Notes for Quantum Mechanics

Richard Seto

```
Date[]
```

{2005, 1, 19, 15, 21, 29.0917952}

{2004, 11, 23, 11, 31, 28.3195952}

Lecture 20

$$a[x_] := \frac{\mathrm{Exp}\left[-\frac{x*x}{2}\right]}{\sqrt[4]{\mathrm{Pi}}}$$

$$b[x_] := \frac{\sqrt{2}\ x*\mathrm{Exp}\left[-\frac{x*x}{2}\right]}{\sqrt[4]{\mathrm{Pi}}}$$

$$e[x_] := \frac{1}{\sqrt{2\,\mathrm{Pi}}}*\mathrm{Exp}[-I*p*x]$$

$$\mathrm{pp1} = \mathrm{Integrate}\left[a[x]*e[x]\Big/\sqrt{2\,\mathrm{Pi}},\ \{x,\ -\infty,\ \infty\}\right]$$

$$\frac{e^{-\frac{p^2}{2}}}{\sqrt{2}\ \pi^{3/4}}$$

phi0 = Simplify[%]

$$\frac{e^{-\frac{p^2}{2}}}{\sqrt{2}\ \pi^{3/4}}$$

```
pp2 = Integrate[b[x] * e[x] / Sqrt[2], {x, -∞, ∞}]
```

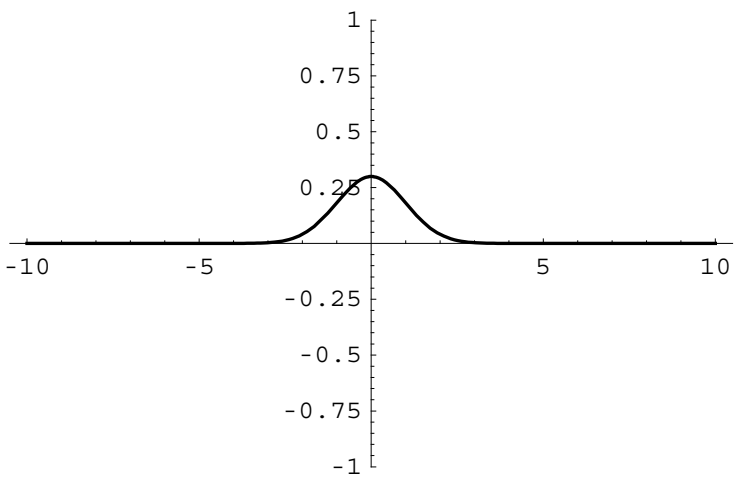
$$-\frac{i\, e^{-\frac{p^2}{2}}\, p}{\sqrt[4]{\pi}}$$

```
phi1 = Simplify[%]
```

$$-\frac{i\, e^{-\frac{p^2}{2}}\, p}{\sqrt[4]{\pi}}$$

$$y[p_] := \frac{e^{-\frac{p^2}{2}}}{\sqrt{2}\, \pi^{3/4}}$$

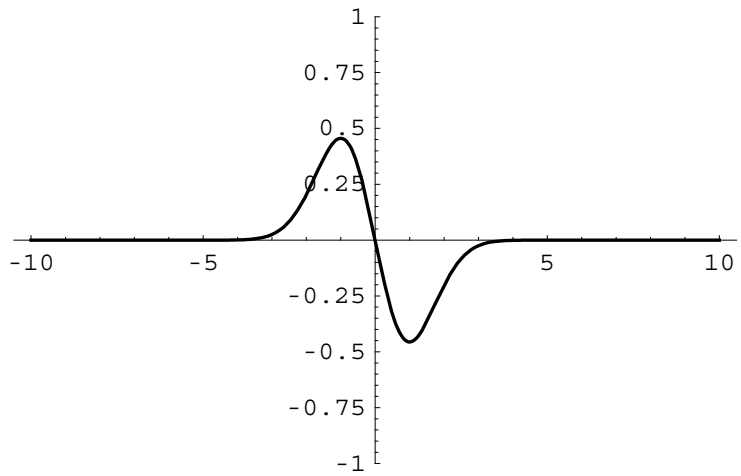
```
Plot[y[x], {x, -10., 10.}, PlotRange → {-1., 1.},
 PlotStyle → {{Thickness[0.005], RGBColor[0, 0, 0]}}]
```



- Graphics -

$$e[p_] := -\frac{i\, e^{-\frac{p^2}{2}}\, p}{\sqrt[4]{\pi}}$$

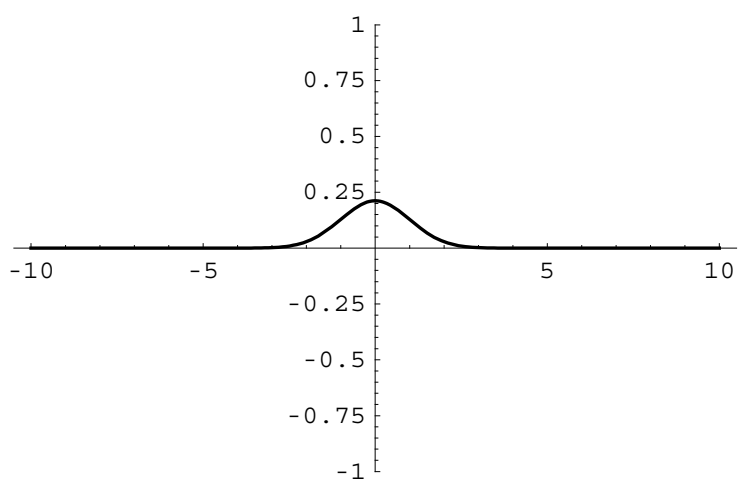
```
Plot[Im[e[p]], {p, -10., 10.}, PlotRange → {-1., 1.},
 PlotStyle → {{Thickness[0.005], RGBColor[0, 0, 0]}}]
```



- Graphics -

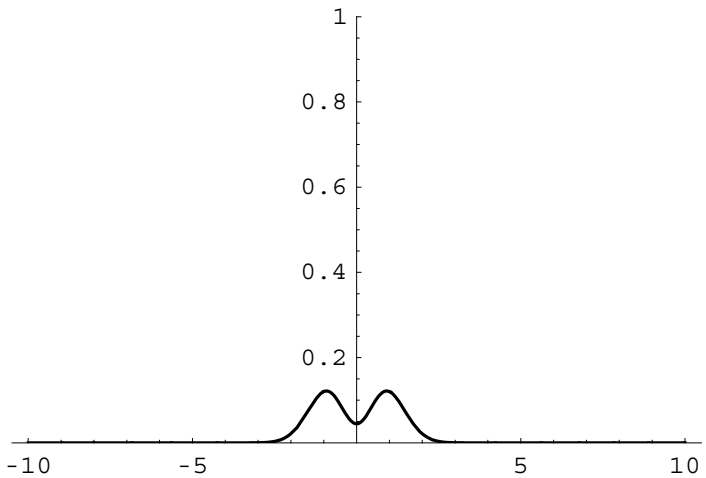
```
f[p_] := (y[p] + e[p]) / Sqrt[2.]
```

```
Plot[Re[f[p]], {p, -10., 10.}, PlotRange → {-1., 1.},
 PlotStyle → {{Thickness[0.005], RGBColor[0, 0, 0]}}]
```



- Graphics -

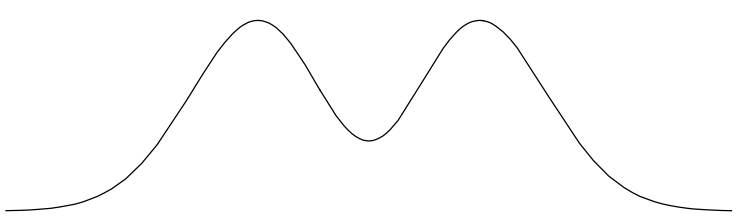
```
c = Plot[Conjugate[f[p]] * f[p], {p, -10., 10.}, PlotRange → {0., 1.},
  PlotStyle → {{Thickness[0.005], RGBColor[0, 0, 0]}}]
```

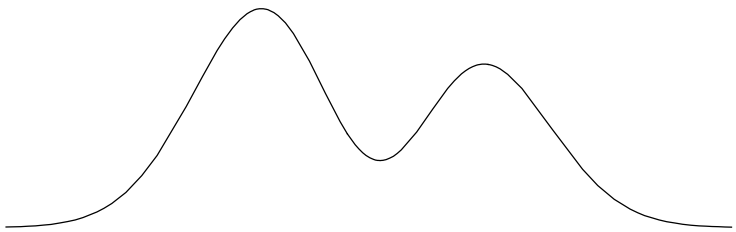


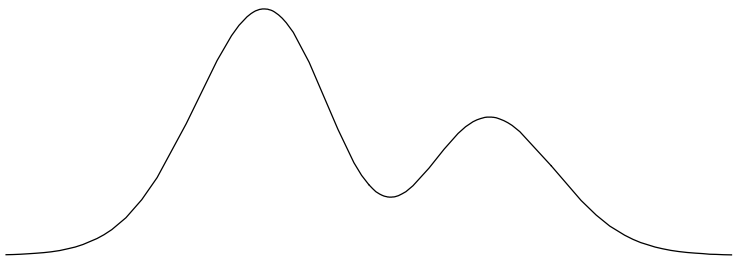
- Graphics -

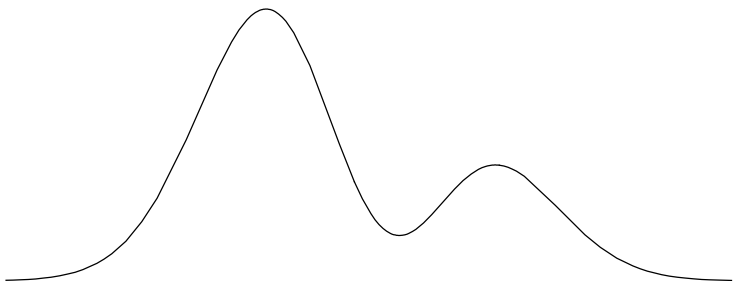
```
g[p_, t_] := Re[Conjugate[y[p] * Exp[-I * (1/2) * t] + e[p] * Exp[-I * (1 + 1/2) * t]] *
    (y[p] * Exp[-I * (1/2) * t] + e[p] * Exp[-I * (1 + 1/2) * t]) / 2.]
```

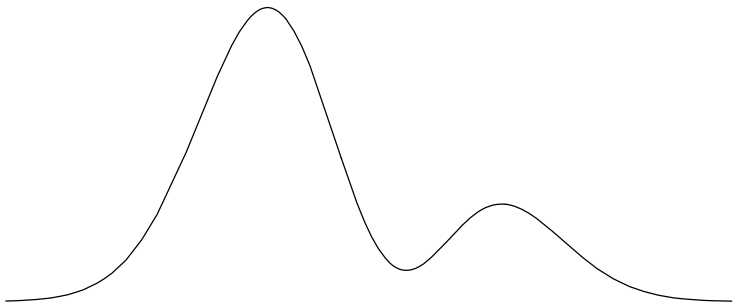
```
Do[Plot[g[p, t], {p, -3, 3}, Axes → False, PlotRange → {0., .3}], {t, 0, 2 * Pi, .2}]
```

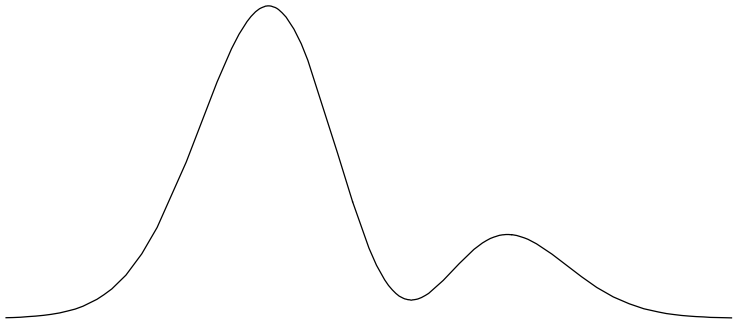


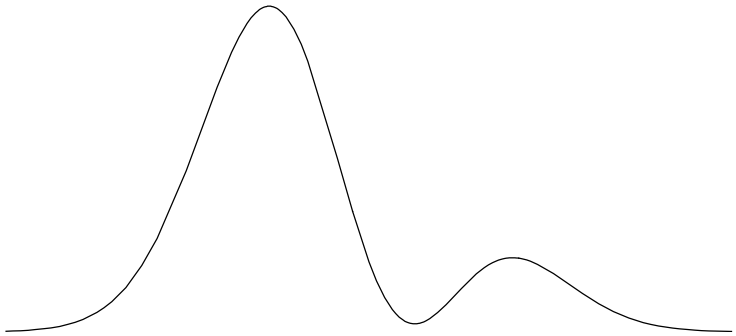


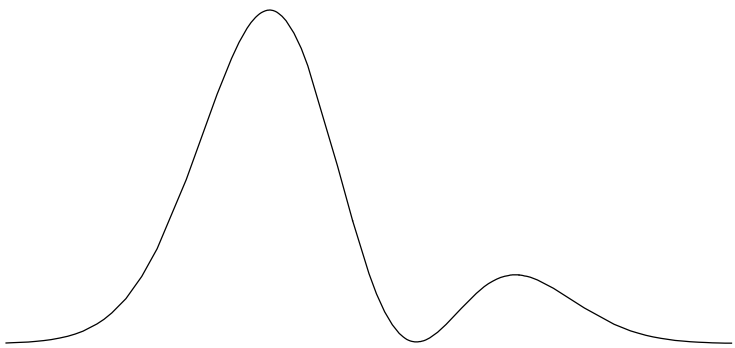


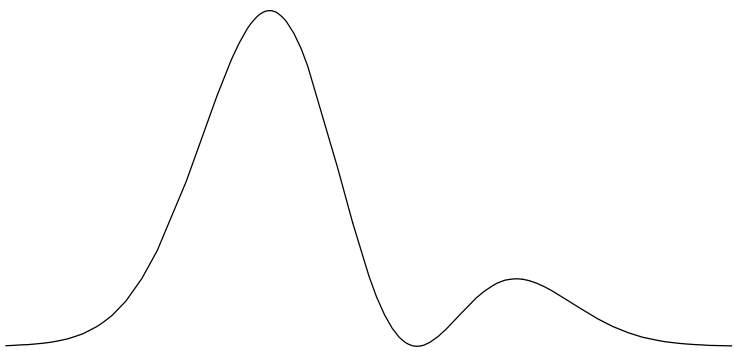


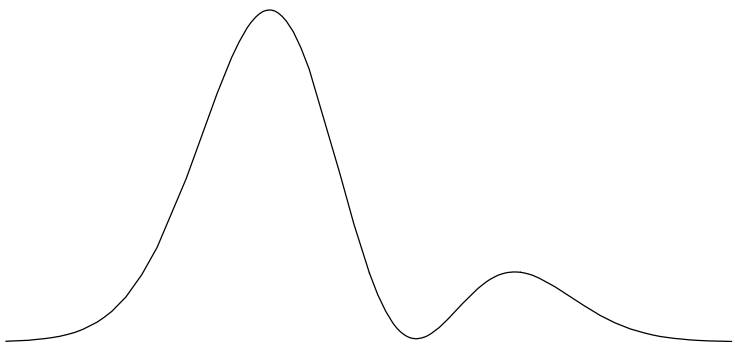


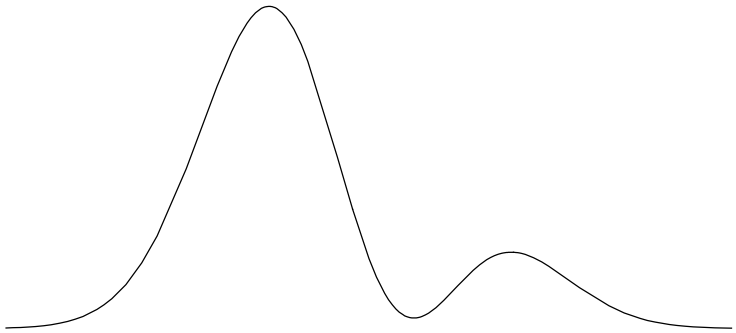


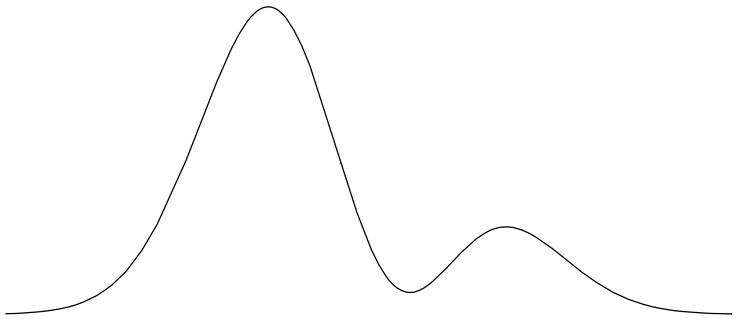


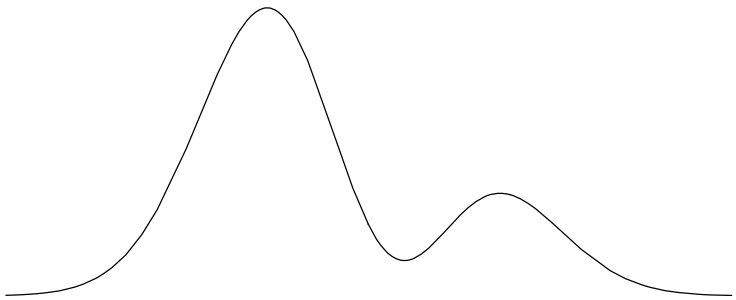


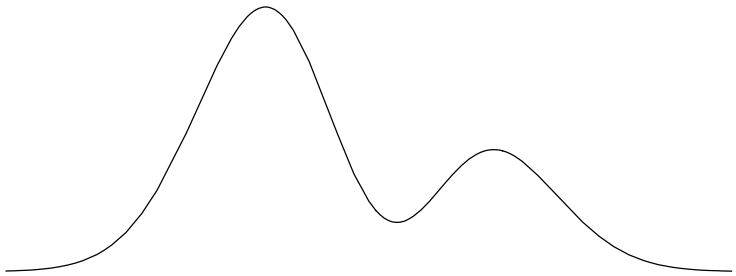


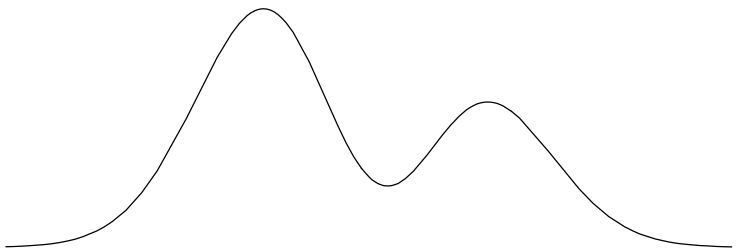


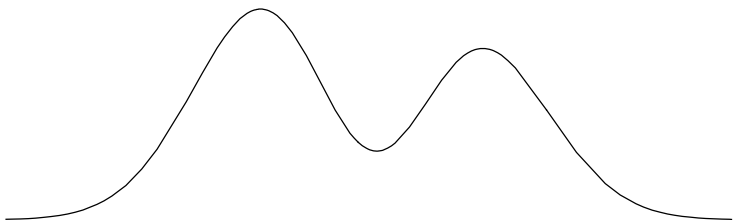


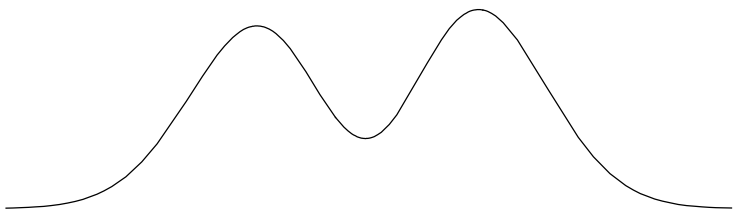


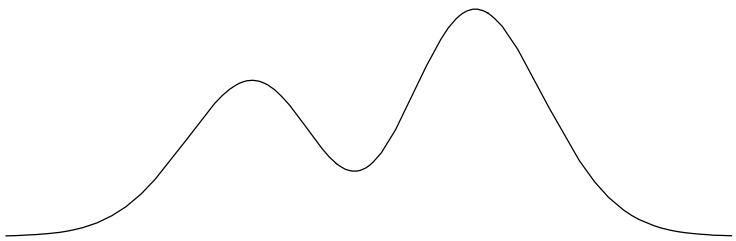


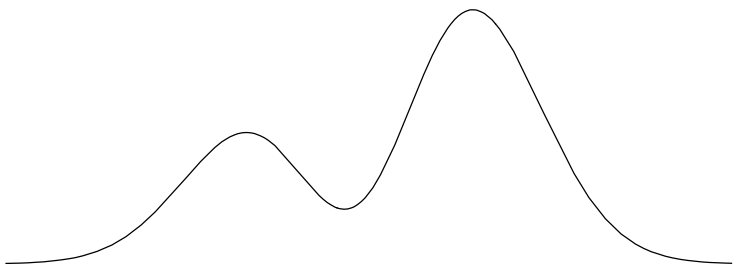


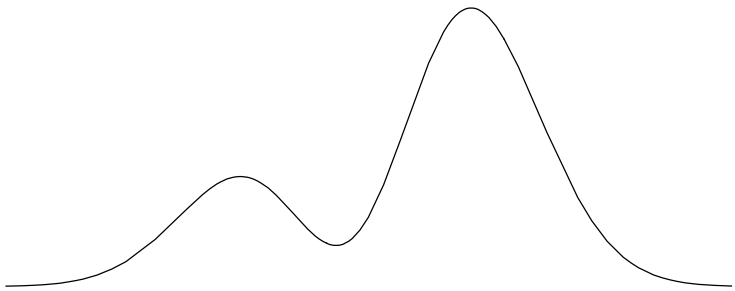


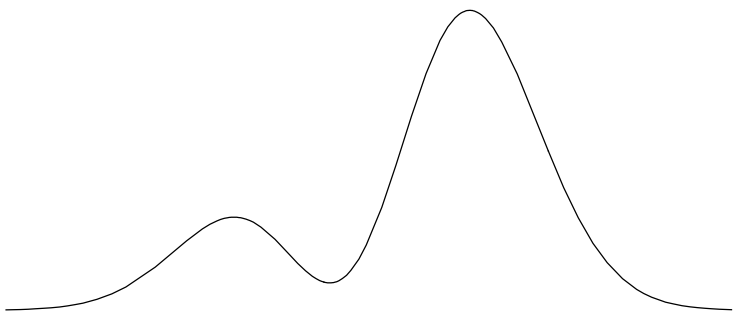


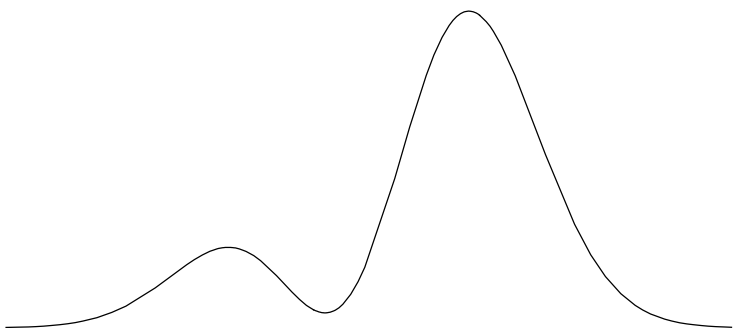


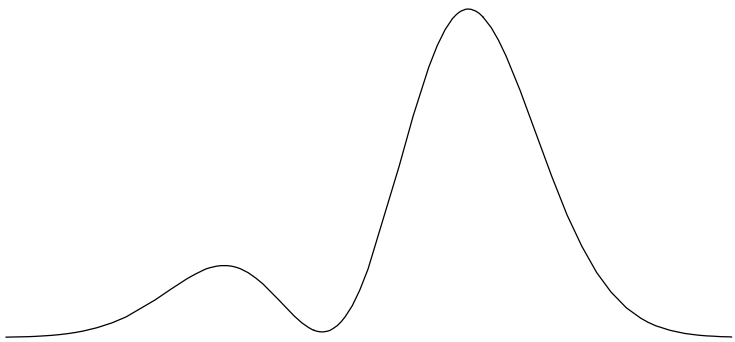


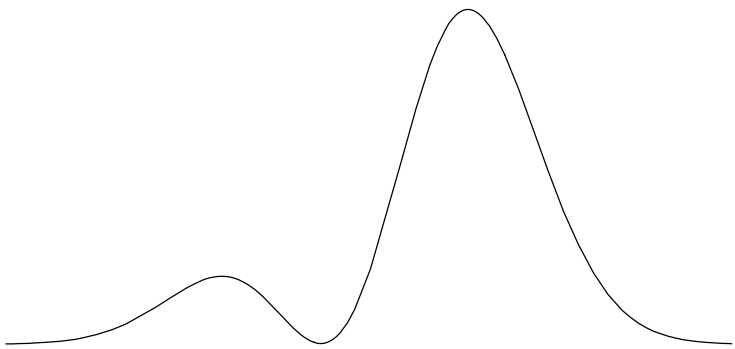


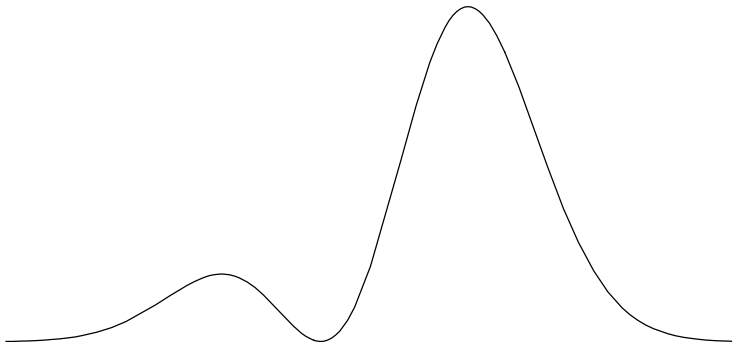


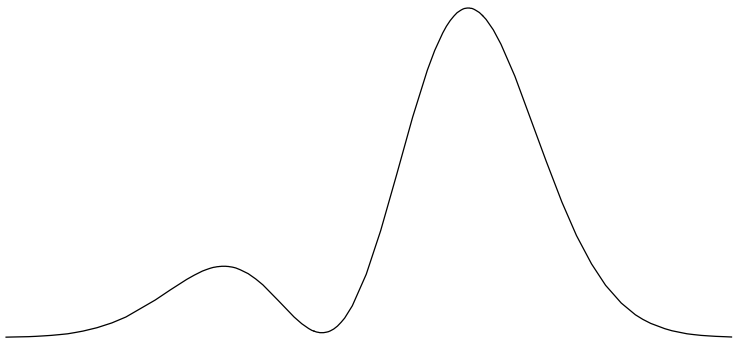


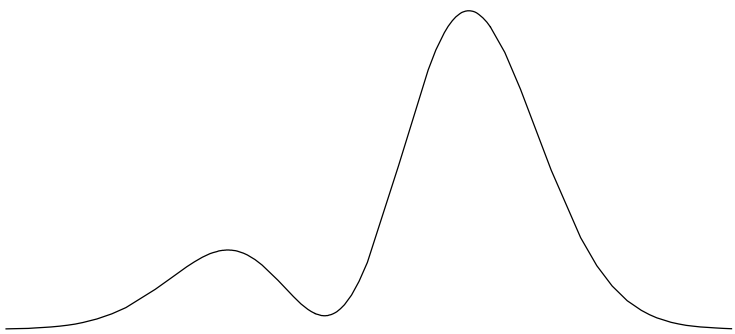


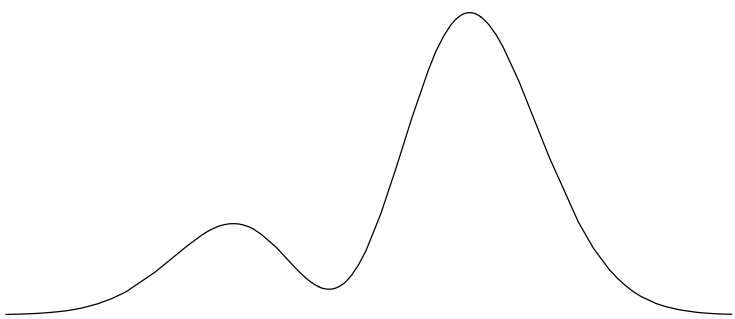


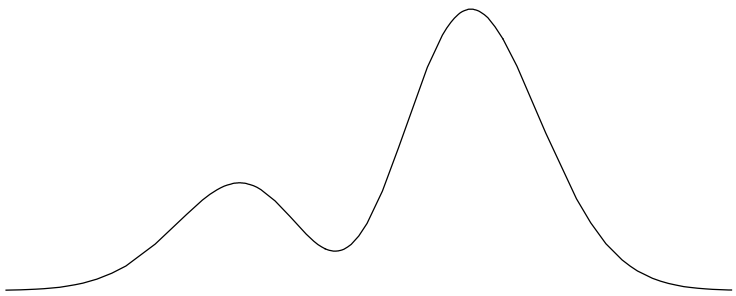


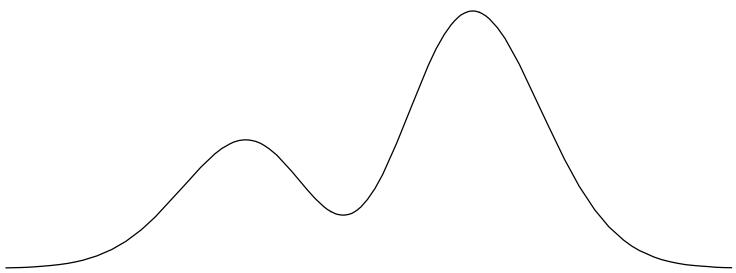


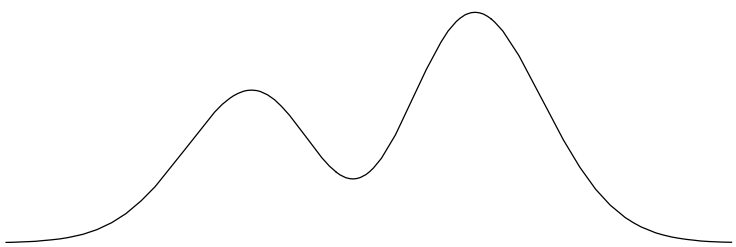


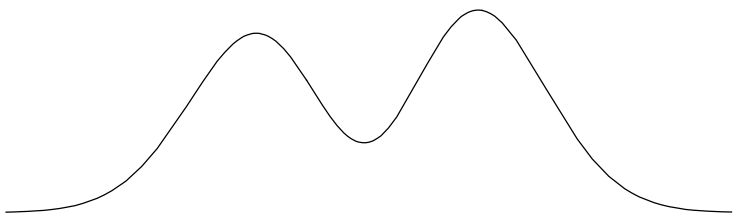












```
psi0 = Exp[-x^2/2] / Sqrt[Sqrt[Pi]]
```

General::spell1 : Possible spelling error: new symbol name "psi0" is similar to existing symbol "phi0". More…

$$\frac{e^{-\frac{x^2}{2}}}{\sqrt[4]{\pi}}$$

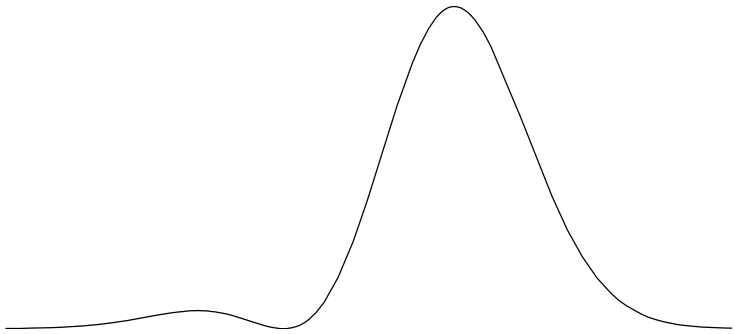
```
psi1 = FullSimplify[x * % - D[%, x]] / Sqrt[2 * 1]
```

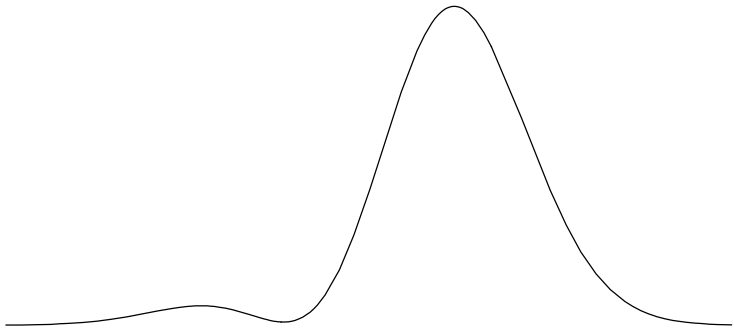
General::spell1 : Possible spelling error: new symbol name "psi1" is similar to existing symbol "phi1". More…

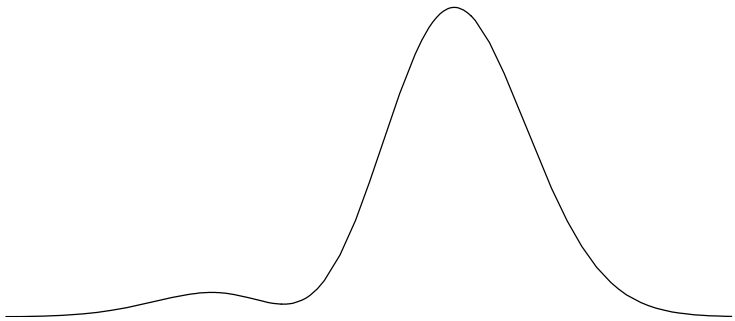
$$\frac{\sqrt{2}\, e^{-\frac{x^2}{2}}\, x}{\sqrt[4]{\pi}}$$

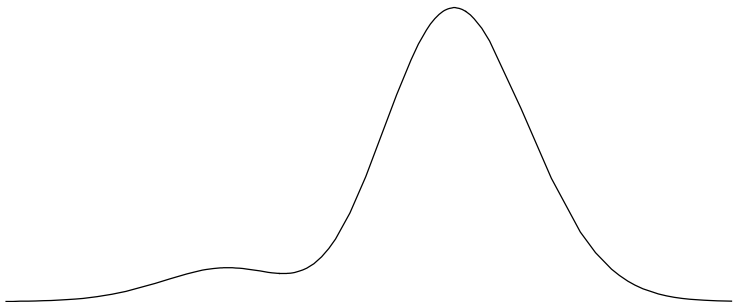
```
h[x_, t_] := Re[(psi0 * Exp[I * (1/2) * t] + psi1 * Exp[I * (1 + 1/2) * t]) *
    (psi0 * Exp[-I * (1/2) * t] + psi1 * Exp[-I * (1 + 1/2) * t]) / 2.]
```

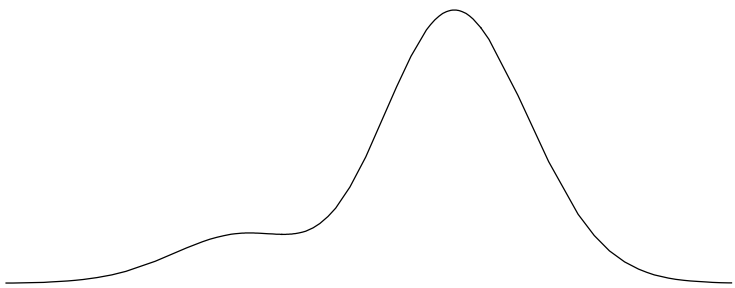
```
Do[Plot[h[x, t], {x, -3, 3}, Axes → False, PlotRange → {0, 1.0}], {t, 0, 2 * Pi, .2}]
```

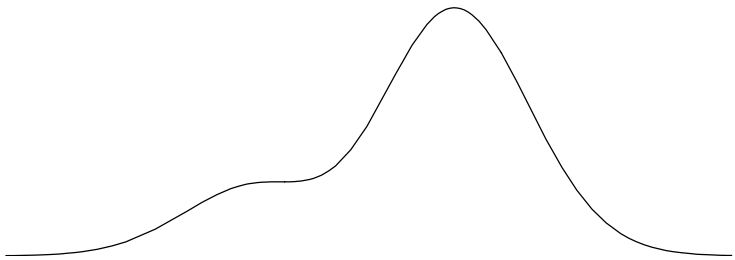


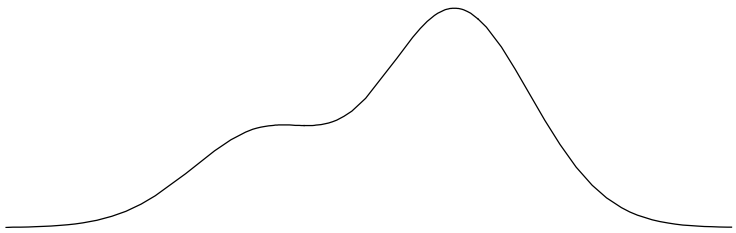


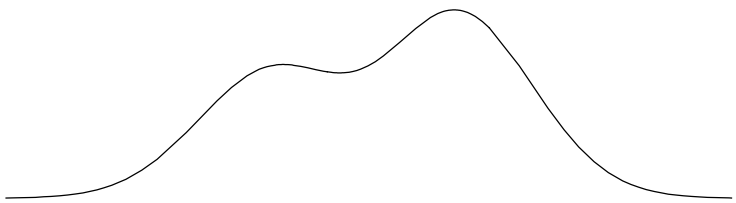


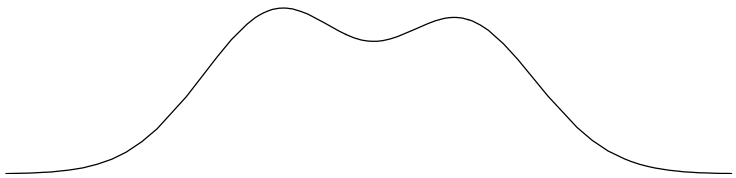


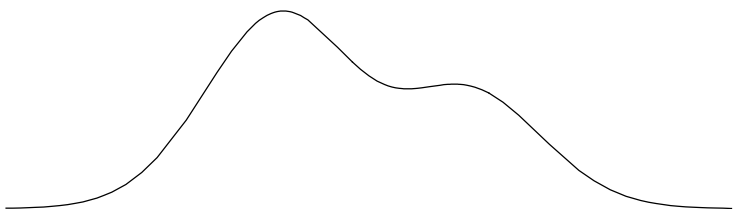


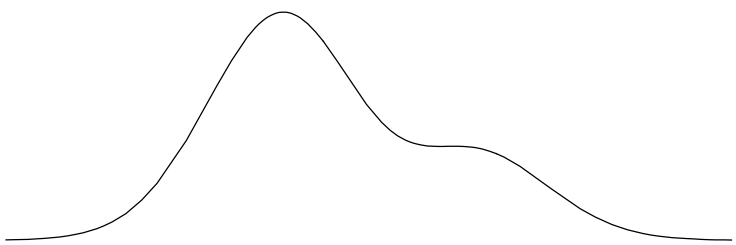


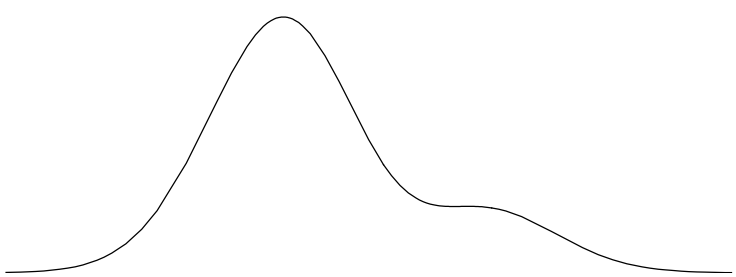


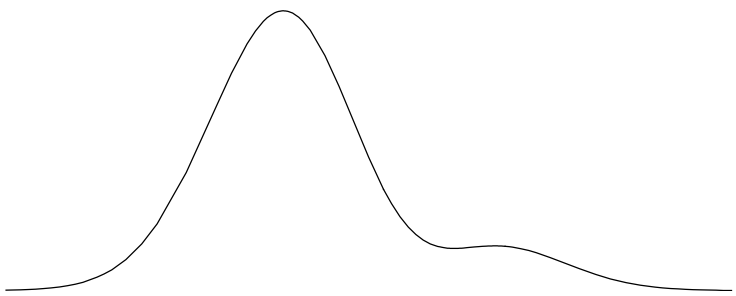


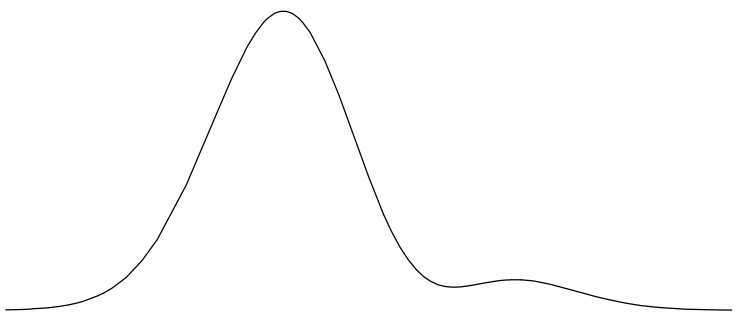


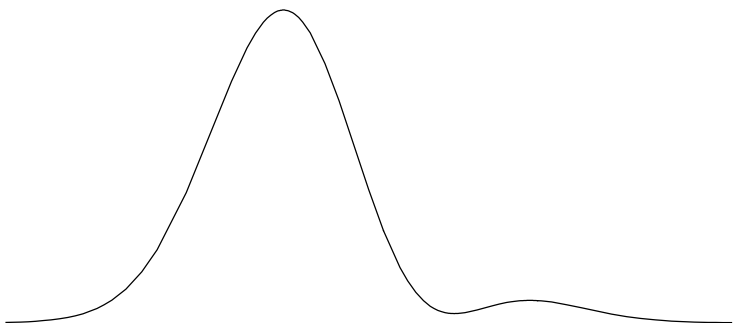


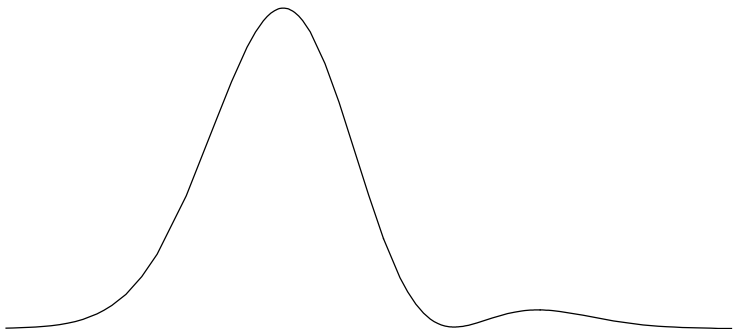


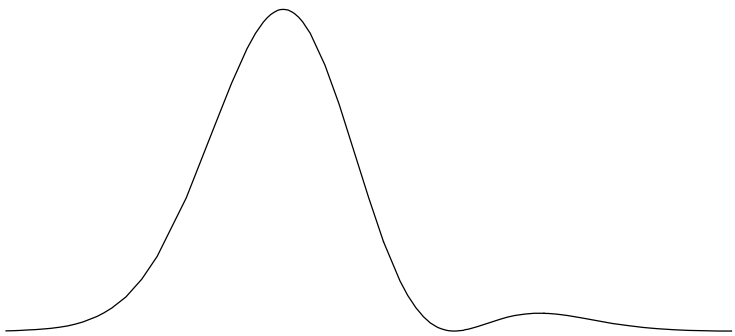


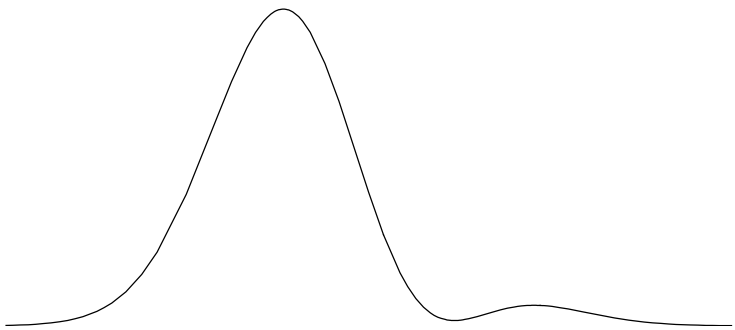


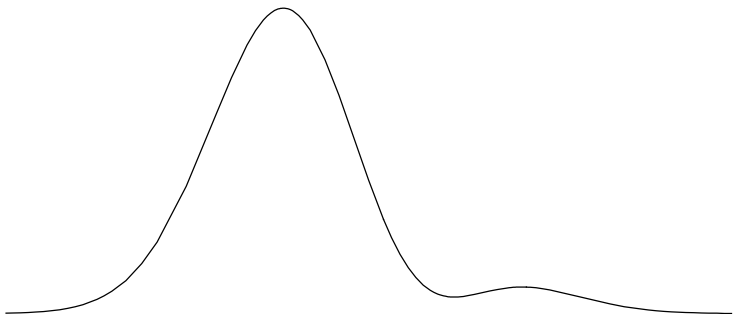


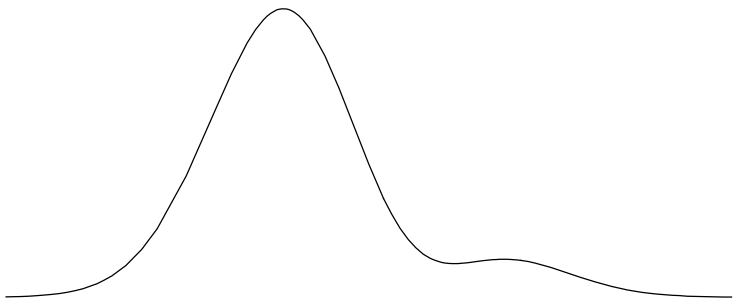


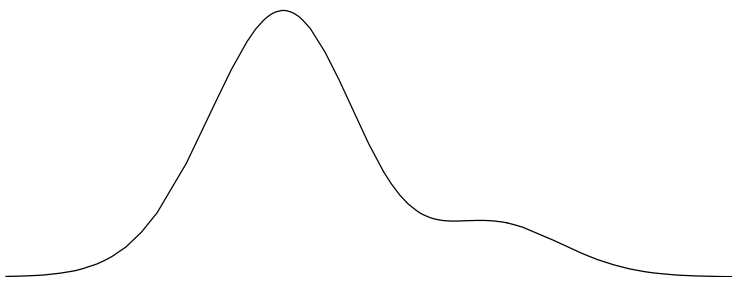


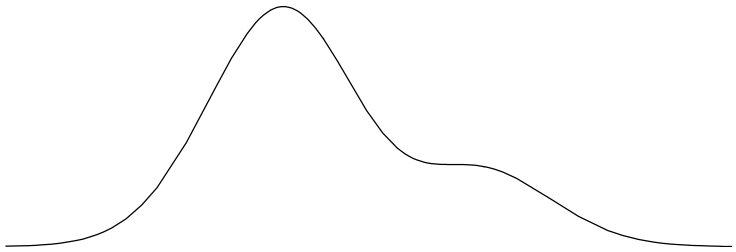


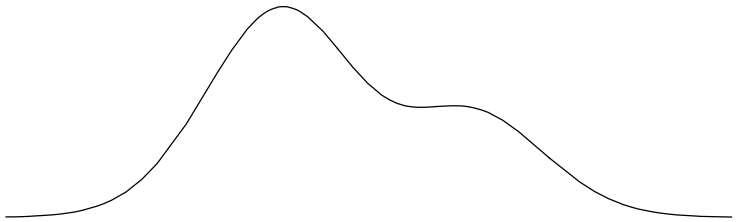


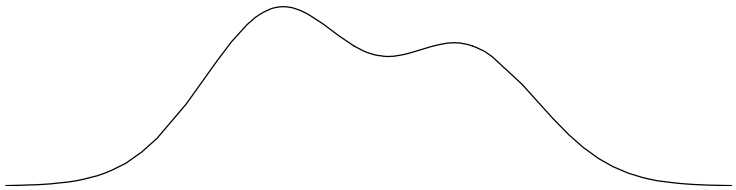


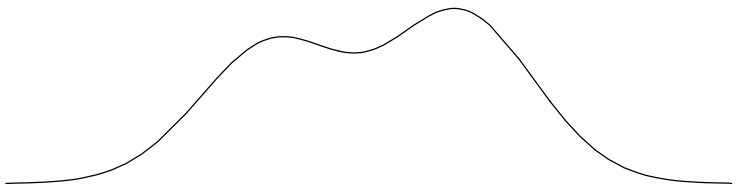


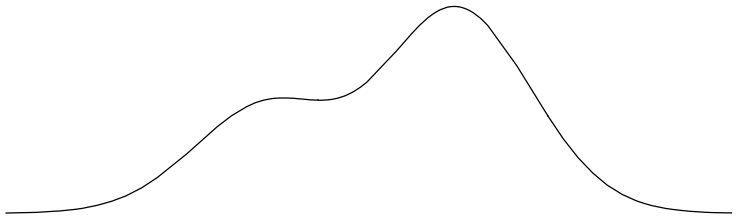


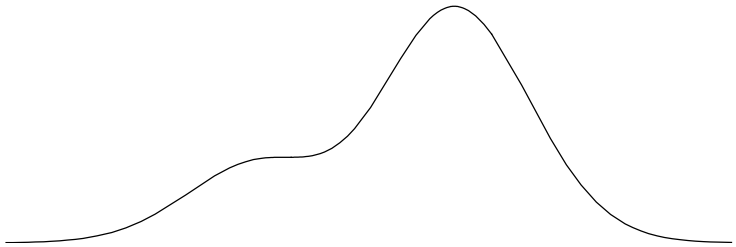


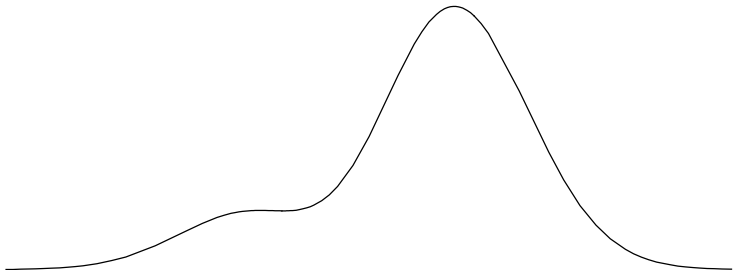


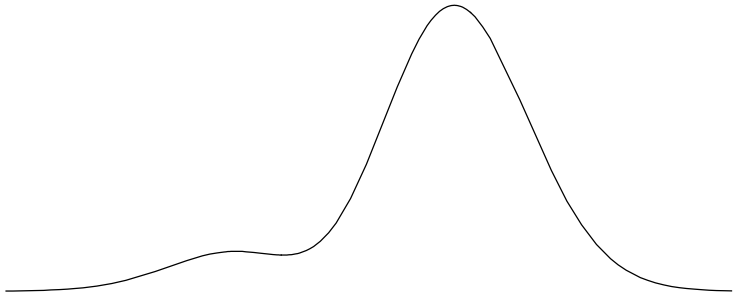


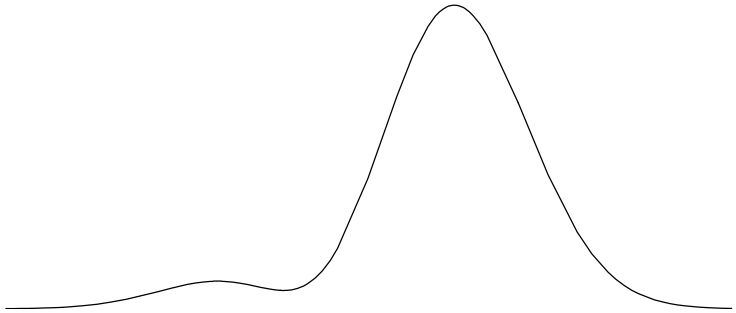


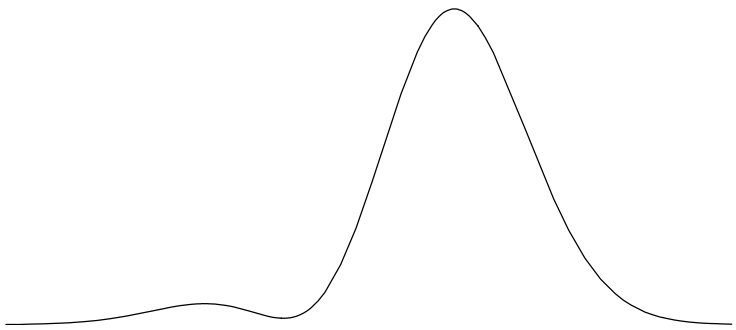


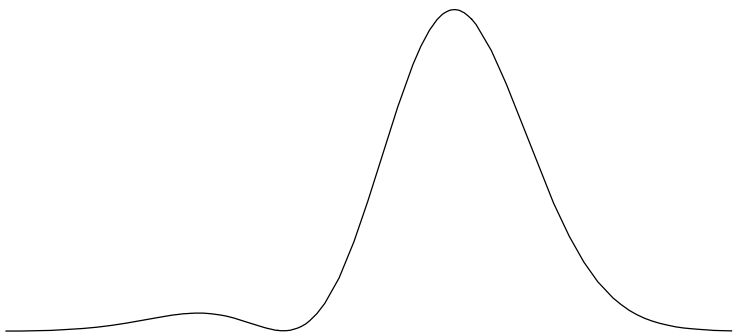




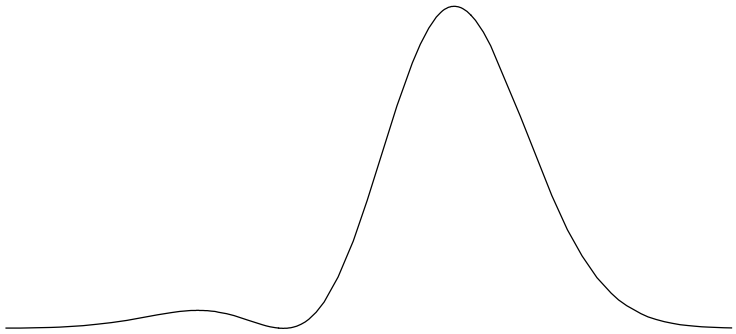


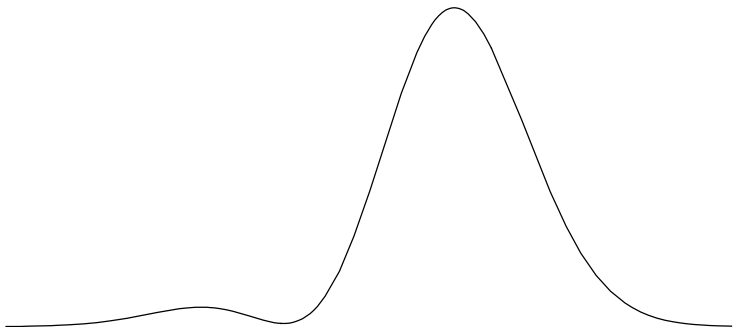


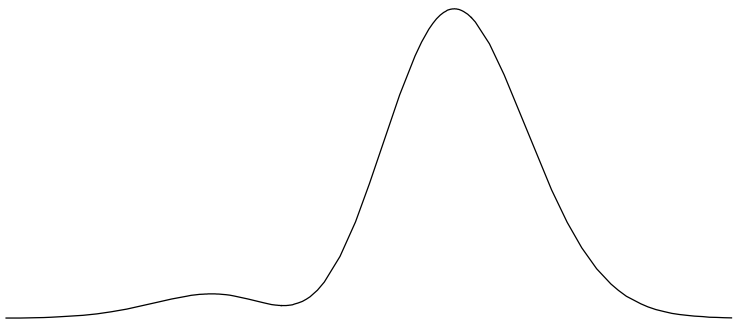


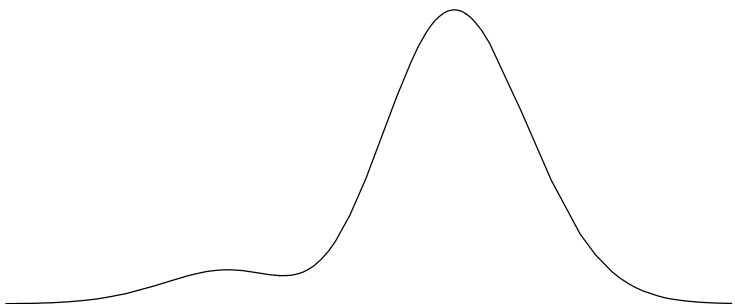


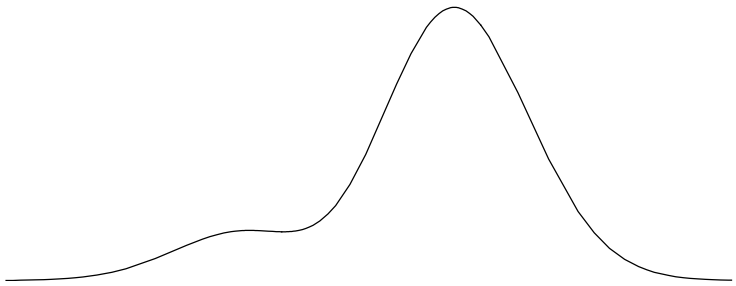
```
Do[Plot[h[x, t], {x, -3, 3}, Axes → False, PlotRange → {0, 1.0}], {t, 0, 2 * Pi, .2}]
```

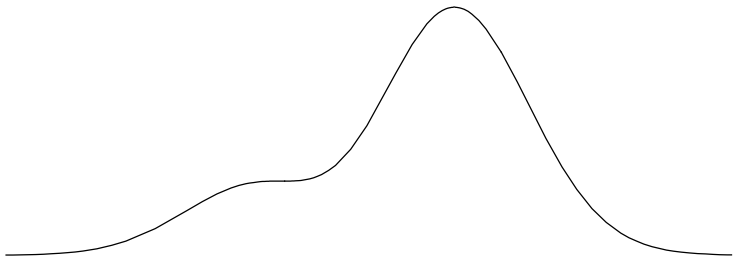


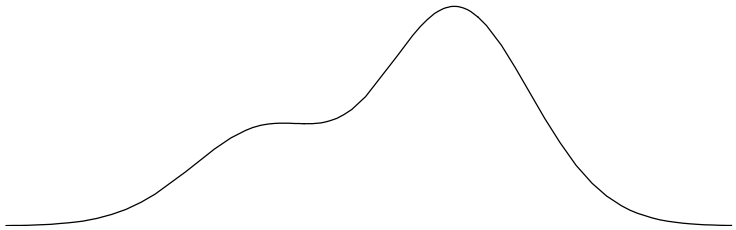


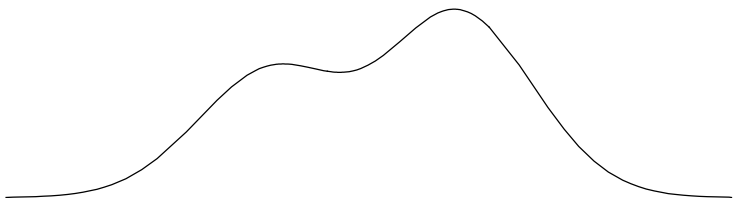


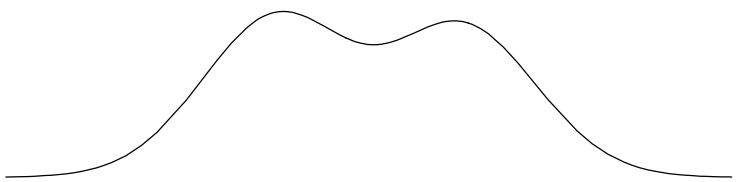


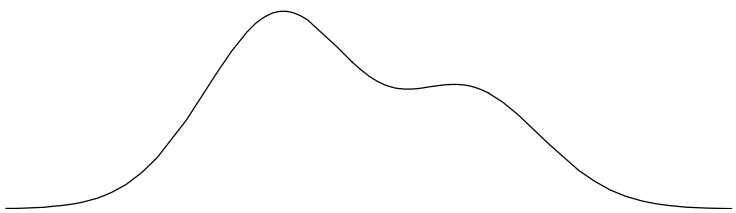


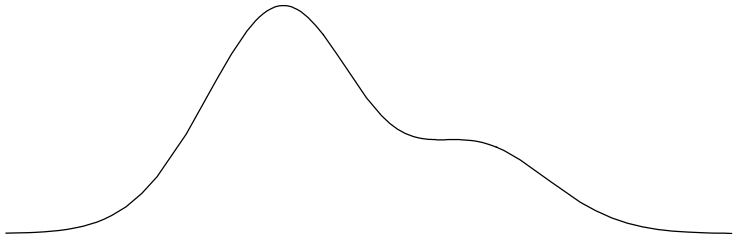


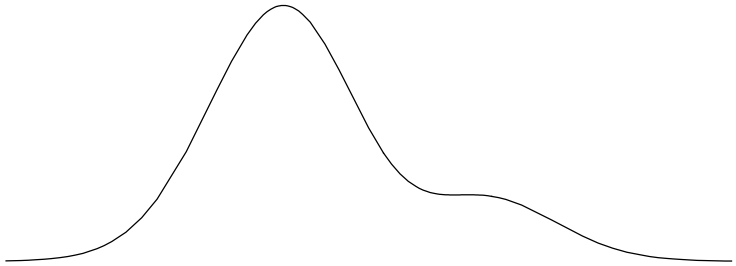


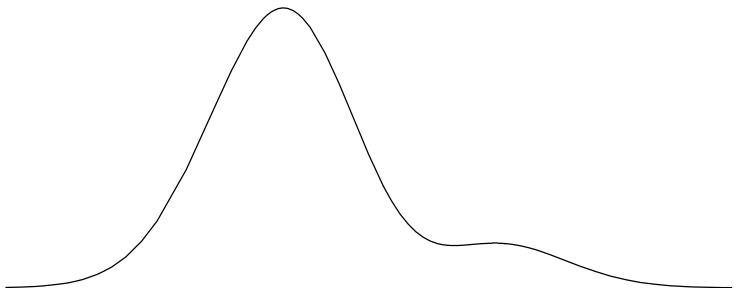


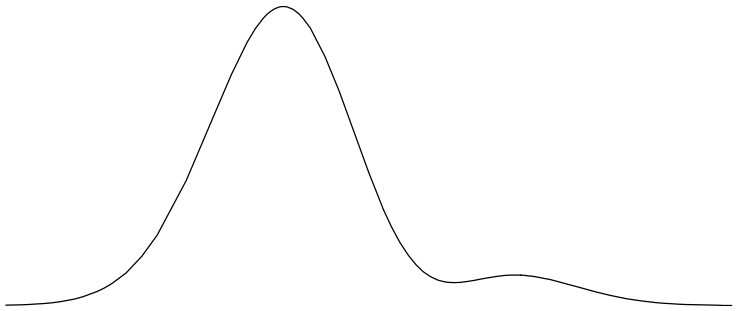


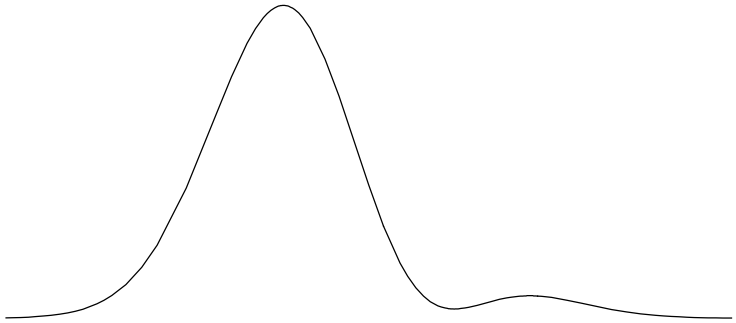


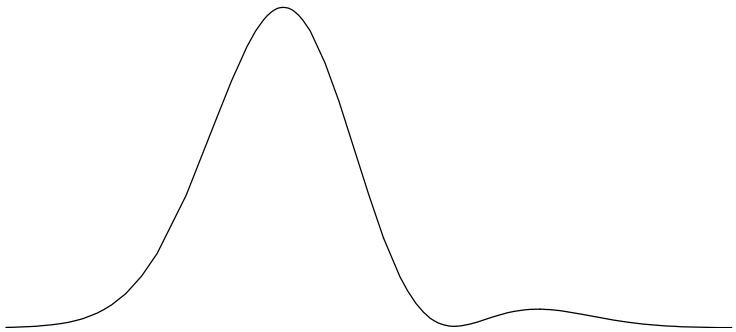


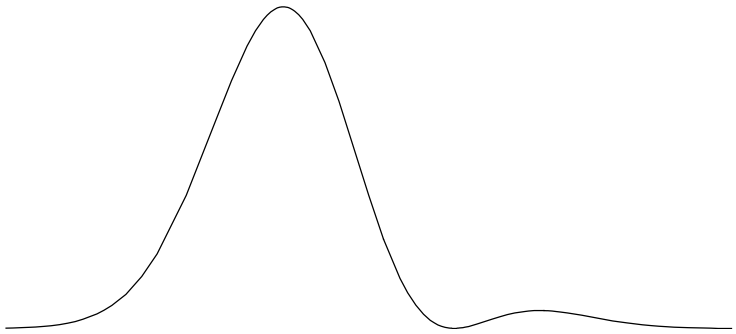


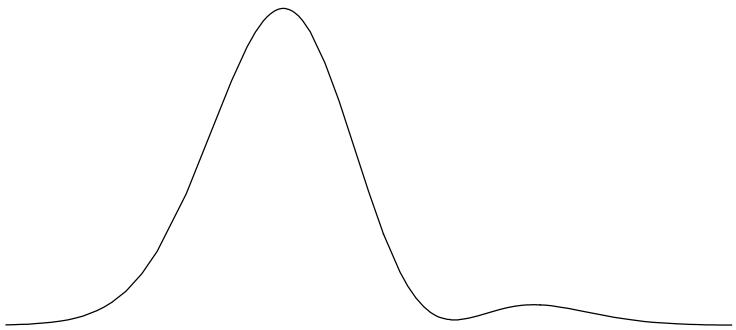


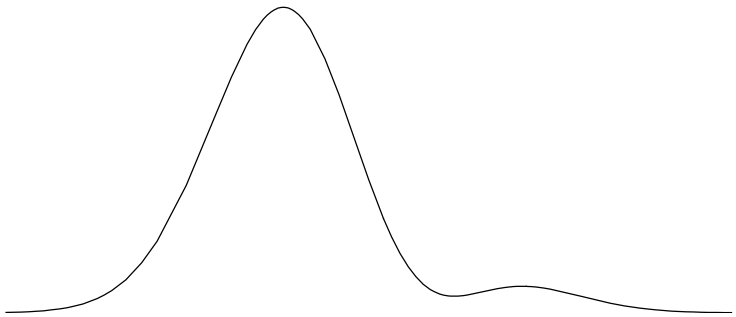


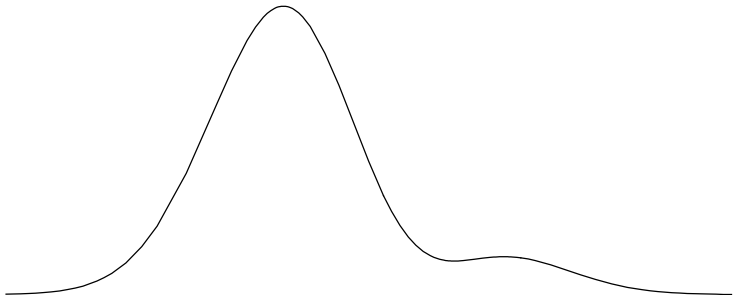


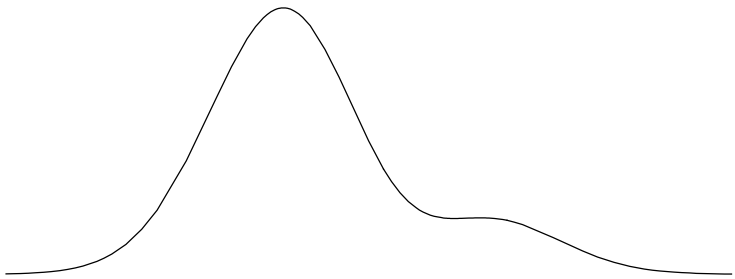


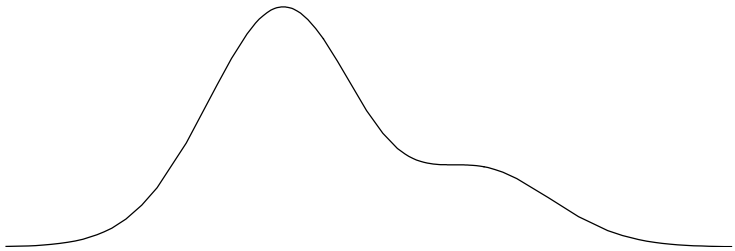


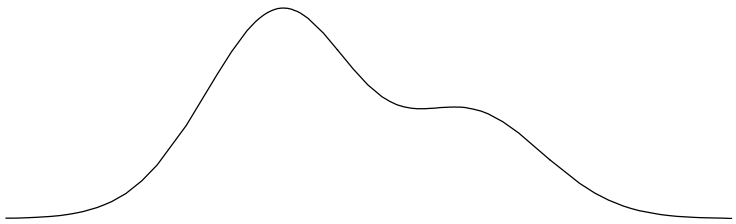


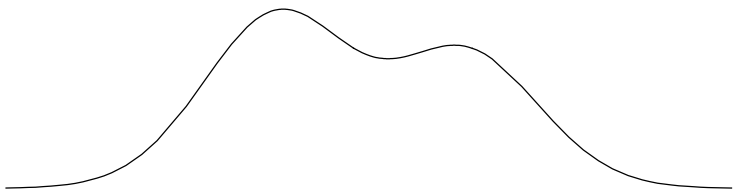


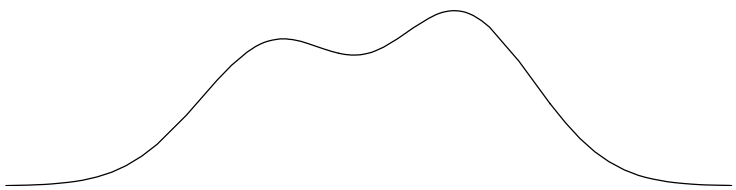


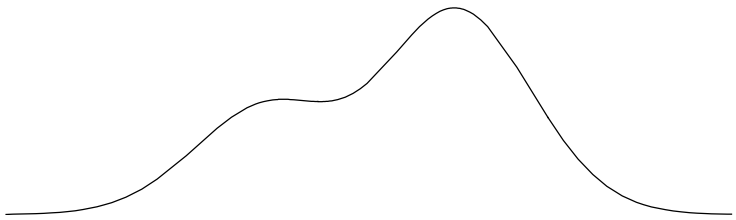


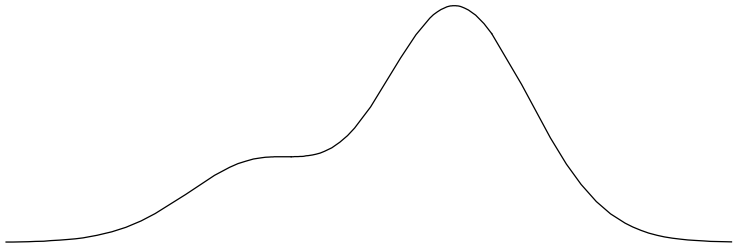


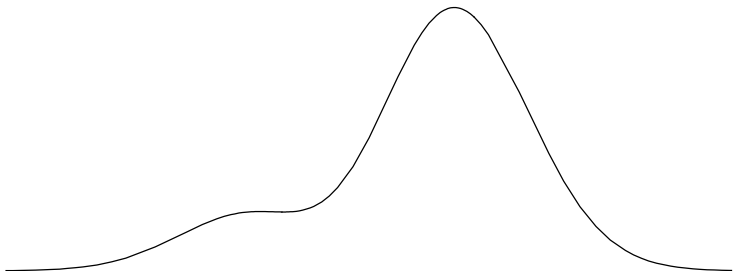


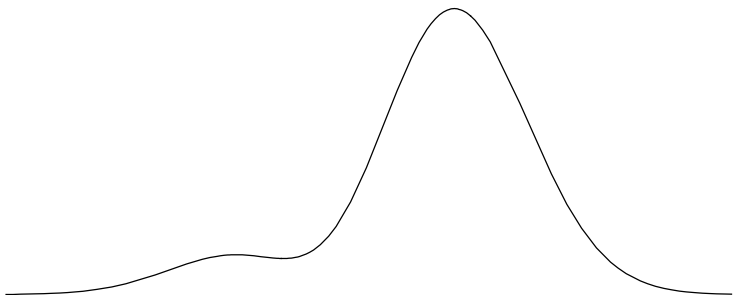


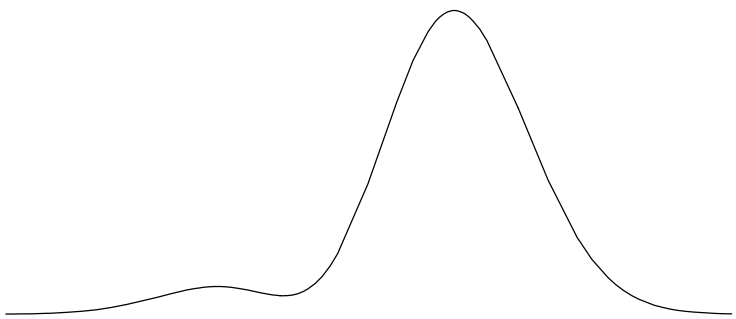


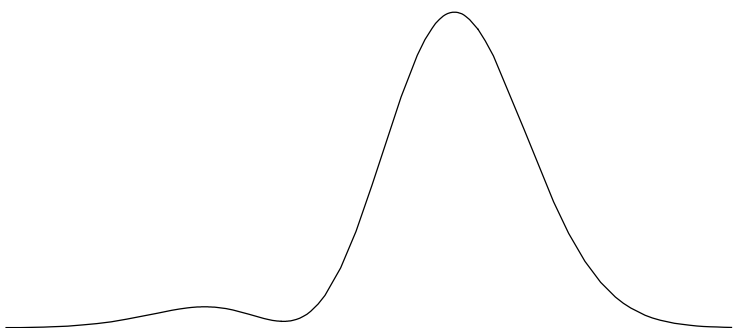


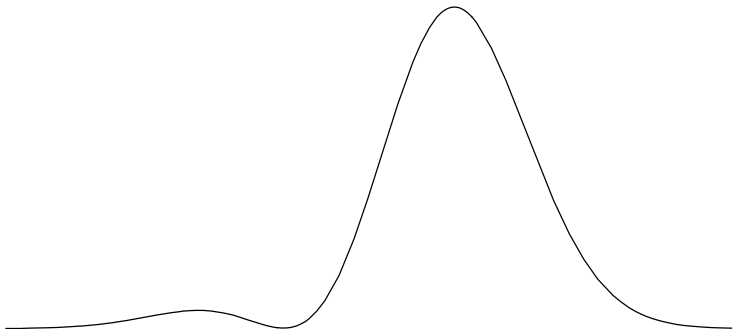




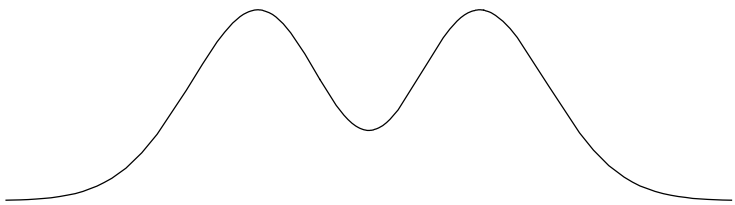


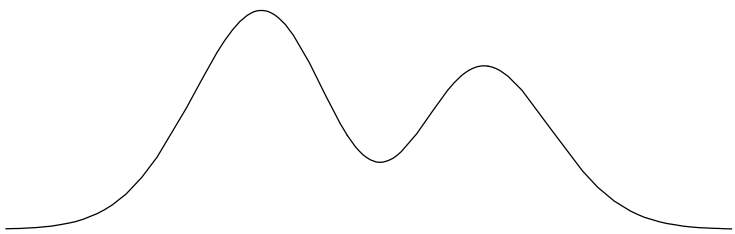


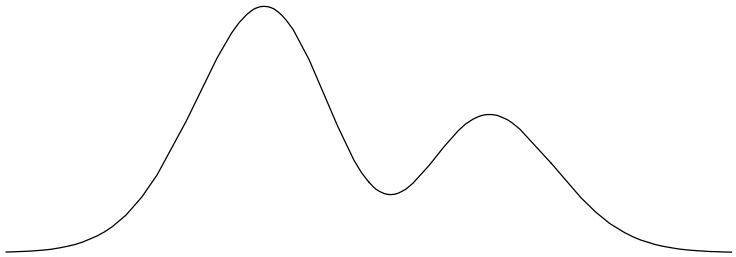


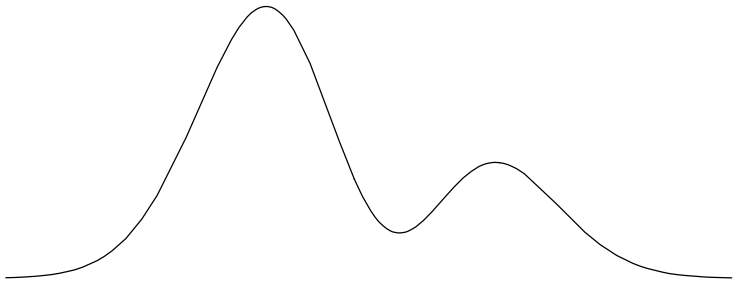


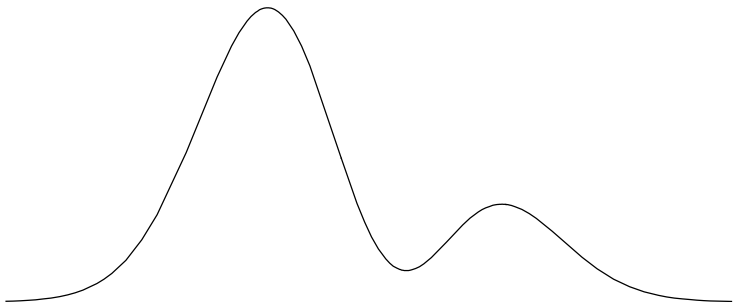
```
Do[Plot[g[p, t], {p, -3, 3}, Axes → False, PlotRange → {0., .3}], {t, 0, 2 * Pi, .2}]
```

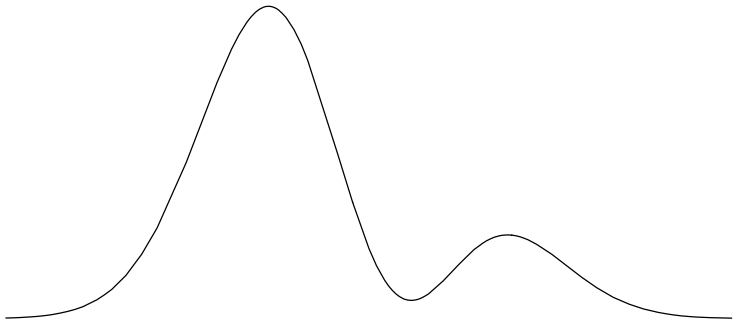


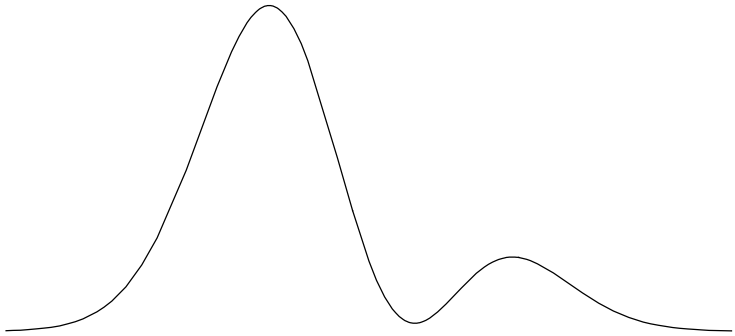


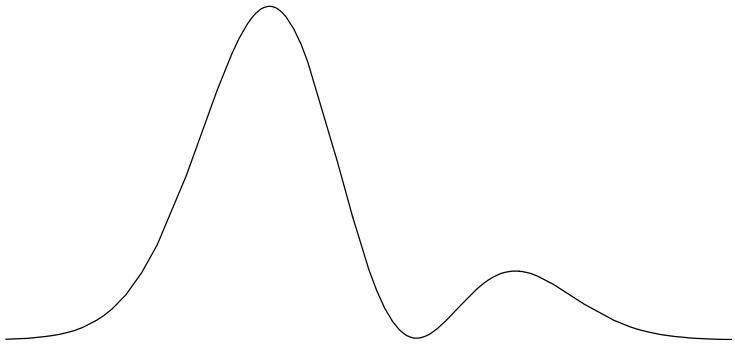


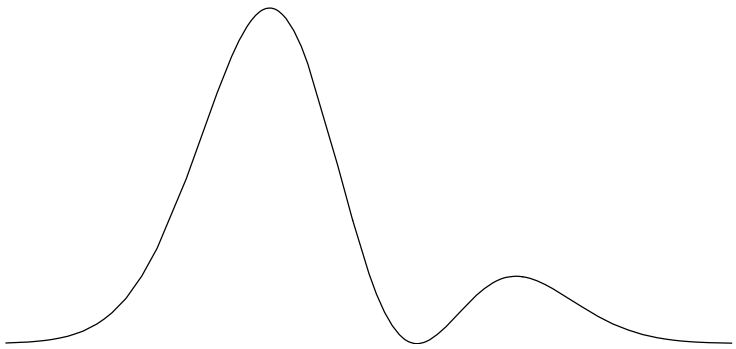


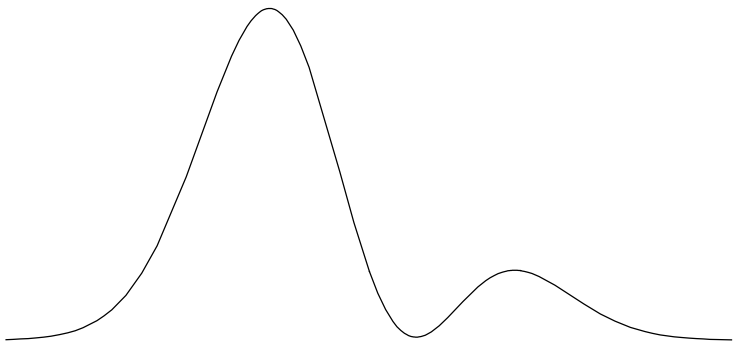


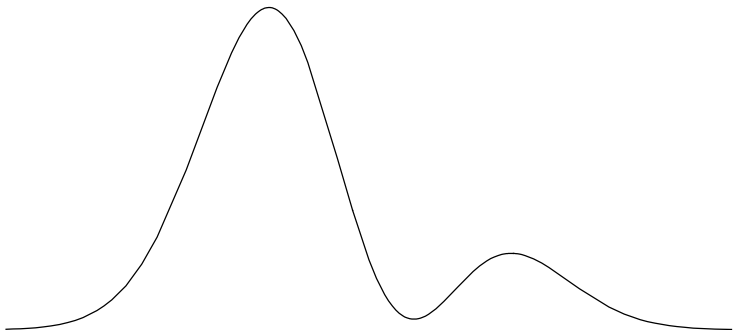


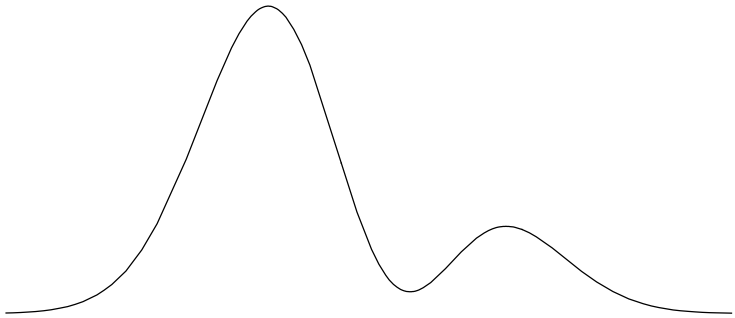


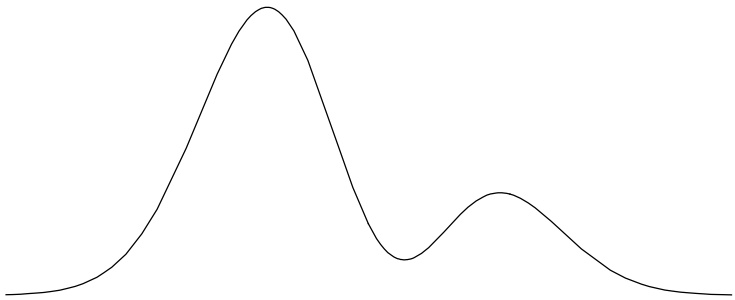


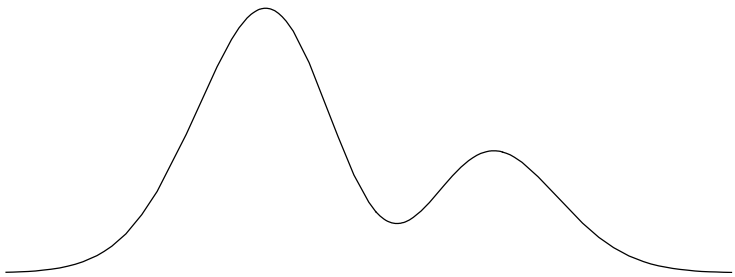


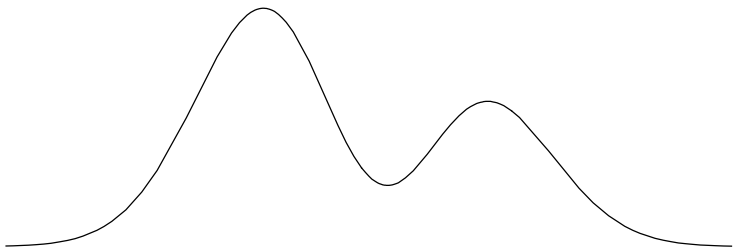


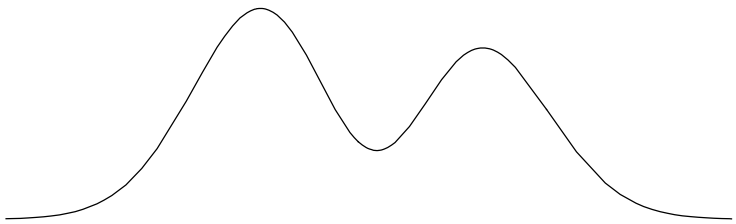


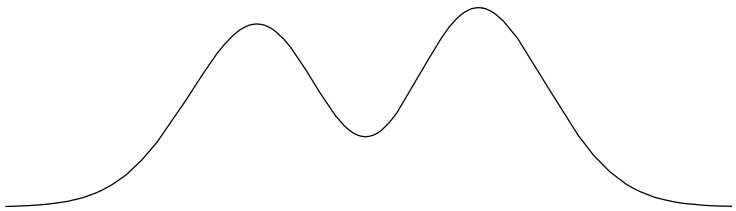


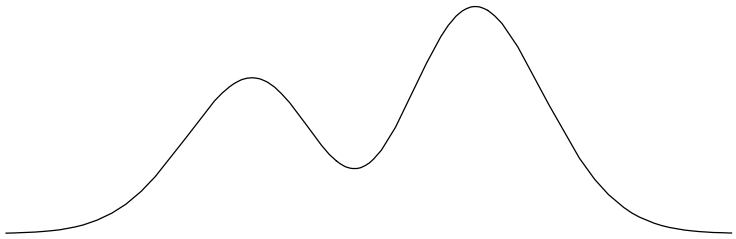


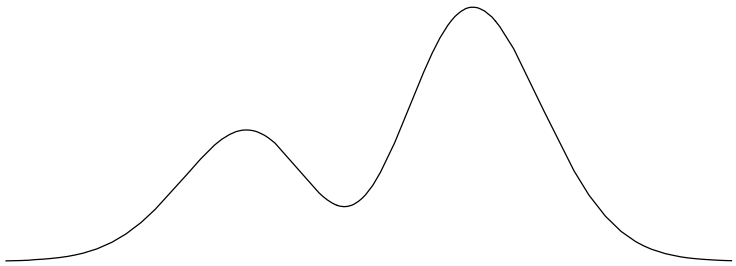


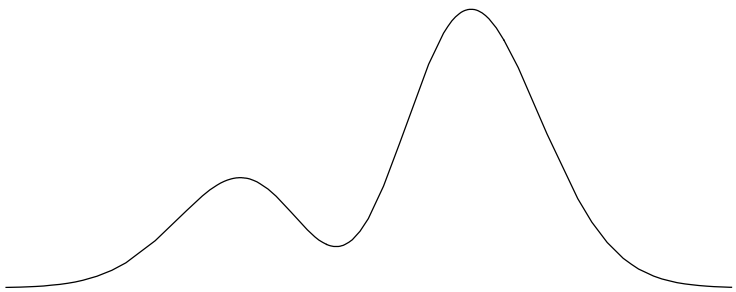


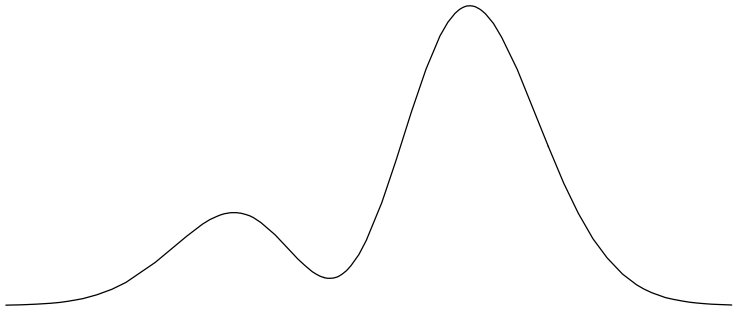


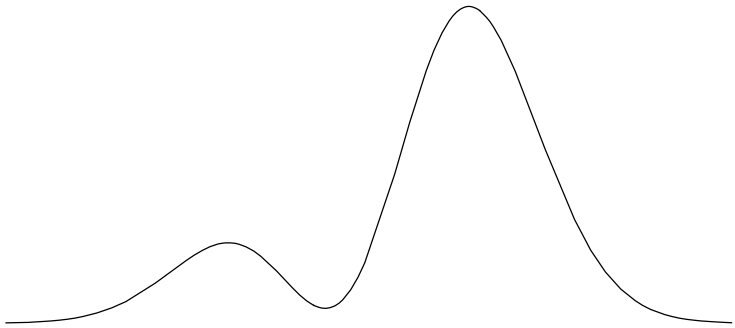


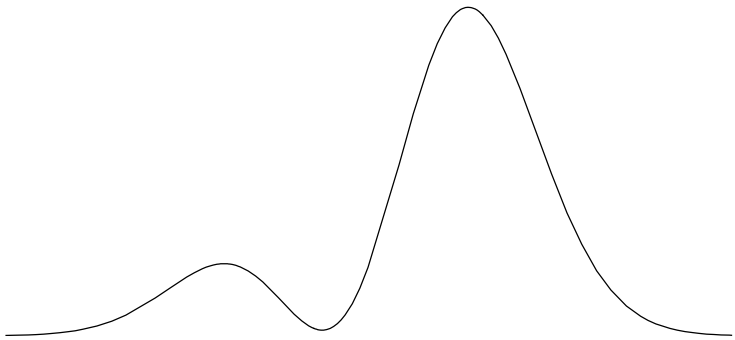


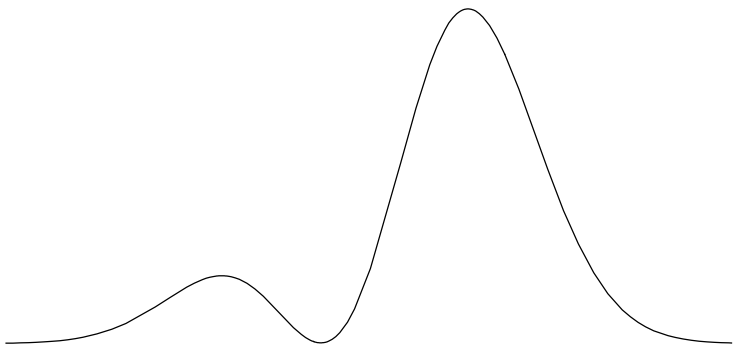


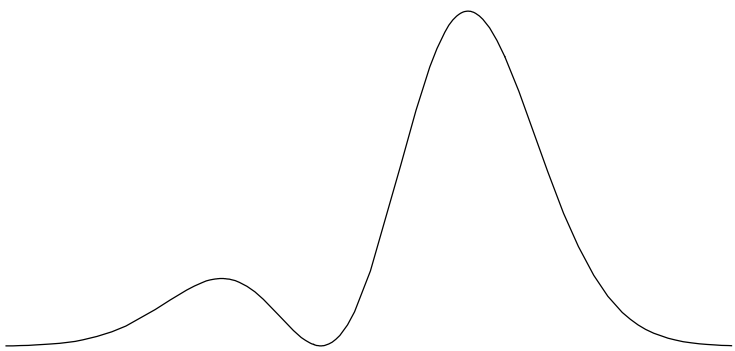


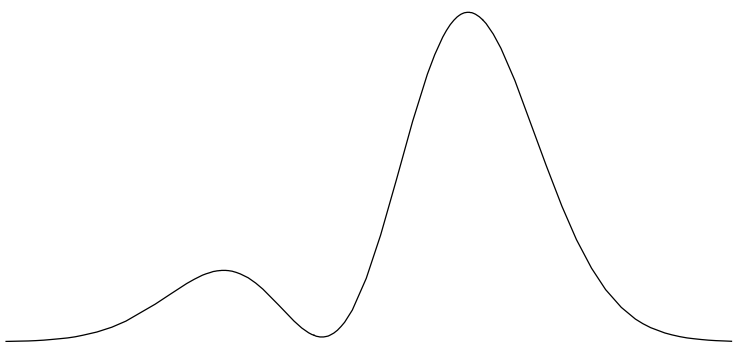


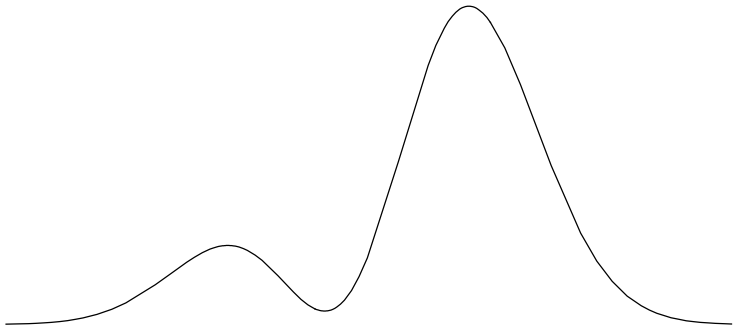


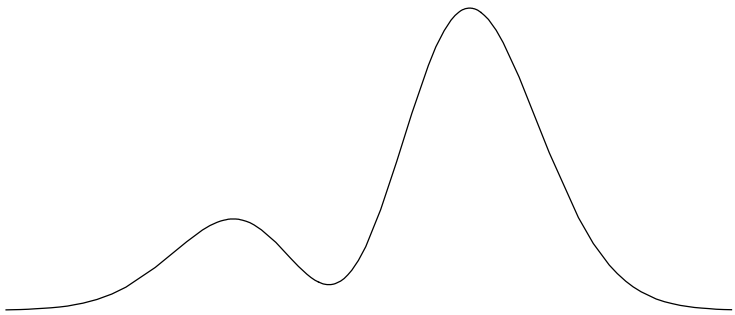


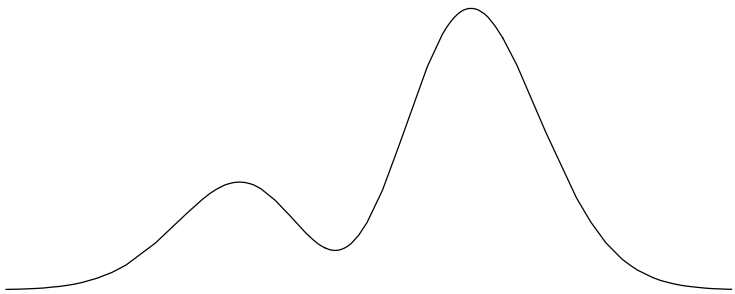


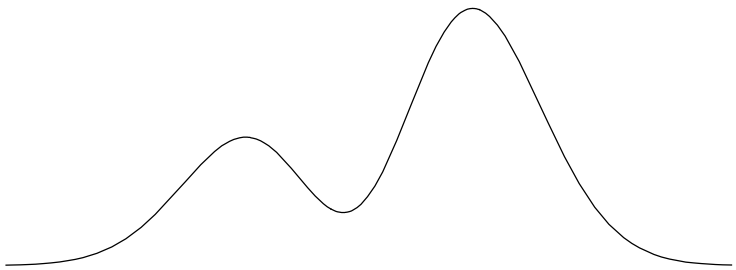


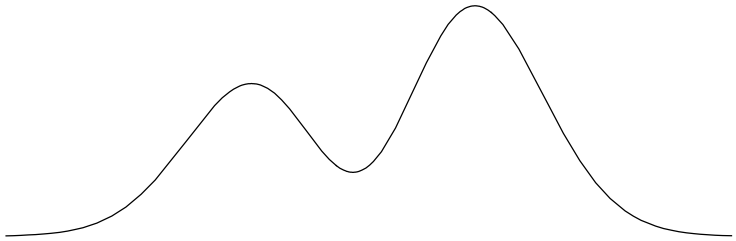


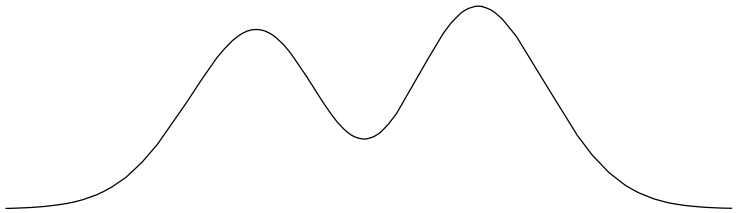




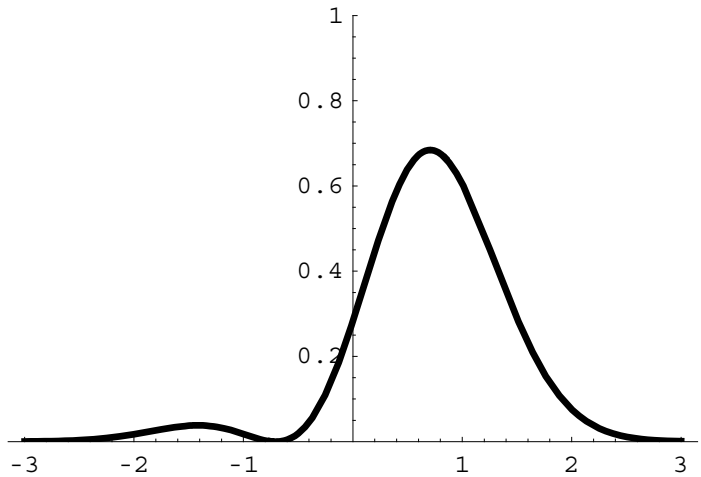


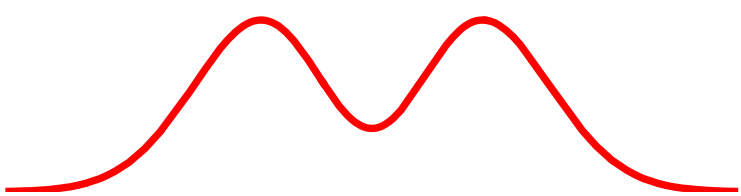


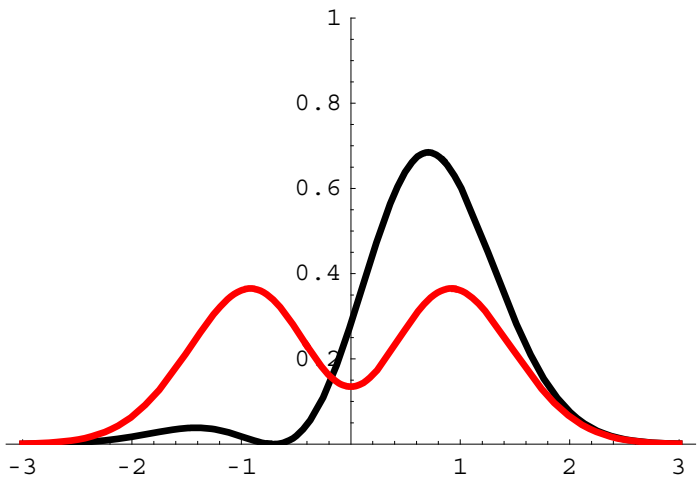


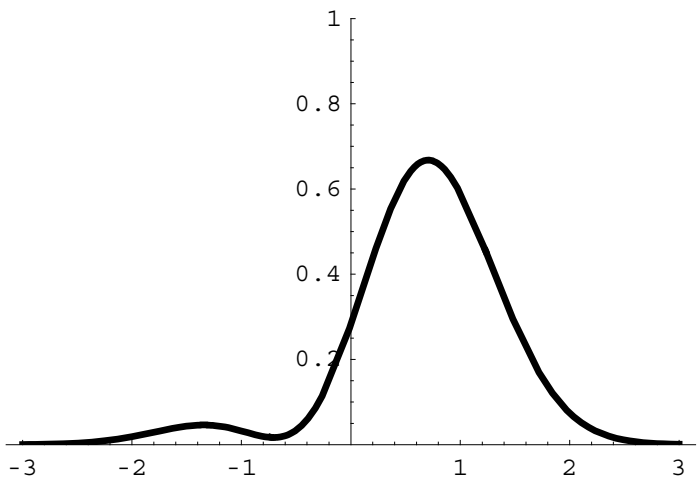


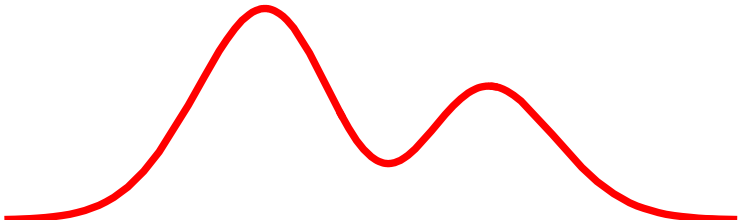
```
For[i = 0, i < 20, i++, {
    t := i * Pi / 10.,
    Show[
      Plot[h(x, t), {x, -3, 3}, PlotStyle → {{Thickness[0.01], RGBColor[0, 0, 0]}}, PlotRange → {0, 1.}],
      Plot[g(p, t) * 3, {p, -3, 3}, Axes → False,
        PlotStyle → {{Thickness[0.01], RGBColor[1, 0, 0]}}, PlotRange → {0., 1.}]
    ]
  }]
```

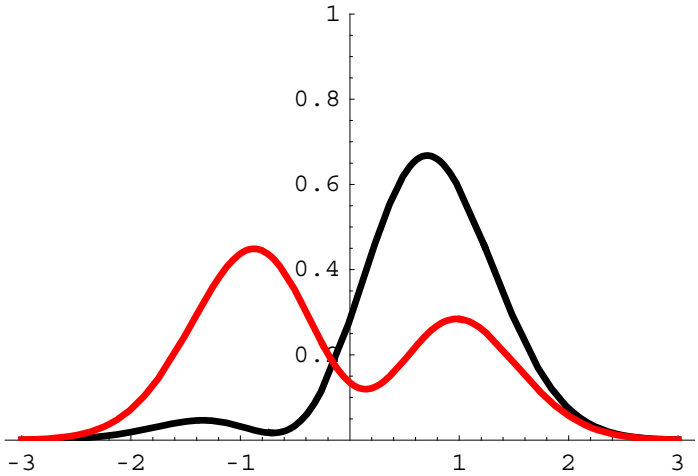


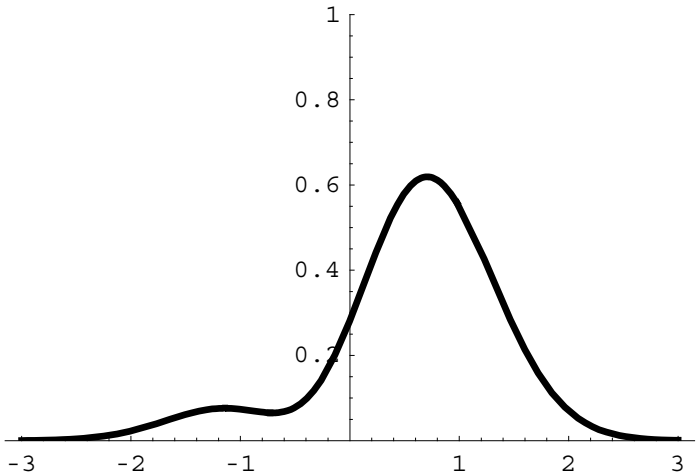


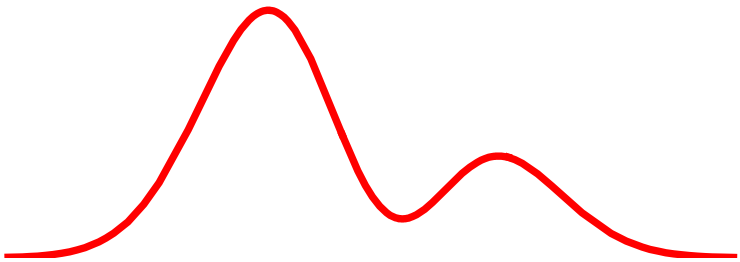

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

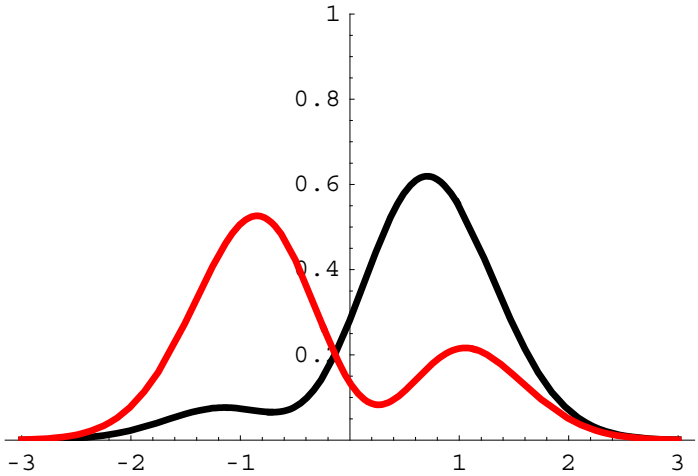

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

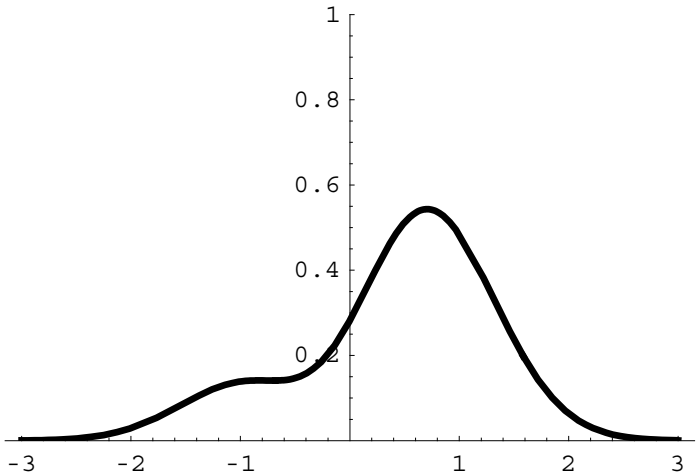


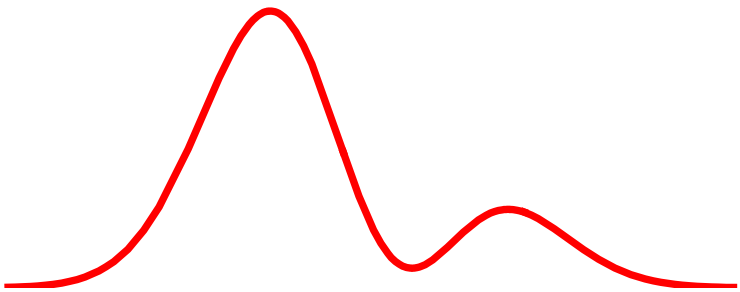

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

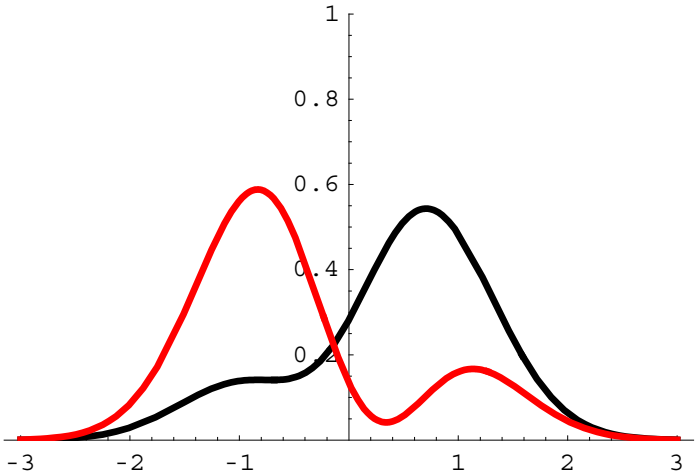

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

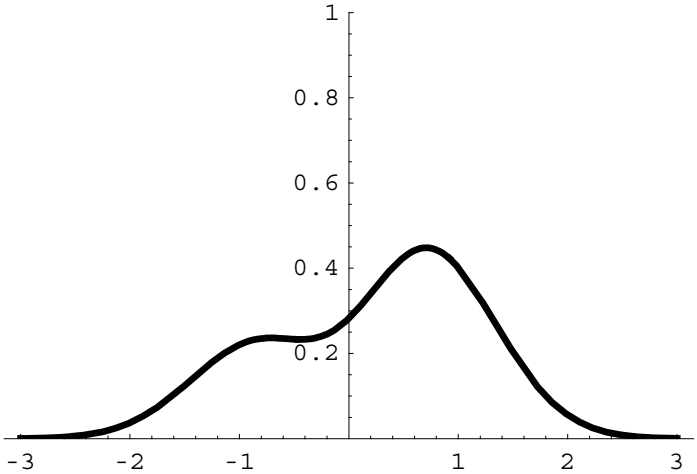


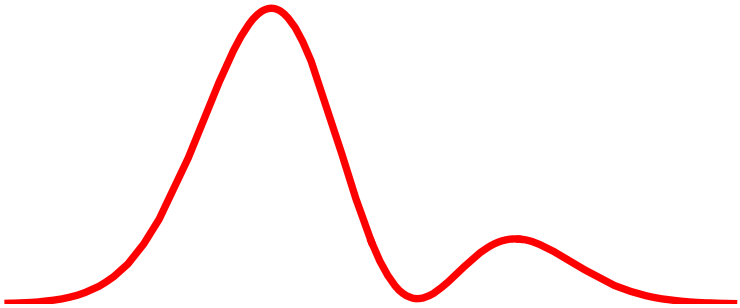

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

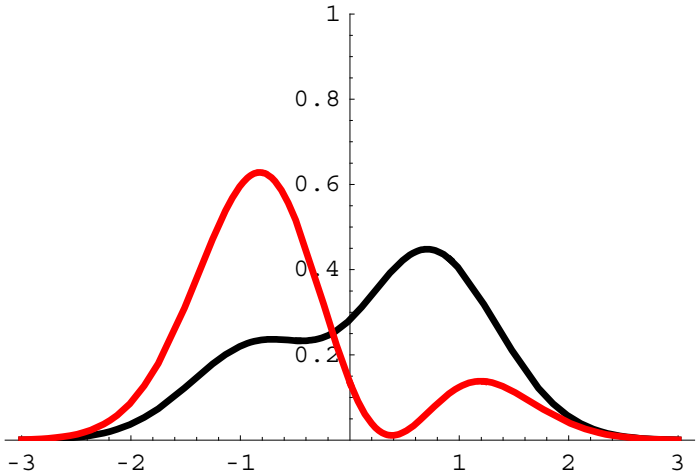

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

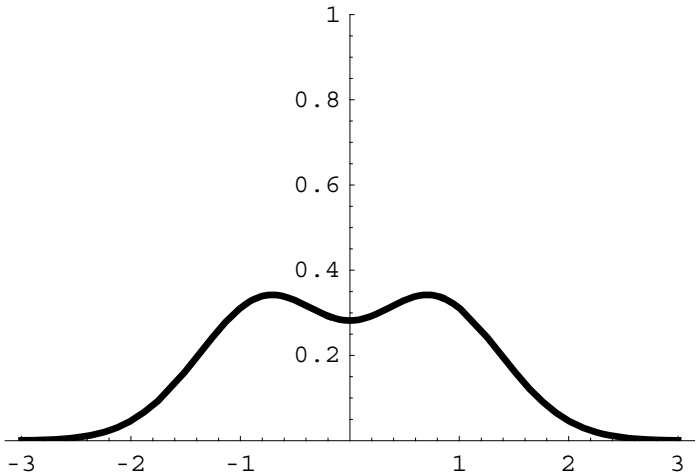


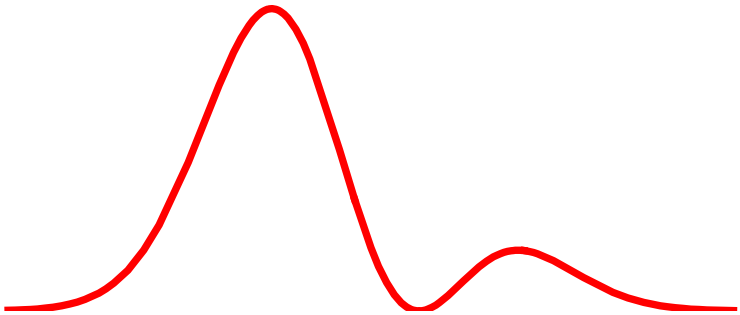

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

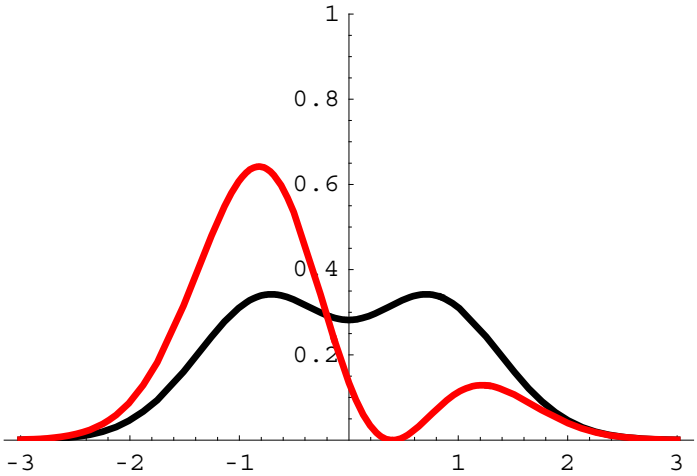

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

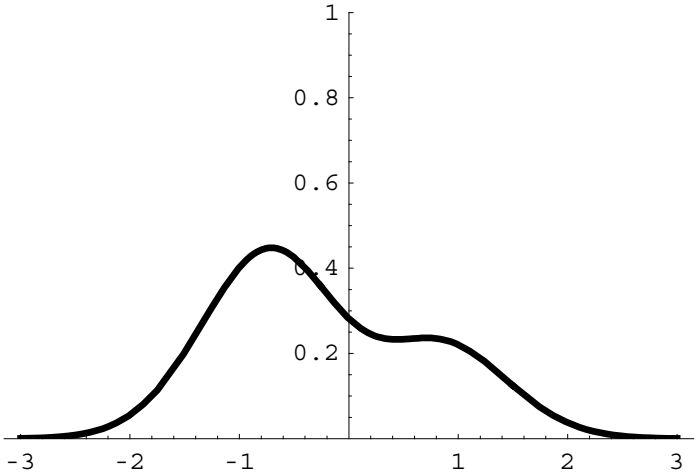


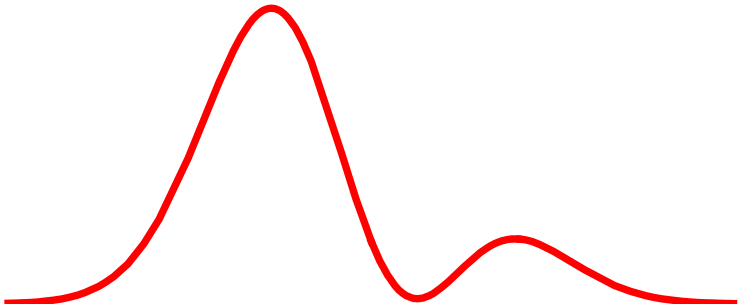

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

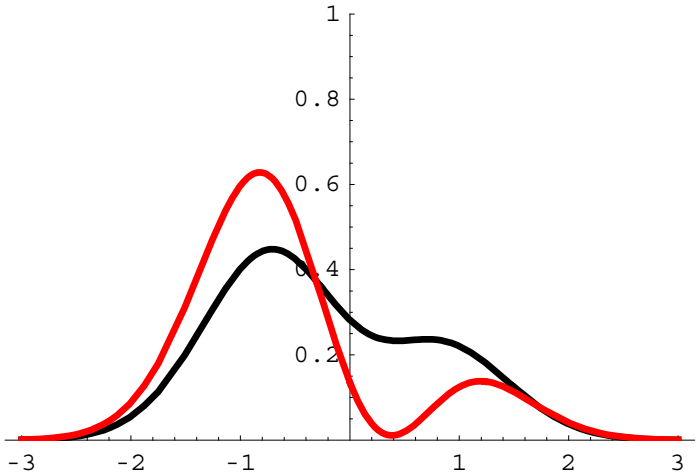

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

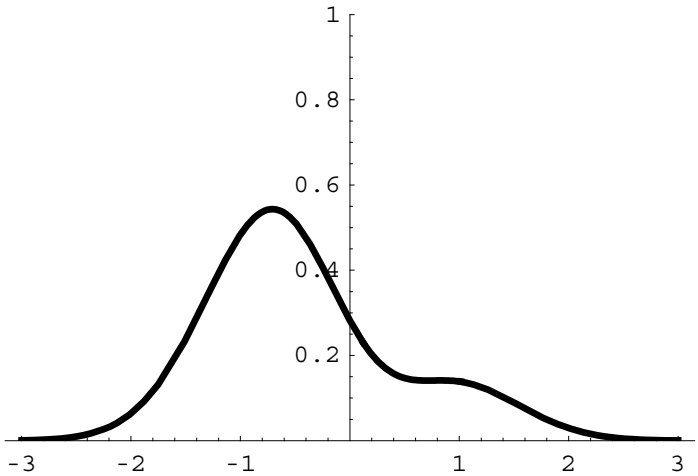


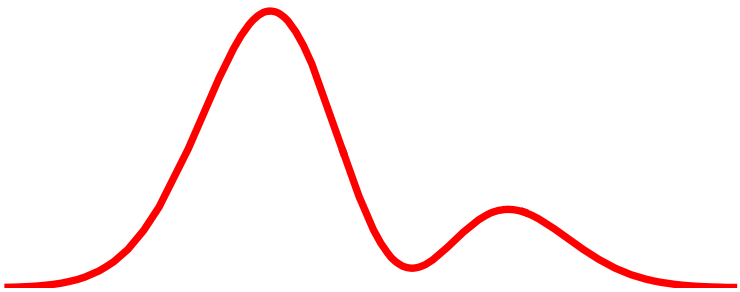

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

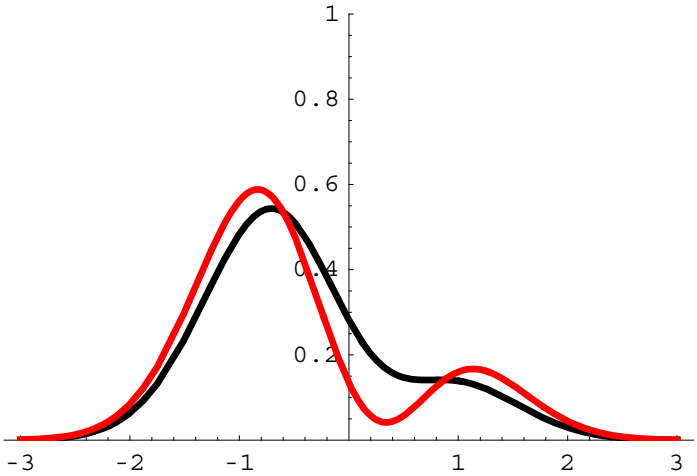

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

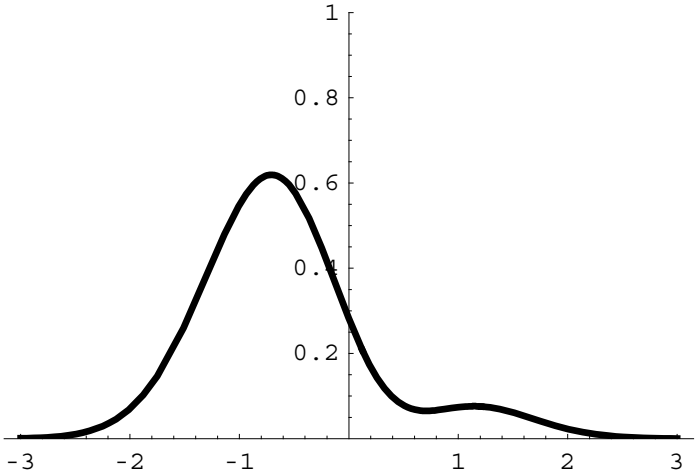


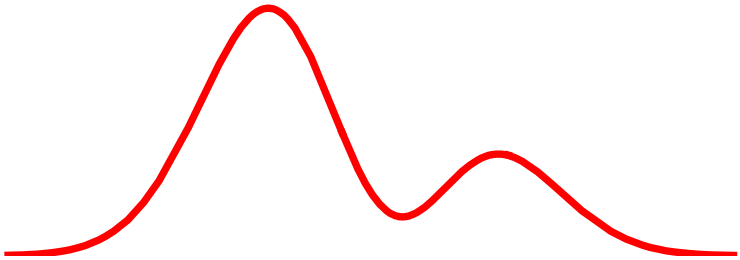

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

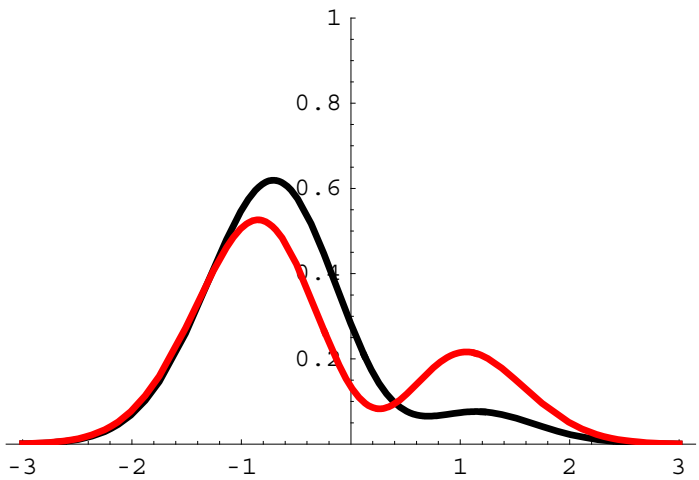

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

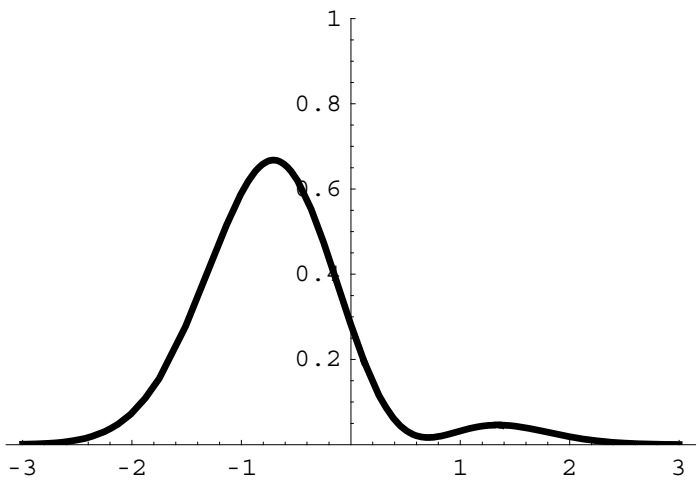


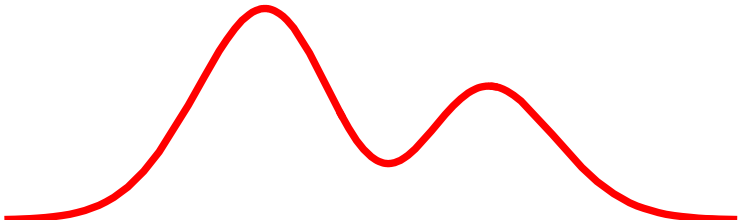

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

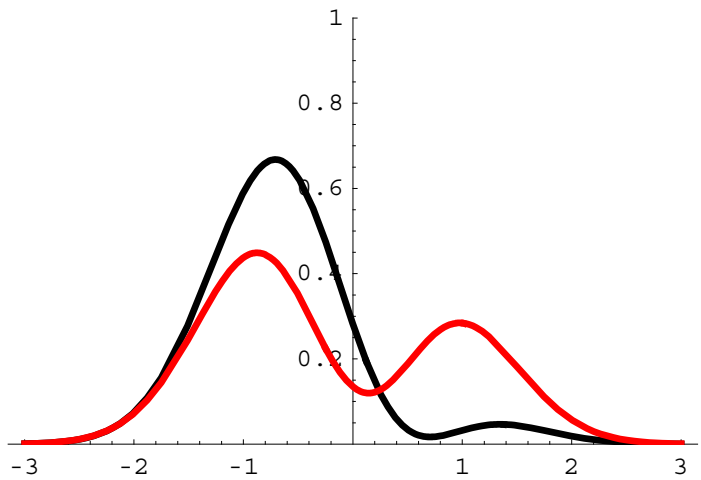

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

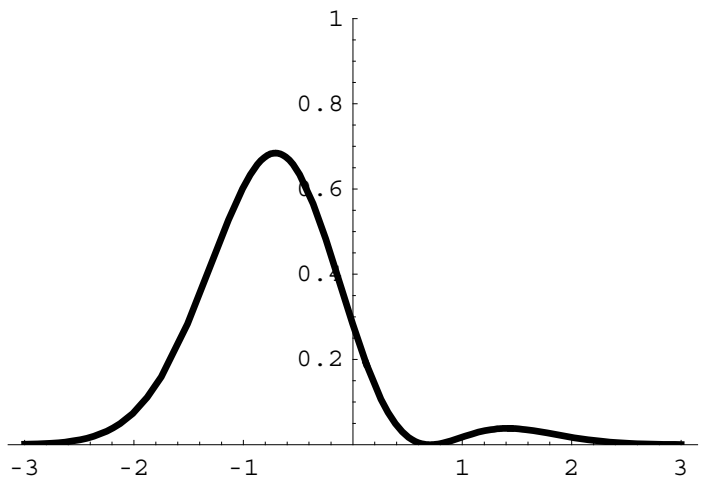


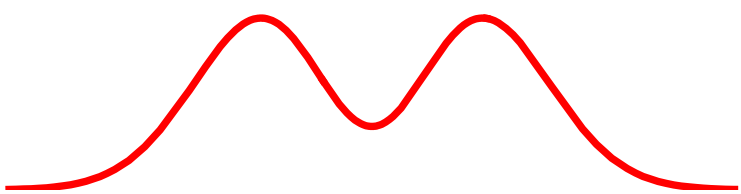

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

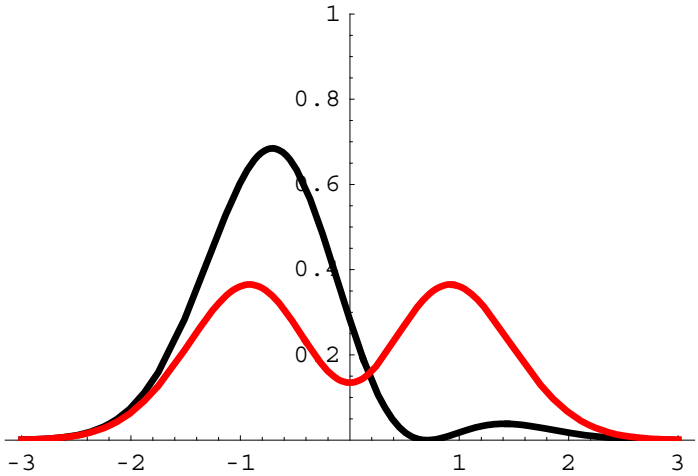

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

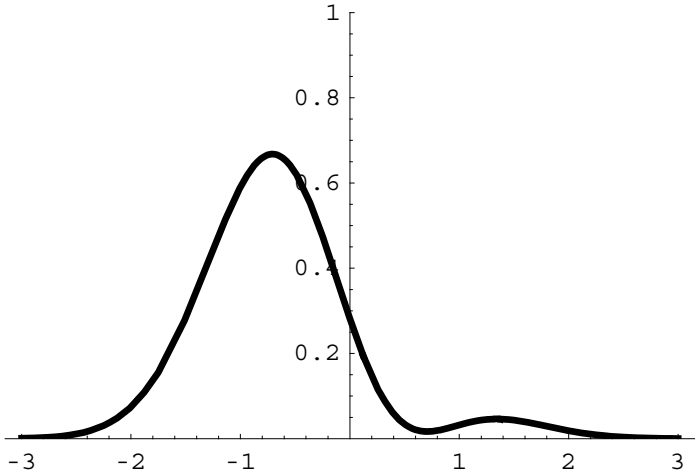


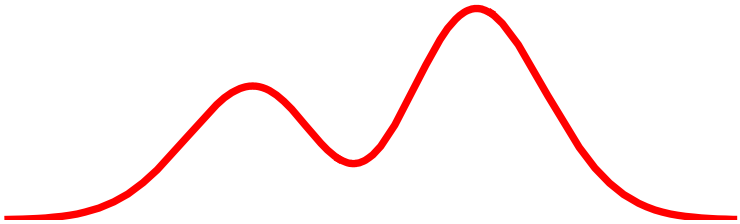

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

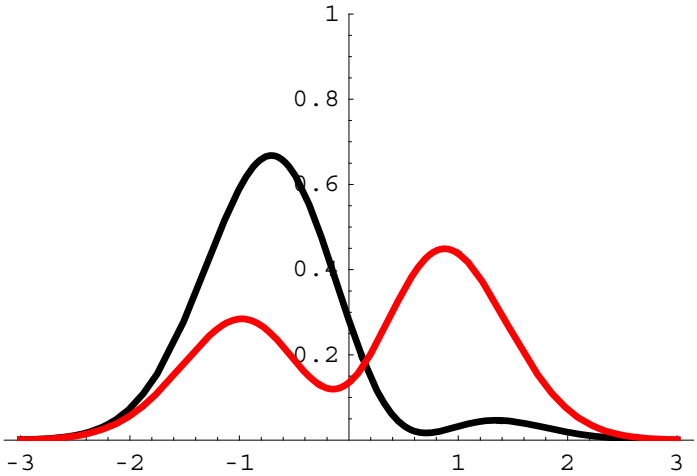

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

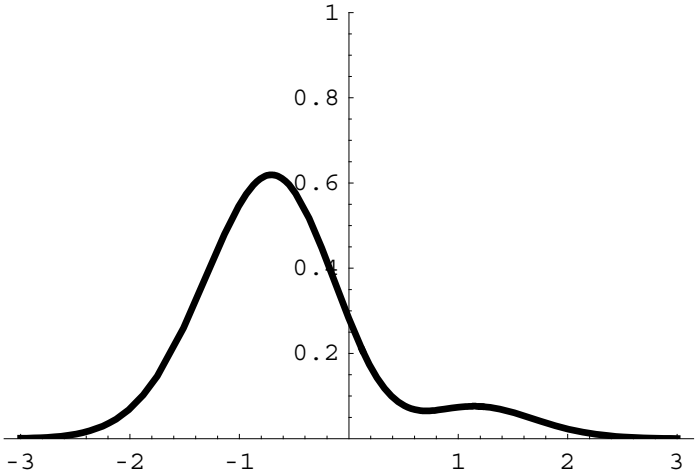


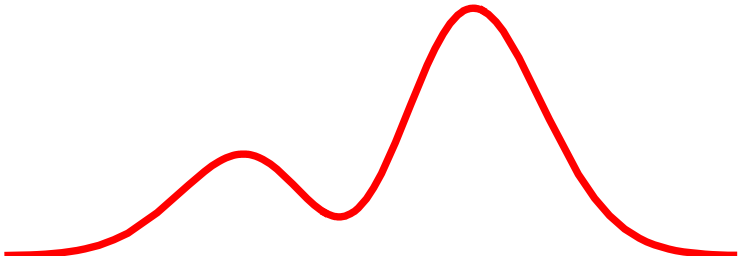

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

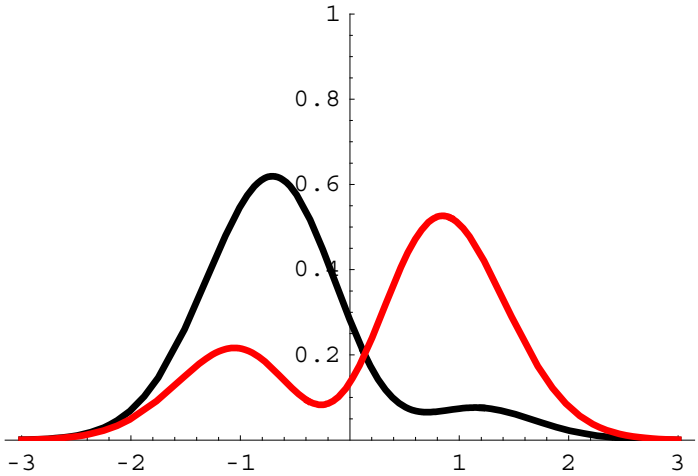

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

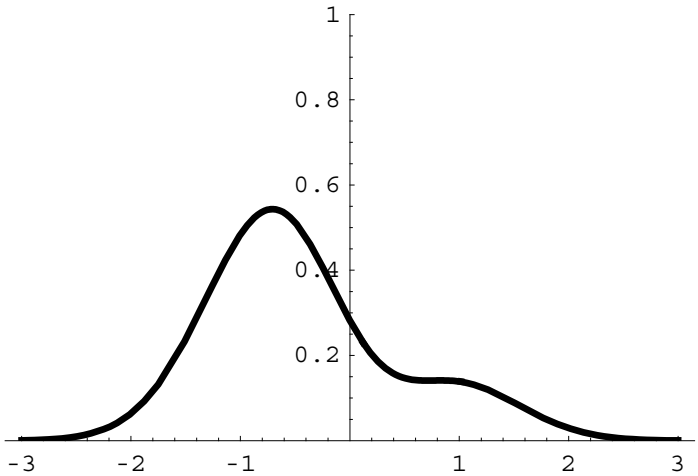


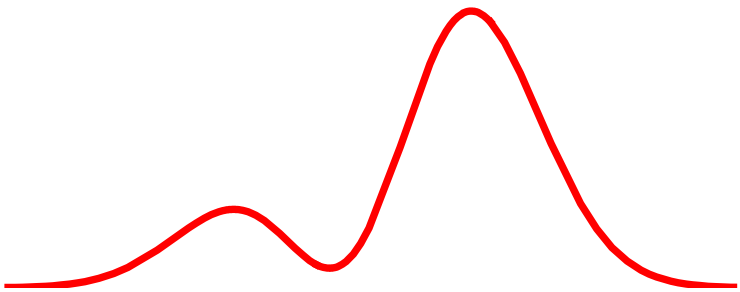

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

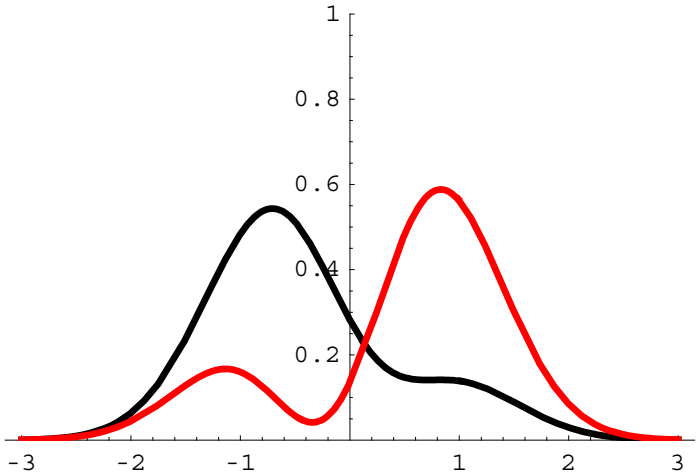

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

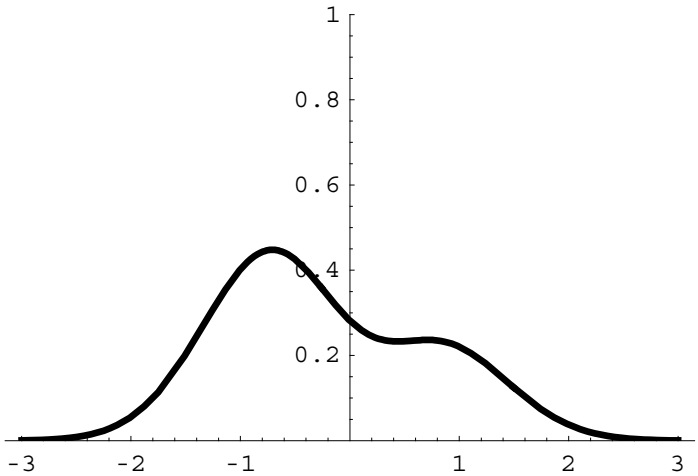


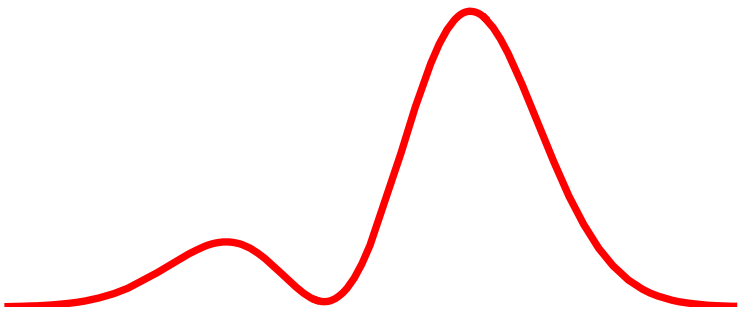

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

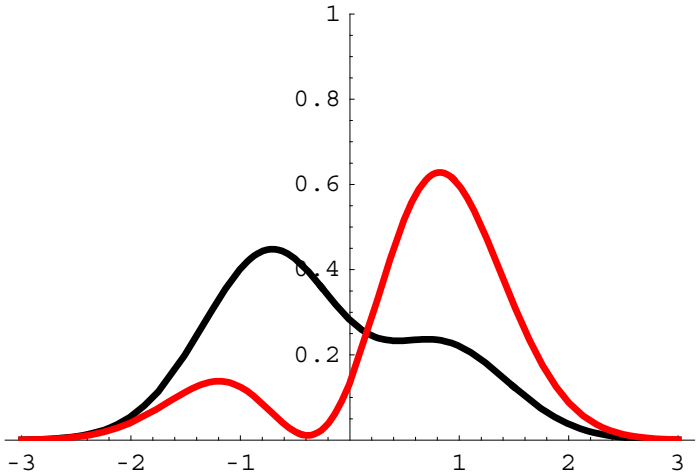

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

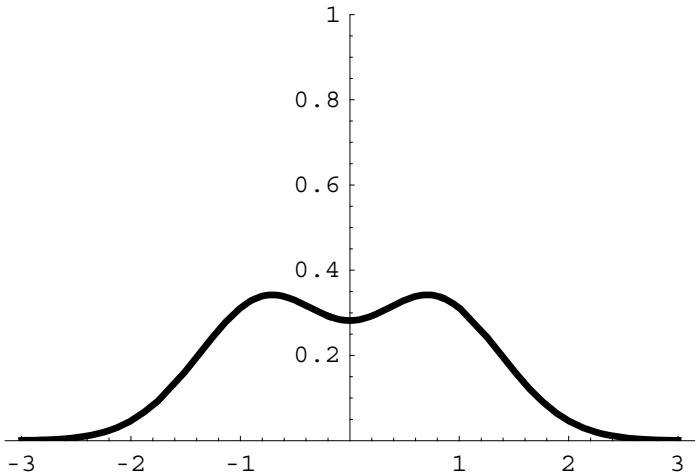


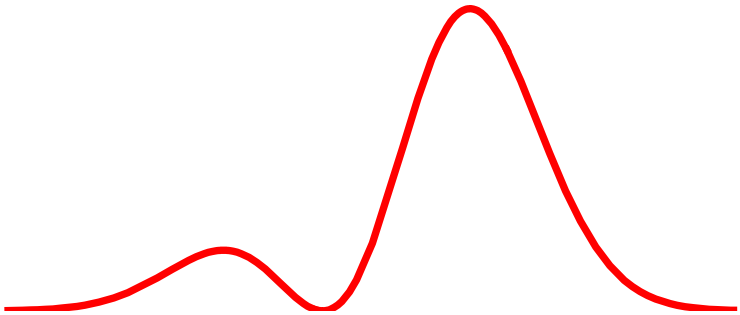

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

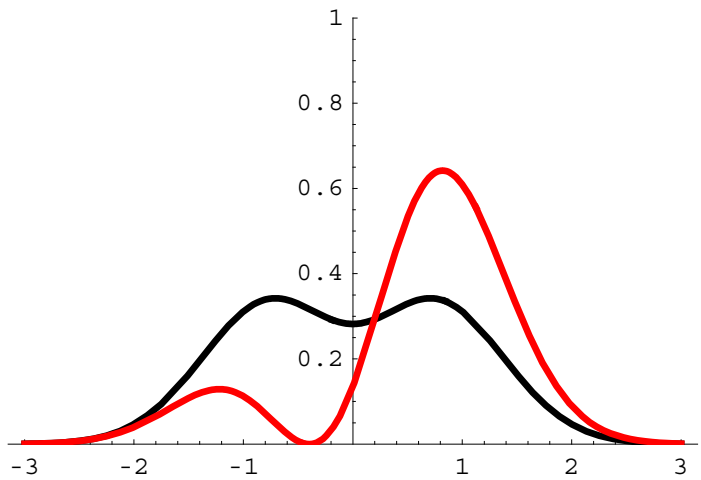

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

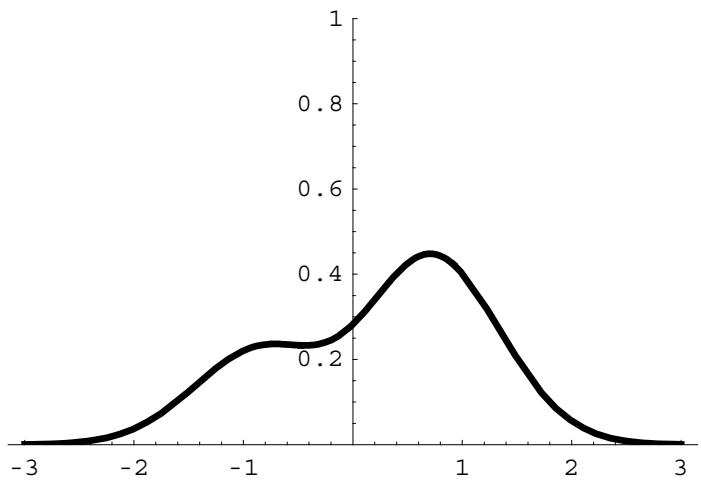


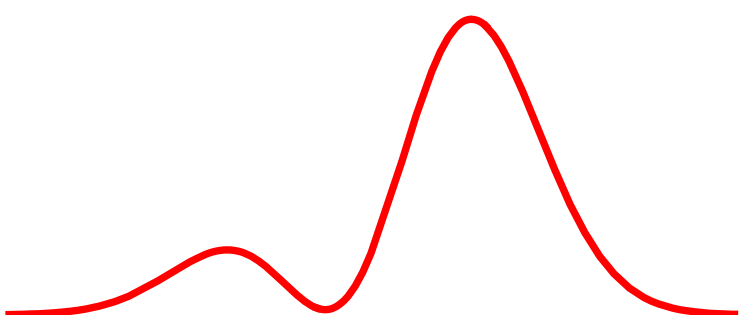

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

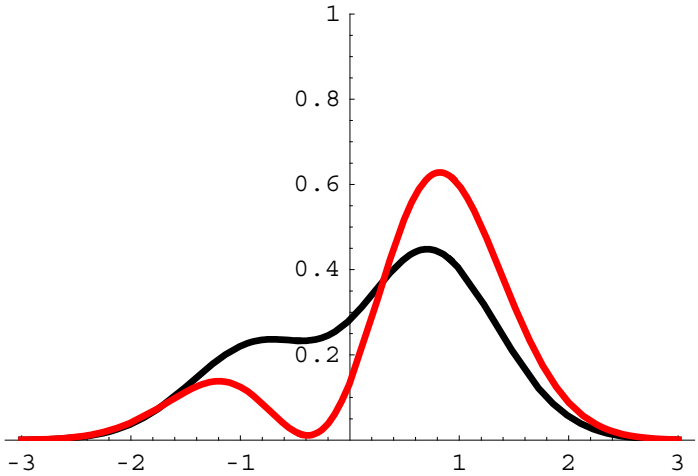

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

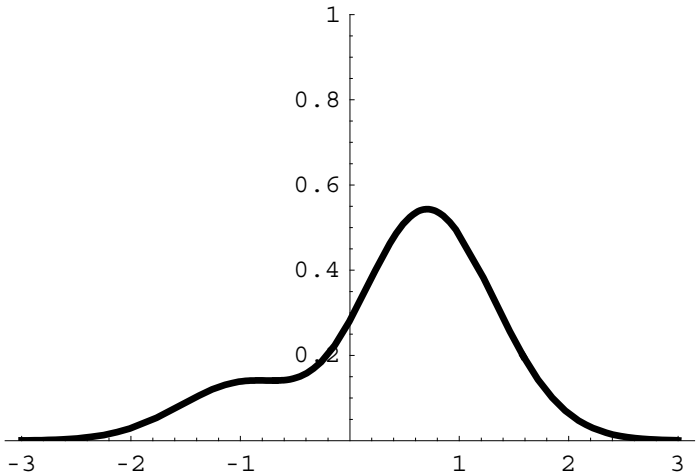


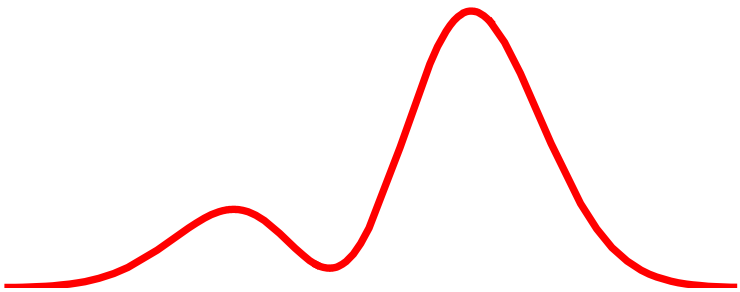

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

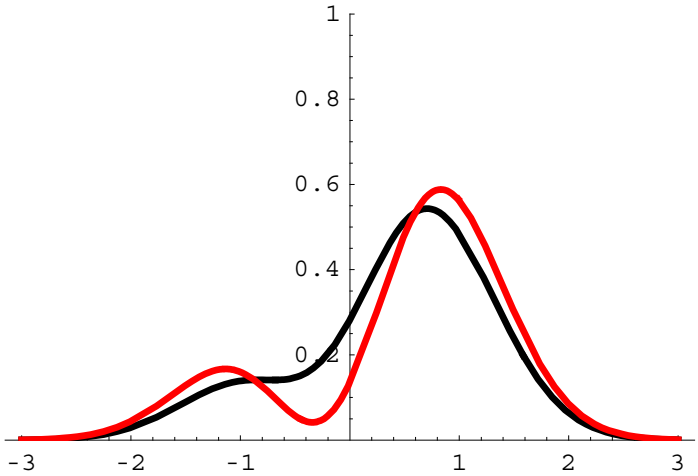

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

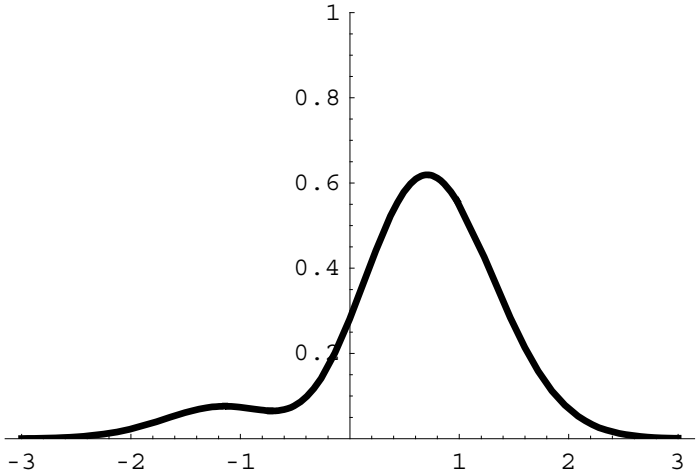


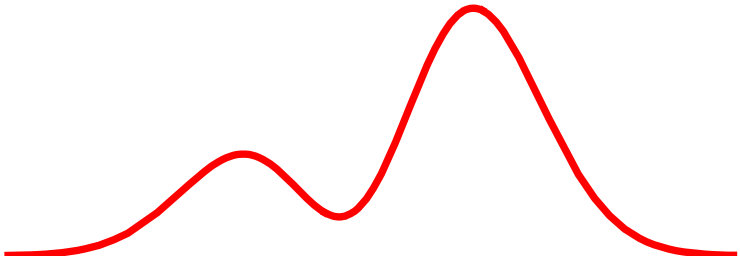

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

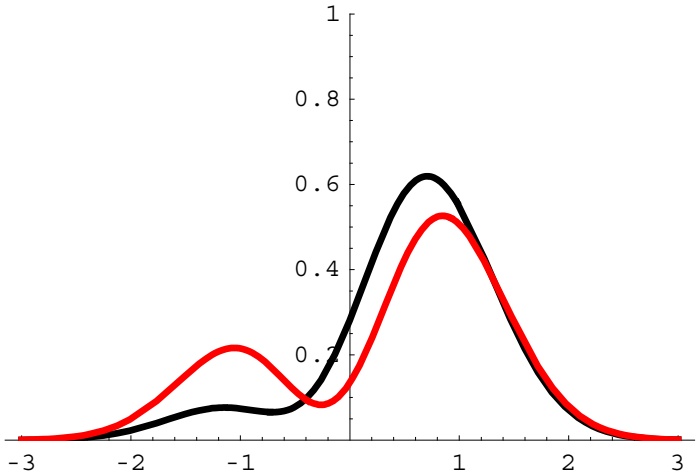

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

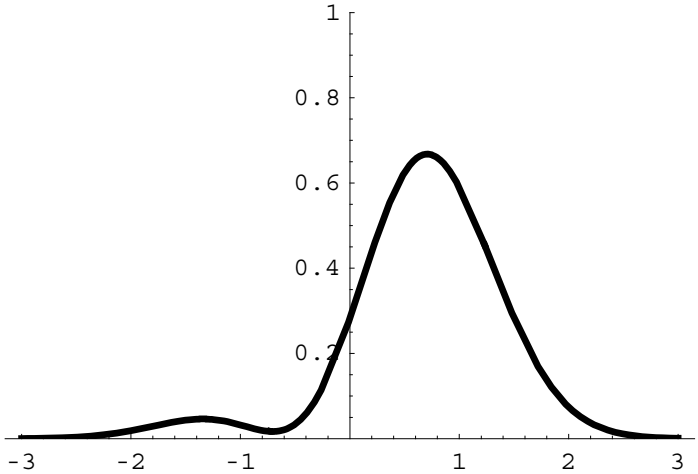


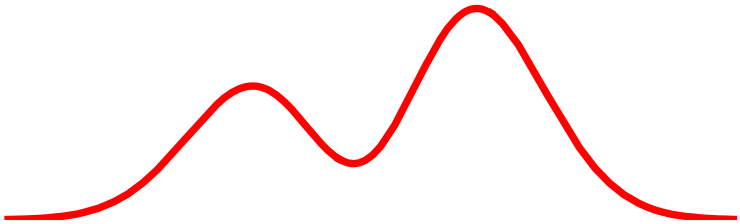

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3

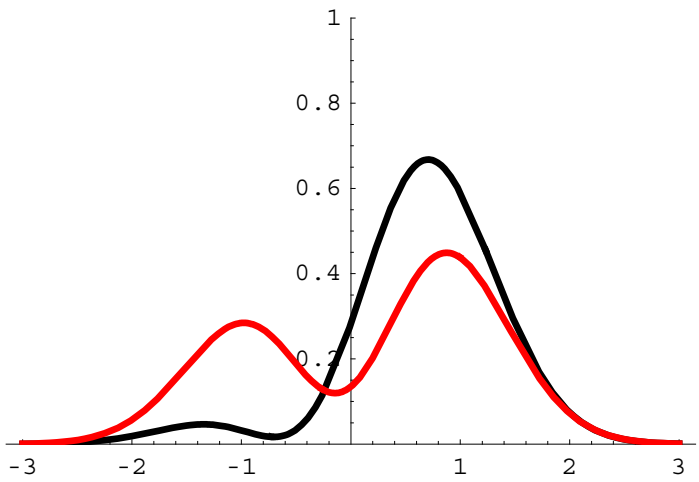

1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3




1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3


1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3




1
0.8
0.6
0.4
0.2
-3
-2
-1
1
2
3