

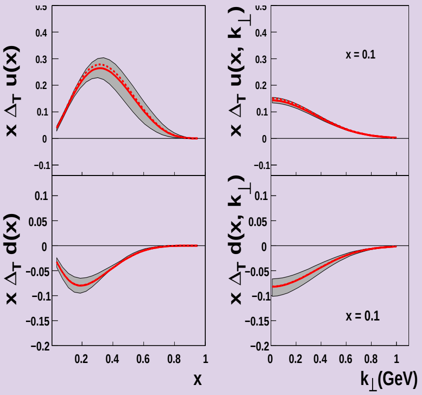


- $\chi^2/\text{d.o.f} = 1.31$
- $\alpha_u \neq \alpha_d$, $\beta_u \neq \beta_d$, γ, δ
  $\chi^2/\text{d.o.f} = 1.33$
- α, β, $\gamma_{fav} \neq \gamma_{unfav}$,
  $\delta_{fav} \neq \delta_{unfav}$
  $\chi^2/\text{d.o.f} = 1.24$
- $\alpha_u \neq \alpha_d$, $\beta_u \neq \beta_d$,
  $\gamma_{fav} \neq \gamma_{unfav}$,
  $\delta_{fav} \neq \delta_{unfav}$
  $\chi^2/\text{d.o.f} = 1.25$

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$



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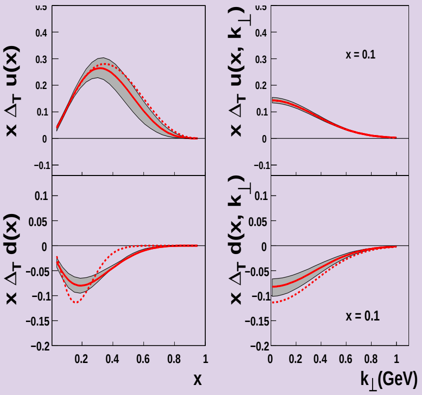

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  χ^2/d.o.f $= 1.25$

Uncertainties of the fit due to parameterization choice

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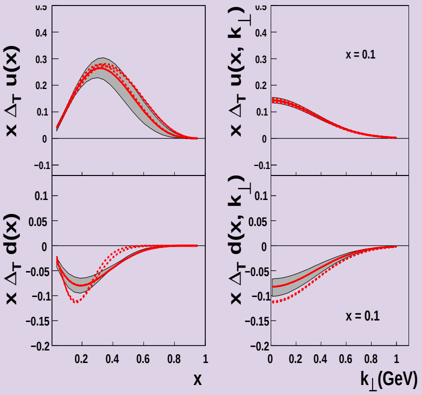
$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$

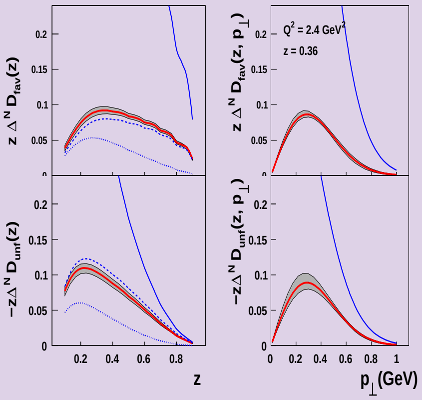


Different parameterizations result in ≈10% change of χ^2/d.o.f that give us an idea of uncertainty due to parameterization choice.

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha,\ \beta_u = \beta_d \equiv \beta,\ \gamma_{fav} = \gamma_{unfav} \equiv \gamma,\ \delta_{fav} = \delta_{unfav} \equiv \delta$



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- $\alpha_u \neq \alpha_d,\ \beta_u \neq \beta_d,\ \gamma,\ \delta$
  $\chi^2/\text{d.o.f} = 1.33$
- $\alpha,\ \beta,\ \gamma_{fav} \neq \gamma_{unfav},$
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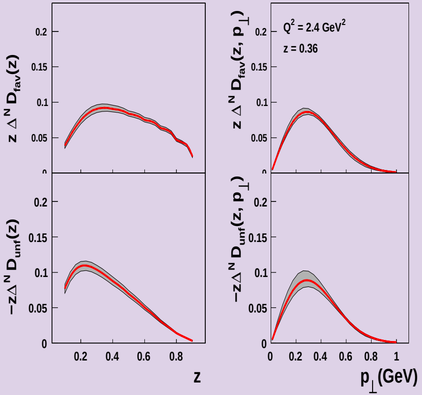
  $\chi^2/\text{d.o.f} = 1.25$

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$



- $\chi^2/\mathrm{d.o.f} = 1.31$
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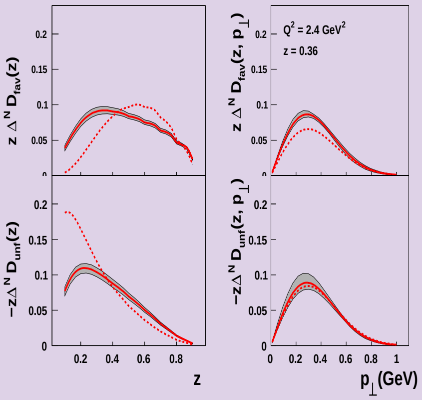
- α, β, $\gamma_{fav} \neq \gamma_{unfav}$, $\delta_{fav} \neq \delta_{unfav}$ $\chi^2/\mathrm{d.o.f} = 1.24$
- $\alpha_u \neq \alpha_d$, $\beta_u \neq \beta_d$, $\gamma_{fav} \neq \gamma_{unfav}$, $\delta_{fav} \neq \delta_{unfav}$ $\chi^2/\mathrm{d.o.f} = 1.25$

Uncertainties of the fit due to parameterization choice

This extraction

$\alpha_u = \alpha_d \equiv \alpha,\ \beta_u = \beta_d \equiv \beta,\ \gamma_{fav} = \gamma_{unfav} \equiv \gamma,\ \delta_{fav} = \delta_{unfav} \equiv \delta$



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Uncertainties of the fit due to parameterization choice

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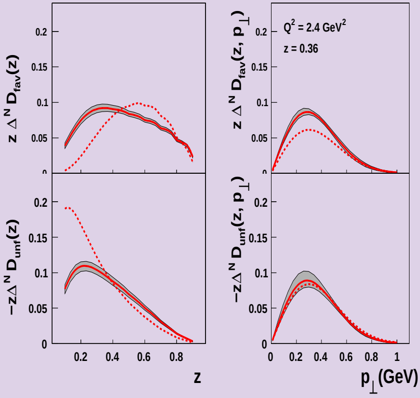
$\alpha_u = \alpha_d \equiv \alpha$, $\beta_u = \beta_d \equiv \beta$, $\gamma_{fav} = \gamma_{unfav} \equiv \gamma$, $\delta_{fav} = \delta_{unfav} \equiv \delta$



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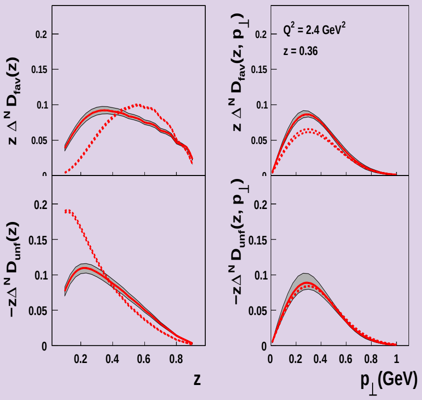

Different parameterizations result in ≈10% change of $\chi^2/\mathrm{d.o.f}$ that give us an idea of uncertainty due to parameterization choice.

Note that such an uncertainty is big for both favoured and unfavoured Collins FF.