Transverse Spin Theory: 3-g Interaction and Transverse Spin Asymmetries

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Based on work done with Kang, Sterman, Vogelsang, and Yuan

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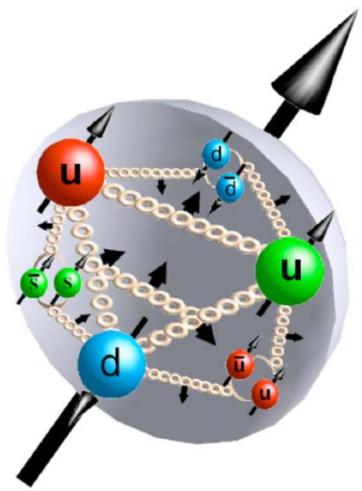
Outline

- ❑ Why single transverse-spin asymmetry (SSA)?
- ❑ SSA in parton model
- ❑ SSA in QCD k_T-factorization approach
- ❑ SSA in QCD collinear factorization approach
- ❑ Twist-3 trigluon correlation functions
- ❑ SSA of D-meson production in SIDIS
- ❑ SSA of D-meson production in hadronic collisions
- ❑ Summary

Single Transverse-Spin Asymmetry (SSA)

- **Definition:**

$$A_i = \frac{\sigma(s) - \sigma(-s)}{\sigma(s) + \sigma(-s)} \qquad i = L, N$$



✧ Parity invariance: $A_L = 0$

✧ PT invariance: $A_N = 0$ for inclusive DIS, but $A_N \neq 0$ in collisions involving two or more hadrons

✧ A_N vanishes in the parton model

✧ A_N is a direct consequence of parton's transverse motion

- **TMD factorization (only Drell-Yan and SIDIS):**

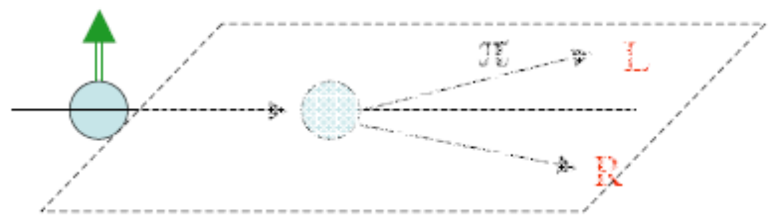
  Spin-dependent TMD distributions, or the Sivers functions

- **Collinear factorization:**

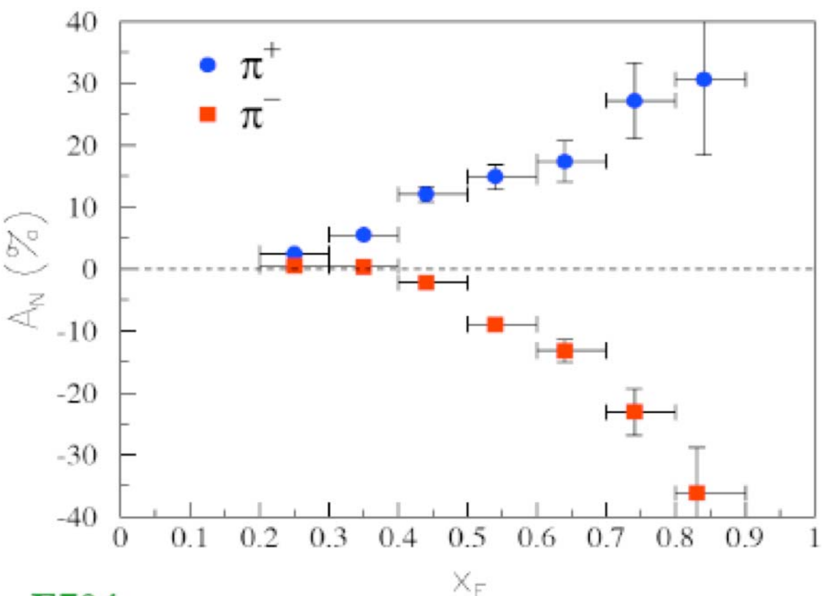
  Twist-3 parton correlation functions, information beyond PDFs

SSA in Hadronic Collisions

- **Hadronic:** $p\uparrow + p \rightarrow \pi(l)X$



If partons are collinear, $A_N \propto \alpha_s m_q$, to be very small



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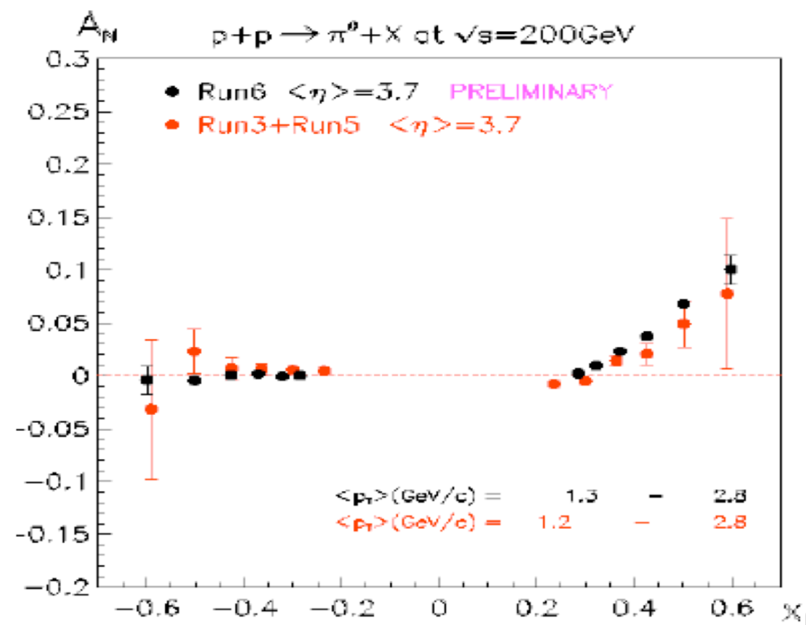
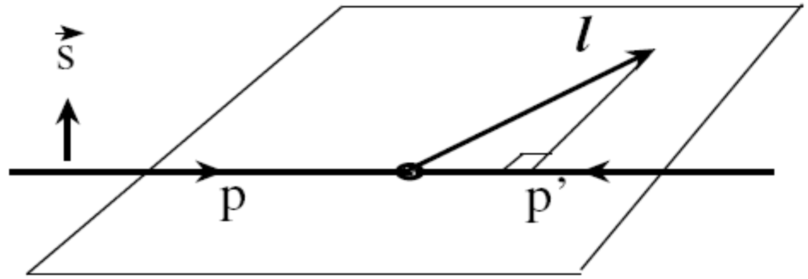


STAR (BRAHMS, too)

A_N=\=0: Result of parton’s transverse motion or correlations!

Single spin asymmetry corresponds to a T-odd triple product

$$A_N \propto i\,\vec{s}_p \cdot (\vec{p} \times \vec{\ell})$$



- the phase "i" is required by time-reversal invariance
- covariant form: $A_N \propto i\epsilon^{\mu\nu\alpha\beta}\, p_\mu s_\nu \ell_\alpha p'_\beta$

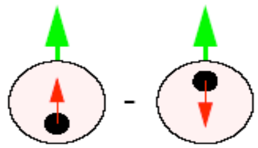
Nonvanishing A_N requires a phase, a spin flip, and enough vectors to fix a scattering plan

- Inclusive DIS does not have enough vectors

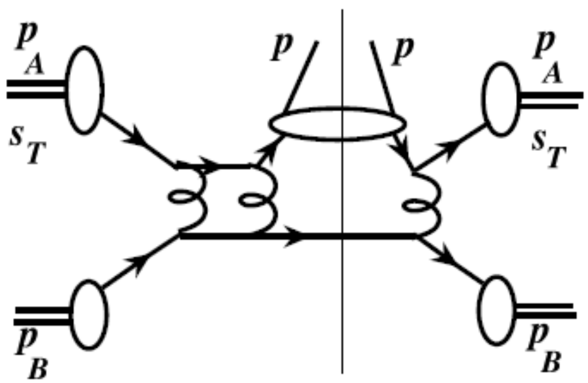
  Note: q and p can only fix a line

SSA in the Parton Model

- transverse spin information at leading twist – transversity:

$\delta q(x) =$  = **Chiral-odd helicity-flip density**

- the operator for δq has even γ's ⟹ quark mass term
- the phase requires an imaginary part ⟹ loop diagram



$\sim m_q \rightarrow 0$

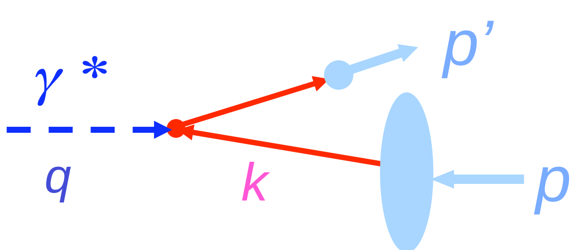
⟹ SSA vanishes in the parton model
connects to parton's transverse motion

TMD Factorization

- **Physical processes with two observed scales: Q and q**

  with the large Q to ensure a hard collision, while
  the $q \sim k_T$ probes a parton's transverse momentum

- **Semi-inclusive DIS:**



Both p and p' are observed
p'_T probes the parton's k_T

Effect of k_T is not suppressed by Q

$$\varphi(x) \Rightarrow \varphi\left(x, k_T^2\right)$$ = TMD parton distributions

Direct information on parton's transverse momentum

TMD parton distributions

- **Transverse momentum dependent (TMD) parton distributions:**

Belitsky, Ji, Yuan, 2003

$$\mathcal{M}_a = \int \frac{P^+ d\xi^-}{\pi} \frac{d^2\xi_\perp}{(2\pi)^2} e^{-ix\xi^- P^+ + i\xi_\perp\cdot k_\perp} \langle PS|\overline{\psi}_a(\xi)\mathcal{L}_v^\dagger(\infty;\xi)\mathcal{L}_v(\infty;0)\psi_a(0)|PS\rangle$$

$$= \frac{1}{2}\left[\underbrace{f_a^{\rm SIDIS}(x,k_\perp)}_{\text{Spin-averaged}}\gamma_\mu P^\mu + \frac{1}{M_P}\underbrace{q_{Ta}^{\rm SIDIS}(x,k_\perp)}_{\text{Spin-dependent Sivers function}}\epsilon_{\mu\nu\alpha\beta}\gamma^\mu P^\nu k^\alpha S^\beta \underbrace{+\ldots}_{\text{Boer, Mudlers, et al functions}}\right]$$

- **Connection to normal parton distributions**

$$q_a(x) = \int d^2k_T\, f_a^{\rm SIDIS}(x, k_T) + {\rm UVCT}$$

- **Spin-dependent TMD parton distributions**

→ Sivers functions: quark or gluon

Measure TMD parton distributions

- **Need processes with two observed momentum scales:**

$$Q_1 \gg Q_2 \begin{cases} Q_1 & \text{necessary for pQCD factorization to have a chance} \\ Q_2 & \text{sensitive to parton's transverse motion} \end{cases}$$

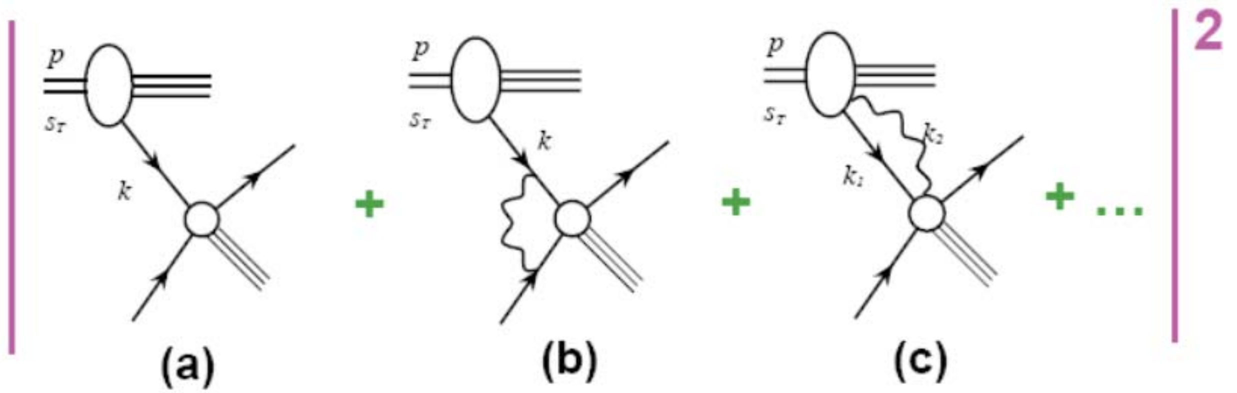
- **Very limited processes with valid TMD factorization**

  - **Drell-Yan transverse momentum distribution:** Q, q_T
    - quark Sivers function
    - low rate
  - **Semi-inclusive DIS for light hadrons:** Q, p_T
    - mixture of quark Sivers and Collins function
  - **Semi-inclusive DIS for heavy mesons (D's):**
    - gluon Sivers function
    - no Collins effect

SSA in collinear factorization

Efremov, Teryaev, 1982, Qiu, Sterman, 1991

- When all observed scales >> Λ_{QCD}, collinear factorization should work:

$\sigma(s_T) \sim$ 

- Leading spin dependent part of the cross section

  ➡ Interference between amplitudes (a) and (b) or (c)

- The hadronic phase – the "i"

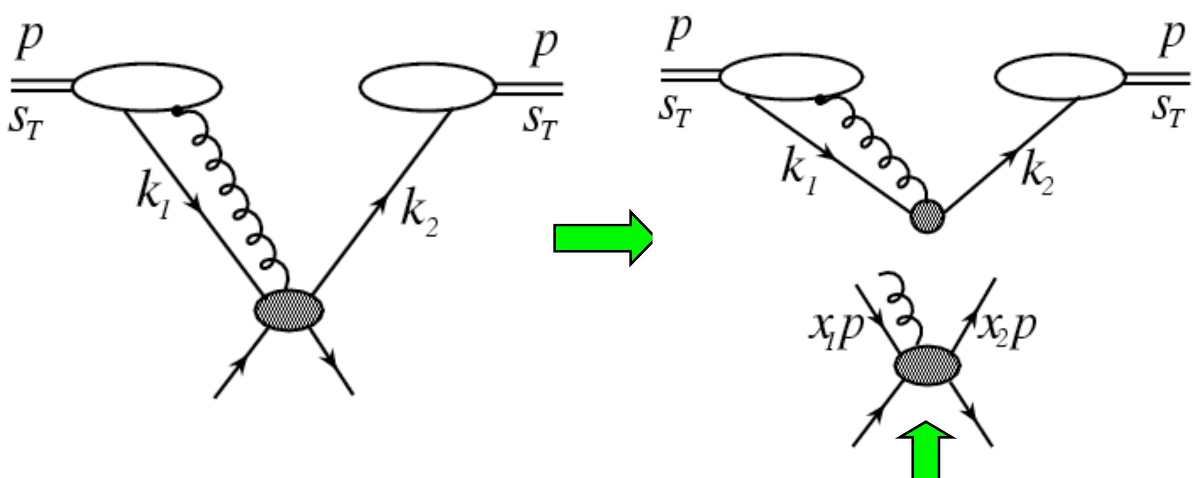
  ➡ $\mathrm{Re}[(a)]$ interferes with $\mathrm{Im}[(b)]$ or $\mathrm{Im}[(c)]$

- $\mathrm{Re}[(a)] \times \mathrm{Im}[(b)] \propto m_Q\ \delta q(s_\perp)$

Twist-3 quark-gluon correlations

❑ Factorization:

Qiu, Sterman, 1991, 1999



$$T_F(x_1, x_2) \propto \langle \bar{\psi}\gamma^+ F^{+\perp}\psi \rangle$$

$$T_D(x_1, x_2) \propto \langle \bar{\psi}\gamma^+ D_\perp \psi \rangle$$

Normal twist-2 distributions

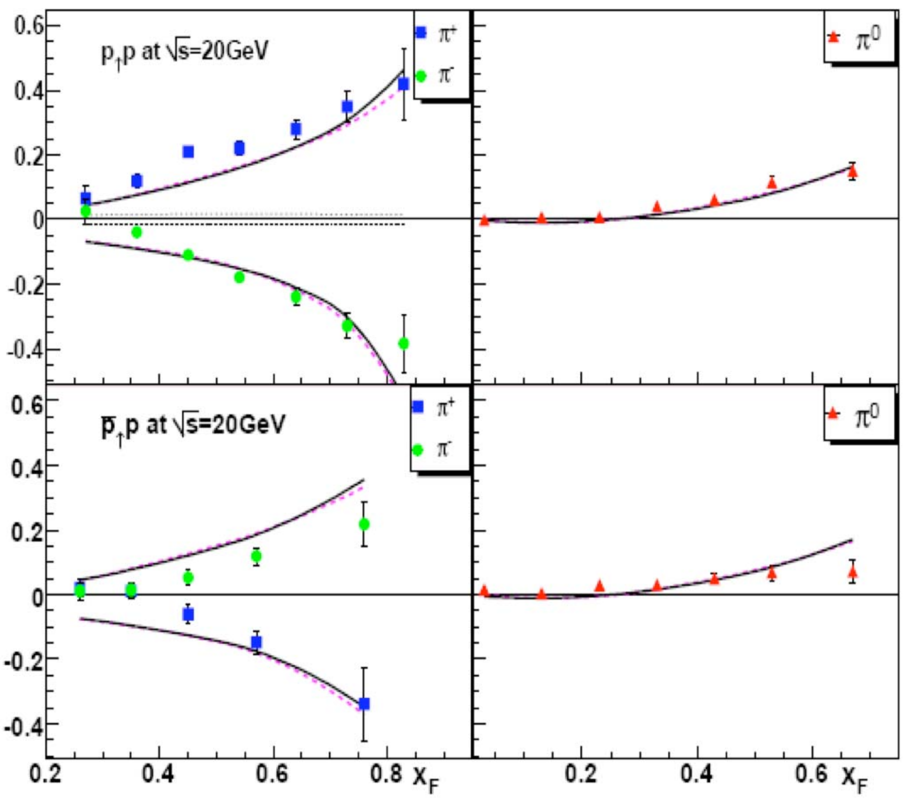
❑ Twist-3 correlation functions:

- Only $T_F(x_1, x_2)$ contributes to SSA due to P and T invariance
- Two gluonic twist-3 correlation functions:

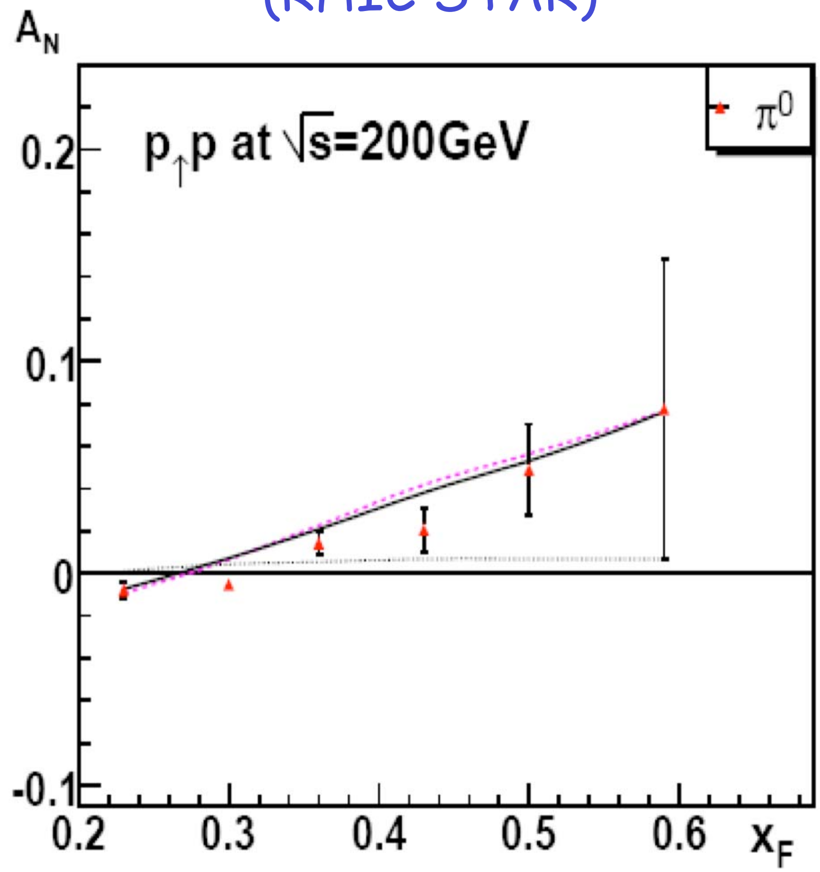
$T_G^{(f)}(x_1, x_2)$ and $T_G^{(d)}(x_1, x_2)$ due two independent color contractions

Asymmetries from the $T_F(x,x)$

(FermiLab E704)



(RHIC STAR)



$T_F(x_1, x_2)$ only in forward region

Kouvaris, Qiu, Vogelsang, Yuan, 2006

Nonvanish twist-3 function ➡ Nonvanish transverse motion

Collinear vs TMD factorization

❑ Relation between TMD distributions and collinear factorized distributions

spin-averaged: $$\int d^2k_T\, f_a^{\rm SIDIS}(x,k_T) + {\rm UVCT}(\mu^2) = q_a(x,\mu^2)$$

Transverse-spin: $$\frac{1}{M_P}\int d^2\vec{k}_\perp \vec{k}_\perp^2\, q_T(x,k_\perp) = T_F(x,x)$$

❑ Relation between two factorization schemes

They are valid for different kinematical regions:

Collinear: $Q_1 \ldots Q_n \gg \Lambda_{QCD}$

TMD: $Q_1 \gg Q_2 > \Lambda_{QCD}$

Common region:

$Q_1 \gg Q_2 \gg \Lambda_{QCD}$

where both schemes are expected to be valid

Ji, Qiu, Vogelsang, Yuan, 2005

Twist-3 trigluon correlation functions

Ji, PLB289 (1992)

- Diagonal tri-gluon correlations:

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-) \right] F^{\alpha +}(y_1^-) | P, s_\perp \rangle$$

- Two tri-gluon correlation functions – color contraction:

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$

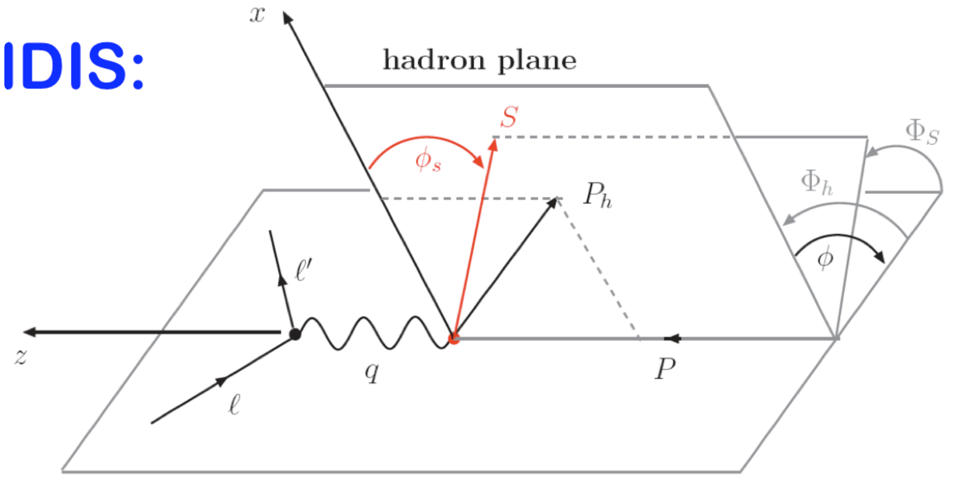
Fermionic correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

- D-meson production in SIDIS:
  - ✧ Clean probe for gluonic twist-3 correlation functions
  - ✧ $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

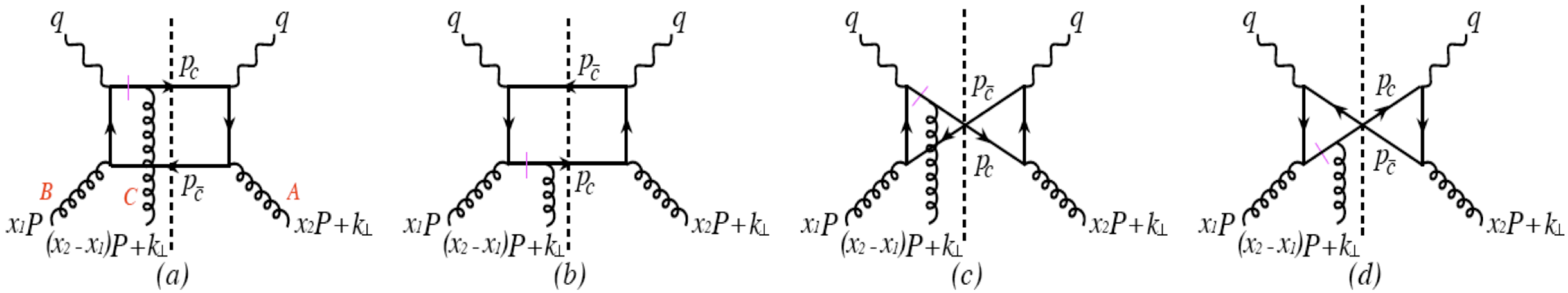
D-meson production in SIDIS

Kang, Qiu, PRD, 2008

❑ Frame for SIDIS:



❑ Dominated by the tri-gluon subprocess:



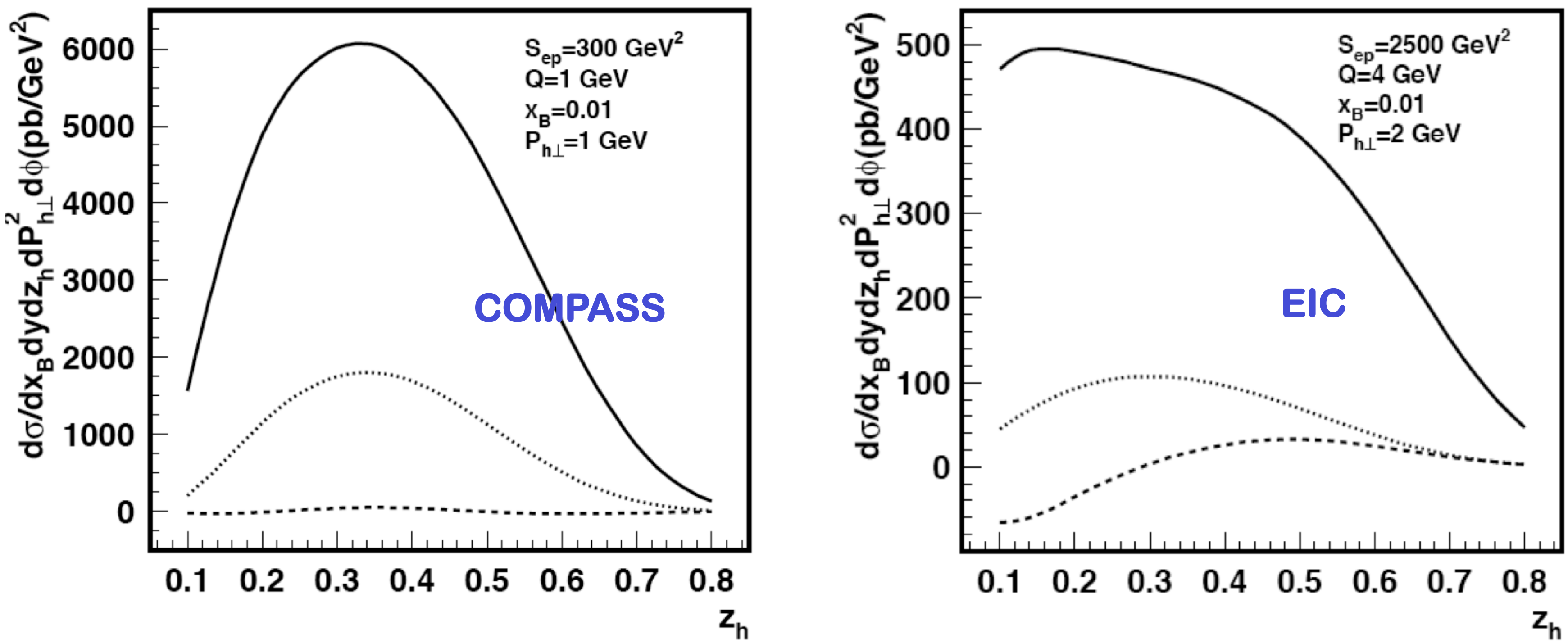
❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

Phenomenology for SIDIS

- Production rate (spin averaged):

$$\frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2\, d\phi} = \sigma_0^U + \sigma_1^U \cos\phi + \sigma_2^U \cos 2\phi$$



Small φ dependence, reasonable production rate

Estimation of the SSA in SIDIS

- **Dependence on tri-gluon correlation functions:**

$$D-\text{meson} \propto T_G^{(f)} + T_G^{(d)} \qquad \overline{D}-\text{meson} \propto T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- **Model for tri-gluon correlation functions:**

$$T_G^{(f,d)}(x,x) = \lambda_{f,d}G(x) \qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07\text{GeV}$$

- **Kinematic constraints:**

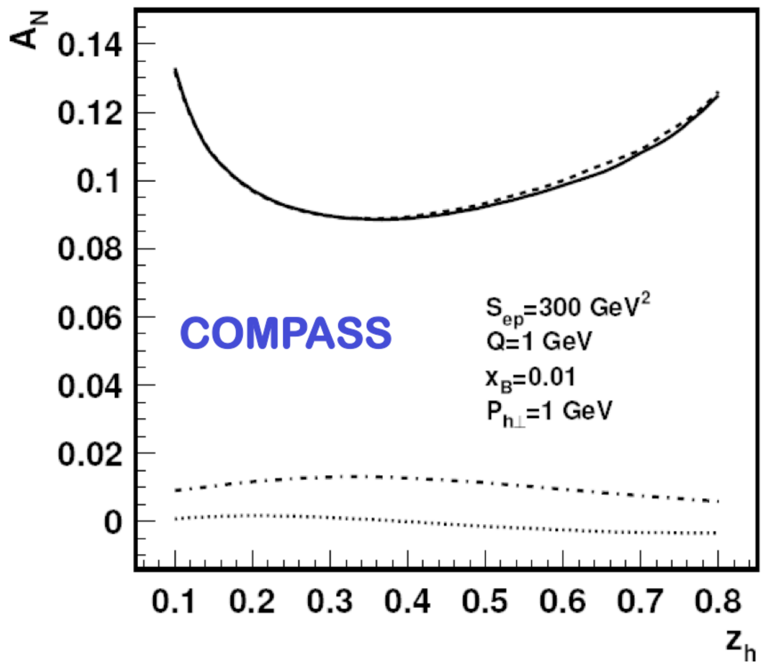
$$x_{min} = \begin{cases} x_B\left[1+\frac{P_{h\perp}^2+m_c^2}{z_h(1-z_h)Q^2}\right], & \text{if } z_h+\sqrt{z_h^2+\frac{P_{h\perp}^2}{m_c^2}} \geq 1 \\ x_B\left[1+\frac{2m_c^2}{Q^2}\left(1+\sqrt{1+\frac{P_{h\perp}^2}{z_h^2 m_c^2}}\right)\right], & \text{if } z_h+\sqrt{z_h^2+\frac{P_{h\perp}^2}{m_c^2}} \leq 1 \end{cases}$$

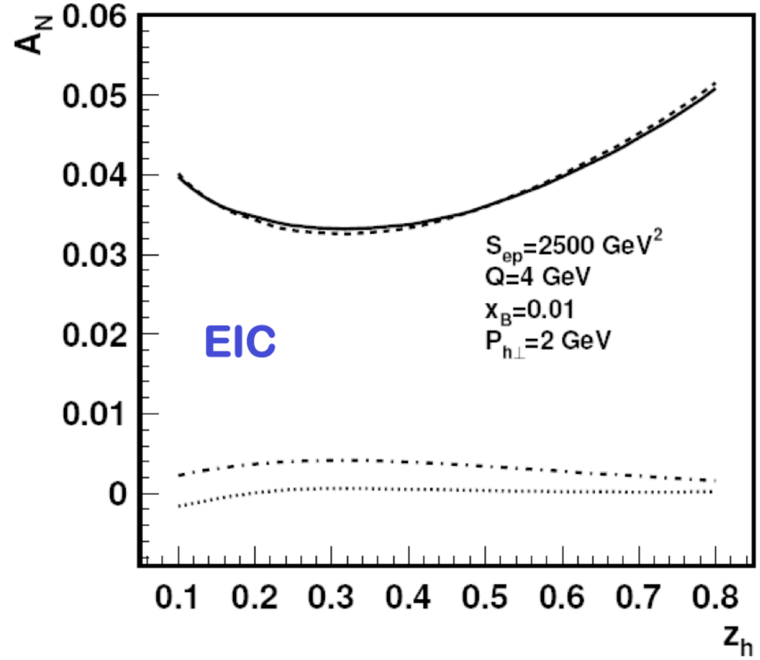
Note: The $z_h(1-z_h)$ has a maximum

SSA should have a minimum if the derivative term dominates

Minimum in the SSA of D-meson production

❑ SSA for D^0 production (λ_f only):





❖ Derivative term dominates, and small φ dependence

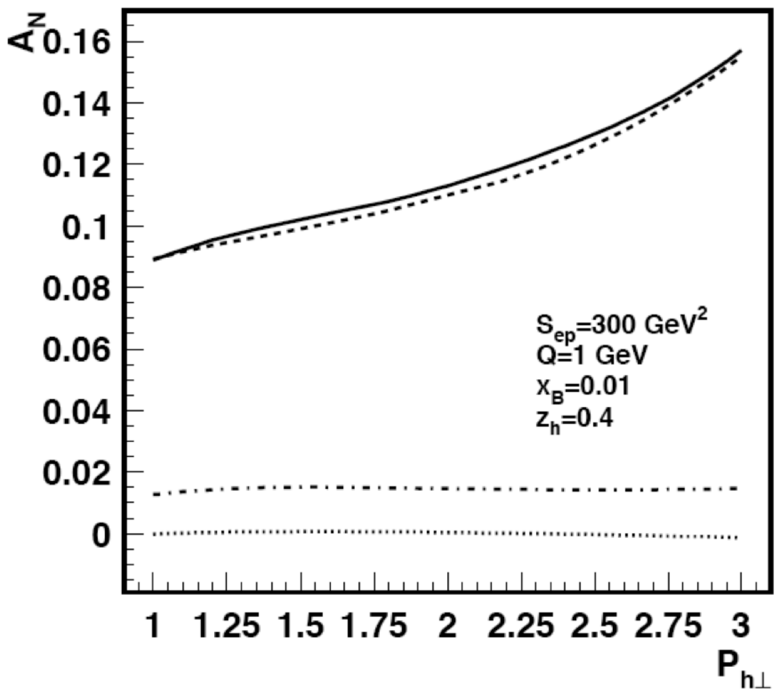
❖ Asymmetry is twice if $T_G^{(f)} = +T_G^{(d)}$, or zero if $T_G^{(f)} = -T_G^{(d)}$

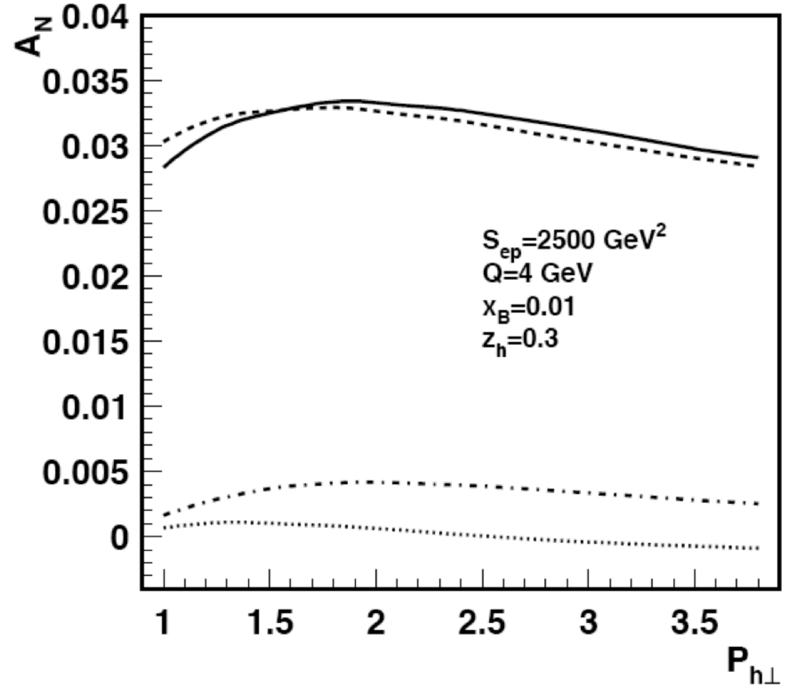
❖ Opposite for the $\bar{D}$ meson

❖ Asymmetry has a minimum ~ z_h ~ 0.5

Maximum in the SSA of D-meson production

- SSA for D^0 production (λ_f only):





- The SSA is a twist-3 effect, it should fall off as $1/P_T$ when $P_T >> m_c$
- For the region, $P_T \sim m_c$,

$$A_N \propto \epsilon^{P_h s_\perp n \bar{n}} \frac{1}{\tilde{t}} = -\sin\phi_s \frac{P_{h\perp}}{\tilde{t}}$$

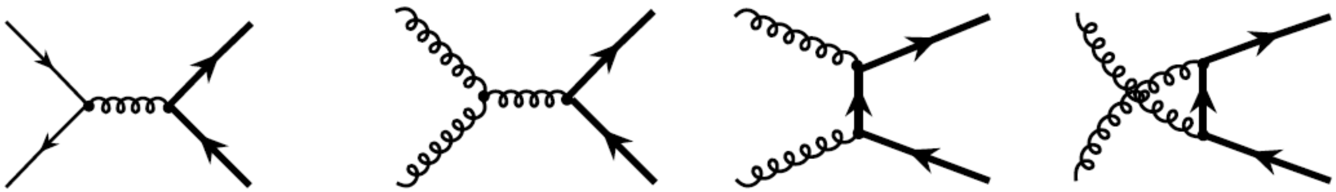
$$\tilde{t} = (p_c - q)^2 - m_c^2 = -\frac{1-\hat{z}}{\hat{x}} Q^2$$

$$\hat{z} = z_h/z, \quad \hat{x} = x_B/x$$

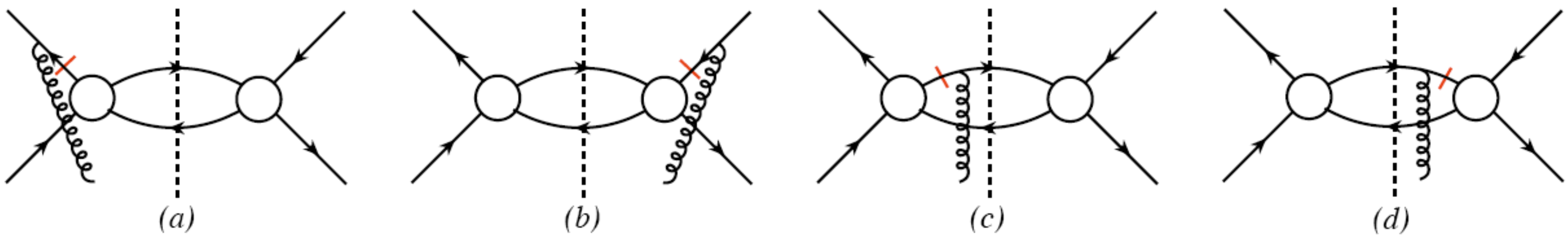
D-meson production in Hadronic Collisions

Kang, Qiu, Vogelsang, Yuan, 2008

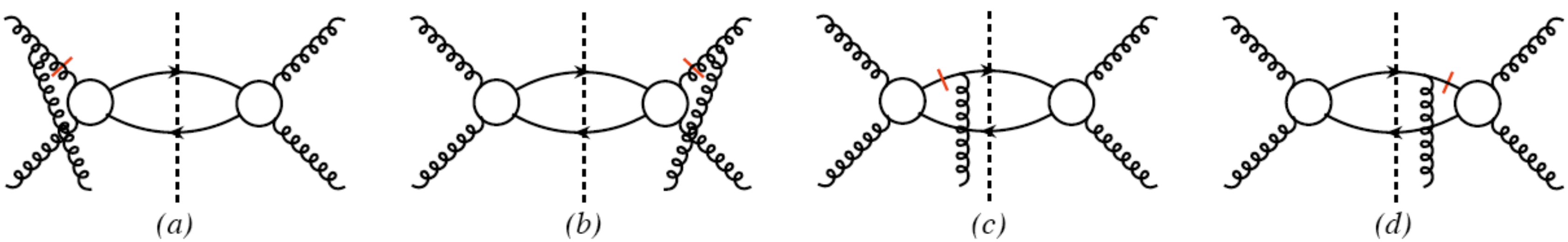
- Two partonic subprocesses:



- Quark-antiquark annihilation:



- Gluon-gluon fusion:



Factorized formula for D-meson production

- Same factorized formula for both subprocesses:

$$E_{P_h}\frac{d\Delta\sigma}{d^3P_h}\Big|_{q\bar{q}\to c\bar{c}} = \frac{\alpha_s^2}{S}\sum_q\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}\phi_{\bar{q}/B}(x')\int\frac{dx}{x}\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{P_h s_T n\bar{n}}}{z\tilde{u}}\right)\delta\left(\tilde{s}+\tilde{t}+\tilde{u}\right)$$
$$\times\left[\left(T_{q,F}(x,x)-x\frac{d}{dx}T_{q,F}(x,x)\right)H_{q\bar{q}\to c}(\tilde{s},\tilde{t},\tilde{u})+T_{q,F}(x,x)\mathcal{H}_{q\bar{q}\to c}(\tilde{s},\tilde{t},\tilde{u})\right],$$

$$E_{P_h}\frac{d\Delta\sigma}{d^3P_h}\Big|_{gg\to c\bar{c}} = \frac{\alpha_s^2}{S}\sum_{i=f,d}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}\phi_{g/B}(x')\int\frac{dx}{x}\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{P_h s_T n\bar{n}}}{z\tilde{u}}\right)\delta\left(\tilde{s}+\tilde{t}+\tilde{u}\right)$$
$$\times\left[\left(T_G^{(i)}(x,x)-x\frac{d}{dx}T_G^{(i)}(x,x)\right)H_{gg\to c}^{(i)}(\tilde{s},\tilde{t},\tilde{u})+T_G^{(i)}(x,x)\mathcal{H}_{gg\to c}^{(i)}(\tilde{s},\tilde{t},\tilde{u})\right],$$

- Hard parts:

$$H_{q\bar{q}\to c}=H_{q\bar{q}\to c}^{I}+H_{q\bar{q}\to c}^{F}\left(1+\frac{\tilde{u}}{\tilde{t}}\right)\qquad H_{gg\to c}^{(i)}=H_{gg\to c}^{I(i)}+H_{gg\to c}^{F(i)}\left(1+\frac{\tilde{u}}{\tilde{t}}\right)$$

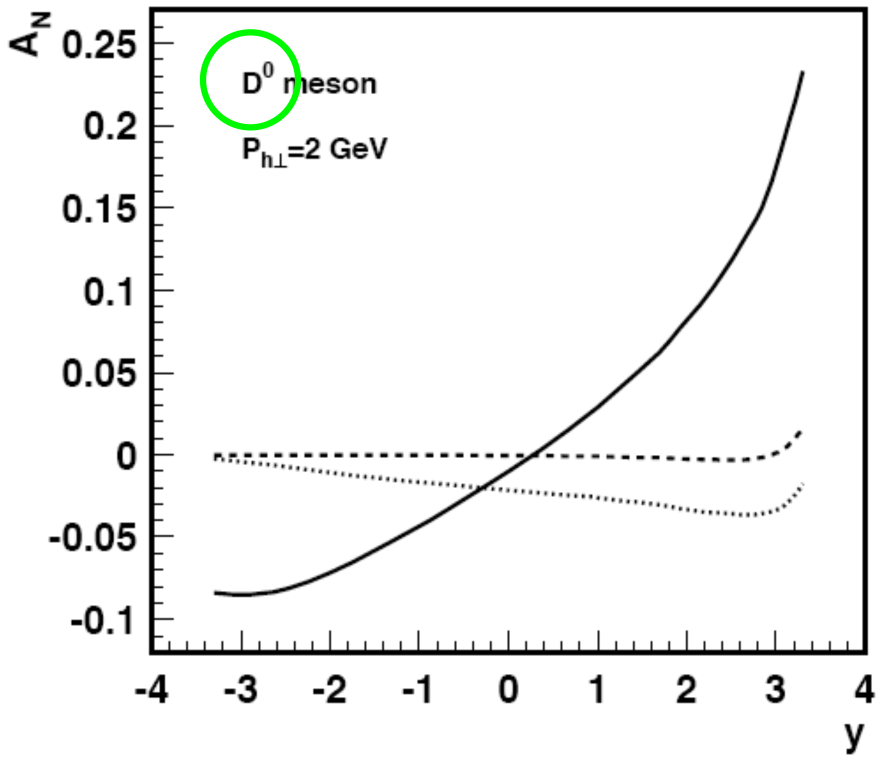
All $\mathcal{H}_{q\bar{q}\to c}$ and $\mathcal{H}_{gg\to c}^{I(i)}$ and $\mathcal{H}_{gg\to c}^{F(i)}$ vanish as $m_c^2\to 0$

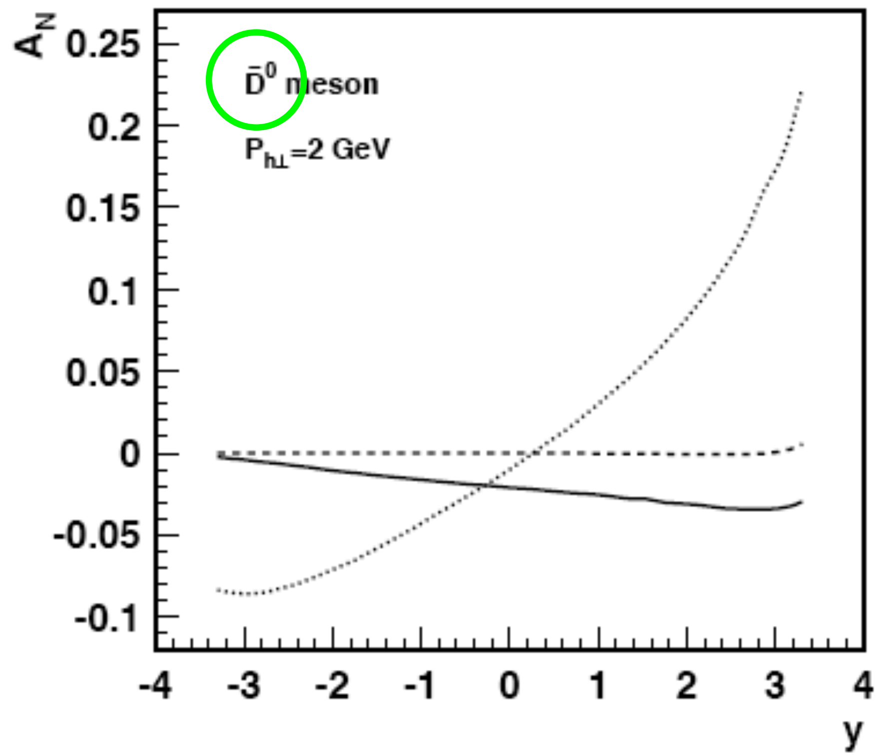
- Hard parts change sign for $T_G^{(d)}(x,x)$ when $c\longrightarrow\bar{c}$

$$H_{gg\to\bar{c}}^{(f)}=H_{gg\to c}^{(f)},\qquad H_{gg\to\bar{c}}^{(d)}=-H_{gg\to c}^{(d)},$$
$$\mathcal{H}_{gg\to\bar{c}}^{(f)}=\mathcal{H}_{gg\to c}^{(f)},\qquad \mathcal{H}_{gg\to\bar{c}}^{(d)}=-\mathcal{H}_{gg\to c}^{(d)}.$$

Rapidity dependence of D-meson production

- SSA at RHIC: $\sqrt{s} = 200\ \text{GeV} \quad \mu = \sqrt{m_c^2 + P_{h\perp}^2} \quad m_c = 1.3\ \text{GeV}$





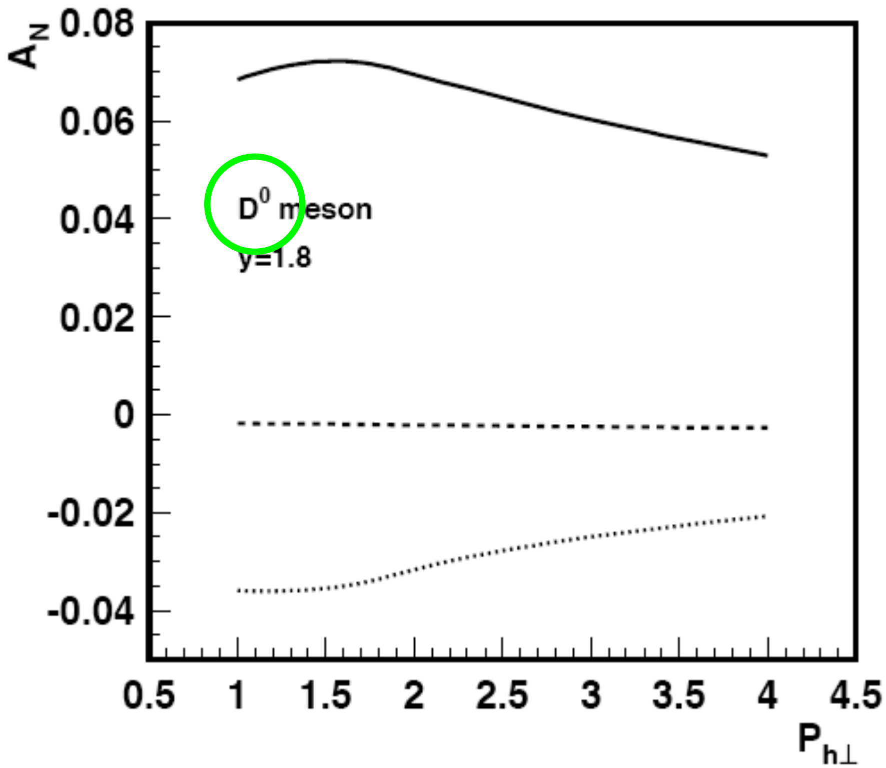
Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = T_G^{(d)}$

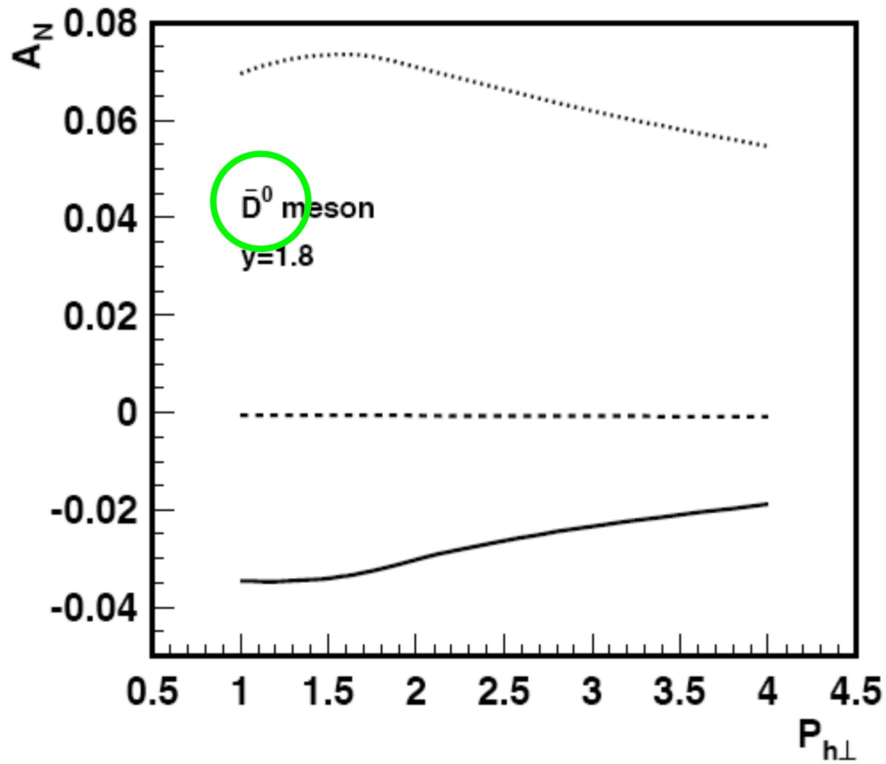
Dashed: $(2)\ \lambda_f = \lambda_d = 0 \qquad T_G^{(f)} = T_G^{(d)} = 0$

Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV} \qquad T_G^{(f)} = -T_G^{(d)}$

P_T-dependence of D-meson production

❑ **SSA at RHIC:** $\sqrt{s} = 200\ \text{GeV}$ $\mu = \sqrt{m_c^2 + P_{h\perp}^2}$ $m_c = 1.3\ \text{GeV}$





Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV}$ $T_G^{(f)} = T_G^{(d)}$

Dashed: $(2)\ \lambda_f = \lambda_d = 0$ $T_G^{(f)} = T_G^{(d)} = 0$

Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV}$ $T_G^{(f)} = -T_G^{(d)}$

Summary

- Single transverse-spin asymmetry is directly connected to the parton's transverse motion (P and T invariance)

  – an excellent probe for the parton's transverse motion

- Two complementary approaches:

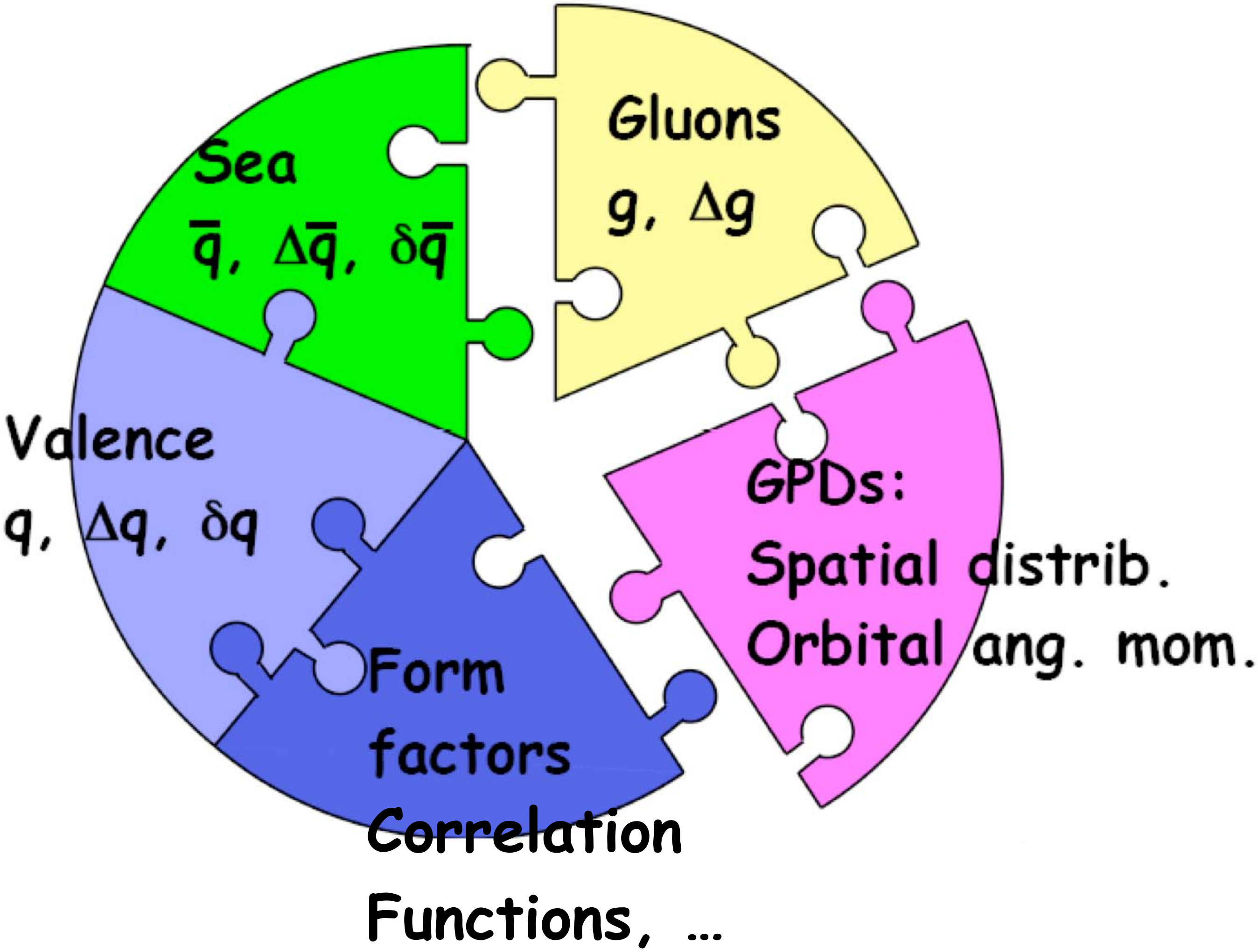
  TMD: direct k_T information – two-scale observables

  Collinear: net spin-dependence of all k_T – single-scale observalbes

- D-meson production in SIDIS, as well as in hadron-hadron collisions, is an excellent observable to measure the new tri-gluon correlation functions

  ⟹ QCD global analysis of twist-3 distributions: $T_F, T_G^{(f)}, T_G^{(d)}$

Challenge: Map out the Nucleon



Several existing and upcoming experiments will help!

Backup slides

What is the $T_F(x,x)$?

- Twist-3 correlation $T_F(x,x)$:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-}$$

$$\times \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

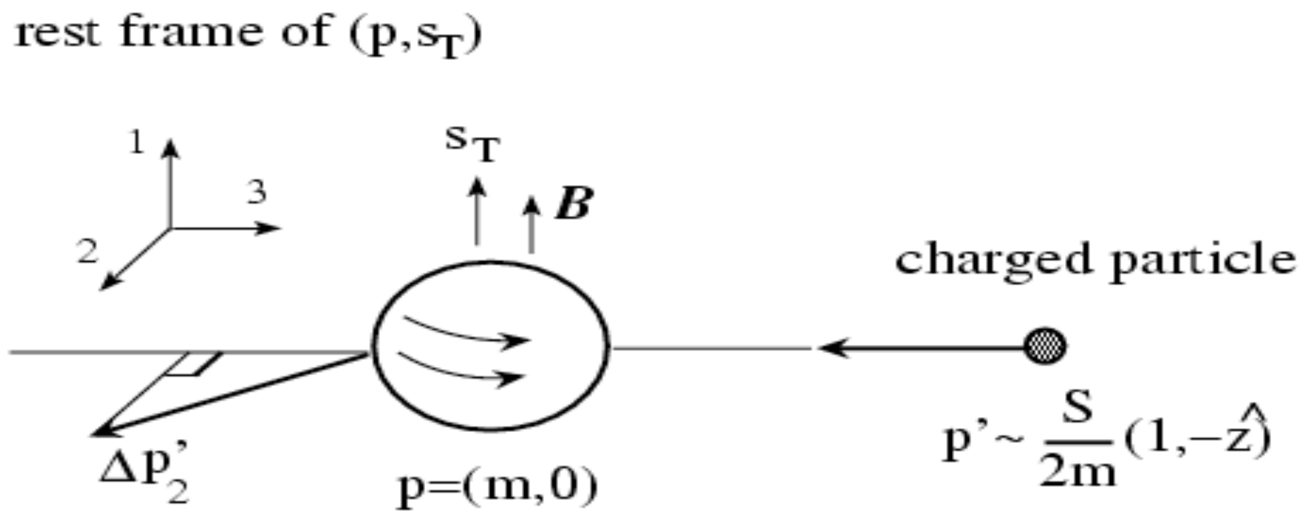
- Twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

$T_F(x,x)$ represents a net spin dependence of a quark's transverse motion via a gluon interaction inside a transversely polarized proton

What the $T_F(x,x)$ tries to tells us?

- **Consider a classical (Abelian) situation:**



– change of transverse momentum

$$\frac{d}{dt}p'_2 = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

– in the c.m. frame

$$(m, \vec{0}) \rightarrow \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \rightarrow n = (0, 1, 0_T)$$

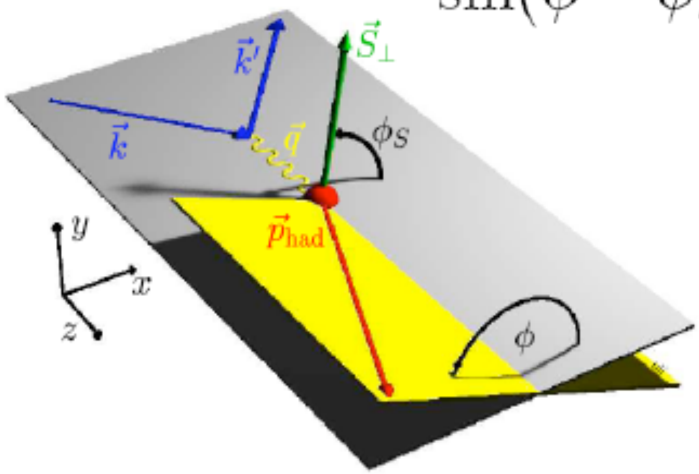
$$\Longrightarrow \frac{d}{dt}p'_2 = e\, \epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^{+}$$

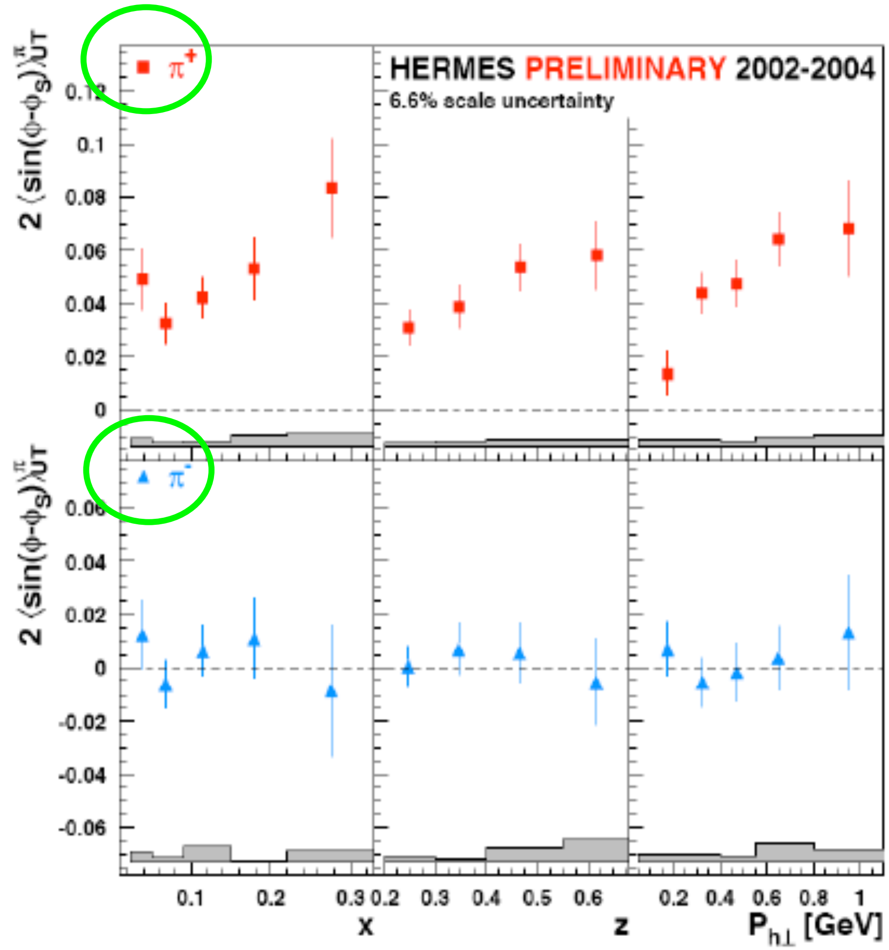
– total change: $\Delta p'_2 = e \int dy^- \epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^{+}(y^-)$

Quark Sivers functions

$\mathbf{e\,p^{\uparrow} \rightarrow e\,\pi\,X}$

$$\sin(\phi - \phi_S)\ \sum_q e_q^2\ f_{1T}^{\perp,q}(x)\ D_q(z)$$





SMC, **HERMES,**
COMPASS, CLAS

Seen !

Nonvanishing Sivers functions