Hadron Physics on the Lattice

J.W. Negele

Future Prospects in QCD at High Energy   BNL   July 21, 2006

Collaborators

MIT
- B. Bistrovic
- J. Bratt
- D. Dolgov
- O. Jahn
- A. Pochinsky
- D. Sigaev

JLab
- R. Edwards
- D. Richards

William & Mary, JLab
- K. Orginos

Arizona
- D. Renner

Yale
- G. Fleming

T. U. Munchen
- Ph. Haegler

DESY Zeuthen
- W. Schroers

U Cyprus
- C. Alexandrou
- G. Koutsou
- Ph. Leontiou

Athens
- A. Tsapalis

ETH, CERN
- Ph. de Forcrand

Julich
- Th. Lippert

Wuppertal
- K. Schilling

Outline

- Introduction
  - QCD
  - Lattice Field Theory
- Highlights from hadron structure
  - Deep inelastic scattering
  - Quark distributions
  - Form factors and generalized form factors
  - Transverse structure
  - Origin of nucleon spin
  - Baryon shapes
  - Exotics
- Insight into how QCD works
- Summary and future challenges

Introduction

- How do hadrons arise from QCD?

- Lagrangian constrained by Lorentz invariance, gauge invariance and renormalizability:

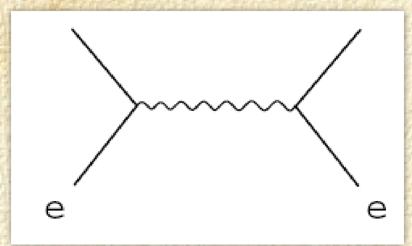
$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F^2_{\mu\nu}$$

$$D_\mu = \partial_\mu - igA_\mu \qquad F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$$

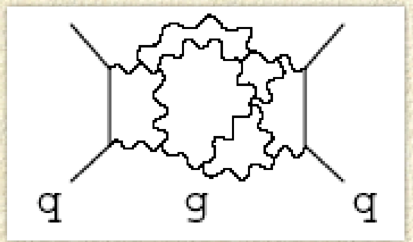
- Deceptively simple Lagrangian produces amazingly rich and complex structure of strongly interacting matter in our universe

Nonperturbative QCD

QED



QCD



- Fundamental differences relative to QED
  - Self-interacting: highly nonlinear
  - Interaction increases at large distance: Confinement
  - Interaction decreases at small distance: Asymptotic Freedom
  - Strong coupling: $\alpha_s \gg \alpha_{em}$
  - Topological excitations
- Solution of QCD
  - Present analytical techniques inadequate
  - Numerical evaluation of path integral on space-time lattice

Profound differences between hadrons and other many-body systems

- Atoms, molecules, nuclei,...
  - Constituents can be removed
  - Exchanged boson generating interactioin may be subsumed into static potential
    - photons → Coulomb potential
    - Mesons → N-N potential
  - Most of mass from fermion constituents
- Nucleons
  - Quarks are confined
  - Gluons are essential degrees of freedom
    - Carry half of momentum
    - Nonperturbative topological excitations
  - Most of mass generated by interactions

Goals

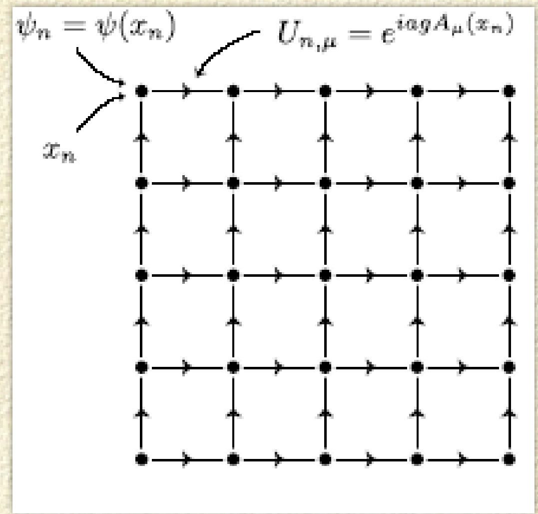
- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment
- Insight into how QCD works
  - Mechanisms
    - Paths that dominate action - instantons
    - Variational wave functions
  - Dependence on parameters
    - N_q, N_f, gauge group
    - m_q

Lattice QCD

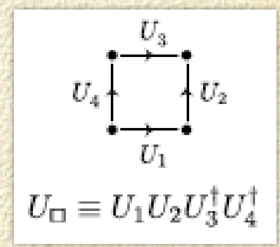
Euclidean time

$$e^{i\int dt d^3x \mathcal{L}} \to e^{-\int d\tau d^3x \mathcal{L}}$$

Discrete space-time lattice



Discrete Action



$$S(U) = \sum_\square \frac{2N}{g^2}(1 - N^{-1}\mathrm{ReTr}U_\square) \to \frac{1}{4}F_{\mu\nu}^2$$

$$\bar\psi M(U)\psi = \sum_n [\bar\psi_n \psi_n + \kappa(\bar\psi_n(1-\gamma_\mu)U_{n,\mu}\psi_{n+\mu} + \bar\psi_{n+\mu}(1+\gamma_\mu)U^\dagger_{n,\mu}\psi_n]$$

$$\to \bar\psi(\partial\!\!\!/ + m + ig A\!\!\!/)\psi$$

Lattice QCD

$$\langle T\, e^{-\beta H}\psi\psi\psi\cdots\bar\psi\bar\psi\bar\psi\rangle$$

$$= \frac{1}{Z}\int \mathcal{D}[\psi]\mathcal{D}[\bar\psi]\mathcal{D}[A]e^{-\int d^4x\left[\bar\psi(\partial\!\!\!/+m+igA\!\!\!/)\psi+\frac{1}{4}F_{\mu\nu}^2\right]}\psi\psi\psi\cdots\bar\psi\bar\psi\bar\psi$$

$$\to \prod_n \frac{1}{Z}\int d\psi_n\, d\bar\psi_n\, dU_n e^{-\sum\limits_n\left[\bar\psi M(U)\psi+S(U)\right]}\psi\psi\psi\cdots\bar\psi\bar\psi\bar\psi$$

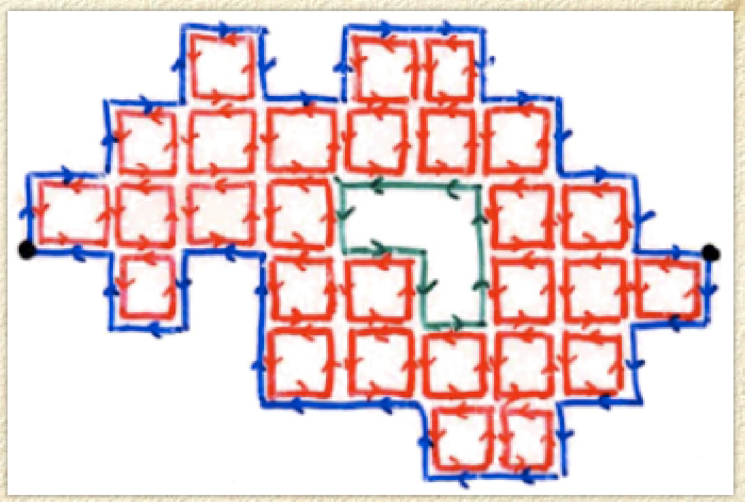
$$= \prod_n\int dU_n\,\underbrace{\frac{1}{Z}\det M(U)e^{-S(U)}}_{\text{Sample with M.C.}}\sum M^{-1}(U)M^{-1}(U)\cdots M^{-1}(U)$$

$$\to \frac{1}{N}\sum_{\substack{i=1\\ U_i\in\frac{\det M(U)}{Z}e^{-S(U)}}}^{N} M^{-1}(U_i)M^{-1}(U_i)M^{-1}(U_i)$$

Lattice QCD - summing over paths

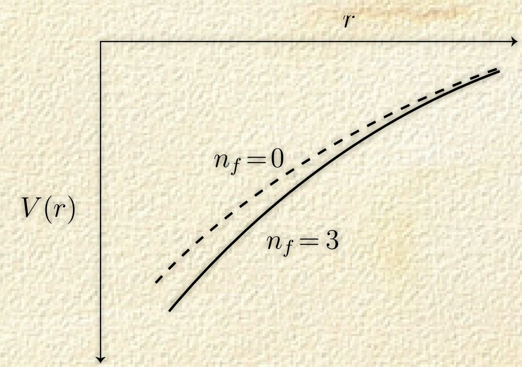
$$\langle T\, e^{-\beta H} \psi\psi\psi \cdots \bar{\psi}\bar{\psi}\bar{\psi} \rangle = \prod_n \int dU_n \frac{1}{Z} \det M(U) e^{-S(U)} \sum M^{-1}(U) M^{-1}(U) \cdots M^{-1}(U)$$

- $M^{-1} = (1 + \kappa U)^{-1}$ connects Ψ's with line of U's
  → Sum over valence quark paths
- det M generates closed loops of U's
  → Sum over sea quark excitations
- S(U) tiles with plaquettes
  → Sum over all gluons
- $32^3 \times 64$ lattices → 10^8 gluon variables



Computational Issues

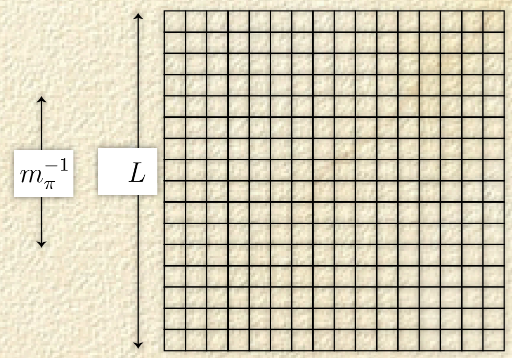
- Fermion determinant - Full QCD



- Small lattice spacing
- Small quark mass
- Large lattice volume

$$\frac{1}{m_\pi} \leq \frac{L}{4}$$

L(fm)	m_π (Mev)
1.6	500
4.0	200
5.7	140



- Cost ~ $(m_q)^{-2.5}(m_\pi)^{-4}$ ~ $(m_\pi)^{-9}$

Current Status

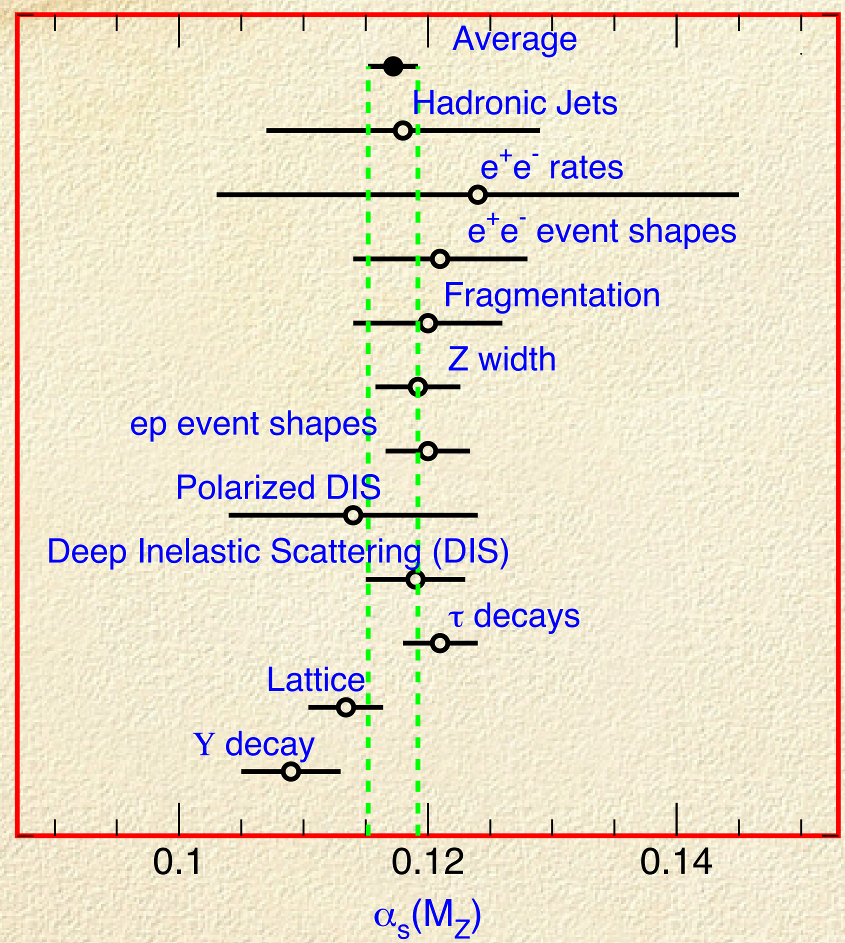
- Include fermion determinant - Full QCD
- Precision results in heavy quark systems
- $(m_\pi)^{-9}$ limited past nucleon structure to "heavy pion world" - $m_\pi \geq$ 500 MeV
- Beginning to explore physical "light pion world" role of chiral symmetry
- Terascale resources required for physical regime
  - DOE: ~8 sustained Teraflops

International Resource Estimates by S. Gottlieb

Final estimates consider multiple users for Julich, KEK, Munich, Edinburgh

Location	type	size	peak	est. perf.	total
Paris-Sud	apeNEXT	1 racks	0.8 TF	0.4 TF	0.4
Bielefeld	apeNEXT	6 (3) racks	4.9 TF	2.5 TF	
DESY (Zeuthen)	apeNEXT	3 racks	2.5 TF	1.2 TF	
Julich	BlueGene/L	8 racks	45.8 TF	11.5 TF $\times 1/2$?	10–15
Munich	SGI Tollhouse	3328 nodes	70 TF	14 TF?? $\times$?	
Rome	apeNEXT	12 (8) racks	9.8 TF	4.9 TF	5
KEK	BlueGene/L	10 racks	57.3 TF	14.3 TF	
Tsukuba	PACS-CS	2560 nodes	14.3 TF	3.3 TF	14–18
KEK	Hitachi		2.1 TF	1 TF ?	
Edinburgh	QCDOC	12 racks	9.8 TF	4.2 TF	4–5
Edinburgh	BlueGene/L	1 racks	5.7 TF	1.4 TF $\times$?	

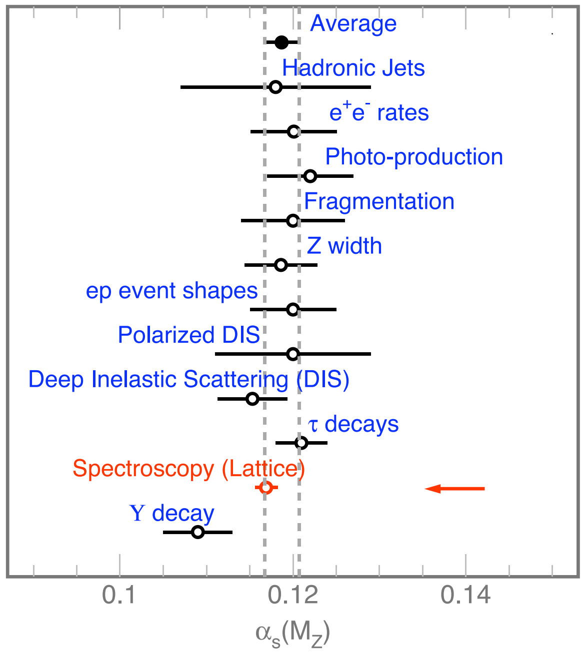
Precision agreement in heavy quark systems



$\alpha_s(M_Z)$ from Particle Data Group

Precision agreement in heavy quark systems

Context:

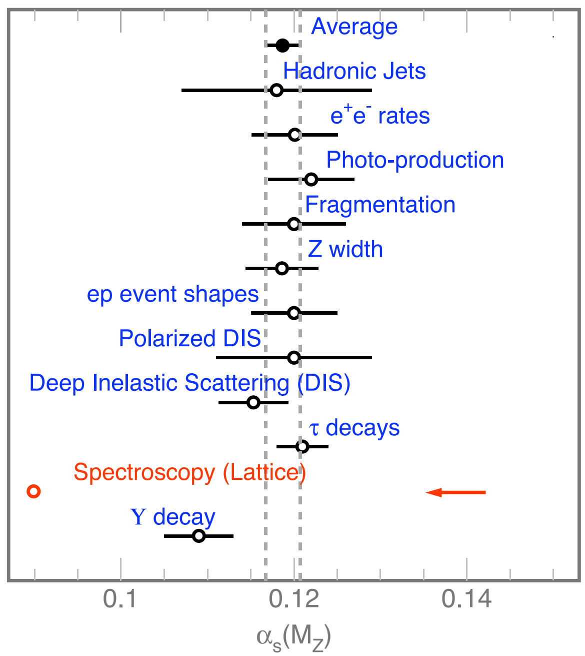


Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004)

$\alpha_s(M_Z)$ from Particle Data Group

Precision agreement in heavy quark systems

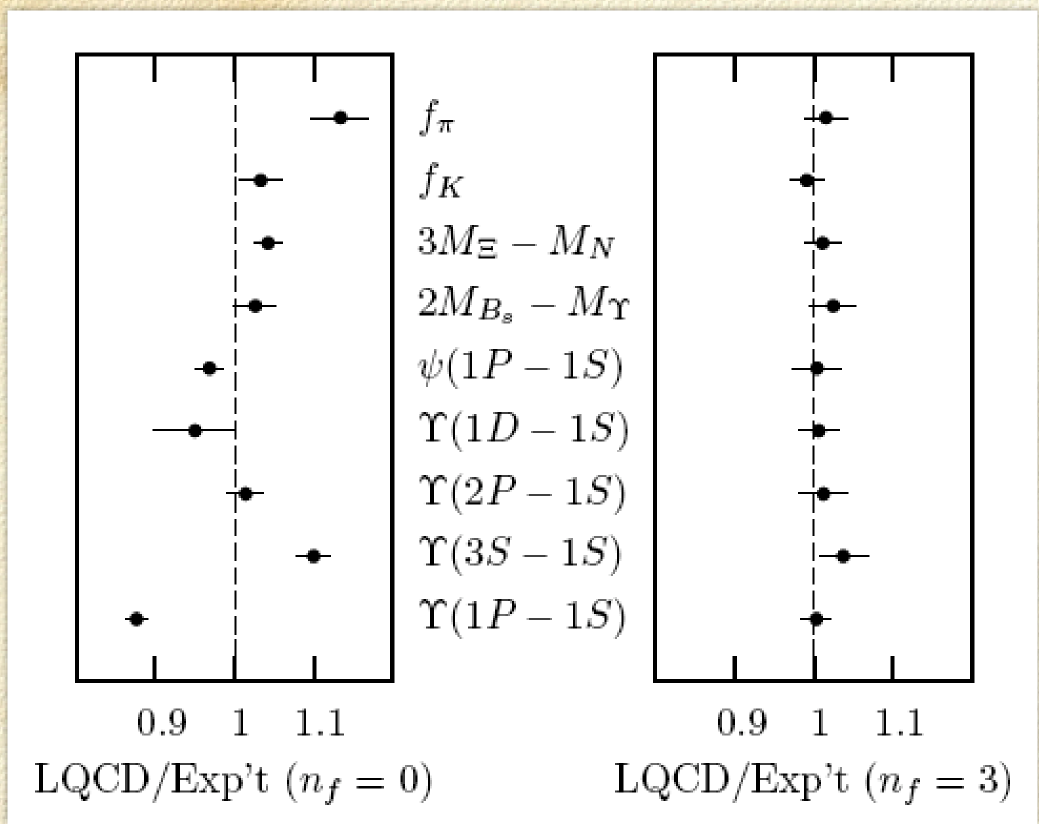
And without light-quark vacuum polarization:



Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004)

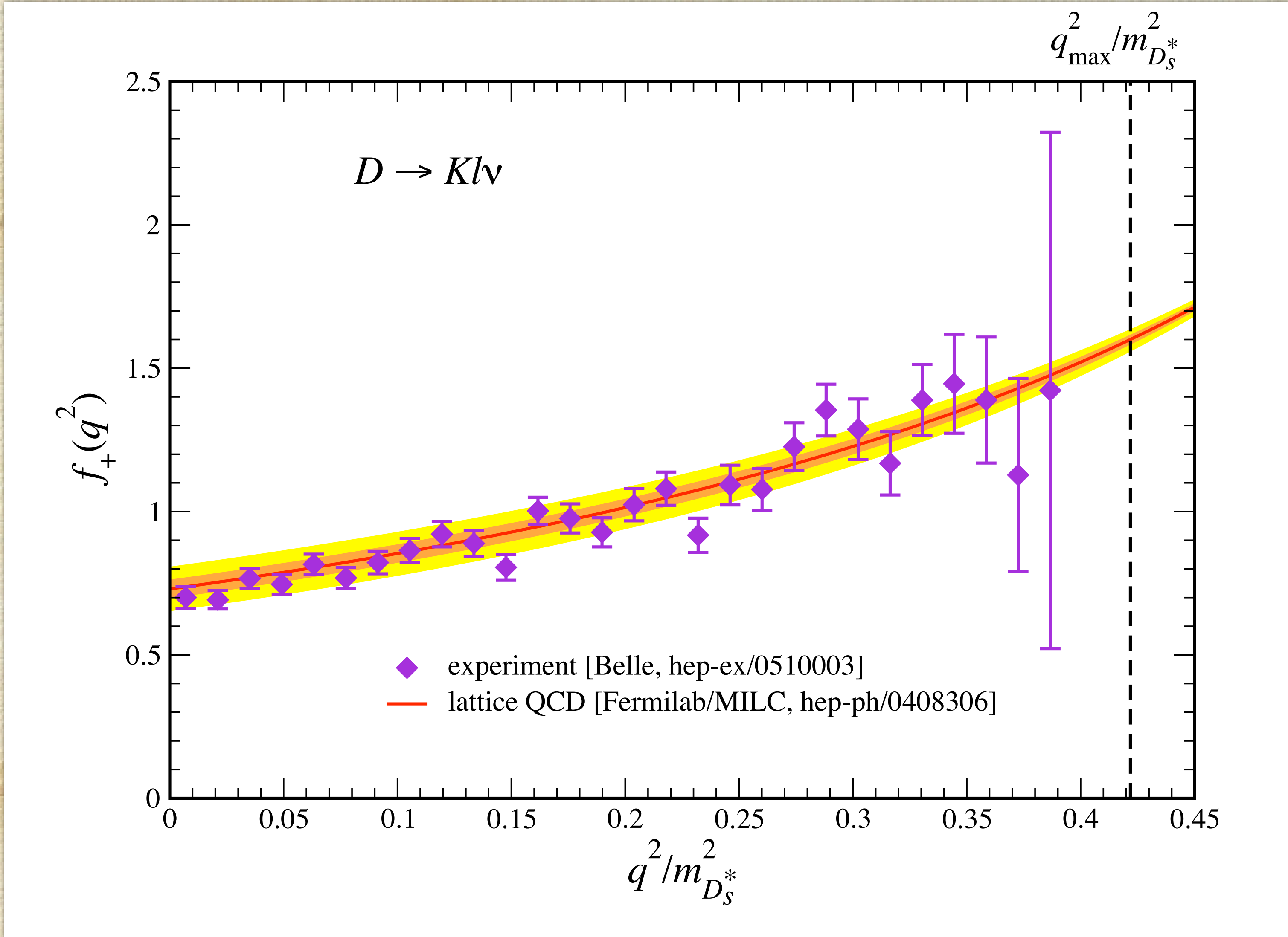
$\alpha_S(M_Z)$ from Particle Data Group

Precision agreement in heavy quark systems

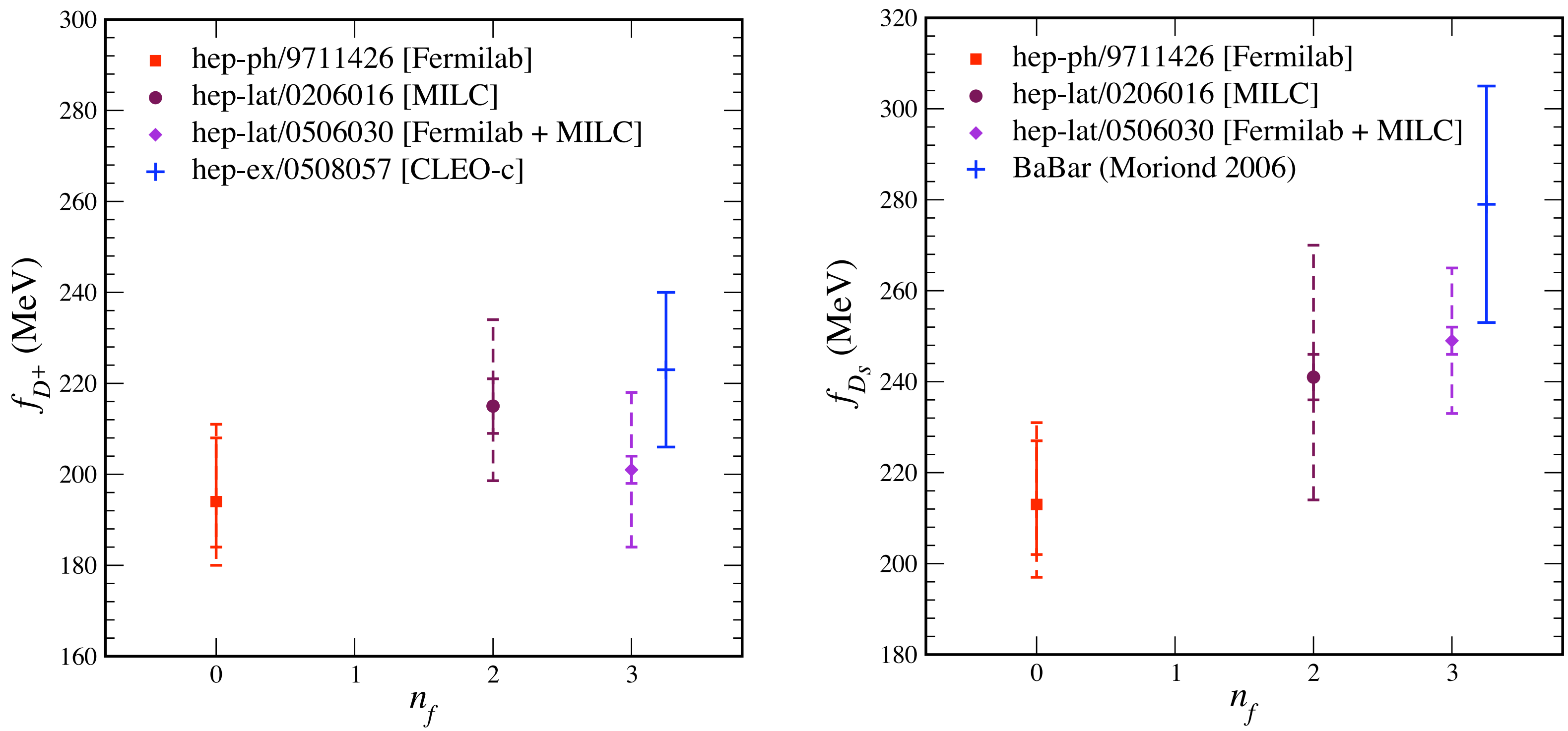


- "Gold Plated Observables" (Davies et. al. hep-lat/0304004)
  - Staggered quarks
  - Asqtad improved action
  - a = 0.13, 0.09 fm

Lattice QCD Predictions



Lattice QCD Predictions



$$\frac{\sqrt{m_D} f_D}{\sqrt{m_{D_s}} f_{D_s}} = 0.786 \pm 0.042 \quad \text{lattice}$$

$$= 0.779 \pm 0.093 \quad \text{CLEO/BaBar}$$

Hadron structure revealed by high energy scattering

- High energy scattering measures correlation functions along light cone
  - Asymptotic freedom: reaction theory perturbative
  - Unambiguous measurement of operators in light cone frame
  - Must think about physics on light cone
- Parton distribution q(x) gives longitudinal momentum distribution of light-cone wave function
- Generalized parton distribution $q(x, r_\perp)$ gives transverse

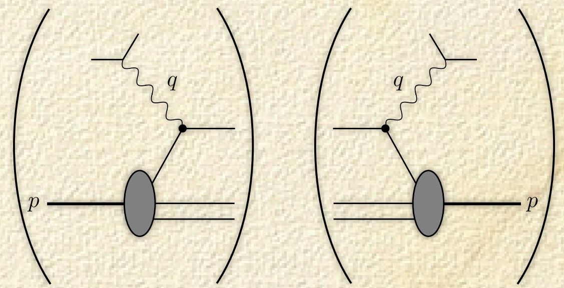
Parton and generalized parton distributions

High energy scattering: light-cone correlation function $(\lambda = p^+ x^-)$

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

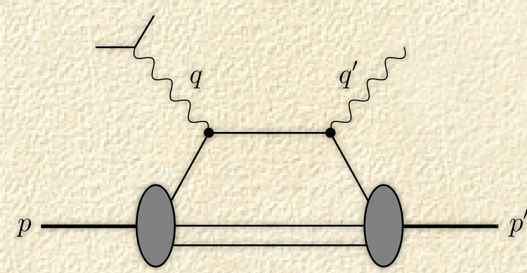
Deep inelastic scattering: diagonal matrix element

$$\langle P|\mathcal{O}(x)|P\rangle = q(x)$$



Deeply virtual Compton scattering: off-diagonal matrix element

$$\langle P'|\mathcal{O}(x)|P\rangle = \langle\gamma\rangle H(x,\xi,t) + \tfrac{i\Delta}{2m}\langle\sigma\rangle E(x,\xi,t)$$
$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n\cdot\Delta/2$$



Moments of parton distributions

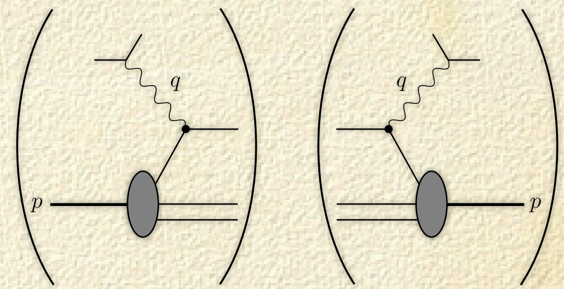
Expansion of $\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$

Generates tower of twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

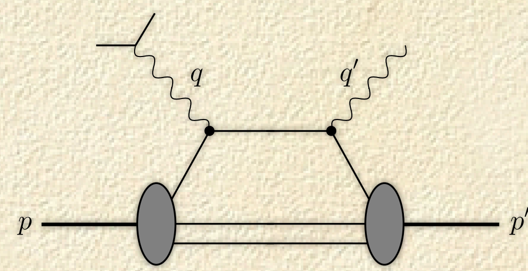
Diagonal matrix element

$$\langle P|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \sim \int dx\, x^{n-1} q(x)$$

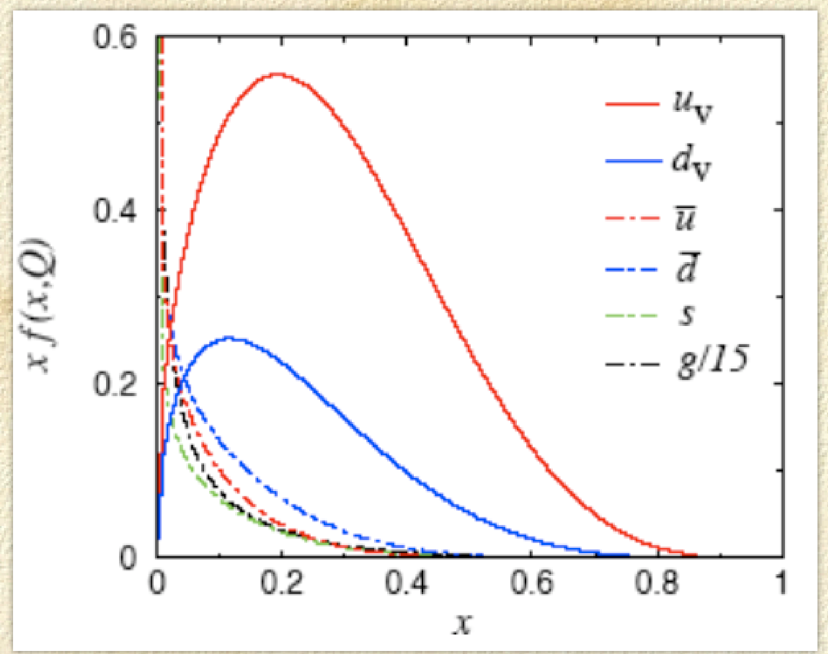


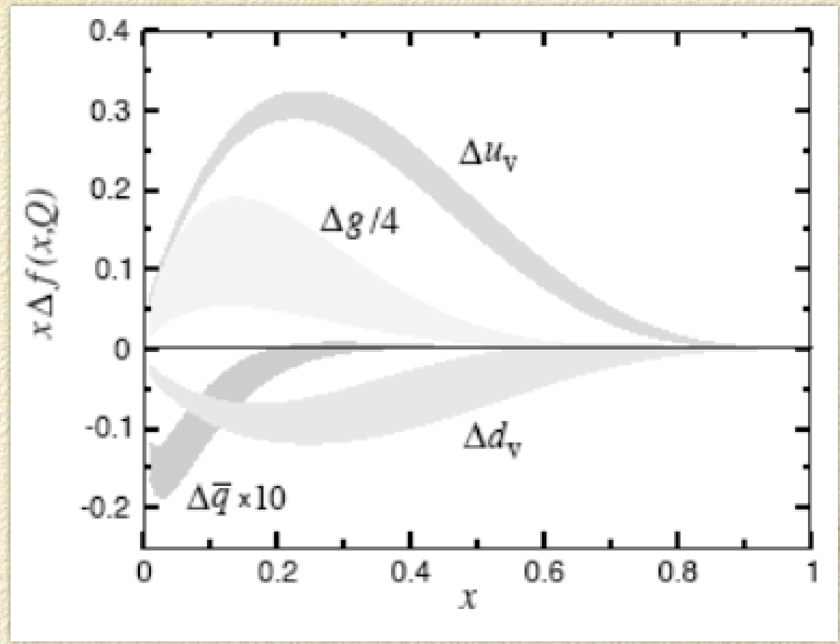
Off-diagonal matrix element

$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle$$
$$\sim \int dx\, x^{n-1} [H(x,\xi,t), E(x,\xi,t)]$$
$$\rightarrow A_{ni}(t), B_{ni}(t), C_n(t)$$



Moments of parton distributions





$$\langle p|\bar{\psi}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_q = \int_0^1 dx x^n[q(x)+(-1)^{(n+1)}\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx x^n[\Delta q(x)+(-1)^{(n)}\Delta\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\sigma_{\mu\nu} D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx x^n[\delta q(x)+(-1)^{(n+1)}\delta\bar{q}(x)]$$

where $\quad q = q_\uparrow + q_\downarrow, \quad \Delta q = q_\uparrow - q_\downarrow, \quad \delta q = q_\top + q_\perp,$

Lattice operators: irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

$\langle x\rangle_q^{(a)}$	6_3^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$
$\langle x\rangle_q^{(b)}$	3_1^+	$\bar\psi\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar\psi\gamma_i\overleftrightarrow{D}_i\psi$
$\langle x^2\rangle_q$	8_1^-	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi - \frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$
$\langle x^3\rangle_q$	2_1^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi + \bar\psi\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi - \{3\leftrightarrow 4\}$
$\langle 1\rangle_{\Delta q}$	4_4^+	$\bar\psi\gamma^5\gamma_3\psi$
$\langle x\rangle_{\Delta q}^{(a)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$
$\langle x\rangle_{\Delta q}^{(b)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$
$\langle x^2\rangle_{\Delta q}$	4_2^+	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$
$\langle 1\rangle_{\delta q}$	6_1^+	$\bar\psi\gamma^5\sigma_{34}\psi$
$\langle x\rangle_{\delta q}$	8_1^-	$\bar\psi\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$
d_1	6_1^+	$\bar\psi\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$
d_2	8_1^-	$\bar\psi\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$

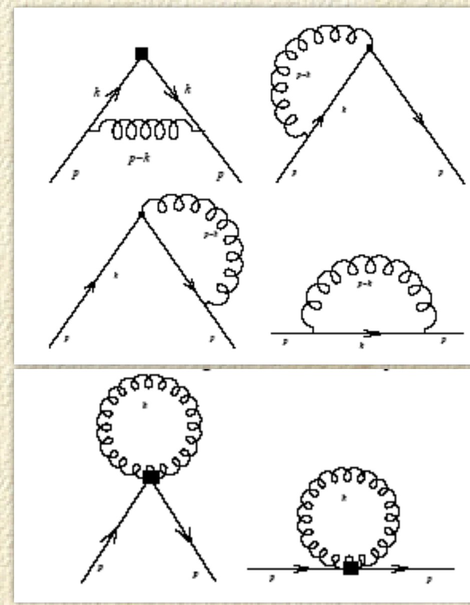
Full QCD Calculations

Collaboration	m_π (MeV)	Gluon action	Quark action
LHPC / SESAM	> 650	Wilson	Wilson
QCDSF / UKQCD	> 550	Wilson	Clover improved Wilson
RBCK	> 500	DBW2	Domain wall
LHPC / MILC	> 350	Asqtad	Staggered sea HYP Domain wall valence

Perturbative renormalization

HYP smeared domain wall fermions - B. Bistrovic

operator	$H(4)$	NOS	HYP	APE
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.792	0.981	1.046
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.847	0.976	0.994
$\bar{q}[\gamma_5]\sigma_{\mu\nu} q$	$6_1^{\mp}$	0.883	0.992	0.993
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^{\pm}$	0.991	0.979	0.954
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^{\pm}$	0.982	0.975	0.951
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$8_1^{\mp}$	1.134	0.988	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	mixing	5.71×10^{-3}	1.88×10^{-3}	8.21×10^{-4}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$4_2^{\mp}$	1.124	0.987	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$	$2_1^{\pm}$	1.244	0.993	0.919
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^{\pm}$	1.011	0.994	0.964
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^{\mp}$	0.979	0.982	0.989
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$	$8_1^{\pm}$	0.955	0.959	0.965



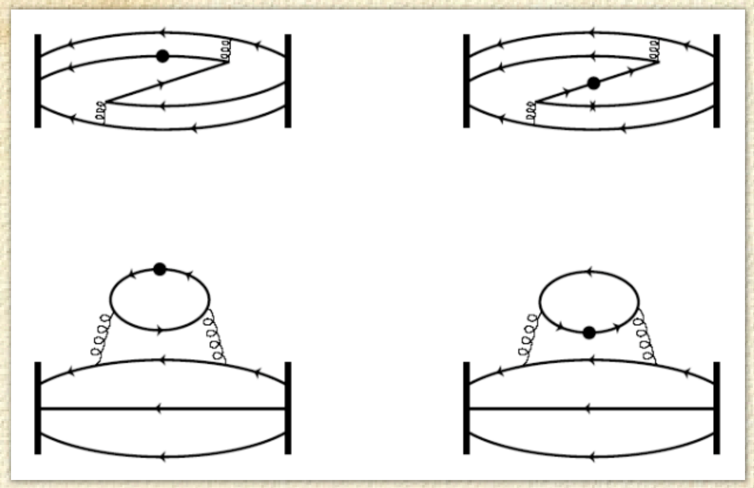
$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2}\frac{N_c^2-1}{2N_c}\left(\gamma_{ij}^{\overline{MS}}\log(Q^2a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}})\right)\right)\cdot O_j^{LATT}(a^2)$$

Numerical calculations

- Heavy quark regime (SESAM configurations)
  - $N_F = 2$, Wilson quarks
  - $a = 0.092$ fm, $16^3 \times 32$ lattice, $L = 1.5$ fm
  - m_π = 744, 831, 897 MeV
- Chiral quark regime (MILC configurations)
  - $N_F = 3$, staggered sea, chiral valence quarks
  - $a = 0.13$ fm, $20^3 \times 32$ ($20^3 \times 32$) lattices, $L = 2.6$ (3.5) fm
  - m_π = 359, 498, 605, 696, 775 MeV
- Collaborators:

B. Bistrovic	K. Orginos	C. Alexandrou
D. Dolgov	A. Pochinsky	Ph. de Forcrand
R. Edwards	D. Renner	Th. Lippert
G. Fleming	D. Richards	K. Schilling
Ph. Haegler	W. Schroers	A. Tsapalis

Hadron matrix elements on the lattice



- Measure $\langle \mathcal{O} \rangle$ for m_q, a, L
- Connected diagrams
- Disconnected diagrams (cancel for $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$)
- Extrapolate $m_q : m_\pi \rightarrow 140$ MeV
  $a \rightarrow \sim 0.05$ fm
  $L \rightarrow \sim 5$ fm

Nucleon axial charge in full lattice QCD

- Why g_A?
- Matrix element of axial current $A_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{\vec{\tau}}{2} q$

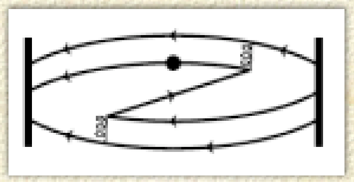
$$\langle N(p+q)|A_\mu|N(p)\rangle = \bar{u}(p+q)\frac{\vec{\tau}}{2}\left[g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\right]u(p)$$

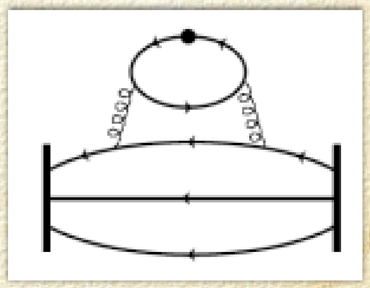
$$g_A(0) = 1.2695 \pm 0.0029$$

- Adler Weisberger $g_A^2 - 1 \sim \int(\sigma_{\pi^+p} - \sigma_{\pi^-p})$
- Goldberger Treiman $g_A \rightarrow f_\pi g_{\pi NN}/M_N$
- Spin content $\langle 1\rangle_{\Delta q} = \int_0^1 dx[\Delta q(x) + \Delta\bar{q}(x)]$

$$g_A = \langle 1\rangle_{\Delta u} - \langle 1\rangle_{\Delta d} \qquad \Sigma = \langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d} + \langle 1\rangle_{\Delta s}$$

Nucleon axial charge

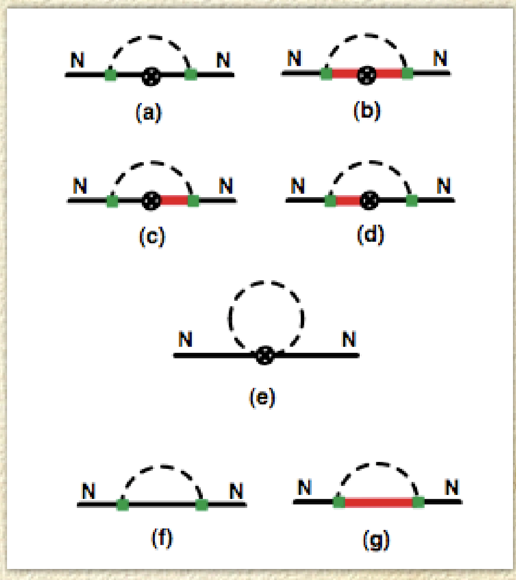




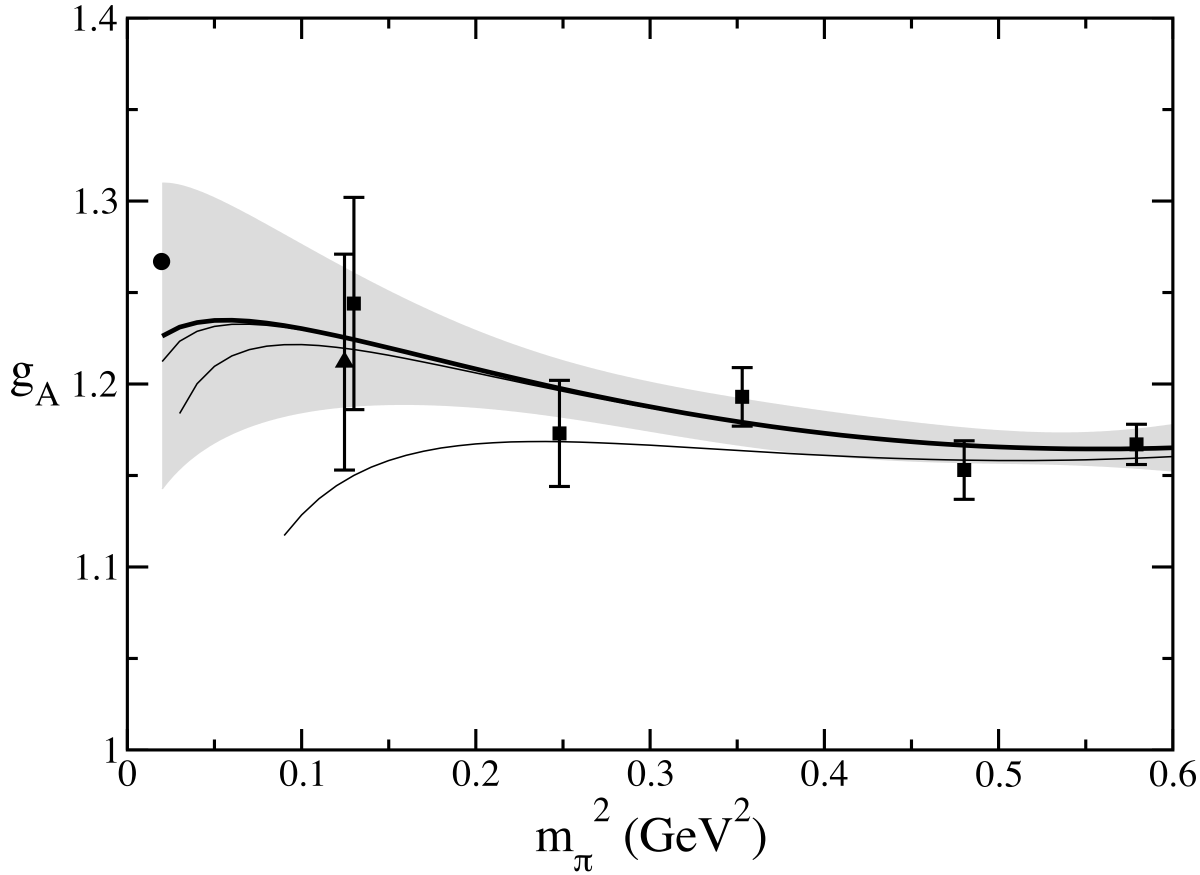
- Gold-Plated observable
  - Accurately measured
  - No disconnected diagrams
  - Chiral perturbation theory for $g_A(m_\pi^2, V)$
  - Renormalization - 5-d conserved current

Nucleon Axial Charge

- Chiral perturbation theory $g_A(m_\pi^2, V)$
  - Beane and Savage hep-ph/0404131
  - Detmold and Lin hep-lat/0501007
- 1-loop theory has 6 parameters
  - Fix $f_\pi,\, m_\Delta - m_N,\, g_{\Delta N}$ (0.3% error)
  - Fit $g_A,\, g_{\Delta\Delta},\, C$
  - Result $g_A(m_\pi = 140) = 1.212 \pm 0.084$

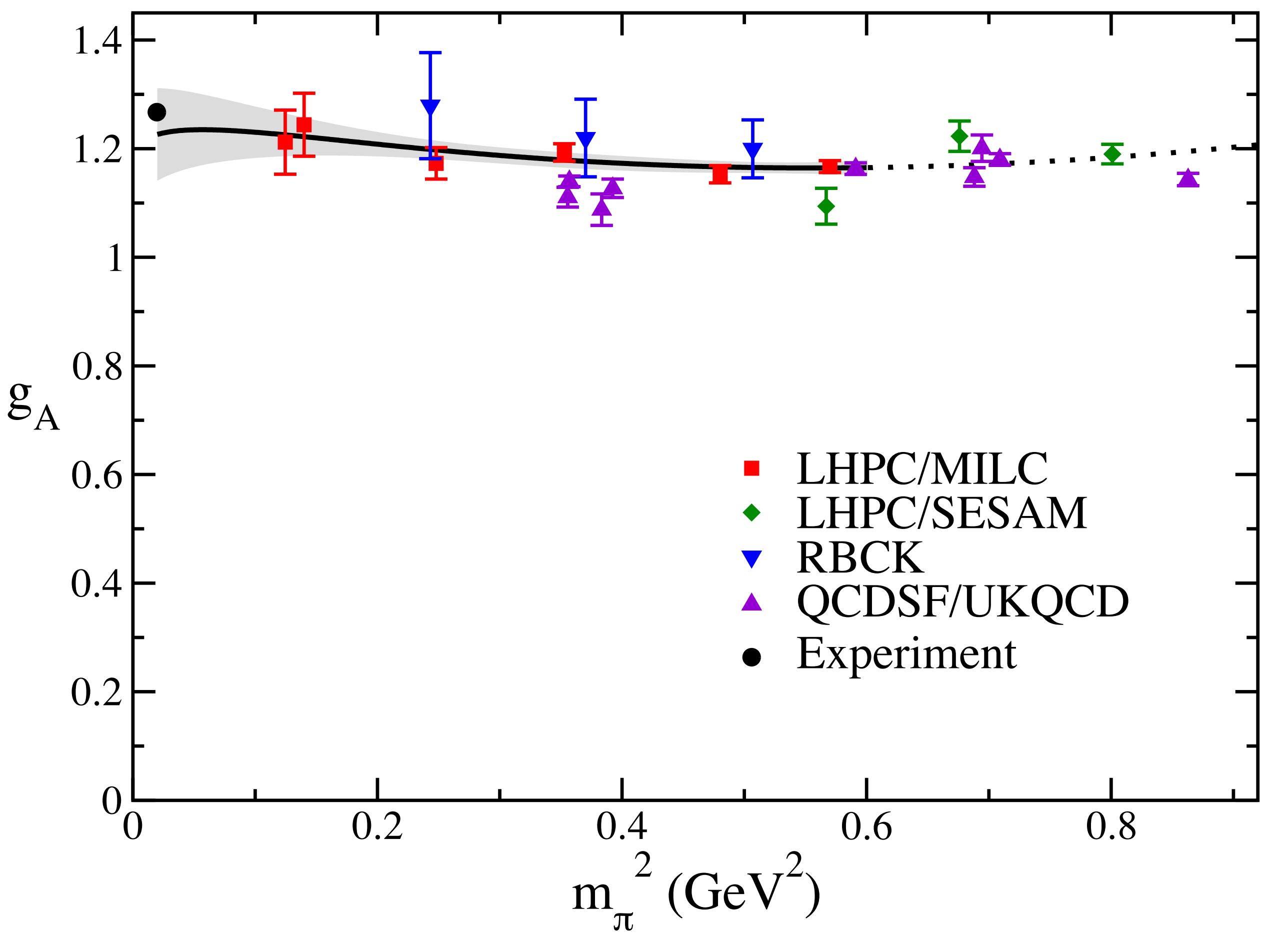


Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



Fit: Beane and Savage hep-ph/0404131

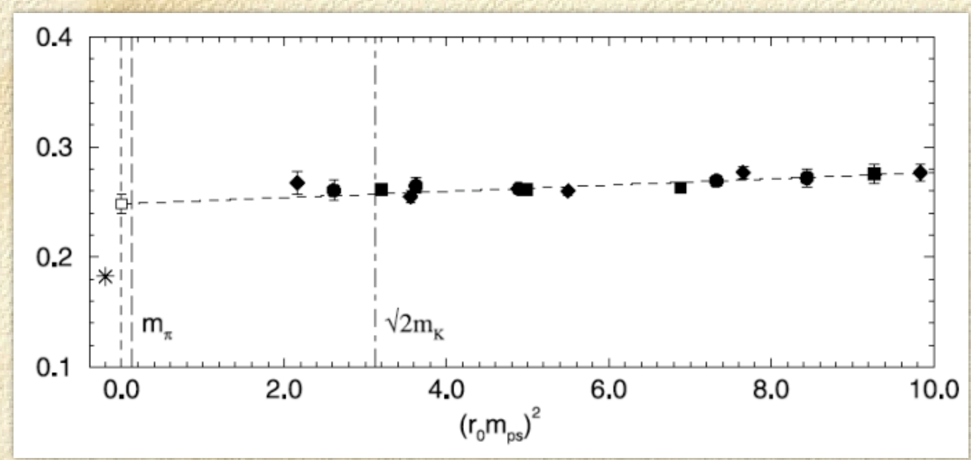
Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$

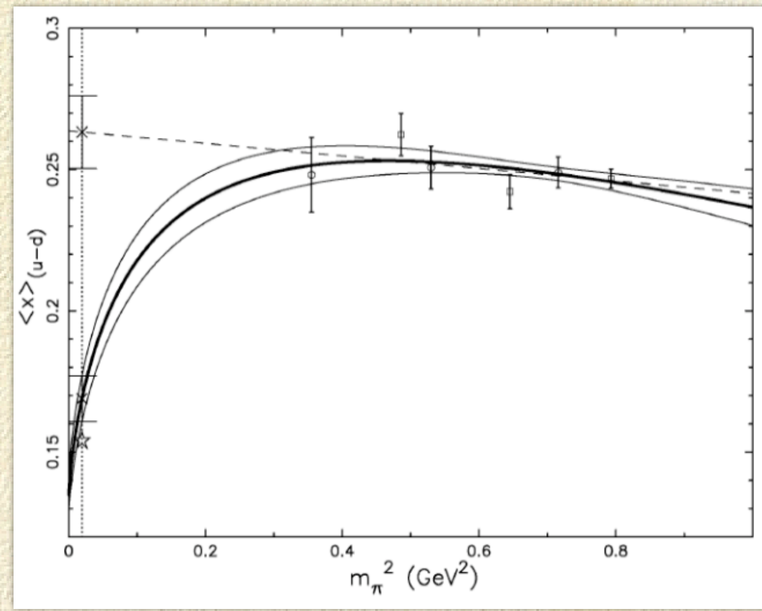


Chiral extrapolation of momentum fraction

Full QCD

Quenched hep-lat/0209160



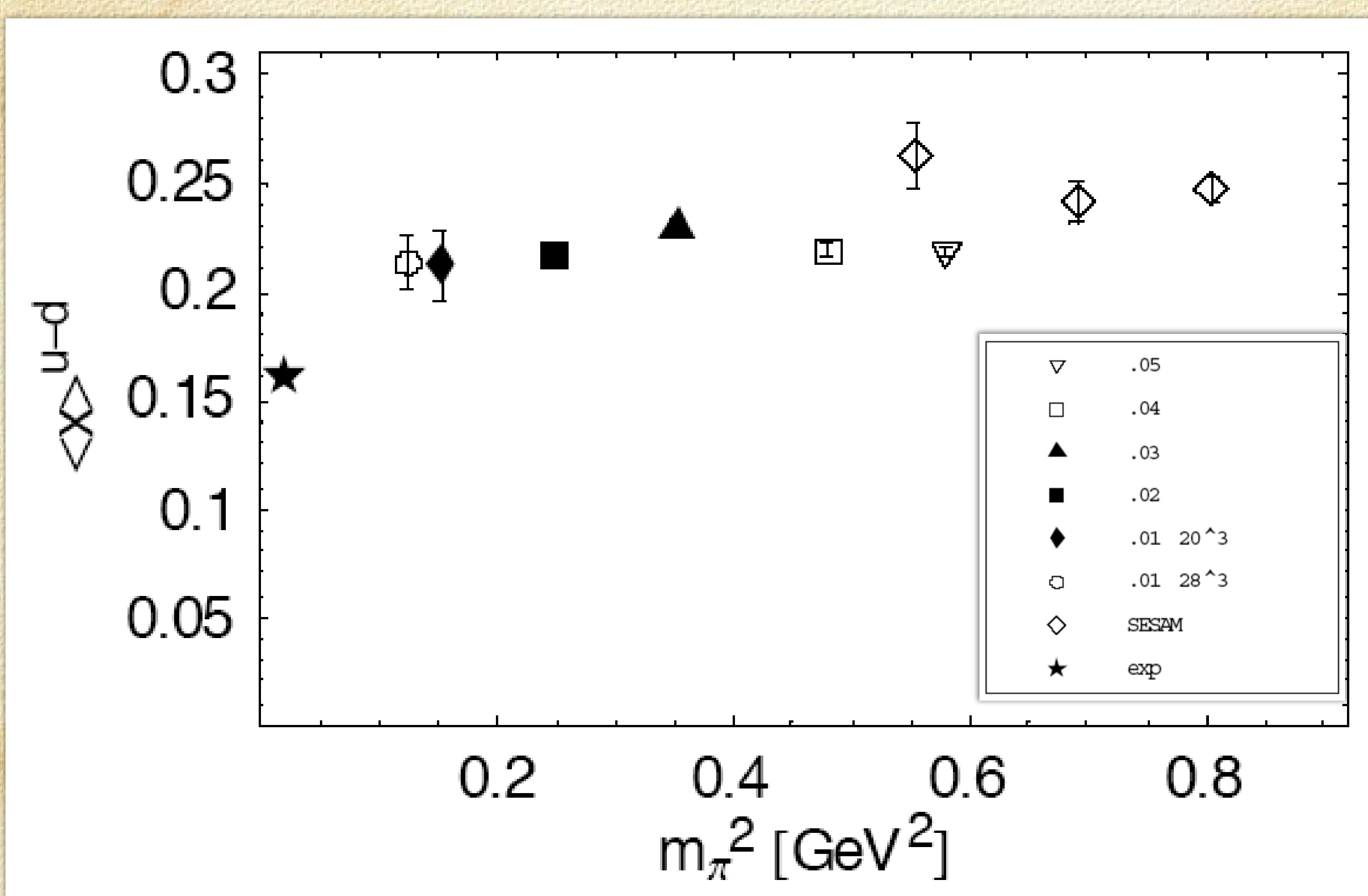


Physical chiral extrapolation formula hep-lat/0103006

$$\langle x^n \rangle_u - \langle x^n \rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2 + 1) m_\pi^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \right] + b_n m_\pi^2$$

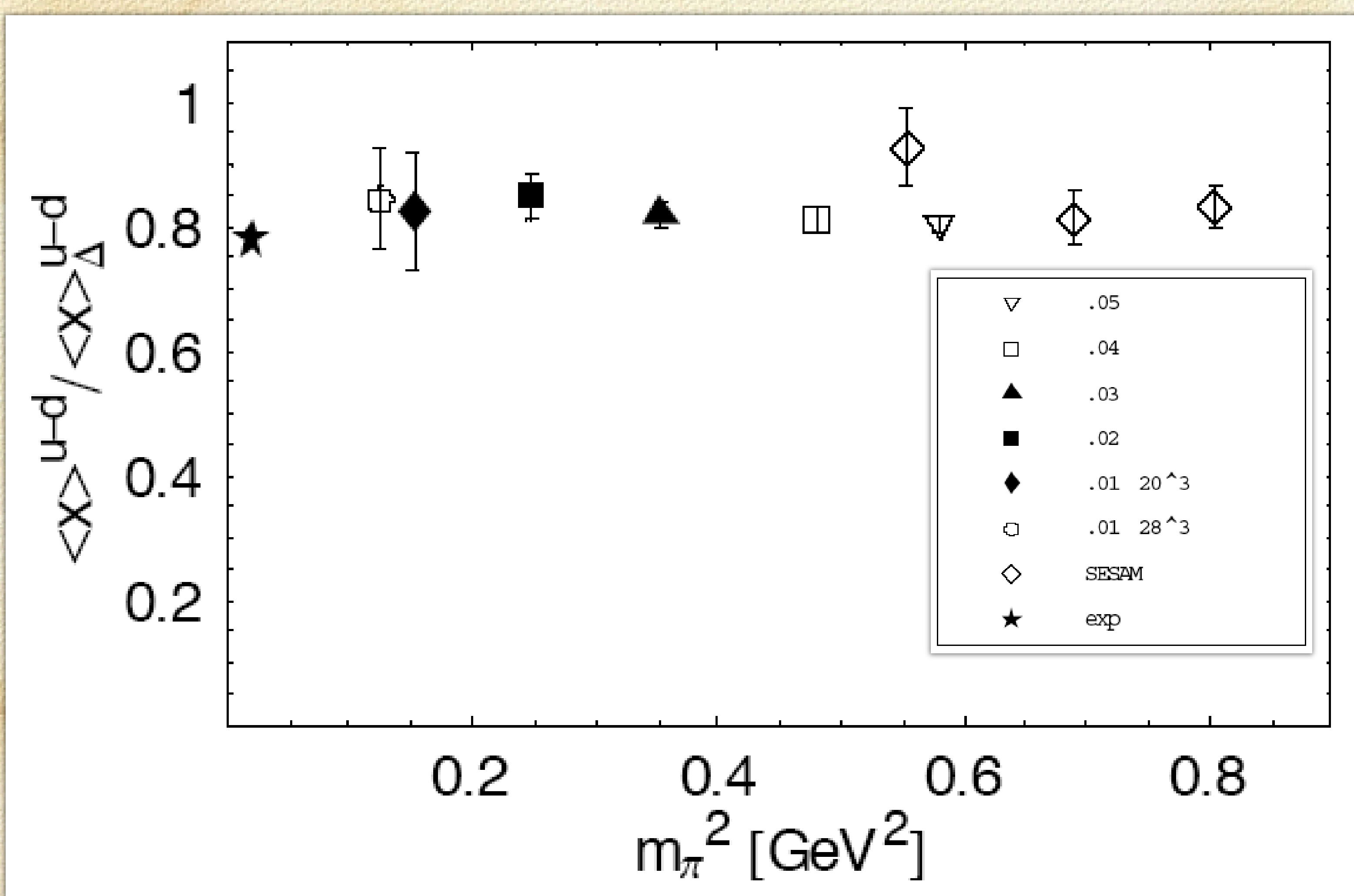
Quark momentum fraction $\langle x \rangle_q^{u-d}$

LHPC/MILC LHPC/SESAM

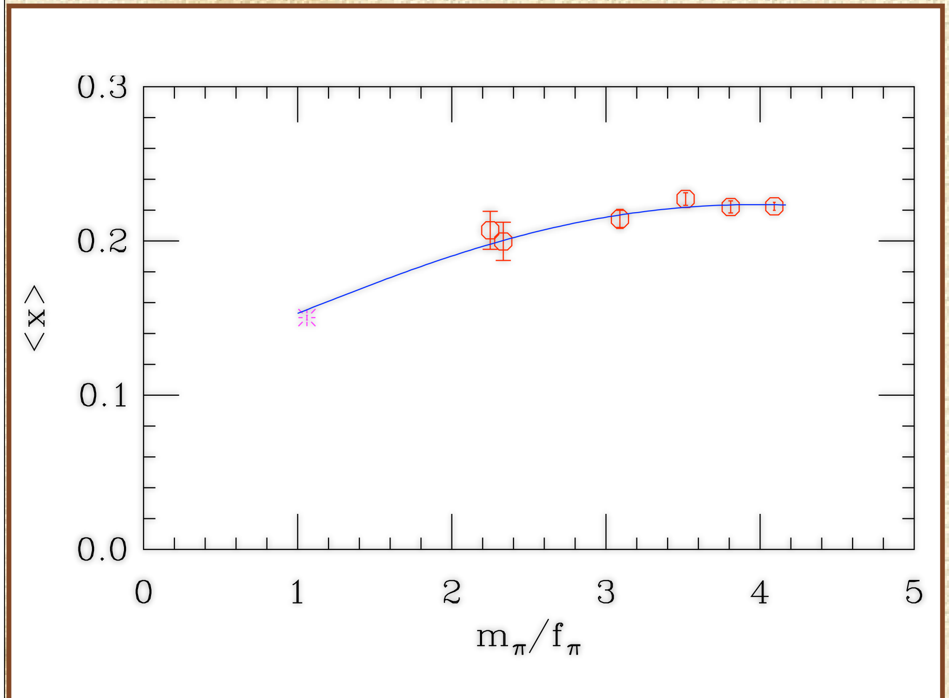


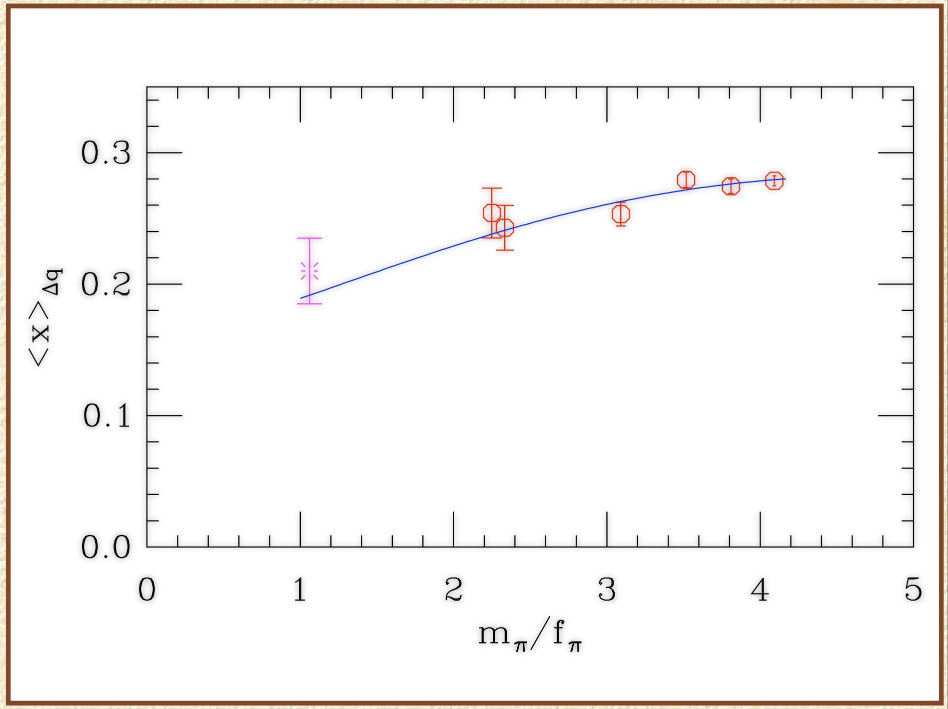
Quark momentum fraction ratio $\langle x \rangle_q^{u-d} / \langle x \rangle_{\Delta q}^{u-d}$

LHPC/MILC LHPC/SESAM



Chiral Extrapolations of $\langle x \rangle_q^{u-d}$ and $\langle x \rangle_{\Delta q}^{u-d}$





Electromagnetic form factors

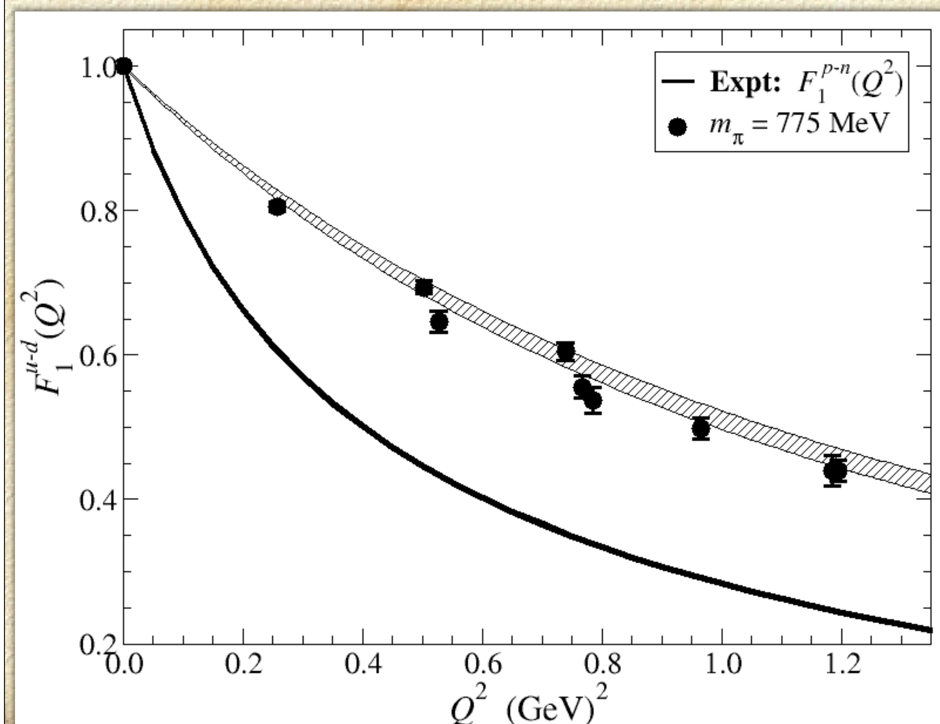
- Simplest off-diagonal matrix element

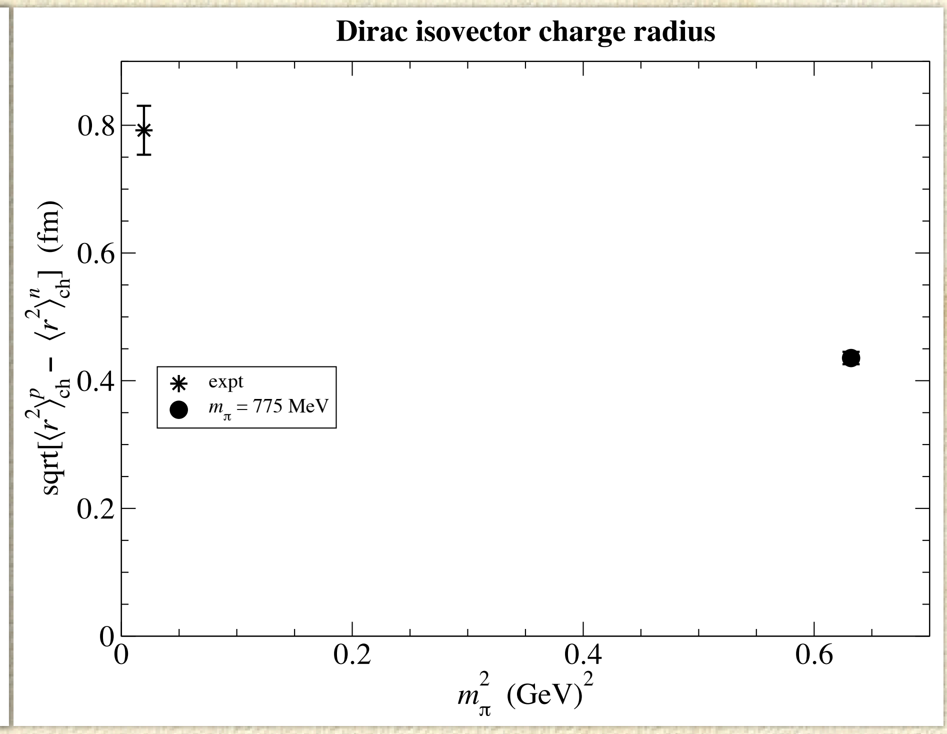
$$\langle p|\bar{\psi}\gamma^{\mu}\psi|p'\rangle = \bar{u}(p)[F_1(q^2)\gamma^{\mu} + F_2(q^2)\frac{i\sigma^{\mu\nu}q_{\nu}}{2m}]u(p')$$

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

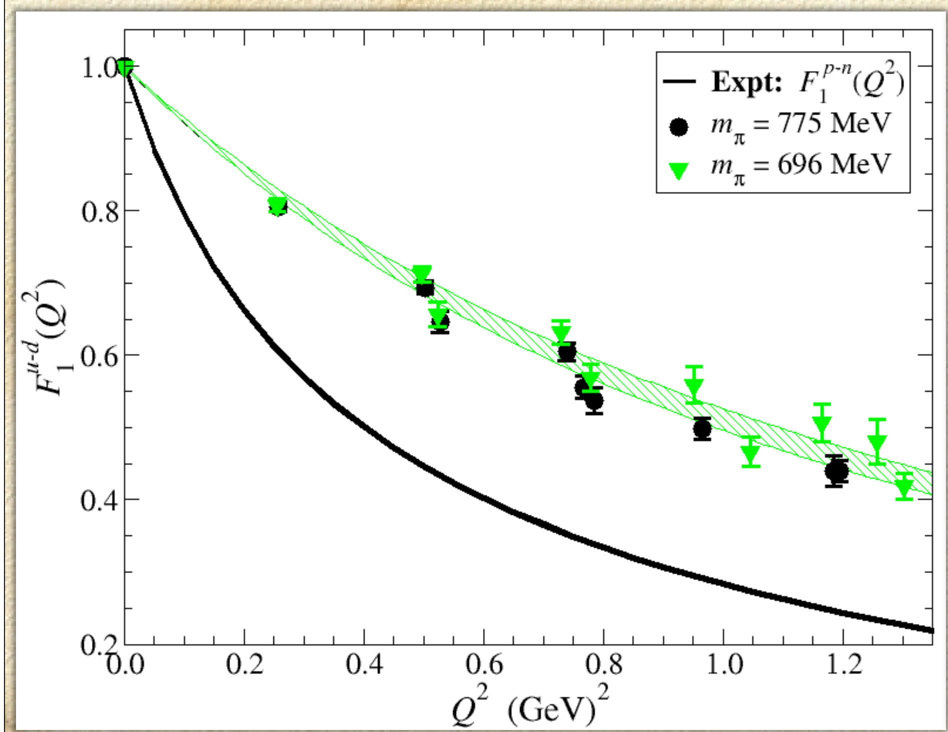
- Fourier transform of charge density if $\mathrm{L_{system}} \gg \mathrm{L_{wavepacket}} \gg \frac{1}{m}$
  - Pb: 5 fm >> 10^{-5} fm, Proton: 0.8 fm ~ 0.2 fm: marginal
  - For transverse Fourier transform of light cone w. f., m → p_+ ~ ∞
- Large q^2: ability of one quark to share q^2 with other constituents to remain in ground state - q^2 counting rules

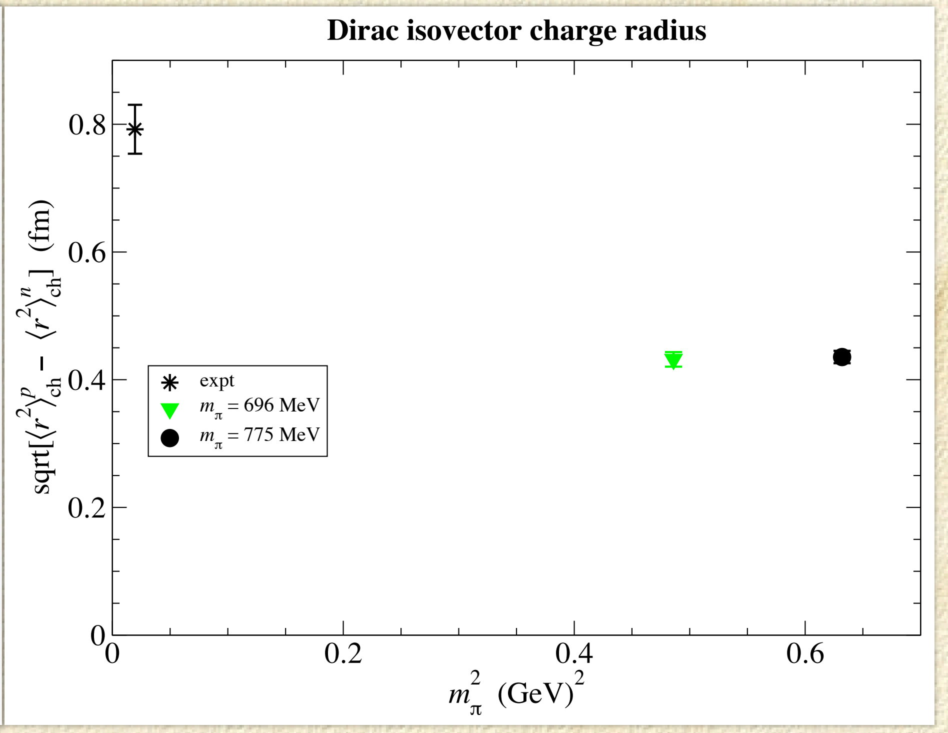
F_1 Isovector Form Factor



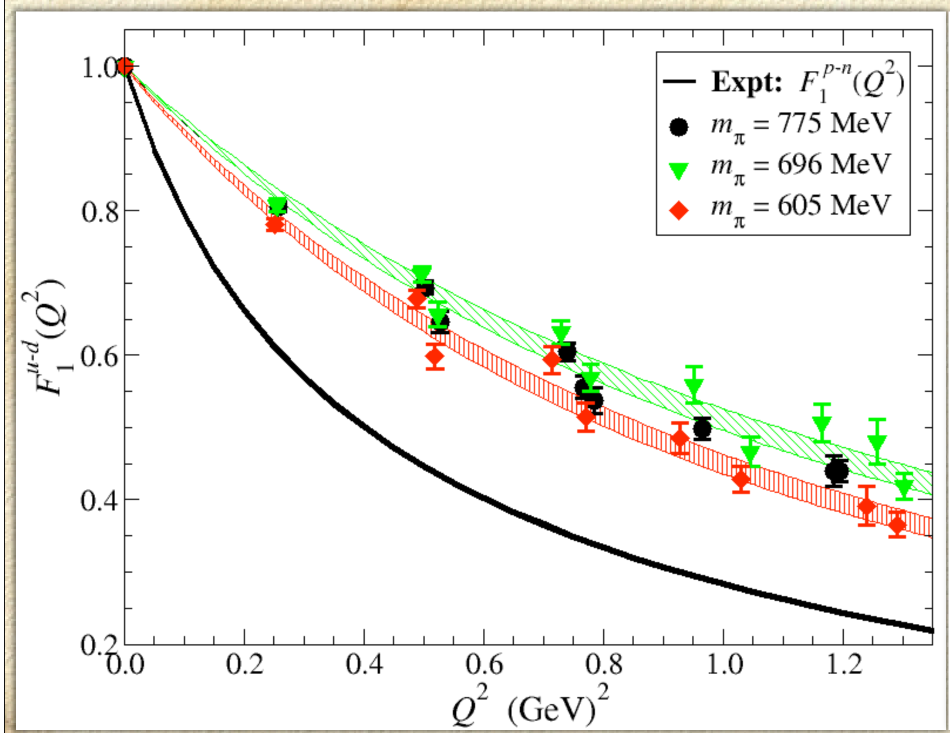


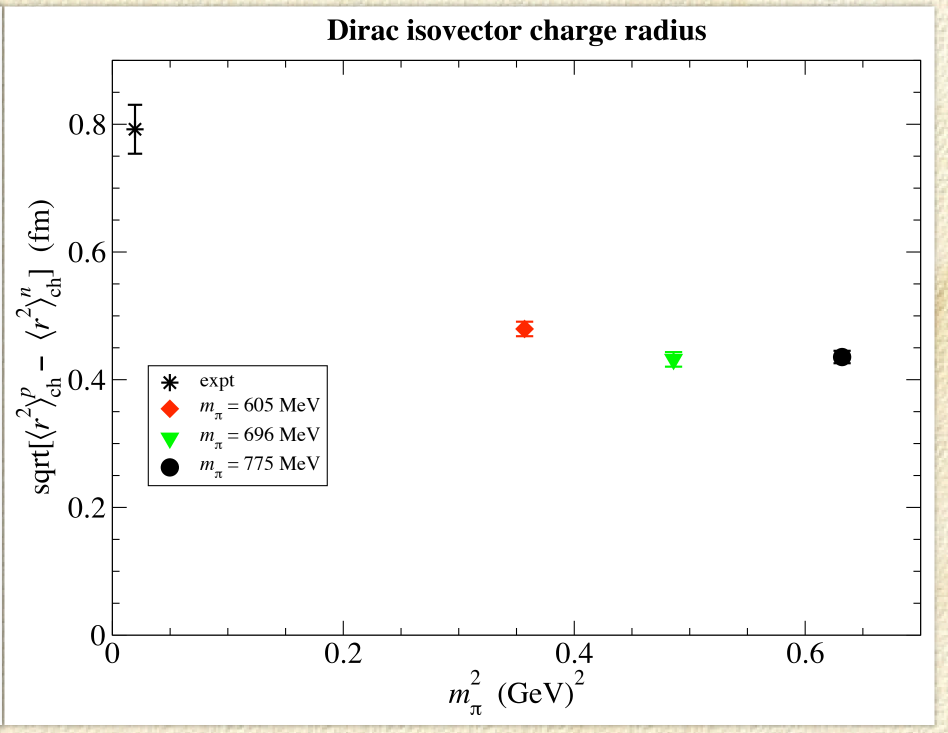
F_1 Isovector Form Factor



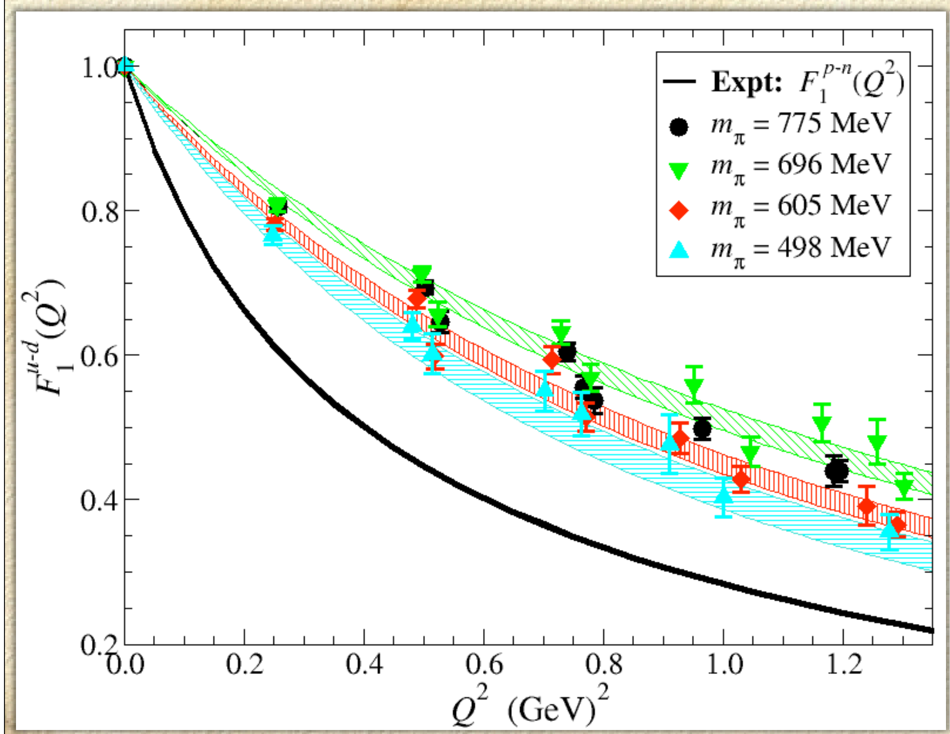


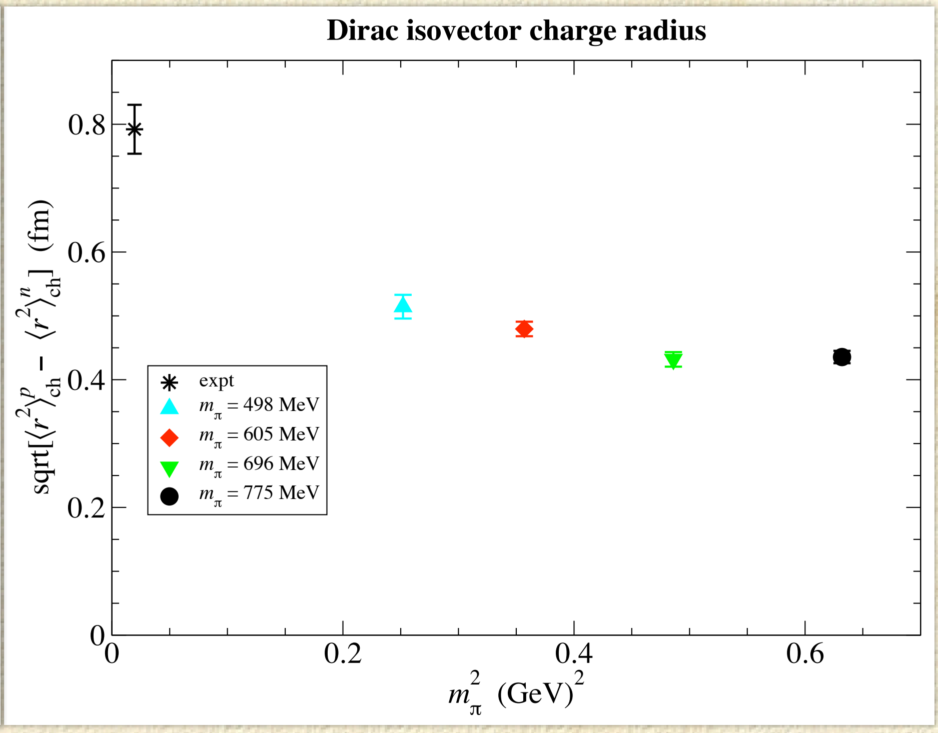
F_1 Isovector Form Factor



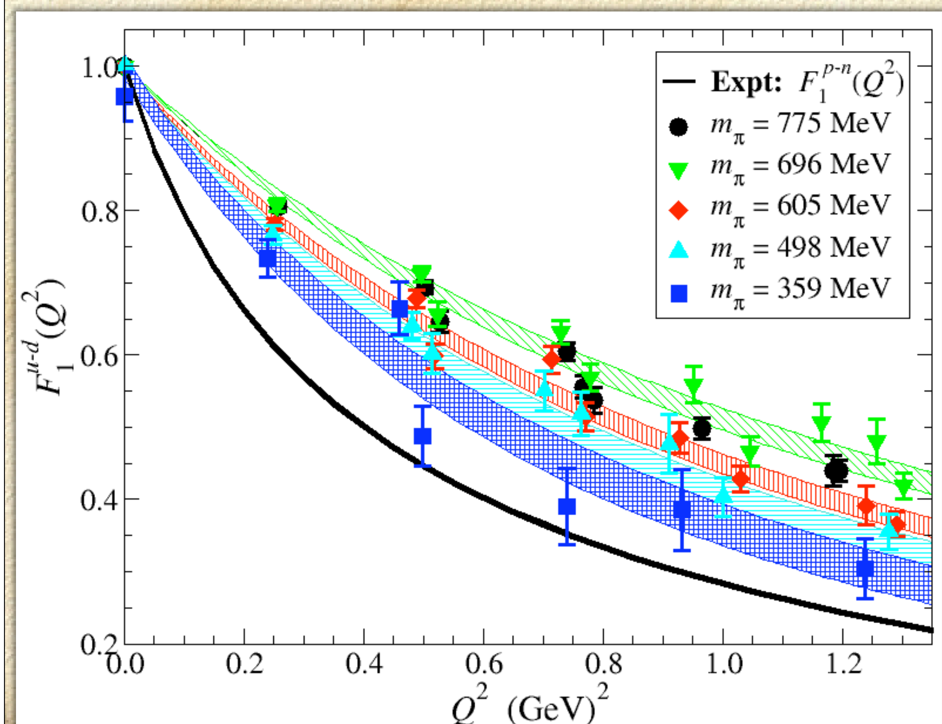


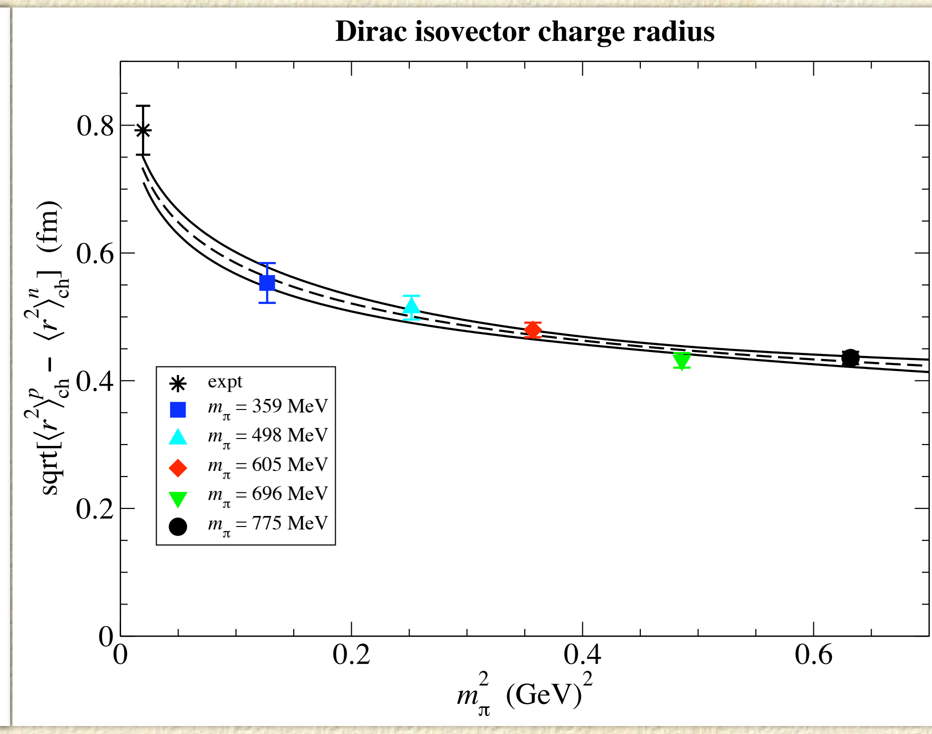
F_1 Isovector Form Factor





F_1 Isovector Form Factor

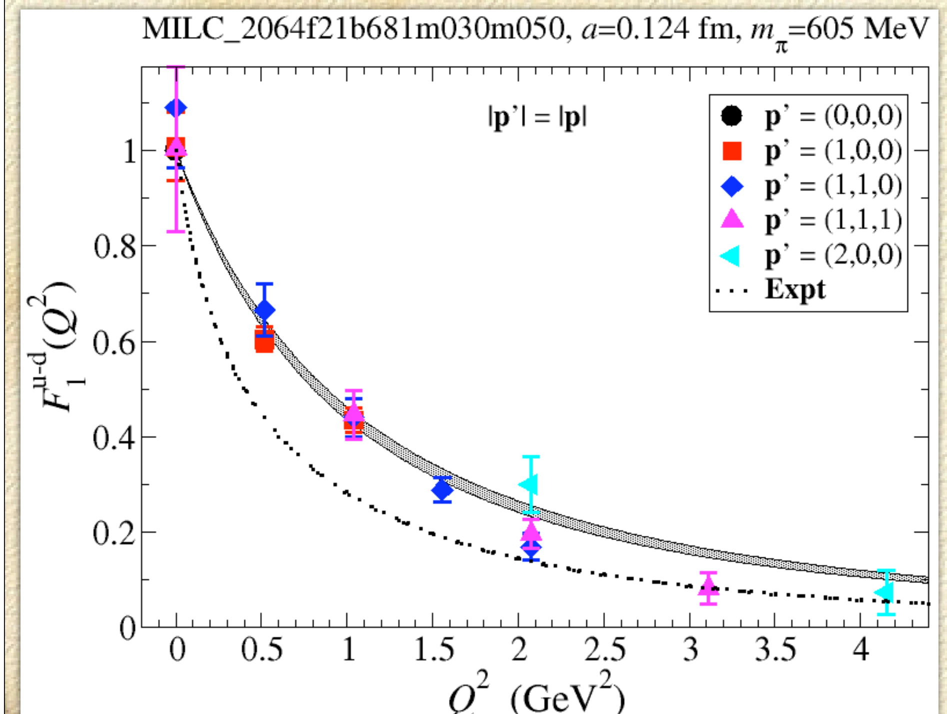


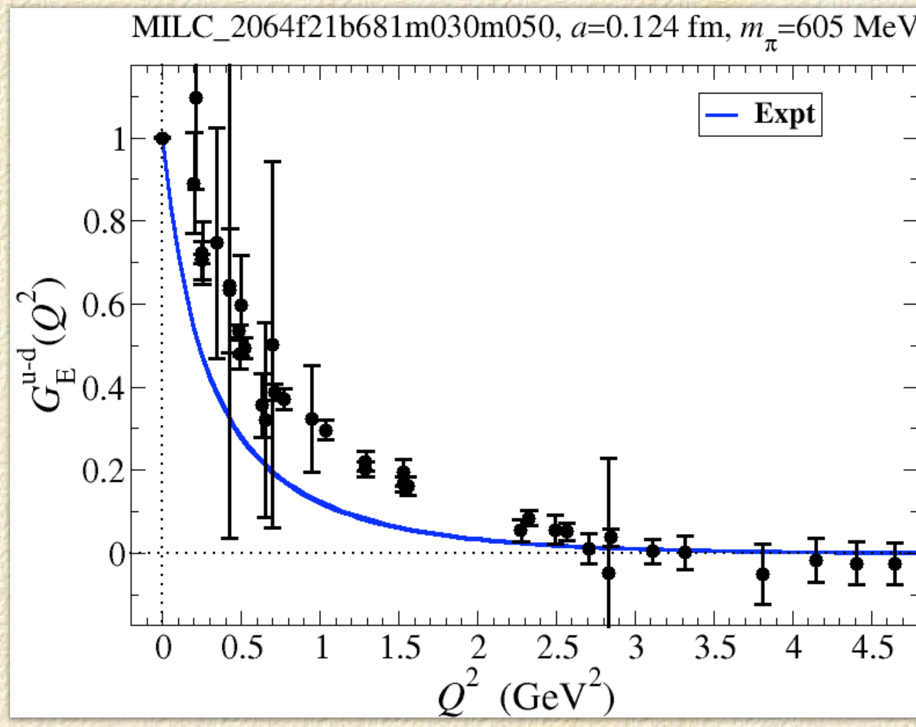


$$\langle r^2\rangle^{u-d} = a_0 - \frac{1+5g_A^2}{(4\pi f_\pi)^2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

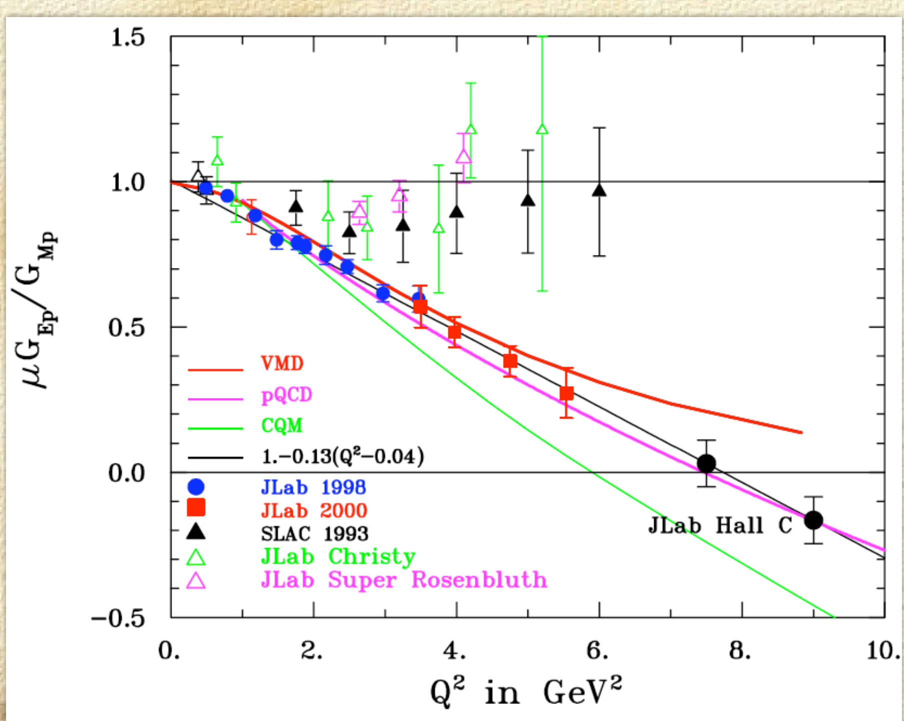
Leinweber, Thomas, Young PRL 86 5011

Isovector Form Factors at higher Q^2

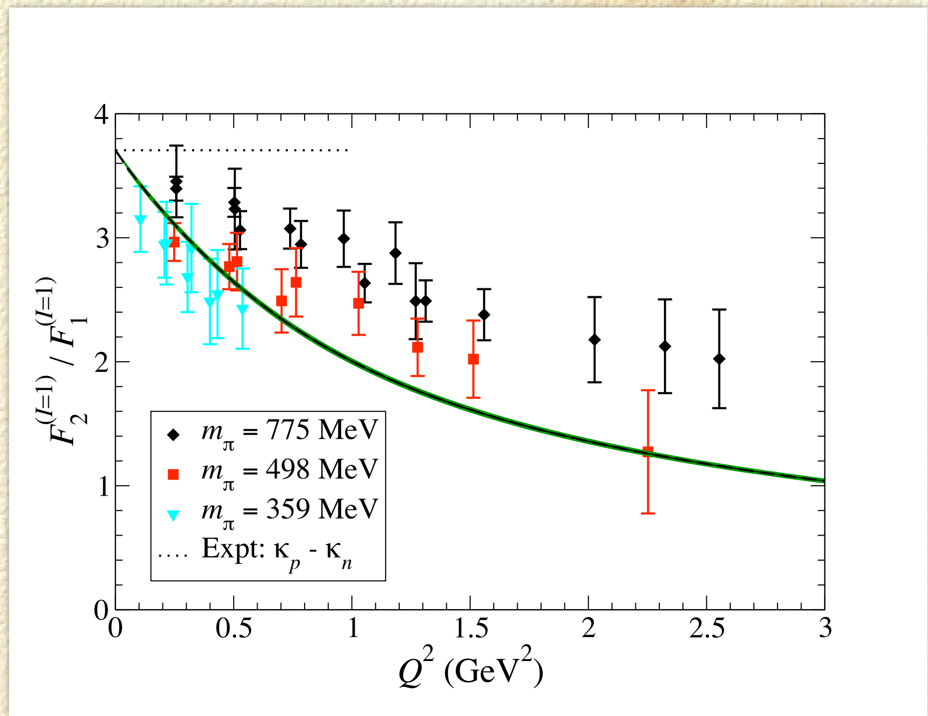




Form factor ratio: F_2 / F_1



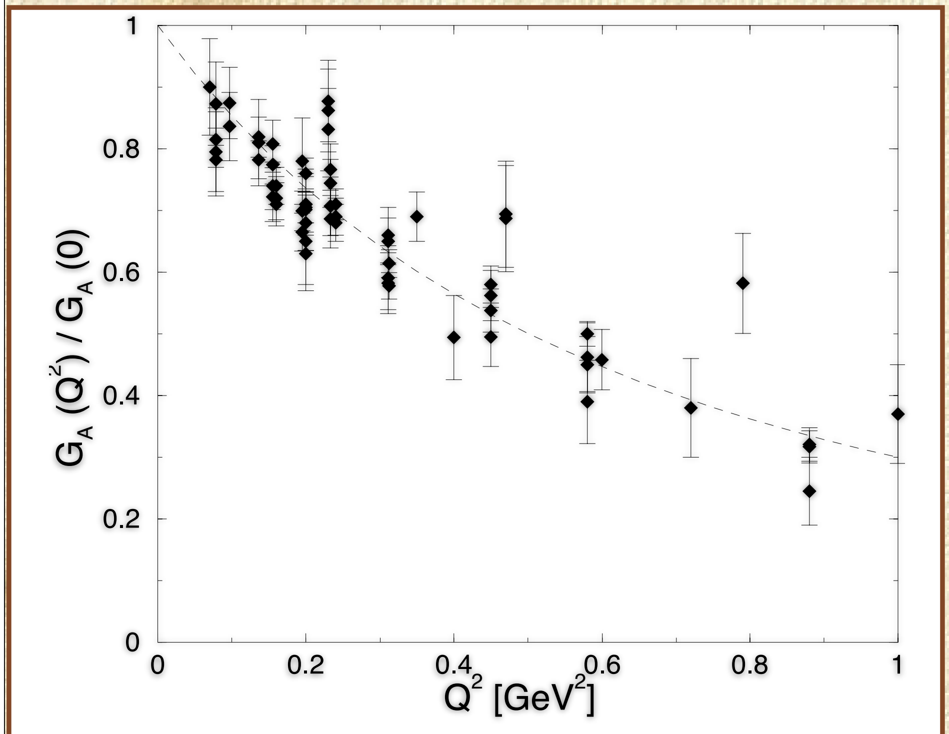
Polarization transfer at JLab

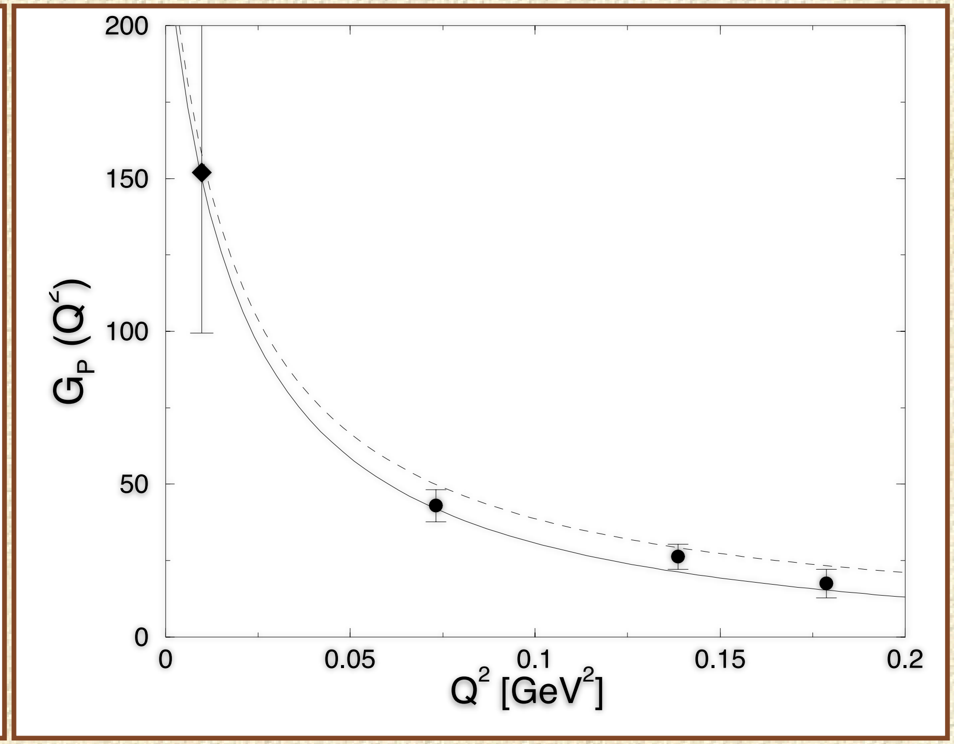


Lattice results

Polarized Nucleon Form Factors G_A and G_P

$$\langle p|\bar{\psi}\gamma^\mu\gamma_5\psi|p'\rangle = \bar{u}(p)[G_A(q^2)\gamma^\mu\gamma_5 + q^\mu\gamma_5 G_P(q^2) + \sigma^{\mu\nu}\gamma_5 q_\nu G_M(q^2)]u(p')$$





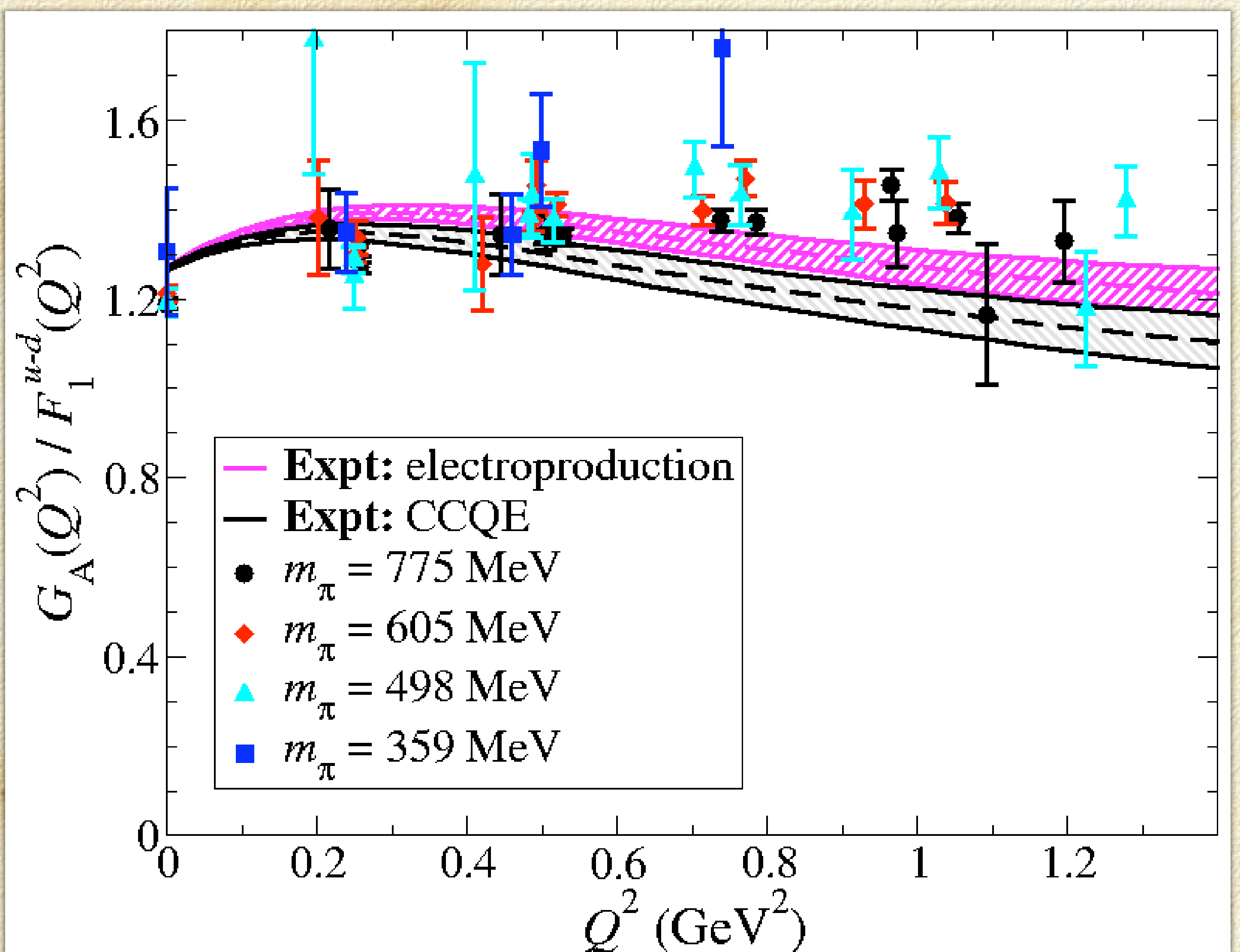
pion electroproduction

$\nu_\mu\, n \rightarrow \mu^-\, p$

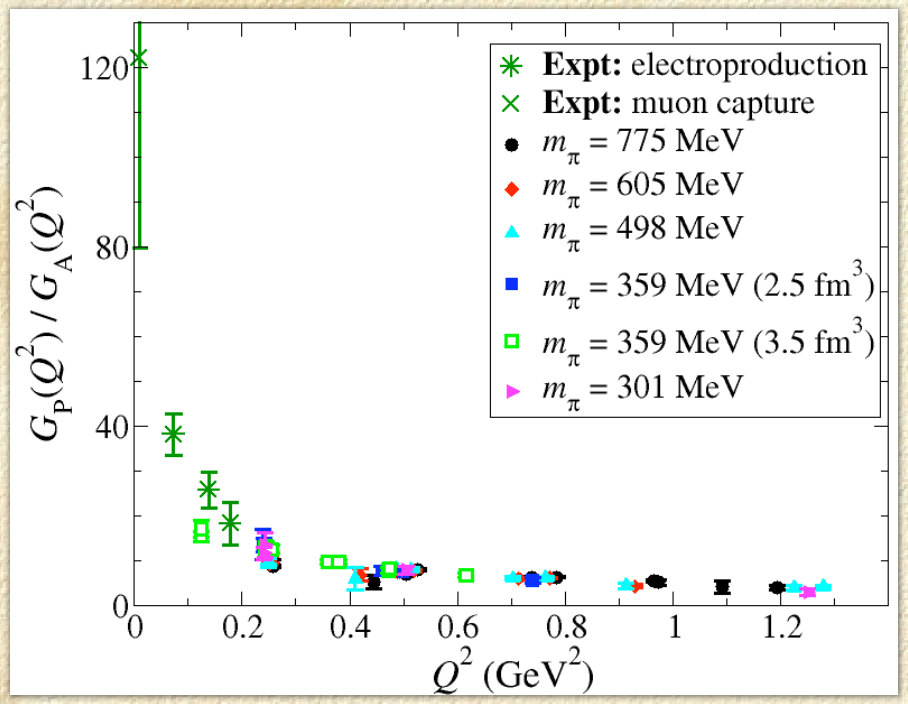
pion electroproduction

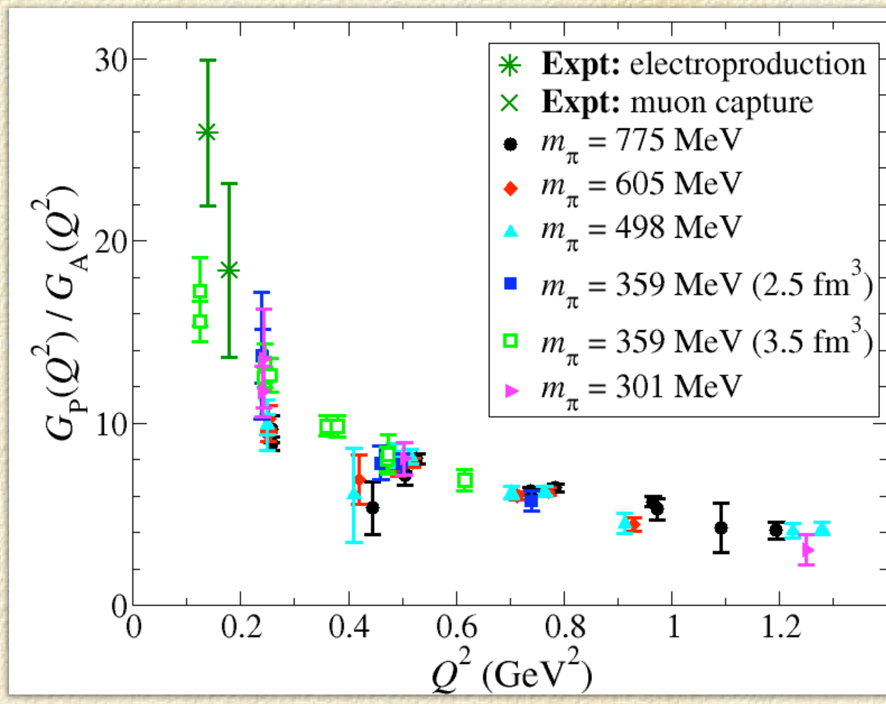
$\mu^-\, p \rightarrow \nu_\mu\, n$

Form factor ratio: G_A/F_1



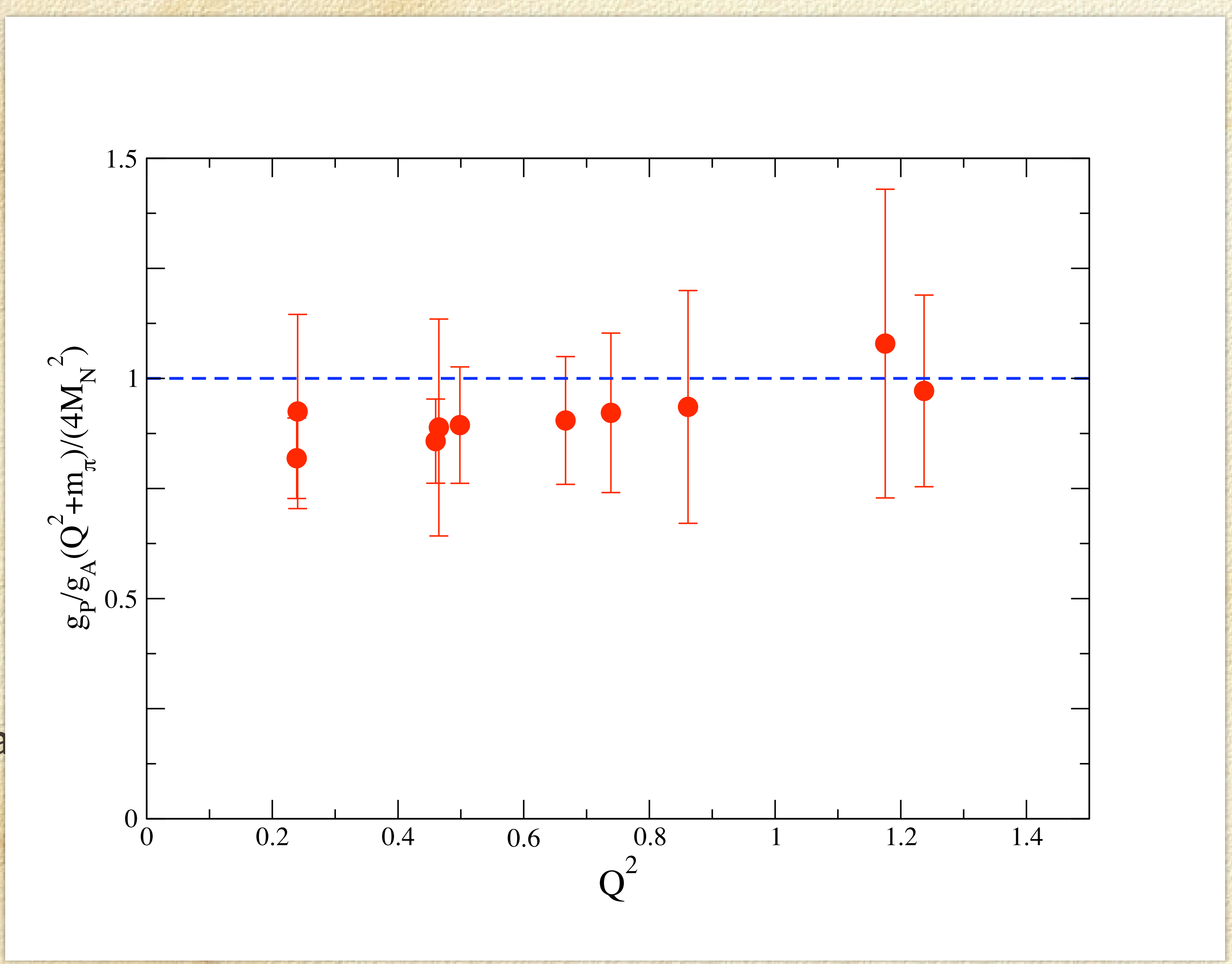
Form factor ratio: G_P/G_A





soft pion pole: $$G_P(q^2) \sim \frac{4M^2 G_A(q^2)}{q^2 - m_\pi^2}$$

Form factor ratio: G_P/G_A



La

Generalized form factors

$$\mathcal{O}_q^{\{\mu_1\mu_2\dots\mu_n\}} = \overline{\psi}_q\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi_q$$

$$\bar{P} = \tfrac{1}{2}(P' + P)$$

$$\Delta = P' - P$$

$$t = \Delta^2$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\mu_1}|P\rangle &= \langle\!\langle\gamma^{\mu_1}\rangle\!\rangle A_{10}(t) \\
&+ \frac{i}{2m}\langle\!\langle\sigma^{\mu_1\alpha}\rangle\!\rangle\Delta_\alpha B_{10}(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle &= \bar{P}^{\{\mu_1}\langle\!\langle\gamma^{\mu_2\}}\rangle\!\rangle A_{20}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\langle\!\langle\sigma^{\mu_2\}\alpha}\rangle\!\rangle\Delta_\alpha B_{20}(t) \\
&+ \frac{1}{m}\Delta^{\{\mu_1}\Delta^{\mu_2\}}C_2(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{30}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{30}(t) \\
&+ \Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{32}(t) \\
&+ \frac{i}{2m}\Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{32}(t),
\end{aligned}$$

Limits of generalized form factors

- Moments of parton distributions $t \to 0$

$$A_{n0} = \int dx x^{n-1} q(x)$$

- Electromagnetic form factors

$$A_{10} = F_1(t), \quad B_{10} = F_2(t)$$

- Total quark angular momentum

$$J_q = \tfrac{1}{2}[A(0)_{20} + B(0)_{20}]$$

Transverse structure of nucleon

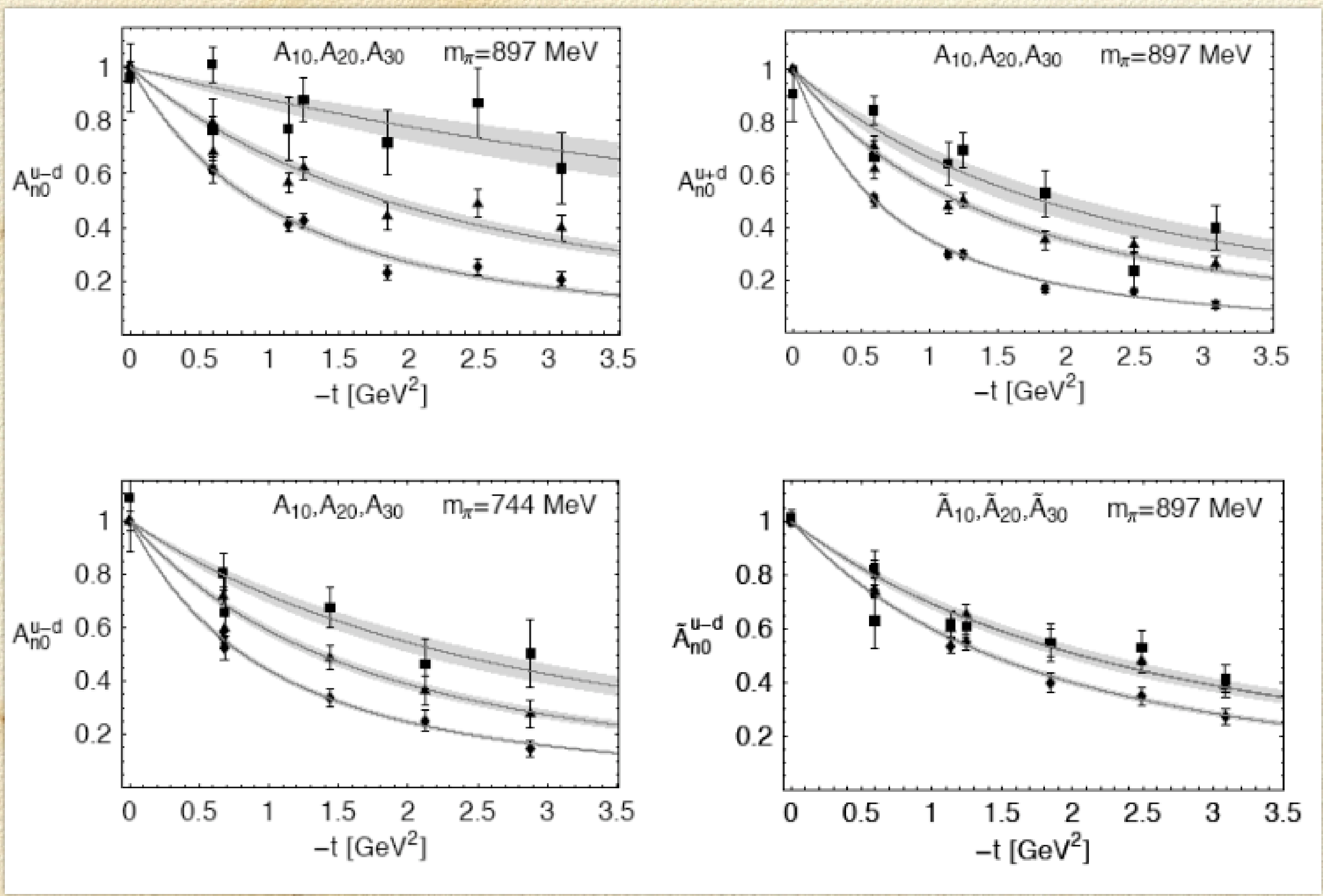
$H(x, 0, -\Delta^2_\perp)$ is transverse Fourier transform of light cone quark distribution $q(x,r_\perp)$ at momentum fraction x

$$q(x, r_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta^2_\perp)\, e^{-ir_\perp \Delta_\perp}$$

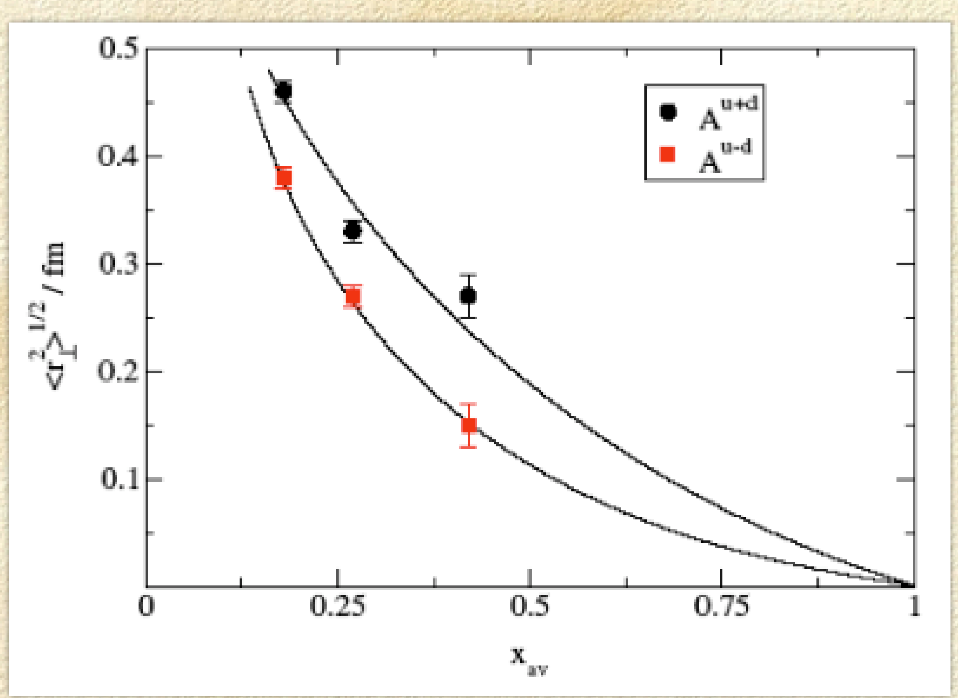
$$\int dx x^{n-1} q(x, r_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} A(-\Delta^2_\perp)\, e^{-ir_\perp \Delta_\perp}$$

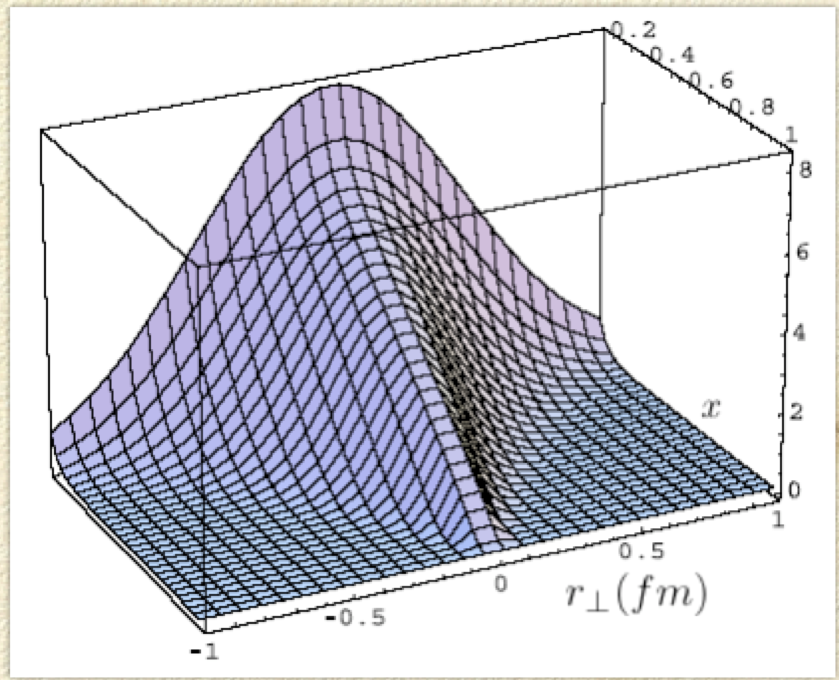
- x → 1: Single Fock space component ⇒ slope → 0
- x ≠ 1: Transverse structure ⇒ slope steeper

Generalized form factors from lattice



Transverse size of light-cone wave function





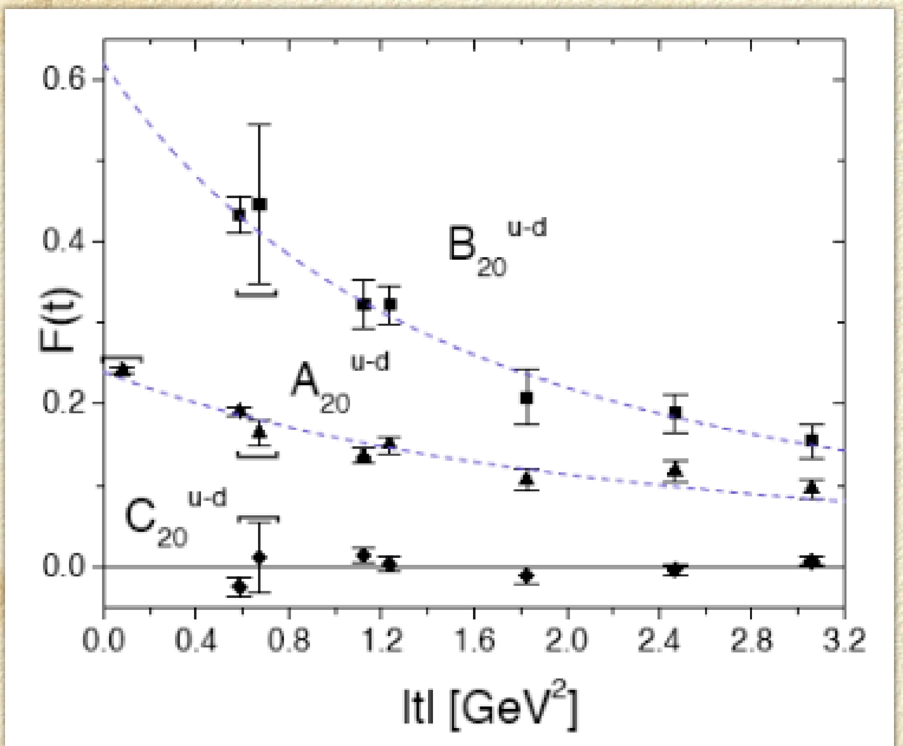
$$x_{\rm av}^n = \frac{\int d^2r_\perp \int dx\, x \cdot x^{n-1} q(x, \vec{r}_\perp)}{\int d^2r_\perp \int dx x^{n-1} q(x, \vec{r}_\perp)}$$

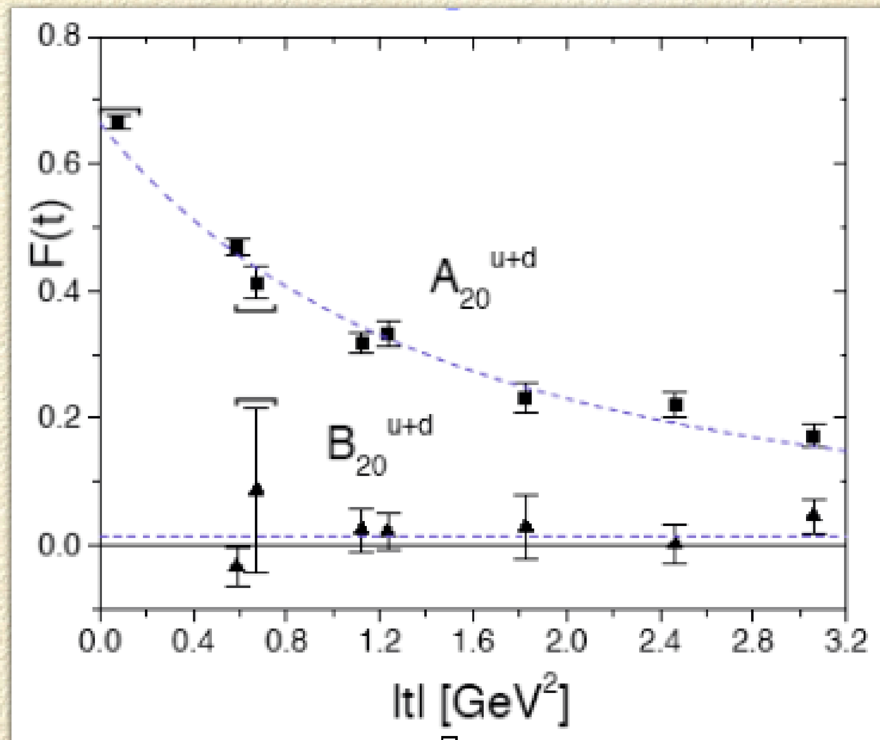
$q(x, \vec{r}_\perp)$ **model** (Burkardt hep-ph/0207047)

First x moments: A_{20}, B_{20}, C_{20}

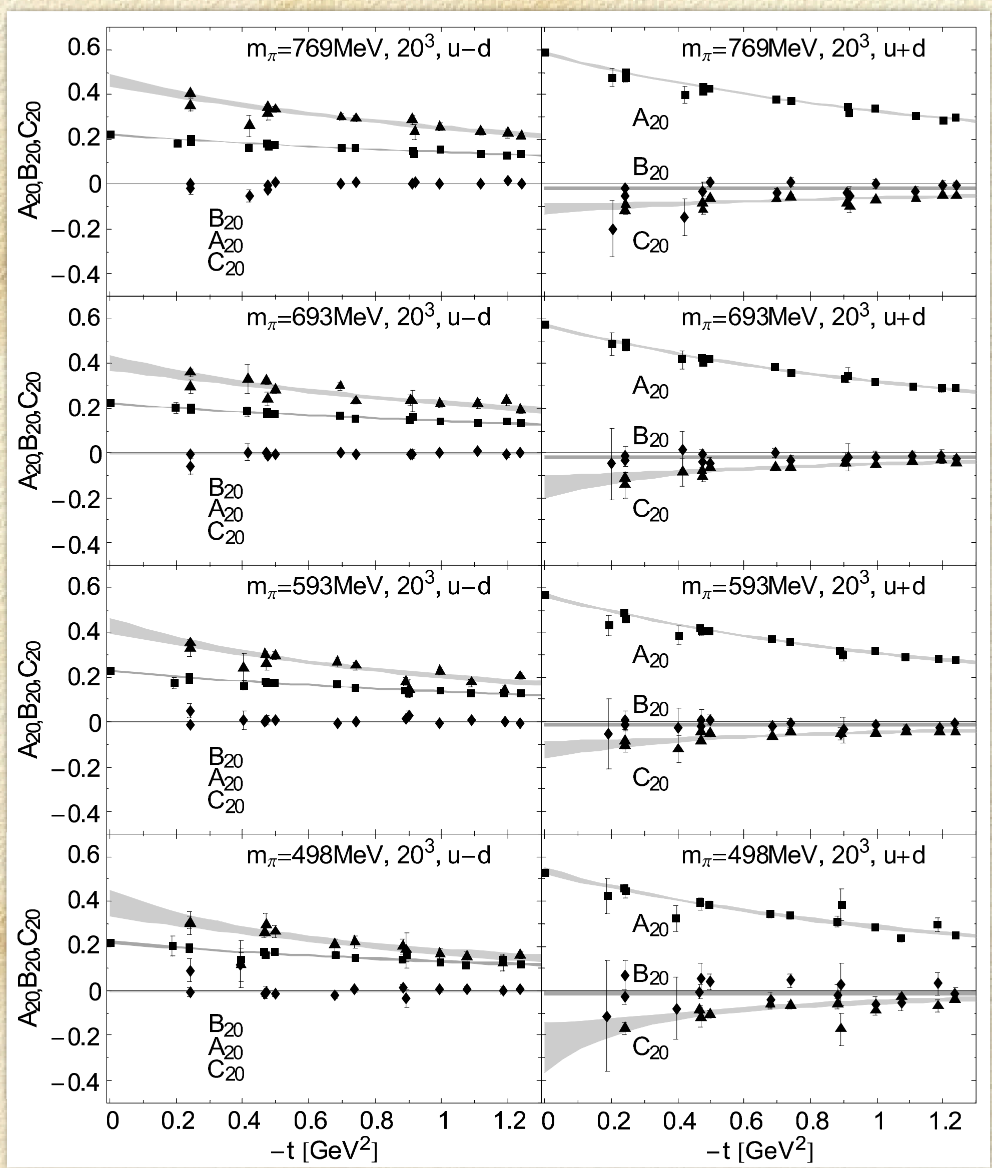
m_π = 897 MeV

LHPC hep-lat/0304018





First x moments: A_{20}, B_{20}, C_{20}



$B_{20}^{u-d} > A_{20}^{u-d}$

$A_{20}^{u+d} > B_{20}^{u+d} \sim 0$

$C_{20}^{u-d} \sim 0$

$C_{20}^{u+d} < 0$

Large N_c behavior

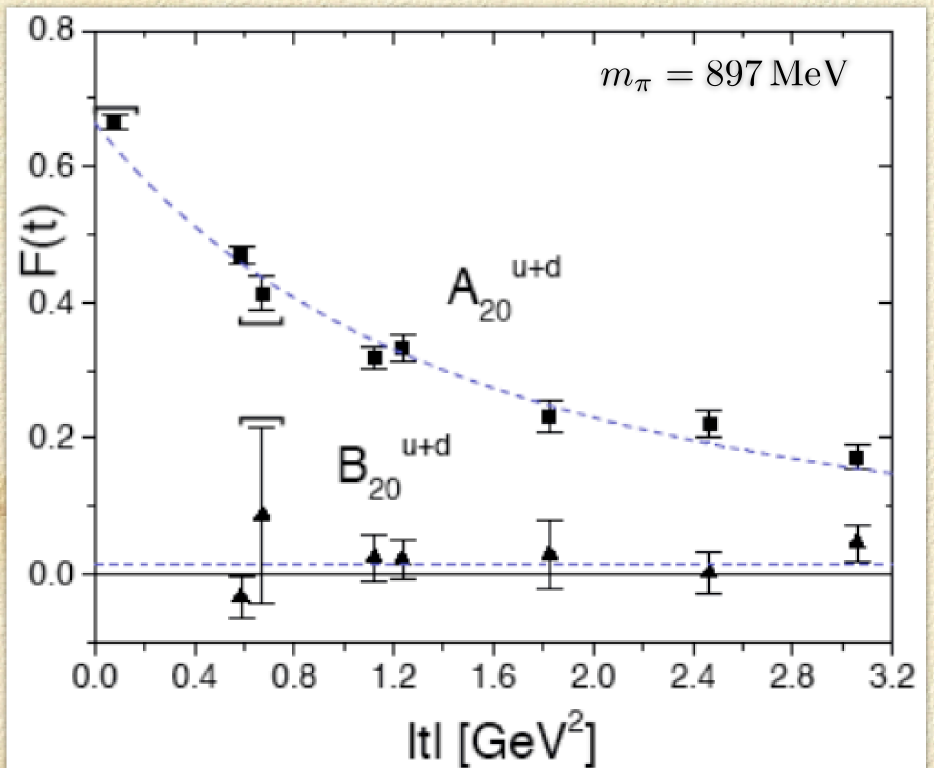
Origin of nucleon spin

"Spin crisis" - only ~ 30% arises from quark spins

quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1 \rangle_{\Delta u + \Delta d} \quad \sim \frac{1}{2}0.682(18)$

total quark contribution (spin plus orbital)

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x \rangle_{u+d} + B_{20}^{u+d}(0)] \quad \sim \frac{1}{2}0.675(7)$$



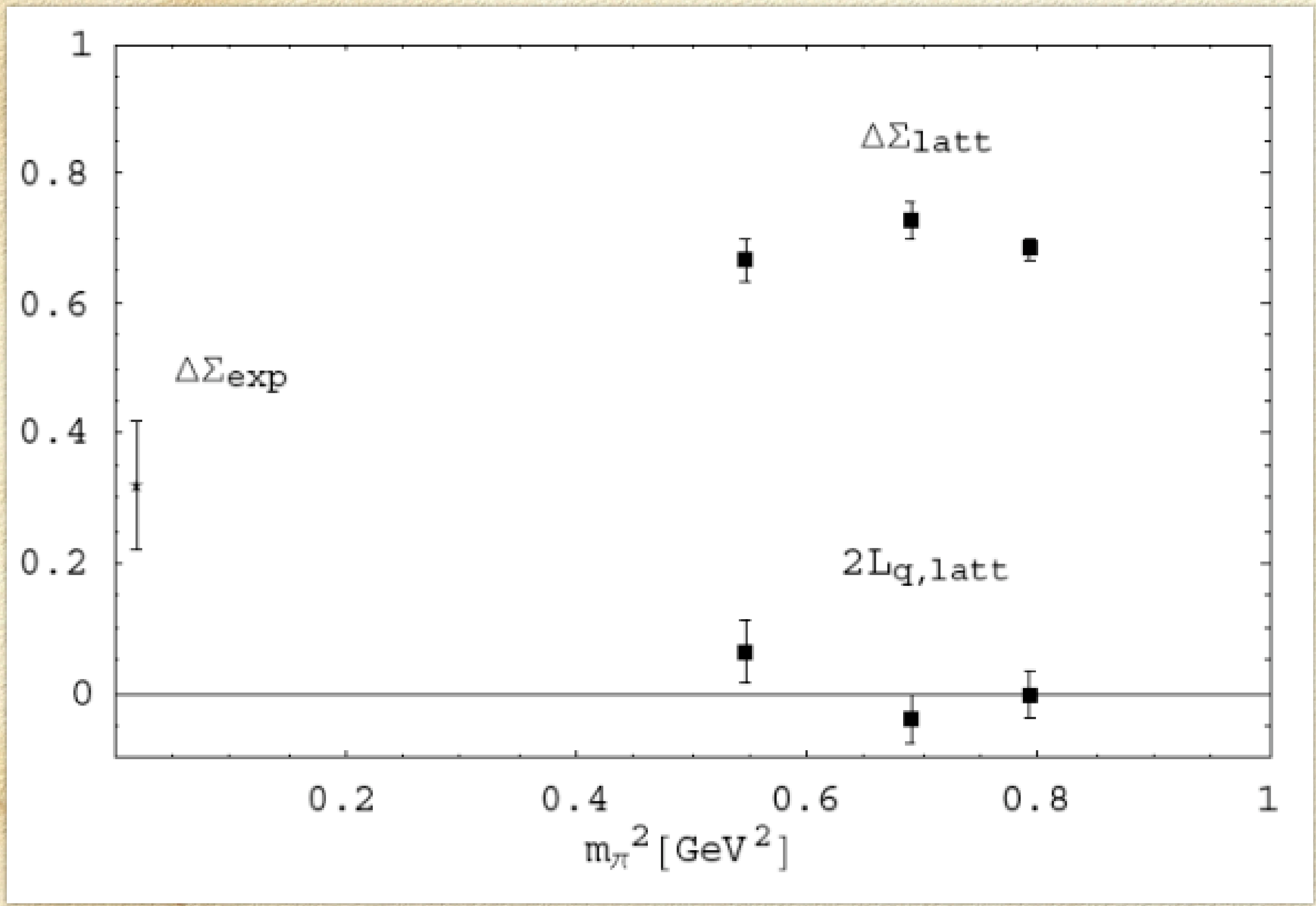
Spin Inventory

68% quark spin

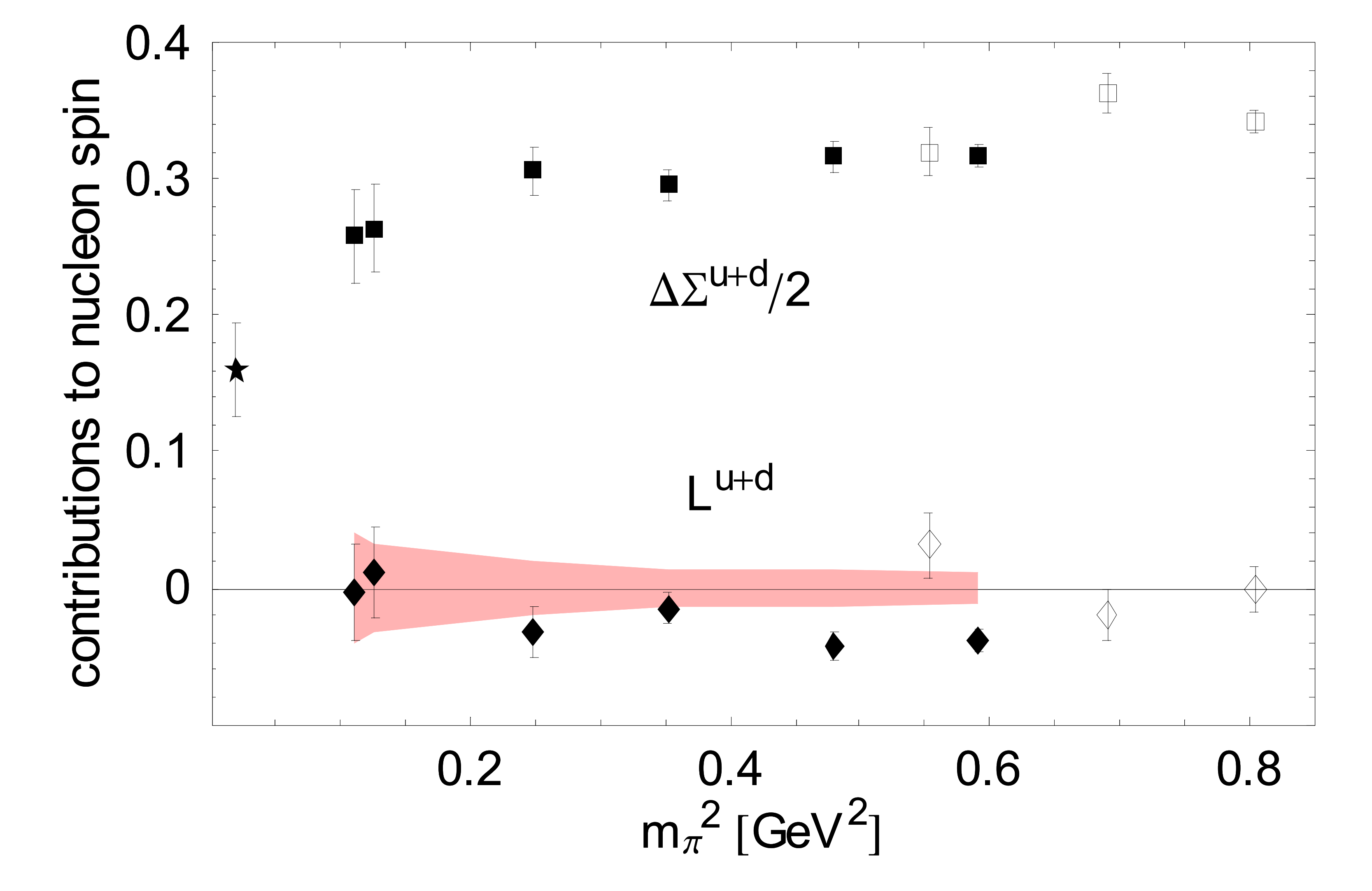
0% quark orbital

32% gluons

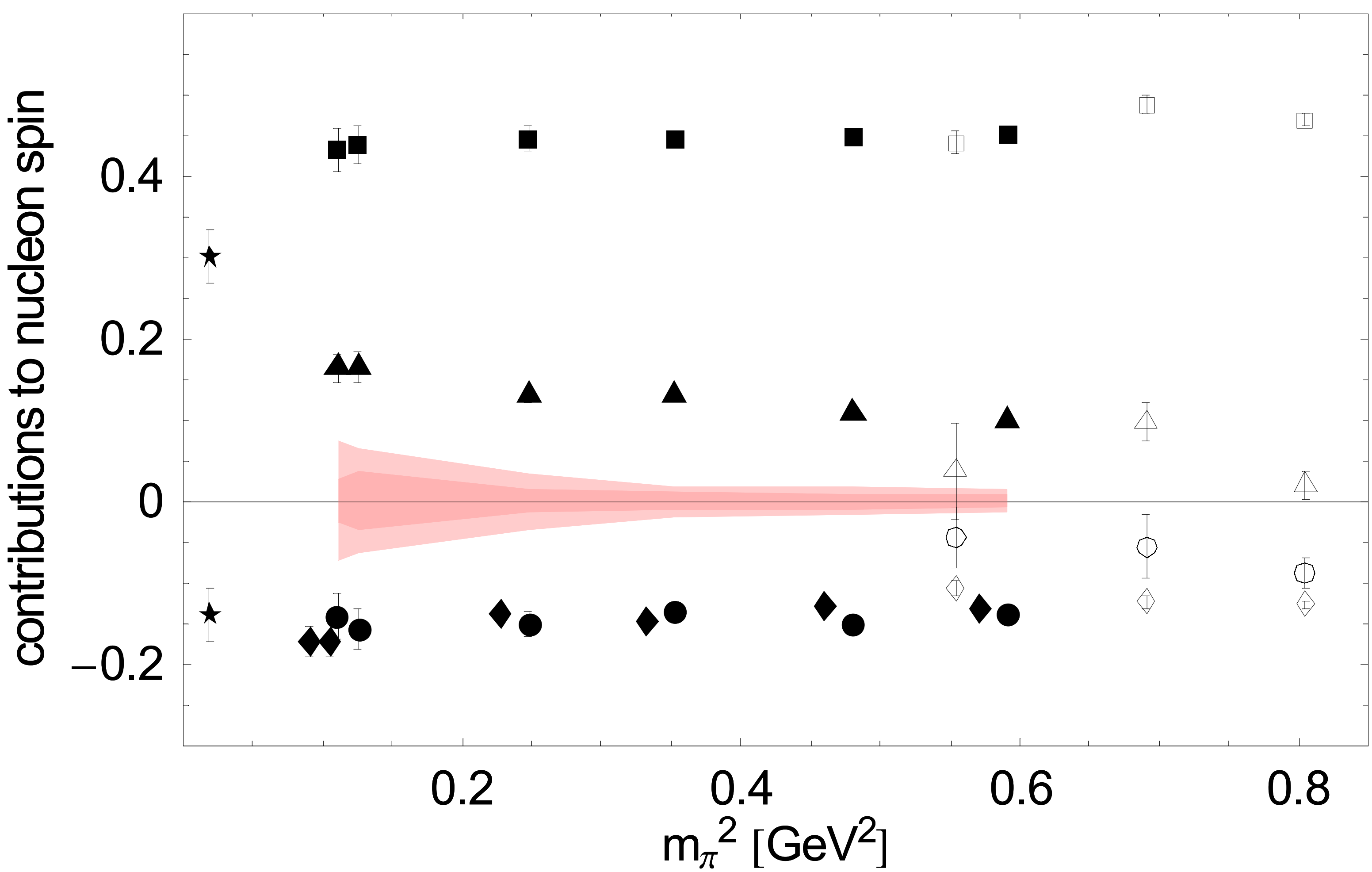
Nucleon spin decomposition



Nucleon spin decomposition



Nucleon spin decomposition

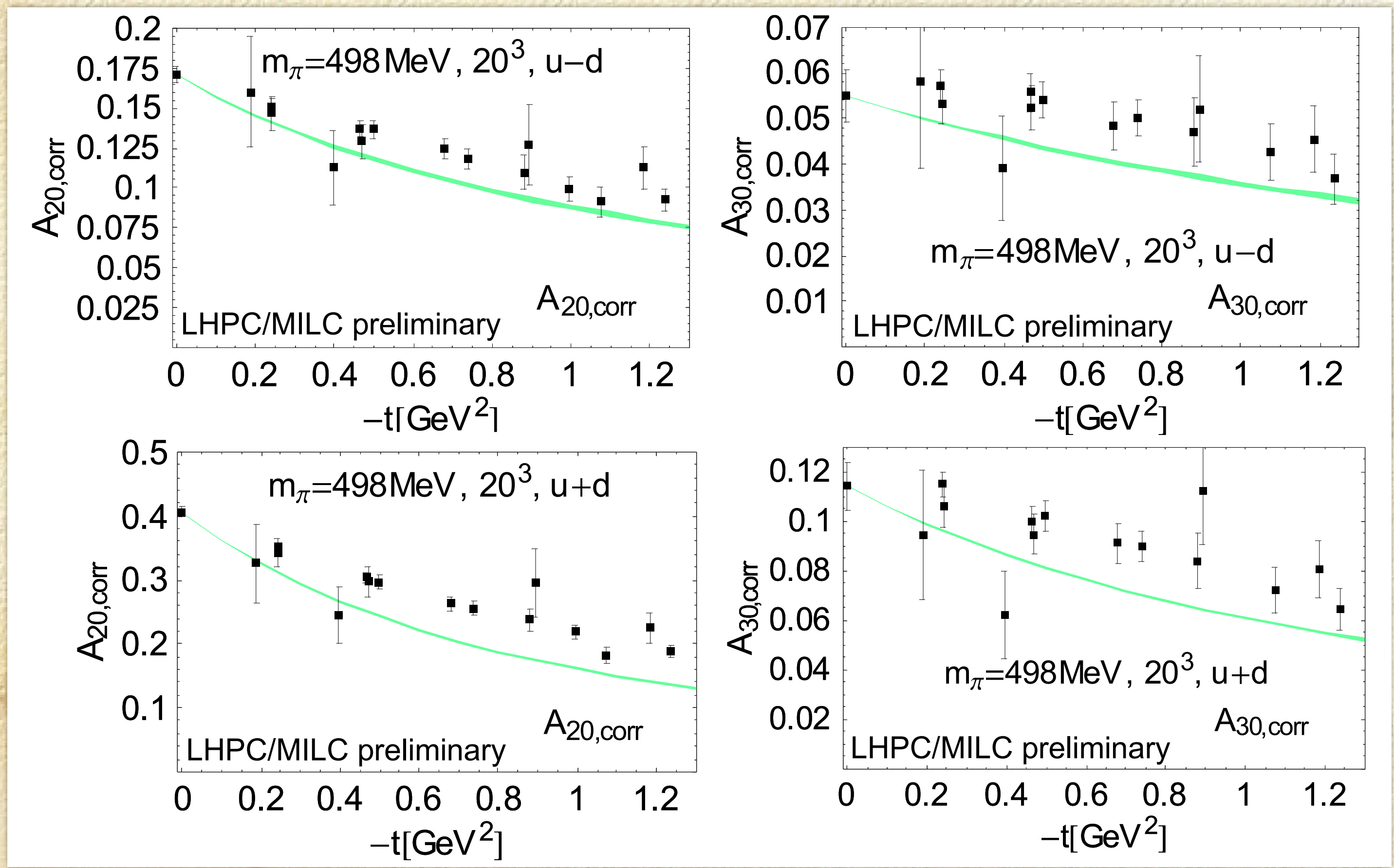


Comparison with Phenomenology

GPD parameterization: Diehl, Feldmann, Jakob, Kroll EPJC 2005

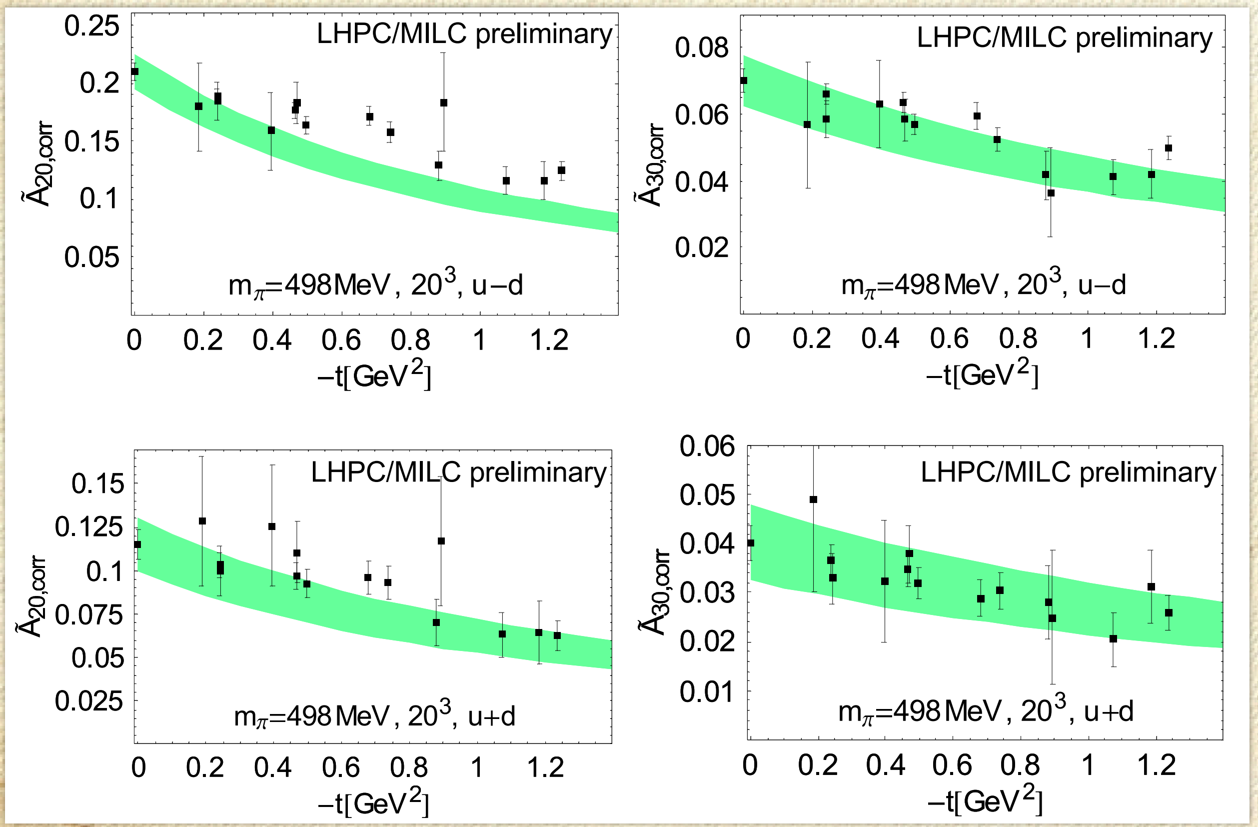
nucleon form factors, CTEQ parton distributions, Regge, Ansatz

$$A_{20} = \int dx\, x\, H(x,0,t) \qquad A_{30} = \int dx\, x^2\, H(x,0,t)$$

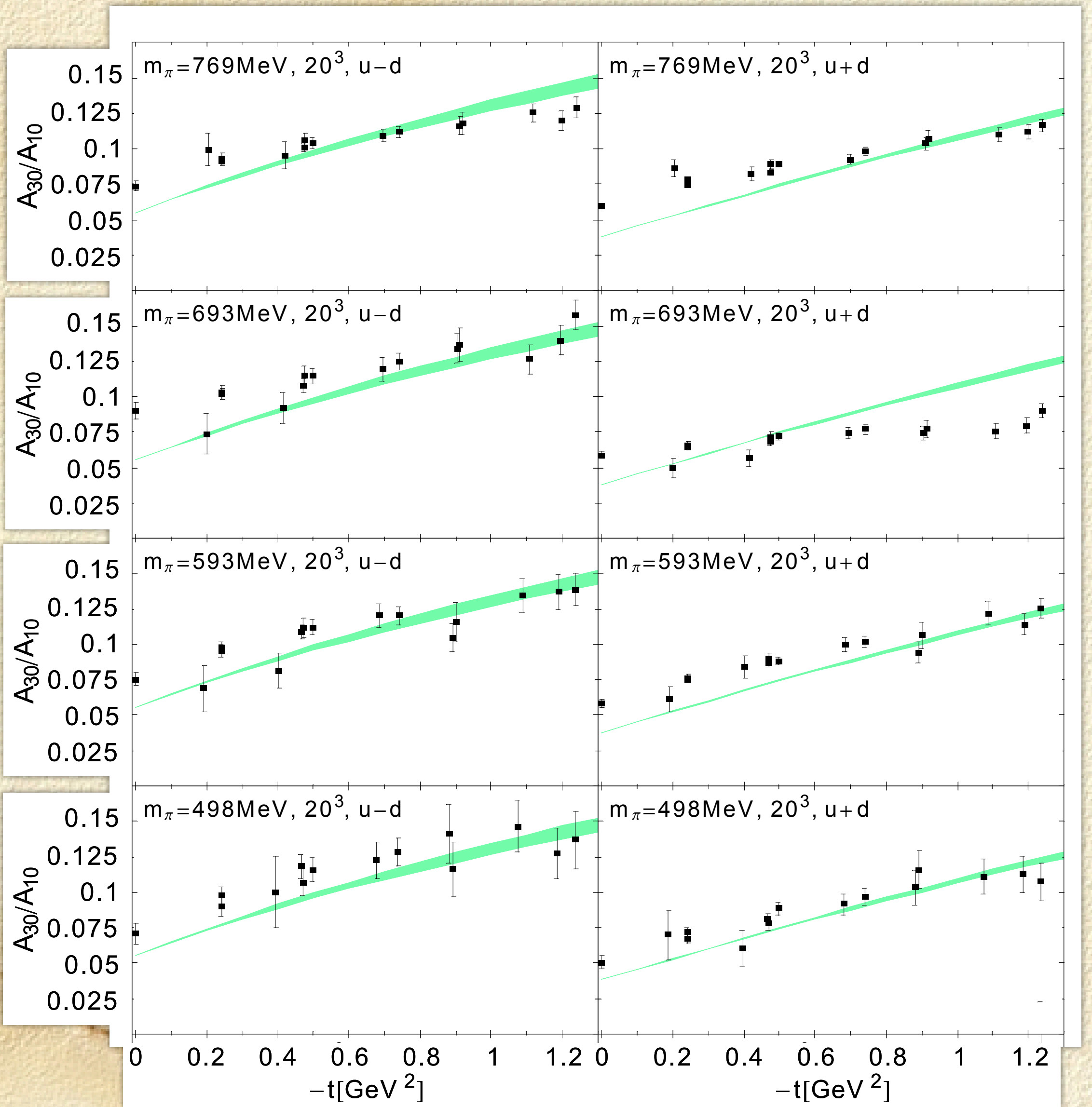


Comparison with Phenomenology

$$\tilde{A}_{20} = \int dx\, x\, \tilde{H}(x,0,t) \qquad \tilde{A}_{30} = \int dx\, x^2\, \tilde{H}(x,0,t)$$

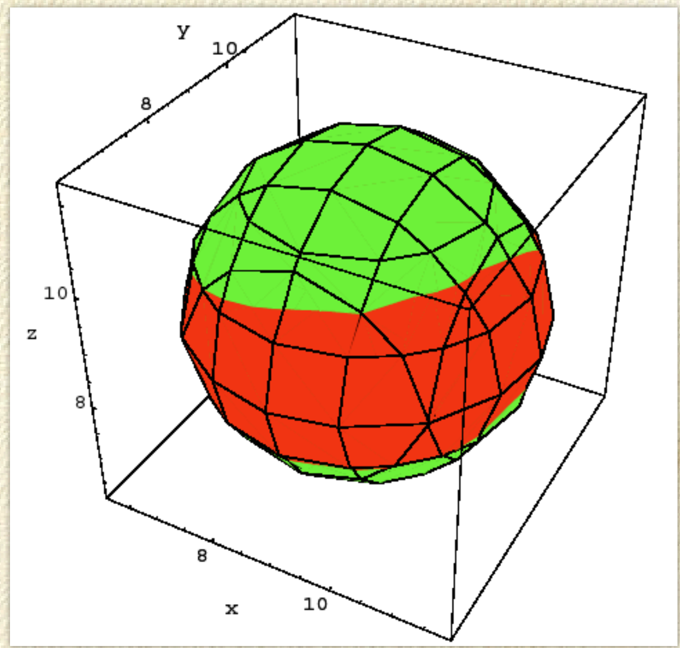


Comparison with Phenomenology

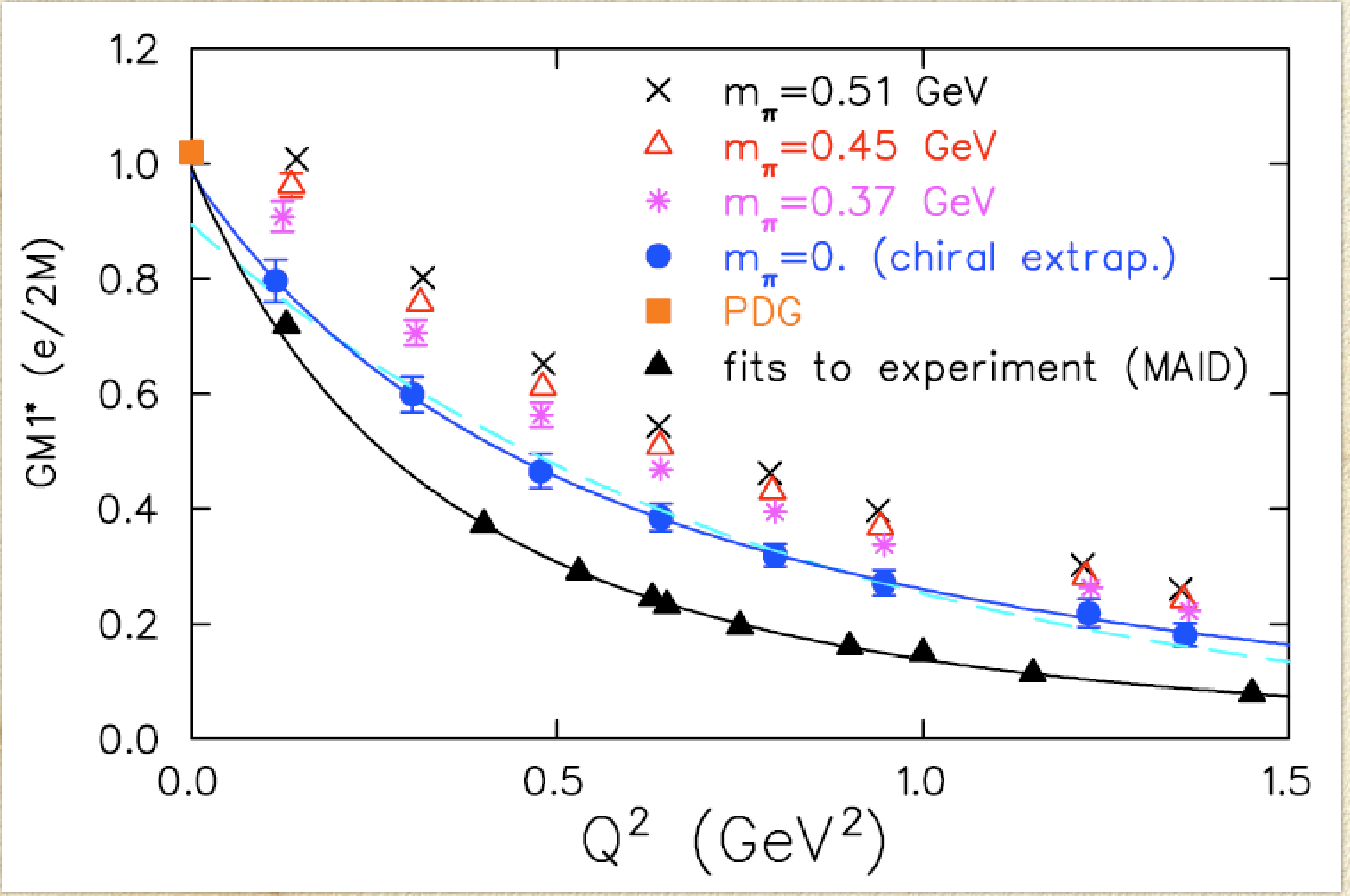


Baryon shapes

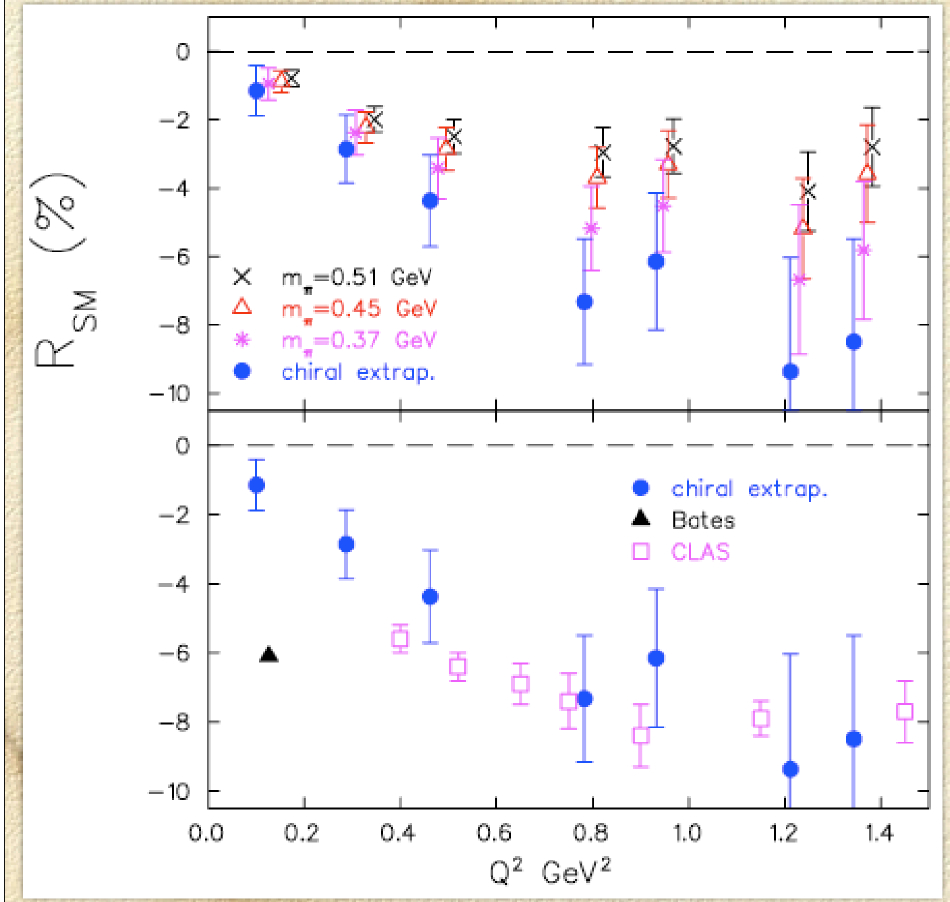
- Observe oblate deformation of spin 1/2 Δ directly on lattice from density-density correlation function (Alexandrou, nucl-th/0311007)
- Infer deformation experimentally from N→Δ transition form factor
  - Dominant transition M1
  - C2 and E2 would vanish if nucleon and Δ spherical



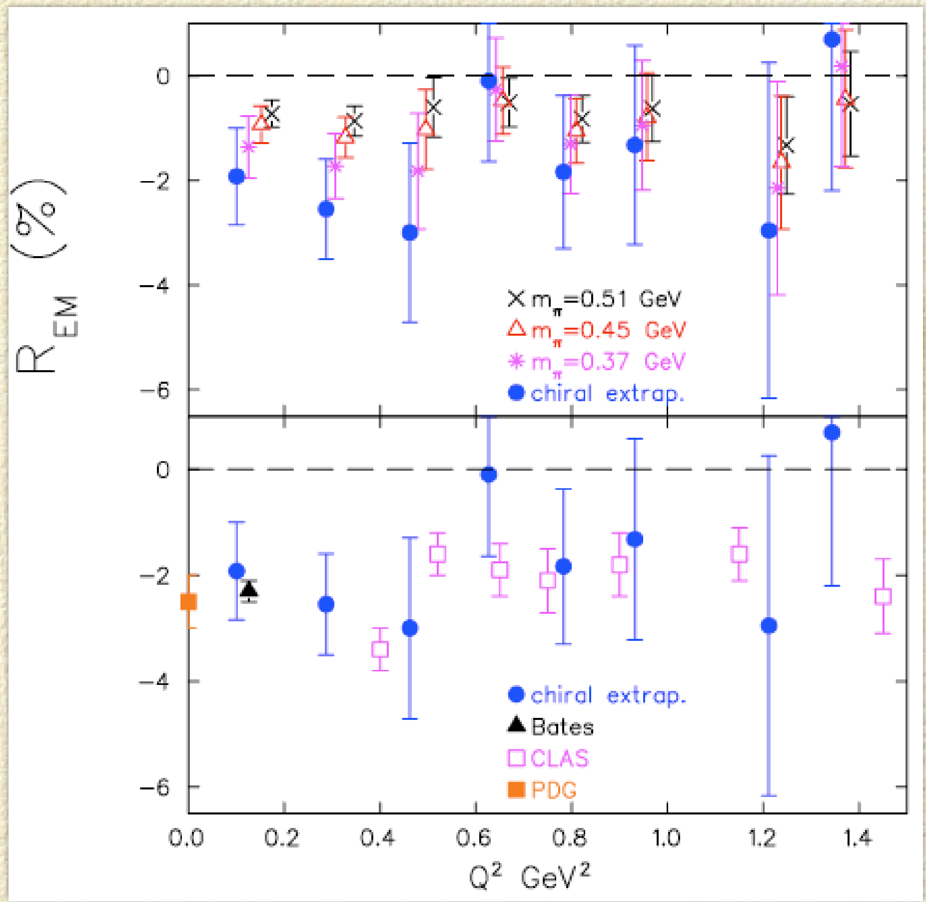
M1 form factor



Electric and Coulomb transitions



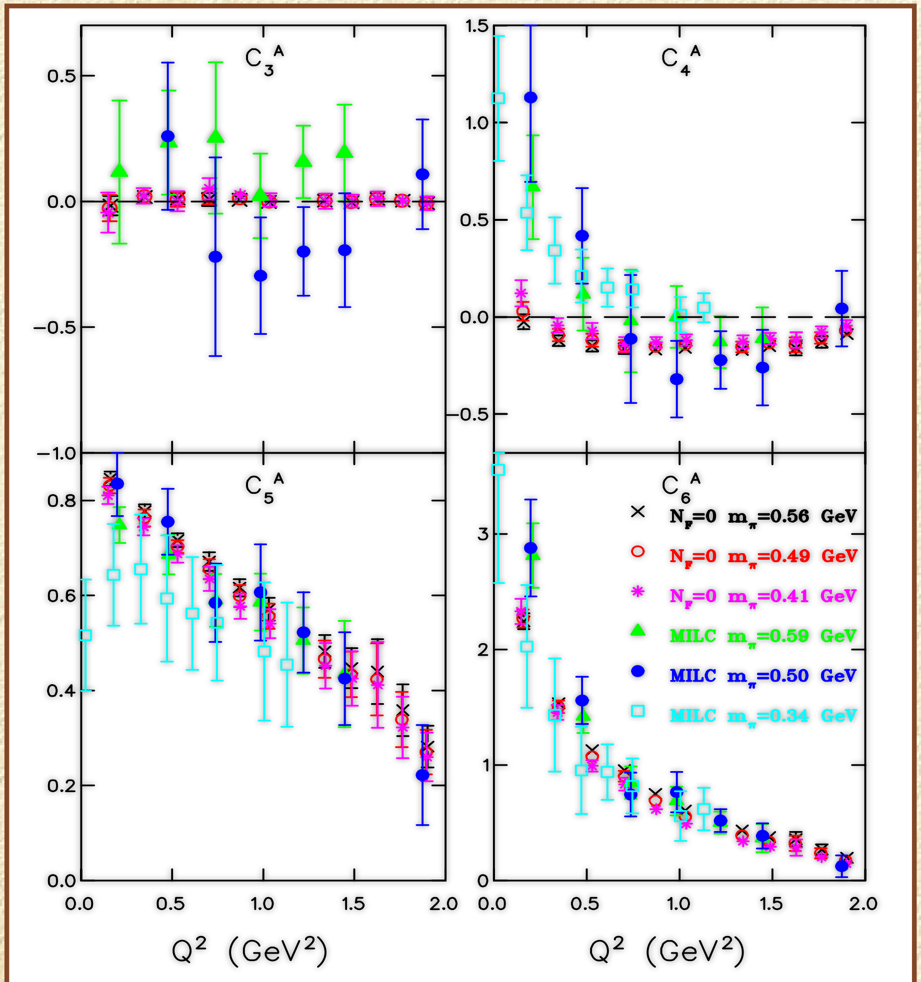
C2/M1



E2/M1

Axial N-Delta transition form factors

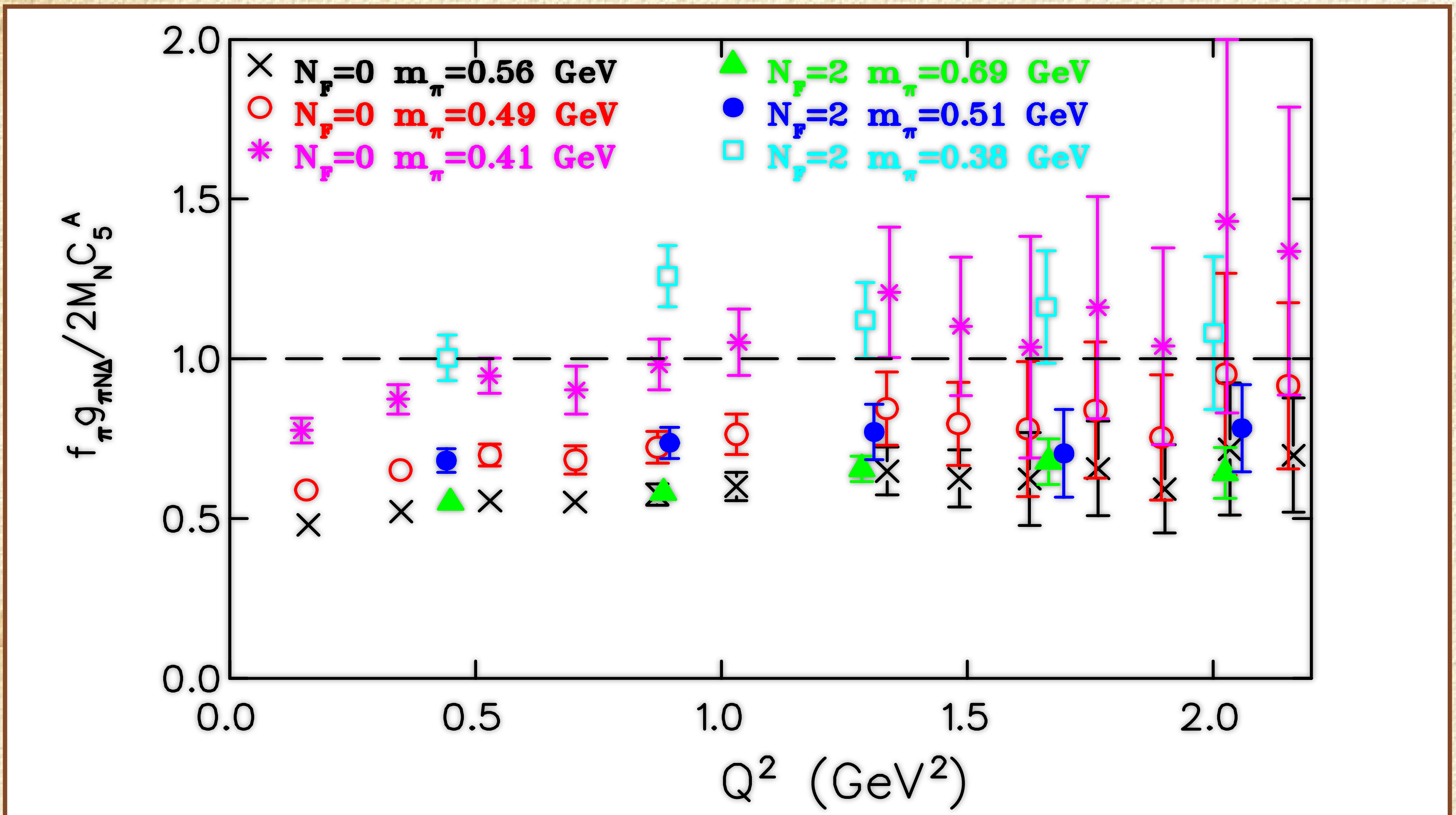
$$\langle \Delta(p',s')|A_\mu|N(p,s)\rangle \propto \bar{u}^\lambda(p',s')\left[\left(\frac{C_3^A(q^2)}{M}\gamma^\nu + \frac{C_4^A(q^2)}{M^2}p'^\nu\right)(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho + C_5^A(q^2)g_{\lambda\mu} + \frac{C_6^A(q^2)}{M^2}q_\lambda q_\mu\right]u(p,s)$$



Alexandrou, Leontiou, Tsapalis, JN

Axial N-Delta transition form factors

Off-diagonal Goldberger-Treiman relation $$C_5^A(q^2) = \frac{F_\pi g_{\pi N\Delta}(q^2)}{2M}$$

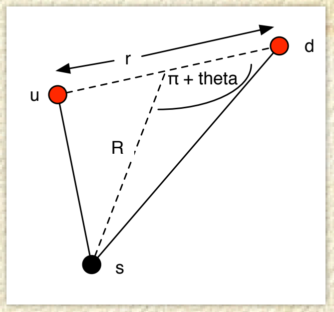


Diquark correlations in heavy light light baryon

Patrick Varilly - senior thesis

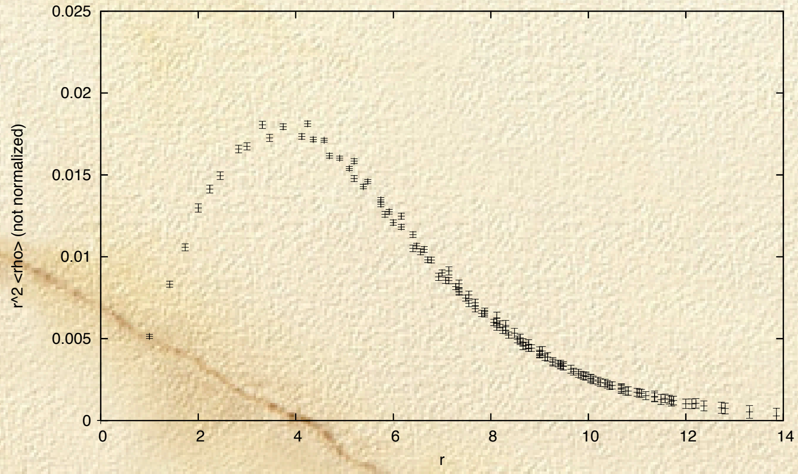
Good diquarks:
color antitriplet
flavor antisymmetric
spin singlet

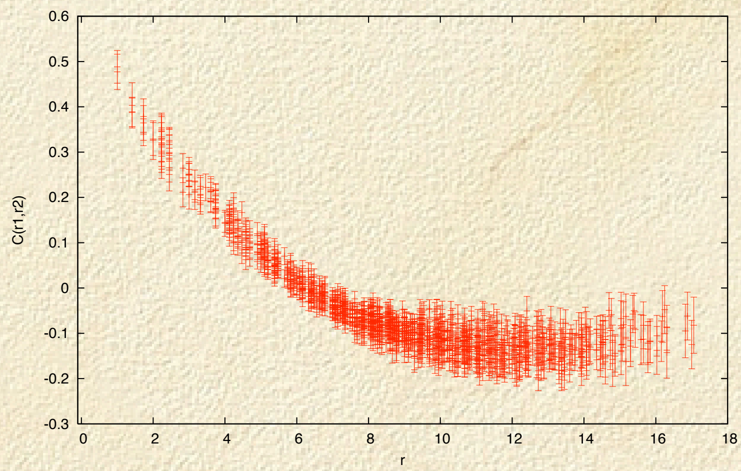
$$(uC\gamma_5 d)h$$



$$\langle \rho(r) \rangle$$

$$C(r_1, r_2) = \frac{\langle \rho(r_1)\rho(r_2)\rangle - \langle \rho(r_1)\rangle\langle \rho(r_2)\rangle}{\langle \rho(r_1)\rangle\langle \rho(r_2)\rangle}$$





Summary

- Entering era of quantitative solution in chiral regime
  - Quark distributions: $g_A, \Sigma, \langle x \rangle$
  - Form factors
  - Transverse structure
  - Origin of nucleon spin
  - Baryon shapes
- Gaining insight into how QCD works
  - Instantons
  - Dependence on m_q, N_c, N_f
  - Diquarks

Future Prospects

- Precision hadron calculations
- Validate by agreement with key experiments
- Theory and experiment work in consort
  - Example: GPD's: Expt. convolution, Theory moments
- Challenges
  - Disconnected diagarams
  - Gluon observables
  - Unstable states
  - Exotic mesons and baryons
  - Evolve quark masses from glueball world to QCD
  - Hadron-hadron phase shifts, adiabatic potentials