Number of binary collisions and total cross section in d+Au at $\sqrt{s_{NN}} = 200$ GeV

David Morrison

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Abstract

The total cross section, $\sigma_{tot}^{\mathrm{d+Au}}$, and average number of binary collisions in a minimum bias sample of deuteron-gold collisions are presented. The method used to estimate these numbers is based on a hybrid between a discrete Monte Carlo and a numerically integrated Glauber model. The main results are: $\sigma_{\mathrm{tot}}^{d+Au} = 2.13 \pm 0.1$ barns; $\langle N_{coll} \rangle = 8.1 \pm 0.5$.

1 The basic approach

I am aware of at least four different efforts to calculate the d+Au total cross section and various related values of interest. Klaus Reygers has continued his work with a Monte Carlo of discrete nucleons on straight line trajectories. Brian Cole, Ron Soltz and Stephen Johnson have obtained results by modifying HIJING to properly include a description of the deuteron. Dima Kharzeev has results based on (I believe) an analytic or semi-analytic integration of thickness functions. A fourth approach, presented here, is a based on a hybrid of a discrete nucleon Monte Carlo for the deuteron and a numerically integrated Glauber model for the gold nucleus. All approaches give similar answers for the d+Au total cross section; they may even agree well at finer levels of comparison, though I haven't done those comparisons yet.

The hybrid approach repeatedly constructs deuterons according to a realistic wavefunction and then samples the thickness function of a gold nucleus. The gold nucleus is described as a continuous Woods-Saxon charge distribution This is a different aproach than I have taken in the past to study Au+Au collisions in which both target and projectile were modeled as continuous distributions. The reason for adjusting the method is that the deuteron is poorly described as a spherical nucleus with a density dependent only on a radial coordinate. It is much better described as two barely conjoined nucleons, something which is straightforward to handle in a discrete Monte Carlo.

The first step in the method is to attempt to model the deuteron correctly.

As have others, I have used a wavefunction due to Hulthèn [3]

$$\phi_{\rm d}(\mathbf{r}_{\rm pn}) = \left(\frac{\alpha\beta(\alpha+\beta)}{2\pi(\alpha-\beta)^2}\right)^{\frac{1}{2}} ({\rm e}^{-\alpha r_{\rm pn}} - {\rm e}^{-\beta r_{\rm pn}})/r_{\rm pn} \tag{1}$$

where $\alpha = 0.228\,{\rm fm}^{-1}$ and $\beta = 1.18\,{\rm fm}^{-1}$. I obtained these values from a talk given by Brian Cole. In these equations, note that $\mathbf{r}_{\rm np}$ refers to the separation between the proton and the neutron. This is an important point to note; $\phi_{\rm d}(\mathbf{r}_{\rm pn})$ is **not** the wavefunction of the reduced 2-body system. Mistaking one for the other gives "factor of 2" mistakes all over the place.

From $\phi_{\rm d}(\mathbf{r}_{\rm pn})$ one obtains the following probability distribution for $\mathbf{r}_{\rm np}$

$$P_{\rm d}(\mathbf{r}_{\rm pn}) = \frac{2\alpha\beta(\alpha+\beta)}{(\alpha-\beta)^2}({\rm e}^{-2\alpha r_{\rm pn}} + {\rm e}^{-2\beta r_{\rm pn}} - 2{\rm e}^{-(\alpha+\beta)r_{\rm pn}}). \tag{2}$$

by multiplying by a 3-space measure, $r^2 \sin\theta dr d\theta d\phi$ and integrating over the angular coordinates.

The Hulthèn wavefunction is fairly old and there have been developments since then [6]. The changes have typically been aimed at better reproducing the high energy scattering data and do not have much effect on the gross size of the deuteron.

To build a deuteron for use in a Monte Carlo, start by obtaining an internucleon separation, $r_{\rm pn}$, based on Eq. 2 and then randomly orienting in 3-space the vector connecting the neutron and proton. (The GSL function `gsl_ran_dir_3d` was used to pick a random orientation with the proper distribution.)

One of the main ingredients in the calculation is the nucleon-nucleon inelastic cross section at $\sqrt{s} = 200\,{\rm GeV}$ which was taken from the PDG plots of p+p reactions [2], as shown in Figure 1. Reading the inelastic cross section off the plot for the appropriate $\sqrt{s} = 200\,{\rm GeV}$, I get $\sigma_{tot} = 42 \pm 1\,{\rm mb}$. Varying this value in the calculation has very little effect.

The gold nucleus is modeled using a Woods-Saxon density distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-c}{a}\right)} \tag{3}$$

where $a = 0.54\,{\rm fm}$ and $c = 1.12 \times A^{1/3} - 0.86 \times A^{-1/3} = 6.40\,{\rm fm}$, a value which agrees well with the measured charge radius of 6.38 fm for gold [1].

2 Total Cross Section

The total cross section, σ_{tot}^{d+Au}, was determined using a Monte Carlo approach. A deuteron was built by sampling an internucleon separation based on the Hulthèn wavefunction above and then choosing an orientation at random. A numerical integration over the transverse plane was done to sample impact parameters. For each impact parameter, the thickness of the gold nucleus seen

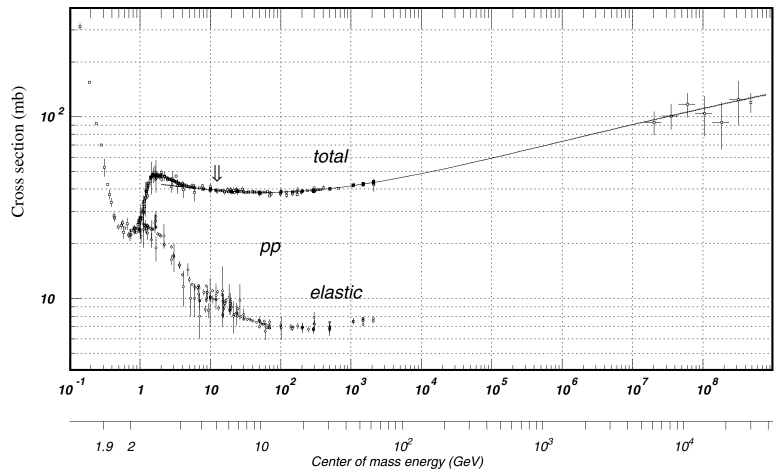

Figure 1: Elastic and total cross sections for p+p reactions. Taken from [2].

by each nucleon of the deuteron was separately calculated. These values were used to determine the probability of d+Au interaction according to the following equation:

$$I = 1.0 - \exp\left(-\sigma_{NN} T_{Au}(b_n)\right) \times \exp\left(-\sigma_{NN} T_{Au}(b_p)\right) \quad (4)$$

where b_n and b_p represent the respective impact parameters of the neutron and the proton with the gold. The term

$$\exp\left(-\sigma_{NN} T_{Au}(b_n)\right) \quad (5)$$

represents the probability that the neutron did *not* interact. Together, the terms

$$\exp\left(-\sigma_{NN} T_{Au}(b_n)\right) \times \exp\left(-\sigma_{NN} T_{Au}(b_p)\right) \quad (6)$$

represent the probability that neutron did not interact *and* the the proton did not interact. Subtracting this from one yields the complementary probability that either the neutron *or* the proton interacted. Something happened and therefore it contributes to the total cross section.

For each sampled deuteron, this probability of interaction is integrated and the total cross section is determined. A histogram is filled with the results for a large ensemble of sampled deuterons, as shown in Fig. 2. The average of this distribution is $\sigma_{tot}^{d+Au} = 2.13$ barns. The other methods of determining the total cross section have all yields slightly higher values for σ_{tot}^{d+Au}, ranging from 2.15 to 2.25 barns. Several things have been investigated to see if the source of the difference can be determined, but so far there is no definitive answer.

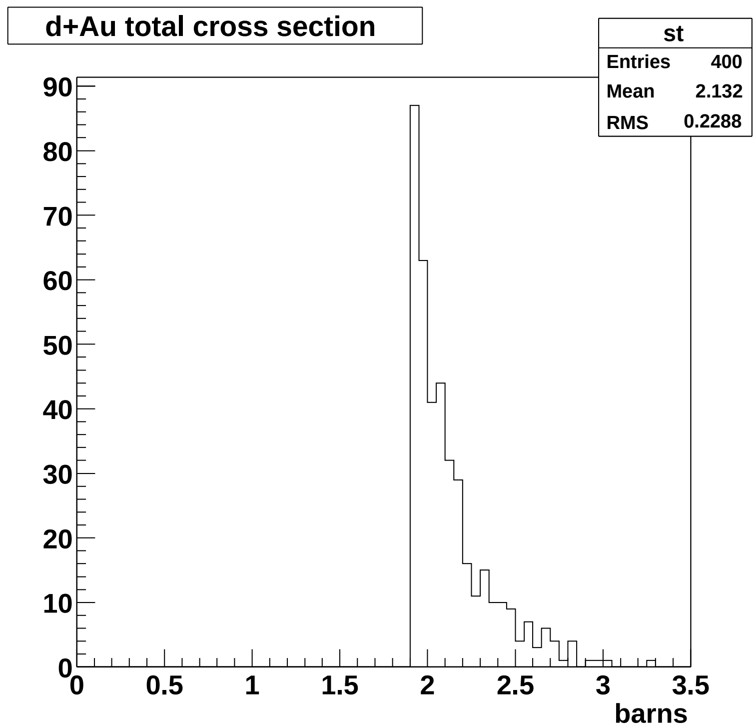

Figure 2: Histogram of $dn/d\sigma_{tot}$ for deuteron-gold collisions.

3 Number of Binary Collisions

A similar procedure is followed to determine the average number of binary collisions. A deuteron is built and for each impact parameter the thickness seen by the nucleons of the deuteron is multiplied by σ_{NN} to obtain the average number of collisions suffered by that nucleon. A Poisson distribution around this average is assumed

$$p(k;\mu) = \frac{\mu^k}{k!}\exp(-\mu) \tag{7}$$

and a value is sampled accordingly. (In the actual code used to calculate this, the GSL routine `gsl_ran_poisson` is used.) The values obtained for the number of binary collisions seen by the proton and the neutron are summed and entered into a histogram. After a large ensemble of deuterons have been built and used to sample the number of binary collisions, the average can be determined from the histogram, as shown in Fig. 3.

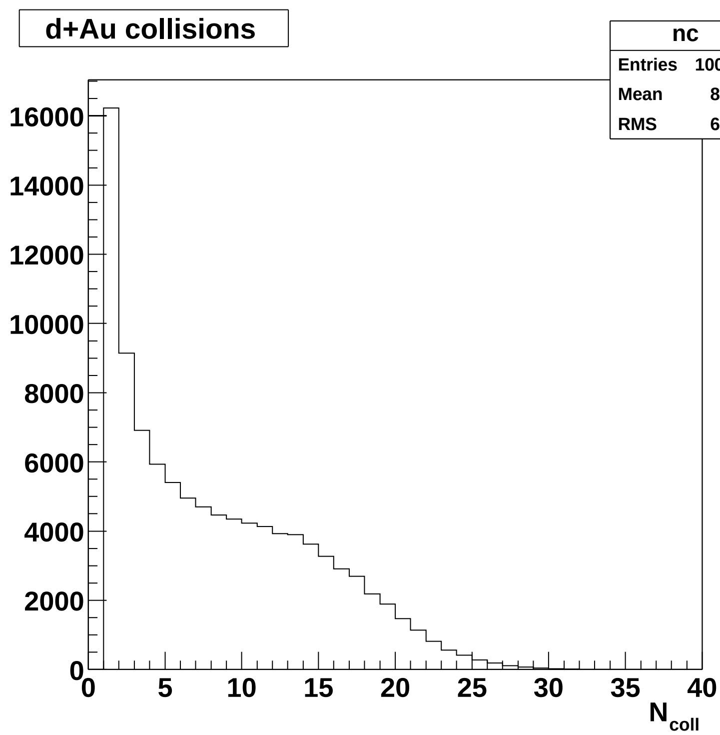

Figure 3: Histogram showing dn/dN_{coll}, the number of binary nucleon-nucleon collisions.

In addition to the total number of binary collisions suffered by the proton and the neutron, one can also look at the probability of n binary collisions for the neutron and m binary collisions for the proton. That is shown in Fig. 4.

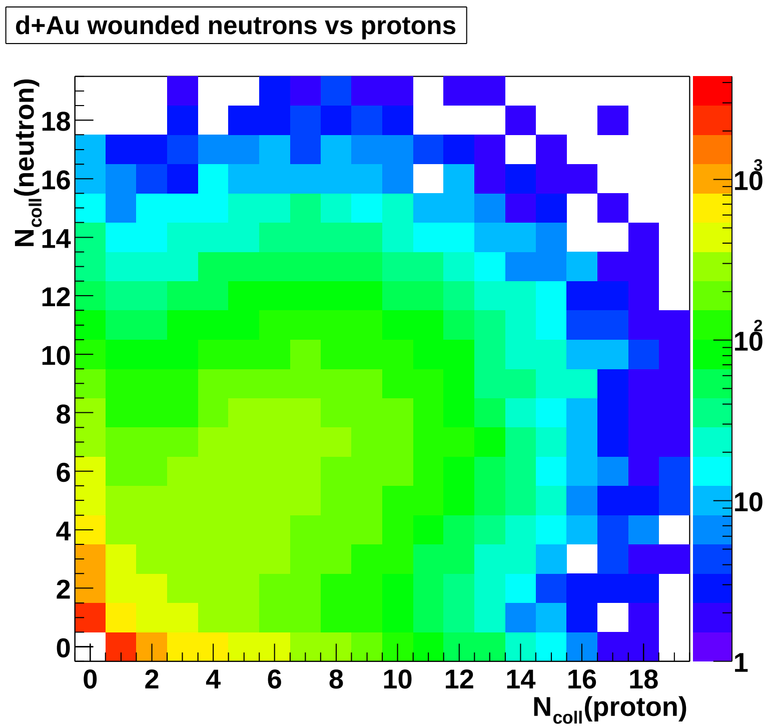

Figure 4: Relative probability n binary collisions for the neutron and m binary collisions for the proton

The joint distribution of the number of binary collisions seen by the proton when the neutron has not suffered any collisions is shown in Fig. 5. As has been pointed out by Brian Cole, the deuteron is so large, that even when the neutron doesn't interact with the gold nucleus, the proton can sweep out essentially the full range of interesting impact parameters with the gold.

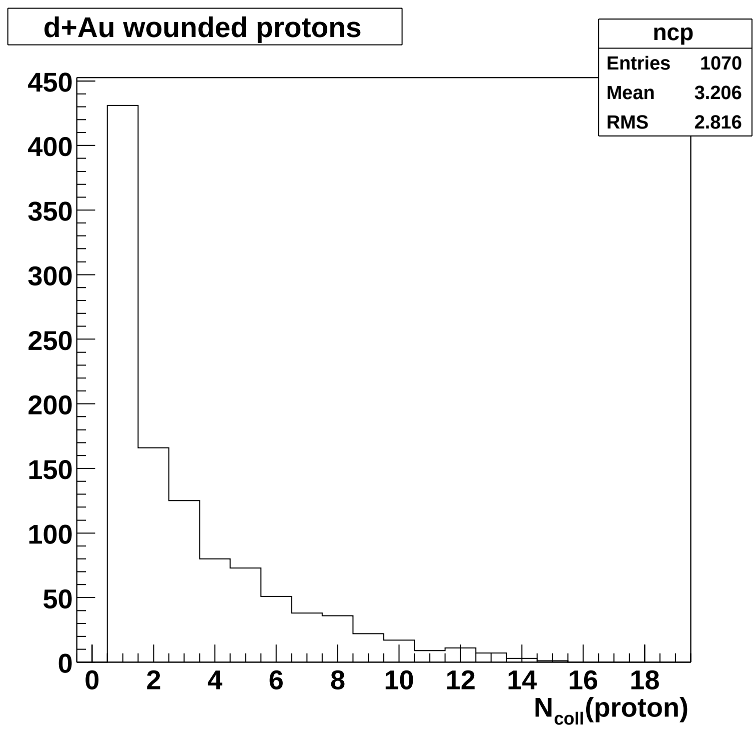

Figure 5: Distribution of the number of binary collision suffered by the deuteron's proton when the neutron sees zero collisions.

4 Nucleon Momentum Distribution in the Deuteron

The nucleons in the have very low average momenta (Fig. 6) which means that the acceptance of the ZDC will be very difference per spectator nucleon than was the case in Au+Au. We have seen preliminary calculations of this acceptance in the global/hadron PWG meetings by Alexei Denisov. The fact that the momentum distribution is weighted toward low values improves the acceptance of the ZDC over what might be naïvely assumed, but this is something that really needs to be determined using something like HIJING or JAM.

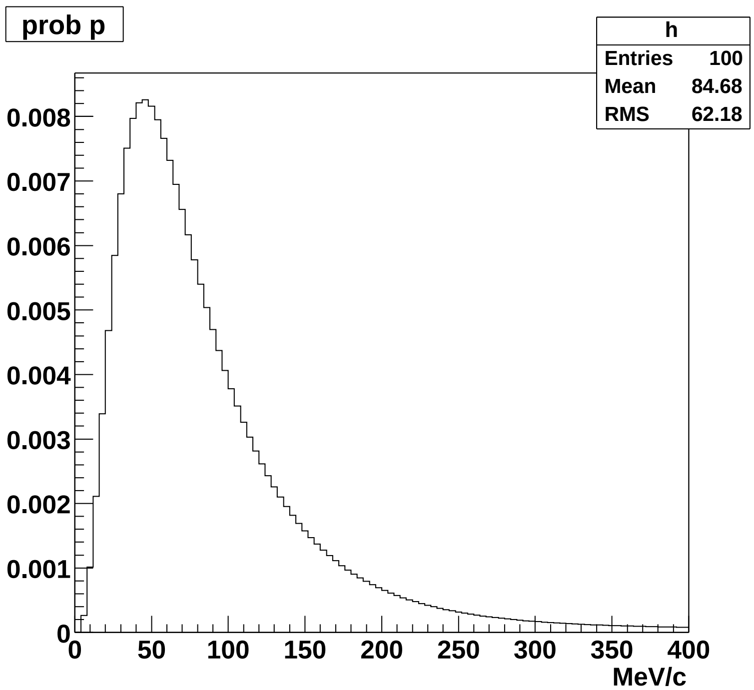

Figure 6: Momentum distribution in the deuteron. From [5]

5 The Code

The code to make all the plots shown in this note is in the main CVS repository as `offline/analysis/glauber`. Just check it out, build the library and run some of the macros contained in there to reproduce these results. In addition, there is an interesting class library called `Pluto`, produced for HADES, which has useful implementations of deuteron wavefunction [4].

6 Summary

These are preliminary numbers and there are others calculating similar quantities, so these shouldn't be taken as the final word on d+Au interactions. In particular, most other calculations tend to get slightly higher (0.05-0.15 barn) total cross sections. At this point the precise reasons for the differences are not known, though they are being investigated. These numbers should be good enough to allow a reasonably solid normalization of early d+Au results in run3.

References

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