Application of the Kalman Filter to PHENIX Charged Particle Tracking

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- **What is the Kalman Filter?**
- **Tracking in PHENIX**
  - fit procedure
  - vertex/momentum fit
  - single particle MC results
  - including multiple scattering

References: R.E. Kalman, Transactions of the ASME, 1960
R. Fruhwirth, NIM A262 (1987) 444-450
P. Billoir, Comp. Phys. Comm. 57(1989) 340

What is the Kalman Filter?

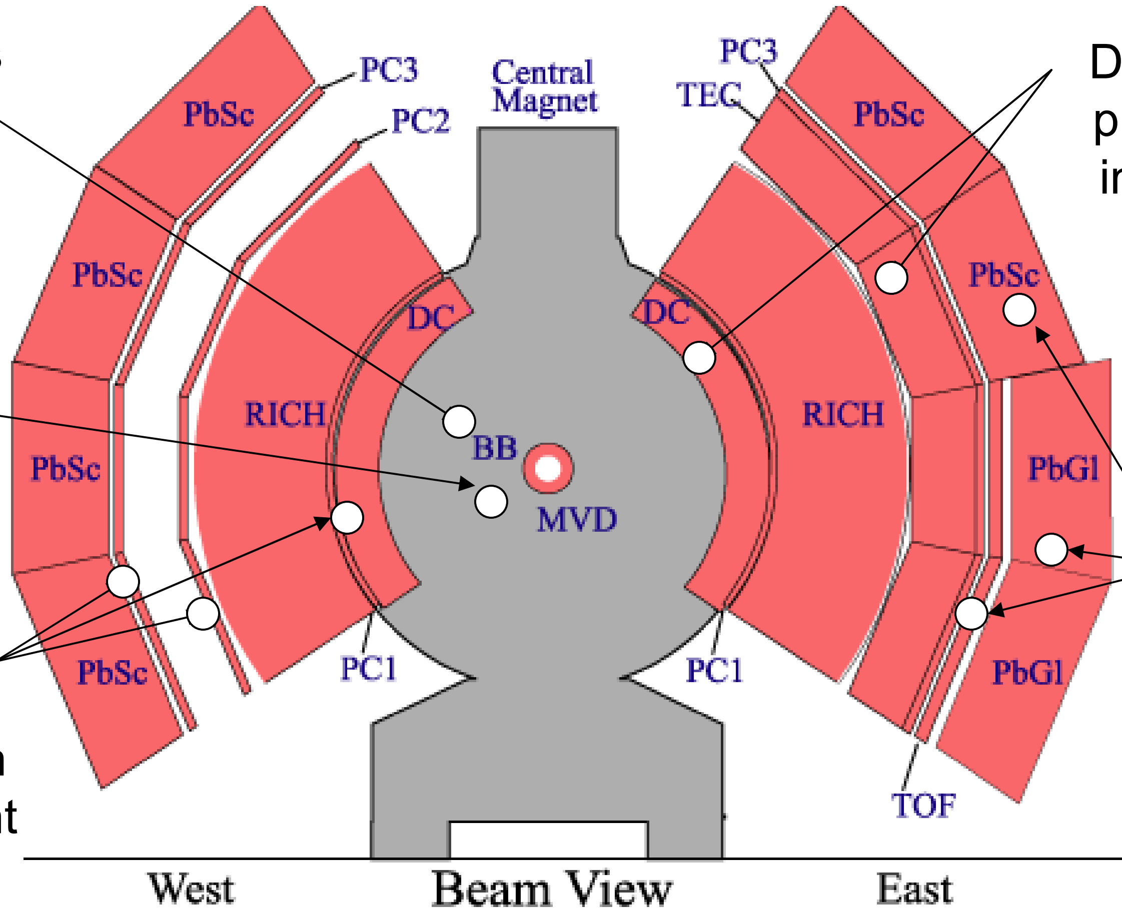
- Recursive technique for obtaining the solution to a least-squares fit.
- Avoids large matrix inversion in traditional fits, does not require analytic fit function.
- Adaptable to include both **measurement** and **process noise**.
- Used for tracking ballistic missiles, facial recognition and robotic piloting. The HEP community has largely adopted it as a standard (BaBar, CLEO, ALICE,…)
- Highly adaptable – can be used for tracking, track fitting, and merging information from different detector systems.

Tracking in PHENIX

Inhomogeneous Magnetic Field
(no analytic track model)

NO tracking in magnetic field.

PC's provide high resolution z-measurement



DC, TEC track predominantly in bend plane

TOF, EMCal can also be included

Momentum reconstruction currently done with only DC and PC1.

Fit Procedure

- Fit track from outside-in (provides best-fit parameters at vertex).
- Approximate PHENIX as a set of measurements made at constant R.
- Seed trajectory is generated from CGL (**C**entral **GL**obal tracking) vertex and momentum, trajectory is described by a parameter vector at each radial position along the track:

$$\eta = (\varphi, z, \varphi_p, \theta_p, p)$$

- The track is transported to location of each hit $\eta \rightarrow \eta'$. This is done numerically for the track parameters and covariance matrix *C* (linearized approximation, known an "extended Kalman fit").

$$T = \frac{\partial \eta'}{\partial \eta} = \text{Transport matrix}$$

$$C' = TCT^T$$

Fit Procedure (cont.)

- Kalman gain matrix is calculated from measurements, measurement error, and state vector covariance matrix.
- Track state vector is updated according to the gain.

$$K = \text{Kalman gain matrix}$$
$$V = \text{covariance matrix of hit to be added}$$
$$H = \text{projects state vector onto measurements}$$
$$m = \text{matrix of measurements}$$
$$K = CH(V + HCH^T)^{-1}$$
$$\eta' = \eta + K(m - H\eta)$$
$$C' = (I - KH)C$$

- Actual measurements directly update the relevant state vector parameters. Derivatives of elements in the state vector are updated through the influence of the covariance and transport matrices).

Vertex and Momentum

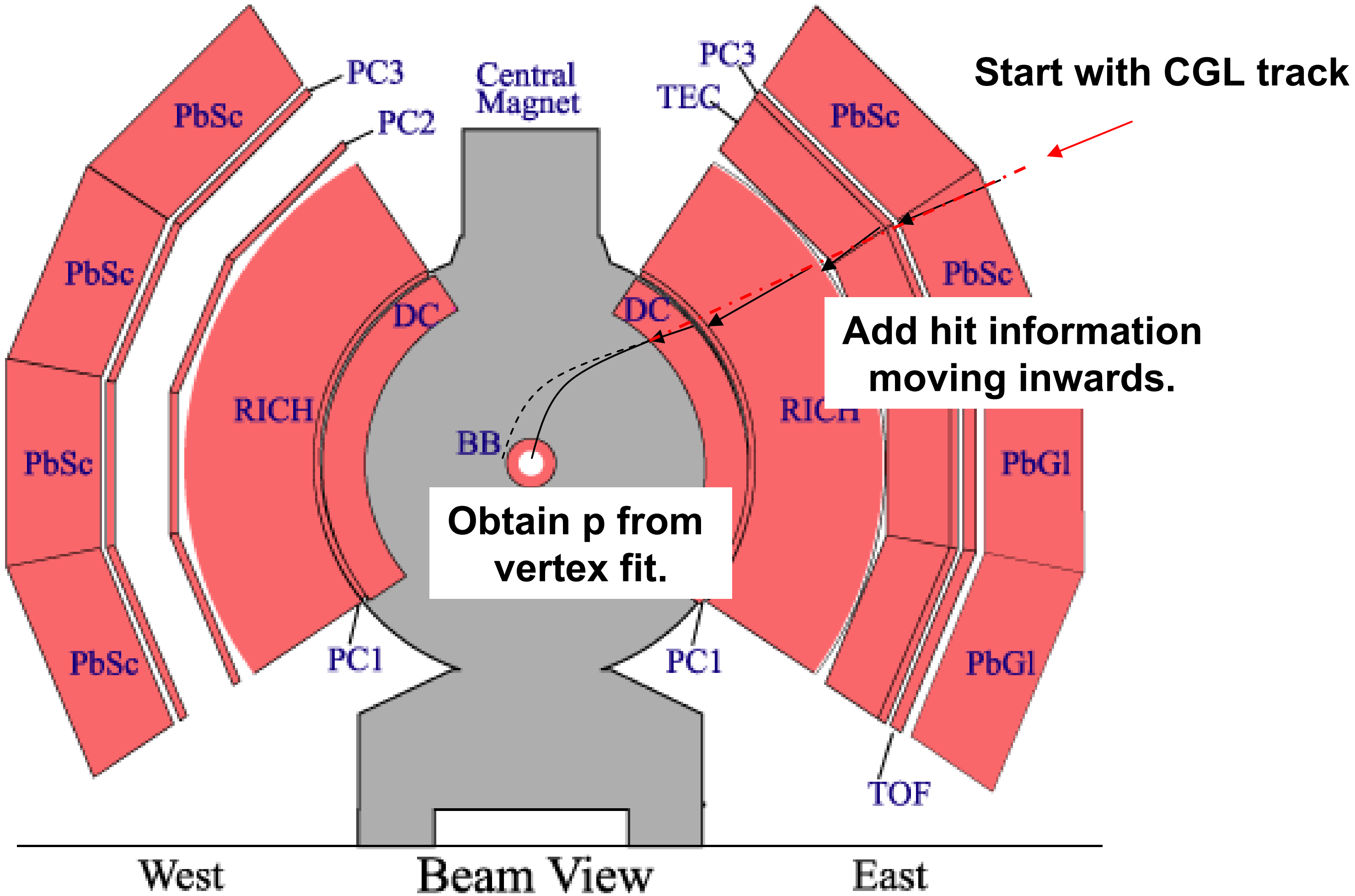
- Once we are through the DC, we have the best estimate of the momentum vector of the track, but we know nothing about the magnitude.
- We have no measurements between the vertex and the DC, so we are forced to assume the track came from the vertex.
- Define a chi-square associated with the vertex as a function of the momentum magnitude:

$$\chi^2_{VTX}(p) = \left(\frac{\Delta r}{\sigma_r}\right)^2 + \left(\frac{\Delta z}{\sigma_z}\right)^2$$

Vertex and Momentum (cont.)

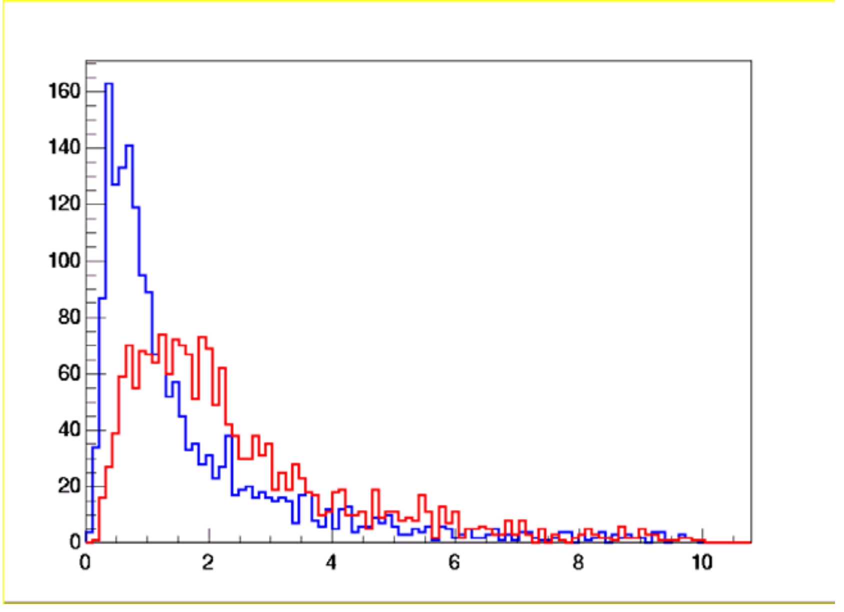
- One-dimensional minimization problem – use the gsl solver to obtain the best-fit momentum.
- Each minimization step involves propagating the particle through the magnetic field to the closest approach to the vertex.
- Note that this procedure fixes the entry vector to the magnetic field, and only varies the total momentum.
- Computationally expensive!

PHENIX Track Fit

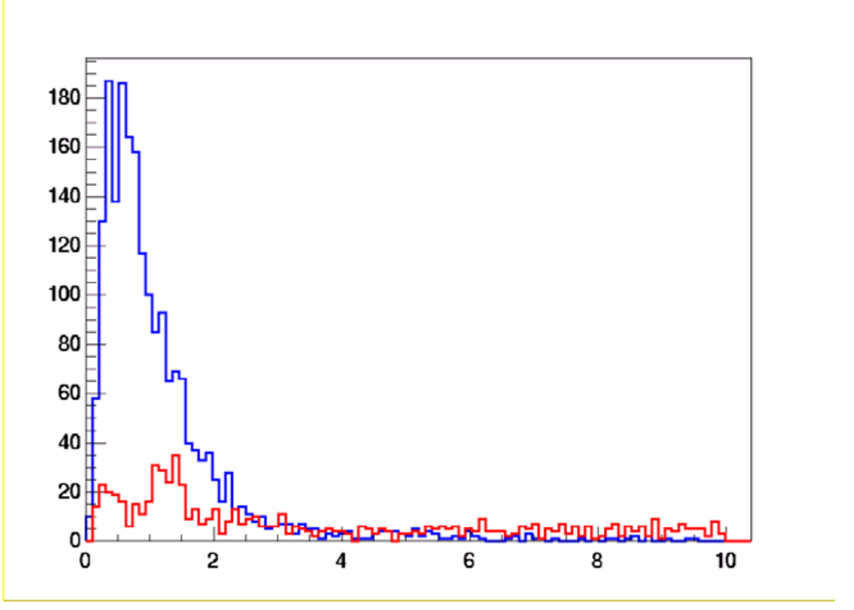


MC χ^2 Distributions

Red: before fit
Blue: after fit



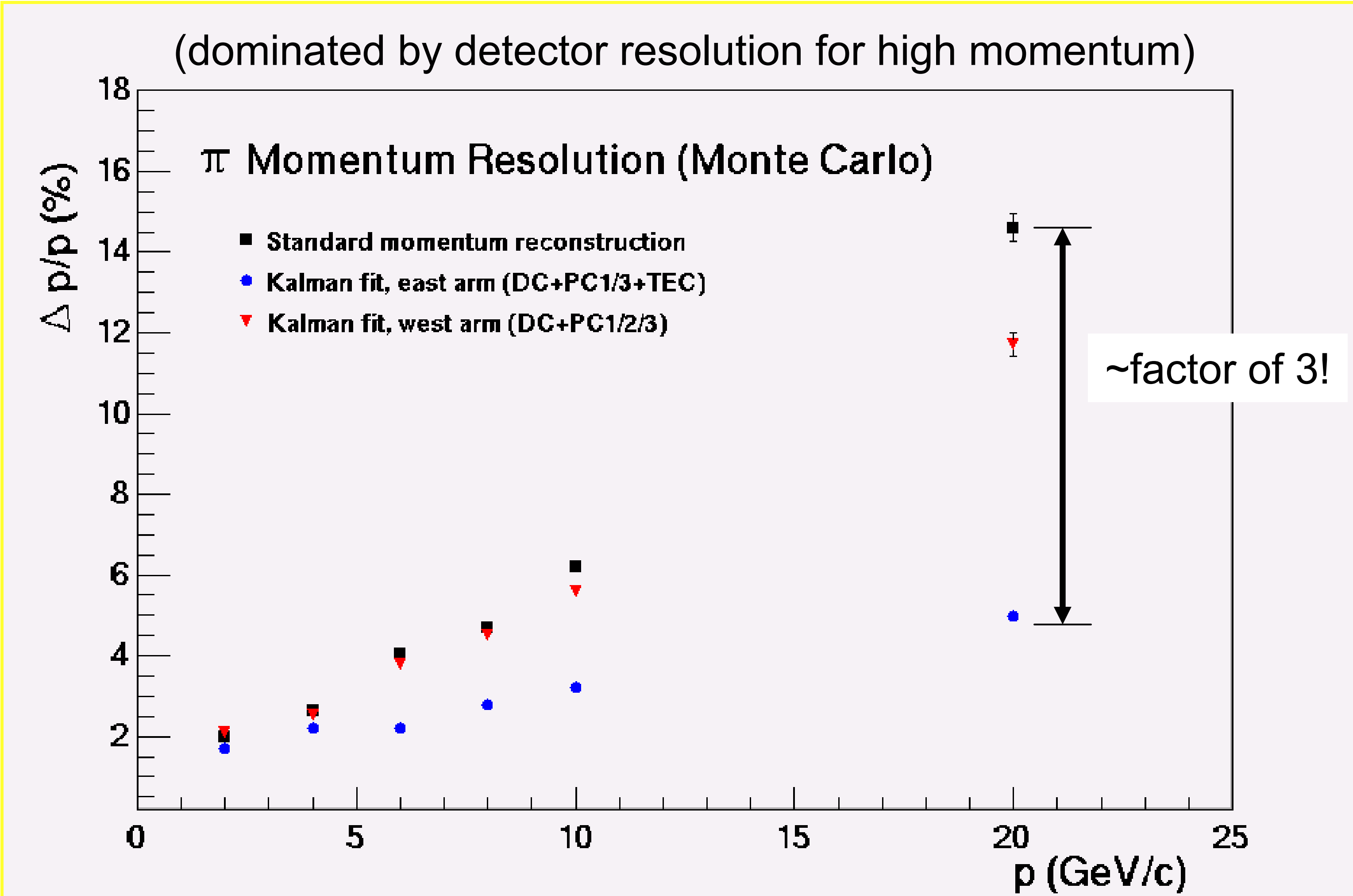
χ^2(phi) per d.o.f



χ^2(z) per d.o.f

- 10 GeV/c π-, east arm (TEC)
- Track consistency with all detectors improved.

Momentum Resolution (p_T>2 GeV/c)



Conclusions

- Significant improvement in momentum resolution at high p_T by including all PHENIX detectors in a Kalman filter.
- Statistically proper chi-square for track selection.
- Extensions for low-p_T possible (multiple scattering and energy loss).

Multiple Scattering

- Multiple scattering introduces a correlation between measurements through the correlation matrix.

- Since we're tracking inwards, we need to **reduce** the covariance matrix as we add subsequent hits. Multiple scattering treated in two planes defined by the momentum vector.

$$\theta_0 = \text{multiple scattering angle}$$

$$V_{\theta_p \theta_p} = \theta_0^{\,2}$$

$$V_{\phi_p \phi_p} = \frac{\theta_0^{\,2}}{\cos^2 \theta_p}$$

- Continuous media treated as a sequence of multiple steps.

Momentum Resolution (p_T=0.5 GeV/c)

(dominated by detector resolution for low momentum)

0.5 GeV/c π^+, East arm

With MS correction resolution approaches that of CGL reconstruction

$(\delta p/p) \sim 1\%$