At the Limits of Perturbative QCD

Workshop on Future Prospects of QCD

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OUTLINE

I. Introduction: factorization and coherence in pQCD

II. Case study: Q_T & and its factorization

III. Threshold resummation

IV. Evolution of color exchange

V. Generalizations and limitations

I. Intro.: factorization & coherence in perturbative QCD

- **From factorization to resummation**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(1/Q^p\right)$$

  - μ = **factorization scale;** m= **IR scale** **(**m *may* **be perturbative)**

  - **New physics in** $\omega_{\rm SD}$**;** $f_{\rm LD}$ **"universal"**

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

PDF f or Fragmentation D

- **Wherever there is evolution there is resummation**

$$\ln \sigma_{\rm phys}(Q, m) = \exp \left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- **Infrared safety & factorization proofs**:
  - **(1) ω_{SD} incoherent with long-distance dynamics**
  - **(2) Mutual incoherence when $v_{\mathrm{rel}} = c$: Jet-jet factorization**
  - **(3) Wide-angle soft radiation sees only total color flow: jet-soft factorization $\Rightarrow$ angular ordering & MLLA**
  - **(4) Dimensionless coupling and renormalizability $\Leftrightarrow$ no worse that logarithmic divergence in the IR: fractional power suppression $\Rightarrow$ finiteness**

II. Vector bosons: Q_T and its factorization

Every final state from a hard scattering carries the imprint of QCD dynamics from at all distance scales

– **Look at transverse momentum distribution at order α_s**

$$q(p_1) + \bar{q}(p_2) \rightarrow \gamma^*(Q) + g(k)\,,$$

– **Treat this $2 \rightarrow 2$ process at lowest order (α_s) "LO" in factorized cross section, so that $\mathbf{k} = -\mathbf{Q}_T$**

– **Factorized cross section at fixed $\mathbf{Q}_T$:**

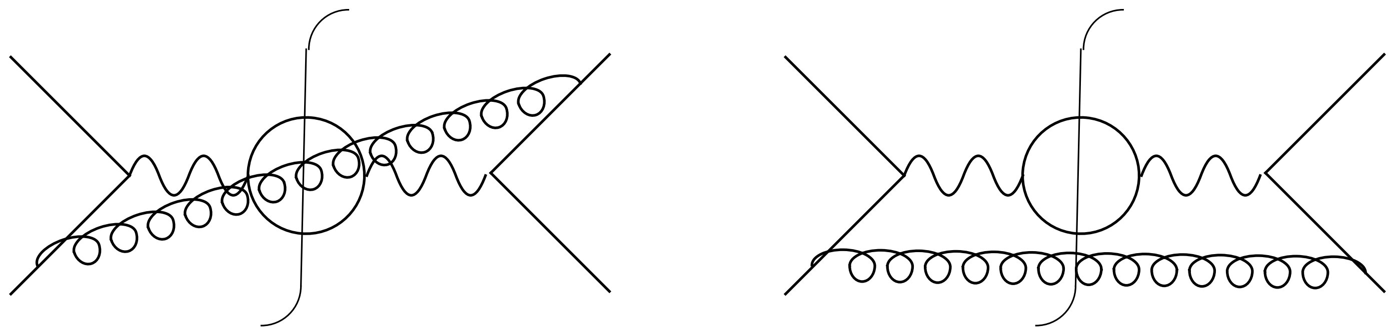
$$\frac{d\sigma_{NN\to\mu^+\mu^-+X}(Q,p_1,p_2)}{dQ^2 d^2\mathbf{Q}_T} = \int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \frac{d\hat{\sigma}_{a\bar{a}\to\mu^+\mu^-(Q)+X}(Q,\mu,\xi_1 p_1,\xi_2 p_2,\mathbf{Q}_T)}{dQ^2 d^2\mathbf{Q}_T} \times f_{a/N}(\xi_1,\mu)\, f_{\bar{a}/N}(\xi_2,\mu)$$

– **Recall:** μ **is the factorization scale that separates IR (f) from UV ($d\hat{\sigma}$) in quantum corrections.**

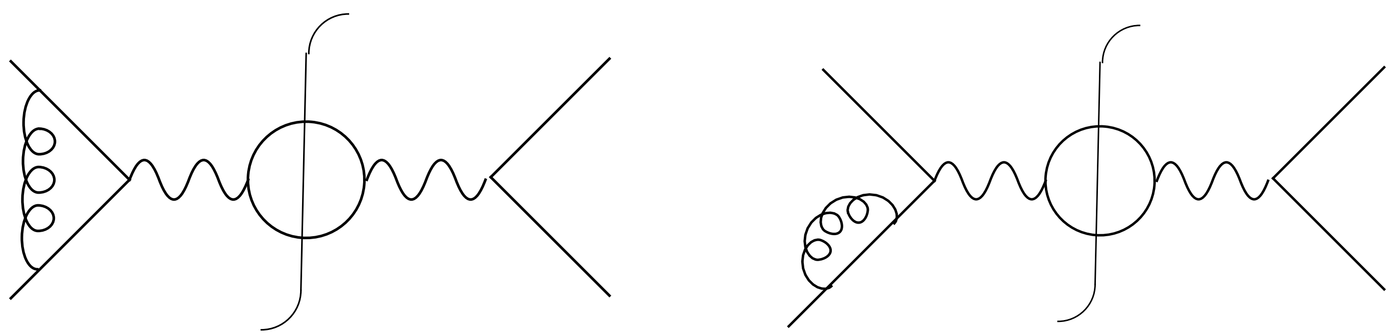
– μ **appears in** $\hat{\sigma}$ **through** $\alpha_s(\mu)$ **and** $\ln(\mu/Q)$ **so choosing** $\mu \sim Q$ **can improve perturbative predictions.**

– **Evolution:** $\mu df(x,\mu)/d\mu = \int_x^1 P(x/\xi)\, f(\xi,\mu)$ **makes energy extrapolations possible.**

– **The diagrams at order α_s**
Gluon emission contributes at $Q_T \neq 0$



Virtual corrections contribute only at $Q_T = 0$



– **The result is finite for $\mathbf{Q}_T \neq 0$ . . .**

$$\frac{d\hat{\sigma}^{(1)}_{q\bar{q}\to\gamma^* g}}{dQ^2\, d^2\mathbf{Q}_T} = \sigma_0 \frac{\alpha_s C_F}{\pi^2} \left(1 - \frac{4\mathbf{Q}_T^2}{(1-z)^2 \xi_1 \xi_2 S}\right)^{-1/2}$$

$$\times \left[\frac{1}{\mathbf{Q}_T^2}\frac{1+z^2}{1-z} - \frac{2z}{(1-z)Q^2}\right]$$

as long as $\mathbf{Q}_T \neq 0$, $z = Q^2/\xi_1\xi_2 S \neq 1$.

Q_T **integral** $\to \frac{\ln(1-z)}{1-z}$**;** z **integral** $\to \frac{\ln \mathbf{Q}_T^2}{\mathbf{Q}_T^2}$**.**

Both singularities cancel in the inclusive cross section.
Both inspire resummation of higher order corrections.

The leading singularity in $\mathbf{Q}_T$

– **As we'll see later:** $1 - z \sim 2k_0/Q \geq 2|\mathbf{k}_T|/Q$

– z **integral: If** Q^2/S **not too big, PDFs nearly constant:**

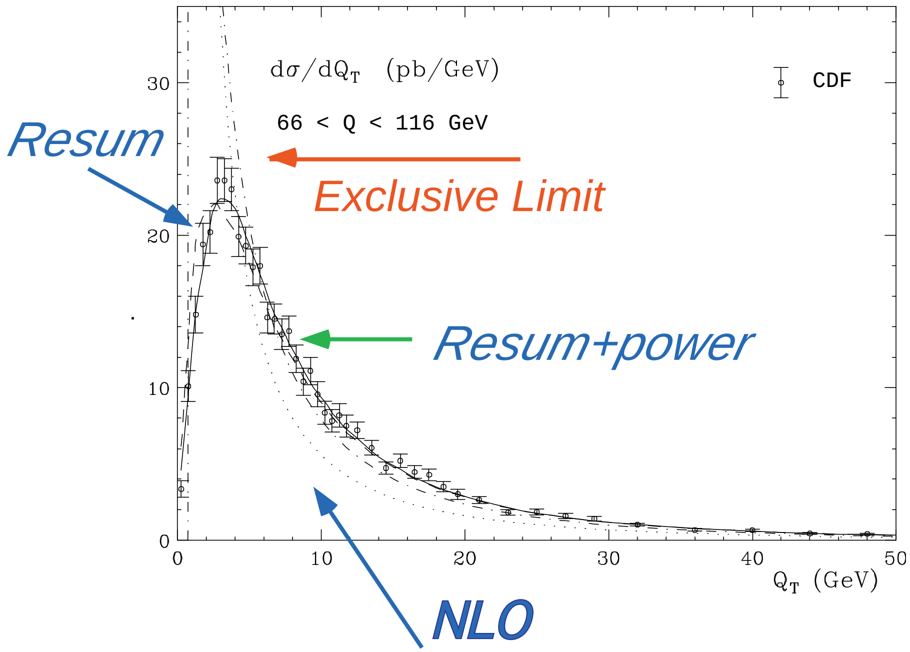
$$\frac{1}{\mathbf{Q}_T^2} \int_{1-Q^2/S}^{1-\mathbf{Q}_T^2/Q^2} \frac{dz}{1-z} = \frac{1}{\mathbf{Q}_T^2} \ln\left[\frac{Q^2}{\mathbf{Q}_T^2}\right]$$

$\Rightarrow$ **Prediction for** Q_T **dependence:**

$$\frac{d\sigma_{NN\to\mu^+\mu^-+X}(Q,\mathbf{Q}_T)}{dQ^2 d^2\mathbf{Q}_T} = \frac{\alpha_s C_F}{\pi} \frac{1}{\mathbf{Q}_T^2} \ln\left[\frac{Q^2}{\mathbf{Q}_T^2}\right]$$

$$\times \sum_{a=q\bar{q}} \int_{\xi_1 \xi_2} \frac{\hat{\sigma}_{a\bar{a}\to\mu^+\mu^-(Q)+X}(Q,\mu)}{dQ^2} f_{a/N}(\xi_1,\mu)\, f_{\bar{a}/N}(\xi_2,\mu)$$

– Compare to: Z p_T from Tevatron Run I



(from Kulesza, G.S., Vogelsang (2002))

– $\ln Q_T/Q_T$ works pretty well for large Q_T

– At smaller Q_T reach a maximum, then a decrease near "exclusive" limit (parton model kinematics)

– Most events are at "low" $Q_T \ll Q = m_Z$.

Getting to $Q_T \ll Q$: Transverse momentum resummation

(Logs of Q_T)/Q_T to all orders

How? Variant factorization and separation of variables

q and $\bar{q}$ "arrive" at point of annihilation with transverse momentum of radiated gluons in initial state.

q and $\bar{q}$ radiate independently (fields don't overlap!).

Final-state QCD radiation too late to affect cross section

$$\frac{d\sigma_{NN\to\mu^+\mu^-+X}(Q,\mathbf{Q}_T)}{dQ^2 d^2\mathbf{Q}_T}$$

Summarized by: Q_T-factorization:

$$\frac{d\sigma_{NN\to QX}}{dQd^2Q_T} \qquad H \times \mathcal{P}_{a/N}(\xi_1, p_1\cdot n, k_{1T})\,\mathcal{P}_{\bar{a}/N}(\xi_2, p_2\cdot n, k_{2T})$$

$$\otimes_{\xi_i,k_{iT}}\; U_{a\bar{a}}(k_{sT}, n)$$

We will solve for the k_T dependence of the $\mathcal{P}$'s.

New factorization variables: n^μ apportions gluons k:

$$p_i\cdot k < n\cdot k \;\Rightarrow\; k \;\in \mathcal{P}_i$$

$$p_a\cdot k, p_{\bar{a}}\cdot k \;>\; n\cdot k \;\Rightarrow\; k \in U$$

Convolution in $k_{i,T}$s $\Rightarrow$ Fourier $\mathrm{e}^{i\vec{Q}_T\cdot\vec{b}}$

The factorized cross section in “impact parameter space”:

$$\frac{d\sigma_{NN\to QX}(Q,b)}{dQ} = \int d\xi_1 d\xi_2$$
$$\times\ H(\xi_1 p_1, \xi_2 p_2, Q, n)_{a\bar{a}\to Q+X}$$
$$\times \mathcal{P}_{a/N}(\xi_1, p_1\cdot n, b)\, \mathcal{P}_{\bar{a}/N}(\xi_2, p_2\cdot n, b)\ U_{a\bar{a}}(b,n)$$

Now we can resum by separating variables!

the LHS independent of $\mu_{\rm ren}$, n $\Rightarrow$ two equations

$$\mu_{\rm ren}\frac{d\sigma}{d\mu_{\rm ren}} = 0 \qquad n^\alpha \frac{d\sigma}{dn^\alpha} = 0$$

Solve and transform back to Q_T: all the (Logs of Q_T)/Q_T:

$$\frac{d\sigma_{NN\mathrm{res}}}{dQ^2\, d^2\vec{Q}_T} = \sum_a H_{a\bar{a}}(\alpha_s(Q^2)) \int \frac{d^2b}{(2\pi)^2}\, e^{i\vec{Q}_T\cdot\vec{b}} \exp\left[\, E^{\mathrm{PT}}_{a\bar{a}}(b,Q,\mu)\,\right]$$

$$\times \sum_{a=q\bar{q}} \int_{\xi_1\xi_2} \frac{d\hat{\sigma}_{a\bar{a}\to\mu^+\mu^-(Q)+X}(Q,\mu)}{dQ^2}\, f_{a/N}(\xi_1,1/b)\, f_{\bar{a}/N}(\xi_2,1/b)$$

"Sudakov" exponent suppresses large $b \leftrightarrow$ small Q_T:

$$E^{\mathrm{PT}}_{a\bar{a}} = -\int_{1/b^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[2A_q(\alpha_s(k_T))\, \ln\left(\frac{Q^2}{k_T^2}\right) + 2B_q(\alpha_s(k_T)) \right]$$

With $B = 2(K+G)_{\mu=p\cdot n}$, and lower limit: $1/b$ (NLL)

$*$ **Comments:**

The functions $A_i(\alpha_s)$ and $B_i(\alpha_s)$ are anomalous dimensions.

And can be calculated by comparison to low orders.

In particular, $A_i(\alpha_s)$ is the numerator of the $1/(1-x)$ term in splitting function $P_{ii}(x)$

because it's the infrared divergent $(x \to 1)$ coefficient of the collinear $b \to \infty$ singularity.

$*$ $A_q(\alpha_s) = \frac{\alpha_s}{\pi} C_q \left(1 \; + \; \frac{\alpha_s}{\pi} K + \ldots\right)$, $K = C_A \left(\frac{67}{18} \; - \; \frac{\pi^2}{6}\right) - \frac{5 n_F}{9}$

* Evaluating a resummed cross sections: re-enter NPQCD.

We start with:

$$E^{\rm PT} = - \int_{1/b^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[2A_q(\alpha_s(k_T)) \ln\left(\frac{Q^2}{k_T^2}\right) + B_q(\alpha_s(k_T)) \right]$$

With running coupling:

$$\alpha_s(k_T) = \frac{\alpha_s(Q)}{1 + \frac{\alpha_s(Q)}{4\pi}\beta_0 \ln\left(\frac{k_T^2}{Q^2}\right)} = \frac{4\pi}{\beta_0 \ln\left(\frac{k_T^2}{\Lambda_{\rm QCD}^2}\right)}$$

Singularity in integral at $b^2 = Q^2 \exp[-4\pi/\beta_0\alpha_s(Q)] \sim \frac{1}{\Lambda^2}$.

* **Problem:** how to do the inverse transform with the running coupling when $k_T^{\min} \sim 1/b$ gets small?

* **At least four approaches:**

  1) Work in Q_T-space directly to some approximation
  The originals: Dokshitzer, Diakanov & Troyan
  Revived by Ellis & Veseli Kulesza & Stirling
  who re-derived it from b-space.

  2) Insert a "soft landing" on the k_T integral by replacing

$$1/b \to \sqrt{1/b^2 + 1/b_*^2}$$

  for some fixed b_*. (CS, CSS "b_*" prescription, ResBos)

3) Extrapolation of $E^{\rm PT}$ into NP region (J.W. Qiu, X.F. Zhan

4) Minimal: avoid the singularity at $1/b = \Lambda_{\rm QCD}$
by monkeying with the b-space contour integral.
(This technique introduced in threshold resummation;
then adapted by Laenen, GS and Vogelsang,
and Bozzi, Catani, de Florian and Grazzini.)

Any of these "define" PT. All will fit the data
qualitatively, and with a little work quantitatively.

But all require new parameters for quantitative fit.
This is not all bad . . . let's see why.

$$
\begin{aligned}
E^{\rm soft} = &\ \frac{1}{2\pi}\int_0^{\mu_I^2}\frac{d^2k_T}{k_T^2}\,A_q(\alpha_s(k_T))\,\ln\left(\frac{Q^2}{k_T^2}\right)\,\left(e^{i\mathbf{b}\cdot\mathbf{k}_T}-1\right)\\
\sim -&\ \int_0^{\mu_I^2}\frac{dk_T^2}{k_T^2}\,(\mathbf{b}\cdot\mathbf{k}_T)^2 A_q(\alpha_s(k_T))\,\ln\left(\frac{Q^2}{k_T^2}\right)+\cdots\\
\sim -&\ b^2\int dk_T^2\,A_q(\alpha_s(k_T))\,\ln\left(\frac{Q^2}{k_T^2}\right)
\end{aligned}
$$

$\theta(k_T-1/b)\Rightarrow(e^{i\mathbf{b}\cdot\mathbf{k}_T}-1)$; **in fact, correct to all orders,**

Note the expansion is for b " small enough" only.

What is $-\, b^2 \int dk_T^2\, A_q(\alpha_s(k_T))\, \ln\left(\frac{Q^2}{k_T^2}\right)$ **?**

Don't really know, but it suggests IR gluons organize to a simple form & give a nonperturbative correction like (exhibiting the μ_I is unconventional)
Perturbative strong coupling is a mirage?

$$E^{\rm NP} = -\, b^2 \mu_I^2 \left(g_1 \ln\left(\frac{Q}{\mu_I}\right) + g_2 \right)$$

Since this is an exponent, whatever the definition of the pertrubative resummed cross section, it is smeared with a Gaussian whose width in b (k_T) space decreases (increases) with $\ln Q$.

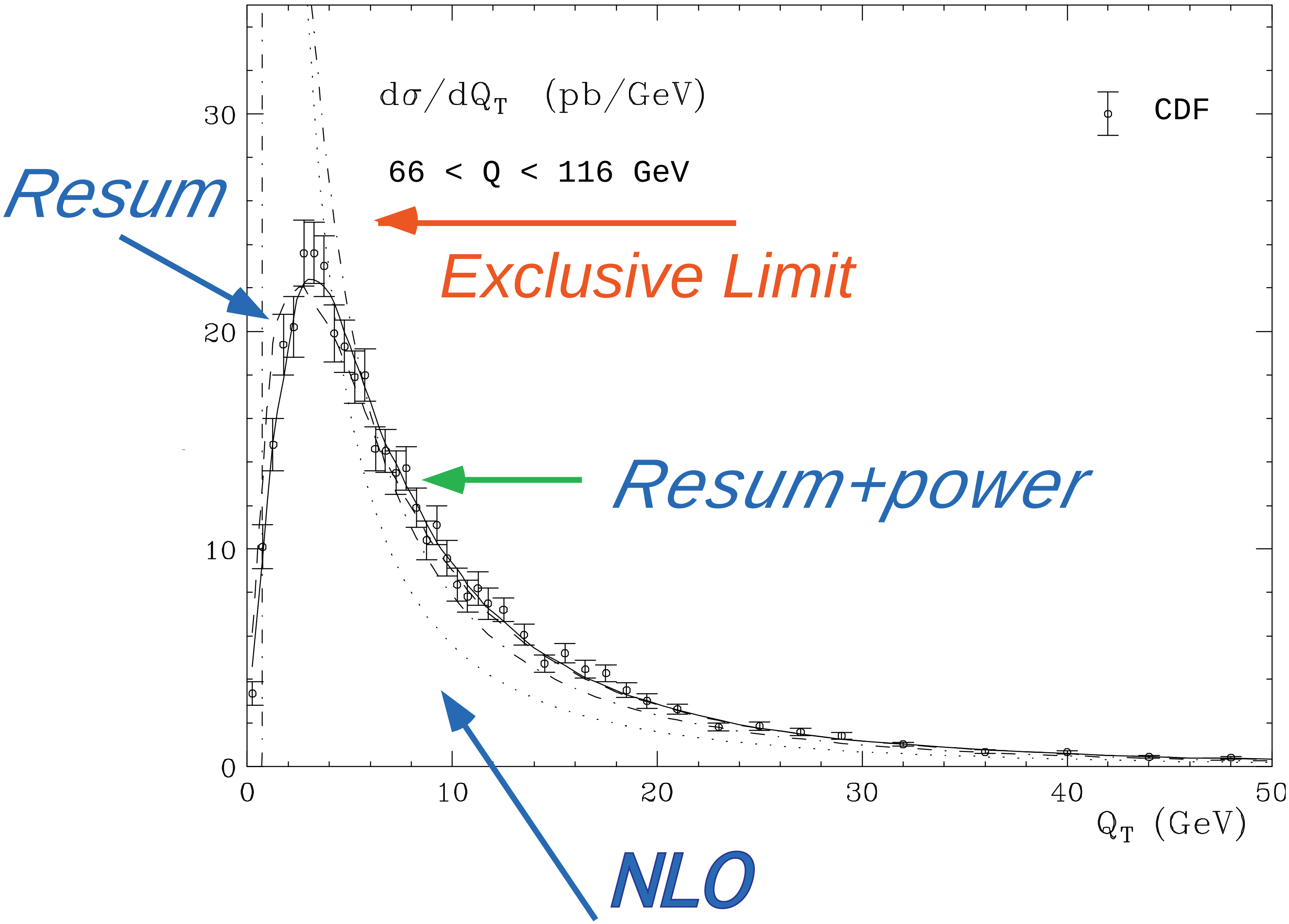
In summary

$$\frac{d\sigma(Q_T)}{dQ^2\, d^2\vec{Q}_T} = \sum_a H_{a\bar{a}}(\alpha_s(Q^2)) \int \frac{d^2b}{(2\pi)^2}\, e^{i\vec{Q}_T\cdot\vec{b}}\, e^{E^{\rm PT}_{a\bar{a}}(b,Q,\mu)}\; e^{\,-\,\mu_I^2 b^2 (g_1\ \ln\left(\frac{Q}{\mu_I}\right)+g_2)}$$

$$\times \sum_{a=q\bar{q}} \int_{\xi_1\xi_2} \frac{d\hat{\sigma}_{a\bar{a}\to\mu^+\mu^-(Q)+X}(Q,\mu)}{dQ^2}\; f_{a/N}(\xi_1, 1/b)\, f_{\bar{a}/N}(\xi_2, 1/b)$$

$$= \pi \int d^2\mathbf{k}_T\; \frac{e^{-k_T^2/4[\mu_I^2(g_2\ln(Q/k_T)+g_2)]}}{\mu_I^2(g_2\ln(Q/k_T)+g_2)}\; \frac{d\sigma_{NN}(\mathbf{Q_T}-\mathbf{k_T})}{dQ^2\, d^2\vec{Q}_T}$$

Which gives curves like the one we saw before.



Successful phenomenology for W and Z.
In principle, can also fit to fixed-target Drell-Yan with the same set of NP parameters.

Qiu and Zhang show that NP corrections are dominant for fixed-target Q^2.

Next – what about those $1/(1-z)$ (soft gluon energy) singularities?

$*$ **Continue with threshold resummation . . .**

IV. Threshold resummation

- **Back to the one-loop DY hard-scattering**

$$\frac{d\hat{\sigma}^{(1)}_{q\bar{q}\to\gamma^* g}}{dQ^2\, d^2\mathbf{Q}_T} = \sigma_0 \frac{\alpha_s C_F}{\pi^2} \left(1 - \frac{4\mathbf{Q}_T^2}{(1-z)^2 \xi_1 \xi_2 S}\right)^{-1/2} \times \left[\frac{1}{\mathbf{Q}_T^2} \frac{1+z^2}{1-z} - \frac{2z}{(1-z)Q^2}\right]$$

- **Factorized cross section at fixed $\mathbf{Q}_T$:**

$$\frac{d\sigma_{NN\to\mu^+\mu^-+X}(Q,p_1,p_2)}{dQ^2 d^2\mathbf{Q}_T} = \int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \frac{d\hat{\sigma}_{a\bar{a}\to\mu^+\mu^-(Q)+X}(Q,\mu,\xi_1 p_1,\xi_2 p_2,\mathbf{Q}_T)}{dQ^2 d^2\mathbf{Q}_T} \times f_{a/N}(\xi_1,\mu)\, f_{\bar{a}/N}(\xi_2,\mu)$$

$z \to 1$ **is called "partonic threshold".**

- **Threshold resummation is resummation for the plus distributions.**

- **Same method as for Q_T, but now fix $k_0^{\rm soft} \sim \frac{1}{2}(1-z)Q$.**

  Laplace or Mellin transform $e^{-N2k_0/Q} \sim z^N$ and $\overline{\rm MS}$ collinear subtraction gives (here NLL accuracy shown) $\exp[\,E_a^{\rm thr}(N,Q)\,]$:

$$E_a^{\rm thr}(N,Q) = \int_{Q^2/N^2}^{Q^2} \frac{du^2}{u^2}\, 2A_a\left(\alpha_s(u)\right)\, \ln\frac{Nu}{Q}$$

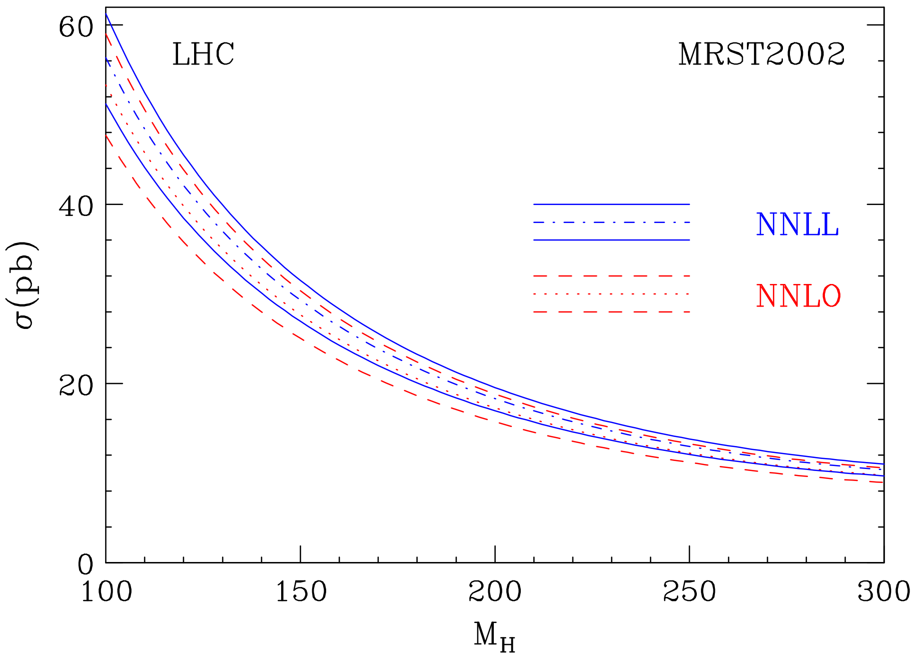
- **Inverse transform to the cross section:**

$$\frac{d\sigma_{NN}^{\rm res}}{dQ^2} = \sum_a \; \hat{\sigma}_a^{(0)}(Q,\mu) \int_{C_N} \frac{dN}{2\pi i} \left(\frac{Q^2}{S}\right)^{-N} \; \exp\left[\, E_a^{\rm thr}(N,Q,\mu) \,\right]$$
$$\times f_{a/N}(N,\mu)\, f_{\bar{a}/N}(N,\mu)$$

Formalism is similar for W, Z, H. "Electroweak annihilation"

Typical collider result . . .

- **Logs: threshold resummation vs. fixed order for H at LHC**



(from Catani, de Florian, Grazzini, Nason (2003))

- **Modest change & decrease in μ-dependence**
  $\longrightarrow$ **increased confidence. But see V.)**

IV. Resummation with Color Exchange

- **Resummed amplitudes in dimensional regularization**

  (Catani (1998) Tejeda-Yeomans & GS (2002) Kosower (2003)) Aybat, Dixon & GS (2006)

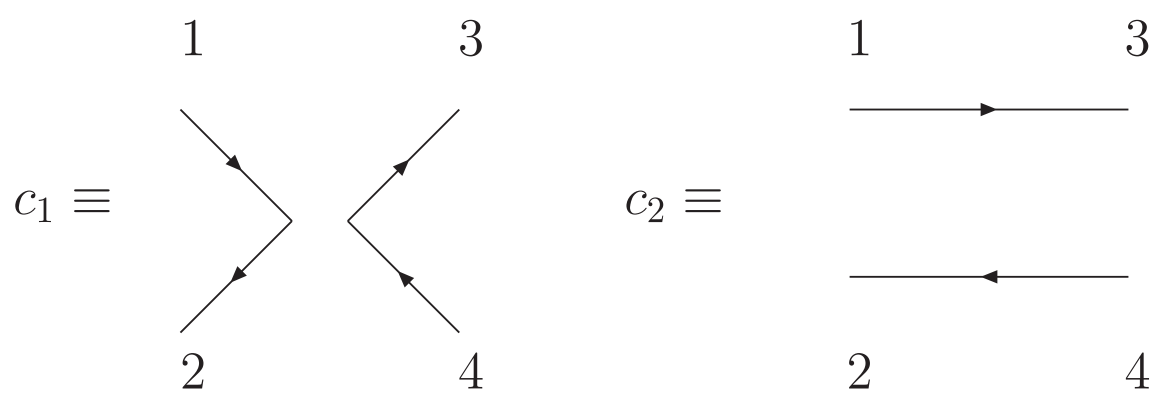
  – **Amplitude for partonic process**

$$\mathrm{f}: \quad f_A(p_A, r_A) + f_B(p_B, r_B) \to f_1(p_1, r_1) + f_2(p_2, r_2)$$

$$\mathcal{M}^{[\mathrm{f}]}_{\{r_i\}}\left(p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \mathcal{M}^{[\mathrm{f}]}_{L}\left(p_j, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) (c_L)_{\{r_i\}}$$

- **Need to control poles in ϵ for factorized calculations at fixed order and for resummation**

Example of $(c_L)_{\{r_i\}}$:



– **Jet/soft factorization** **(Sen (1983)):**

$$\mathcal{M}_L^{[\mathrm{f}]}\left(p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \prod_{i=A,B,1,2} J_i^{[\mathrm{virt}]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

$$\times \mathbf{S}_{LI}^{[\mathrm{f}]}\left(p_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) h_I^{[\mathrm{f}]}\left(\wp_i, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

- **Soft function labelled by color exchange (singlet, octet . . .)**

- **Factors require dimensional regularization**

- **Same factorization $\rightarrow$ resummation**

- **Poles at 2- and higher loops . . .**

- **Relation to supersymmetric Yang-Mills theories**

  Bern, Dixon, Kosower & Smirnov (2004) verified structure to 3 loops

– **Dimensionally-regularized jets**

(Magnea & GS (1990))

$$
\begin{aligned}
J_i\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right) = \exp\Bigg\{ \frac{1}{4}\int_0^{-Q^2}\frac{d\xi^2}{\xi^2}\Bigg[&\mathcal{K}^{[i]}(\alpha_s(\mu^2),\epsilon) \\
&+\mathcal{G}^{[i]}\left(-1,\bar{\alpha}_s\left(\frac{\mu^2}{\xi^2},\alpha_s(\mu^2),\epsilon,\right)\epsilon\right) \\
&+\frac{1}{2}\int_{\xi^2}^{\mu^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\gamma_K^{[i]}\left(\bar{\alpha}_s\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\Bigg]\Bigg\}.
\end{aligned}
$$

– γ_K, $\mathcal{K}$ **related to** A **above,** $\mathcal{G}+\mathcal{K}$ **to** B

– **Dimensionally-regularized S**

$$\mathbf{S}^{[\mathrm{f}]}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\epsilon\right)$$

$$= \mathrm{P}\ \exp\left[-\frac{1}{2}\int_0^{-Q^2}\frac{d\tilde{\mu}^2}{\tilde{\mu}^2}\mathbf{\Gamma}^{[\mathrm{f}]}\left(\bar{\alpha}_s\left(\frac{\mu^2}{\tilde{\mu}^2},\alpha_s(\mu^2),\epsilon\right)\right)\right]$$

$\Gamma^{[\mathrm{f}]}$: **anomalous dimension; color mixing**

- **Color Mixing** (Date (1983) Sen (1983) . . .

  Kidonakis & GS (1996) Bonciani et al. (1998,2003) Dokshitzer & Marchesini (2005)

  Aybat, Dixon, GS (2006))

  – Cross sections & amplitudes:
    NLL exponentiation in basis that diagonalizes Γ

$$\exp \int \frac{dm}{m} \left[\lambda^{(\mathrm{f})}(\alpha_s(m))\right]$$

  – [f] color exchange basis, λs: eigenvalues of Γ_S

– **Example: f:** $g + g \to g + g$

$$\mathrm{Tr}\,[\,T_{a_1}T_{a_2}T_{a_3}T_{a_4}\,]\ \text{and 5 perms}$$

$$[\,T_{a_1}T_{a_2}]\ \mathrm{Tr}\,[T_{a_3}T_{a_4}]\ \text{and 2 perms}$$

– **Color mixing governed by a 9×9 matrix**

- $g + g \to g + g$

(Kidonakis, Oderda, GS (1998), Bern, Dixon, Kosower (2000) Anastasiou, Glover, Oleari, Tejeda-Yeomans (2

$$\Gamma^{(1)}_{S'} = C_A \begin{pmatrix} T & 0 & 0 & 0 & 0 & 0 & -\frac{U}{N_c} & \frac{T-U}{N_c} & 0 \\ 0 & U & 0 & 0 & 0 & 0 & 0 & \frac{U-T}{N_c} & -\frac{T}{N_c} \\ 0 & 0 & T & 0 & 0 & 0 & -\frac{U}{N_c} & \frac{T-U}{N_c} & 0 \\ 0 & 0 & 0 & (T+U) & 0 & 0 & \frac{U}{N_c} & 0 & \frac{T}{N_c} \\ 0 & 0 & 0 & 0 & U & 0 & 0 & \frac{U-T}{N_c} & -\frac{T}{N_c} \\ 0 & 0 & 0 & 0 & 0 & (T+U) & \frac{U}{N_c} & 0 & \frac{T}{N_c} \\ \frac{T-U}{N_c} & 0 & \frac{T-U}{N_c} & \frac{T}{N_c} & 0 & \frac{T}{N_c} & 2T & 0 & 0 \\ -\frac{U}{N_c} & -\frac{T}{N_c} & -\frac{U}{N_c} & 0 & -\frac{T}{N_c} & 0 & 0 & 0 & 0 \\ 0 & \frac{U-T}{N_c} & 0 & \frac{U}{N_c} & \frac{U-T}{N_c} & \frac{U}{N_c} & 0 & 0 & 2U \end{pmatrix}$$

– **New result for all massless $2 \to n$ processes** (Aybat, Dixon, GS (2006))

$$\Gamma_S = \frac{\alpha_s}{\pi}\left(1 + \frac{\alpha_s}{\pi} K\right)\, \Gamma^{(1)}_{S'} \ + \ \cdots$$

$\Gamma^{(2)} = (K/2)\Gamma^{(1)}$ **with same K as in the DGLAP splitting.**

Related to the "CMW" or bremsstrahlung scheme.

(Catani, Marchesini & Webber (1990))

To NNLO, "single-web" exchange generalizes single gluon.

(C.F. Berger, 2002)

Another indication of simplification for IR gluons.

V. Generalizations and limitations

A) Factorization with no hard scattering: BFKL

(Sen (1980) Balitsky (1996) Kúcs (2003) . . . related to saturation, colored glass condensate)

– **Regge limit in PT for elastic scattering:** $p_A + p_B \rightarrow p'_A + p'_B$

$$M(t,s): \quad -(p'_A - p'_B)^2 - t \ll s = (P_A + p_B)^2$$

$$M(t,s) = \sum_{m,\ell} \int \left(\prod_{i=1}^{m-1} \mathrm{d}^{D-2} k_{i\perp} \right) \left(\prod_{j=1}^{\ell-1} \mathrm{d}^{D-2} p_{j\perp} \right)$$

$$\times \Gamma_A^{(m)\, a_1 \dots\, a_m}(p_A, q, n, k_{1\perp}, \dots, k_{m\perp})$$

$$\times S'^{\,(n,\ell)}_{a_1 \dots\, a_n,\, b_1 \dots\, b_e ll}(q, n; k_{1\perp}, \dots, k_{n\perp}; p_{1\perp}, \dots, p_{m\perp})$$

$$\times \Gamma_B^{(\ell)\, b_1 \dots\, b_m}(p_B, q, n; p_{1\perp}, \dots, p_{\ell\perp})$$

– **Factorization at fixed rapidity separation:**

Jets, $\Gamma_{A,B}$ & soft, S; no H. Introduce vector n^μ as above.
– **Evolution equations (in $\ln s \sim$ rapidity $\sim \ln(1/x)$) give**
– **generically m convolutions at $N^m LL$**

$$\left(p_A \cdot n \frac{\partial}{\partial p_A \cdot n} - 1 \right) \Gamma_A^{(\ell)\ a_1 \dots a_\ell}(p_A, q, n; k_{1\perp}, \dots, k_{\ell\perp}) =$$

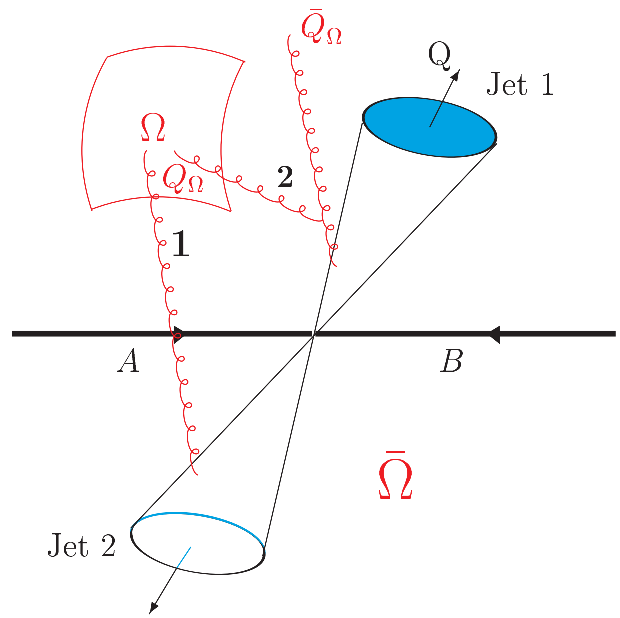
$$\sum_m \int \prod_{j=1}^m \mathrm{d}^{D-2} l_{j\perp}\, \mathcal{K}^{(\ell,m)}_{a_1 \dots a_n; b_1 \dots b_m}(k_{1\perp}, l_{1\perp}, \dots; q, n)$$

$$\times \Gamma_B^{(m)\ b_1 \dots b_m}(p_A, q, n; l_{1\perp} \dots)$$

– **Can project onto different color exchange:**
octet, $m = 0$ LL reggeized gluon
singlet, $m = 1$, BFKL LL pomeron ordered in rapidity, not k_T . . .

B) Non-global logs: color and energy flow

(Dasgupta & Salam (2001) . . .)



– **Simplest cases: 2 jets. Measure distribution** $\Sigma_\Omega(E)$

– **Very interesting case: energy flow between jets in** WW **fusion to** H.

– **Choices for Cross Section:**

– **a) Inclusive in $\bar{\Omega}$ $\longrightarrow$ Number of jets not fixed!**

– **b) Correlation with event shape τ_a . . . :**
fixes number of jets $\longrightarrow$ factorization

(Berger, Kúcs, GS (2003), Dokshitzer, Marchesini (2003), Banfi, Salam, Zanderighi (2004,5))

– **Contrast: for number of jets not fixed: nonlinear evolution! The approximate evolution equation for Σ :**

(Banfi, Marchesini, Smye (2002)) LL in E/Q, large-N_c) **Define:** $\partial_\Delta \equiv E\,(\partial/\partial E)$

$$\partial_\Delta \Sigma_{ab}(E) = -\partial_\Delta R_{ab}\, \Sigma_{ab}(E) + \int_{k \text{ not in } \Omega} dN_{ab\to k}\, \left(\Sigma_{ak}\Sigma_{kb} - \Sigma_{ab}\,\right)$$

$$dN_{ab\to k} \equiv \frac{d\Omega_k}{4\pi}\, \frac{\beta_a \cdot \beta_b}{\beta_k \cdot \beta_b\, \beta_k \cdot \beta_a} \quad (\text{“}\textit{dipole source}\text{”})$$

$$R_{ab} \equiv \int_E^Q \frac{dE'}{E'} \int_\Omega dN_{ab\to k}\,, \quad (\textit{suppression due to uncancelled virtual gluons})$$

– **Origin of the nonlinearity**

* ∂_E **can come from unobserved "hard" gluon** $G(k)$**.**

* **New hard gluon** $G(k)$ **acts as new, recoil-less source.**

* **Large-**N_c **limit:** $\bar{q}(a)G(k)q(b)$ **sources** $\longrightarrow \bar{q}(a)q(k) \oplus \bar{q}(k)q(a)$**.**

* **"Global" event shapes don't allow an extra hard gluon.**
(observed everywhere), but fixing an event shape may limit the number of events.

* **We are far from a full understanding.**
Forshaw, Kyrieleis, Seymour, 2006: "superleading" logs

C) Large threshold effects in observed hadrons

– Pions at fixed target and RHIC (Vogelsang and de Florian, 2004)

$$\frac{p_T^3\, d\sigma(x_T)}{dp_T} = \sum_{a,b,c} \int_0^1 dx_1\, f_{a/H_1}\left(x_1, \mu_F^2\right) \int_0^1 dx_2\, f_{b/H_2}\left(x_2, \mu_F^2\right)$$

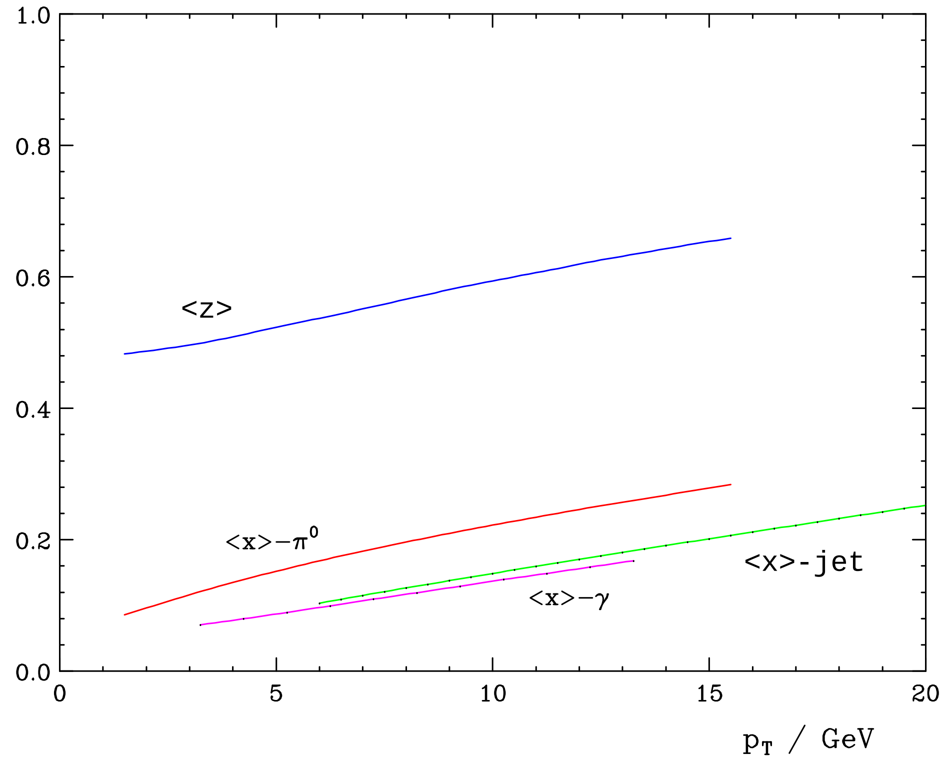
$$\times \int_0^1 dz\, z^2\, D_{h/c}\left(z, \mu_F^2\right)$$

$$\times \int_0^1 d\hat{x}_T\, \delta\left(\hat{x}_T - \frac{x_T}{z\sqrt{x_1 x_2}}\right) \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta}\, \frac{\hat{x}_T^4\, \hat{s}}{2} \frac{d\hat{\sigma}_{ab\to cX}(\hat{x}_T^2, \hat{\eta})}{d\hat{x}_T^2 d\hat{\eta}}$$

$\hat{\eta}$: **pseudorapidity at parton level**

$$\hat{\eta}_+ = -\hat{\eta}_- = \ln\left[(1 + \sqrt{1 - \hat{x}_T^2})/\hat{x}_T\right]$$

– **Averages for distribution x and fragmentation z's**



RHIC 200 GeV midrapidity average z for pions, and average x for pions, photons, jets at (NLO). Thanks to Werner Vogelsang.

– **Large z enhances threshold singularities.**

– **Singularities at one loop:**

$$\frac{\hat{s}\, d\hat{\sigma}^{(1)}_{ab\to cX}(v,w)}{dv\, dw} \approx \frac{\hat{s}\, d\hat{\tilde{\sigma}}^{(0)}_{ab\to cd}(v)}{dv} \left[A'\,\delta(1-w) + B' \left(\frac{\ln(1-w)}{1-w}\right)_+ + C' \left(\frac{1}{1-w}\right)_+ \right]$$

– **For resummation, take x_T^{2N} moments $\to$ factorization:**

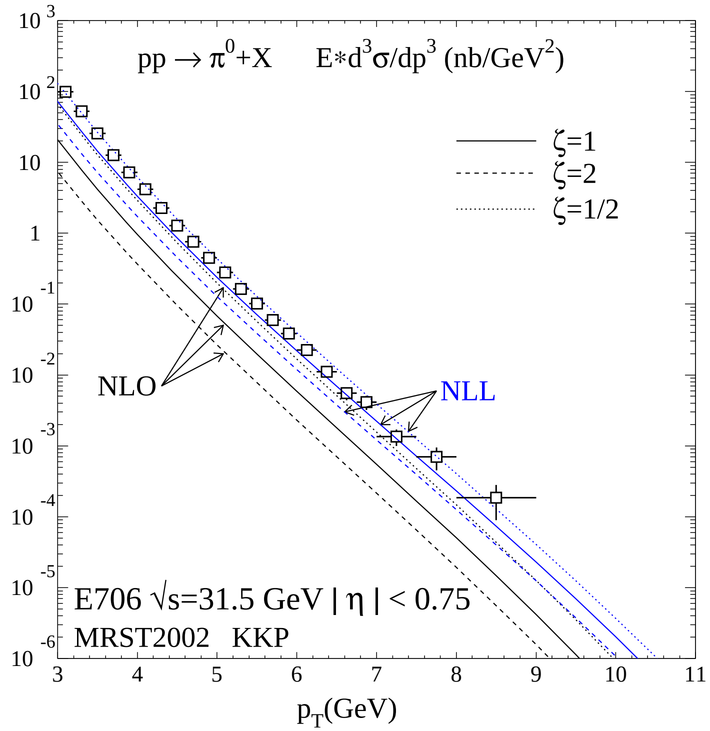
$$\hat{\sigma}^{(\mathrm{res})}_{ab\to cd}(N) = C_{ab\to cd}\, \Delta^a_N\, \Delta^b_N\, \Delta^c_N\, J^d_N \left[\sum_I G^I_{ab\to cd}\, \Delta^{(\mathrm{int})ab\to cd}_{I\,N} \right] \hat{\sigma}^{(\mathrm{Born})}_{ab\to cd}(N)$$

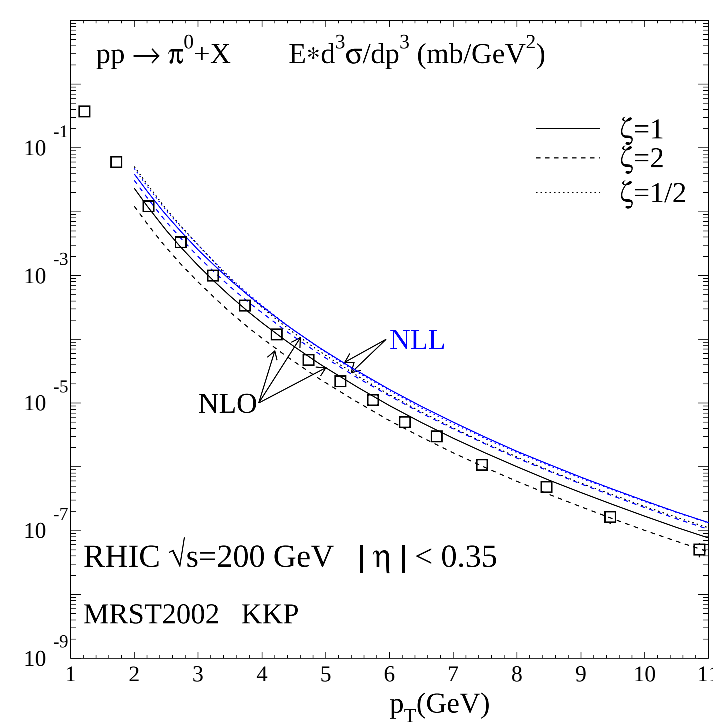
– **A typical NLL resummed factor:**

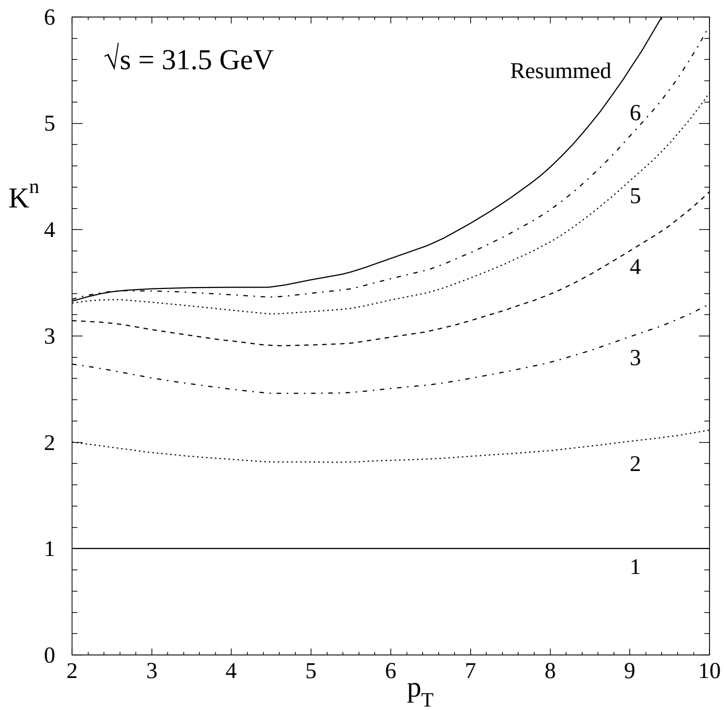
$$\Delta^a_N = \exp\left[\int_0^1 \frac{z^{N-1}-1}{1-z} \int_{\mu^2_{FI}}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} A_a(\alpha_s(q^2)) \right]$$

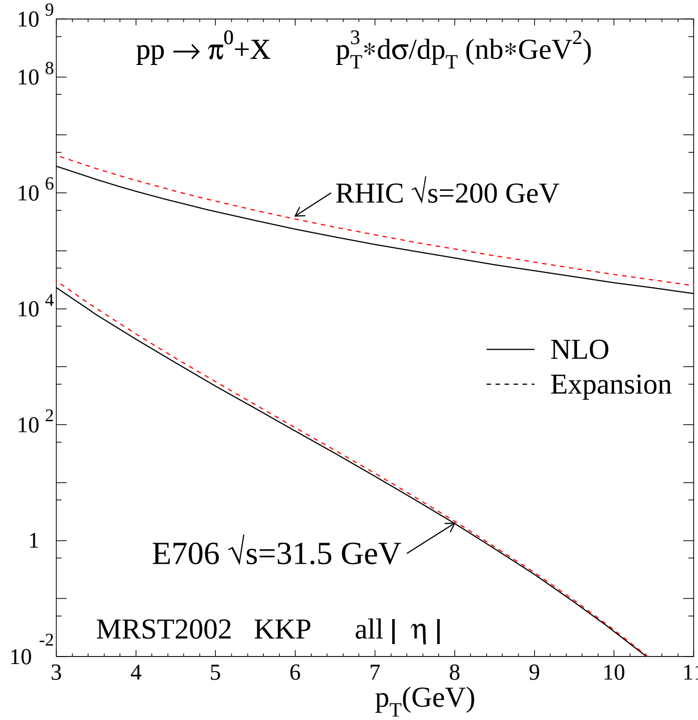
$$A = C_F(\alpha_s/\pi)(1 + K(\alpha_s/\pi)) \; + \; \ldots$$

– **Invert the moments: resolve a long-standing fixed-target vs. collider puzzle.**









- **Left: expansion of resummed cross section to fixed orders.**
- **Right: exact NLO vs. NLO expansion.**
- **Shows in π^0 1PI cross sections threshold resummation is more accurate and more important in fixed target range.**

Conclusions/Summary

- Resummation is absolutely necessary for many distributions (Q_T, event shape) just one step away from inclusive cross sections, because most events are found in regions with ordered scales ($Q_T \ll Q$, $m_{\rm jet} \ll E_{\rm jet}$).
- It induces the form of certain NP effects.
- Resummations reflect quantum incoherence.
- Resummation with color exchange displays surprising simplicity, at least at NNLL/pole. (Hint of duality?)
- The soft limit of multigluon exchange may be simpler than suggested by the growing perturbative coupling.
- Many puzzles remain, especially connected to energy flow for non-global cross sections.