E_T Distributions and other Event-by-Event Fluctuations

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625 TRACKS, 733 EMCAL CLUSTERS IN $\Delta\phi = \pi$, $\delta\eta = 0.70$

CURRENT AND FUTURE DIRECTIONS AT RHIC, AUGUST 8, 2002

Some Favorite Statistics

•In the theory of probability and statistics, a statistic is a quantity computed entirely from the sample, i.e. a statistic is any function of the observed sample values.

•Two of the most popular statistics are the sum and the average:

$$S_n \equiv \sum_{i=1}^{n} x_i \tag{1}$$

$$\bar{x}_{(n)} \equiv \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n} S_n \tag{2}$$

where the x_i are n samples from a the same population or probability density function, $f(x)$.

• From the theory of mathematical statistics, the probability distribution of a random variable S_n, which is itself the sum of n independent random variables with a common distribution $f(x)$:

$$S_n = x_1 + x_2 + \cdots + x_n \tag{3}$$

is given by $f_n(x)$, the n-fold convolution of the distribution $f(x)$:

$$f_n(x) = \int_0^x dy\, f(y)\, f_{n-1}(x-y) \qquad . \tag{4}$$

The mean, $\mu_n = \langle S_n \rangle$, and standard deviation, σ_n, of the n-fold convolution obey the familiar rule

$$\mu_n = n\mu \qquad \sigma_n = \sigma\sqrt{n} \quad , \tag{5}$$

where μ and σ are the mean and standard deviation of the distribution $f(x)$.

• A complementary case is that of a random variable Z_n, which is the sum of n random variables with a common distribution $f(x)$—which are themselves 100% correlated—for example:

$$Z_n = x + x + \cdots + x = nx \qquad . \tag{6}$$

This is just a scale transformation. The behavior of the mean and the standard deviation for a scale transformation is $\mu \rightarrow n\mu$, $\sigma \rightarrow n\sigma$, which is quite different than the behavior of the standard deviation under convolution (Eq. 5).

Some Favorite Functions

The Gamma distribution is an example of a probability density function (pdf) which has particularly simple properties under convolutions and scale transformations. The Gamma distribution is a function of a continuous variable x and has paramters p and b

$$f(x) = f_\Gamma(x, p, b) = \frac{b}{\Gamma(p)} (bx)^{p-1} e^{-bx} \tag{7}$$

where

$$p > 0, \quad b > 0, \quad 0 \leq x \leq \infty$$

$\Gamma(p) = (p-1)!$ if p is an integer, and $f(x)$ is normalized, $\int_0^\infty f(x)dx = 1$. The mean and standard deviation of the distribution are

$$\mu \equiv \langle x \rangle = \frac{p}{b} \qquad \sigma \equiv \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{\sqrt{p}}{b} \qquad \frac{\sigma^2}{\mu^2} = \frac{1}{p} \qquad . \tag{8}$$

The n-fold convolution of the Gamma distribution (Eq. 7) is simply given by the function

$$f_n(x) = \frac{b}{\Gamma(np)} (bx)^{np-1} e^{-bx} = f_\Gamma(x, np, b) \tag{9}$$

i.e. $p \rightarrow np$ and b remains unchanged. Note that the mean and standard deviation of Eq. 9

$$\mu_n = \frac{np}{b} \qquad \sigma_n = \frac{\sqrt{np}}{b} \qquad \frac{\sigma_n}{\mu_n} = \frac{1}{\sqrt{np}} \tag{10}$$

when compared to Eq. 8 explicitly obey Eq. 5. The result of a scale transformation $x \rightarrow nx$ for a Gamma distribution (Eq. 7) is simply $b \rightarrow b/n$, with p remaining unchanged. **To summarize, the n-th convolution of the Gamma distribution $f_\Gamma(x, p, b)$ is $f_\Gamma(x, np, b)$; the scale transformation $x \rightarrow nx$ of $f_\Gamma(x, p, b)$ is $f_\Gamma(x, p, b/n)$.**

The p_T distribution is a Gamma Distribution

The principal advantage of the Gamma distribution for the present problem is that it is one of the standard representations of the inclusive single particle p_T distribution:

$$\frac{d\sigma}{p_T dp_T} = b^2 e^{-bp_T} \tag{11}$$

$$\frac{d\sigma}{dp_T} = b^2 p_T e^{-bp_T} \quad . \tag{12}$$

Clearly, Eq.s 11, 12 represent a Gamma distribution with $p = 2$, $\langle p_T \rangle = 2/b$, where typically $b = 6$ (GeV/c)$^{-1}$ for p-p collisions. The 'inverse slope parameter' $1/b$ is sometimes referred to as the 'Temperature parameter'.

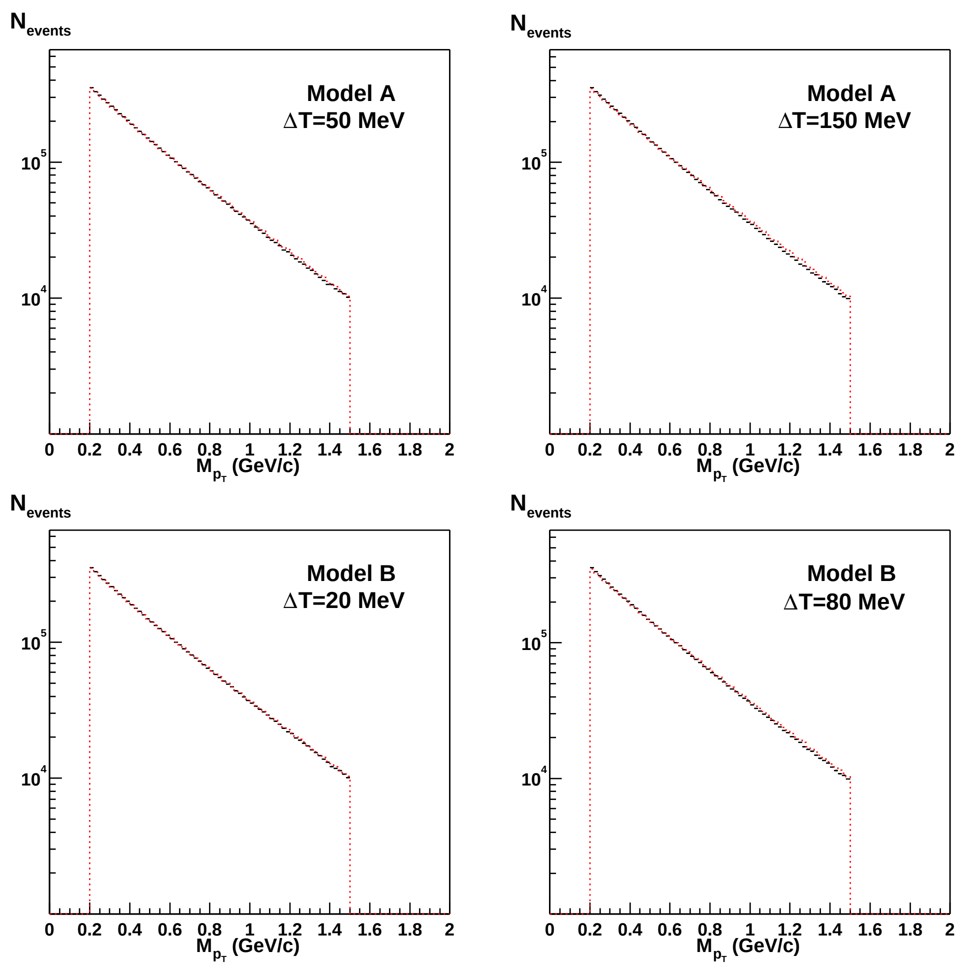

Figure 1: PHENIX inclusive p_T **Not** M_{p_T} distribution for 5% most central collisions(black). Red are 2-component models to be discussed later. Note the limited range of p_T used.

Event by Event Distribution
It's a not a Gaussian, it's a Gamma Distribution

$$M_{p_T} = \overline{p_T}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} p_{T_i} = \frac{1}{n}E_{Tc}$$

$$E_{Tc} = \sum_{i=1}^{n} p_{T_i} \qquad E_T = \sum_{i=1}^{n} e_{T_i}$$

Analytical formula for statistically independent emission

For statistical independent emission an analytical formula for the distribution in M_{p_T} can be obtained. It depends on the 4 semi-inclusive parameters $\langle n\rangle$, $1/k$, b and p which are derived from the quoted means and standard deviations of the semi-inclusive p_T and multiplicity distributions. The result is in excellent agreement with the NA49 Pb+Pb-central measurement.

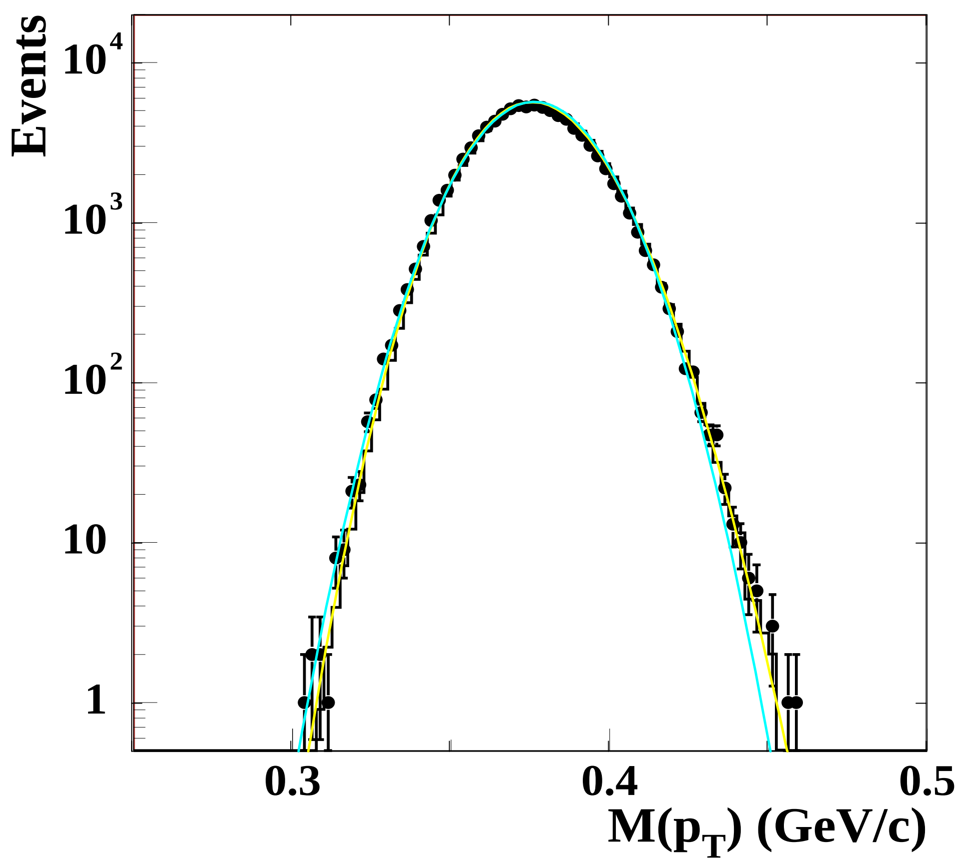

Figure 2: Gamma Distribution for M_{p_T} (yellow) compared Gaussian with same μ and σ (light blue) for NA49 measurement (filled points) and mixed event distribution (histogram).

see M. J. Tannenbaum, Phys. Lett. B498, 29 (2001)

It's a not a Gaussian, it's a Gamma Distribution

PHENIX M_{p_T} vs. Centrality
Now Gamma Distribution Shape is Obvious

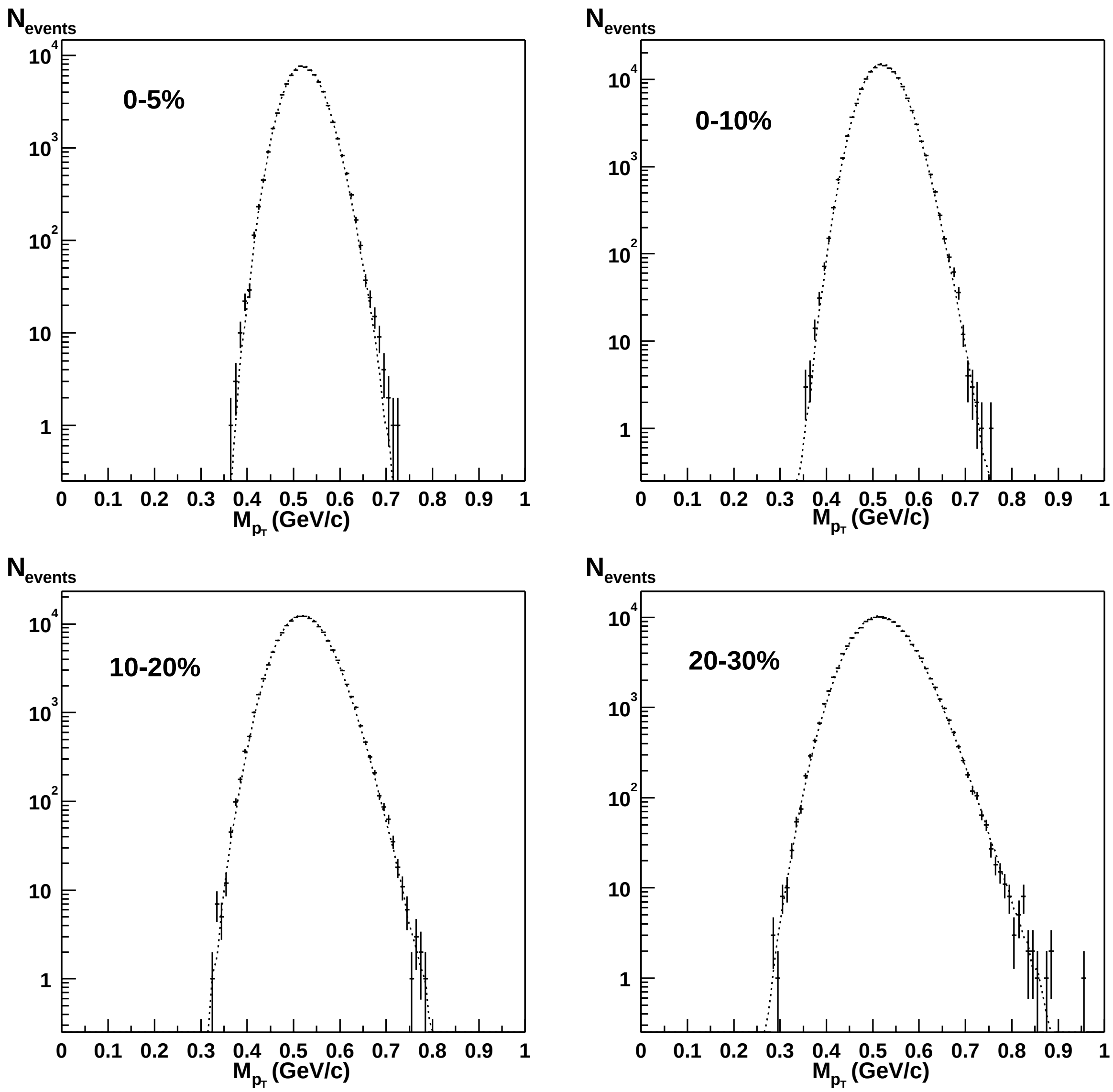

Figure 3: PHENIX M_{p_T} data for Ai+Au at $\sqrt{s_{NN}}$ = 130 GeV vs centrality. The dotted curves are mixed event distributions used as the random baseline.

- Use Mixed Events as a random baseline reference since analytical formula doesn't work in general.
- For Mixed Events **must use exactly the same n distribution as the data and match the inclusive $\langle p_T \rangle$ to high precision.**
- Data indicate very small, if any, non random effect **How to Quantify?**

Most Groups Use Moments (fortunately just μ and σ)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\langle \bar{x} \rangle = \langle x \rangle \equiv \mu$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\sigma_{\bar{x}}^2 = \langle \bar{x}^2 \rangle - \langle \bar{x} \rangle^2$$

•For Statistically Independent Emission

$$\sigma_{\bar{x}}^2 = \left\langle \frac{\sigma_x^2}{n} \right\rangle$$

Typical Measures of Sensitivity

$$\frac{\text{Measured -Random}}{\text{Random}}$$

$$(\frac{\sigma_{\bar{x}}^2}{\mu^2} - \frac{1}{n}\frac{\sigma_x^2}{\mu^2})/\frac{1}{n}\frac{\sigma_x^2}{\mu^2}$$

$$(\frac{\sigma_{\bar{x}}}{\mu} - \frac{1}{\sqrt{n}}\frac{\sigma_x}{\mu})/\frac{1}{\sqrt{n}}\frac{\sigma_x}{\mu} = F$$

•For small effects these measures are equivalent

$$\frac{\Delta\sigma^2}{\sigma^2} = 2\,\frac{\Delta\sigma}{\sigma} = 2F$$

•PHENIX uses Mixed Events as Random Baseline with exactly the same n distribution as the data and with the inclusive $\langle p_T \rangle = \mu$ matched to high precision.

$$(\frac{\sigma_{\bar{x}}}{\mu} - \frac{\sigma_{\bar{x}-\text{mixed}}}{\mu})/\frac{\sigma_{\bar{x}-\text{mixed}}}{\mu} = F$$