Glauber Monte-Carlo Calculations for Au+Au Collisions at $\sqrt{s_{NN}} = 200$ GeV

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Abstract

The results of the Glauber calculations for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are summarized. For the total geometrical Au+Au cross section we find a value of $\sigma_{geo}^{Au+au} = (6847 \pm 542)$ mb. All results are available in electronic form at [1].

Contents

1 Glauber Model

The Glauber model is based on a simple geometrical picture of a nucleus-nucleus collision. Nucleons are assumed to travel on straight line trajectories, independent of whether they collide with other nucleons or not. After a nucleon is struck by another nucleon, one definitely has a highly excited baryonic object. It is assumed in the Glauber model that the cross section for the interaction of this excited object with other ground state or excited nucleons is identical to the ordinary inelastic nucleon-nucleon cross section σ_{nn}.

Quantities in the Glauber model can be calculated based on analytic expressions, see e.g. [5]. However, modeling e.g. smearing effects introduced by the experimental centrality selection is much easier in a Monte-Carlo framework. In the MC framework the nucleons of the two Au nuclei are distributed in space according to the nucleon density profile. Then an impact parameter b of the nucleus-nucleus collision is chosen randomly. A collision of two nucleons takes place if their distance d satisfies

$$d < \sqrt{\sigma_{nn}/\pi}. \tag{1}$$

Details of the PHENIX Glauber Monte-Carlo code have been discussed in [2, 3].

2 Quantities Calculated with the Glauber Model

2.1 N_{part}, N_{coll}, Nuclear Overlap Function T_{AB}, and Impact Parameter b

A participant is defined as a nucleon that has suffered at least one inelastic nucleon-nucleon collision. The number of participants N_{part} is frequently used to characterize the centrality of a nucleus-nucleus collision, especially when reactions of nuclei with different mass number A are compared. The number of inelastic nucleon-nucleon collisions N_{coll} and the nuclear overlap function T_{AB} for a given centrality class are related quantities, see e.g. [4]. We use the relation

$$T_{AB} = \langle N_{coll} \rangle / \sigma_{nn} \tag{2}$$

to calculate the nuclear overlap function for a given centrality class. Another quantity that can be calculated in the Glauber model is the average impact parameter $\langle b \rangle$ of a centrality class. Figure 12 depicts the impact parameter distribution for Au+Au collisions with at least one inelastic nucleon-nucleon collision.

2.2 Eccentricity

The overlap zone of the two nuclei in a heavy ion reaction with non-zero impact parameter has an almond-like shape in the transverse plane (see Fig. 1). The deviation of that shape from a spherical shape can be described by the eccentricity ε under the assumption that the position of all participating nucleons is known:

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} = \frac{\sum_i y_i^2 - \sum_i x_i^2}{\sum_i y_i^2 + \sum_i x_i^2} \tag{3}$$

The variable i in this equation runs over all participating nucleons. The quantities x_i and y_i denote the position of a nucleon in the transverse plane. The coordinate system is chosen such that the centers of the two nuclei are at $(-b/2, 0)$ and $(b/2, 0)$, respectively, where b denotes the impact parameter.

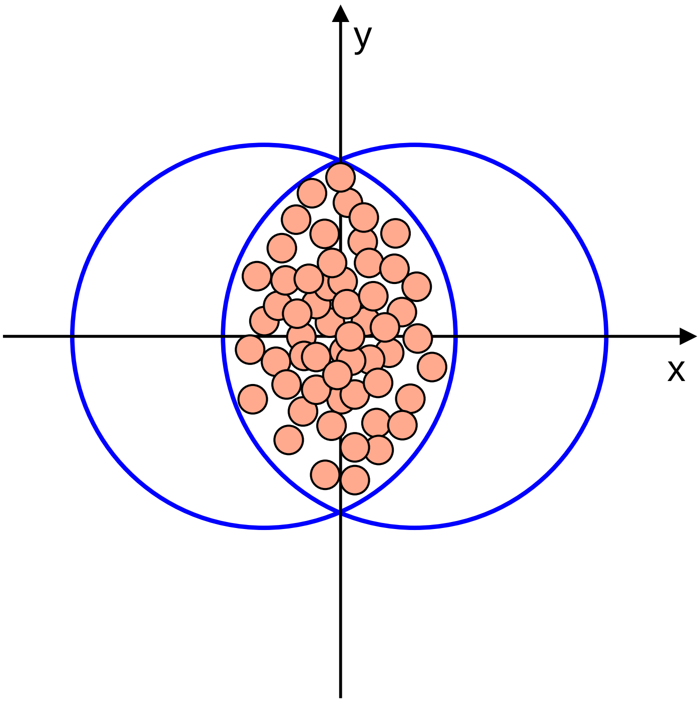

Figure 1: Sketch of the coordinate system used for the calculation of the eccentricity in eq. 3. The big circles indicate the two Au nuclei in the transverse plane. The impact parameter vector $\vec{b}$ is parallel to the x-axis. The participating nucleons are depicted as filled disks.

3 Model Parameters

The nuclear density profile is parameterized by a Woods-Saxon function

$$\rho(r) = \rho_0 \cdot \frac{1}{1 + \exp(\frac{r-R}{a})} \tag{4}$$

For gold nuclei the parameters

$$R = 6.38\,\text{fm} \quad \text{and} \quad a = 0.54\,\text{fm} \tag{5}$$

are used [6]. For a center of mass energy of $\sqrt{s_{nn}} = 200$ GeV an inelastic nucleon-nucleon cross section of $\sigma_{nn} = 42$ mb was used in the Glauber calculations. The parameters of the Glauber calculations are passed to the MC code as input text files. The input file for the default calculation and for the variations discussed in section 4 are available at [1].

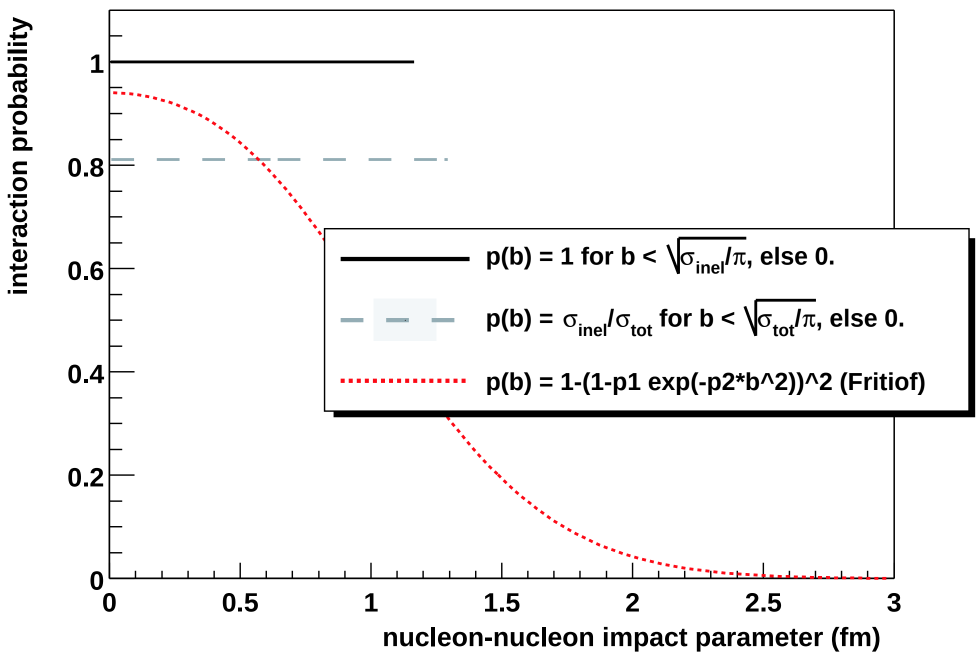

Figure 2: Different nucleon-nucleon overlap functions used in the Glauber Monte-Carlo Calculations. The default calculation uses the 'black disk' overlap function. The gray disk overlap function (dashed line) and the Gaussian overlap function (dotted line) are used in calculation 9 and 10, respectively. The variable b in the legend denotes the impact parameter of the nucleon-nucleon collision.

4 Estimation of Systematic Errors

In order to estimate the systematic errors of the calculated quantities we run the Glauber calculations with varying model parameters. There are 15 different Glauber calculations in total. The following list gives a brief explanation of each calculation.

1. Default calculation ($R = 6.38$ fm, $a = 0.54$ fm, $\sigma_{nn} = 42$ mb)
2. $\sigma_{nn} = 39$ mb instead of $\sigma_{nn} = 42$ mb.
3. $\sigma_{nn} = 45$ mb instead of $\sigma_{nn} = 42$ mb.
4. Different Woods-Saxon parameters ($R = 6.65$ fm, $a = 0.55$ fm)
5. Different Woods-Saxon parameters ($R = 6.25$ fm, $a = 0.53$ fm)
6. In the default calculation nucleons are allowed to overlap. In this calculation nucleons are simulated with a hard core. The radius of the hard core was taken as $r = 0.4$ fm (as in the event generator VENUS), such that the distance between the centers of two nucleons is always greater than $2r = 0.8$ fm.
7. Due to the formation of deuterons and other processes only a fraction of all spectator neutrons end up in the ZDCs. This fraction is centrality dependent and is described by a neutron loss function in the simulation. In this calculation a different neutron loss function was used.
8. A different smearing function that describes the BBC resolution was used in this simulation.
9. Gray disk nucleon-nucleon overlap function: Nucleon-nucleon interaction probability given by $p(b) = \sigma_{inel}/\sigma_{tot}$ for $b < \sqrt{\sigma_{tot}/\pi}$, where b denotes the nucleon-nucleon impact parameter here and $\sigma_{inel} = \sigma_{nn} = 42$ mb (cf. Figure 2).
10. Gaussian nucleon-nucleon overlap function: Nucleon-nucleon interaction probability given by $p(b) = 1 - (1 - p_1 \exp(-p_2 b^2))^2$ taken from [7] ((cf. Figure 2). The parameters are $p_1 = 0.755$ and $p_2 = 0.89$ fm^{-2}.
11. For the centrality selection with the 'clock'-method the angle φ of a given event in the BBC-ZDC plane has to be calculated. This angle is calculated with respect to the point $(0.2 \cdot BBC/BBC_{max}, 0)$ in the BBC-ZDC plane both for real data and in the simulation. In this simulation the origin was chosen as $(0.5 \cdot BBC/BBC_{max}, 0)$.
12. In all calculations so far it is assumed that the experimental centrality cuts are perfect in the sense that the selected data sample exactly corresponds to the nominal fraction of the cross section (i.e. it is e.g. assumed that the experimental data sample of the 5% most central events exactly corresponds to a cross section of $0.05 \cdot \sigma_{inel}$). This knowledge is limited by the uncertainty of the minimum bias trigger efficiency. This uncertainty was estimated as $(92.2^{+2.5}_{-3.0})\%$ [8, 9]. This uncertainty is translated into an error of the extracted quantities. For this purpose

the percentiles of the cross section in the simulation were modified by a fraction 2.5/92.2, such that a slightly more central event sample was selected for each centrality class. N_{coll} is e.g. determined in the simulation for the $0-9.73\%$ most central events instead of the $0-10\%$ most central events.

13. As in calculation 12, but a slightly less central event sample for each centrality class was selected. E.g. the $0-10.34\%$ most central events instead of the $0-10\%$ most central events were evaluated.

14. Even if the BBC trigger efficiency was exactly known, there still would be an uncertainty related to the experimental event selection. This uncertainty can be estimated by considering the number of selected events in a given class for different run periods. An analysis by Gabor David reveals that there are variations beyond the expected statistical fluctuations. Based on this analysis the error of the experimental centrality class limits were estimated as shown in Figure 3. According to the error estimate given by Figure 3 e.g. the $60-70\%$ centrality class might actually be the $59.28-69.23\%$ class or the $60.72-70.77\%$ class. In this calculation we assume that the experimental centrality selection has a bias towards more central events compared to the default Glauber calculation.

15. Same as calculation 14, but the centrality classes are modified such that less central events are selected.

The final systematic error of an extracted quantity for a given centrality class is the quadratic sum of all differences compared to the default calculation:

$$\sigma_{sys} = \sqrt{\sum_i (x_i - x_{default})^2}, \tag{6}$$

where x stands for extracted quantity. e.g. N_{coll}. If a parameter is changed in both directions (e.g. the inelastic cross section σ_{nn}) then only the higher of the two deviations is used in eq. 6. As an example the results for N_{coll} for the different model calculations along with the estimated systematic error are shown in Figure 4 (central collisions) and in Figure 5 (peripheral collisions).

5 Results

5.1 Total Inelastic Cross Section

For the total inelastic Au+Au cross section at $\sqrt{s_{NN}} = 200$ GeV we obtain

$$\sigma_{geo}^{Au+Au} = (6847 \pm 542) \text{ mb}. \tag{7}$$

The result for the different model assumptions are shown in Figure 11. The inelastic Au+Au section as a function of the impact parameter b is depicted in Figure 12.

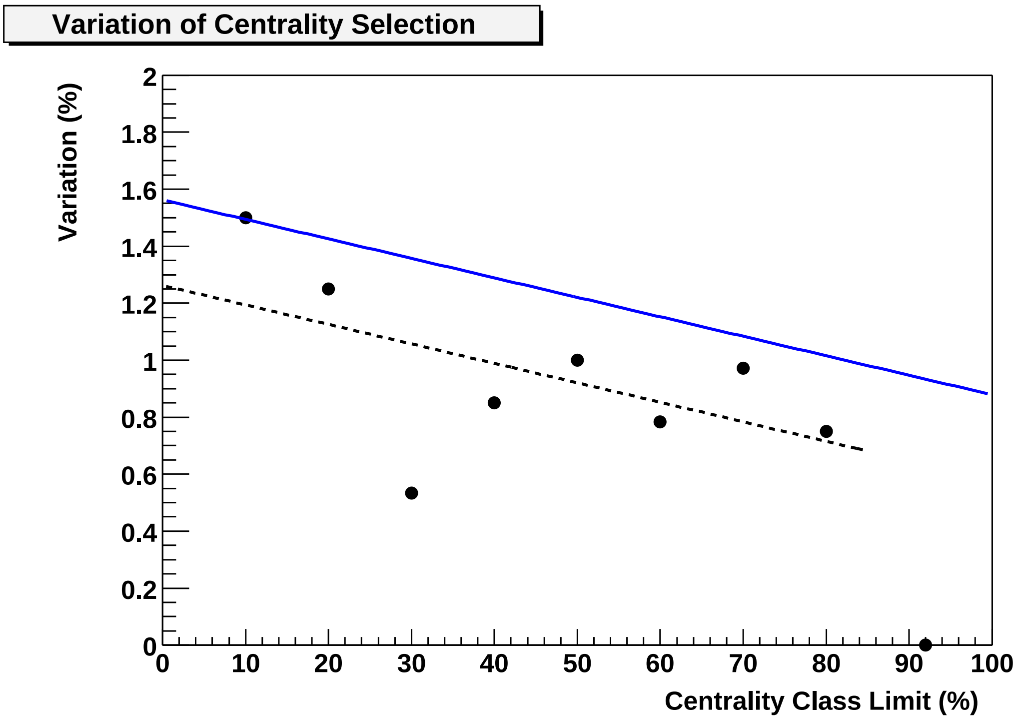

Figure 3: Estimated uncertainty of the centrality class limits (see calculation 14 and 15). The data points reflect the fluctuations in the number of selected events when different run periods are considered. The dotted line is a fit with a linear function. The solid line, which is the fit result plus an extra offset, represents the actual error estimate.

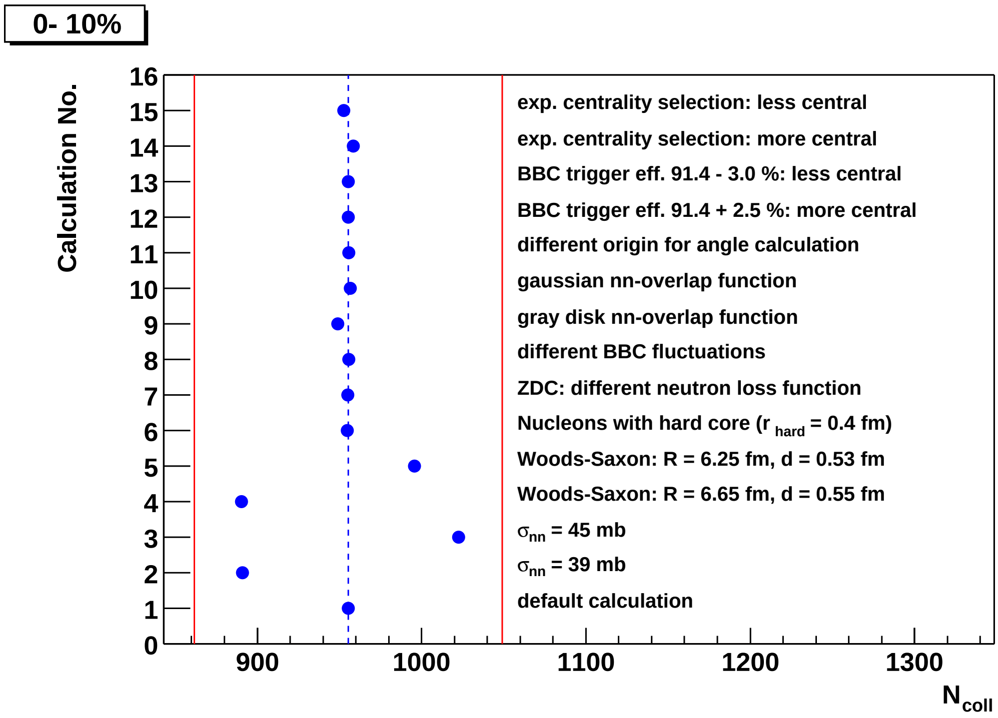

Figure 4: Results of the different calculations for N_{coll} for central Au+Au collisions $(0-10\%$ of the total inelastic cross section). The solid lines indicate the systematic error.

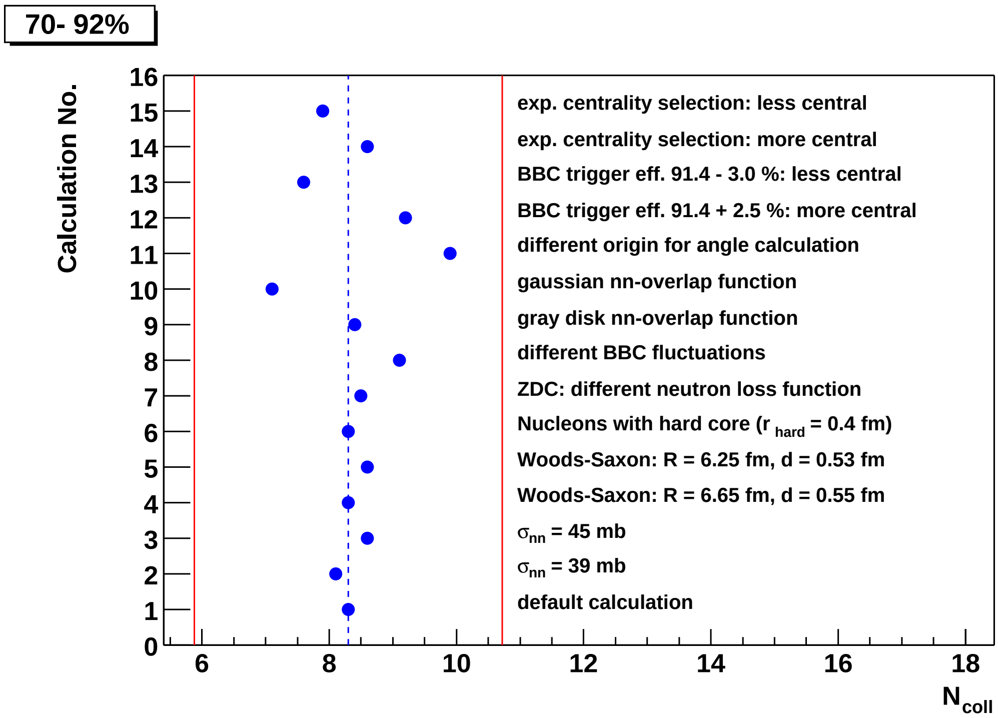

Figure 5: Results of the different calculations for N_{coll} for peripheral Au+Au collisions $(70-92.2\%$ of the total inelastic cross section).

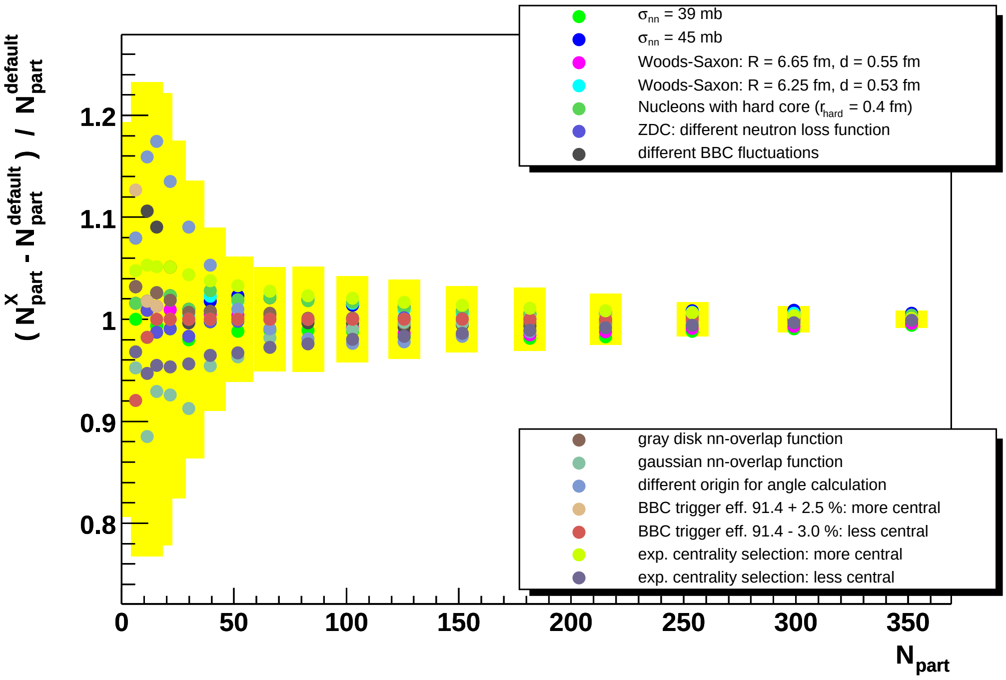

Figure 6: Systematic error of N_{part} vs. N_{part}.

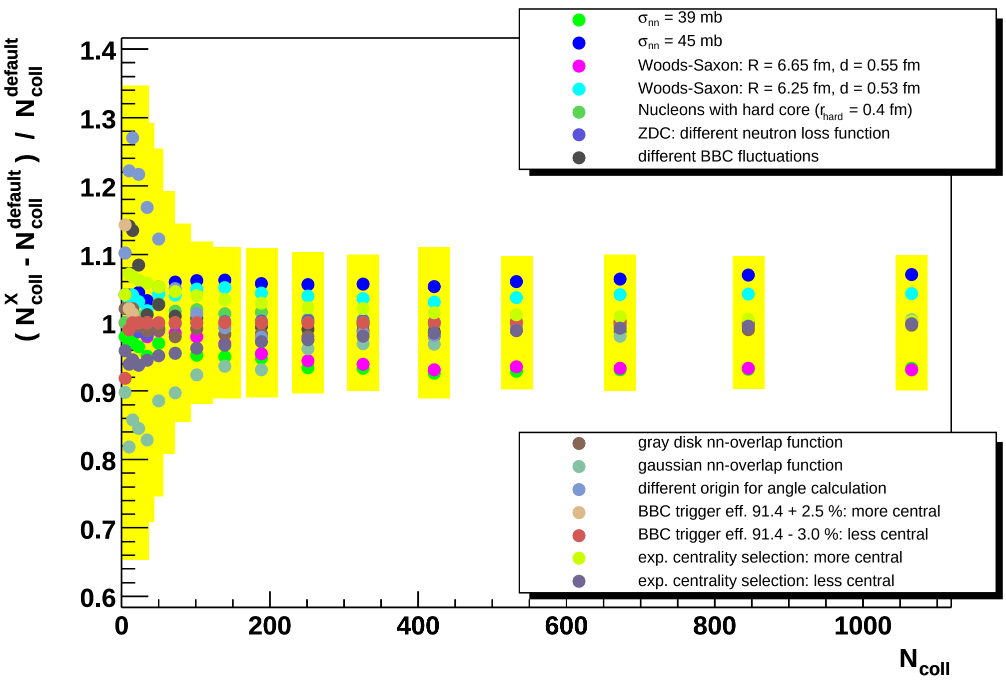

Figure 7: Systematic error of N_{coll} vs. N_{coll}.

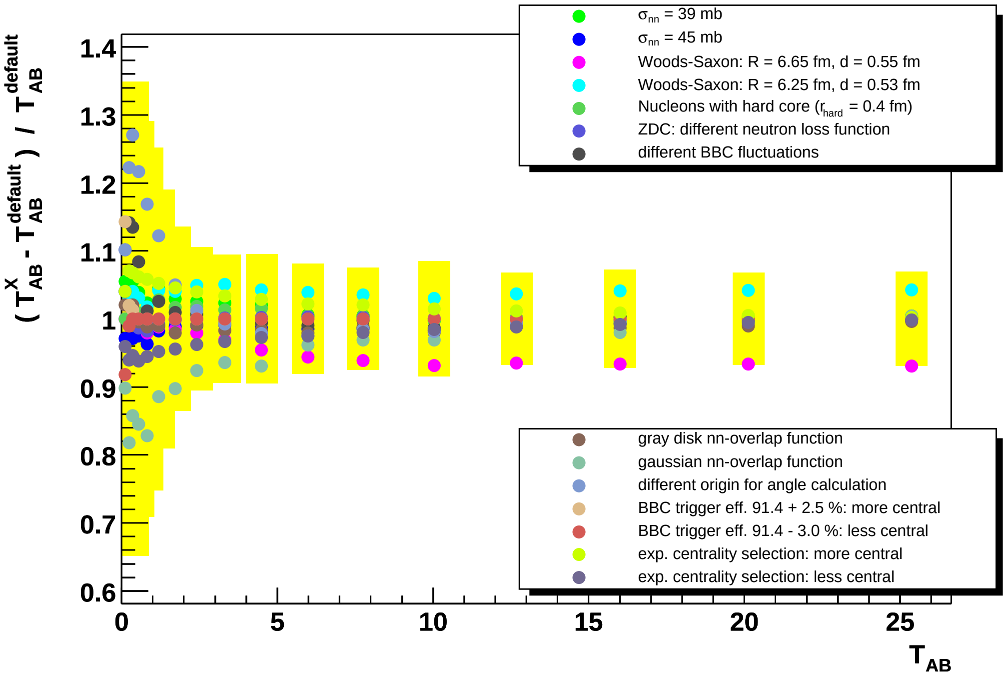

Figure 8: Systematic error of T_{AB} vs. T_{AB}.

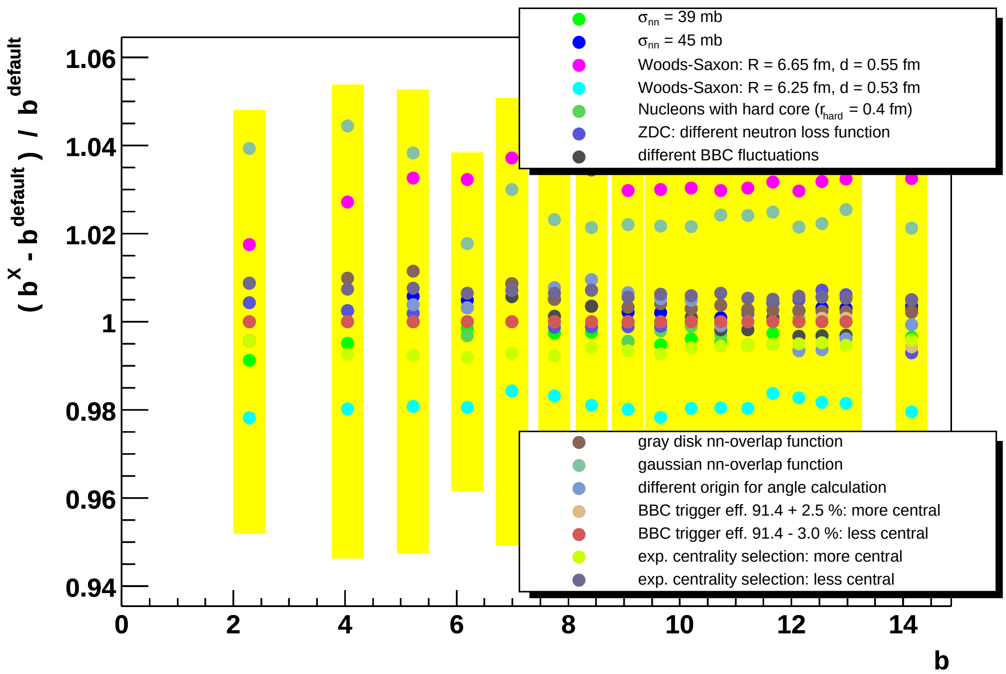

Figure 9: Systematic error of b vs. b.

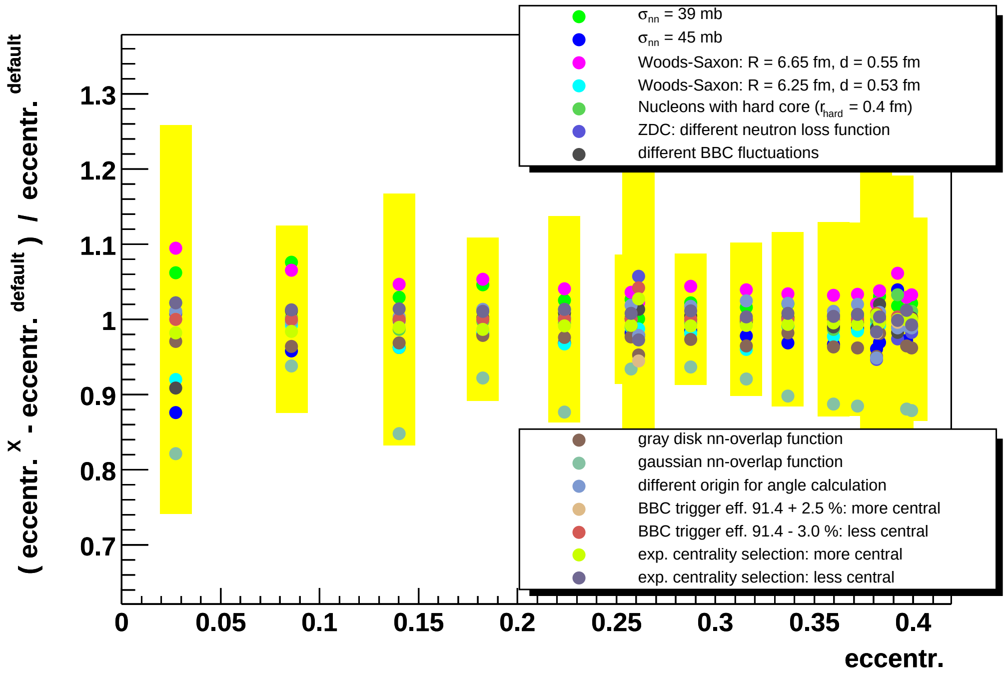

Figure 10: Systematic error of the eccentricity ε vs. ε.

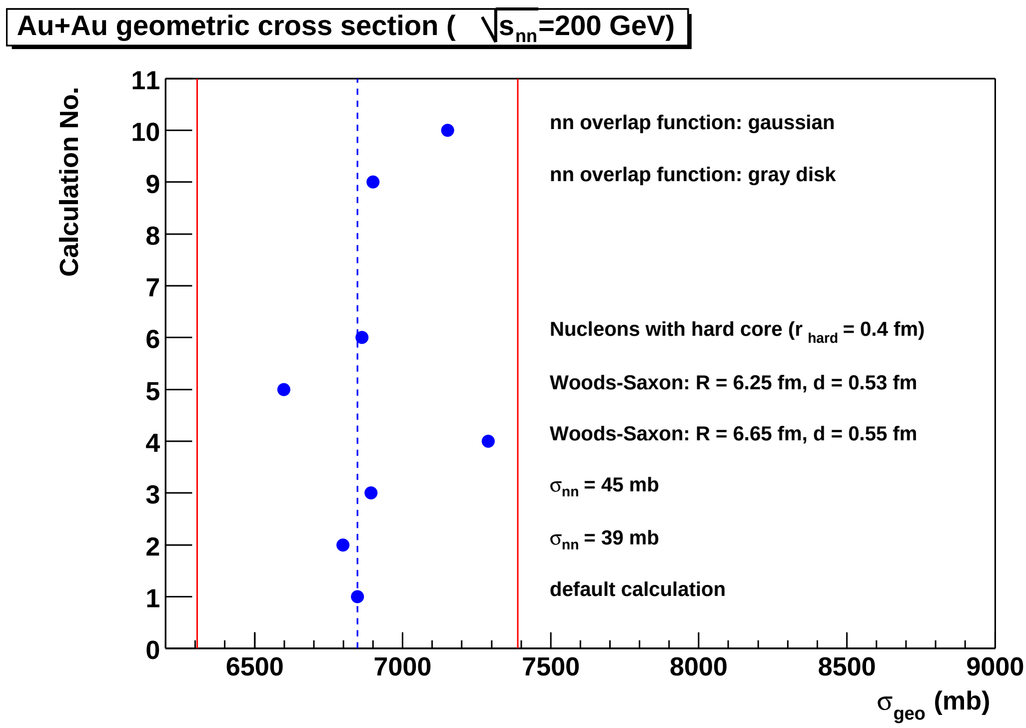

Figure 11: Total geometrical cross section for Glauber calculation with different parameters.

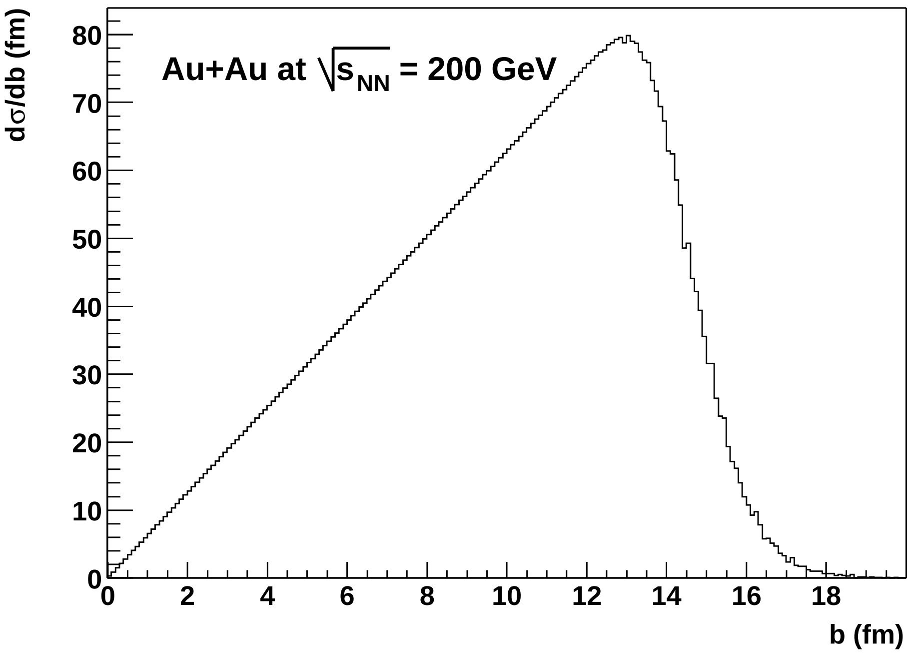

Figure 12: Differential Au+Au inelastic cross section $d\sigma/db$.

5.2 N_{part}, N_{coll}, T_{AB}, b, ε

The results of the Glauber calculation along with the estimated systematic error are given in Table 1 for various centrality classes. It is assumed that the experimental centrality selection was done with the 'clock-method' (see e.g. [2]).

References

[1] `https://www.phenix.bnl.gov/phenix/WWW/p/draft/reygers/glauber`

[2] K. Reygers, *Number of Nucleon-Nucleon Collisions in Central and Peripheral Au+Au Reactions*, PHENIX analysis note AN078

[3] S. Kelly, D. Morrison, J. Nagle, K. Reygers, *Calculation of the Number of Participating Nucleons for Centrality Classes Defined with the ZDC and the BBC*, PHENIX analysis note AN033

[4] M. J. Tannenbaum, *Thickness function and Pointlike Scaling for Hard Scattering*, PHENIX analysis note AN057

[5] C.Y. Wong, *Introduction to High-Energy Heavy-Ion Collisions*, World Scientific

[6] H. R. Collard, L. R. B Elton, R. Hofstadter, *Landolt-Börnstein, Numerical Data and Functional Relationsships in Science and Technology, New Series, Volume 2, Nuclear Radii*, Springer-Verlag, 1967

[7] H. Pi, *An Event Generator for Interactions between Hadrons and Nuclei – FRITIOF Version 7.0*, e.g. available at http://wwwinfo.cern.ch/asdoc/psdir/fritiof.ps.gz

[8] J.L. Nagle, M. Chiu, J. Frantz, K. Reygers, *Minimum Bias Trigger Selection and Centrality Note*, PHENIX AN113

[9] https://www.phenix.bnl.gov/phenix/WWW/p/lists/phenix-p-l/msg03797.html

class	$\langle N_{part} \rangle$	sys. err.	$\langle N_{coll} \rangle$	sys. err.	T_{AB} (mb^{-1})	sys. err.	$\langle b \rangle$ (fm)	sys. err.	ε	sys. err.
0- 5%	351.4	2.9	1065.4	105.3	25.37	1.77	2.3	0.1	0.027	0.007
5- 10%	299.0	3.8	845.4	82.1	20.13	1.36	4.1	0.2	0.086	0.011
10- 15%	253.9	4.3	672.4	66.8	16.01	1.15	5.2	0.3	0.140	0.023
15- 20%	215.3	5.3	532.7	52.1	12.68	0.86	6.2	0.2	0.182	0.020
20- 25%	181.6	5.6	421.8	46.8	10.04	0.85	7.0	0.4	0.224	0.031
25- 30%	151.5	4.9	325.6	32.4	7.75	0.58	7.8	0.3	0.257	0.022
30- 35%	125.7	4.9	251.0	25.9	5.98	0.48	8.4	0.4	0.287	0.025
35- 40%	102.7	4.3	188.6	20.6	4.49	0.43	9.1	0.4	0.315	0.032
40- 45%	82.9	4.3	139.4	15.4	3.32	0.31	9.7	0.4	0.337	0.039
45- 50%	65.9	3.4	101.3	12.1	2.41	0.25	10.2	0.4	0.360	0.046
50- 55%	51.6	3.2	72.1	10.5	1.72	0.23	10.7	0.4	0.372	0.048
55- 60%	39.4	3.5	49.9	9.6	1.19	0.23	11.2	0.4	0.383	0.069
60- 65%	29.8	4.1	34.4	8.7	0.82	0.21	11.7	0.5	0.397	0.052
65- 70%	21.5	3.8	22.6	6.6	0.54	0.16	12.1	0.5	0.399	0.054
70- 75%	15.5	3.4	14.8	5.1	0.35	0.12	12.6	0.5	0.392	0.075
75- 80%	11.3	2.6	9.9	3.3	0.24	0.08	13.0	0.6	0.381	0.115
80- 92.2%	6.3	1.2	4.9	1.2	0.12	0.03	14.1	0.6	0.261	0.082
0- 10%	325.2	3.3	955.4	93.6	22.75	1.56	3.2	0.2	0.057	0.008
10- 20%	234.6	4.7	602.6	59.3	14.35	1.00	5.7	0.3	0.161	0.021
20- 30%	166.6	5.4	373.8	39.6	8.90	0.72	7.4	0.3	0.241	0.026
30- 40%	114.2	4.4	219.8	22.6	5.23	0.44	8.7	0.4	0.301	0.028
40- 50%	74.4	3.8	120.3	13.7	2.86	0.28	9.9	0.4	0.348	0.042
50- 60%	45.5	3.3	61.0	9.9	1.45	0.23	11.0	0.4	0.377	0.058
60- 70%	25.7	3.8	28.5	7.6	0.68	0.18	11.9	0.5	0.398	0.053
70- 92.2%	9.5	1.9	8.3	2.4	0.20	0.06	13.5	0.5	0.317	0.084
00- 15%	301.5	3.7	861.1	83.4	20.50	1.40	3.9	0.2	0.085	0.013
15- 60%	112.9	3.4	231.3	22.0	5.51	0.40	8.9	0.4	0.302	0.036
60- 80%	19.5	3.3	20.4	5.9	0.49	0.14	12.3	0.5	0.392	0.072
80- 92.2%	6.3	1.2	4.9	1.2	0.12	0.03	14.1	0.6	0.261	0.082
0- 20%	279.9	4.0	779.0	75.2	18.55	1.27	4.4	0.2	0.109	0.014
20- 60%	100.2	3.4	193.7	19.1	4.61	0.36	9.3	0.4	0.317	0.038
60- 92.2%	14.5	2.5	14.5	4.0	0.35	0.10	13.0	0.5	0.342	0.074
00- 30%	242.1	4.2	644.0	62.9	15.33	1.06	5.4	0.3	0.153	0.018
30- 60%	78.0	3.4	133.7	14.0	3.18	0.28	9.9	0.4	0.342	0.042
60- 70%	25.7	3.8	28.5	7.6	0.68	0.18	11.9	0.5	0.398	0.053
70- 80%	13.4	3.0	12.4	4.2	0.30	0.10	12.8	0.5	0.387	0.092
80- 92.2%	6.3	1.2	4.9	1.2	0.12	0.03	14.1	0.6	0.261	0.082
0- 92.2%	109.1	4.1	257.8	25.4	6.14	0.45	9.5	0.4	0.281	0.045

Table 1: Result of the Glauber calculations for various centrality classes. These results are available as an ASCII file at [1].